$\Rightarrow Q_k \cong \mathcal{O}(e_r \otimes \Lambda^{k-1}\mathbb{C}^{r-1})$

$$\begin{tikzcd} Q_k = \frac{E_k}{F_k} \arrow[r, "\partial"] \arrow[d, "\cong"] & Q_{k-1} = \frac{E_{k-1}}{F_{k-1}} \arrow[d, "\cong"] \\ \mathcal{O}(e_r\otimes\Lambda^{k-1}\mathbb{C}^{r-1}) \arrow[r, "\partial"] & \mathcal{O}(e_r\otimes\Lambda^{k-2}\mathbb{C}^{r-1}) \end{tikzcd}$$

$e_J\wedge e_r + F_k \in Q_k \longmapsto e_r\otimes e_J \in \mathcal{O}(e_r\otimes\Lambda^{k-1}\mathbb{C}^{r-1})$

$Q_{k-1} \ni \partial e_J\wedge e_r \pm e_J f_r + F_{k-1} = \partial e_J\wedge e_r + F_{k-1} \longmapsto e_r\otimes\partial e_J \in \mathcal{O}(e_r\otimes\Lambda^{k-2}\mathbb{C}^{r-1})$, $\#J = k-2$

$$\begin{aligned} \partial(e_J\wedge e_r) &= \partial e_J\wedge e_r \pm e_J\wedge\partial e_r \\ &= \partial e_J\wedge e_r \pm e_J\wedge f_r = \partial e_J\wedge e_r \pm f_r e_J. \end{aligned}$$

Thus $Q.$ is again a Koszul complex, to which the induction assumption applies.

$Q_k \cong \mathcal{O}(e_r\otimes\Lambda^{k-1}\mathbb{C}^{r-1}) \cong \mathcal{O}\otimes_{\mathbb{C}}\Lambda^{k-1}\mathbb{C}^{r-1} \Rightarrow$ Put $Q'_{k-1} = Q_k. \Rightarrow Q'.$ is a Koszul complex

We now examine the lower right-hand corner

$$\begin{tikzcd} & E_1 \arrow[r] \arrow[d] & I_r \arrow[r] & 0 \\ Q_2 \arrow[r, "\partial"] & Q_1 \arrow[r, "\alpha"] \arrow[d] & I_r/I_{r-1} \arrow[r] \arrow[d] & 0 \\ & 0 & 0 & \end{tikzcd}$$

JONG IE WAH YUN PILL