$r \in E$ in the exceptional divisor to the point of intersection of $\mathbb{P}^3$ with $< L, r > = \mathbb{P}^2$.

$\Rightarrow$ $f_L(B_L)$ is a measure zero set in $\mathbb{P}^3$, and since $f_L(\tilde{X}) = \mathbb{P}^3$ ($\because$ $\tilde{X}$ is 3-dim and $\mathbb{P}^3$ is irreduble), for generic point $p \in \mathbb{P}^3$, $f_L^{-1}(p)$ is a single point $\Rightarrow$ f_L is generically one to one.

$\Rightarrow$ f_L is birational by the result on P493. $\lrcorner$

A closer examination of f_L, in fact, tells us a good deal more about X. To begin with, note that if

$$\pi : \tilde{X}_L \longrightarrow X$$

is the blow-up of X along L and $F = \pi^{-1}(L) \subset \tilde{X}_L$ the exceptional divisor of the blow-up, then f_L may be extended to a holomorphic map

$$\tilde{f}_L : \tilde{X}_L \longrightarrow \mathbb{P}^3$$

by sending a point $(p, \eta) \in F$, corresponding to the normal vector η to L at p, to the point of intersection of V_3 with the 2-plane spanned by L and any line through p representing the vector η — since η is defined as a tangent vector to X at p modulo tangent vectors to L at p, this is well-defined.