to show that $M-W-f^{-1}(V)$ is biholomorphic to $\mathbb{P}^n-V-g^{-1}(W)$, by the propositon on P19.

$$f: M-W-f^{-1}(V) \longrightarrow \mathbb{P}^n-V-g^{-1}(W)$$
$$z \longmapsto [1, f_1(z), \cdots f_n(z)].$$

$\Rightarrow$ f gives a local coordinate on open dense subset $f^{-1}(U_0 \cap (\mathbb{P}^n-V-g^{-1}(W)))$.

Conversely, suppose that $\exists$ n meromorphic functions $f_1, \cdots f_n$ on M providing local coordinates almost everywhere.

$\sqcap$ Let $M \xrightarrow{f} \mathbb{P}^n$ be defined by
$$z \longmapsto [1, f_1(z), \cdots f_n(z)].$$

$\Rightarrow$ According to the argument on P491,
$$M-V \xrightarrow{f} \mathbb{P}^n \text{ is holomorphic.}$$

By the assumption, $M-V \xrightarrow{f} \operatorname{im} f \subset \mathbb{C}^n$ is bihomorphic, where $\mathbb{P}^n - f(M-V)$ is of measure zero.

$\Rightarrow$ $\exists$ $g: \mathbb{P}^n-W \longrightarrow M$ holorphic s.t $g \circ f$ & $f \circ g$ are identies as rational maps.

Example. $\mathbb{C} \xrightarrow{f} \mathbb{C}$
$$z \longmapsto z^2.$$

$\Rightarrow$ $\mathbb{C}-\{0\} \xrightarrow{f} \mathbb{C}-\{0\}$ is locally biholomorphic, but f is not birational, since $\exists$ no rational inverse. $\lrcorner$

Note that a rational map $f: M \longrightarrow N$ is birational if and only if it is generically one to one: if, for generic