Date

$$\frac{1}{2\pi i}\int_{\partial\Delta}\pi\cdot\eta = \sum \mathrm{Res}_{s_\lambda}(\pi\cdot\eta)$$

$$\Rightarrow \quad \mathrm{Res}_{s_\lambda}(\pi\cdot\eta) = \frac{1}{2\pi i}\int_{B_\epsilon(s_\lambda)}\pi\cdot\eta .$$

$\Rightarrow$ Around s_λ. $\quad \eta = \frac{a_{-1}}{z} + a_0 + \cdots$

and $\pi(z) = \pi(s_\lambda) + z(b_1 + \cdots)$.

$$\Rightarrow \quad \pi\cdot\eta = \frac{\pi(s_\lambda)}{z}\cdot a_{-1} + \cdots$$

$$\Rightarrow \quad \frac{1}{2\pi i}\int_{B_\epsilon(s_\lambda)}\pi\cdot\eta = \frac{1}{2\pi i}\int_{B_\epsilon(s_\lambda)} a_{-1}\frac{\pi(s_\lambda)}{z} = \pi(s_\lambda)\cdot\frac{1}{2\pi i}\int_{B_\epsilon(s_\lambda)}\frac{1}{z}\times a_{-1}$$

$$= \pi(s_\lambda)\, a_{-1} = \pi(s_\lambda)\,\mathrm{Res}_{s_\lambda}(\eta).$$

$$\Rightarrow \quad \int_{\partial\Delta}\pi\cdot\eta = \sum\pi(s_\lambda)\,\mathrm{Res}_{s_\lambda}(\eta) = \sum \mathrm{Res}_{s_\lambda}(\eta)\int_{p_0}^{s_\lambda}\omega .$$

On the other hand, we can compute the integral of $\pi\cdot\eta$ around $\partial\Delta$ explicitly by considering together the contributions of the pair of sides of $\partial\Delta$ corresponding to δ_i and δ_i^{-1}: since, for points $p\in\delta_i$ and $p'\in\delta_i^{-1}$ identified on S, the difference $\pi(p')-\pi(p)$ is a constant Π^{g+i}, we see that

$$\int_{\delta_i+\delta_i^{-1}}\pi\cdot\eta = -\Pi^{g+i}\cdot\int_{\delta_i}\eta = -\Pi^{g+i}\cdot N^i .$$

The difference $\pi(p')-\pi(p)$ is a constant Π^{g+i} by P229.

$$\int_{\delta_i+\delta_i^{-1}}\pi\cdot\eta = \int_{\delta_i}\pi\cdot\eta + \int_{\delta_i^{-1}}\pi\cdot\eta .$$