The bundle $[H]$ associated to a hyperplane in $\mathbb{P}^n$ is called the hyperplane bundle: its inverse, $J = [H]^* = [-H]$, is called the universal bundle on $\mathbb{P}^n$.

$\ulcorner$

$$\begin{array}{ccccccc} H^1(\mathbb{P}^n, \mathcal{O}) & \longrightarrow & H^1(\mathbb{P}^n, \mathcal{O}^*) & \xrightarrow[\cong]{c_1} & H^2(\mathbb{P}^n, \mathbb{Z}) & \longrightarrow & H^2(\mathbb{P}^n, \mathcal{O}) \\ \| & & \| & & & & \| \\ 0 & & \mathrm{Pic}(\mathbb{P}^n). & & & & 0 \end{array}$$

$$\begin{array}{ccc} \mathrm{Div}(\mathbb{P}^n) & \longrightarrow & H^1(\mathbb{P}^n, \mathcal{O}^*) \\ \| & & \| \ \mathrm{Pic}(\mathbb{P}^n) \\ H^0(\mathbb{P}^n, \mathcal{M}^*/\mathcal{O}^*) & & \\ \cup & & \\ D & \longmapsto & [D] \end{array}$$

$$0 \longrightarrow \mathcal{O}^* \longrightarrow \mathcal{M}^* \longrightarrow \mathcal{M}^*/\mathcal{O}^* \longrightarrow 0$$

$$\begin{array}{ccccccc} H^0(\mathbb{P}^n, \mathcal{O}^*) & \longrightarrow & H^0(\mathbb{P}^n, \mathcal{M}^*) & \longrightarrow & H^0(\mathbb{P}^n, \mathcal{M}^*/\mathcal{O}^*) & \longrightarrow & H^1(\mathbb{P}^n, \mathcal{O}^*) \\ & & & & \| & & \downarrow \\ & & & & \mathrm{Div}(\mathbb{P}^n) & & H^1(\mathbb{P}^n, \mathcal{M}^*) \end{array}$$

$$\begin{array}{ccc} H^1(\mathbb{P}^n, \mathcal{O}^*) & \xrightarrow{c_1} & H^2(\mathbb{P}^n, \mathbb{Z}) \\ \cup & & \cup \\ L = [D] & \longmapsto & c_1(L) \\ & & \updownarrow \quad \text{Poincare-dual} \\ & & (D) \in H_{2n-2}(\mathbb{P}^n, \mathbb{Z}) \end{array}$$

$\Rightarrow$ Since $H_{2n-2}(\mathbb{P}^n, \mathbb{Z})$ is generated by (H), $(D) = \alpha\,(H)$ for $\alpha \in \mathbb{Z}$. $\lrcorner$

We can give a direct geometric construction of the universal bundle J on $\mathbb{P}^n$ as follows: Let $\mathbb{P}^n \times \mathbb{C}^{n+1}$ be the trivial bundle of rank $(n+1)$ on $\mathbb{P}^n$, with all fibers identified with $\mathbb{C}^{n+1}$. Then the universal bundle is just the subbundle J of $\mathbb{P}^n \times \mathbb{C}^{n+1}$ whose fiber at each point $Z \in \mathbb{P}^n$ is the line in $\mathbb{C}^{n+1}$