

sequence  $C$  may be defined as the number of non-zero entries.

$$n-k=C_1, \quad n-k-C_i = C_1 - C_i, \quad n-k-C_1 = 0.$$

$$\Rightarrow (*) \delta(a, b; c) = \#(\sigma_a \cdot \sigma_b \cdot \sigma_{c_1-c_k} \dots \sigma_{c_1-c_n}) \text{ in } G(l(c), l(c)+C_1).$$

As an immediate consequence, we see that  $\delta(a, b; c) = 0$  if  $\sigma_a$  or  $\sigma_b$  is null in  $G(l(c), l(c)+C_1)$ , i.e.,  $\delta(a, b; c) = 0$  if either

1.  $c_1 < a_1$  or  $c_1 < b_1$ , or
2.  $l(c) < l(a)$  or  $l(c) < l(b)$ .

$\nVdash \sigma_a$  is null in  $G(l(c), l(c)+C_1)$  if  $a_i > l(c)+C_1 - l(c)$  or  $l(c) < l(a) \Rightarrow a_i > c_1 \Rightarrow a_1 > c_1$

Thus  $\sigma_a = 0$  in  $G(l(c), l(c)+C_1)$  if  $a_1 > c_1$  or  $l(c) < l(a)$

See P199

Next, note that for any vector space  $W$  of dimension  $n$ , we have a natural isomorphism

$$*: G(k, W) \longrightarrow G(n-k, W^*)$$

defined by

$$*\Lambda = \text{Ann}(\Lambda) = \{l \in W^* : l(\Lambda) = 0\}.$$

$\nVdash \Lambda \in G(k, W) \Rightarrow *\Lambda = \{l \in W^* : l(\Lambda) = 0\}$  has the same dimension as  $\dim \Lambda^\perp = n-k$ .

$$G(k, W) \xrightarrow{*} G(n-k, W^*) \xrightarrow{L} G(k, W)$$

Here  $L : G(n-k, W^*) \longrightarrow G(k, W)$  is defined by  $L(\tau) = \{ \Lambda \in G(k, W) \text{ or } \Lambda \subset W \mid l(\Lambda) = 0 \text{ for all } l \in \tau \}$