Since

$$\det\begin{pmatrix} 1 & 0 & \cdots & 0 & z_{1j} & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & z_{2j} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 1 & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & & \boxed{z_{ij}} & 0 & & \vdots \\ \vdots & \vdots & & & \vdots & 1 & \ddots & \vdots \\ 0 & 0 & \cdots & & z_{kj} & 0 & \cdots & 1 \end{pmatrix} = \pm z_{ij},$$

$$p(z_{11}, z_{12} \cdots z_{1,n-k}, \cdots \cdots z_{k,n-k})$$
$$= (\ast \cdots \pm z_{11} \cdots \pm z_{12} \cdots \pm z_{1,n-k}, \ast \cdots \ast \cdots z_{k,n-k}) \quad \text{and}$$

$$Dp = \begin{pmatrix} \ast & 1 & \ast \cdots \ast & 0 & \ast & \cdots & \ast & \ast & 0 & \ast & & \ast \\ \vdots & 0 & \ast & 1 & \ast & \cdots & \vdots & \ast & 0 & \ast & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \ast & \ast & \vdots & \ast & & \vdots \\ \ast & 0 & \ast \cdots \ast & 0 & \ast & \cdots & \ast & \ast & 1 & \ast & & \ast \end{pmatrix}.$$

$\Rightarrow$ Dp has rank at least $k(n-k)$. $\Rightarrow$ $\operatorname{rank}(Dp) = k(n-k)$.
$\Rightarrow$ p is an embedding.

For example.

$$\begin{tikzcd} \begin{pmatrix} 1, & z_{11}, & 0 & z_{12} \\ 0, & z_{21}, & 1 & z_{22} \end{pmatrix} \arrow[r, mapsto, "p"] \arrow[d, leftrightarrow] & (z_{21}, 1, z_{22}, z_{11}, z_{11}z_{22} - z_{21}z_{12}, -z_{12}) \\ (z_{11}, z_{12}, z_{21}, z_{22}) \arrow[ur, "p"] & \end{tikzcd}$$

$\Rightarrow$ Dp has rank 4.

Now we shall determine equations which define the Plücker image of $G(k, V)$ in $\mathbb{P}(\Lambda^k V)$. What we are asking for are the conditions that a multivector $\Lambda \in \Lambda^k V$ be decomposable, i.e., of the