and

$$c(T'(W_2)) = 1 + 3\ell + 3\ell^2.$$

$\ulcorner$

$$W_2 \hookrightarrow W$$

See P409

$$\begin{tikzcd} T'(W)|_{W_2} \arrow[r] \arrow[d] & T'(W) \arrow[d] \\ W_2 \arrow[r, hook, "i"] & W \end{tikzcd}$$

$c(T'(W)|_{W_2}) = i^*(c(T'(W))) = i^*((1+w)^6)$
$= i^*(1 + 6w + 15w^2)$
for, for example
$i^*w^3 = w^3|W_2 = 0\ (H \cap H \cap H \cap W_2 = \mathbb{P}^2 \cap W_2 = 0)$
$\Rightarrow\ i^*w^i = 0$ for $i \geq 3$.
Since $i^*w = w|W_2$ and $i^*w^2 = (i^*w)^2 = w^2|W_2$,
$c(T'(W)|_{W_2}) = i^*(1 + 6w + 15w^2) = 1 + 6w|W_2 + 15w^2|W_2$
$= 1 + 12\ell + 60\ell^2$.
$c(T'(W_2)) = c(T'(\mathbb{P}^2)) = (1+w_0)^3 = 1 + 3w_0 + 3w_0^2 + w_0^3$
$= 1 + 3\ell + 3\ell^2 + \ell^3$, since $w_0 = \mathbb{P}^1$ a line in $W = \mathbb{P}^2$,
and $\ell^3 = 0 \in H^6(\mathbb{P}^2) = 0$.

$\lrcorner$

From the C^∞ decomposition of vector bundles

$$T'(W)|_{W_2} = T'(W_2) \oplus N_{W_2|W}$$

we obtain