Finally, it remains to show that for any two n-planes $\Lambda, \Lambda' \subset F$, the dimension of their intersection is congruent to n mod 2 if and only if they belong to the same family. Again, we proceed by induction: the statement is trivially true for $n=0$ (and more visibly for $n=1$); assume it for all $m<n$.

$n=0 \Rightarrow F = \{X_0^2 + X_1^2 = 0\} \subset \mathbb{P}^1.$

$\Rightarrow F = \{\underset{\Lambda_1}{\underset{\shortparallel}{[1, i]}}, \underset{\Lambda_2}{\underset{\shortparallel}{[1, -i]}}\}$

$\dim(\Lambda_1 \cap \Lambda_1) = 0 \equiv 0\,(2) \qquad \dim(\Lambda_1 \cap \Lambda_2) \overset{= -1 \text{ (guess.)}}{<} 0$ See P237 bottom line

$n=1 \Rightarrow F = \{X_0^2 + X_1^2 + X_2^2 = 0\} \subset \mathbb{P}^3.$

$\Rightarrow$ F contains 1-dimensional families of 1-planes, since $F = \mathbb{P}^1 \times \mathbb{P}^1$. In other words,

$\{\{x\} \times \mathbb{P}^1, \mathbb{P}^1 \times \{y\}\}. \Rightarrow x_1 \times \mathbb{P}^1 \cap x_2 \times \mathbb{P}^1 = \phi$

$\Rightarrow \dim \phi = -1. \Rightarrow -1 \equiv 1\,(2)$

$x \times \mathbb{P}^1 \cap \mathbb{P}^1 \times y = x \times y \Rightarrow \dim(x \times y) = 0 \not\equiv 1\,(2).$

Refer to P478 ~ P479.

Suppose first that Λ and Λ' intersect, and let p be any point of $\Lambda \cap \Lambda'$. Let $\mathbb{P}^{2n-1}$ be any hyperplane in $\mathbb{P}^{2n}$ not containing p; by what we have seen, the intersection $F \cap T_p(F)$ of F with its tangent plane at p is just the cone through p over the smooth, $(2n-2)$-dimensional quadric $\tilde{F} = F \cap$