

If $f(z)=0$, then $a_d(f(z))=0 \Rightarrow a_d(f(z))$ is divided by f_i for some i , by Corollary (Weak Nullstellensatz) on P11. $\Rightarrow h^d(z) \in \{f_1, \dots, f_n\}$. Here we may assume that f_i is irreducible, since we can apply the Corollary \Rightarrow for f_i^* where $f_i^* = f_{i,1} \dots f_{i,r}$ irreducible

(f) Finally, we can give one more interpretation of the local intersection number $(D_1, \dots, D_n)_{z=0}$, where $D_i = (f_i)$. Let $\mathcal{O} = \mathcal{O}_z$ and $I \subset \mathcal{O}$ be the ideal defined by $f^*(m_w)$ where $m_w \subset \mathcal{O}_w$ is the maximal ideal of functions $h(w)$ with $h(0)=0$. Then we have

\mathcal{O}/I is a finite-dimensional complex vector space, and $\dim_{\mathbb{C}}(\mathcal{O}/I) = (D_1, \dots, D_n)_{z=0}$.

$\mathbb{F} \quad f^*(w_1) = f_1, \dots, f^*(w_n) = f_n$ defined \equiv generated

$$I = I(f_1, \dots, f_n)$$

\Rightarrow Consider \mathcal{O}/I . \Rightarrow For any $h+I$, by the argument above, we have the expression

$$h^d + a_1(w) h^{d-1} + \dots + a_d(w) \equiv 0, \text{ where } a_1(0) = a_d(0) = 0.$$

$\Rightarrow (h+I)^d \in I$, since $a_1(w), \dots, a_d(w) \in I(f_1, \dots, f_n)$.

$\frac{\mathcal{O}_w}{m_w}$ is a field which is isomorphic to \mathbb{C} .

$f^* : \frac{\mathcal{O}_w}{m_w} \longrightarrow \frac{\mathcal{O}}{I(f_1, \dots, f_n)}$ is well-defined, since

$f^*(m_w) \subset I(f_1, \dots, f_n)$. Furthermore, f^* is one to one, for if $f^*(g) \in I(f_1, \dots, f_n)$, where $g \in \mathcal{O}_w$, then

$$(f^*g)(z) = \sum h_i f_i(z) \Rightarrow z=0, (f^*g)(0)=0 \text{ since } f_i(z)=0.$$