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with equality if and only if C is normal.

Proof. Let D be the hyperplane section of C. Then

$$\dim |D| = h^0(D) - 1 \geq n > \frac{d}{2},$$

and so by Clifford's theorem D is nonspecial.

$\ulcorner$

Suppose $D' \sim D$ and D' effective divisor on C. $\Rightarrow$ Since D' is on C in $\mathbb{P}^n$, $\exists$ at most $(n+1)$-linearly independent points in D'. Let $p_1, \cdots p_{n+1}$ be the points in D, s.t every other point in D is a linear combination of $p_1, p_2, \cdots p_n$. Consider $\sigma = x_1\sigma_0 + \cdots + x_n\sigma_n$, where $(\sigma_0 = 0) = (Z_0 = 0)$ $\cdots$ $(\sigma_n = 0) = (Z_n = 0)$. $\Rightarrow$ $H^0(\mathbb{P}^n, \mathcal{O}(H)) = \langle \sigma_1, \sigma_2, \cdots \sigma_n \rangle$ since every hyperplane in $\mathbb{P}^n$ is expressed as a linear combination $a_0 Z_0 + \cdots + a_n Z_n = 0$.

$$\begin{aligned}
\sigma(p_1) &= x_0\sigma_0(p_1) + \cdots + x_n\sigma_n(p_1) = 0 \\
\sigma(p_2) &= x_0\sigma_0(p_2) + \cdots + x_n\sigma_n(p_2) = 0 \quad \cdots \circledast \\
&\vdots \\
\sigma(p_{n+1}) &= x_0\sigma_0(p_{n+1}) + \cdots + x_n\sigma_n(p_{n+1}) = 0.
\end{aligned}$$

$\Rightarrow$ Since $\operatorname{rank}\{(\sigma_0(p_1), \cdots \sigma_n(p_1)), \cdots (\sigma_0(p_{n+1}), \cdots \sigma_n(p_{n+1}))\}$ $= \operatorname{rank}\{p_1, \cdots p_{n+1}\}$, we can have nontrivial solution for $\circledast$. In other words, $(\sigma = 0) \cap C = D'$, since every other point is expressed as a linear combination of $p_1 \cdots p_n$, and $(\sigma = 0)$ contains every point in D', where D' is an effective divisor of distinct points.

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