We want to show

$$\begin{array}{ccc} \mathcal{H}^p(\Omega^*(*D))_x & \longrightarrow & \mathcal{H}^p(\mathcal{A}^*(*D))_x = H^p_{DR}(P^*(k.n)) \\ \varphi + d\Omega^{q-1} & \longmapsto & 0 \end{array}$$

is injective. ┘

Before giving the proof, we remark that on two previous occasions. we have proved the isomorphism

$$H^*(M.\mathbb{C}) \cong H^*_{DR}(M.\mathrm{hol})$$

for a complex manifold $M$ satisfying

$$H^q(M.\Omega^p) = 0 \;,\quad q > 0.$$

┌ See p449 ┘

Since this latter is true for $M = P^*(k.n)$, we may write $\varphi = d\eta$ where $\eta$ is holomorphic in $P^*(k.n)$ but may have an essential singularity on the divisor $(z_1 \cdots z_k) = 0$.

┌ By the remark on p27, $H^{p.q}_{\bar\partial}(P^*(k.n)) = H^q(P^*(k.n), \Omega^p) = 0$. ┘

By being careful we must show that $\eta$ may be taken to be meromorphic.

Proof. The argument is not difficult but is a little long. We shall concentrate on writing $\varphi = d\eta$, where $\eta$ has at most a pole on the divisor $(z_1 \cdots z_k) = 0$. The argument will also show that