

since $i(l)\Lambda = 0$. \cup

Consequently, the condition $\Lambda \wedge w = 0$ for all $w \in W$ is equivalent to $i(\bar{Z})\Lambda \in \Lambda^\perp$ for all \bar{Z} , which is in turn equivalent to

$$(*) \quad i(i(\bar{Z})\Lambda)\Lambda = 0 \quad \text{for all } \bar{Z} \in \Lambda^{k+1}V^*.$$

\Rightarrow We have to show that $i(\bar{Z})\Lambda \in \Lambda^\perp$, given an element $\bar{Z} \in \Lambda^{k+1}V^*$. To do this, we have only to prove that $\langle i(\bar{Z})\Lambda, w \rangle = 0$ for all $w \in W$. $\Rightarrow \langle i(\bar{Z})\Lambda, w \rangle = \langle \bar{Z}, \Lambda \wedge w \rangle = 0$ since $\Lambda \wedge w = 0$ for all $w \in W$. //

\Leftarrow Given $w \in W$, we have to show that $\Lambda \wedge w = 0$. To show this, we have only to show that, for all $\bar{Z} \in \Lambda^{k+1}V^*$, $\langle \bar{Z}, \Lambda \wedge w \rangle = 0$ since $\Lambda^{k+1}V^* \rightarrow \Lambda^{k+1}W^*$ is a natural projection. $\langle \bar{Z}, \Lambda \wedge w \rangle = \langle i(\bar{Z})\Lambda, w \rangle = 0$ since $i(\bar{Z})\Lambda \in \Lambda^\perp = \text{Ann}(W)$.

$$i(\bar{Z})\Lambda \in \Lambda^\perp = \{v^* \in V^* \mid i(v^*)\Lambda = 0\}$$

$$\Leftrightarrow i(i(\bar{Z})\Lambda)\Lambda = 0. \quad \cup$$

The left-hand side of (*) gives $\binom{n}{k+1}$

quadratic forms in the homogeneous coordinates Λ_i of $p(G(k, V))$; setting them equal to zero gives the classical Plücker relations.