To verify this assertion it will suffice to exhibit a point of $I-J$; that is, six conics $C_1, C_2 \cdots C_5$ and C' such that $V_{C_1}, \cdots V_{C_5}$ meet transversely at C'. But this is clear: if C' is any smooth conic, $C_1, \cdots C_5$ conics simply tangent to C' at distinct points $p_1, \cdots p_5$, then the tangent hyperplanes $T_{C_i}(V_{C'}) = H_{p_i}$ are independent.

Γ Suppose we can find a smooth conic C s.t C is tangent to C' only at a given arbitrary point p.

$\Rightarrow$ C is a smooth point of $V_{C'}$, and H_p is the tangent plane of $V_{C'}$ at C, by the result on P150, (and H_p is the tangent plane of V_C at C').

And assume that H_{p_i}'s are linearly dependent.

Let $\langle \sigma_1, \cdots \sigma_6 \rangle = H^0(\mathbb{P}^2, \mathcal{O}(2H))$.

$\Rightarrow$ $\{ (\sigma_1(p_1), \cdots \sigma_6(p_1)) = v_1, \cdots (\sigma_1(p_5), \cdots \sigma_6(p_5)) = v_5 \}$ is a set of linearly dependent vectors.

$\Rightarrow$ $\exists$ two distinct nontrivial solution a_i's & b_i's s.t

$\tau_1 = a_1 \sigma_1 + \cdots + a_6 \sigma_6 = 0$ for all p_i's.

$\tau_2 = b_1 \sigma_1 + \cdots + b_6 \sigma_6 = 0$ "

$\Rightarrow$ $\#(\tau_1 \cdot \tau_2) \geq 5$ $\Rightarrow$ It is impossible, since $\#(\tau_1 \cdot \tau_2) = 4$ ($\because$ $\deg \tau_1 = \deg \tau_2 = 2$).

$\Rightarrow$ $T_{C_i}(V_{C_i}) = H_{p_i}$'s are linearly independent. $\Rightarrow$