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by the property of transverality, $\exists$ open set $\mathcal{O} \ni (C_1, \cdots C_5)$ in $(W)^5$ s.t, for all $(C_1', \cdots C_5') \in \mathcal{O}$, $V_{C_1'}, \cdots V_{C_5'}$ meet transversely away from the subvariety of singular conics.

⊛ Since we assumed that assertion 1 is not true. $\exists$ a set $K \subset (W)^5$ s.t $K \subset (U)^5$. K^c is not a generic set in some open set in $(U)^5$ and for all $(C_1'', \cdots C_5'') \in K$, $V_{C_1''}, \cdots V_{C_5''}$ does not meet transversely at some point in U.

Observe the following:

① Since $\pi_1 : I \longrightarrow (W)^5$ is generically finite-to one, by Sard's theorem, π_1 is a covering map generically, or $\pi_1(I)$ is a set of measure zero.

② By the assumption that assertion 1 is not valid, ⊛ and ①, $\dim J = \dim I$.

$\Rightarrow$ Since I is irreducible, $I = J$. $\Rightarrow$ Since π_1 is generically finite to one, $\dim \pi_1(J) = \pi_1(I) \overset{\dim}{=} (W)^5$.

Note that, if $V_{C_1} \cap \cdots \cap V_{C_5} \cap U = \phi$, V_{C_i}'s meet transversely. $\Rightarrow$ For generic $C_1, \cdots C_5$, V_{C_i}'s meet transversely away from the subvariety of singular conics. $\Rightarrow$ It is done.