$\Rightarrow \quad \pi' \circ i_{\mathcal{O}(2)} \circ \varphi : \mathbb{P}^2 \longrightarrow \mathbb{P}^5 \longrightarrow \mathbb{P}^3$

$$[(X_0, X_1, X_2)] \longmapsto [(X_1^2 X_2^2, \ X_0^2 X_1 X_2, \ X_0 X_1^2 X_2, \ X_0 X_1 X_2^2)].$$

$$\| \quad [(X_1 X_2, \ X_0^2, \ X_0 X_1, \ X_0 X_2)]$$

I think

"$\Rightarrow \pi' \circ i_{\mathcal{O}(2)} \circ \varphi$ defines a unique holomorphic map from $\tilde{\mathbb{P}}^2 \xrightarrow{\sigma} \mathbb{P}^2$ to $\mathbb{P}^3$."

I think : This is what the authors meant.
I don't understand rational maps yet.
Let's stop here.

$$\mathbb{P}^5 \longrightarrow \mathbb{P}^3$$

$$[(X_0, X_1, X_2, X_3, X_4, X_5)] \longmapsto [(X_0, X_1, X_2, a_0 X_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_4 + a_5 X_5)]$$

where $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \end{pmatrix}$ has rank 4.

$$\tau(X_0, \cdots X_5) = a_0 X_1^2 X_2^2 + a_1 X_0^2 X_1^2 + \cdots + a_5 X_0^2 X_1 X_2.$$

For rational maps, refer to P495 ~ P497.
Specially, see P495 example 2. & P510

Extensions of Modules

Ext^1 and Extensions – Local Case.

We consider again the local ring $\mathcal{O} = \mathbb{C}\{z_1, \cdots z_n\}$ and finitely generated modules over $\mathcal{O}$.