hyperplane containing V_3 would be tangent to G.

⌈ The arguments above prove that $\{L_i\}$ & $\{L_i'\}$ span a unique 4-plane.

First, assume that $\{L_i\}$ & $\{L_i'\}$ span a 4-plane. $\Rightarrow$ $\{L_i\}$ & $\{L_i'\}$ lie in a 3-plane $\Rightarrow$ $Q = G \cap H \cap H' = G \cap V_3$ contain $\{L_i\}$ & $\{L_i'\}$ s.t

$$L_i \cap L_j' \neq \phi \quad i \neq j$$
$$L_i \cap L_i' = \phi .$$

(i) Q smooth.

$\Rightarrow$ Since G is a quadric hypersurface in $\mathbb{P}^5$, $G \cap V_3$ is a smooth quadric in $V_3 = \mathbb{P}^3$.

$\Rightarrow$ By the results on P478 ~ P479, Q has two families A, B of lines in Q.

If $L_1 \in A$, then $L_2', L_3', L_4' \in B$, and $L_1' \in A$.

What about L_2 ? $\Rightarrow$ $L_2 \cap L_3' \neq \phi \Rightarrow L_2 \in A$.

But $L_1' \cap L_2 = \phi \Rightarrow L_2 \in B \Rightarrow$ Contradiction.

$\Rightarrow$ Q can not be smooth.

(ii) Q rank 3.

$\Rightarrow$ By changing coordinates, $Q = X_0^2 + X_1^2 + X_2^2$

$\Rightarrow Q$ is the cone through $[0, 0, 0, 1]$ over $X_0^2 + X_1^2 + X_2^2$ in $\mathbb{P}^2$. Any line in $\mathbb{P}^2 \cap Q$ is a form of $a_0 X_0 + a_1 X_1 + a_2 X_2 = 0$. $\Rightarrow$ Clearly, $[0, 0, 0, 1]$ lies on $a_0 X_0 + a_1 X_1 + a_2 X_2 = 0$. Obviously, any line in Q (outside $\mathbb{P}^2$) passes $[0, 0, 0, 1]$.