2. $E \otimes F$, given by transition functions

$$\tilde{j}_{\alpha\beta} = g_{\alpha\beta}(x) \otimes h_{\alpha\beta}(x) \in GL(\mathbb{C}^k \otimes \mathbb{C}^l).$$

3. $\Lambda^r E$, given by transition functions

$$\tilde{j}_{\alpha\beta}(x) = \Lambda^r g_{\alpha\beta}(x) \in GL(\Lambda^r \mathbb{C}^k)$$

In particular, $\Lambda^k E$ is a line bundle given by

$$\tilde{j}_{\alpha\beta}(x) = \det g_{\alpha\beta}(x) \in GL(1, \mathbb{C}) = \mathbb{C}^*.$$

called the determinant bundle of E.

Point: $\quad L(u) \wedge L(v) = (\det L)(u \wedge v)$, $\quad u, v \in \mathbb{C}^2$,

$u = A(e_1) \quad v = A(e_2)$

$\Rightarrow \quad u \wedge v = A(e_1) \wedge A(e_2) = \det A (e_1 \wedge e_2)$

$L(u) \wedge L(v) = L \circ A(e_1) \wedge L \circ A(e_2) = \det L \det A \; e_1 \wedge e_2$

Def: A subbundle $F \subset E$ of a bundle E is a collection $\{F_x \subset E_x\}_{x \in M}$ of subspaces of the fibers E_x of E s.t $F = \bigcup F_x \subset E$ is a submanifold of E; F is clearly a vector bundle itself.

The condition that $F \subset E$ is a submanifold is equivalent to saying that for every $x \in M$, $\exists$ a nbd U of x in M and a trivialization

$$\varphi_U : E_U \longrightarrow U \times \mathbb{C}^k \cong \mathbb{C}^n \times \mathbb{C}^k$$

s.t

$$\varphi_U|_{F_U} : F_U \longrightarrow U \times \mathbb{C}^l \subset U \times \mathbb{C}^k.$$

$$\varphi_U(F_U) \subset U \times \mathbb{C}^k \cong \mathbb{C}^n \times \mathbb{C}^k \qquad U \overset{\varphi}{\cong} \mathbb{C}^n$$

$\cong \mathbb{C}^m \times \mathbb{C}^l$. but since for all $(v, 0)$. $U \times 0 \subset \varphi_U(F_U)$.