

$$\{v \in \mathbb{C}^n \mid \|v\| = 1\} = K$$

$\Rightarrow f'$ has maximum or minimum.

$$\Rightarrow \exists \mu_0 \in \mathbb{R} \text{ s.t.}$$

$$0 < \mu_0 + f(v) = \mu_0 + \frac{\langle Bv, v \rangle}{\langle Av, v \rangle}$$

for all v with $\|v\|=1$.

$$\Rightarrow \text{For } \forall v \in \mathbb{C}^n - \{0\}, \quad \frac{\langle B \frac{v}{\|v\|}, \frac{v}{\|v\|} \rangle}{\langle A \frac{v}{\|v\|}, \frac{v}{\|v\|} \rangle} = \frac{\langle B \frac{v}{\|v\|}, \frac{v}{\|v\|} \rangle}{\langle A \frac{v}{\|v\|}, \frac{v}{\|v\|} \rangle}$$

$$\Rightarrow \mu_0 + f(v) > 0 \text{ for all } v \in \mathbb{C}^{n-1,1}$$

$$\Rightarrow \mu + f(v) > 0 \text{ if } \mu \geq \mu_0, \text{ for all } v \in \mathbb{C}^n \text{ s.t. } |v| = 1.$$

$$\Rightarrow \langle \mu A v + B v, v \rangle \geq 0 \quad \text{if } \mu \geq \mu_0. \quad v \in \mathbb{C}^n \setminus \{0\}$$

$\Rightarrow \mu A + B$ is positive definite $\quad \square$

Indeed, the proof of Theorem B is essentially the same as that of Kodaira's theorem, the only difference being that now we must associate a definite sign to the curvature operator on a general vector bundle.

First, by Kodaira-Serre duality,

$$H^q(M, \mathcal{O}(L^\mu \otimes E)) \cong H^{n-q}(M, \mathcal{O}(L^{-\mu} \otimes E^* \otimes K_M)),$$

so it will be sufficient to prove that for any E ,
there exists μ s.t.