$\mathcal{S}_n$.
③ We have dealt with $\mathbb{P}^2$.

orphic line bundle with two sections s, s' such that the simultaneous equations $s=0$, $s'=0$ define a zero-dimensional subscheme Z of S.

⌜ See P697. $Z = \{ z \in S : s = 0 = s' \}$ whose structure sheaf $\mathcal{O}_Z = \mathcal{O}/\mathcal{I}$ is a sheaf of rings possibly with nilpotent elements. ⌟

There is an ideal sheaf $\mathcal{J} \subset \mathcal{O}$ with $\mathcal{O}_Z = \mathcal{O}/\mathcal{J}$; in fact, $\mathcal{J}$ is the image under the mapping

$$\mathcal{O}(L^*) \otimes \mathcal{O}(L^*) \longrightarrow \mathcal{O}$$

given by $(f, f') \longmapsto fs + f's'$.

⌜ See P697. The authors explain what is $\mathcal{J}$.
$\mathcal{J} = \operatorname{Im} \alpha$, where

$$\mathcal{O}(L^*) \otimes \mathcal{O}(L^*) \xrightarrow{\alpha} \mathcal{O}$$

$$\mathcal{O}(L^*)(U) \otimes \mathcal{O}(L^*)(U) \xrightarrow{\alpha_U} \mathcal{O}(U)$$

$$(f, f') \longmapsto f \otimes s + f' \otimes s'$$

Try to think everything simply. ⌟