$(b_1, b_2, b_3) \in \mathbb{C}^3$ s.t $b_1\sigma_1 + b_2\sigma_2 + b_3\sigma_3 = 0$ is nonsingular on $M - (B_1 \cap B_2 \cap B_3)$.

Let $S = \{ (p, [(a_1, a_2, a_3)]) \in M \times \mathbb{P}^2 \mid$
$a_1\sigma_1 + \cdots + a_3\sigma_3 = 0$ at p
$a_1 \nabla\sigma_1 + a_2\nabla\sigma_2 + a_3\nabla\sigma_3 = 0$ at p. $\}$.

$\Rightarrow$ S is an analytic subvariety of $M \times \mathbb{P}^2$.
$\Rightarrow$ By the proper mapping property, $\pi_2 : M \times \mathbb{P}^2 \longrightarrow \mathbb{P}^2$
$\pi_2(S)$ is an analytic subvariety of $\mathbb{P}^2$.
But since $\exists$ $\pi_2(S) \not\ni [(b_1, b_2, b_3)]$ by the argument above, $\pi_2(S)$ is a proper subvariety of $\mathbb{P}^2$. $\Rightarrow$
The generic element of W is nonsingular on $M - (B_1 \cap B_2 \cap B_3)$.

Here ⊛ follows from the fact that $M - B$ has a countable basis, since M has a countable basis. See P191 Munkres, Topology a first course.

This is another wrong direction !!!

Here is a correct proof.

Let $V = \{\sigma_3 = 0\}$.
$\Rightarrow$ By the argument on P137 ~ P138, we can show that for $^\forall (a_1, a_2) \in U \subset \mathbb{C}^2$, U open dense subset

$a_1\sigma_1 + a_2\sigma_2 = 0$ has nonzero gradient on $V - \{\sigma_1 = 0\} \cap \{\sigma_2 = 0\}$. In other words,
$S = \{ \sigma_3 = 0,\ a_1\sigma_1 + a_2\sigma_2 = 0,\ a_1\nabla\sigma_1 + a_2\nabla\sigma_2 = 0 \} \subset M \times \mathbb{P}^2$
$\Rightarrow$ $\pi_2(S)$ is a proper algebraic variety of $\mathbb{P}^2$.