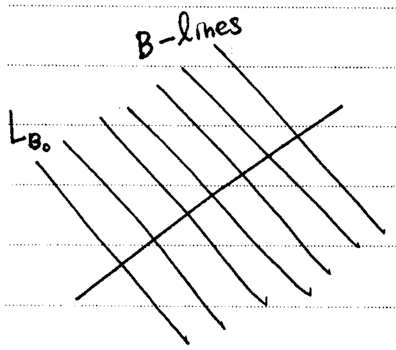

$L_0 \cong \mathbb{P}^1 \quad \Rightarrow \quad S \supset \mathbb{P}^1 \times \mathbb{P}^1$

$\Rightarrow S$ is irreducible $\Rightarrow S \cong \mathbb{P}^1 \times \mathbb{P}^1$.

We can obtain another description of a quadric surface S by projecting from a point p of S onto a plane H in $\mathbb{P}^3$. Of course, the projection map $\pi_p : S - \{p\} \longrightarrow H$ is not defined at, and does not extend over, the point p.

Γ $\langle p, H \rangle = \mathbb{P}^3$

$\Rightarrow \exists$ an isomorphism $\phi : \mathbb{P}^3 \longrightarrow \mathbb{P}^3$ s.t

$$p \longmapsto [(0,0,0,1)]$$
$$H \longmapsto \{[(*,*,*,0)]\} = \mathbb{P}^2$$

$\Rightarrow$

$$\begin{tikzcd} \phi(S) - [(0,0,0,1)] \arrow[d, "\pi_{p_0}"'] \\ \mathbb{P}^2 \end{tikzcd}$$

is well-defd, but not defined at, and does not extend over, the point p_0.

$\Rightarrow$

$$\begin{tikzcd} S - p \arrow[r, leftrightarrow, "\phi"] \arrow[d, "\pi_p"] & \phi(S) - \underbrace{[(0,0,0,1)]}_{p_0} \arrow[d, "\pi_p"] \\ H \arrow[r, leftrightarrow] & \mathbb{P}^2 \end{tikzcd}$$

$\Rightarrow \pi_p$ is not defined at, and does not extend over, the point p.