Date

$\vdots$

$$h^0(mD) \geq \frac{m(m+1)}{2}(n-1) + m+1$$

$$h^0((l+m)D) \geq \frac{m(m+1)}{2}(n-1) + m+1 + ld.$$

$\ulcorner$ $h^0(D) \geq n+1$ by the proof of corollary P251

$h^0(2D) - h^0(D) \geq 2(n-1)+1$

$\vdots$

$h^0(mD) - h^0((m-1)D) \geq m(k-1)+1$

$h^0((m+1)D) - h^0(mD) = d.$ $\lrcorner$

$\vdots$

But now for l sufficiently large, the divisor $(l+m)D$ will be nonspecial.

$\ulcorner$ $\deg(K-(l+m)D) = \deg K - (l+m)d < 0$. for l suffiently large. $\Rightarrow$ $h^0(K-(l+m)D) = 0$. $\Rightarrow$ $(l+m)D$ is not special. $\lrcorner$

By Riemann-Roch, then

$$h^0((l+m)D) = (l+m)d - g+1,$$

so that

$$(*) \qquad g \leq (l+m)d \; \frac{m(m+1)}{2}(n-1) - m - 1 - ld + 1$$

$$= \frac{m(m-1)}{2}(n-1) + m(d - m(n-1)-1).$$

$\ulcorner$ By R-R, $h^0((l+m)D) = (l+m)d - g + 1 + \underset{0}{\underset{\shortparallel}{h^0(K-(l+m)D)}}$

$\Rightarrow$ By $h^0((l+m)D) \geq \frac{m(m+1)}{2}(n-1) + m+1+ld,$

$$(l+m)d - g+1 \geq \frac{m(m+1)}{2}(n-1) + m+1+ld.$$