$$\Rightarrow \quad P^3(A) = \sum_{\# I = 3} \det(A_{I,I}).$$

Here, $\epsilon(\sigma) = \text{sign}\{\sigma(3), \sigma(4), \sigma(5)\}$, since $\sigma(1) = 1$, $\sigma(2) = 2$.

$$\Lambda^3 A : \Lambda^3 \mathbb{R}^5 \longrightarrow \Lambda^3 \mathbb{R}^5$$
$$e_i \wedge e_j \wedge e_k \longmapsto Ae_i \wedge Ae_j \wedge Ae_k.$$

$$\Lambda^3 A(e_1 \wedge e_2 \wedge e_3) = Ae_1 \wedge Ae_2 \wedge Ae_3$$
$$= (a_{11} e_1 + a_{21} e_2 + a_{31} e_3) \wedge (a_{12} e_1 + a_{22} e_2 + a_{32} e_3)$$
$$\wedge (a_{13} e_1 + a_{23} e_2 + a_{33} e_2)$$
$$= (a_{11} a_{22} a_{33} - a_{21} a_{12} a_{33} + a_{21} a_{32} a_{13} - a_{11} a_{32} a_{23}$$
$$\pm a_{31} a_{12} a_{23} \pm a_{31} a_{22} a_{13}) \, e_1 \wedge e_2 \wedge e_3 + \cdots$$
$$= \det(A_{\{1,2,3\},\{1,2,3\}}) \, e_1 \wedge e_2 \wedge e_3 + \cdots$$
$$\Rightarrow \quad \Lambda^3 A(e_i \wedge e_j \wedge e_k) = \cdots + \det(A_{\{i,j,k\},\{i,j,k\}}) \, e_i \wedge e_j \wedge e_k + \cdots$$

$$\Rightarrow \quad \text{trace}(\Lambda^3 A) = \sum_{\# I = 3} \det(A_{I,I})$$

Practice : $Ae_1 \wedge Ae_2 \wedge Ae_3$

$$= a_{11} a_{22} a_{33} \, e_1 \wedge e_2 \wedge e_3 + a_{\sigma(1)1} e_{\sigma(1)} \wedge a_{\sigma(2)2} e_{\sigma(2)} \wedge a_{\sigma(3)3} e_{\sigma(3)}$$
$$+ \cdots$$
$$= \sum \epsilon(\sigma) \, a_{\sigma(1)1} a_{\sigma(2)2} a_{\sigma(3)3} \, e_1 \wedge e_2 \wedge e_3 + \boxed{}$$
$$= \det(A_{\{1,2,3\},\{1,2,3\}}) \, e_1 \wedge e_2 \wedge e_3 + \boxed{}$$

끝

The polynomials P^i are called the elementary invariant polynomials. In fact, any holomorphic function f on M_n invariant under conjugation is expressible as a power series in the P^i: If we set