$$\varphi \in H^0(M, \Omega^q) = H^0(\{U_i\}, \Omega^q) = H^0(\{V_i\}, \Omega^q)$$

is given in U_i by

$$(\varphi|U_i) = \varphi = \sum_J \varphi_{iJ}(z)\, dz_{i,J}, \longrightarrow dz_{i,j_1} \wedge \cdots \wedge dz_{i,j_p}.$$

where $\varphi_{i,J}(z) \in \mathcal{O}(U_i)$.

We define the norm

$$\|\varphi\| = \sum_{i,J} \sup |\varphi_{i,J}(z)|.$$

This norm is finite, and since (1) $H^0(\{V_i\}, \Omega^q) \cong H^0(\{U_i\}, \Omega^q)$ and any sequence of analytic functions

$\psi_\nu \in \mathcal{O}(U_i)$ satisfying $\sup_{z \in V_i} |\psi_\nu(z) - \psi_\mu(z)| \longrightarrow 0$ has

a subsequence converging uniformly to a holomorphic function $\psi \in \mathcal{O}(V_i)$, (See p201. Th 8.6 Silverman).

we deduce that with this norm $H^0(M, \Omega^q)$ is a complete Banach space. By the Montel theorem (P 300. Th 14.6 R & C Rudin) or (Silverman) given a sequence $\varphi_\nu \in H^0(M, \Omega^q)$ with $\|\varphi_\nu\| \leq 1$,

we may extract a subsequence whose coefficient $\varphi_{\nu,i,J}(z) \in \mathcal{O}(U_i)$ converge uniformly to some $\varphi_{i,J}(z) \in \mathcal{O}(V_i)$. Thus the unit ball in this Banach space is compact, and by a result in Banach space theory, this implies that it is finite dimensional.

$\lceil$ (Gleason) If V is locally compact, then V is finite dimensional. $\rfloor$