$$A = \begin{pmatrix} v_{11}, & \cdots & v_{1n} \\ \vdots & & \\ \vdots & & \\ v_{n1}, & \cdots & v_{nn} \end{pmatrix} \in GL(n)$$

According to Fiedorowicz lecture note P55.6 (K-theory), $GL(n, \mathbb{C})$ is homotopy equivalent to $U(n, \mathbb{C})$.
By Husemoller P92. $\pi_0(U(n, \mathbb{C})) = 0$. i.e. $U(n, \mathbb{C})$ is path-connected.
$\Rightarrow \exists\ \alpha : I \longrightarrow GL(n)$ s.t $\alpha(0) = I$. $\alpha(1) = A$.

$$\begin{array}{ccc} A : G(k,n) & \longrightarrow & G(k,n) \\ \Psi & & \Psi \\ W_a & \longmapsto & \sigma_a(V) \end{array}$$

But since $A \simeq I$ (homotopic to identity), P59.
$H_*(G(k,n)) \xrightarrow{A_* = id_*} H_*(G(k,n))$. See Greenberg.

The subvarieties $\sigma_a(V)$ are called the Schubert cycles of the Grassmannian.
The simplest example of a Grassmannian different from projective space is the $G(2,4)$ of 2-planes in $\mathbb{C}^4$. The Schubert cycles on $G(2,4)$ are

codim 1 : $\sigma_{1,0}(V_2) = \{\Lambda : \dim(\Lambda \cap V_2) \geqq 1\}$,
codim 2 : $\sigma_{1,1}(V_3) = \{\Lambda : \Lambda \subset V_3\}$,
$\sigma_{2,0}(V_1) = \{\Lambda : \Lambda \supset V_1\}$,
codim 3 : $\sigma_{2,1}(V_1, V_3) = \{\Lambda : V_1 \subset \Lambda \subset V_3\}$.

Γ $n - k + i - a_i = b_i$ $b_1 = 2 + 1 - 1 = 2$, $b_2 = 2 + 2 - 0 = 4$
$\dim(\Lambda \cap V_4) = 2$ codimension $= 2 \cdot (a_1 + a_2) = 2$ (real)
$\Rightarrow$ complex codim. is 1.