Changing y into $y-k$ corresponds to changing

$$\frac{Z_2}{Z_0} \longmapsto \frac{Z_2}{Z_0} - k = \frac{Z_2 - kZ_0}{Z_0}.$$

$$\Rightarrow \quad \begin{aligned} Z_0 &\longmapsto Z_0 \\ Z_1 &\longmapsto Z_1 \\ Z_2 &\longmapsto Z_2 - kZ_0 \end{aligned} \quad \Rightarrow \text{ linear transformation on } \mathbb{P}^2.$$

$y^2 = x^3 + ax + b \Rightarrow x^3 + ax + b = 0$ can not have a zero of order ≥ 2, since

if $y^2 - (x-\alpha)^2(x-\beta) = f(x,y)$.

$$\Rightarrow \frac{\partial f}{\partial x} = -2(x-\alpha)(x-\beta) - (x-\alpha)^2 \overset{1}{\overset{\shortparallel}{(x-\beta)'}}$$

$$\frac{\partial f}{\partial y} = 2y$$

at $(\alpha, 0)$, $\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = 0$. $*$ to the embedding. $\Rightarrow$ $x \longmapsto (\beta-\alpha)x + \alpha = x'$

$$Z_1/Z_0 \longmapsto (\beta-\alpha)\frac{Z_1}{Z_0} + \alpha\frac{Z_0}{Z_0}$$

$$= \frac{(\beta-\alpha)Z_1 + \alpha Z_0}{Z_0}$$

$$\Rightarrow \quad \begin{aligned} Z_0 &\longmapsto Z_0 \\ Z_1 &\longmapsto (\beta-\alpha)Z_1 + \alpha Z_0 \\ Z_2 &\longmapsto Z_2. \end{aligned}$$

$\Rightarrow$ We have $y^2 = (\beta-\alpha)^3 x(x-1)(x-\lambda)$

$\Rightarrow$ $y \mapsto$ something