~~pf) Since $H^q(U_{\alpha_0} \cap \cdots \cap$~~

pf) Since $H^r(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p}, \mathcal{O}) = 0$, we have

$$H^r(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p}, \mathcal{O}) = H^r(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p}, \Omega^0) = H^{0,r}_{\bar\partial}(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p})$$

$= 0$ and so $\quad Z^{0,r}_{\bar\partial}(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p}) = \bar\partial \mathcal{A}^{0,r-1}(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p})$.

$$\overset{\|}{Z^{0,r}_{\bar\partial}}(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p}) = \bar\partial A^{0,r-1}(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p})$$

$\Rightarrow$ We have exact sequences of cochain groups

$$0 \longrightarrow C^p(\underline{U}, Z^{0,r-1}_{\bar\partial}) \longrightarrow C^p(\underline{U}, \cancel{Z_{\bar\partial}}\, \mathcal{A}^{0,r-1}) \longrightarrow C^p(\underline{U}, Z^{0,r}_{\bar\partial}) \longrightarrow 0$$

which gives exact sequences

$$\cdots \to \underset{\overset{\|}{0}}{H^p(\underline{U}, \mathcal{A}^{0,r-1})} \longrightarrow H^p(\underline{U}, Z^{0,r}_{\bar\partial}) \longrightarrow H^{p+1}(\underline{U}, Z^{0,r-1}_{\bar\partial}) \longrightarrow \underset{\overset{\|}{0}}{H^{p+1}(\underline{U}, \mathcal{A}^{0,r-1})} \to$$

Since $H^p(\underline{U}, \mathcal{A}^{0,r}) \overset{(P42)}{=} 0$ for $p > 0$,

$$\begin{aligned} H^p(\underline{U}, Z^{0,r}_{\bar\partial}) &\cong H^{p+1}(\underline{U}, Z^{0,r-1}_{\bar\partial}) \\ &\cong H^{p+2}(\underline{U}, Z^{0,r-2}_{\bar\partial}) \\ &\;\;\vdots \\ &\cong H^{p+r}(\underline{U}, Z^{0,0}_{\bar\partial}) = H^{p+r}(\underline{U}, \overset{\Omega^{0,0}}{\overset{\|}{\mathcal{O}}}) \end{aligned}$$

Put $p + r = q$. $\Rightarrow$ $H^{q-r}(\underline{U}, Z^{0,r}_{\bar\partial}) \cong H^q(\underline{U}, \mathcal{O})$.

Put. $r = q-1$. $\Rightarrow$ $H^1(\underline{U}, Z^{0,q-1}_{\bar\partial}) \cong H^q(\underline{U}, \mathcal{O})$.

Then from the sequence

$$H^0(\underline{U}, Z^{0,q-1}_{\bar\partial}) \longrightarrow H^0(\underline{U}, \mathcal{A}^{0,q-1}_{\bar\partial}) \to H^0(\underline{U}, Z^{0,q}_{\bar\partial}) \to H^1(\underline{U}, Z^{0,q-1}_{\bar\partial}) \to \underset{\overset{\|}{0}}{H^1(\underline{U}, \mathcal{A}^{0,q-1}_{\bar\partial})}$$

$$H^1(\underline{U}, Z^{0,q-1}_{\bar\partial}) \cong \frac{H^0(\underline{U}, Z^{0,q}_{\bar\partial})}{\bar\partial^* H^0(\underline{U}, \mathcal{A}^{0,q-1})} = \frac{\cancel{Z^{0,q}_{\bar\partial}(\underline{U})}\; Z^{0,q}_{\bar\partial}(M)}{\bar\partial \mathcal{A}^{0,q-1}(M)} = \frac{Z^{0,q}_{\bar\partial}(M)}{\bar\partial A^{0,q-1}(M)} = H^{0,q}_{\bar\partial}(M)$$