Note that by the compatibility relations, we can give such a collection $\{e_\lambda\}$ by specifying e_{λ_α} for some basis $\{\lambda_\alpha\}$ for Λ so long as the functions $\{e_{\lambda_\alpha}\}$ satisfy

$$(*) \qquad e_{\lambda_\alpha}(z+\lambda_\beta)\, e_{\lambda_\beta}(z) = e_{\lambda_\beta}(z+\lambda_\alpha)\, e_{\lambda_\alpha}(z).$$

$\ulcorner$ $e_{\lambda_\alpha}(z+\lambda_\beta)\, e_{\lambda_\beta}(z) = e_{\lambda_\alpha+\lambda_\beta}(z)$ by the compatibility relations. $\Rightarrow$ Given any $\lambda \in \Lambda$, we can express $e_\lambda(z)$ in terms of e_{λ_α}'s since $\lambda = a_\alpha \lambda_\alpha$, $a_\alpha \in \mathbb{Z}$.
For example,

$$e_{2\lambda_\alpha}(z) = e_{\lambda_\alpha}(z+\lambda_\alpha)\, e_{\lambda_\alpha}(z).$$

Note that $e_0(z) = 1$. $\Rightarrow$ $e_\lambda(z-\lambda)\, e_{-\lambda}(z) = 1 = e_0(z)$
$\Rightarrow$ $e_{-\lambda}(z) = 1/e_\lambda(z-\lambda)$. $\lrcorner$

Our aim now is to show that any line bundle $L \longrightarrow M$ can be given by multipliers $\{e_\lambda(z)\}$ of a very simple character. We will do this in two stages: first, we will construct line bundles having arbitrary positive Chern class, using elementary functions e_λ; then we will show that any positive line bundle $L \longrightarrow M$ is determined, up to translation in M, by its Chern class.

One simplification is immediate: If $\{\lambda_1, \cdots \lambda_{2n}\}$ is any basis for Λ over $\mathbb{Z}$ with $\lambda_1, \cdots \lambda_n$ linearly independent over $\mathbb{C}$, then we have