Proof. Recall that the $\bar\partial$-Poincare lemma gives exact sheaf sequences

$$0 \to \mathcal{Z}_{\bar\partial}^{n,n-p-1} \longrightarrow \mathcal{A}^{n,n-p-1} \xrightarrow{\bar\partial} \mathcal{Z}_{\bar\partial}^{n,n-p} \longrightarrow 0,$$

where $\mathcal{A}^{p,q}$ is the sheaf of C^∞ (p,q) forms and $\mathcal{Z}_{\bar\partial}^{p,q} \subset \mathcal{A}^{p,q}$ is the subsheaf of $\bar\partial$-closed forms.

By $\bar\partial$-Poincare lemma on P25.

Since the sheaves $\mathcal{A}^{p,q}$ have no higher cohomology, the Dolbeault isomorphism is a composition of isomorphisms

$$i_p : H^p(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p-1}) \xrightarrow{\sim} H^{p-1}(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p})$$

obtained from coboundary maps in the exact cohomology sequence of the above sheaf sequence. For $p=1$, the right-hand side is to be replaced by

$$H^0(U^*, \mathcal{Z}_{\bar\partial}^{n,n-1}) / \bar\partial H^0(U^*, \mathcal{A}^{n,n-2}) = H_{\bar\partial}^{n,n-1}(U^*).$$

$$H^{p-1}(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p-1}) \to H^{p-1}(U^*, \mathcal{A}^{n,n-p-1}) \to H^{p-1}(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p}) \to H^p(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p-1})$$
$$\to H^p(U^*, \mathcal{A}^{n,n-p-1})$$

$\Rightarrow$ If $p-1 \geq 1$, $H^{p-1}(U^*, \mathcal{A}^{n,n-p-1}) = H^p(U^*, \mathcal{A}^{n,n-p-1}) = 0$ by P42.

$\Rightarrow$ $H^{p-1}(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p}) \cong H^p(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p-1})$.

$\Rightarrow$ $H^p(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p-1}) \cong H^{p-1}(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p}) \cong H^{p-2}(U^*, \mathcal{Z}_{\bar\partial}^{n,n-p+1}) \cong \cdots = H^1(U^*, \mathcal{Z}_{\bar\partial}^{n,n-2})$.

Consider $0 \to \mathcal{Z}_{\bar\partial}^{n,n-2} \longrightarrow \mathcal{A}^{n,n-2} \longrightarrow \mathcal{Z}_{\bar\partial}^{n,n-1} \longrightarrow 0$

$$H^0(U^*, \mathcal{Z}_{\bar\partial}^{n,n-2}) \longrightarrow \underset{\mathcal{A}^{n,n-2}(U^*)}{\underset{\shortparallel}{H^0(U^*, \mathcal{A}^{n,n-2})}} \xrightarrow{\bar\partial} \underset{\mathcal{Z}_{\bar\partial}^{n,n-1}(U^*)}{\underset{\shortparallel}{H^0(U^*, \mathcal{Z}_{\bar\partial}^{n,n-1})}} \longrightarrow H^1(U^*, \mathcal{Z}_{\bar\partial}^{n,n-2})$$
$$\to H^1(U^*, \mathcal{A}^{n,n-2}) = 0$$

$$\Rightarrow H^1(U^*, \mathcal{Z}_{\bar\partial}^{n,n-2}) = \mathcal{Z}_{\bar\partial}^{n,n-1}(U^*) / \bar\partial \mathcal{A}^{n,n-2}(U^*) = H_{\bar\partial}^{n,n-1}(U^*).$$