Conversely, if ω does not have the hyperplane at infinity as a component of its polar divisor,

$\sum d_i - (n+1) - \ell \geq 0$, $\Leftarrow\!\!>$ $\deg(g) \leq d_1 + \cdots + d_n - (n+1)$. ⌟

((Here the hyperplane is $\{[(0, X_1, X_2 \cdots X_n)] \mid X_1 \cdots X_n \in \mathbb{C}\}$, $x_1 \to \infty$ $\left(x_1 = \frac{X_1}{X_0}\right)$))

The global residue theorem then gives

$$(*) \qquad \sum_{\nu} \mathrm{Res}_{p_\nu} \left\{ \frac{g(x)\, dx_1 \wedge \cdots \wedge dx_n}{f_1(x) \cdots f_n(x)} \right\} = 0.$$

⌜ $\mathbb{P}^n = \overline{B(0,r)} \cup \overline{B(0,r)}^c$ $\qquad \overline{B(0,r)} \subset \mathbb{C}^n$.

$$\int_{\mathbb{P}^n} \omega = 0 = \int_{\overline{B(0,r)}} \omega + \cancel{\int_{B(0,r)^c} \omega}$$

$$= \sum_{\nu} \mathrm{Res}_{p_\nu} \left\{ \frac{g(x)\, dx_1 \wedge \cdots \wedge dx_n}{f_1 \cdots f_n} \right\}$$

$$= \int_{\mathbb{C}^n} \omega = \int_{\mathbb{C}^n} \frac{g(x)\, dx_1 \wedge \cdots \wedge dx_n}{f_1 \cdots f_n}$$

⌟

In the case where the D_i meet transversely at $d_1 \cdots d_n$ distinct points, $(*)$ reduces to the Jacobi relation

$$\sum_{\nu} \frac{g(P_\nu)}{\left(\partial(f_1 \cdots f_n)/\partial(x_1 \cdots x_n)\right)(P_\nu)} = 0, \qquad \deg(g) \leq \sum d_i - (n+1)$$

proved by him in 1834. For $n=1$, we obtain the Lagrange interpolation formula

$$\sum_{\nu} \frac{g(P_\nu)}{f'(P_\nu)} = 0, \qquad \deg(g) \leq \deg(f) - 2;$$

It was in this context that Jacobi was led to his formula.