

\mathbb{F} $x \in G = G(2, 4)$ and x means l_x , see P197.

By Pieri's Formula on P203, since, if $a = a, 0, 0, \dots$

$$\sigma_a \cdot \sigma_b = \sum_{\substack{b_i \leq c_i \leq b_{i+1} \\ \sum c_i = a + \sum b_i}} \sigma_c,$$

$$\textcircled{1} \quad \sigma_1 \cdot \sigma_1 = ? \quad a = 1, 0 \quad b = \begin{smallmatrix} 1 \\ b_1 \end{smallmatrix}, \begin{smallmatrix} 0 \\ b_2 \end{smallmatrix} \quad \text{and, by P202, } b_0 = n - k = 4 - 2 = 2.$$

$$\Rightarrow C_1 + C_2 = 2$$

$$\Rightarrow \sigma_1 \cdot \sigma_1 = \sigma_2 + \sigma_{1,1}.$$

$$\textcircled{2} \quad \sigma_1 \cdot \sigma_2 = ?$$

$$a = 1, 0 \quad b = 2, 0 \Rightarrow C_1 + C_2 = 3 \Rightarrow 2 \leq C_1 \leq 2 \Rightarrow C_1 = 2$$

$$\Rightarrow C_2 = 1 \Rightarrow \sigma_1 \cdot \sigma_2 = \sigma_{2,1}$$

$$\textcircled{3} \quad \sigma_1 \cdot \sigma_{1,1} = ?$$

$$a = 1, 0, \quad b = 1, 1 \Rightarrow C_1 + C_2 = 3 \quad 1 \leq C_1 \leq 2 \Rightarrow \text{If } C_1 = 2,$$

$$\Rightarrow C_2 = 1. \quad \text{Otherwise, if } C_1 = 1, \text{ then } C_2 = 2$$

$$\Rightarrow \text{Contradiction to the fact that } C_1 \geq C_2.$$

$$\Rightarrow \text{Thus } \sigma_1 \cdot \sigma_{1,1} = \sigma_{2,1}$$

$$\textcircled{4} \quad \sigma_2 \cdot \sigma_2 = ?$$

$$a = \begin{smallmatrix} 2 \\ a_1 \end{smallmatrix}, 0 \quad b = 2, 0 \Rightarrow C_1 + C_2 = 4, \quad 2 \leq C_1 \leq 2 \Rightarrow C_1 = 2$$

$$\Rightarrow C_2 = 2. \Rightarrow \sigma_2 \cdot \sigma_2 = \sigma_{2,2} = 1$$

$$\textcircled{5} \quad \sigma_{1,1} \cdot \sigma_{1,1} = ?$$

$$a = 1, 1 \quad b = 1, 1 \Rightarrow \begin{smallmatrix} 1 \\ a_1 \end{smallmatrix} = 2 - \begin{smallmatrix} 1 \\ b_2 \end{smallmatrix} \quad \begin{smallmatrix} 1 \\ a_2 \end{smallmatrix} = 1 = 2 - \begin{smallmatrix} 1 \\ b_1 \end{smallmatrix}$$

$$\Rightarrow \text{By the result on P198, } \sigma_{1,1} \cdot \sigma_{1,1} = 1$$

$$\textcircled{6} \quad \sigma_1 \cdot \sigma_{2,1} = ?$$

$$a = 1, 0 \quad b = 2, 1 \quad C_1 + C_2 = 4 \quad 2 \leq C_1 \leq 2 \quad C_1 = 2$$