Definition. Given finitely generated $\mathcal{O}$-modules M and N,

$$\begin{cases} \mathrm{Ext}^n_{\mathcal{O}}(M,N) = H^n(\mathrm{Hom}_{\mathcal{O}}(E.(M), N)), \\ \mathrm{Tor}_n^{\mathcal{O}}(M,N) = H_n(E.(M) \otimes_{\mathcal{O}} N). \end{cases}$$

We shall derive the main properties of Ext, for the most part leaving the analogous properties of Tor to the reader. We first note that by 2 and 3 Ext is well-defined independently of the projective resolution $E.(M)$. More generally, maps

$$\varphi : M \longrightarrow M', \quad \psi : N \longrightarrow N'$$

induce

$$\Phi^* : \mathrm{Ext}_{\mathcal{O}}(M', N) \longrightarrow \mathrm{Ext}_{\mathcal{O}}(M,N),$$
$$\Psi_* : \mathrm{Ext}_{\mathcal{O}}(M, N) \longrightarrow \mathrm{Ext}_{\mathcal{O}}(M,N'),$$

with functoriality properties such as

$$\begin{tikzcd} M'' \arrow[r, "\gamma"'] \arrow[rr, bend left, "\lambda"] & M' \arrow[r, "\varphi"'] & M \end{tikzcd}$$

$$\Downarrow$$

$$\Lambda^* = \Phi^* \circ \Gamma^*$$

Thus, $\mathrm{Ext}^*_{\mathcal{O}}(M,N)$ is a functor contravariant in M and covariant in N.

$\ulcorner$ We want to show that $\Lambda^* = \Phi^* \circ \Gamma^*$ where

$$\begin{tikzcd} M'' \arrow[r, "\gamma"'] \arrow[rr, bend left, "\lambda"] & M' \arrow[r, "\varphi"'] & M \end{tikzcd}$$