

We define the residue map

see p 233

$$R: H^1(\Omega(*)) \longrightarrow \bigoplus_{D \in \text{Div}(M)} \mathbb{C}_D$$

by $R(\varphi) = \bigoplus_i \lambda_i \cdot 1_{D_i}$

\mathbb{C}_D is the constant sheaf concentrated on divisor D
 $\Rightarrow \mathbb{C}_D(U) \cong \begin{cases} \mathbb{C} & \text{if } U \cap D \neq \emptyset, U \text{ connected.} \\ 0 & \text{otherwise.} \end{cases}$

More precisely, \mathbb{C} is given the discrete topology

$$\mathbb{C}_D(U) = \{ f: U \longrightarrow \mathbb{C} \text{ continuous s.t. } f=0 \text{ on } U-D \}$$

$$\Rightarrow \mathbb{C}_D(U \cap V) \ni \sigma = \tau \Rightarrow \exists ? \rho \in \mathbb{C}_D(U \cup V) \text{ s.t.} \\ \rho|_U = \sigma \quad \rho|_V = \tau. \text{ Define } \rho: U \cup V \longrightarrow \mathbb{C} \text{ by} \\ \rho(x) = \sigma(x) \text{ if } x \in U \\ \rho(x) = \tau(x) \text{ if } x \in V. \quad \square$$

The notation means that $R(\varphi)$ is the constant λ_i on the divisor D_i .

$$\begin{array}{ccc} \mathbb{F} & R_u: & H^1(\Omega(*))(U) \longrightarrow \bigoplus_{D \in \text{Div} M} \mathbb{C}_D(U) \\ & \downarrow & \\ & \varphi & \longmapsto \frac{1}{2\pi\sqrt{-1}} \int_{\gamma_0} \varphi \cdot 1_D \end{array}$$

where $1_D: U \longrightarrow \mathbb{C}$ defined by
 if $x \in U \cap D, 1_D(x) = 1 \in \mathbb{C}$
 if $x \notin U \cap D, 1_D(x) = 0 \in \mathbb{C}.$ \square