We first check that no line l_x is singular at more than one point, i.e., that if $\sigma(p)$ is tangent to F at x, then for $q \neq p \in l_x$, $\sigma(q)$ can not also be tangent to F at x. But $\sigma(p) \cap \sigma(q) = \{x\}$, so the linear span of $\sigma(p)$ and $\sigma(q)$ in $\mathbb{P}^5$ is all of $T_x(G)$; thus $\sigma(p)$ and $\sigma(q)$ can not both be contained in $T_x(F) \neq T_x(G)$.

$\ulcorner$ $\langle \sigma(p), \sigma(q) \rangle \neq T_x(G) \Rightarrow \langle \sigma(p), \sigma(q) \rangle \subset \mathbb{P}^3$
$\Rightarrow$ $\sigma(p) \cap \sigma(q)$ contains a line., which contradicts to the fact $\sigma(p) \cap \sigma(q) = \{x\}$. $\Rightarrow \langle \sigma(p), \sigma(q) \rangle = T_x(G)$.
Since F intersects G transversely, $T_x(F) \neq T_x(G)$. $\Rightarrow$ $\sigma(q)$ can be tangent to F at x. $\lrcorner$

We can therefore define a map

$$\pi : \Sigma \longrightarrow S$$

sending each $x \in \Sigma$ to the unique $p \in l_x$ for which $\sigma(p)$ is tangent to F at x.

$\ulcorner$

$$\begin{aligned} \pi : \Sigma &\longrightarrow S \\ x &\longmapsto p \end{aligned}$$

. p unique s.t $p \in l_x$. where $\sigma(p)$ is tangent to F at x.

$\lrcorner$

By what was said above, π is one to one and surje-