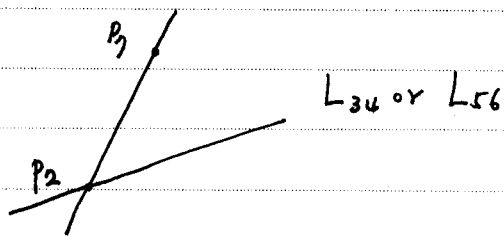


⌈ The cubic  $L_{27} + L_{34} + L_{56}$  is singular at  $p_2 \Rightarrow$



⌋

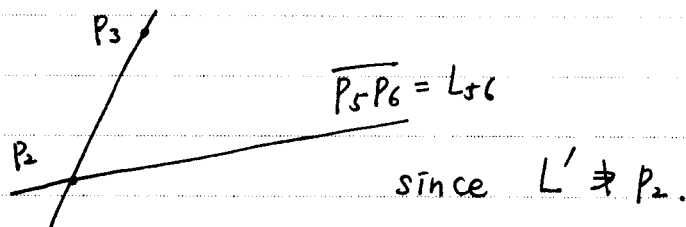
In this case, take  $L'$  any line through  $p_7$  missing all the other points  $p_i$ ; the cubic

$$L_{23} + L_{56} + L'$$

contains  $p_1$ ; thus  $p_2$  lies on  $L_{56}$ .

⌈ Since  $L_{34}$  contains  $p_2$ ,  $L_{23} \ni p_4$ .

$\Rightarrow L_{23} + L_{56} + L' \ni p_2, p_3, p_4, p_5, p_6, p_7 \Rightarrow$  It contains  $p_1$ . Again this cubic is singular at  $p_2$ .



⌋

But then since

$$L_{27} + L_{35} + L_{46}$$

contains  $p_1$ , either  $L_{35}$  or  $L_{46}$  must pass through  $p_2$ , and in either case it follows that  $p_2, p_3, p_4, p_5$ , and  $p_6$  are collinear.