Thus $h^0(\mathcal{J}_{P_0}(4)) = \frac{(n+1)(n+2)}{2} - 12 + h^1(\mathcal{J}_{P_0}(4))$

$= 15 - 12 + h^1(\mathcal{J}_{P_0}(4)) = 3 + h^1(\mathcal{J}_{P_0}(4))$

$\Rightarrow \quad |\mathcal{J}_{P_0}(4)| = 2$ or 3. $\qquad = 3$ or 4.

$\lrcorner$

To see when this happens we consider the triangle Δ with vertices p_1, p_2, p_3. This is a plane cubic with double points at the points p_i, and we shall say that our configuration is in special position in case the defining equation of Δ is in the ideal $\mathcal{J}_{P_i}$ at each vertex. We now prove that: The configuration is in special position if and only if, the points of Γ are distinct and collinear.

Proof. If the points of Γ are distinct and collinear, then some member of E of the pencil $|C + \lambda C'|$ will have this line L_0 as tangent, from which it follows that

$$E = \Delta + L_0,$$

and so $\Delta \in |\mathcal{J}_{P_0}(3)|$ and the configuration is in special position.

$\ulcorner$ C & C' are represented by f & g respectively. Since the points of Γ are distinct, C meets with C' transversely.