$$1 = L \cdot L = D \cdot D' > 1.$$

$\ulcorner$ Since $h^0(L) = 3$, $\dim |L| = 2$. Let $H^0(M, \mathcal{O}(L)) = \langle \sigma_1, \sigma_2, \sigma_3 \rangle$ and $(\sigma_1 = 0) = D$. If $p \in (\sigma_1 = 0)$ or $(\sigma_2 = 0)$, it is done. If not so, $(a_1\sigma_1 + a_2\sigma_2)(p) = 0 \Rightarrow$ $a_1\sigma_1(p) + a_2\sigma_2(p) = 0 \Rightarrow$ We can find nontrivial solutions $a_1, a_2$ (both not zero). $\Rightarrow$ For such $(a_1, a_2)$, let $\sigma = a_1\sigma_1 + a_2\sigma_2$. $\Rightarrow$ Let $D' = (a_1\sigma_1 + a_2\sigma_2 = 0)$. $\Rightarrow$ $D' \cap D \ni p$ $\Rightarrow$ $\#(D' \cap D) > 1$ since $p$ is a singular point, and $D$ does not meet $D'$ transversely. See P 62. $\Rightarrow$ $1 = L \cdot L = D \cdot D' > 1$ Contradiction. $\Rightarrow$ $D$ must be smooth. $\lrcorner$

The genus of the curve $D$ is given by the adjunction formula:

$$\pi(D) = \frac{D \cdot D + D \cdot K}{2} + 1 = \frac{1-3}{2} + 1 = 0,$$

i.e., $D \cong \mathbb{P}^1$.

$\ulcorner$ See P216 & P471. Since $D$ is smooth and irreducible,

$$g(D) = \frac{D \cdot D + D \cdot K}{2} + 1 = \frac{1-3}{2} + 1 = 0$$

$\Rightarrow$ By P222, $D \cong \mathbb{P}^1$. $\lrcorner$

The restriction $L|_D$ is then the hyperplane (i.e. point) bundle $H_{\mathbb{P}^1}$; and from the cohomology of the exact