By the formal properties 1-3 of smoothing, a harmonic distribution T satisfies $T_\delta = T$ for $\delta > 0$. More precisely, by property 3,

$$\Delta T_\epsilon = (\Delta T)_\epsilon = 0,$$

so that $T_\epsilon = T_{\psi_\epsilon}$ for a harmonic function ψ_ϵ.

F Suppose T is a harmonic distribution, i.e., $\Delta T = 0$.
Consider the distribution T_ϵ on p374.
$\Rightarrow$ By property 3, on p374, $\Delta T_\epsilon = (\Delta T)_\epsilon = 0$

$$T_\epsilon(\phi) = \int_{\mathbb{R}^n} T_\epsilon(x)\, \phi(x)\, dx.$$

$$(\Delta T_\epsilon)(\phi) = 0 = \pm \int_{\mathbb{R}^n} T_\epsilon(x)\, \Delta\phi(x)\, dx$$

$$\pm T_\epsilon(\Delta\phi) = \int_{\mathbb{R}^n} \Delta T_\epsilon(x)\, \phi(x)\, dx. \quad \text{(by integration by parts)}$$

$\Rightarrow \Delta T_\epsilon = 0$ since $\int_{\mathbb{R}^n} \Delta T_\epsilon(x)\, \phi(x)\, dx$ for all $\phi \in C_c^\infty(\mathbb{R}^n)$.

$\Rightarrow T_\epsilon(x)$ is a harmonic function

$\Rightarrow T_\epsilon = T_{\psi_\epsilon}$ where $\psi_\epsilon(x) = T_\epsilon(x)$ 」

Then

$$(T_\epsilon)_\delta = (T_{\psi_\epsilon})_\delta = T_{\psi_\epsilon} = T_\epsilon,$$

and for $\varphi \in C_c^\infty(\mathbb{R}^n)$,