Since $\varphi - \varphi = 0 = \alpha_{-1} \circ \partial_N$, take $\alpha_{-1} = 0$.

$$\begin{tikzcd} E_0 \arrow[r] \arrow[d, "\Phi_0, \Psi_0"] & M \arrow[d, "\varphi"] \\ F_0 \arrow[r] & N \end{tikzcd}$$

$\Rightarrow$ $\partial_N \circ \Phi_0 = \varphi \circ \partial_M$

$\partial_N \circ \Psi_0 = \varphi \circ \partial_M$

$\Rightarrow$ $\partial_N \circ (\Phi_0 - \Psi_0) = 0$.

$\Downarrow$ $\operatorname{Im}(\Phi_0 - \Psi_0) \subset \ker \partial_N$.

Consider

$$\begin{tikzcd} & E_0 \arrow[dl, dashed, "\alpha_0"'] \arrow[d, "\Phi_0 - \Psi_0"] & \\ F_1 \arrow[r] & \ker \partial_N \arrow[r] & 0 \end{tikzcd}$$

$\Rightarrow$ By the definition of projective, $\exists$ α_0 s.t $\Phi_0 - \Psi_0 = \partial_N \circ \alpha_0$. -- ⊛

$$\begin{tikzcd} E_2 \arrow[r] \arrow[d, "\Phi_2, \Psi_2"] & E_1 \arrow[r] \arrow[d, "\Phi_1, \Psi_1"] \arrow[dl, dashed, "\alpha_1 (?)"'] & E_0 \arrow[d, "\Phi_0, \Psi_0"] \arrow[dl, "\alpha_0"'] \\ F_2 \arrow[r] & F_1 \arrow[r] & F_0 \end{tikzcd}$$

We have to find α_1 s.t $\Phi_1 - \Psi_1 = \alpha_0 \circ \partial_M + \partial_N \circ \alpha_1$, i.e.,

$$\partial_N \circ \alpha_1 = \Phi_1 - \Psi_1 - \alpha_0 \circ \partial_M.$$

To use projectiveness,

$$\begin{aligned} \partial_N \circ (\Phi_1 - \Psi_1 - \alpha_0 \circ \partial_M) &= \partial_N \circ \Phi_1 - \partial_N \circ \Psi_1 - \partial_N \circ \alpha_0 \circ \partial_M \\ &= \Phi_0 \circ \partial_M - \Psi_0 \circ \partial_M - \partial_N \circ \alpha_0 \circ \partial_M \quad \text{by commutativity} \\ &= (\Phi_0 - \Psi_0) \circ \partial_M - \partial_N \circ \alpha_0 \circ \partial_M \\ &= \partial_N \circ \alpha_0 \circ \partial_M - \partial_N \circ \alpha_0 \circ \partial_M \quad \text{by ⊛ above} \\ &= 0 \end{aligned}$$

$\Rightarrow$ $\operatorname{Im}(\Phi_1 - \Psi_1 - \alpha_0 \circ \partial_M) \subset \ker \partial_N$.