Then since the base locus of σ_i's is empty, i.e. $\{X_0^3 = X_1^3 = X_2^3 = X_3^3 = 0\} = \phi$, by the Bertini's theorem, $\exists$ a nonsingular cubic in $\mathbb{P}^3$. $\Rightarrow$
$\pi_2(\mathcal{S}) = V$ is a proper subvariety of $\mathbb{P}^{19}$.
$\Rightarrow$ $W - V$ is open and connected. $\Rightarrow$ $W - V$ is path-connected. $\Rightarrow$ We have a smooth curve γ joining S_0 & S. ⏎

Let $\pi : X \longrightarrow W$ be the projection map on the first factor. The inverse image $X' = \pi^{-1}(\gamma(I)) \subset X$ is a smooth manifold, and the map $\gamma^{-1} \circ \pi : X' \longrightarrow I$ is smooth.

For simplicity, assume σ_1, σ_2 form a basis for $H^0(\mathbb{P}^3, \mathcal{O}(3H))$.
$\Rightarrow$ $\gamma(t) = a_1(t)\, \sigma_1(x, y, z) + a_2(t)\, \sigma_2(x, y, z) \overset{\text{actually}}{=} \gamma(t, x, y, z)$
Consider the following map, locally, to use the inverse function theorem

$$(t, x, y, z) \longmapsto (t, x, y, \gamma(t, x, y, z))$$

and assume $\dfrac{\partial \gamma}{\partial z} \neq 0$ since $\gamma(t, x, y, z)$ is nonsingular. $\Rightarrow$ The Jacobian is nonzero.
$\Rightarrow$ $G(t, x, y, \gamma(t, x, y, z)) = (t, x, y, z)$.
$\Rightarrow$ If $\gamma(t, x, y, z) = 0$, $z = g(t, x, y)$ for some smooth function g. Here we assumed x, y, z are real.
$\Rightarrow$ $\{\gamma(t, x, y, z) = 0\}$ is a smooth manifold, and $X' = \pi^{-1}(\gamma(I)) = \{\gamma(t, x, y, z) = 0\}$ is a smooth manifold.