

The reader is referred to P. 146 for the definition and intersection numbers of the Schubert cycles $\sigma_1(h_0)$, $\sigma_2(p_0)$, $\sigma_{1,1}(h_0)$, and $\sigma_{2,1}(p_0, h_0)$ on G .

Now, since the wedge product

$$\Lambda: \Lambda^2 \mathbb{C}^4 \times \Lambda^2 \mathbb{C}^4 \longrightarrow \Lambda^4 \mathbb{C}^4 \cong \mathbb{C}$$

is a non degenerate pairing, every hyperplane in $\mathbb{P}(\Lambda^2 \mathbb{C}^4)$ is of the form

$$H_{\omega_0} = \{ \omega : \omega \wedge \omega_0 = 0 \}.$$

□ We have only to show the following lemma:

Let V be a vector space with an inner product $\langle \cdot, \cdot \rangle$. Assume that $\langle \cdot, \cdot \rangle$ is nondegenerate.

Then every hyperplane W in V is of the form $W = \{ v \in V : \langle v, w \rangle = 0 \text{ for some } w \in V \}.$

Proof).

Let $\{v_1, \dots, v_n\}$ be a basis for V . Since we may assume that $W = \langle v_1, \dots, v_{n-1} \rangle$, any hyperplane W is expressed as $\{ a_1 v_1 + \dots + a_n v_n \mid a_1 b_1 + \dots + a_n b_n = 0 \text{ for some } (b_1, b_2, \dots, b_n) \}.$

Let $w = a_1 v_1 + \dots + a_n v_n$.

$$W^\perp = \{ v \in V \mid \langle v, w \rangle = 0 \}$$

$$\begin{aligned} \langle v, w \rangle &= \langle \sum_i x_i v_i, \sum_j a_j v_j \rangle = \sum_{i,j} x_i \langle v_i, v_j \rangle a_j \\ &= {}^t x \bar{V} a, \quad x = \begin{pmatrix} x_1 \\ \vdots \end{pmatrix}, \quad \bar{V}_{ij} = \langle v_i, v_j \rangle, \quad a = \begin{pmatrix} a_1 \\ \vdots \end{pmatrix}. \end{aligned}$$