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$$\Rightarrow f_L : \bigcup_{L' \in B_L - L} L' \longrightarrow \mathbb{P}^3$$
$$\quad\quad L' \longmapsto f_L(L')$$

$\Rightarrow$ $B_L - L \longleftrightarrow E_L$ is isomorphic.
$L' \longleftrightarrow f_L(L')$

$V_2 \cap E_L \ni y = f_L(L') \Rightarrow L' \cap L \neq \phi$ and by the note P993. since $L' \subset \langle L, y\rangle \subset \langle L, L'\rangle \subset \langle L, V_2\rangle = \overline{L, V_2}$ which is a hyperplane in $\mathbb{P}^5$ ($\because$ $L \cap V_2 \doteqdot \phi$ and $\overline{L, V_2} = \mathbb{P}^4$).

$\lrcorner$

But for generic V_2, the hyperplane section $\overline{L, V_2} \cap X$ is the smooth intersection of two quadrics in 4-space, and we have seen that each of the 16 lines on such a surface meets exactly five other lines on the surface.

$\ulcorner$ Let $K = \{ H \mid H$ hyperplane in $\mathbb{P}^5$ containing $L \}$.
$\Rightarrow$ $\dim K = 3$ i.e. $K = \mathbb{P}^3$.
Let $Q = \{ H \mid H$ hyperplane in $\mathbb{P}^5$, $H \supset T_x(X)$ for some $x \in L \}$ $\Rightarrow$ Since x varies over L and, for a fixed $x \in L$, $\dim \{ H \mid H \supset T_x(X) \} = 1$, $\dim Q = 2$. $\Rightarrow$ $K - Q \neq \phi$
Thus, for generic $H \supset L$, $H \cap X$ is smooth on L since $H + T_x(X) = \mathbb{P}^5$.