Now let $C^s(T)$ be the functions of class ~~$\mathcal{S}$~~ s on T.
A function $\varphi \in C^\circ(T)$ has a Fourier expansion $\sum \varphi_\xi e^{i\langle \xi, x\rangle}$, where

$$\varphi_\xi = \int_T \varphi(x)\, e^{-i\langle \xi, x\rangle}\, dx. \quad \left(dx = \frac{dx_1 \wedge \cdots \wedge dx_n}{(2\pi)^n} \right)$$

We have Parseval's identity

$$\int_T |\varphi|^2 = \int_T \left(\sum \varphi_\xi e^{i\langle \xi, x\rangle}\right) \left(\sum \overline{\varphi}_{\xi'} e^{-i\langle \xi', x\rangle} \right)$$

$$= \int_T \varphi_\xi \overline{\varphi}_{\xi'}\, e^{i\langle \xi - \xi', x\rangle} dx = \int_T \sum_\xi |\varphi_\xi|^2 dx = \sum_\xi |\varphi_\xi|^2 = \|\varphi\|_0^2$$

so that $C^\circ(T)$ maps into H_0 injectively with $\|\ \|_0$ as L^2-norm on $C^\circ(T)$.

⌈

$$C^\circ(T) \xrightarrow{\phi} H_0$$

$$\varphi \longmapsto \sum_{\xi \in \mathbb{Z}^n} \varphi_\xi e^{i\langle \xi, x\rangle} = \overline{\varphi}.$$

Question: $\|\overline{\varphi}\|_0 < \infty$?

$$\int_T |\varphi|^2 = \|\overline{\varphi}\|_0^2 < \infty$$

$< \infty$. ⌋

⌈ Injectiveness.

$$\varphi \longmapsto \overline{\varphi}. \quad \Rightarrow \quad \overline{\varphi} = 0 \quad \Rightarrow \|\overline{\varphi}\|_0 = 0$$

$$\Rightarrow \varphi_\xi = 0 \quad \Rightarrow \quad \int_T |\varphi|^2 = 0 \quad \Rightarrow \quad |\varphi|^2 = 0 \quad \Rightarrow \varphi = 0$$

since φ is continuous.
The justification of this interchange of limits is done by using partial sums and the Cauchy-Schwartz inequality. ⌋ Here, actually they assume the powerful thing, which says that, if $f \in L^2(T)$, the Fourier series of f converges to f in the L^2-sence.