

$$\mathcal{F} \xrightarrow{\theta} \mathcal{F}^+$$

Injectiveness of  $\theta$  is clear.  
We need to show  $\theta$  is surjective.

$$\text{Given } \tau \in \mathcal{F}^+(U), \quad \tau: U \longrightarrow \bigcup_{p \in U} \mathcal{F}_p.$$

$\Rightarrow$  For each  $p \in U$ ,  $\exists p \in V_p$  s.t.  $V_p \subset U$ .  
and  $\sigma_p \in \mathcal{F}(V_p)$ .  $\sigma(p)_q = \tau(q)$ .

$\Rightarrow$  Obviously,  $\sigma(p)|_{V_p \cap V_{p'}} = \sigma(p')|_{V_p \cap V_{p'}}$  if  $V_p \cap V_{p'} \neq \emptyset$ .  
 $\Rightarrow \exists \sigma \in \mathcal{F}(U)$  s.t.  $\sigma|_{V_p} = \sigma(p)$ .

$$\begin{aligned} \Rightarrow \sigma_p &= \sigma(p)_p. & \sigma_q &= \sigma(p)_q. & \text{for } q \in V_p. \\ \Rightarrow \sigma_p &= \tau(p). & \text{for all } p. & \Rightarrow \theta(\sigma) = \tau. \end{aligned}$$

Def: A subsheaf of a sheaf  $\mathcal{F}$  is a sheaf  $\mathcal{F}'$  s.t. for every open set  $U \subseteq X$ ,  $\mathcal{F}'(U)$  is a subgroup of  $\mathcal{F}(U)$ , and the restriction maps of the sheaf  $\mathcal{F}'$  are induced by those of  $\mathcal{F}$ .

It follows that for any point  $p$ , the stalk  $\mathcal{F}'_p$  is a subgroup of  $\mathcal{F}_p$ .

If  $\varphi: \mathcal{F} \rightarrow \mathcal{G}$  is a morphism of sheaves, we define the kernel of  $\varphi$ , denoted by  $\ker \varphi$ , to be the presheaf kernel of  $\varphi$  (which is a sheaf). Thus  $\ker \varphi$  is a subsheaf of  $\mathcal{F}$ .

We say that a morphism of sheaves  $\varphi: \mathcal{F} \rightarrow \mathcal{G}$  is injective if  $\ker \varphi = 0$ .

Thus  $\varphi$  is injective  $\Leftrightarrow$  the induced map  $\varphi(U): \mathcal{F}(U) \rightarrow \mathcal{G}(U)$  is injective for every open set  $U$  of  $X$ .