$\Rightarrow (\rho_1=0)\cap(\rho_2=0)\supset l \Rightarrow$ Contradiction.
If $F = l l'$, since no three of p_i's are collinear, l and l' must contain two of p_i's. ⏎

The singular conics passing through $\{p_i\}$ are therefore

$$\overline{p_1p_2}+\overline{p_3p_4},\quad \overline{p_1p_3}+\overline{p_2p_4} \quad\text{and}\quad \overline{p_1p_4}+\overline{p_2p_3}.$$

⌈ A singular conic in $\mathbb{P}^2$ is of form $X_0^2+X_1^2=0$ or $X_0^2=0$. $X_0^2=0$ is not the case since $(X_0=0)$ contains all p_i's. $\Rightarrow (X_0^2+X_1^2=0) = (X_0+iX_1=0)\cup(X_0-iX_1=0)$
$\Rightarrow$ A singular conic is a union of two lines.
$\Rightarrow$ The number of choices is ${}_4C_2/2 = 6/2 = 3$,
more precisely,
$\overline{p_1p_2}+\overline{p_3p_4}$, $\overline{p_1p_3}+\overline{p_2p_4}$, and $\overline{p_1p_4}+\overline{p_2p_3}$ ⏎

So we see again that L contains three singular conics.

4. Alternatively, note that $W_1\subset W$ is the image of $\mathbb{P}^{2*}\times\mathbb{P}^{2*}$ under the map f sending a pair of lines (l_1, l_2) to the conic $l_1+l_2\in W$.

⌈ Since any singular conic in $\mathbb{P}^2$ is a union of two lines as we saw above,

$$\begin{array}{rccc} f: & \mathbb{P}^{2*}\times\mathbb{P}^{2*} & \longrightarrow & W \\ & (l_1, l_2) & \longmapsto & l_1+l_2 \end{array}$$