

$$= - \int_{|z|=r} \frac{f(z)}{z} dz + 2\pi\sqrt{-1} f(0),$$

$$\text{since } f(z) = -C_2 \int_{\mathbb{C}} \phi_i \circ \psi_i^{-1}(\omega) \varphi(z, \omega) \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega},$$

$$\begin{aligned} (**) &= C_2 \int_{|z_1|=r} \left(\int_{\mathbb{C}} \phi_i \circ \psi_i^{-1}(\omega) \varphi(z_1, \omega) \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega} \right) \frac{dz_1}{z_1} \\ &\quad + 2\pi\sqrt{-1} (-C_2) \int_{\mathbb{C}} \phi_i \circ \psi_i^{-1}(\omega) \varphi(0,0) \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega}. \end{aligned}$$

\Rightarrow For a fixed z_1 with $|z_1|=r$,

$$\begin{aligned} &\int_{\mathbb{C}} \phi_i \circ \psi_i^{-1}(\omega) \varphi(z_1, \omega) \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega} \\ &= \int_{|z_2| \leq r} \phi_i \circ \psi_i^{-1}\left(\frac{z_2}{z_1}\right) \varphi(z_1, z_2) dz_2 \wedge d\bar{z}_2 \end{aligned}$$

Forget all the arguments above.

$$\begin{aligned} \widetilde{\mathbb{C}}^n &\xrightarrow{\pi'} \mathbb{C}^n \xrightarrow{P} \mathbb{P}^{n-1} \\ \widetilde{\Delta} = \{(z_1, \dots, z_n), \ell\} &\xrightarrow{\pi'^{-1}} \pi'^{-1}(B[r]) \longrightarrow B[r]. \end{aligned}$$

where $(z_1, \dots, z_n) \in \ell$.

$$\begin{aligned} \int_{B[r]} \bar{\partial} \varphi \wedge \beta &= \int_{\pi'^{-1}(B[r])} \pi'^*(\bar{\partial} \varphi \wedge \beta) = \int_{\pi'^{-1}(B[r])} \pi'^*(\bar{\partial} \varphi) \wedge \pi'^*\beta \\ &= \int_{|\lambda| \leq r, \ell \in \mathbb{P}^{n-1}} \bar{\partial}(\pi'^*\varphi) \wedge C_n \theta \wedge \pi'^*((P^*\Omega)^{n-1}) = - \int_{|\lambda| \leq r} C_n \theta \left(\int_{\mathbb{P}^{n-1}} \bar{\partial}_\lambda(\pi'^*\varphi) \right. \\ &\quad \left. \wedge \pi'^*((P^*\Omega)^{n-1}) \right) = - \int_{|\lambda| \leq r} C_n \theta \wedge \bar{\partial}_\lambda \left(\int_{\mathbb{P}^{n-1}} \pi'^*\varphi \wedge \pi'^*((P^*\Omega)^{n-1}) \right) \end{aligned}$$