

of independent relations among the row vectors
 $= g - \#$ of linearly independent row vectors.
 $= \dim$ of a set of vectors perpendicular to all row vectors. \Downarrow

We have proved the Riemann - Roch for effective divisors, and hence for all divisors of degree $\geq g$.

Γ Suppose D is a divisor of degree $\geq g$.

By the corollary of Jacobi inversion, D is linearly equivalent to an effective divisor. \Downarrow

For a general D of degree $\leq g-2$, we apply the formula to $K-D$ to obtain

$$h^0(K-D) = (2g-2-d) - g+1 + h^0(D) \\ \Rightarrow h^0(D) = d-g+1 + h^0(K-D).$$

Γ $\deg(K-D) = 2g-2-d \Rightarrow 2g-2-d \geq 2g-2-g+2 = g \Rightarrow K-D \sim$ effective divisor \Rightarrow We have the formula to apply to $K-D$. \Downarrow

Finally, if $\deg D = g-1$ and neither D nor $K-D$ is linearly equivalent to an effective divisor, then $h^0(D) = h^0(K-D) = 0$, and the formula again holds.

Γ $h^0(D) = \dim H^0(S, \mathcal{O}(D)) \neq 0 \Rightarrow \exists \sigma \in H^0(S, \mathcal{O}(D))$
 $\Rightarrow (\sigma=0) \sim D \Rightarrow$ Since $(\sigma=0)$ is an effective divisor, D must be linearly equivalent to an effective divisor. \Downarrow

