$$(\nabla_{v_j}(\nabla_{v_i} f))_\alpha = (f_{\alpha,i})_j = f_{\alpha,i,j}$$
$$= v_j(f_{\alpha,i}) + A^0(f\, i)$$
$$= v_j(v_i(f_\alpha) + A^0(f)) + A^0(\text{first order of } f_\alpha\text{'s and } f_\alpha\text{'s})$$
$$= v_j v_i(f_\alpha) + v_j(A^0(f)) + A^0(\longrightarrow)$$

In the same way.

$$v_i(v_j(f_\alpha)) + v_i(A^0(f)) + A^0(\text{first order} + f_\alpha\text{'s})$$
$$= f_{\alpha,j,i} = v_i(v_j(f_\alpha)) + A^1(f) + A^0(f)$$

$$\Rightarrow \quad f_{\alpha,j,i} - f_{\alpha,i,j} = v_i(v_j(f_\alpha)) - v_j(v_i(f_\alpha)) + A^1(f)$$

$$[\nabla_i, \nabla_j] f_\alpha = A^1(f)$$
$$\|$$
$$\nabla_i \nabla_j f_\alpha - \nabla_j \nabla_i f_\alpha = \left((\nabla_{v_i}(\nabla_{v_j} f))_\alpha - (\nabla_{v_j}(\nabla_{v_i} f))_\alpha\right)$$

Suppose now that E and $T(M)$ have metrics and that $\{e_\alpha\}$, $\{v_i\}$ are orthonormal frames.
The global Sobolev s-norm of sections $f \in C^\infty(M, E)$ is defined by

$$\|f\|_s^2 = \sum_{k \le s} \int_M \|\nabla^k f\|_0^2 \, dx.$$

where $\nabla^k f = \underbrace{\nabla(\nabla(\cdots(\nabla f)\cdots))}_{k \text{ times}}$.

Denote by $H_s(M, E)$ the completion of $C^\infty(M, E)$ in this norm. Since by our remarks at the