we call this limit group E_∞ and say that the spectral sequence $\{E_r\}$ converges to E_∞.

Proposition. Let K^* be a filtered complex. Then there exists a spectral sequence $\{E_r\}$ with

$$E_0^{p,q} = \frac{F^p K^{p+q}}{F^{p+1} K^{p+q}},$$

$$E_1^{p,q} = H^{p+q}(Gr^p K^*),$$

$$E_\infty^{p,q} = Gr^p(H^{p+q}(K^*)).$$

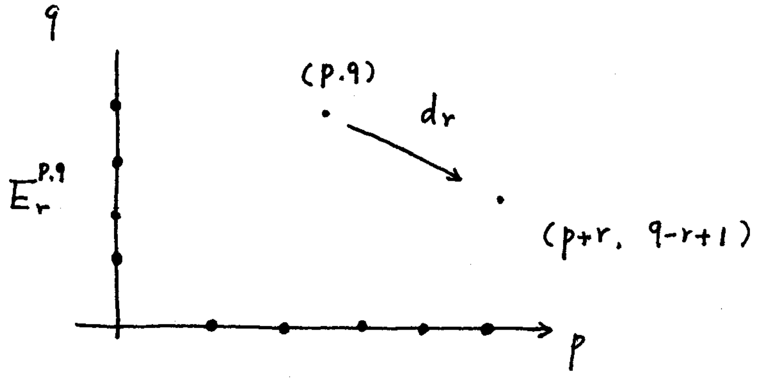

Figure 3.

The last statement is usually written

$$E_r \Rightarrow H^*(K^*)$$

and we say that the spectral sequence abuts to $H^*(K^*)$.

Proof. The initial term has been defined, and

$$\begin{array}{ccc} d_0 : E_0^{p,q} & \longrightarrow & E_0^{p,q+1} \\ \| & & \| \\ F^p K^{p+q}/F^{p+1}K^{p+q} & \longrightarrow & F^p K^{p+q+1}/F^{p+1}K^{p+q+1} \end{array}$$