Consider now the set of k-planes on F passing through the points p_i and p_{ij}. Take $\mathbb{P}^{n-3}_{ij}$ an $(n-3)$-plane lying in the intersection $T_{p_i}(F) \cap T_{p_{ij}}(F)$ and missing the line $\overline{p_i p_{ij}}$; $\mathbb{P}^{n-3}_{ij}$ intersects F in a smooth quadric F_{ij}, with $F \cap T_{p_i}(F) \cap T_{p_{ij}}(F)$ the cone through the line $\overline{p_i p_{ij}}$ over F_{ij}.

$\ulcorner$ Since $F \cap T_{p_i}(F)$ is a cone through p_i over F_i, and $F \ni p_{ij}$, $\overline{p_i p_{ij}} \subset F$. $\Rightarrow$ $\overline{p_i p_{ij}} \subset T_{p_i}(F) \cap T_{p_{ij}}(F)$.

Without loss of generality, we may assume that $p_i = [\,1, 0, \cdots 0\,]$ $p_{ij} = [\,0, 1, \cdots 0\,]$ $T_{p_i}(F) = \{X_{n+1} = 0\}$ $T_{p_{ij}}(F) = \{X_n = 0\}$. and $\mathbb{P}^{n-3}_{ij} = \{[\,0, 0, X_2, \cdots X_{n-1}\,]\}$.

Since $T_{p_i}(F) = (X_{n+1} = 0)$, by P175 ~ P176,

$$\left\{\sum \frac{\partial F}{\partial X_i}(p_i) X_i = 0\right\} = \{X_{n+1} = 0\}. \Rightarrow \text{ If we let}$$

$$F = \sum_{i,j=0}^{n+1} q_{ij} X_i X_j, \qquad \frac{\partial F}{\partial X_i}(p_i) = 0 = \sum_{j=0}^{n+1} q_{ij} X_j (p_i)$$

$$= q_{i0} \text{ for all } i = 0, \cdots n. \qquad \frac{\partial F}{\partial X_{n+1}}(p_i) \neq 0 \Rightarrow q_{n+1,0} \neq 0.$$

Thus

$$F|_{T_{p_i}(F)} \text{ has } Q = \left(\begin{array}{c|ccc|c} 0 & 0 & \cdots & 0 & q_{0,n+1} \\ \hline 0 & * & & * & * \\ \vdots & & (n+1)\times(n+1) & & \vdots \\ 0 & * & \cdots & * & \\ \hline q_{0,n+1} & & & & * \end{array}\right).$$

nonsingular

$q_{0,n+1} = q_{n+1,0}$

Similarly, since $T_{p_{ij}}(F) = (X_n = 0)$, we have