┌ $M_0 = M - V$ is smooth $\Rightarrow$ $M_0^* = M_0$ is irreducible $\Rightarrow$ $\{(p, x) \mid f(p) = x,\ p \in M_0\}$ is irreducible $\Rightarrow$ Its closure is also irreducible. Refer to P130.

Suppose V is connected and smooth.

Claim: $\overline{V}$ is irreducible.

Suppose $\overline{V} = V_1 \cup V_2$. $V_1 - (V_1 \cap V_2)$ & $V_2 - V_1 \cap V_2$ are disjoint. $\Rightarrow$ $(V_1 - (V_1 \cap V_2)) \cap V$ & $(V_2 - (V_1 \cap V_2)) \cap V$ are disjoint. $\Rightarrow$ Since $(V_1 \cap V_2) \cap V = \phi$ ($\because$ any point $p \in V_1 \cap V_2$ is not smooth point of V), those form a seperation for V. $\Rightarrow$ V is disconnected. ┘

Conversely, suppose $\Gamma \subset M \times \mathbb{P}^n$ is any k-dimensional irreducible analytic subvariety having intersection number

$$\#(\Gamma, \{p\} \times \mathbb{P}^n) = 1$$

with the fibers of $M \times \mathbb{P}^n$ over M. For each $p \in M$, Γ will either meet the fiber $\{p\} \times \mathbb{P}^n$ transversely in a single point $(p, f(p))$ — in which case by the implicit function theorem Γ is the graph of a holomorphic map near p — or have at least a curve in common with it. The former is clearly generically the case.

┌ $\Gamma^* \ni p$ $\Rightarrow$ $\exists$ open set U in Γ^* s.t $U \ni p$ $U \overset{\varphi}{\cong} \mathbb{C}^k$. Consider the following

$$\begin{tikzcd} & U \arrow[dl, "\varphi"'] \arrow[dr, "\pi"] & \\ \mathbb{C}^k \arrow[rr] & & \mathbb{C}^k, \end{tikzcd}$$

$\pi : M \times \mathbb{P}^n \longrightarrow M$.