

distinct points  $x_m \in G$  without limit point in  $G$ , such that  $\phi_m(x_m) \neq 0$  ( $m=1, 2, 3, \dots$ ). Let  $W$  be the set of all  $\phi \in C_c^0(G)$  that satisfy

$$|\phi(x_m)| < m^{-1} |\phi_m(x_m)| \quad (m=1, 2, 3, \dots).$$

Since each  $K$  contains only finitely many  $x_m$ , it is easy to see that  $W$  is open in  $C_c^0(G)$ . Since  $\phi_m \notin mW$ , no multiple of  $W$  contains  $E$ . This shows that  $E$  is not bounded.

¶  $W$  is open in  $C_c^0(G)$ .  $\phi \in W \Rightarrow |\phi(x_m)| < m^{-1} |\phi_m(x_m)|$ .  $\psi \in N(\phi, \delta) \Rightarrow \|\phi - \psi\|_0 < \delta$ . (?)

$W$  is not open in  $C_c^0(G)$  w.r.t  $\|\cdot\|_0$ .

We made a mistake in understanding the topology  $C_c^0(G)$ . Let's try it.

$C_c^p(G)$ .  $K$  compact subset of  $G$ .

$$\Rightarrow V_N = \{ \varphi \in C^\infty(G) : \|\varphi\|_p < \frac{1}{N} \text{ on } K_N \}.$$

where  $\|\varphi\|_p < \frac{1}{N}$  on  $K_N$  means that

$$\max \{ |D^\alpha \varphi(x)| : x \in K_N \}, \text{ and } K_N \subset K_{N+1}, \cup K_N = G.$$

$\{V_N\}$  forms a local convex base for  $C^p$  topology.

On  $\mathcal{D}_K$ , we take the induced topology  $\tau_K$ .

$\Rightarrow \beta =$  collection of all convex balanced sets  $W \subset C_c^p(G)$  s.t.  $\mathcal{D}_K \cap W \in \tau_K$  for every compact set  $K \subset G$ .

Specially  $p=0$ .  $W \cap \mathcal{D}_K$  is open in  $\mathcal{D}_K$  for

$$W \cap \mathcal{D}_K \ni \phi \Rightarrow \phi + \{ \psi \in C_c^0(G) : \|\psi\|_0 < \delta \}$$

on  $K_N \supset K \subset W \cap \mathcal{D}_K \Rightarrow E$  is not bounded.)