

be a vertex of some quadric in N_p .

Let $N_p = \langle F_1, F_2, F_3 \rangle$, F_i smooth since N_p is a generic net. Suppose $F = aF_1 + bF_2 + cF_3$ has p as a vertex.

\Rightarrow Since p is singular, at p

$$0 = \frac{\partial F}{\partial x_0} = a \frac{\partial F_1}{\partial x_0} + b \frac{\partial F_2}{\partial x_0} + c \frac{\partial F_3}{\partial x_0}$$

$$0 = \frac{\partial F}{\partial x_1} = a \frac{\partial F_1}{\partial x_1} + b \frac{\partial F_2}{\partial x_1} + c \frac{\partial F_3}{\partial x_1}$$

$$0 = \frac{\partial F}{\partial x_2} = a \frac{\partial F_1}{\partial x_2} + b \frac{\partial F_2}{\partial x_2} + c \frac{\partial F_3}{\partial x_2}$$

$$0 = \frac{\partial F}{\partial x_3} = a \frac{\partial F_1}{\partial x_3} + b \frac{\partial F_2}{\partial x_3} + c \frac{\partial F_3}{\partial x_3}$$

In a matrix form

$$(a, b, c) \begin{pmatrix} \frac{\partial F_1}{\partial x_0} & \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial x_3} \\ \frac{\partial F_2}{\partial x_0} & \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial x_3} \\ \frac{\partial F_3}{\partial x_0} & \frac{\partial F_3}{\partial x_1} & \frac{\partial F_3}{\partial x_2} & \frac{\partial F_3}{\partial x_3} \end{pmatrix} = 0 \dots (*)$$

Since p is a smooth point for F_1, F_2 & F_3 , and $T_p F_1 \cap T_p F_2 \cap T_p F_3 = \{p\}$ ($\because \langle F_1, F_2, F_3 \rangle = N_p$ is a generic net),

$$\begin{pmatrix} \frac{\partial F_1}{\partial x_0} & \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial x_3} \\ \frac{\partial F_2}{\partial x_0} & \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial x_3} \\ \frac{\partial F_3}{\partial x_0} & \frac{\partial F_3}{\partial x_1} & \frac{\partial F_3}{\partial x_2} & \frac{\partial F_3}{\partial x_3} \end{pmatrix} \text{ has rank } 3.$$