Applying this in particular to the canonical curve, we have

Noether's Theorem. For any curve nonhyperelliptic C, the map
$$\mathrm{Sym}^{\ell} H^0(C, \mathcal{O}(K)) \longrightarrow H^0(C, \mathcal{O}(\ell K))$$
is surjective for all ℓ.

┌ Since $H^0(\mathbb{P}^{g-1}, \mathcal{O}(H)) \longrightarrow H^0(C, \mathcal{O}(H))$ is surjective and injective, $H^0(\mathbb{P}^{g-1}, \mathcal{O}(H))$ can be identified with $H^0(C, \mathcal{O}(H)) = H^0(C, \mathcal{O}(K))$. ┘

Castelnuovo's inequality can be inverted in two ways to give an upper bound on n in terms of d and g and a lower bound on d in terms of n and g. Without going through the manipulation, we have

$$n \leq \frac{2(\ell(d-1) - g)}{\ell(\ell+1)}, \qquad \ell = \left[\frac{2(g-1)}{d} + 1\right],$$

$$d \geq \frac{(j+1)}{2}(n-1) + \frac{g}{j} + 1, \qquad j(j-1) < \frac{2g}{n-1} \leq j(j+1).$$

┌ Foget the computation. I hate this because I can not get the right answer without struggling for several hours. We will continue this some time later. ┘

93. 7. 1

$$g \leq \frac{m(m-1)}{2}(n-1) + m(d - m(n-1) - 1)$$