

$$\gamma(t) = \left\{ (\cos 2\pi t) f + \sin 2\pi t \right\}_{0 \leq t \leq \frac{1}{4}}.$$

$$\gamma(0) = (f) \quad \gamma\left(\frac{1}{4}\right) = (1) = \phi \quad \Downarrow$$

" $(f)_0 - (f)_\infty$

Examples

1. In case M is a compact connected Riemann surface, a divisor D on M is just a finite sum

$$D = \sum n_i p_i$$

of points $p_i \in M$ with multiplicities n_i . The degree of D is defined to be its fundamental class $(D) \in H_0(M, \mathbb{Z}) \cong \mathbb{Z}$; clearly

$$\deg D = \sum n_i, \text{ since } (D) = \sum n_i (p_i) = \left(\sum n_i\right) (*)$$

(since M is path-connected) \Downarrow

By the above proposition, if Θ is the curvature form of a connection in the line bundle $[D]$,

$$\frac{i}{2\pi} \int_M \Theta = \langle C_1([D]), [M] \rangle = \deg D.$$

$$\Gamma \quad C_1([D]) \equiv \frac{i}{2\pi} \Theta. \quad \text{by the proposition.}$$

$$C_1([D]) \equiv \eta_D'' \in H_{DR}^2(M).$$

$$\int_M C_1([D]) \equiv \frac{i}{2\pi} \int_M \Theta = \#(D \cdot M) = \deg D.$$

$$\langle C_1([D]), [M] \rangle \equiv \int_M C_1([D]) \quad \Downarrow$$