locally $\sigma \times \tau \in R^n \times R^n$. $\quad \sigma$. k-vectors
τ $(n-k)$-vectors in R^n.

$\sigma_1 = (\sigma_{11}, \sigma_{12} \cdots \sigma_{1n})$ $\qquad \tau_1 = (\tau_{11}, \tau_{12} \cdots \tau_{1n})$

$\sigma_2 = (\sigma_{21}, \sigma_{22} \cdots \sigma_{2n})$ $\qquad \tau_2 = (\tau_{21}, \tau_{22}, \cdots \tau_{2n})$

$\vdots \qquad \vdots$

$\sigma_k = (\sigma_{k1}, \sigma_{k2} \cdots \sigma_{kn})$ $\qquad \tau_{n-k} = (\tau_{n-k,1}\ \tau_{n-k,2} \cdots \tau_{n-k,n})$

$\sigma_i \times \tau_j = (\underbrace{\sigma_{i1} \cdots \sigma_{in}, \tau_{j1}, \tau_{j2} \cdots \tau_{jn}}_{2n})$

$$\begin{pmatrix} \sigma_{11}, \sigma_{12} \cdots \sigma_{1n}, \tau_{11} \cdots \tau_{1n} \\ \sigma_{21}\ \sigma_{22} \cdots \sigma_{2n}, \tau \end{pmatrix}$$

$\Rightarrow$ Orientation on $\sigma \times \tau$ is $(\sigma_{11}\ \sigma_{12} \cdots \sigma_{1n}, 0 \cdots 0)$
$\cdots$ $(\sigma_{k1}, \sigma_{k2}, \cdots \sigma_{kn}, 0 \cdots 0)$, $(0, 0 \cdots 0\ \tau_{11}, \tau_{12} \cdots \tau_{1n})$
$\cdots$ $(0, 0 \cdots 0, \tau_{n-k,1} \cdots \tau_{n-k,n})$.

Then since Δ has an orientation which is
$(1, 0 \cdots 0\ 1\ 0 \cdots 0)$ $\quad (0, 1\ 0 \cdots \overset{n}{0}\ 0\ 1\ 0 \cdots 0) \cdots (\overset{n}{0 \cdots 1} | \overset{n}{0 \cdots 1})$.

$$\bar{\iota}_{(p,p)}(\sigma \times \tau, \Delta) = \text{sign of } \det \begin{array}{c} k \\ n-k \\ n \end{array} \begin{pmatrix} \sigma & 0 \\ 0 & \tau \\ I & I \end{pmatrix} \begin{array}{c} \\ \\ n \end{array}$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad n \qquad n$$

$$\Rightarrow \det \begin{pmatrix} \sigma & 0 \\ 0 & \tau \\ I & I \end{pmatrix} = \det \begin{pmatrix} \sigma & 0 \\ -\tau & 0 \\ I & I \end{pmatrix} \text{ by subtracting.}$$