the eigenspace for h with eigenvalue $(n-p)$ will be $H^p(M)$. Applying our results on finite-dimensional representations of sl_2 to this representation yields the

Hard Lefshetz Theorem.

The map $L^k : \underset{= V_k}{H^{n-k}(M)} \longrightarrow \underset{\| V_{-k}}{H^{n+k}(M)}$ is an isomorphism;

and if we define the primitive cohomology

$$P^{n-k}(M) = \ker L^{k+1} : H^{n-k}(M) \longrightarrow H^{n+k+2}(M)$$
$$= \ker \Lambda \cap H^{n-k}(M).$$ see p120 (*).

then we have $H^m(M) = \bigoplus_k L^k P^{m-2k}(M)$, called

the Lefschetz decomposition.

$\ulcorner$ $V_m = (V_m \cap W_1) \oplus (V_m \cap W_2)$
$= (W_1)_m \oplus (W_2)_m$

$$(W_1)_m \underset{\Lambda^m}{\overset{L^m}{\rightleftarrows}} (W_1)_{-m}$$

Let $V = W_1 \oplus W_2$

$(W_1)_k \oplus (W_1)_{k-2} \oplus \cdots \oplus (W_1)_n \oplus \cdots \oplus (W_1)_{-n} \oplus \cdots \oplus (W_1)_{-k}$
$(W_2)_n \oplus \cdots \oplus (W_2)_{-n}$

$\Rightarrow$ $V_m = (W_1)_m \oplus (W_2)_m = \bigoplus_{l=m}^{k} Y^{\frac{l-m}{2}} (\ker X \cap V_l)$

$\Rightarrow$ Let $\frac{l-m}{2} = k$. $Y = L$. $X = \Lambda$.

$\Rightarrow$ $V_m = \bigoplus L^k (\ker \Lambda \cap V_{2k+m})$