$$B_n = \int_{\|x\|=\epsilon} \eta\sigma \times \frac{n-2}{C_n}$$

$$\Rightarrow \lim_{\epsilon\to 0} B_\epsilon = \frac{n-2}{C_n}\eta(0) \quad \text{by } 371.$$

Thus if we start with $\dfrac{1}{\|x-y\|^{n-2}} \dfrac{1}{n-2}$,

we get the precise formula.

"Comment"

$$G(x,y) \text{ should be } \begin{cases} \dfrac{1}{2-n}\dfrac{1}{\|x-y\|^{n-2}}, & n\geq 3 \\ \log\|x-y\|, & n=2. \end{cases}$$

and

$$\rho(x) = -C_n \int_{y\in\mathbb{R}^n} \frac{\eta(y)\,dy}{\|x-y\|^{n-2}} \cdot \frac{1}{2-n}$$

$\Rightarrow$ $\Delta\rho = \eta$. Everything follows exactly."

by integration by parts and def. of deriv. of distribution

$$\Delta T = T_\eta = T_{\Delta\psi} = \Delta T_\psi \Rightarrow \Delta(T - T_\psi) = 0$$

$\Rightarrow$ By the lemma P376, $\exists\ \varphi \in C^\infty(\mathbb{R}^n)$ s.t

$T - T_\psi = T_\varphi \Rightarrow T = T_\psi + T_\varphi = T_{\psi+\varphi}$. $\lrcorner\!\lrcorner$

Regularity also works locally: