larity in u'. ⌟

Finally, we consider $\tilde{\tilde{\varphi}} = \tilde{\varphi} - d\gamma' = \zeta'' \wedge du_k/u_k$.

⌜ $\tilde{\tilde{\varphi}} = \tilde{\varphi} - d\gamma' = \tilde{\varphi} - \zeta' = \zeta' + \zeta'' \wedge \frac{du_k}{u_k} - \zeta'$
$= \zeta'' \wedge du_k/u_k$. ⌟

Writing $(\Delta^*)^k = (\Delta^*)^{k-1} \times \Delta^*$ and using Künneth,
$\tilde{\tilde{\varphi}} = 0$ in $H^q_{DR}((\Delta^*)^k) \Rightarrow \zeta'' = 0$ in $H^{q-1}_{DR}((\Delta^*)^{k-1})$.

⌜
$$H^q_{DR}((\Delta^*)^k) \cong \bigoplus_{i+j=q} H^i_{DR}((\Delta^*)^{k-1}) \otimes H^j_{DR}(\Delta^*)$$
$$= H^{q-1}_{DR}((\Delta^*)^{k-1}) \otimes H^1_{DR}(\Delta^*) \oplus \cdots$$

$$\begin{array}{ccccc} H^{q-1}_{DR}((\Delta^*)^{k-1}) & \otimes & H^1_{DR}(\Delta^*) & \xrightarrow[\text{injective}]{} & H^q_{DR}((\Delta^*)^k) \\ \Downarrow & & & & \\ \zeta'' & \otimes & \frac{du_k}{u_k} & \longmapsto & \zeta'' \wedge \frac{du_k}{u_k} \\ & & & & \parallel \\ & & & & \tilde{\tilde{\varphi}} = 0 \end{array}$$

see p 56.

$\Rightarrow$ Since $\frac{du_k}{u_k}$ is a generator of $H^1_{DR}(\Delta^*)$,
ζ'' must be 0. ⌟

Then $\zeta'' = d\gamma''$ where γ'' has at most a pole in u', and $\tilde{\tilde{\varphi}} = d(\gamma'' \wedge du_k/u_k)$.

Q.E.D.

⌜ ζ'' is a form in only $u' = (u_1 \cdots u_{k-1})$. By induction,
$\zeta'' = d\gamma''$. $\Rightarrow$ $\tilde{\tilde{\varphi}} = d\gamma'' \wedge du_k/u_k$
$= d(\gamma'' \wedge du_k/u_k) \Rightarrow \varphi = d(\eta + \gamma'' \wedge du_k/u_k)$. $\Rightarrow$