$$\varphi = \frac{h(z)}{z-a_i}\,dz = \frac{g(w)}{w}\,dw \Rightarrow \frac{h(z)}{z-a_i} = \frac{g(w)}{w}\frac{\partial w}{\partial z}$$

$\Rightarrow \lim_{z\to a_i} h(z)\frac{w}{z-a_i} = g(0)\frac{\partial w}{\partial z}(a_i) = h(a_i)\frac{\partial w}{\partial z}(a_i) \Rightarrow h(a_i) = g(0) \Rightarrow$ This implies

the independence of the residue of the choice of the coordinate charts.

$$\int_{S-\cup B_\epsilon(a_i)} d\varphi = 0$$ since φ is holomorphic on $S-\cup B_\epsilon(a_i)$, and we can make $S-\cup B_\epsilon(a_i)$ into simply connected pieces whose piece is homeomorphic to a disk in the complex plane.

$$-\int_{S-\cup_i B_\epsilon(a_i)} d\varphi = \int_{\partial(\cup_i B_\epsilon(a_i))} \varphi \quad \text{by Stokes' theorem.}$$

Applying this to $\varphi = df/f$ shows again that meromorphic function f on S has the same number of zeros as poles.

$$0 = \sum_i \mathrm{Res}_{a_i}(df/f) = \#\text{ of zeros} \quad \#\text{ of poles}$$

Point is here.

f has zeros at z_0

$\Rightarrow f(z) = (z-z_0)^n g(z)$. $g(z_0) \neq 0$.

$$\Rightarrow f'(z) = n(z-z_0)^{n-1}g(z) + (z-z_0)^n g'(z)$$

$$\Rightarrow \frac{f'(z)}{f(z)} = \frac{n}{z-z_0} + \frac{g'(z)}{g(z)}$$

$$\Rightarrow \text{Residue of } \frac{f'(z)}{f(z)} \text{ at } z_0 = n.$$

We return to our Riemann surface S of genus 1. A noted before, there are no nonconstant meromorphic functions on S with only a single pole at p. On the other hand, by the vanishing theorem

$$H^1(S, \mathcal{O}(p)) = 0,$$

and so the exact sequence

$$0 \to \mathcal{O}(p) \to \mathcal{O}(2p) \to \mathbb{C}_p \to 0$$