

in Σ are all smooth rational curves, having by adjunction self-intersection -2 on Σ , we see from our discussion of isolated singularities of surfaces that the points of R are all ordinary double points of S .

Let C be a smooth irreducible curve on Σ .

\Rightarrow By the argument on P471,

$$g(C) = \frac{K_{\Sigma} \cdot C + C \cdot C}{2} + 1$$

Thus if $C \cong \mathbb{P}^1$ & $C \cdot C = -1$, i.e., Σ is minimal,

$$g(C) = 0 = \frac{0 - 1}{2} + 1, \text{ which is impossible.}$$

$\Rightarrow \Sigma$ is minimal.

$\pi: \Sigma \rightarrow S$ is one to one and onto $S-R$, which is the set of smooth points of S . It remains to show that π is holomorphic. Given $x \in \Sigma \subset \mathbb{P}^5$,

$x = [X_0, X_1, \dots, X_5]$, l_x is given by, for example,

$$l_x = \begin{pmatrix} 1, 0, z_1(x), z_2(x) \\ 0, 1, z_3(x), z_4(x) \end{pmatrix}, \text{ where}$$

z_i 's are holomorphic in x . Any point p in l_x is represented by $v_x = (1, 0, z_1(x), z_2(x))$

+ $\alpha (0, 1, z_3, z_4)$, $\alpha \in \mathbb{P}^1$.

$\Rightarrow \sigma(p) = \{ \tilde{\phi}(\overline{y v_x}) \}$, $y \neq v_x$, $y \in \mathbb{P}^3$.

$\Rightarrow \tilde{\phi}(\overline{y v_x}) = (y_0 e_1 + y_1 e_2 + y_2 e_3 + y_3 e_4) \wedge$

$$(e_1 + \alpha e_2 + (z_1 + \alpha z_3) e_3 + (z_2 + \alpha z_4) e_4)$$