Date

$$0 = \sum \int_{\delta_i + \delta_i^{-1}} \psi + \int_{\delta_{g+i}+\delta_{g+i}^{-1}} \psi + \int_{\beta_i+\beta_i^{-1}} \psi$$

$$= -\sum \left(\int_{\delta_i} d\log f\right)\left(\int_{\delta_{g+i}} d\log g\right) + \sum \left(\int_{\delta_{g+i}} d\log f\right)\left(\int_{\delta_i} d\log g\right)$$

$$+ 2\pi\sqrt{-1} \sum \mathrm{ord}_{q_i}(g)\cdot \log f(q_i) \quad \text{---} \quad (2)$$

$$\Rightarrow (2) - (1) \Rightarrow 2\pi\sqrt{-1}\left(\sum \mathrm{ord}_{q_i}(g) \log f(q_i) - \sum \mathrm{ord}_{p_i}(f)\log g(p_i)\right)$$

$$= 2\left(\sum \left(\int_{\delta_i} d\log f\right)\left(\int_{\delta_{g+i}} d\log g\right) - \sum \left(\int_{\delta_{g+i}} d\log f\right)\left(\int_{\delta_i} d\log g\right)\right)$$

But $\int_{\delta_i} d\log f$ is always an integral multiple of $2\pi\sqrt{-1}$; thus the right-hand term above is an integral multiple of $(2\pi\sqrt{-1})^2$ and we see that

$$\sum \mathrm{ord}_{q_i}(g) \log f(q_i) - \sum \mathrm{ord}_{p_i}(f)\cdot \log g(p_i) \in 2\pi\sqrt{-1}\,\mathbb{Z}.$$

Exponentiating, we obtain

$$\prod_i f(q_i)^{\mathrm{ord}_{q_i}(g)} = \prod_i g(p_i)^{\mathrm{ord}_{p_i}(f)}$$

as desired. Q.E.D.

$\ulcorner$ $\int_{\delta_i} d\log f = 2\pi\sqrt{-1}\cdot n, \quad n \in \mathbb{Z}$.
$\int_{\delta_i} d\log g = 2\pi\sqrt{-1}\cdot m, \quad m \in \mathbb{Z}$

since the residues are integers ($\because$ residues are orders of f & g) and $\int_{\delta_i} d\log f = \log f(p') - \log f(p)$ = the differe-