$\Rightarrow$ V' is irreducible by Proposition on P21. $\lrcorner$

As suggested in the last chapter, the holomorphic Euler characteristic $\chi(\mathcal{O}_M)$ of M is itself expressible as a polynomial in the Chern classes of M: the formula

$$\chi(\mathcal{O}_M) = \frac{1}{12}(c_1(M)^2 + c_2(M))$$

$$= \frac{1}{12}(K \cdot K + \chi_2(M))$$

is called Noether's formula. We defer the proof until the last section of this chapter.

$\ulcorner$ $c_1(M) = c_1(TM) = -c_1(T^*M) = -K \Rightarrow c_1(M)^2 = K \cdot K$ $\chi_2(M) = c_2(TM) = c_2(T'M)$ see P17. $\lrcorner$

We observe that the Riemann-Roch theorem for line bundles gives a direct proof of the index theorem for divisors, as follows: let E be a positive divisor on S. We have seen (P. 164) that the intersection pairing on the group $H^{1,1}(S) \cap H^2(S, \mathbb{Z})$ of divisors modulo homology is nondegenerate: if it had two positive eigenvalues, it would of course have at least one in the orthogonal complement of the class of E; i.e., we could find a divisor D with

$$D \cdot E = 0 \text{ and } D \cdot D = d > 0.$$

$\ulcorner$ $H^{1,1}(S) \cap H^2(S, \mathbb{Z}) = \dfrac{\{\text{divisors on } M\}}{\text{homological equivalence}}$ by Lefschetz theorem on $(1,1)$ classes on P163 ~ P164.