$$\mathrm{Res}_{\{0\}}\left(\frac{h(x,y)\,dx\wedge dy}{x\,y^2}\right) = \mathrm{Res}_{\{0\}}\left(\frac{\Delta(x,y)\,h(x,y)\,dx\wedge dy}{x\,f(x,y)^2}\right).$$

「 See P 657 ~ P658. 」

By the Cauchy formula the left-hand side is $h_y(0,0)$.

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$$\mathrm{Res}_{\{0\}}\left(\frac{h(x,y)\,dx\wedge dy}{x\,y^2}\right) = \left(\frac{1}{2\pi\sqrt{-1}}\right)^2 \int_{|x|=\epsilon}\int_{|y|=\epsilon} \frac{h(x,y)}{x\,y^2}\,dx\,dy$$

$$= \left(\frac{1}{2\pi\sqrt{-1}}\right)^2 \int_{|y|=\epsilon}\frac{1}{y^2}\int_{|x|=\epsilon}\frac{h(x,y)}{x}\,dx\ dy = \frac{1}{2\pi\sqrt{-1}}\int_{|y|=\epsilon}\frac{1}{y^2}\,h(0,y)\,dy$$

Since $h(0,y) = h(0,0) + h_y(0,0)\cdot y + \cdots$,

$$\frac{1}{2\pi\sqrt{-1}}\int_{|y|=\epsilon}\frac{h(0,y)}{y^2}\,dy = h_y(0,0).$$

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Taking $h = P/\Delta$, we obtain

$$\mathrm{Res}_{\{0\}}\left(\frac{P(x,y)\,dx\wedge dy}{x\,f(x,y)^2}\right) = \frac{P_y(0)}{f_y(0)^2} - \frac{P(0)\,f_{yy}(0)}{f_y(0)^3},$$

since $f_y(0) = b$ and $f_{yy}(0) = e$. Q.E.D. for Lemma.

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$$\mathrm{Res}_{\{0\}}\left(\frac{P(x,y)\,dx\wedge dy}{x\quad f(x,y)^2}\right) = \mathrm{Res}_{\{0\}}\left(\frac{P/\Delta\ \ dx\wedge dy}{x\,y^2}\right)$$

$$= \left.\frac{\partial(P/\Delta)}{\partial y}\right|_{(0,0)} = \left.\frac{\Delta\frac{\partial P}{\partial y} - P\frac{\partial \Delta}{\partial y}}{\Delta^2}\right|_{(0,0)} = \frac{P_y(0)}{b^2} - \frac{P(0)\,be}{b^4}$$