$$\int_\alpha \eta_\sigma \wedge \eta_\tau = \#(\gamma \cdot \alpha) \quad \Rightarrow \quad \text{What is the } \gamma \in H_{k+l-n}(M) \,?$$

Consider the inclusion $i : \alpha \hookrightarrow M$.

$$\int_\alpha i^*\eta_\sigma \wedge i^*\eta_\tau = \int_\alpha i^*(\eta_\sigma \wedge \eta_\tau) = \#(\gamma \cdot \alpha)$$

since $\int_\alpha i^*(\eta_\sigma \wedge \eta_\tau) = \int_\alpha \eta_\sigma \wedge \eta_\tau$.

What is the Poincare-duals of $i^*\eta_\sigma$ and $i^*\eta_\tau$?

β ; $(n-k)$-cycle in α

$$\int_\beta i^*\eta_\sigma = \int_\beta \eta_\sigma = \#(\sigma \cdot \beta) = \#(\sigma \cap \alpha \cdot \beta) \quad \text{since } \beta \subset \alpha.$$

$\Rightarrow$ $\sigma \cap \alpha$ is the Poincare-dual to $i^*\eta_\sigma$, similarly,
$\tau \cap \alpha$ 〃 $i^*\eta_\tau$.

Then by P59. (intersection of cycles in homology is Poincaré-dual to wedge product in cohomology),

$$\int_\alpha i^*(\eta_\sigma \wedge \eta_\tau) = \int_\alpha \eta_\sigma \wedge \eta_\tau = \#((\sigma \cap \alpha) \cdot (\tau \cap \alpha))$$

$$= \#((\sigma \cap \tau) \cdot \alpha)$$

$\Rightarrow$ $\gamma = \sigma \cap \tau$ (homologous)

Thus since ω is Poincare-dual to H,
ω^2 is Poincare-dual to $H \cap H = \mathbb{P}^{N-2}$
$\vdots$
ω^k is Poincare-dual to $\overbrace{H \cap \cdots \cap H}^{k} = \mathbb{P}^{N-k}$

Note that in this interpretation, the primitive cohomology $P^{n-k}(M)$ of M corresponds via the the isomorphisms