$$\begin{array}{ccccc} T_x^{*'} \otimes L_x & = & \mathrm{Hom}(T_x', L_x) & \overset{\varphi_\alpha}{\cong} & \mathrm{Hom}(T_x', x \times \mathbb{C}) \\ & & \Downarrow & & \Downarrow \\ & & ds & \longmapsto & ds_\alpha \end{array}$$

$$\begin{array}{ccccc} T_x^{*} \otimes L_x & = & \mathrm{Hom}(T_x', L_x) & \overset{\varphi_\beta}{\cong} & \mathrm{Hom}(T_x', x \times \mathbb{C}) \\ & & \Downarrow & & \Downarrow \\ & & ds & \longmapsto & ds_\beta \end{array}$$

$g_{\alpha\beta}\, ds_\beta = ds_\alpha \Rightarrow ds$ is well-defined. --- ⊛

(ii). $\mathcal{U} \not\ni x. \Rightarrow d_x^{\mathcal{U}}$ is zero homomorphism.

Given $\eta \in T_x^{*'} \otimes L_x$, by a trivialization φ_α,

$\eta_\alpha \in \mathrm{Hom}(T_x', \mathbb{C}) \Rightarrow \exists\ s \in H^0(M, \mathcal{J}_x(L))$
s.t $ds_\alpha = \eta_\alpha$.

By another trivialization φ_β, $\eta_\beta \in \mathrm{Hom}(T_x', \mathbb{C})$
$\Rightarrow \eta_\beta = g_{\beta\alpha}\, \eta_\alpha$.
$\Rightarrow$ By ⊛ above, $\eta_\beta = g_{\beta\alpha}\, \eta_\alpha = g_{\beta\alpha}\, ds_\alpha = ds_\beta$.
$\Rightarrow d_x$ is a well-defined surjective sheaf map.

Note that (**) is the limiting case of (*) when $y \to x$.

⌈ Given $s \in H^0(M, \mathcal{O}(L))$, $s(x) \in L_x$ $s(y) \in L_y$.
As $y \to x$, if φ_α is a trivialization of L near x, $s_\alpha(y) \doteq s_\alpha(x) + \underline{ds_\alpha(v)}$ roughly, where v is a multiple of $y - x$.

$v \in T_x'M$, $ds_\alpha(v) \in L_x$.