

Now, suppose $p \in S$ is any point, $h = T_p(S)$ its tangent plane, and let p^* and h^* be the hyperplane and point in \mathbb{P}^{3*} dual to p and h , respectively.

Γ p^* is the ^{set of} hyperplanes containing p , and h^* be the set of hyperplanes containing h . $\Rightarrow p^*$ is the hyperplane in \mathbb{P}^{3*} and h^* is the point. \Rightarrow

The dual lines $\{l_x^*\}$ to the pencil of lines $\{l_x\}$ in \mathbb{P}^3 through p and lying in h form the pencil of lines in \mathbb{P}^{3*} containing h^* and lying in p^* , and they are all tangent to S^* .

Γ $p \in l_x \subset h = T_p(S) \Rightarrow l_x$ is tangent to S
 $\Rightarrow p^* \supset l_x^* \supset h^* \Rightarrow$ By the result above, l_x^* is tangent to S^* . \Rightarrow

Every element of the pencil $\{l_x^* \cdot S^*\}$ they cut out on the curve $p^* \cap S^*$ is therefore singular, and so by Bertini they are all singular at the base locus h^* of $\{l_x^*\}$, i.e., $h^* \in S^*$ and $p^* = T_{h^*}(S^*)$. We see, then, that

S and S^* are dual surface,