If $V \subset M$ complex submanifold, we define the normal bundle $N_{V/M}$ to V in M to be the quotient of the tangent bundle to M, restricted to V, by the subbundle,

$$T'(V) \hookrightarrow T'(M)/_V.$$

The conormal bundle $N^*_{V/M}$ of to V in M is the dual of the normal bundle.

Metrics, Connections, and Curvature.

$E \to M$ complex vector bundle.

Def: A hermitian metric on E = hermitian inner product on each fiber E_x of E, varying smoothly with $x \in M$ – i.e. s.t if $\zeta = \{\zeta_1, \cdots \zeta_k\}$ is a frame for E, then the functions

$$h_{i\bar{j}}(x) = (\zeta_i(x), \zeta_j(x)) \quad \text{are } C^\infty.$$

A frame ζ for E is called unitary if $\zeta_1(x), \cdots \zeta_k(x)$ is an orthonormal basis for E_x. for each x; unitrary frames always exist locally, since we can take any frame and apply the Gram–Schmidt process.
If E is a bundle with hermitian metric, $F \subset E$ a subbundle, then the subspace $\{F_x^\perp \subset E_x\}$ form a subbundle of E, C^∞ isomorphic to the quotient bundle E/F.
(See Milnor. Charateristic classes p28.)

A holomorphic vector bundle with a hermitian metric is called a hermitian vector bundle.