Date

ed special; a special divisor whose associated linear system is larger than that of the generic divisor of its degree — i.e., such that $h^0(K-D) > g-d$ — is called irregular. A linear system is called special or irregular if its individual divisors are.

$\ulcorner$ D special divisor of degree d.
G generic divisor of deg d.

① $d \le g$

$h^0(D) > 1 = h^0(G)$

$\Rightarrow d-g+1+h^0(K-D) > 1 \Rightarrow h^0(K-D) > g-d.$

② $d > g.$

$h^0(D) > h^0(G) = d-g+1$

$\Rightarrow d-g+1+h^0(K-D) > d-g+1 \Rightarrow h^0(K-D) > -1$

$\Rightarrow$ If $h^0(K-D) > g-d$, the associated linear system of D is larger than that of the generic divisor of deg(D).

It should be mentioned at this point that the Riemann-Roch formula can be given a sheaf-theoretic proof. In general, if $E \longrightarrow M$ is a holomorphic vector bundle on a compact complex manifold M, we define the holomorphic Euler characteristic of E to be

$$\chi(E) = \sum (-1)^p h^p(M, \mathcal{O}(E));$$

we usually write $\chi(\mathcal{O}_M)$ for the holomorphic Euler chara