$\Rightarrow \quad a^*_{a_i} \geq i.$ for all i.

Now we want to compute $\sum_{i=1}^{n-k} a^*_i$ where $\{a^*_i\}$ is the smallest sequence s.t $a^*_{a_i} \geq i$.

$\Rightarrow$ The smallest sequence a^* is as follows.

$$a^*_1 \geq \cdots \geq \underset{\substack{\| \\ m_\ell}}{a^*_{a_{m_\ell}}} \geq \cdots \geq \underset{\substack{\| \\ m_{\ell-1}}}{a^*_{a_{m_{\ell-1}}}} \geq \cdots \geq \underset{\substack{\| \\ m_{\ell-2}}}{a^*_{a_{m_{\ell-2}}}} \geq \cdots \geq \underset{\substack{\| \\ m_1}}{a^*_{a_{m_1}}} \geq \cdots a^*_{n-k}$$

$$a^*_1 = \cdots = a^*_{a_{m_\ell}} = m_\ell \quad \Rightarrow \quad \sum a^*_i = m_\ell \cdot a_{m_\ell}$$
$$a^*_{a_{m_\ell}+1} = \cdots = a^*_{a_{m_{\ell-1}}} = m_{\ell-1} \quad \Rightarrow \quad \sum a^*_i = m_{\ell-1}\,(a_{m_{\ell-1}} - a_{m_\ell})$$
$$a^*_{(a_{m_{\ell-1}}-1)} = \cdots = a^*_{a_{m_{\ell-2}}} = m_{\ell-2} \quad \Rightarrow \quad \sum a^*_i = m_{\ell-2}\,(a_{m_{\ell-2}} - a_{m_{\ell-1}})$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \qquad \vdots \qquad \vdots$$
$$a^*_{a_{m_2}-1} = \cdots = a^*_{a_{m_1}} = m_1 \quad \Rightarrow \quad \sum a^*_i = m_1\,(a_{m_1} - a_{m_2})$$
$$a^*_{a_{m_1}-1} = \cdots = a^*_{n-k} = 0 \quad \Rightarrow \quad \sum a^*_i = 0.$$

$$\Rightarrow \quad \sum a^*_i = m_\ell\, a_{m_\ell} + m_{\ell-1}\,(a_{m_{\ell-1}} - a_{m_\ell}) + m_{\ell-2}(a_{m_{\ell-2}} - a_{m_{\ell-1}}) + \cdots + m_1\,(a_{m_1} - a_{m_2})$$
$$= (m_\ell - m_{\ell-1})\, a_{m_\ell} + (m_{\ell-1} - m_{\ell-2})\, a_{m_{\ell-1}} + \cdots + (m_2 - m_1)\, a_{m_2} + m_1\, a_{m_1} = \sum a_i$$

$$\Rightarrow \dim \sigma_a = \dim \sigma_{a^*} = k(n-k) - \sum_{i=1}^{k} a_i = k(n-k) - \sum_{i=1}^{n-k} a^*_i.$$