$$\Rightarrow \quad \epsilon_{IJ}\,\epsilon_{I^\circ J^\circ} = \epsilon_{IJ}^{2}\,(-1)^{(p+q)(n-p+n-q)} = (-1)^{(p+q)(2n-p-q)}$$

$$(p+q)(2n-p-q) \equiv -(p+q)^2 \equiv (p+q)$$

$$\Rightarrow \quad \epsilon_{IJ}\,\epsilon_{I^\circ J^\circ} = (-1)^{p+q} \quad \text{//}.$$

In terms of star, the adjoint operator is

$$\bar\partial^{*} = -*\,\bar\partial\,*.$$

pf). For $\psi \in A^{p,q-1}(M)$ and $\eta \in A^{p,q}(M)$

$$(\bar\partial\psi, \eta) \equiv \int_M \bar\partial\psi \wedge *\eta = (-1)^{p+q}\int_M \psi\wedge\bar\partial*\eta + \int_M \bar\partial(\psi\wedge*\eta).$$

$$\bar\partial(\psi\wedge*\eta) = \bar\partial\psi\wedge*\eta + (-1)^{p+q-1}\psi\wedge\bar\partial*\eta$$

$$\Rightarrow \quad \bar\partial\psi\wedge*\eta = \bar\partial(\psi\wedge*\eta) - (-1)^{p+q-1}\psi\wedge\bar\partial*\eta$$

$$= \bar\partial(\psi\wedge*\eta) + (-1)^{p+q}\psi\wedge\bar\partial*\eta.$$

$$d(\psi\wedge\eta) = d\psi\wedge\eta + (-1)^{\deg\psi}\psi\wedge d\eta$$

$$\bar\partial(\psi\wedge\eta)+\partial(\psi\wedge\eta) = (\bar\partial\psi+\partial\psi)\wedge\eta + (-1)^{\deg\psi}\psi\wedge(\bar\partial\eta+\partial\eta)$$

$$= \underbrace{\bar\partial\psi\wedge\eta + (-1)^{\deg\psi}\psi\wedge\bar\partial\eta} + \underbrace{\partial\psi\wedge\eta + (-1)^{\deg\psi}\psi\wedge\partial\eta}$$

Since $\bar\partial = d$ on forms of type $(n, n-1)$, the second term on the right,

$$\int_M d(\psi\wedge*\eta) = 0 \quad \text{by Stoke's theorem.}$$

Thus, for all ψ,

$$(\bar\partial\psi, \eta) = -\int_M \psi\wedge*(*\bar\partial*\eta) = \int_M \psi\wedge(-**\bar\partial*\eta)$$

$$= \int_M \psi\wedge(*(-*\bar\partial*\eta)) = \int_M \psi\wedge*(\bar\partial^*\eta) = (\psi, \bar\partial^*\eta).$$