$\Rightarrow$ Any quadric is of form $X_0^2+X_1^2+X_2^2=0$ or $X_0^2+X_1^2=0$ in $\mathbb{P}^2$ $\Rightarrow$ Any curve which is singular has a double point and no other singularities.
$\Rightarrow$ L is a Lefschetz pencil of quadrics on $\mathbb{P}^2$.
Since any element in L may be expressed as aS_1+bS_2, $S_1, S_2 \in L$, $a, b \in \mathbb{C}$, $(S_1=0)\cap(S_2=0)$ is the base points.
$\Rightarrow \#((S_1=0)\cap(S_2=0)) = F_\lambda \cdot F_\lambda$, where F_λ is a generic element. Of course, we count multiplicities.
$\Rightarrow$ Since F_λ is a quadric, $F_\lambda \cdot F_\lambda = 4$, by the Bezout theorem on P670, & P172. ◻

Since $\chi(F_\lambda)=2$ and $n=4$, this yields

$$3 = 2\cdot 2 + \mu - 4,$$

i.e., $\mu=3$ and W_1 is cubic.

$\ulcorner$ F_λ is a quadric, and if it is smooth, by the genus formula, see P221 & P216, $0 = \frac{-\chi(F_\lambda)+2}{2} \Rightarrow$ $\chi(F_\lambda)=2$. $\Rightarrow$ $3 = \chi(\mathbb{P}^2) = \dim H_0(\mathbb{P}^2) + \dim H_2(\mathbb{P}^2) + \dim H_4(\mathbb{P}^2) = 4+\mu-4$ $\Rightarrow$ $\mu=3$. $\Rightarrow \#(L\cap W_1)=3$ $\Rightarrow$ W_1 is cubic in W. (# of singular cubics in L). ◻

3. Letting L again be a generic pencil of W, L will have four base points p_1, p_2, p_3, p_4 and will consist of all conics in $\mathbb{P}^2$ passing through the points $\{p_i\}$. (See Figure 1.)