and Θ_t is 2-form.

Thus we have

$$\frac{\partial}{\partial t} P(\Theta_t) = k\, \tilde{P}(d\eta, \Theta_t, \cdots \Theta_t)$$
$$+ k(k-1)\, \tilde{P}(\eta, \Theta_t \wedge (\tilde{\theta} + t\eta) - (\tilde{\theta} + t\eta) \wedge \Theta_t, \cdots \Theta_t).$$

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$$\frac{\partial}{\partial t} P(\Theta_t) = k\, \tilde{P}(d\eta, \cdots \Theta_t) - k\, \tilde{P}(\tilde{\theta} \wedge \eta + \eta \wedge \tilde{\theta}, \cdots \Theta_t)$$
$$- 2kt\, \tilde{P}(\eta \wedge \eta, \cdots \Theta_t)$$

$$= k\, \tilde{P}(d\eta, \cdots \Theta_t) + k(k-1)\, \tilde{P}(\eta, \Theta_t \wedge \tilde{\theta} - \tilde{\theta} \wedge \Theta_t, \cdots \Theta_t)$$
$$- t\, k(k-1)\, \tilde{P}(\eta, \eta \wedge \Theta_t - \Theta_t \wedge \eta, \cdots \Theta_t)$$

$$= k\, \tilde{P}(d\eta, \cdots \Theta_t) + k(k-1)\, \tilde{P}(\eta, \Theta_t \wedge \tilde{\theta} - \tilde{\theta} \wedge \Theta_t - t\eta \wedge \Theta_t$$
$$+ t\, \Theta_t \wedge \eta, \cdots \Theta_t)$$

$$= k\, \tilde{P}(d\eta, \cdots \Theta_t) + k(k-1)\, \tilde{P}(\eta, \Theta_t \wedge (\tilde{\theta} + t\eta) - (\tilde{\theta} + t\eta) \wedge \Theta_t,$$
$$\cdots, \Theta_t)$$

But for any connection θ with curvature Θ, $d\Theta = \theta \wedge \Theta - \Theta \wedge \theta$ and consequently

$$d\Theta_t = (\tilde{\theta} + t\eta) \wedge \Theta_t - \Theta_t \wedge (\tilde{\theta} + t\eta);$$

so we can write, finally

$$\frac{\partial}{\partial t} P(\Theta_t) = k\, \tilde{P}(d\eta, \cdots \Theta_t) - k(k-1)\, \tilde{P}(\eta, d\Theta_t, \cdots \Theta_t)$$
$$= k\, d\tilde{P}(\eta, \Theta_t, \cdots \Theta_t).$$

Q.E.D.