Γ Measure zero is not enough for the existence of such an arc $\gamma(t)$, $0 \le t \le \varepsilon$. We need more stronger condition. Actually, the critical values of $f: U \to \mathbb{C}^n$ form an algebraic variety in $\mathbb{C}^n$, for

$$K = \left\{ J_f(z) = \left| \frac{\partial(f_1, \cdots f_n)}{\partial(z_1, \cdots z_n)} \right| = 0 \right\} = \text{set of all critical}$$

points in U, which is an algebraic subvariety of U. ?

? ⇒ Since we don't know whether $f|_K$ is proper or not, we can not say that $f(K)$ is an algebraic subvariety of $\mathbb{C}^n$. ///

But we know that by Sard's theorem $f(K)$ has measure zero, and for any $z \notin f(K)$ $\exists$ an open set U_z s.t $U_z \ni z$ and $U_z \cap f(K) = \phi$.

(i) $z \in f(U) - f(K)$.

⇒ Still we have a problem, i.e., we can't go on further.

$f: U \longrightarrow \mathbb{C}^n$, $f^{-1}(0) = (0)$ ⇒ $f \neq 0$ on ∂U.

⇒ $\|f\| \ge \delta$ on ∂U. Consider $B(0, \frac{\delta}{2})$ in $\mathbb{C}^n$.

Claim: $f(K)^c \cap B(0, \frac{\delta}{2})$ is open.

pf). $z \in f(K)^c \cap B(0, \frac{\delta}{2})$ ⇒ $f^{-1}(z)$ is a finite set of points interior to U, since $f^{-1}(z)$ is an algebraic variety in U, and $f^{-1}(z) \subset V \subset \overline{V} \subset U$. ⇒ Let $f^{-1}(z) = \{w_1, \cdots w_l\}$. ⇒ $J_f(w_i) \neq 0$ for $i = 1, \cdots l$. ⇒ $\exists$ open set O_i s.t $f| : O_i \overset{\subset U}{\longrightarrow} f(O_i)$ is biholomorphic. ⇒ Let $W = \bigcap_{i=1}^{l} f(O_i)$. ⇒ W does not contain a critical value. ⇒ $W \cap f(K) = \phi$ ⇒ $f(K)^c \cap B(0, \frac{\delta}{2})$ is open. ⇒ By com-