

Suppose  $h[a] = 0$  in  $H^{p,q}(E_r)$ .

$\Rightarrow a = dl + dx + y$  where

$$dl \in d(F^{p-r}K^{p+q-1}) \cap F^pK^{p+q}$$

$$dx \in d(F^{p-r+1}K^{p+q-1})$$

$$y \in F^{p+1}K^{p+q}.$$

$\Rightarrow$  Since  $d dl = 0$ ,  $[dl] = 0$  in  $B$ .

$$da = dy \in F^{p+r+1}K^{p+q+1}$$

$\Rightarrow d(dx+y) \in F^{p+r+1}K^{p+q+1}$  and

$$dx = a - dl - y \in F^pK^{p+q} \Rightarrow dx+y \in F^pK^{p+q}, \text{ and}$$

$$dx+y \in A \Rightarrow [a] = 0 \text{ in } B.$$

$\Rightarrow h$  is one to one.

Clearly  $h$  is onto.  $\Rightarrow h$  is isomorphic.

$$\Rightarrow H^{p,q}(E_r) \cong E_{r+1}^{p,q} \cong \frac{\{a \in F^pK^{p+q}; da \in F^{p+r+1}K^{p+q+1}\}}{(d(F^{p-r}K^{p+q-1}) + F^{p+1}K^{p+q}) \cap \downarrow}$$

□

For  $r$  sufficiently large,

$$E_r^{p,q} = \frac{\{a \in F^pK^{p+q}; da=0\}}{dK^{p+q-1} + F^{p+1}K^{p+q}}$$

$$\cong \frac{F^p H^{p+q}(K^*)}{F^{p+1} H^{p+q}(K^*)}$$

$$= Gr^p H^{p+q}(K^*).$$

This completes the proof of the proposition. Q.E.D.

□ For  $r$  sufficiently large,  $F^{p+r}K^* = \{0\}$  by the def.  
 $F^{p+r}K^{p+q+1} = \{0\}$ , and  $F^{p-r+1}K^* = K^*$ ,  $F^{p-r+1}K^{p+q-1} =$