$$gAg^{-1} = \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & \ddots & \\ & & \lambda_n \end{pmatrix}, \quad \lambda_i \neq 0 \text{ for all } i.$$

$\Rightarrow$ $\exists$ a path $\alpha_i(t) \neq 0$ s.t $\alpha_i(0) = \lambda_i$ $\alpha_i(1) = 1$

$\Rightarrow$ Consider $A_t = g^{-1} \begin{pmatrix} \alpha_1(t) & 0 & & 0 \\ 0 & \alpha_2(t) & & \\ & & \ddots & \\ 0 & & & \alpha_n(t) \end{pmatrix} g$

$\Rightarrow$ This proves that K is path-connected.

For dense & open, roughly speaking, since the minimal polynomial of a matrix A has the distinct roots if and only if A is diagonalizable, and the set of minimal polynomials which have a double root, is smaller than the set of minimal polynomials which have distinct roots, i.e., "some sort of" nondiagonalizable matrices form a subvariety in diagonalizable matrices.

$$\begin{pmatrix} a_{11}-x, & a_{12} \\ a_{21}, & a_{22}-x \end{pmatrix} \Rightarrow \begin{pmatrix} 0, & a_{12} \\ a_{21} - \dfrac{(a_{22}-x)(a_{11}-x)}{a_{12}}, & a_{22}-x \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0, & a_{12} \\ a_{21} - \dfrac{(a_{22}-x)(a_{11}-x)}{a_{12}}, & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a_{12}, & 0 \\ 0, & a_{12} - \dfrac{(a_{22}-x)(a_{11}-x)}{a_{21}} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a_{12}, & 0 \\ 0, & \frac{1}{a_{12}} \{ (a_{12}a_{21} - a_{22}a_{11} - (a_{11}+a_{22})x + x^2) \} \end{pmatrix}$$

$$\| $$

$\frac{1}{a_{12}}$ characteristic polynomial