$$\lim_{\epsilon\to 0}\int_{\|u\|=\epsilon}\frac{\sum\overline{\Phi_i(u)}\wedge\Phi(u+z)}{\|u\|^{2n}}\wedge\varphi(\overset{u+z}{\omega})$$

$$=\lim_{\epsilon\to 0}\int_{\|u\|=\epsilon}\frac{\sum\overline{\Phi_i(u)}\wedge\Phi(u)}{\|u\|^{2n}}\wedge\varphi(u+z)$$

Since $\Phi(u+z)=d(u_1+z_1)\wedge\cdots\wedge d(u_n+z_n)$ and we only need $du_1\wedge\cdots\wedge du_n$.

$\Rightarrow$ By P371 ~ P372, $\displaystyle\lim_{\epsilon\to 0}\int_{\|u\|=\epsilon}\frac{\sum\overline{\Phi_i(u)}\wedge\Phi(u)}{\|u\|^{2n}}\wedge\varphi(u+z)=\varphi(z)$

$$\Rightarrow\int_{\mathbb{C}^n}\psi(z)\wedge\lim_{\epsilon\to 0}\int_{\|\omega-z\|=\epsilon}k(z,\omega)\wedge\varphi(\omega)=\int_{\mathbb{C}^n}\psi(z)\wedge\varphi(z)=T_\varphi(\psi)$$

$\Rightarrow\quad \bar\partial K_\varphi+(-1)^q K_\varphi\bar\partial=T_\varphi$ i.e

$$(\bar\partial K_\varphi)(\psi)+(-1)^q K_\varphi\circ\bar\partial(\psi)=T_\varphi(\psi)\quad\text{for all }\psi\in A_c^{n,n-q}(\mathbb{C}^n)$$

$\lrcorner$

A first application of the homotopy formula is another proof of the $\bar\partial$-Poincaré lemma for smooth forms. Given a $\bar\partial$-closed form $\varphi\in A^{0,q}(U)$, where $U\subset\mathbb{C}^n$ is an open set, we may find a relatively compact open set $V\subset U$ and bump function $\rho\in C_c^\infty(U)$ with $\rho\equiv 1$ on V. Then $\rho\varphi\in A_c^{0,q}(\mathbb{C}^n)$, and

$$(\rho\varphi)(z)=\bar\partial(K_{\varphi\rho})(z)+K(\bar\partial(\rho\varphi))(z).$$

$\ulcorner$ $\rho\varphi\in A_c^{0,q}(\mathbb{C}^n)$.

By the above, $(\bar\partial K_{\rho\varphi})(\psi)+(-1)^q K_{\rho\varphi}\circ(\bar\partial(\psi))=T_{\rho\varphi}(\psi)$,