Thus we proved the generic element for each pencil contained in the system is smooth away from the base locus of the pencil. Even more, except a finite # of effective divisors in the pencil, rest of them are smooth away from the base locus of the pencil. The finite # of effective divisors may be smooth away from the base locus of the system. (this is not correct).

Two things are not clear.

① Why does it suffice to prove for a pencil?
② Is connected components of $V-B$ discrete? (or finite)

$V-B = \bigcup C_\lambda$. where C_λ is a connected component. (path-connected) since $V-B$ is an analytic variety, and so locally R^k for some k.
For each $p \in V-B$, $\exists$ a local defining function f_p.
$\Rightarrow$ By Weierstrass Preparation Theorem, $\exists$ an open set $U \ni p$. s.t $U \cap C \neq \phi$, where C is the connected component of p.

$\Rightarrow$ With respect to subspace topology, C_λ is open. for each λ.
$\Rightarrow$ Choose $a_\lambda \in C_\lambda$.
$\Rightarrow$ If $\{a_\lambda\}$ is infinite, $\exists$ a limit point a_0 in $\overline{V-B}$, where the closure in M. $\Rightarrow$ $a_0 \in V-B$ or $a_0 \in B$.

If $a_0 \in V-B$, for any open set $U \ni a_0$, $\exists$ infinitely many a_λ's in U. $\Rightarrow$ Contradiction to discreteness.

If $a_0 \in B$, $\exists$ $\{a_i\}$ convergent to a_0

$\Rightarrow$ $\frac{S_1}{S_2} = f$ meromorphic function. $\Rightarrow$ $f(a_i) \in \mathbb{P}^1$.

$\lim f(a_i) \overset{?}{=} f(a_0)$ $\qquad a_0 \in B = \bigcap_{i=1}^{2} D_{\lambda_i}$

$\Rightarrow$ $f(a_0)$ have to have two diff't values.