┌ $E = \mathbb{P}(N_{W_2|W}) \xrightarrow{\pi} W_2$ rank of $N_{W_2|W} = 3$

$\Rightarrow$ By the Proposition on P606,

$$\zeta^3 - c_1(N_{W_2|W})\,\zeta^2 + c_2(N_{W_2|W})\,\zeta - c_3(N_{W_2|W}) = 0$$

Here $c_1(N_{W_2|W})\,\zeta^2 = \pi^*(c_1(N_{W_2|W}))\,\zeta^2$, and since, by the result on P751, $c_1(N_{W_2|W}) = 9\ell$, $c_2(N_{W_2|W}) = 30\ell^2$ and $c_3(N_{W_2|W}) \in H^6(W_2, \mathbb{Z}) = H^6(\mathbb{P}^2, \mathbb{Z}) = 0$, we have

$\zeta^3 - 9\tilde{\ell}\,\zeta^2 + 30\,\tilde{\ell}^2\zeta = 0$, where $\tilde{\ell} = \pi^*\ell$.

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Now we have seen that the tautological bundle restricts to the universal bundle $[-H]$ on each fiber E_p of $E \to W_2$, and so

$$\zeta^2 \cdot \tilde{\ell}^2 = c_1(T|_{E_p})^2 = 1.$$

┌ $T|_{E_p} \ni v \Rightarrow$ The vector space $\langle v \rangle$ expresses a line $\mathbb{P}(\langle v \rangle) \in E_p$, projective space.

$$\zeta^2 \cdot \tilde{\ell}^2 = \zeta^2 \cdot \pi^*(\ell)^2 = \zeta^2 \cdot \pi^*(\ell^2) = \zeta^2 \cdot \pi^*(p)$$
$$= c_1(T)^2 \cdot \pi^*(p)^2 = c_1(T)^2|_{E_p} = (c_1(T)|_{E_p})^2 = c_1(T|_{E_p})^2$$
$$= c_1(-H)^2 = (-1)^2 = 1, \text{ since } T|_{E_p} = [-H].$$

$\pi^*(p) = E_p$, and $c_1(T)|_{E_p} = c_1(T|_{E_p})$. ($\because$ $i: E_p \to E$ and $i^* c_1(T) = c_1(T)|_{E_p} = c_1(T|_{E_p})$.)

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