are distinct finite points, $\exists$ no double point, which implies $\frac{\partial f}{\partial y} = 0$ at the points $L \cdot C$. $\Rightarrow$ We may assume

$$f(x, y) = ax + by + \frac{cx^2}{2} + dxy + \frac{ey^2}{2} + \cdots, \quad b \neq 0,$$

since $\left.\frac{\partial f}{\partial y}\right|_{(0,0)} = b \neq 0$.

⌟

Consider the ideals

$$I = \{x, y^2\}, \quad I' = \{x, f(x,y)^2\}.$$

For a suitable function $g(x,y)$ holomorphic in a nbd of the origin,

$$f(x,y)^2 = g(x,y)\,x + (b^2 + bey + \cdots)\,y^2.$$

Consequently, $I' \subset I$ with transformation matrix

$$A = \begin{pmatrix} 1, & g(x,y) \\ 0, & b^2 + bey + \cdots \end{pmatrix}.$$

⌜

$$x = 1\,x + 0 \cdot y^2$$
$$f(x,y)^2 = g(x,y)\,x + (b^2 + bey + \cdots)\,y^2.$$

⌟

Note that the determinant $\Delta = b^2 + bey + \cdots$ is nonzero at the origin.

⌜ Note that $f(x,y)^2$ has the only one $b^2 y^2$ as a term y^2, i.e., the coeff. of y^2 in $f(x,y)^2$ is b^2. ⌟

For $h(x,y)$ holomorphic, the transformation law gives