

$$a \in K^{p,q} \text{ s.t. } \delta a = 0 \quad d(a + \delta K^{p,q-1}) \subset \delta K^{p+1,q-1} \\ \Rightarrow da \in \delta K^{p+1,q-1}.$$

$$'E_1^{p,q} = \frac{\ker \delta}{\text{im } \delta} \xrightarrow{d} 'E_1^{p+1,q}$$

$$\Rightarrow \text{im } d = \frac{d(\ker \delta) + \delta(K^{p,q-1})}{\delta(K^{p,q-1})}$$

$$\ker d = \frac{\{a \in \ker \delta : da \in \delta(K^{p+1,q-1})\} + \delta(K^{p,q-1})}{\delta(K^{p,q-1})}$$

$$\Rightarrow \text{But since } \delta^2 = 0 \text{ and } d\delta + \delta d = 0,$$

$$\delta(K^{p,q-1}) \subset \{a \in \ker \delta : da \in \delta(K^{p+1,q-1})\}.$$

$$\Rightarrow \frac{\ker d}{\text{im } d} = \frac{\{a \in \ker \delta : da \in \delta(K^{p+1,q-1})\}}{d(\ker \delta) + \delta(K^{p,q-1})} = 'E_2^{p,q}.$$

Thus  $\delta K^{p,q-1} + (d K^{p-1,q}) \cap \ker \delta$  must be corrected into  $\delta K^{p,q-1} + d \ker \delta^{p-1,q}$ , for

by the relation  $\delta d + d\delta = 0$ ,

$$\delta K^{p,q-1} + d \ker \delta^{p-1,q} \subset \delta K^{p,q-1} + (d K^{p-1,q}) \cap \ker \delta$$

but we can not conclude

$$\delta K^{p,q-1} + (d K^{p-1,q}) \cap \ker \delta \subset \delta K^{p,q-1} + d \ker \delta^{p-1,q}. \quad \sqcup$$

Examples.

If  $M$  is a complex manifold and

$$\begin{cases} K^{p,q} = A^{p,q}(M), \\ d = \partial \text{ and } \delta = \bar{\partial}, \end{cases}$$