Date

we can show that $\exists$ a hyperplane $\mathbb{P}^{n-1}$ s.t $\mathbb{P}^{n-1}$ intersect $V \cap \mathbb{P}^{n-1}$ transversally.

Continue this process, we have

$V \cap \underbrace{\mathbb{P}^{n-1} \cap \cdots \cap \mathbb{P}^{n-1}}_{k} = V \cap \mathbb{P}^{n-k} =$ set of isolated points, since V & $\mathbb{P}^{n-k}$ have complementary dimensions.

Here is some missing point, what if $\mathbb{P}^{n-1} = \mathbb{P}^{n-1}$?

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$(n-k)$ linearly independent vectors
$(a_{1,0}\ a_{1,1} \cdots a_{1n}) \cdots (a_{n-k,0},\ a_{n-k,1} \cdots a_{n-k,n})$
determine a k-plane $\mathbb{P}^k$ in $\mathbb{P}^n$.

Let $U \subset \{z_0 \neq 0\}$ be an open set in $\mathbb{P}^n$
s.t $U = \{f_1 = 0\} \cap \cdots \cap \{f_k = 0\}$

$$\Rightarrow \quad \begin{aligned} a_{1,0} z_0 + a_{11} z_1 + \cdots + a_{1n} z_n &= 0 = P_1([z_0, z_1 \cdots z_n]) \\ a_{2,0} z_0 + a_{21} z_1 + \cdots + a_{2n} z_n &= 0 = P_2([(z_0 \cdots z_n)]) \\ &\ \vdots \\ a_{n-k,0} z_0 + a_{n-k,1} z_1 + \cdots + a_{n-k,n} z_n &= 0 = P_{n-k}([z_0, z_1 \cdots z_n]) \\ f_1 &= 0 \\ &\ \vdots \\ f_k &= 0 \end{aligned}$$