$(R_{g*}\sigma)(v_1, v_2)$. $\quad (v_1, v_2) = v_1 \frac{\partial}{\partial x} + v_2 \frac{\partial}{\partial y}$ at (x, y).

$$\sigma(R_{g*}(v_1, v_2)) = \frac{(a_{11}x + a_{12}y)\,dy - (a_{21}x + a_{22}y)\,dx}{(a_{11}x + a_{12}y)^2 + (a_{21}x + a_{22}y)^2}\left\{(a_{11}v_1 + a_{12}v_2)\frac{\partial}{\partial x}\right.$$

$$\begin{pmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{pmatrix}, \quad \begin{pmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} a_{11}v_1 + a_{12}v_2 \\ a_{21}v_1 + a_{22}v_2 \end{pmatrix}$$

$$\left. + (a_{21}v_1 + a_{22}v_2)\frac{\partial}{\partial y}\right)$$

$$= \frac{1}{x^2 + y^2}\left\{(a_{11}x + a_{12}y)(a_{21}v_1 + a_{22}v_2) - (a_{21}x + a_{22}y)(a_{11}v_1 + a_{12}v_2)\right\}$$

$$= \frac{1}{x^2+y^2}\left(\cancel{a_{11}a_{21}xv_1} + a_{12}a_{21}yv_1 + a_{11}a_{22}xv_2 + \cancel{a_{12}a_{22}yv_2} - \cancel{a_{21}a_{11}xv_1} - a_{22}a_{11}yv_1 - \cancel{a_{22}a_{12}yv_2}\right)$$

$$= \frac{1}{x^2+y^2}(-yv_1 + xv_2) = \sigma(v_1, v_2) = \sigma\left(v_1\frac{\partial}{\partial x_1} + v_2\frac{\partial}{\partial x_2}\right)$$

$$\Rightarrow \quad R_{g*}\sigma = \sigma$$

As we can see clearly, $R_g^*\sigma$ is the same as

$$\frac{x'dy' - y'dx'}{x'^2 + y'^2}, \quad \text{where } \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

$$\int_{\|x-y\|=\epsilon} \sigma_y(x) = \int_{\|x-y\|=\epsilon} \sigma(x-y) = \int_{\|t\|=\epsilon} \sigma(t) = 1$$

The invariant volume form means that when we translate $\sigma_y(x)$ through an orientation preserving isometry on the sphere $\|x-y\| = \epsilon$, $\sigma_y(x)$ does not change its form. (?). Strange expression.

Since the Laplacian is invariant under translations and prop-