$$0 \longrightarrow V \otimes \Lambda^{p-1}\mathbb{C}^{n-1} \longrightarrow \Lambda^p \mathbb{C}^n \xrightarrow{\pi} \Lambda^p \mathbb{C}^{n-1} \longrightarrow 0$$

is exact, where $\pi : \mathbb{C}^n \longrightarrow \mathbb{C}^{n-1}$ is defined by

$\pi(e_i) = e_i$ if $i \leq n-1$

$\pi(e_n) = 0$ $\qquad \dim V = 1.$

Clearly $V \otimes \Lambda^{p-1}\mathbb{C}^{n-1} \xrightarrow{f \wedge id} \Lambda^p \mathbb{C}^n$ &

$v \otimes \omega \longmapsto f(v) \wedge \omega$

$$\pi : \Lambda^p \mathbb{C}^n \longrightarrow \Lambda^p \mathbb{C}^{n-1}$$
$$\omega_1 \wedge \cdots \wedge \omega_p \longmapsto \pi(\omega_1) \wedge \cdots \wedge \pi(\omega_p)$$

$\Rightarrow$ Obviously, $\pi : \Lambda^p \mathbb{C}^n \longrightarrow \Lambda^p \mathbb{C}^{n-1}$ is onto.

$v \otimes \omega_1 \wedge \cdots \wedge \omega_{p-1} \longmapsto f(v) \wedge \omega_1 \wedge \cdots \wedge \omega_{p-1} \longmapsto \pi f(v) \wedge \pi(\omega_1) \wedge \cdots$
$\wedge \pi(\omega_{p-1}) = 0$, since $\pi f(v) = 0.$ $\Rightarrow$ $\ker \pi \supset Im(f \wedge id).$

linearly independent

$$\pi\Big(\sum_{\{i_1 \cdots i_p\} \not\ni n} a_{i_1 \cdots i_p} e_{i_1} \wedge \cdots \wedge e_{i_p} + \sum_{\{i_1 \cdots i_p\} \ni n} a_{i_1 \cdots i_p} e_{i_1} \wedge \cdots \wedge e_{i_p} \Big)$$

$$= \sum_{\{i_1 \cdots i_p\} \not\ni n} a_{i_1 \cdots i_p} e_{i_1} \wedge \cdots \wedge e_{i_p} = 0 \quad (\text{since } \pi(e_n) = 0)$$

$\Rightarrow$ $a_{i_1 \cdots i_p} = 0$ if $\{i_1 \cdots i_p\} \not\ni n.$

some $\alpha \neq 0$

$\Rightarrow$ $e_{i_1} \wedge \cdots \wedge e_{i_p} = \pm \alpha f(v) \wedge e_i \wedge \cdots \wedge e_i.$

$\Rightarrow$ $\ker \pi \subset Im(f \wedge id).$ $\Rightarrow$ $\ker \pi = Im(f \wedge id).$

Suppose $\sum a_{i_1 \cdots i_{p-1}} f(v) \wedge e_{i_1} \wedge \cdots \wedge e_{i_{p-1}} = 0.$

$\Rightarrow$ Since $\pi(f(v)) = 0,$ $f(v) = k e_n$, $k \neq 0.$

$\Rightarrow$ $\sum a_{i_1 \cdots i_{p-1}} k e_n \wedge e_{i_1} \cdots \wedge e_{i_{p-1}} = 0.$