We extend the connection D to an operator $D: \mathcal{A}^q(E) \to \mathcal{A}^{q+1}(E)$ by forcing Leibnitz' rule; that is, by setting, for $\xi \in \mathcal{A}^0(E)$ and η a q-form,

$$D(\eta \otimes \xi) = d\eta \otimes \xi + (-1)^q \eta \wedge D\xi \in \mathcal{A}^{q+1}(E).$$

We then define the curvature operator Θ by

$$\Theta = D^2 : \mathcal{A}^q(E) \longrightarrow \mathcal{A}^{q+2}(E).$$

In terms of a trivialization φ_α, we have

$$(\Theta \xi)_\alpha = \Theta_\alpha \xi_\alpha,$$

where Θ_α is the matrix of 2-forms

$$\Theta_\alpha = d\theta_\alpha - \theta_\alpha \wedge \theta_\alpha ;$$

Θ_α is called the curvature matrix of D in terms of φ_α. If φ_β is another trivialization with $\varphi_\alpha = g_{\alpha\beta}\varphi_\beta$,

$$\Theta_\alpha = g_{\alpha\beta}\, \Theta_\beta\, g_{\alpha\beta}^{-1}.$$

This transition rule just expresses the directly verifiable fact that Θ is linear over $C^\infty(M)$, i.e. that $\Theta \in A^2(Hom(E,E))$.

┌ $\Theta_\alpha(f\cdot(v_1(x), \cdots v_n(x))) = g_{\alpha\beta}\, \Theta_\beta\, g_{\alpha\beta}^{-1}(f\cdot(v_1(x)\cdots v_n(x)))$

$= f \quad \Theta_\alpha(v(x)) \qquad\qquad \| \quad f\, g_{\alpha\beta}\, \Theta_\beta\, g_{\alpha\beta}^{-1}(v(x))$

See P75 for the linearity of D^2. ┘

In the case of E a line bundle the curvature matrix is, according to the transition rule above, a global 2-form, and we have seen that the cohomology class $[(\sqrt{-1}/2\pi)\Theta]$, the Chern class of E reflects the topological structure of E.