We concluded, then, if l_x did not lie on any pair of confocal pencils, and if $T_x(X) \cap X$ contained no multiple components, that l_x met S in four distinct points.

☞ See P764

We have since seen that the degree of S is indeed four, and so we can now invert our argument to obtain the characterization:

For any $x \in X$, the line l_x will be tangent to S if and only if either
1. l_x is a singular line, i.e., it is held in common by two confocal pencils; or
2. the intersection $T_x(X) \cap X$ contains a multiple component.

We have already seen that the locus $\Sigma \subset X$ of singular lines is the smooth intersection of X with a quadric hypersurface in $\mathbb{P}^5$: we turn our attention now to the second possibility.

☞ l_x is tangent to $S \Leftarrow l_x$ is held in common by two confocal pencils. See P764 ~ P765