Let $\{\varphi_1, \varphi_2 \cdots \varphi_n\}$ be a unitary local frame around z_0.
s.t $d\varphi_i$'s $=0$ at z_0.

Define e_K. $\bar{e}_K$, $\bar{\iota}_K$. ι_K as follows;

$$e_K(\varphi_I \wedge \bar{\varphi}_J) = \varphi_K \wedge \varphi_I \wedge \bar{\varphi}_J$$
$$\bar{e}_K(\varphi_I \wedge \bar{\varphi}_J) = \bar{\varphi}_K \wedge \varphi_I \wedge \bar{\varphi}_J.$$

and ι_K, $\bar{\iota}_K$ are adjoints of e_K. $\bar{e}_K$ respectively.

Then $L(\eta) = \eta \wedge \omega$ can be expressed as

$\frac{i}{2} \sum e_K \bar{e}_K$. . Its adjoint Λ may be expres ed as $-\frac{i}{2} \sum \bar{\iota}_K \iota_K$.

Define ∂_K and $\bar{\partial}_K$ on $A^{p,q}(M)$ by

$$\partial_K\left(\sum \eta_{I\bar{J}} \varphi_I \wedge \bar{\varphi}_J\right) = \sum v_K(\eta_{I\bar{J}}) \varphi_I \wedge \bar{\varphi}_J$$

and $$\bar{\partial}_K\left(\sum \eta_{I\bar{J}} \varphi_I \wedge \bar{\varphi}_J\right) = \sum \bar{v}_K(\eta_{I\bar{J}}) \varphi_I \wedge \bar{\varphi}_J.$$

⇒ Note that ∂ and $\bar{\partial}$ commute with e_l. $\bar{e}_l$. ι_l and $\bar{\iota}_l$ and with each other.

$\iota_K e_K + e_K \iota_K = 2$
$e_K \iota_l + \iota_l e_K = 0$ if $k \neq l$
$e_K \bar{\iota}_l + \bar{\iota}_l e_K = 0$ for all k, l.

* If we assume that the adjoint of ∂_K is $-\bar{\partial}_K$, the adjoint of $\bar{\partial}_K$ is assumed to be $-\partial_K$.

⇒ By the computation on P114, we know that
$[\Lambda, \bar{\partial}'] = -i\partial^*$, where $\bar{\partial}' = \sum \bar{\partial}_K e_K$, $\partial^* = -\sum \partial_K \bar{\iota}_K$,