$\mathbb{Z} \oplus 5\mathbb{Z} = 6\mathbb{Z} \quad \Rightarrow \quad \chi(U) = 1 + 1 + 6 = 8.$

In fact, this last argument gives us something more: it tells us that S is smooth away from the locus R. To see this, let q be any point of S. Then the hyperplane sections

$$\{ T_x(G) \cap X \}_{x \in \sigma(q)}$$

form a linear system on X; by Bertini's theorem, for generic $x \in \sigma(q)$ the surface $U_x = T_x(G) \cap X$ will be smooth away from the base locus $X_q = \sigma(q) \cap X$ of the linear system.

First of all, the base locus is

$$\bigcap_{x \in \sigma(q)} (T_x(G) \cap X) = \Big(\bigcap_{x \in \sigma(q)} (T_x(G) \cap G) \Big) \cap F$$

$$= \bigcap_{x \in \sigma(q)} \Big(\bigcup_{p \in l_x} \sigma(p) \Big) \cap F \quad \text{by P258}$$

$$= \sigma(q) \cap F, \quad \text{since} \quad \bigcap_{x \in \sigma(q)} \Big(\bigcup_{p \in l_x} \sigma(p) \Big) = \sigma(q) \quad (\because\ l_x \ni q \text{ for all } x \in \sigma(q))$$

$$= \sigma(q) \cap G \cap F = \sigma(q) \cap X \quad \text{since} \quad \sigma(q) \subset G.$$

Next, $T_x(G) \cap X = T_x(G) \cap G \cap F$.

$\Rightarrow$ $T_x(G) \cap G = \sigma(l_x)$ by P258 & P257.

Assume that $q = [1, 0, 0, 0]$.

$\Rightarrow$ By P256, $T_x(G) = H_{w_x} = \{ w \wedge e_1 \wedge \overset{=0}{v_x} \}$, $w_x = e_1 \wedge v_x$.