

since $\sigma(p', h)$ & $\sigma(p', h')$ are special, and
 $T_x(X) \cap X$ contains 3 lines ~~for~~ $\sigma(p')$. DATE ()

Consider $\overline{p_L, q'} \in L_2 = \sigma(p', h')$. \Rightarrow By the argument above,
 $p' = q'$.

$\Rightarrow \overline{p_L, q'} \in \sigma(q', h)$ and $\overline{p_L, q'} \in \sigma(q', h')$.

\Rightarrow If $h \neq h'$, then consider $x \in \sigma(q', h) \cap \sigma(q', h')$
 and $T_x(X) \cap X$. $\Rightarrow T_x(X) \cap X$ contains two lines
 from $\sigma(p)$ and $\sigma(h)$ respectively. \Rightarrow Contradiction.

$\Rightarrow h = h' \Rightarrow$ Since h contains q' , $p_L, p_L', h = h_L$.

Thus $\overline{p_L, q'}$ and $\overline{p_L', q'}$ must lie in $\sigma(q', h_L)$, which
 is special. But since $\sigma(h_L) \cap X = L \cup L'$, $\sigma(q', h_L)$
 is either L or L' . This is impossible since, by the
 assumption, L and L' are not special.

Thus $\overline{p_L, q'} = \overline{p_L', q'}$ or one of them are singular. \square

On the other hand, through a point $q' \notin R$ in S'
 there is only one singular line of X , and so
 we must have $\overline{p_L, q'} = \overline{p_L', q'}$.

\square If $q' \notin R$ in S , $\sigma(q') \cap X = \sigma(q', h_1) \cup \sigma(q', h_2)$
 $h_1 \neq h_2$. $\Rightarrow \exists$ only one singular line $h_1 \cap h_2$, since
 $\pi: \Sigma \rightarrow S$ is one to one & onto $S-R$. \Rightarrow

Thus by the result above, say $\overline{p_L, q'}$, it is
 singular $\Rightarrow \overline{p_L, q'} = \overline{p_L', q'}$ for,

① $\overline{p_L', q'}$ nonsingular

\Rightarrow As we saw above, $\overline{p_L', q'} \in \sigma(q', h)$ special

$\Rightarrow \sigma(q') \cap X = \sigma(q', h) \cup \sigma(q', \tilde{h})$

\Rightarrow Since $\overline{p_L, q'}$ is singular, $\overline{p_L, q'}$ and $\overline{p_L', q'}$ lie

$h \cap \tilde{h} = h_1 \cap h_2 = \overline{p_L, q'} \in \sigma(q') \cap X$