$\left(-\frac{1}{\pi}\frac{\partial^2}{\partial z_i \partial \bar{z}_j}\log|s|^2\right)$ is positive. definite matrix.

by $P_2$9. (see the def.)

$\Rightarrow \left(-\frac{\partial^2}{\partial z_i \partial \bar{z}_j}\log|s|^2\right)$ is positive definite matrix

$\Rightarrow \left(\frac{\partial^2}{\partial z_i \partial \bar{z}_j}\log|s|^2\right)$ is negative definite matrix.

$\Rightarrow$ The Hessian $H(\varphi)$

$$\begin{pmatrix} \frac{\partial^2\varphi}{\partial x_i \partial x_j}, & \frac{\partial^2\varphi}{\partial x_i y_j} \\ \frac{\partial^2\varphi}{\partial y_i \partial x_j} & \frac{\partial^2\varphi}{\partial y_i \partial y_j} \end{pmatrix}$$

has at least n negative eigenvalues.

Γ lemma: Suppose. $A+B+i(D-C)$ is negative definite hermitian. where $A.B$ $n\times n$ matrix $^tA = A$ $^tB = B$. $^tC = D$.

Then prove that $\begin{pmatrix} A & D \\ C & B \end{pmatrix}$ has at least

n negative eigen values.

pf). Since $^tC = D$, $C+D$ is real symmetric.

$\Rightarrow$ $C+D$ has real n - eigenvalues.

Let $\{X_{+1}, X_{+2} \cdots X_{+l}\}$ be eigenvectors with + eigenvalues.

$\{X_{01}\ X_{02} \cdots X_{0m}\}$ 〃 0 〃

$\{X_{-1}\ X_{-2} \cdots X_{-k}\}$ 〃 − 〃

where $l+m+k = n$.