In particular, we can discuss the operator

$$D^2 : \mathcal{A}^0(E) \longrightarrow \mathcal{A}^2(E).$$

First, D^2 is linear over $\mathcal{A}^0$. i.e. $\sigma \in \mathcal{A}^0(E)(U)$ and f C^∞ function,

$$D^2(f\sigma) = D(df \otimes \sigma + f \cdot D\sigma) = -df \wedge D\sigma + df \wedge D\sigma + f\, D^2\sigma = f\, D^2\sigma.$$

Consequently, the map $D^2 : \mathcal{A}^0(E) \longrightarrow \mathcal{A}^2(E)$ is induced by a bundle map $E \longrightarrow \Lambda^2 T^* \otimes E$, or in other words, D^2 corresponds to a global section Θ of the bundle

$$\Lambda^2 T^* \otimes \mathrm{Hom}(E, E) = \Lambda^2 T^* \otimes E^* \otimes E.$$

⌈ Define $D^2 : E \longrightarrow \Lambda^2 T^* \otimes E$ by $D^2 v \equiv (D^2\sigma)(x)$ where σ is a section of E s.t $\sigma(x) = v \in E_x$.

To check the well-definedness of D^2, $\sigma(x) = 0 \overset{?}{\Rightarrow} (D^2\sigma)(x) = 0$. Since D^2 is a local operator, we have only to consider D^2 an open set containing x.

$U \ni x$, $E|_U \cong U \times \mathbb{C}^n$, $\Rightarrow$ $\sigma = \sum f_i \sigma_i$, where σ_i's is a local frame for $E|_U$.

Suppose $\sigma(x) = 0$. $\Rightarrow$ $f_i(x) = 0$ for all i.

$\Rightarrow$ $D^2\sigma = \sum D^2(f_i \sigma_i) = \sum f_i (D^2\sigma_i)$

$\Rightarrow$ $(D^2\sigma)(x) = \sum f_i(x)\, (D^2\sigma_i)(x) = \sum 0\; D^2\sigma_i(x) = 0.$ ⌋

⌈ Given a bundle map $f : E \longrightarrow F$,

$$\begin{tikzcd} E \arrow[r, "f"] \arrow[dr] & F \arrow[d] \\ & M \end{tikzcd}$$

$\Rightarrow$ For each $x \in M$, $\exists$ $f_x \in \mathrm{Hom}(E_x, F_x) = \mathrm{Hom}(E, F)_x$,