

$$\frac{f_1 \cdots f_n dz_1 \cdots dz_n}{f_n(z)}).$$

Confusion again as on p426

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The Global Residue Theorem

Suppose that $M \subset M'$ are complex n -manifolds, where M is relatively compact with smooth boundary $\partial M = \bar{M} - M$. The case that $M = M'$ is itself compact will be in many respects the most interesting situation. Suppose that D_1, \dots, D_n are effective divisors defined in some nbd U of \bar{M} in M' and whose intersection $D_1 \cap \dots \cap D_n$ is a discrete - hence finite - set of points in M . By analogy with the previous notation, we set

$$D = D_1 + \dots + D_n,$$

$$U^* = U - (D_1 \cap \dots \cap D_n),$$

$$U_i = U - D_i$$

so that $\underline{U} = \{U_i\}$ is an open covering of U^* . Suppose that

$$\omega \in H^0(U, \Omega^n(D))$$

is a meromorphic n -form on U with polar divisor D .

For each point $p \in U_1 \cap \dots \cap U_n$ we may restrict ω to a nbd U_p of p and define the residue

$$\text{Res}_p \omega$$

as in the previous section. On the other hand,

$$\omega \in C^{\infty}(U, \Omega^n)$$

defines a class $[\omega] \in H^{n-1}(U^*, \Omega^n)$ that has a Dolbeault representative.