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$$\Rightarrow \sum_{p\in Z} \langle \psi_p, e_p \rangle = \sum_{p \in Z} \mathrm{Res}_p\left(\frac{\psi}{s}\right)$$

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Corollary (Cayley - Bacharach for Vector Bundles).
If Z consists of distinct simple points, then each $D \in |K \otimes \det E|$ that passes through all but one point of Z necessarily contains that remaining point.

$\ulcorner$ Since this is a corollary of Residue Theorem for Vector Bundles, $\exists$ a section $s \in H^0(M, \mathcal{O}(E))$ s.t $(s=0) = Z$. Let $\psi \in H^0(M, \mathcal{O}(K \otimes \det E))$ s.t $(\psi = 0) = D$. Let p be the remaining point. Let $q \in Z$, $q \neq p$.

$\Rightarrow$

$$\mathrm{Res}_q\left(\frac{\psi}{s}\right) = 0.$$

But, by the Residue Theorem for Vector Bundles,

$$\sum_{p \neq q \in Z} \mathrm{Res}_q\left(\frac{\psi}{s}\right) + \mathrm{Res}_p\left(\frac{\psi}{s}\right) = 0.$$

$\Rightarrow$ $\mathrm{Res}_p\left(\frac{\psi}{s}\right) = 0$ $\Rightarrow$ This implies that $\psi(p) = 0$, otherwise $\mathrm{Res}_p\left(\frac{\psi}{s}\right) \neq 0$. $\Rightarrow$ D passes p.

$\lrcorner$

The result at the end of the preceding section may be rephrased as:

Corollary. On an algebraic surface S given a set Z of isolated points and holomorphic line bundle L, there