genus 0. Then

$$h^1(S, \mathcal{O}) = h^0(S, \Omega^1) = 0$$

and so, for $L = [p]$ the line bundle associated to any point $p \in S$ we see from the long exact cohomology sequence associated to the sequence

$$0 \to \mathcal{O}_S \longrightarrow \mathcal{O}_S(L) \longrightarrow L_p \to 0$$

that L has a global section nonzero at p, i.e., there exists a nonconstant meromorphic function f on S, holomorphic away from p and having only a simple pole at p.

Γ $\quad H^{1,0}(S) = \overline{H^{0,1}(S)} = H^0(S, \Omega^1) = \overline{H^1(S, \mathcal{O})}$

$\Rightarrow$ Since $g(S) = h^0(S, \Omega^1) = \dim H^0(S, \Omega^1)$, and the assumption $g = 0$, $\quad h^1(S, \mathcal{O}) = h^0(S, \Omega^1) = 0$.

From the page 138 ~ 139,

$$0 \longrightarrow \mathcal{O}_M(E \otimes [-D]) \longrightarrow \mathcal{O}_M(E) \longrightarrow \mathcal{O}_D(E|_D) \longrightarrow 0$$

Let $E = L$, $D = p$ $\Rightarrow$ $\mathcal{O}_S(L \otimes [-p]) = \mathcal{O}_S(S \times \mathbb{C})$
$= \mathcal{O}_S$. $\quad \mathcal{O}_p(L|_p) = L_p$.

$\Rightarrow$ $\quad 0 \longrightarrow \mathcal{O}_S \longrightarrow \mathcal{O}_S(L) \longrightarrow L_p \longrightarrow 0$

$\Rightarrow$

$$H^0(S, \mathcal{O}) \longrightarrow H^0(S, \mathcal{O}_S(L)) \longrightarrow \underset{\overset{\|}{L_p}}{H^0(S, L_p)} \longrightarrow \underset{\overset{\|}{0}}{H^1(S, \mathcal{O})}$$

$\Rightarrow$ $\quad H^0(S, \mathcal{O}) \longrightarrow H^0(S, \mathcal{O}(L)) \longrightarrow L_p \longrightarrow 0$

Choose $0 \neq v \in L_p$. $\Rightarrow$ we have $\sigma \in H^0(S, \mathcal{O}(L))$ s.t $\sigma(p) = v$. $\Rightarrow$ Consider a section s_0 of L s.t $(s_0 = 0) = (p)$ $\Rightarrow$ σ/s_0 is a meromorphic function on S, which satisfies the condition.