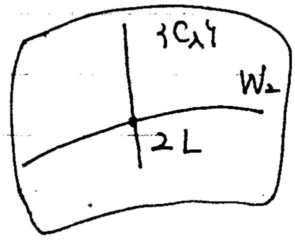
sponding to v if and only if the line $\{C_\lambda\}$ has intersection multiplicity 3 or more with V_c at $2L$; it will thus suffice to show that for any point $2L \in W_2$ and any line $\{C_\lambda\}$ through $2L$ but not tangent to W_2 at $2L$, there exists a conic C such that

$$\mathrm{mult}_{2L}(V_c, \{C_\lambda\}) = 2.$$

⌈

$$\begin{tikzcd} \tilde{W} \arrow[r, "\pi"] & W \\ \cup & \cup \\ E \arrow[r] & W_2 \end{tikzcd}$$



W

$\tilde{V}_c = \pi^* V_c - \mathrm{mult}_{W_2}(V_c) E$ by P605

Since $\mathrm{mult}_{W_2}(V_c) = 2$ by the result on P750,

$\tilde{V}_c = \pi^* V_c - 2E$.

$\Rightarrow \quad \tilde{V}_c = \pi^* V_c - 2E \ni [v] \Leftrightarrow (\pi^* V_c)_{2L} - 2E_{2L} \ni [v]$

This implies that V_c intersects with $\{C_\lambda\}$ more than 2 times at $2L$, i.e., $\mathrm{mult}_{2L}(\{C_\lambda\}, V_c) \geq 3$, since $(\pi^* V_c)_{2L} = m E_{2L}$, $m \geq 3$.

Conversely, if $\{C_\lambda\}$ has intersection multiplicity 3 or more with V_c at $2L$, then $\tilde{V}_c = \pi^* V_c - 2E \ni [v]$.