by linearity. see P 403. in the proof of lemma.

$$= (-1)\binom{n}{r}\sum_{i=1}^{n-r}\tilde{P}(E,\cdots,\iota(v)\Theta,\cdots E,\Theta,\cdots\Theta)$$

$$= (-1)\binom{n}{r}\cdot(n-r)\quad\tilde{P}(\underbrace{E,\cdots E}_{n-r-1},\iota(v)\Theta,\Theta,\cdots\Theta)$$

$$= (-1)\ {}_nC_r\cdot(n-r)\quad\tilde{P}(E,\cdots E,\iota(v)\Theta,\cdots\Theta)$$

$$= (-1)\ (r+1)\ {}_nC_{r+1}\quad\tilde{P}(E,\cdots E,\iota(v)\Theta,\cdots\Theta)$$

$$= (-1)\ {}_nC_{r+1}\quad\iota(v)\ \tilde{P}(\underbrace{E,\cdots E}_{n-r-1},\underbrace{\Theta,\cdots\Theta}_{r+1})$$

$$= (-1)\ \iota(v)\left\{\binom{n}{r+1}\tilde{P}(E\cdots E,\Theta,\cdots\Theta)\right\} = (-1)\ \iota(v)\ P_{r+1}(E,\Theta).$$

Let $\omega\in A^{1,0}(M^*)$ be the form dual to v under the metric on M; set

$$\Phi_r = \omega\wedge(\bar\partial\omega)^{n-r-1}\wedge P_r(E,\Theta)\in A^{n,n-1}(M^*).$$

We have, for $0\le r\le n-1$,

$$\bar\partial\Phi_r = (\bar\partial\omega)^{n-r}\wedge P_r(E,\Theta)+\omega\wedge(\bar\partial\omega)^{n-r-1}\wedge\iota(v)P_{r+1}(E,\Theta);$$

since $\iota(v)\omega = 1$,

$$0 = \iota(\bar\partial v)(\omega)+\iota(v)\bar\partial\omega \Rightarrow \iota(v)\bar\partial\omega = 0,$$

and so

$$\iota(v)\bar\partial\Phi_r = (\bar\partial\omega)^{n-r}\wedge\iota(v)P_r(E,\Theta)+(\bar\partial\omega)^{n-r-1}\wedge\iota(v)P_{r+1}(E,\Theta).$$

$$\bar\partial\Phi_r = (\bar\partial\omega)^{n-r}\wedge P_r(E,\Theta)+\overset{(-1)^{2(n-r)+1+1}}{\vee}\omega\wedge(\bar\partial\omega)^{n-r-1}\wedge\iota(v)P_{r+1}(E,\Theta)$$

$$1 = \iota(v)(\omega) = \omega(v)$$

$$0 = \bar\partial(\omega(v)) = \underbrace{(\bar\partial\omega)(v)+\omega(\bar\partial v)} = \iota(v)(\bar\partial\omega)$$

since $\bar\partial v = 0$ ($\because$ v is holomorphic)