$\Rightarrow$ Since $(\delta(df))_{\alpha\beta\gamma} = (df)_{\beta\gamma} - (df)_{\alpha\gamma} + (df)_{\alpha\beta}$ $= df_{\beta\gamma} - df_{\alpha\gamma} + df_{\alpha\beta} = 0$, $df = (df_{\alpha\beta}) \in Z^1(M, Z^1_d)$.

$\Rightarrow$ In $C^0(\underline{U}, \mathcal{A}^1_d) \longrightarrow C^1(\underline{U}, \mathcal{A}^1_d) \xrightarrow{\delta} C^2(\underline{U}, \mathcal{A}^1_d)$;

$df = (df_{\alpha\beta}) \in \ker\delta \subset C^1(\underline{U}, \mathcal{A}^1_d)$.

$\Rightarrow$ Again P42, since $H^1(M, \mathcal{A}^1_d) = 0$, $\exists\ (\omega_\alpha) \in C^0(\underline{U}, \mathcal{A}^1_d)$, s.t. $(\delta\omega)_{\alpha\beta} = df_{\alpha\beta}$ $= \omega_\beta - \omega_\alpha$. $\omega_\alpha \in A^1(U_\alpha)$ ⌟

The global 2-form $d\omega_\alpha = d\omega_\beta$ then represents the image of z in $H^2_{DR}(M, \mathbb{C})$.

⌈ Remember or see P44.

$\check{H}^2(M, \mathbb{R}) \cong H^1(M, Z^1_d)$

From $0 \to H^0(M, \mathcal{A}^1) \xrightarrow{d^*} H^0(M, Z^2) \longrightarrow H^1(M, Z^1) \to 0$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad // \ H^1(M, \mathcal{A}^1)$ ⌟

$$H^1(M, Z^1) = \frac{H^0(M, Z^2)}{d^* H^0(M, \mathcal{A}^1)} = \frac{Z^2(M)}{d A^1(M)}$$

$$= \frac{Z^2(M)}{d A^1(M)} = H^2_{DR}(M, R).$$

The above process traces these isomorphisms.

Obviously $(d\omega)_\alpha = d\omega_\alpha \in A^2(U_\alpha)$ and $dd\omega_\alpha = 0$ & $d\omega_\alpha = d\omega_\beta \Rightarrow [d\omega] \in H^2_{DR}(M, \mathbb{C})$ ⌟