

$$H_2(M, \mathbb{Z}) \times H_2(M, \mathbb{Z}) \rightarrow \mathbb{Z}$$

is symmetric and non degenerate.

⌈ Nondegenerate means that $H_2(M, \mathbb{Z}) / \text{torsion} \times H_2(M, \mathbb{Z}) / \text{torsion}$

$\rightarrow \mathbb{Z}$ is nondegenerate, I think. I don't see

$H_2(M, \mathbb{Z})$ has no torsion for an algebraic surface M .

Refer to P110. and P53 (Poincaré duality)

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For divisors D and D' on M we define the intersection number $D \cdot D'$ of D and D' to be simply the intersection number of their fundamental classes $(D), (D') \in H_2(M, \mathbb{Z})$. Similarly, if $L \rightarrow M$ and $L' \rightarrow M$ are two line bundles, we take the intersection number $L \cdot L'$ of L and L' to be given by

$$L \cdot L' = (C_1(L) \cup C_1(L')) [M],$$

and likewise we define the intersection number $L \cdot D$ of a line bundle L with a divisor D to be just the value of the Chern class $C_1(L) \in H^2(M, \mathbb{Z})$ on the fundamental class $(D) \in H_2(M, \mathbb{Z})$ of D .

Since intersection of cycles is Poincaré dual to cup product, all these definitions are consistent with the correspondence between divisors and line bundles; i.e., if $L = [D]$ and $L' = [D']$, then $D \cdot D' = L \cdot D' = L' \cdot D = L \cdot L'$.

⌈ P53 ~ P59. & P60 ~ P61

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