「Remark; The pullback divisor is well-defined as long as $\pi(M) \not\subset D$, for. $\exists$ at least one point s.t at that point, $f_\alpha(\pi(x)) \neq 0$. for some α.

$\Rightarrow \{\pi^* f_\alpha\}$ defines a global section, for, if $\pi^* f_\alpha \equiv 0$ on $\pi^{-1}(U_\alpha)$ $\Rightarrow$ $\pi^* f_\beta \equiv 0$ on $\pi^{-1}(U_\beta)$. $(U_\alpha \cap U_\beta \neq \phi)$,
since $\dfrac{f_\beta \circ \pi}{f_\alpha \circ \pi} \in \mathcal{O}^*(U_\alpha \cap U_\beta)$ $(\Leftrightarrow f_\beta \circ \pi = h \cdot f_\alpha \circ \pi)$
where h is non-zero holomorphic on $U_\alpha \cap U_\beta$, and by the identity theorem.
Since $\{\pi^{-1}(U_\alpha)\}$ covers M, $\pi^* f_\alpha \equiv 0$ for all α.
$\Rightarrow$ This implies $\pi(M) \subset D$.
Similarly we can show this for pole cases. Take the reciprocal. 」

「I think "the multiplicities may occur" means that $\pi(M)$ may be tangent to V. $\Rightarrow$ This might not be true.
Suppose π is 2-fold covering map.

$\Downarrow$
Multiplicity means that in $f = f_1^{a_1} f_2^{a_2}$, a_1, a_2.

$$\begin{array}{ccc} M & \xrightarrow{\pi} & M = N \\ \cup & & \cup \\ V & \longrightarrow & 2V \end{array}$$

Example

$\pi : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$ defined by
$(z_1, z_2) \longmapsto (z_1^2, z_2^2)$.

Consider a section $\mathbb{C}^2$ as follows;

$$f : \mathbb{C}^2 \longrightarrow \mathbb{C}$$
$$(z_1, z_2) \longmapsto z_1$$

$\Rightarrow \pi^* f = f^2$ $\quad (\because \pi^* f(z_1, z_2) = f(z_1^2, z_2^2) = z_1^2$
$= f^2(z_1, z_2)$

$\Rightarrow$ The corresponding divisor to f $\pi^* f = 2(\{0\} \times \mathbb{C})$
while $f = \{0\} \times \mathbb{C}$ and $\pi^{-1}(\{0\} \times \mathbb{C}) =$ simply $\{0\} \times \mathbb{C}$
not $2 \times (\{0\} \times \mathbb{C})$. 」