1022
97.1.1

「 See P227 for a canonical basis. See P231 for a basis $\omega_1, \cdots \omega_g$ for $H^0(S, \Omega^1)$ s.t

$$\int_{\delta_i} \omega_\alpha = \delta_{i\alpha}, \quad 1 \leq i, \alpha \leq g;$$

$\Omega = (I, Z)$.

By P231 ~ P232, Z is symmetric and $\operatorname{im} Z = Y > 0$.

(Also, for $\delta_\alpha = 1$ for all α, by using R.C III, Z symmetric」 and $\operatorname{im} Z = Y > 0$ are proved.

Thus $J(S)$ is an Abelian variety, and moreover has a principal polarization given in terms of the basis $\{dx_i\}$ for $H^1(J(S), \mathbb{Z})$ dual to $\{\lambda_i\} \in H_1(J(S), \mathbb{Z})$ by

$$\omega = \sum dx_\alpha \wedge dx_{n+\alpha}.$$

「 Since $J(S) = \dfrac{\mathbb{C}^g}{\mathbb{Z}\{\lambda_1, \cdots \lambda_{2g}\}}$ has the period matrix $\Omega = (I, Z)$ with Z symmetric and $\operatorname{im} Z > 0$, by P306, Riemann Conditions III, $J(S)$ is an Abelian variety. Furthermore,

$$\omega = \sum_{i=1}^{n} dx_i \wedge dx_{n+i}. \text{ is a principal polarization.}$$

Here

$$\int_{\lambda_j} dx_i = \delta_{ij}, \text{ and } \omega(\lambda_i, \lambda_{i+n}) = \omega\left(\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_{i+n}}\right)$$

$= 1$. (The reason why ω is principal polarization is that $\Omega = (I, Z)$, Z symmetric and $\operatorname{im} Z > 0$ assures $\omega = \sum dx_i \wedge dx_{i+n}$ is (1, 1) type and positive. See P303 ~ P304, and combine 」) P231 and P232