$\eta \in A^1(End(E,E))$. $\quad \eta = \eta_{\alpha\beta}\, e_\alpha^* \otimes e_\beta$ ← one form.

$\sigma = \sigma_\alpha e_\alpha$.

$\Rightarrow (D_t\eta)(\sigma)$

$= D_t(\eta(\sigma)) + \eta(D_t(\sigma))$

$D_t(\eta(\sigma)) = (D_t\eta)(\sigma) + (-1)^{\deg\eta}\eta(D_t(\sigma))$

Consider $A^k$ ← matrix forms, $k$ ← vector forms.

① $D_t(\eta(\sigma))$

$= D_t(\eta_{\alpha\omega}\sigma_\alpha e_\omega)$

$= d(\eta_{\alpha\omega}\sigma_\alpha)\otimes e_\omega - \eta_{\alpha\omega}\sigma_\alpha \wedge (D_t e_\omega)$

$= d(\eta_{\alpha\omega}\sigma_\alpha)\otimes e_\omega - \eta_{\alpha\omega}\sigma_\alpha \wedge (\tilde\theta + t\eta)_{\omega\beta} \overset{\otimes}{\vee} e_\beta$

$= d(\eta_{\alpha\omega}\sigma_\alpha)\otimes e_\omega - \eta_{\alpha\omega}\wedge(\tilde\theta + t\eta)_{\omega\beta}\, \sigma_\alpha \overset{\otimes}{\vee} e_\beta$

$= \{ d(\eta_{\alpha\omega}\sigma_\alpha) - (\eta\wedge\tilde\theta + t\,\eta\wedge\eta)_{\alpha\omega}\sigma_\alpha \} \overset{\otimes}{\wedge} e_\omega$

$= d(\eta^{\sigma_\alpha}_{\alpha\omega}) - (\eta\wedge\tilde\theta + t\,\eta\wedge\eta)_{\alpha\omega}\ \sigma_\alpha \otimes e_\omega$

② $\eta(D_t(\sigma))$

$= \eta\{(d\sigma_\alpha + (\tilde\theta + t\eta)_{\beta\alpha}) \overset{\sigma_\beta}{\vee} \otimes e_\alpha\}$

$= \eta_{\alpha\omega}\{d\sigma_\alpha + (\tilde\theta + t\eta)_{\beta\alpha}\overset{\sigma_\beta}{\vee}\}\otimes e_\omega$

$= \{\eta_{\alpha\omega}\, d\sigma_\alpha + \eta_{\alpha\omega}\wedge(\tilde\theta + t\eta)_{\beta\alpha}\overset{\sigma_\beta}{\vee}\} \otimes e_\omega$

$= \{\eta_{\alpha\omega}\, d\sigma_\alpha - (\tilde\theta + t\eta)_{\beta\alpha}\wedge\eta_{\alpha\omega}\overset{\sigma_\beta}{\wedge}\} \otimes e_\omega$

$= \{\eta_{\alpha\omega}\, d\sigma_\alpha - ((\tilde\theta + t\eta)\wedge\eta)_{\beta\omega}\overset{\sigma_\beta}{\vee}\} \otimes e_\omega$

$= \{\eta_{\alpha\omega}\, d\sigma_\alpha - (\tilde\theta\wedge\eta + t\,\eta\wedge\eta)_{\beta\omega}\overset{\sigma_\beta}{\vee}\} \otimes e_\omega$

$= \{\eta_{\alpha\omega}\, d\sigma_\alpha - (\tilde\theta\wedge\eta + t\,\eta\wedge\eta)_{\beta\omega}\sigma_\beta\} \otimes e_\omega.$