Specializing still further, let M be a complex manifold of dimension n, $E = T'(M) \longrightarrow M$ its holomorphic tangent bundle and v a C^∞ section of E having nondegenerate zeros at $p_\nu \in M$. Let $z = (z_1, \cdots, z_n)$ be local holomorphic coordinates centered around p_ν, and write

$$z_i = x_{2i-1} + \sqrt{-1}\, x_{2i}$$

so that $x = (x_1, \cdots x_{2n})$ is an oriented real coordinate system for M near p_ν. Then, if

$$v(z) = \sum (a^\nu_{jk} + \sqrt{-1}\, b^\nu_{jk})\, z_j \frac{\partial}{\partial z_k} + [2]$$

and if $A_{p_\nu} = (A^\nu, B^\nu)$ as above, we have

$$C_n(M) = \sum (-1)^{n(n-1)/2} \operatorname{sgn} \det (A_{p_\nu}).$$

Now let

$$v'(z) = \frac{1}{2} (v(z) + \overline{v(z)})$$

be the real vector field obtained from v by the real projection $T'(M) \longrightarrow T_{\mathbb{R}}(M)$.

$$T_{\mathbb{R}}(M) = \mathbb{R} \left\{ \frac{\partial}{\partial x_{2i}}, \frac{\partial}{\partial x_{2i-1}} \right\}$$

$$\begin{array}{ccc} T'(M) & \longrightarrow & T_{\mathbb{R}}(M) \\ \Downarrow & & \Downarrow \\ \dfrac{\partial}{\partial z_i} & \longmapsto & \dfrac{1}{2} \dfrac{\partial}{\partial x_{2i-1}}. \\ \| & & \end{array}$$

$$\frac{1}{2} \left(\frac{\partial}{\partial x_{2i-1}} + (-\sqrt{-1}) \frac{\partial}{\partial x_{2i}} \right)$$

Then

$$v'(z) = \frac{1}{2} \sum (a^\nu_{jk} + \sqrt{-1}\, b^\nu_{jk})\, z_j \cdot \left(\frac{\partial}{\partial x_{2k-1}} - \sqrt{-1} \frac{\partial}{\partial x_{2k}} \right)$$

$$+ \frac{1}{2} \sum (a^\nu_{jk} - \sqrt{-1}\, b^\nu_{jk}) \cdot \bar{z}_j \cdot \left(\frac{\partial}{\partial x_{2k-1}} + \sqrt{-1} \frac{\partial}{\partial x_{2k}} \right) + [2]$$