$g^{-1}(\alpha_0\phi_0 + W)$ is open. $\Rightarrow$ This means that the scalar multiplication is continuous. //

Note: From now on, the symbol $\mathcal{D}(\Omega)$ will denote the topological vector space $(\mathcal{D}(\Omega), \tau)$ that has just been described. All topological concepts related to $\mathcal{D}(\Omega)$ will refer to this topology τ.

6.5 Theorem

(a) A convex balanced subset V of $\mathcal{D}(\Omega)$ is open $\Leftrightarrow V \in \beta$.

(b) The topology τ_K of any $\mathcal{D}_K \subset \mathcal{D}(\Omega)$ coincides with the subspace topology that $\mathcal{D}_K$ inherits from $\mathcal{D}(\Omega)$.

(c) If E is a bounded subset of $\mathcal{D}(\Omega)$, then $E \subset \mathcal{D}_K$ for some $K \subset \Omega$, and there are numbers $M_N < \infty$ such that every $\phi \in E$ satisfies the inequalities

$$\|\phi\|_N \le M_N \qquad (N = 0, 1, 2, \cdots).$$

(d) $\mathcal{D}(\Omega)$ has the Heine-Borel property.

(e) If $\{\phi_i\}$ is a Cauchy sequence in $\mathcal{D}(\Omega)$, then $\{\phi_i\} \subset \mathcal{D}_K$ for some compact $K \subset \Omega$, and

$$\lim_{i,j\to\infty} \|\phi_i - \phi_j\|_N = 0 \qquad (N = 0, 1, 2 \cdots).$$

(f) If $\phi_i \to 0$ in the topology of $\mathcal{D}(\Omega)$, then there is a compact $K \subset \Omega$ which contains the support of every ϕ_i, and $D^\alpha \phi_i \longrightarrow 0$ uniformly, as $i \longrightarrow \infty$, for every multiindex α.

(g) In $\mathcal{D}(\Omega)$, every Cauchy sequence converges.

Remark. In view of (b), the necessary conditions expressed by