of S.

Thus $\quad C(S) = \bigcup\limits_{\substack{p \in L_0 \\ q \in \mathbb{P}^2}} \overline{f(p), f(q)}$,

from which we see that $C(S)$ is of dimension at most four.

To show $C(S) = \bigcup\limits_{\substack{p \in L_0 \\ q \in \mathbb{P}^2}} \overline{f(p), f(q)}$, we have only to show

that $\quad \overline{f(u), f(u')} \ni {}^{\forall} x, \quad \Rightarrow \quad x \in \bigcup\limits_{\substack{p \in L_0 \\ q \in \mathbb{P}^2}} \overline{f(p), f(q)}$.

Given $x \in \overline{f(u), f(u')}$, by the previous conclusion,

$\overline{f(u_0), x}$ is a chord of S, where $u_0 \in L_0 \cap L = \overline{u, u'}$.

$\Rightarrow \quad x \in \overline{f(u_0), f(q)}, \quad q \in \mathbb{P}^2, \quad$ since $\overline{f(u_0), f(q)} = \overline{f(u_0), x}$.

$$\begin{aligned} \mathbb{P}^2 \times L_0 &\longrightarrow \mathbb{P}^5 \\ (q, p) &\longmapsto \overline{f(p), f(q)} \end{aligned}$$

$\dim C(S) \leq \dim (\mathbb{P}^2 \times L_0 \times \mathbb{P}^1) = 4$.

Explicitly, we describe $C(S)$ as the locus

$\{ \alpha \cdot f(s, t) + (1 - \alpha) \cdot f(0, t') \} = \{ [1, \alpha s, \alpha t + (1 - \alpha) t', \alpha s^2, \alpha s t, \alpha t^2 + (1 - \alpha) t'^2] \}$.