$\tilde{\omega}^5 = 1,$

$\tilde{\omega}^4 \cdot e = ((2\tilde{\ell})^4)_E = 0,$

$\tilde{\omega}^3 \cdot e^2 = ((2\tilde{\ell})^3 \cdot \zeta)_E = 0,$

$\tilde{\omega}^2 \cdot e^3 = ((2\tilde{\ell})^2 \cdot \zeta^2)_E = 4(\tilde{\ell}^2 \cdot \zeta^2)_E = 4$

$\tilde{\omega} \cdot e^4 = (2\tilde{\ell} \cdot \zeta^3)_E = 18,$

$e^5 = \zeta^4 = 51,$

and so

$$(6\tilde{\omega} - 2e)^5 = 6^5\tilde{\omega}^5 - 5 \cdot 6^4 \cdot 2\,\tilde{\omega}^4 e + 10 \cdot 6^3 \cdot 2^2 \cdot \tilde{\omega}^3 \cdot e^2$$
$$- 10 \cdot 6^2 \cdot 2^3 \cdot \tilde{\omega}^2 \cdot e^3 + 5 \cdot 6 \cdot 2^4 \cdot \tilde{\omega} \cdot e^4 - 2^5 e^5$$
$$= 6^5 - 10 \cdot 6^2 \cdot 2^3 \cdot 4 + 5 \cdot 6 \cdot 2^4 \cdot 18 - 2^5 \cdot 51$$
$$= 7776 - 11520 + 8640 - 1632$$
$$= 3264.$$

The answer, then, is that

For a generic choice of five conic curves in $\mathbb{P}^2$, there will be exactly 3264 smooth conics tangent to all five.

$\ulcorner$ $\tilde{\omega}^4 \cdot e = (2\tilde{\ell})^4 \cdot e = (2\tilde{\ell}^4)_E = 0$, since $\tilde{\ell}^3 = 0$ ($\because$ $\ell^3 = 0$). Similarly, $\tilde{\omega}^3 \cdot e^2 = 0$.

$\tilde{\omega}^2 \cdot e^3 = ((2\tilde{\ell})^2 \cdot e^2)_E = (2\tilde{\ell})_E^2 \cdot (e|_E)^2 = ((2\tilde{\ell})^2 \cdot \zeta^2)_E = 4$

$\tilde{\omega} \cdot e^4 = (2\tilde{\ell} \cdot \zeta^3)_E = 18$

$e^5 = (e|_E)^4 = \zeta^4 = 51.$

$\lrcorner$

We now go back and verify the transversality