

Thus if we have a smaller subspace  $W' \subset W$ , then  $\exists \omega_0 \in W - W'$ . Let  $\omega_0^*$  be a dual basis element.  
 $\Rightarrow \omega_0^*(\omega_0) = 1 \Rightarrow \omega_0^* \notin \Lambda^\perp = \{v^* \in V^* : i(v^*)\Lambda = 0\}$   
 $\Rightarrow i(\omega_0^*)\Lambda \neq 0 \Rightarrow \Lambda$  is not in the image of  $\Lambda^k W' \rightarrow \Lambda^k V$ . More precisely,  
 $W = \text{Ann}(\Lambda^\perp) = \{v \in V : v^*(v) = 0 \text{ for all } v^* \in \Lambda^\perp\}$   
 $W \supset W' \quad \omega_0 \in W - W' \Rightarrow \text{Suppose } \omega_0^* \in \Lambda^\perp.$   
 $\Rightarrow \omega_0^*(\omega_0) = 0$  since  $\omega_0^* = 0$  on  $W \supset W'$ .  
 $\Rightarrow$  We can conclude that  $W$  is the minimal subspace.  $\cup$

We now define  $W' = \{\omega \in W : \omega \wedge \Lambda = 0\}$ .  
 If  $\Lambda$  is decomposable, then clearly  $W' = W$ .  
 $\square$  If  $\Lambda$  is decomposable,  $\Lambda = v_1 \wedge \dots \wedge v_k \Rightarrow$   
 $W' \supset \langle v_1, \dots, v_k \rangle$ , and  $\dim W = k \Rightarrow W = W'$  (Remember  $v_1, \dots, v_k \in W$ ).  $\cup$

Conversely, if  $\Lambda$  is not decomposable so that  $\dim W = l > k$ , then since the pairing  $\Lambda^k W \otimes \Lambda^{l-k} W \rightarrow \Lambda^l W$  is nondegenerate we deduce that  $W' \neq W$ . So  $\Lambda$  is decomposable  $\Leftrightarrow W = W'$ .

$\square$  Clearly,  $\Lambda^k W \otimes \Lambda^{l-k} W \rightarrow \Lambda^l W$  is nondegenerate,  
 given  $a \in \Lambda^k W$ ,  $a = \sum a_i e_i$ ,  $\sum a_i e_{i_1} \wedge \dots \wedge e_{i_k}$ , assume  $a_1 \neq 0 \Rightarrow$  Consider  $e_{i_1}, i_1 = 1, \dots, l-k$ .  
 $a \wedge e_{i_1} = \pm a_1 e_{i_1} \wedge \dots \wedge e_{i_k}$ , where  $W = \langle e_1, e_2, \dots, e_l \rangle$ .  
 This implies that  $\exists \omega_0 \in W$  s.t.  $\Lambda \wedge \omega_0 \neq 0$ ,  
 since  $\Lambda \in \Lambda^k W \Rightarrow W \neq W'$ . Here we used the fact above that  $l \geq k$  with equality holding  $\Leftrightarrow \Lambda$  is decomposable.  $\cup$