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By cases, then:

1. If no integer $c_i - i$ lies in the interval $[-d-1, a_d - d-1]$, then

$$c_i - i \in [a_i - i, a_{i-1} - i],$$

and σ_c can appear only in the last term of the sum (*).

⌈ The last term of (*) is $(-1)^d \sigma_{a_1, a_2, \cdots a_{d-1}} \cdot \sigma_{a_d}$.
If $j < d$, and suppose such σ_c appear in the j-th term of the sum (*),

$$\Rightarrow \sigma_{a_1, \cdots a_{j-1}, a_{j+1}-1, \cdots a_d-1} \cdot \sigma_{a_j + d-j} = \sigma_{a_j+d-j} \cdot \sigma_{a_1, a_2 \cdots a_{j-1}, a_{j+1}-1, \cdots a_d-1}$$

$$= \sum_{\substack{a_i \le c_i \le a_{i-1} \quad i \le j-1 \\ \sum c_i = a + \sum_{b_i = a_i} b_i}} \sigma_c \qquad \begin{matrix} a_{i+1}-1 \le c_i \le a_i - 1 \quad i > j \\ a_{j+1} - 1 \le c_j \le a_{j-1}. \end{matrix}$$

$$\Rightarrow c_j - j \in [a_{j+1} - (j+1), a_{j-1} - j]$$

$$\Rightarrow c_j - j \in [a_{j+1} - (j+1), a_j - (j+1)]$$

$$\text{or } [a_j - j, a_{j-1} - j] \text{ so}$$

$$\Rightarrow c_j - j \notin [a_{j+1} - (j+1), a_j - (j+1)], \text{ and so } c_j - j \in [a_j - j, a_{j-1} - j]$$

$$= [-d-1, a_d - (d+1)]$$

By the assumption, $j = d$. $\Rightarrow$ Contradiction.
$\Rightarrow \sigma_c$ can appear only in the last term. ⌋

But since $c_i \ge a_i$ and $\sum c_i = \sum a_i$, it follows that $c = a$.

⌈ From $c_i - i \in [a_i - i, a_{i-1} - i]$ for all i,