⌈ $d(\arg f_1) \wedge \cdots \wedge d(\arg f_n) = - d(\arg f_2) \wedge d(\arg f_1) \wedge \cdots \wedge d(\arg f_n)$. ⌋

Third, we shall say that $f = (f_1, \cdots f_n)$ is nondegenerate in case the Jacobian determinant

$$\mathcal{J}_f(0) = \left| \frac{\partial(f_1, \cdots f_n)}{\partial(z_1, \cdots z_n)} (0) \right| \neq 0$$

is nonzero at the origin. Later on we shall see that the Jacobian is not identically zero.

⌈ Already we saw the Jacobian is not identically zero on note P519. ⌋

In the nondegenerate case we find that

$$\mathrm{Res}_{\{0\}} = \frac{g(0)}{\mathcal{J}_f(0)}.$$

To prove this, consider the mapping $w = f(z)$, which by the inverse function theorem is biholomorphic in a nbd of the origin. Set

$$G(w) = g(f^{-1}(w)),$$

$$K(w) = \frac{dw_1}{w_1} \wedge \cdots \wedge \frac{dw_n}{w_n} \quad (\text{Cauchy kernel}),$$

and

$$\mathcal{J}_f(w) = \mathcal{J}_f(f^{-1}(w)).$$

Then $\omega = f^*\left(\frac{GK}{\mathcal{J}}\right)$,