$\deg \check{S} = {}^{\#}(l_x \cap \check{S}) = {}^{\#}\{p : X_p \text{ is singular}\}$
is just the number μ of singular curves in this pencil.

$\ulcorner$ $l_x \cap \check{S} \ni p \Leftrightarrow X_p$ is singular, $p \in l_x$. $\Rightarrow$
$\mu =$ number of singular curves in $\{X_p\}_{p \in l_x} = {}^{\#}(l_x \cap \check{S})$
$= \deg \check{S}$ $\lrcorner$

Now the generic curve X_p is a smooth conic, with Euler characteristic 2, and if we take l_x disjoint from R, all the singular curves X_p in our pencil will consist of two distinct lines, i.e., the pencil will be Lefschetz. By the general formula

$$\chi(U) = 2\chi(X_p) - n + \mu$$

of Section 2, Chapter 4, we have

$$\chi(U) = 4 + \mu.$$

$\ulcorner$ $\deg X_p = 2 \Rightarrow$ By the genus formula, $g(X_p) = 0$
$\Rightarrow X_p \cong \mathbb{P}^1 \Rightarrow$ The Euler characteristic of X_p is 2.
By the guess on P263, we assume for the time being that R is a finite points set. $\Rightarrow$ We can choose l_x s.t $l_x \cap R = \phi$. $\Rightarrow$ X_p is the union of two distinct lines $\Rightarrow \exists$ only one double point which is the only singular set in X_p. $\Rightarrow$ By the definition of P509, $\{X_p\}_{p \in l_x}$ is Lefschetz.
$\Rightarrow$ By the proposition on P509, $\chi(U) = 2\chi(X_p) + \mu - n$,