We also know that ω is unique up to an exact form.

A special case when all this has been made quite explicit is when M is complex manifold and Γ is the cycle carried by an analytic subvariety V of codimension 1. If V is given locally as the divisor of $f_\alpha \in \mathcal{O}(U_\alpha)$, and if we have chosen positive functions h_α in U_α with $h_\alpha / h_\beta = |f_\alpha / f_\beta|^2$ in $U_\alpha \cap U_\beta$ then $\omega = dd^c \log h_\alpha$ is the Chern class of the line bundle $[V]$.

☞ See P 148

Especially noteworthy is the case when $[V]$ is positive in the sense that, with a suitable choice of the metric that in $[V]$, the real $(1,1)$-form ω is positive.

We shall introduce a formalism that includes both cycles and smooth forms. This will lead to a cohomology theory to which both the ordinary singular and de Rham's theorie map, and both maps will be isomorphisms.

Definitions; Residue Formulas

We may our definitions first on $\mathbb{R}^n$. Let $C_c^\infty(\mathbb{R}^n)$ be the vector space of compactly supported smooth functions on $\mathbb{R}^n$. If $x = (x_1, x_2 \cdots x_n)$ are coordinates on $\mathbb{R}^n$, we let $D_i = \partial / \partial x_i$ and $D^\alpha = D_1^{\alpha_1} \cdots D_n^{\alpha_n}$ for $(\alpha_1, \cdots \alpha_n) = \alpha \in (\mathbb{Z}^+)^n$. The C^p-topology is defined on $C_c^\infty(\mathbb{R}^n)$ by saying that a sequence $\varphi_n \longrightarrow 0$ in case there is a compact set K with all $\operatorname{supp} \varphi_n \subset K$ and with

$$D^\alpha \varphi_n(x) \longrightarrow 0$$

uniformly for $x \in K$ and all α satisfying $[\alpha] = \alpha_1 + \cdots + \alpha_n \leq p$. The C^∞ topology is defined by saying that $\varphi_n \longrightarrow 0$ in case all $\operatorname{supp} \varphi_n \subset K$ and $\varphi_n \longrightarrow 0$ in the C^p topology for each p.