

$\Rightarrow \varphi_{\Delta} = \sum (-1)^{p+q} \varphi_{p,q,\mu,\mu}$. Here we assumed a lot of things. \Rightarrow
See the next paragraph. \Rightarrow

This tells us nothing essentially new: in case M is Kähler, this follows from the ordinary Lefschetz fixed-point formula and the Hodge decomposition; in general, it follows from the Lefschetz formula and the Fröhlicher spectral sequence relating Dolbeault and de Rham cohomology given in Section 5 of this chapter. To obtain finer information about the action of f on the Dolbeault groups of M , let $\eta_{\Delta}^{p,q}$ be the (p,q) th component of the class η_{Δ} under the decomposition into bitype.

$$H_{\bar{\partial}}^{n,n}(M \times M) = \bigoplus_{p,q} (\pi_1^* H_{\bar{\partial}}^{p,q}(M) \otimes \pi_2^* H_{\bar{\partial}}^{n-p,n-q}(M)),$$

and set

$$\eta_{\Delta}^{\circ} = \sum_q \eta_{\Delta}^{\circ,q}.$$

See p 103 bottom. \square

η_{Δ}° is then represented by the form

$$\varphi_{\Delta}^{\circ} = \sum_{q,\mu} (-1)^q \varphi_{0,q,\mu,\mu},$$

and so the value of η_{Δ}° on the cycle P_f is given by

$$\eta_{\Delta}^{\circ}(P_f) = \int_{P_f} \varphi_{\Delta}^{\circ} = \sum_q (-1)^q \int_M \sum_{\mu} \varphi_{0,q,\mu} \wedge f^* \psi_{n,n-q,\mu}^*$$