NO.

DATE . . ( )

$\Rightarrow \#(S^* \cap l_x^*) = \#$ of lines in X passing through x
$= 4$

Note: Let K be a pencil in $\mathbb{P}^{3*}$.
$\Rightarrow \exists$ two linearly independent hyperplanes
$p \cdot X = 0$, & $p' \cdot X = 0$, where $X = [X_0, X_1, X_2, X_3]$
$p = [p_0, p_1, p_2, p_3]$ & $p' = [p_0', p_1', p_2', p_3']$.
$\Rightarrow (\lambda p + \lambda' p') \cdot X = 0$ contains a line for all
$\lambda, \lambda' \in \mathbb{C}$. The line is expressed as l_x, $x \in G$.
$\Rightarrow K = l_x^*$ as above.

Likewise, the second argument goes over, and indeed establishes an important point: given any line $l_x \subset \mathbb{P}^3$ not passing through any points of R or lying in any hyperplanes of R^*, we have two pencils on the surface $U = T_x(G) \cap X$:

$$\{X_p = \sigma(p) \cap U\}_{p \in l_x} \text{ and } \{X_h = \sigma(h) \cap U\}_{h \supset l_x}.$$

$\ulcorner$ $h \longmapsto X_h$ is 'some sort of' linear.

Both are Lefschetz, and so the number of singular fibers in each is $\chi(U) - 4$.

$\ulcorner$ $\{X_p\}$ is Lefschetz by P765. Since $l_x^* \cap R^* = \phi$,