$\Rightarrow$ The inverse of μ is holomorphic $\Rightarrow$ μ is biholomorphic. ┘

We have thus established the fundamental fact that the nonsingular cubic curves in $\mathbb{P}^2$ are the same as the compact Riemann surfaces $\mathbb{C}/\Lambda$ for a suitable lattice Λ in the complex plane. It follow that every such curve C has a group structure; we want to briefly discuss this.

First, recall that at the end of the previous section we constructed meromorphic functions F and F' on a Riemann surface C of genus 1, having a double and triple pole, respectively, at a base point $p_0 \in C$ and holomorphic elsewhere. (See P223) We chose F and F' so that in terms of a local coordinate w around p_0,

$$F(w) = \frac{1}{w^2} + [1]$$

and

$$F'(w) = \frac{1}{w^3} + [1]$$

and

$$dF = F' \cdot \omega,$$

where ω is a global nonzero holomorphic 1-form on C.

Pf ω is nonvanishing. By P223, $F' = \lambda \frac{dF}{\omega} + \lambda' F + \lambda''$

Let $\omega' = \frac{\omega}{\lambda}$.

$$\Rightarrow F' = \frac{dF}{\omega'} + \lambda' F + \lambda'' \Rightarrow (F' - \lambda' F - \lambda'')\omega' = dF.$$