┌ For the general case, let $n=4$. $\dim V = 2$.
$\Rightarrow$ $\mathbb{C}^4 - \mathbb{C}^2 = \{ (z_1, z_2, z_3, z_4) :$ both z_4 & z_3 can not be zero simultaneously. $\}$
As before, assume that D does not contain the z_4-axis. Since $H^1(\mathbb{C}^4 - \mathbb{C}^2, \mathcal{O}) = H^2(\mathbb{C}^4 - \mathbb{C}^2, \mathbb{Z}) = 0$, $H^1(\mathbb{C}^4 - \mathbb{C}^2, \mathcal{O}^*) = 0$. $\Rightarrow$ $\exists$ $h \in \mathcal{O}(\mathbb{C}^4 - \mathbb{C}^2)$ s.t $(h=0) = D$.

" $H^1(\mathbb{C}^4 - \mathbb{C}^2, \mathcal{O}) = H^{0,1}_{\bar\partial}(\mathbb{C}^4 - \mathbb{C}^2) = 0$ need not be true. See P49 example 4. $H^1(\mathbb{C}^2 - \{0\}, \mathcal{O}) = \infty$
We can not apply Hodge Decomposition Theorem to $\mathbb{C}^2 - \{0\}$.
If we can do it, $H^1(\mathbb{C}^2 - \{0\}, \mathcal{O})$
$= H^{0,1}_{\bar\partial}(\mathbb{C}^2 - \{0\}) = H^{0,1}(\mathbb{C}^2 - \{0\}) \subset H^1(\mathbb{C}^2 - \{0\}) = H^1(S^3) = 0$."
$\Rightarrow$ Contradiction. Fail, again.

The point is here: $n = 4$. $\dim V = 2$.
For a point $(z_1, z_2, 0, 0) \in V$, $\exists$ an open set U in $\mathbb{C}^4$ s.t $U \cap D = (h=0)$, h is holomorphic on U. $\Rightarrow$ We have only to show that $\bar{D}$ is analytic in U. We don't have to worry about the line bundle business. The whole argument above is true since $\Delta^n - V$ is open.
Wrong Again, since D is analytic in $\Delta^n - V$.

Let's try again!