is a curve. ⇒ By the classical Plücker formula, $g(\tilde{B}_L) = 2$, since C_L is a quartic plane curve with one ordinary double point. ⇒ $g(B_L) = g(\tilde{B}_L) = 2$.
⇒ By the Lemma on P505, and $B_L \cdot B_L = 2$,

$$\pi(B_L) = \frac{K_A \cdot B_L + B_L \cdot B_L}{2} + 1 = 2, \text{ and } \pi(B_L) = g(B_L)$$

which implies that B_L is smooth.
By P505, $g(B_L) \leq \pi(B_L) = 2 \Rightarrow g(B_L) = g(C_L) \leq 2$.
If h_L is not tangent to S, then $h_L \cap S$ is smooth.

$$\Rightarrow g(C_L) = \frac{(4-1)(4-2)}{2} = 3 \text{ by the genus formula.}$$

⇒ Contradiction to the fact that $g(C_L) \leq 2$.
We may apply this argument to S^* to get that $p_L \cap S^*$ is singular, i.e., p_L is tangent to S^*, I guess, at least I hope. If not, "S and S^* are dual" can not be proved this way. ⌟

Now, since B_L is a positive divisor on A by the Lefschetz theorem the inclusion map on integral homology

$$i_* : H_1(B_L, \mathbb{Z}) \twoheadrightarrow H_1(A, \mathbb{Z})$$

is surjective.

⌜ $[B_L]$ is positive line bundle on A ⇒ By P159,
$i_* : H_1(B_L, \mathbb{Z}) \twoheadrightarrow H_1(A, \mathbb{Z})$ is onto since $1 = \overset{n}{2} - 1$. ⌟

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