its Chern class, $\pi_1^* L$ is trivial.

If we then take a trivialization $\tilde{\varphi} : \pi_1^* L \to (\mathbb{C}^*)^n \times \mathbb{C}$ and choose our trivialization $\varphi$ of $\pi^* L$ to extend $\tilde{\varphi}$, we have

$$e_{\lambda_\alpha}(z) \equiv 1, \quad \alpha = 1, 2 \cdots n.$$

$$\begin{tikzcd}
\pi^* L \arrow[r] \arrow[d] & \pi_1^* L \arrow[r] \arrow[d] & L \arrow[d] \\
V \arrow[r] & V/\mathbb{Z}\{\lambda_1 \cdots \lambda_n\} \arrow[r, "\pi_1"] & M
\end{tikzcd}$$

$$\begin{tikzcd}
(z, \ell) \arrow[r, mapsto, "\circledast"] & ([z], \ell) \\[-20pt]
\rotatebox{90}{$\in$} & \rotatebox{90}{$\in$} \\[-20pt]
V \times \mathbb{C} \arrow[r] \arrow[d] & V/\mathbb{Z}\{\lambda_1 \cdots \lambda_n\} \times \mathbb{C} \arrow[d] \\
V \arrow[r, "p"] & V/\mathbb{Z}\{\lambda_1, \cdots \lambda_n\}
\end{tikzcd}
\quad \Rightarrow \quad V \times \mathbb{C} = p^*(V/\mathbb{Z}\{\lambda_1 \cdots \lambda_n\} \times \mathbb{C})$$

We can choose a trivialization $\varphi$ of $\pi^* L$ so that the following diagram is commutative

$$\begin{tikzcd}
V \times \mathbb{C} \arrow[rr, "\cong"] \arrow[rr, bend right, leftarrow, "\circledast"'] \arrow[ddr] & & V/\mathbb{Z}\{\lambda_1, \cdots \lambda_n\} \times \mathbb{C} \arrow[ddr] & & \\
& \pi^* L \arrow[ul, "\varphi"'] \arrow[rr] \arrow[d] & & \pi_1^* L \arrow[ul, "\tilde{\varphi}"'] \arrow[r] \arrow[d] & L \arrow[d] \\
& V \arrow[rr] & & V/\mathbb{Z}\{\lambda_1 \cdots \lambda_n\} \arrow[r] & V/\Lambda
\end{tikzcd}$$