$\dim |\mathcal{J}_{P_0}(4)| = 2 + \dim H^0(\mathbb{P}^2; \mathcal{J}_P(H)). \geq 3.$

$\Rightarrow \dim H^0(\mathbb{P}^2; \mathcal{J}_P(H)) \geq 1$ $\Rightarrow$ This implies that $\exists$ a section l s.t $l = 0$ on P. $\Rightarrow$ P lie on l, which is a line in $\mathbb{P}^2$. $\Rightarrow$ P is a set of collinear 4 points.

Assume $l = \{[0, z_1, z_2] \in \mathbb{P}^2\}$.

Let f_1 be a homogeneous polynomial of deg 4 for C.
" f_2 " for C'.

$$\Rightarrow \quad f_1(z_0, z_1, z_2) = z_0\, g_1(z_0, z_1, z_2) + h_1(z_1, z_2)$$
$$f_2(z_0, z_1, z_2) = z_0\, g_2(z_0, z_1, z_2) + h_2(z_1, z_2)$$

where g_1, g_2 homogeneous polynomials of deg 3
h_1, h_2 " of deg 4

with $h_i \nmid z_0$.

$\Rightarrow \{f_1 = 0\} \cap \{f_2 = 0\} = P_0 + P$. and $\{f_1 = 0\} \cap \{f_2 = 0\} \cap l = P$ $\Rightarrow$ $\{h_1 = 0\} \cap \{h_2 = 0\} = P$.

$\Rightarrow$ Since $\deg h_1 = \deg h_2 = 4$ and $\# P = 4$, $h_1 = \alpha h_2$ for some conetant $\alpha \in \mathbb{C}$, ($\because$ h_1 & h_2 can be considered as a polynomial of one variable.

$$z_1^4 h_1(1, \tfrac{z_2}{z_1}) = h_1(z_1, z_2) \qquad h_2(z_1, z_2) = z_1^4 h_2(1, \tfrac{z_2}{z_1})$$

and $h_1(w)$ is divisible by $h_2(w)$. $w = z_2/z_1$.
if $h_1 \nmid z_1$.
If h_1 / z_1, then h_2 / z_1. etc.).

$$\Rightarrow \quad f_1 - \alpha f_2 = z_0(g_1 - g_2)$$

$\Rightarrow$ Since $f_1 = f_2$ on P_0, $z_0(g_1 - g_2) = 0$ on P_0