Since $\phi_m \notin mW$, no multiple of W contains E. This shows that E is not bounded.

⸢ If $\phi_m \in mW$, $|\phi_m(x_m)| < m^{-1}|\phi_m(x_m)|$ which is impossible.
$\Rightarrow$ $\phi_m \notin mW$. $mW \not\supset E$ since $E \ni \phi_m$. ⸥

It follows that every bounded subset E of $\mathcal{D}(\Omega)$ lies in some $\mathcal{D}_K$. By (b), E is then a bounded subset of $\mathcal{D}_K$. Consequently

(5) $$\sup \{\|\phi\|_N : \phi \in E\} < \infty \qquad (N = 1, 2, \cdots).$$

This completes the proof of (c).

⸢ Since E is bounded in $\mathcal{D}_K$, $\exists$ t s.t $tV_N \supset E$.
$\Rightarrow \frac{1}{t}\|\phi\|_N < \frac{1}{N}$ for all $\phi \in E$
$\Rightarrow \|\phi\|_N < \frac{t}{N}$. see Th. 1.37. ⸥

Statement (d) follows from (c), since $\mathcal{D}_K$ has the Heine-Borel property.

⸢ E closed & bounded $\Rightarrow$ $\exists$ $\mathcal{D}_K$ s.t $E \subset \mathcal{D}_K$.
$\Rightarrow$ E is closed & bounded in $\mathcal{D}_K$ $\Rightarrow$ Since $\mathcal{D}_K$ has the Heine-Borel property by P 33 ~ P34,
($\because$ $\mathcal{D}_K$ is a closed subset of $C^\infty(\Omega)$ and C^∞ has the Heine-Borel property by P33 ~ P34), $\mathcal{D}(\Omega)$ has the Heine-Borel property. ⸥

Since Cauchy sequences are bounded (Section 1.29), (c) implies that every Cauchy sequence $\{\phi_i\}$ in $\mathcal{D}(\Omega)$ lies in some $\mathcal{D}_K$. ⸢ p 22 ⸥