⌈ Misunderstanding so far.

$$(\nabla^2 f)(v_1, v_2) = \nabla_{v_1}(\nabla_{v_2} f)$$

$$(\nabla^2 f)(v_1, v_2) \neq \nabla_{v_1}(\nabla_{v_2} f) - \nabla_{v_2}(\nabla_{v_1} f) - \nabla_{[v_1, v_2]} f.$$

⌋

Thus, we have a norm equivalent to the usual Sobolev norm on sections compactly supported in a nbd of a point.

⌋

⌈ Suppose M can covered by two open charts,

$$M = U_1 \cup U_2.$$

and suppose $E|U_1 \cong U_1 \times \mathbb{R}^n$. $E|U_2 \cong U_2 \times \mathbb{R}^n$.

$\Rightarrow \exists$ V_1, V_2 open and $\overline{V_i} \subset U_i$ for $i=1,2$
s.t $V_1 \cup V_2 = M$.

$\Rightarrow \exists$ a partition of unity $\{\varphi_1, \varphi_2\}$. s.t $\sum_{i=1}^{2} \varphi_i = 1$
$\operatorname{supp} \varphi_i \subset U_i$ and $\varphi_i > 0$ on $\overline{V_i}$ for $i=1,2$.

(Sobolev. lemma)

Given a section $\sigma \in \mathcal{H}_{[n/2]+1+s}(M, E)$,

$$\sigma = \sum \varphi_i \sigma = \varphi_1 \sigma + \varphi_2 \sigma.$$

First, consider $\varphi_1 \sigma$. $\Rightarrow$ $\varphi_1 \sigma \in \mathcal{H}_{[n/2]+1+s}(M, E)$.

$\varphi_1 \sigma \in \Gamma(E|U_1)$ $\Longrightarrow$ We can consider it, as a section on U_1.

$\Rightarrow$ Also, we can think $\varphi_1 \sigma$ as a map from ~~the~~ a ~~two~~ torus to $\mathbb{R}^n$. $\Rightarrow$ By local Sobolev lemma, $\varphi_1 \sigma \in C^s(U, E)$.