

$$\Rightarrow \left. \begin{array}{l} b_{11} x'_{11} + x'_{1n} b_n = 0 \\ x'_{22} b_2 + x'_{2n} b_n = 0 \\ \vdots \\ x'_{n-1,n-1} b_{n-1} + x'_{n-1,n} b_n = 0 \end{array} \right\} \textcircled{*} \Rightarrow \text{We have nonzero } b_i \text{'s satisfying } \textcircled{*}.$$

Here we may have $x'_{ii} = 0$, and $x'_{in} = 0$ by the Cayley - Bacharach property. Still we have nonzero b_i 's. \Rightarrow We proved that \exists bivectors $0 \neq \tau_p$ s.t

$$\sum_{p \in \mathbb{Z}} \langle \psi, \tau_p \rangle = 0 \quad \text{for all } \psi \in H^0(S, \Omega^2(L)),$$

$$\tau_p \in \mathcal{O}_{Z,p} \otimes \Omega_p^2(L^*).$$

$$\text{From } 0 \longrightarrow \mathcal{I}_Z \longrightarrow \mathcal{O} \longrightarrow \mathcal{O}_Z \longrightarrow 0$$

$$\text{Ext}^1(S; \mathcal{I}_Z, L^*) \longrightarrow \text{Ext}^1(S; \mathcal{O}_Z, L^*) \longrightarrow \text{Ext}^1(S; \mathcal{O}, L^*)$$



$$\text{Ext}^1(S; \mathcal{I}_Z, L^*)^* \longleftarrow$$

$$H^0(S, \mathcal{O}_Z \otimes \Omega^2(L)) \longleftarrow H^0(S, \Omega^2(L))$$

$$\underset{\parallel}{=} \bigoplus_{p \in \mathbb{Z}} \mathcal{O}_{Z,p} \otimes \Omega_p^2(L)$$

□

Finally, the Cayley - Bacharach property may be given a nice geometric interpretation in case the linear system $|K \otimes L|$ gives a base-point-free mapping