

the  $\psi_\alpha(\omega)$  are the same for  $N$ .

⌈ The metric on  $M \times N$  is  $\ll \gg_{M \times N} = \ll \gg_M + \ll \gg_N$  ⌋

Using the formula  $\bar{\partial}^* = - * \bar{\partial} *$  and the lemma, we find that

$$\begin{cases} \bar{\partial}_{M \times N}^* = \bar{\partial}_M^* + \bar{\partial}_N^* \\ \bar{\partial}_M \bar{\partial}_N^* + \bar{\partial}_N^* \bar{\partial}_M = 0 = \bar{\partial}_M^* \bar{\partial}_N + \bar{\partial}_N \bar{\partial}_M^* . \end{cases}$$

⌈  $\langle \bar{\partial}_{M \times N} \varphi, \psi \rangle = \langle \varphi, \bar{\partial}_{M \times N}^* \psi \rangle$

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$$\begin{aligned} \langle \bar{\partial}_M \varphi + \bar{\partial}_N \varphi, \psi \rangle &= \langle \bar{\partial}_M \varphi, \psi \rangle + \langle \bar{\partial}_N \varphi, \psi \rangle \\ &= \langle \varphi, \bar{\partial}_M^* \psi \rangle + \langle \varphi, \bar{\partial}_N^* \psi \rangle = \langle \varphi, (\bar{\partial}_M^* + \bar{\partial}_N^*) \psi \rangle \\ \Rightarrow \bar{\partial}_{M \times N}^* &= \bar{\partial}_M^* + \bar{\partial}_N^* . \end{aligned}$$

Since  $\bar{\partial}_{M \times N}^* = - * \bar{\partial}_{M \times N} * = - *_{M \times N} \bar{\partial}_{M \times N} *_{M \times N}$ ,

$$\langle \varphi, - *_{M \times N} \bar{\partial}_{M \times N} *_{M \times N} \psi \rangle = \langle \varphi, (\bar{\partial}_M^* + \bar{\partial}_N^*) \psi \rangle . \quad \text{---} \textcircled{*}$$

We have only to check for decomposable forms since they are dense.

Let  $\varphi = \varphi_M \wedge \varphi_N$   $\psi = \psi_M \wedge \psi_N$ .

$$\begin{aligned} \Rightarrow \langle \varphi_M \wedge \varphi_N, - *_{M \times N} \bar{\partial}_{M \times N} *_{M \times N} \psi_M \wedge \psi_N \rangle \\ &= \langle \varphi_M \wedge \varphi_N, - *_{M \times N} \bar{\partial}_{M \times N} \epsilon_1 *_{M \times N} \psi_M \wedge *_{M \times N} \psi_N \rangle \\ &= \langle \varphi_M \wedge \varphi_N, - *_{M \times N} \epsilon_1 (\bar{\partial}_M *_{M \times N} \psi_M \wedge *_{M \times N} \psi_N + (-1)^{\deg *_{M \times N} \psi_M} \bar{\partial}_N *_{M \times N} \psi_M \wedge *_{M \times N} \psi_N) \rangle \\ &= \langle \varphi_M \wedge \varphi_N, \epsilon_1 \end{aligned}$$