$$\lim_{r\to 0}\int_{Z[r]} \Omega^{n-1} = 1.$$ ~~since it is the volume of the origin.~~

Since the Lelong number is semicontinuous, it follows that

$$\Theta(T_Z, p') \geq 1$$

for $p' \in Z$.

$\Theta(T_Z, p') = \mathrm{mult}_{p'}(Z) \geq 1$. for all smooth points p''s, and for any point p'. since $\mathrm{mult}_{p'}(Z) \geq 1$.
I don't understand why they need the semicontinuity of the Lelong number.

Building on previous work by several authors, Siu has recently proved that for a general closed positive current T the set of points where

$$\Theta(T, p') \geq \epsilon > 0$$

is supported in a codimension-p analytic subvariety.*
We shall not use this result, but it is worthwhile to keep in mind when we discuss the proper mapping theorem in the section after next, where in fact a special case of Siu's theorem will be proved.

$\Theta(T_Z, p') > 1$. $\Rightarrow$ p' is not a smooth point.
$\Rightarrow$ Z_s is a subvariety of Z with codimension ≤ 1, by P21 Proposition.