" Suppose $\sum_i P(A_1, \cdots, gA_i - A_i g, \cdots A_K) = 0$ for $g \in GL_n$.
For $h \in GL_n$ with $\|g-h\| < \epsilon$, ϵ sufficiently small.

By $(I+g')^{-1} = I - g' + [\partial]$,
$\Rightarrow$ $h - g = (hg^{-1} - I)g = h'g$ g fixed.
$\Rightarrow$ $(I+h')^{-1} = I - h' + [\partial]$.
$\Rightarrow$ $(I+h')g = g + h'g = h = g - h'g + [\partial]^{(\text{of } h'_{ij}\text{'s})}$

Using coordinates on GL_n the entries $h'g = h-g$,
we compute the linear term f_1 of the power series
expansion for f around g.

$$f(h) = P(hA_1h^{-1}, \cdots hA_ih^{-1}, \cdots hA_K h^{-1})$$
$$= P((I+h')gA_1g^{-1}(I+h')^{-1}, \cdots (I+h')gA_ig^{-1}(I+h')^{-1}, \cdots (I+h')gA_Kg^{-1}(I+h')^{-1})$$

$\cdots$

$$= P((I+h')gA_1g^{-1}(I-h'), \cdots (I+h')gA_Kg^{-1}(I-h')) + [\partial]$$
$$= P(gA_1g^{-1}, \cdots, gA_Kg^{-1}) + \sum_i P(gA_1g^{-1}, \cdots h'gA_ig^{-1} - gA_ig^{-1}h', \cdots gA_Kg^{-1}) + [\partial]$$

Thus if P is invariant, $f(h) = P(A_1, \cdots A_K)$
$= P(gA_1g^{-1}, \cdots gA_Kg^{-1})$ and
$$\sum_i P(gA_1g^{-1}, \cdots h'gA_ig^{-1} - gA_ig^{-1}h', \cdots gA_Kg^{-1}) = 0.$$

$$\Rightarrow \sum_i P(gA_1g^{-1}, \cdots, h'gA_ig^{-1} - gA_ig^{-1}h', \cdots gA_Kg^{-1})$$

$$= \sum_i P(A_1, \cdots, g^{-1}h'gA_i - A_ig^{-1}h'g, \cdots A_K) = 0$$