

$$\sigma_a(V') = \{ \Lambda \in G(k, n+1) \mid \dim(\Lambda \cap V'_{n+1-k+i-a_i}) \geq i \}$$

$$\text{We claim: } \sigma_a(V) = \bar{\iota}_1^{-1}(\sigma_a(V'))$$

$$\begin{aligned} \Lambda \in \sigma_a(V) &\Rightarrow \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i \Rightarrow \text{Since} \\ \dim(\Lambda \cap V'_{n+1-k+i-a_i}) &\geq \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i, (V'_{n+1-k+i-a_i} \supset V_{n-k+i-a_i}) \\ \bar{\iota}_1(\Lambda) = \Lambda \in \sigma_a(V') &\Rightarrow \sigma_a(V) \subset \bar{\iota}_1^{-1}(\sigma_a(V')) \\ \Lambda \in \bar{\iota}_1^{-1}(\sigma_a(V')) &\Rightarrow \bar{\iota}_1(\Lambda) \in \sigma_a(V') \Rightarrow \Lambda \in \sigma_a(V') \\ \Rightarrow \dim(\Lambda \cap V'_{n+1-k+i-a_i}) &\geq i \Rightarrow \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i. \end{aligned}$$

Non sense

$$V = (0 \subset \mathbb{C}^1 \subset \mathbb{C}^2 \subset \dots \subset \mathbb{C}^n) = (0 \subset \underbrace{\langle e_1 \rangle}_{V_1} \subset \underbrace{\langle e_1, e_2 \rangle}_{V_2} \subset \dots \subset \langle e_1, \dots, e_n \rangle)_{V_n}$$

$$V' = (0 \subset \underbrace{\langle e_{n+1} \rangle}_{V'_1} \subset \underbrace{\langle e_1, e_{n+1} \rangle}_{V'_2} \subset \underbrace{\langle e_1, e_2, e_{n+1} \rangle}_{V'_3} \subset \dots \subset \underbrace{\langle e_1, \dots, e_{n+1} \rangle}_{V'_{n+1}})$$

$$\sigma_a(V) = \{ \Lambda \in G(k, n) \mid \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i \}$$

$$\sigma_a(V') = \{ \Lambda \in G(k, n+1) \mid \dim(\Lambda \cap V'_{n+1-k+i-a_i}) \geq i \}$$

$$\text{If } \Lambda \in \sigma_a(V), \dim(\Lambda \cap V_{n-k+i-a_i}) = \dim(\Lambda \cap V'_{n+1-k-i-a_i})$$

$$\text{Since } \Lambda \in G(k, n), \text{ and so } \Lambda \cap V_{n-k+i-a_i} = \Lambda \cap V'_{n+1-k-i-a_i}.$$

$$\Rightarrow \Lambda = \bar{\iota}_1(\Lambda) \in \sigma_a(V') \Rightarrow \bar{\iota}_1^{-1}(\sigma_a(V')) \ni \Lambda.$$

$$\Rightarrow \sigma_a(V) \subset \bar{\iota}_1^{-1}(\sigma_a(V'))$$

$$\text{If } \Lambda \in \bar{\iota}_1^{-1}(\sigma_a(V')), \bar{\iota}_1(\Lambda) = \Lambda \in \sigma_a(V') \Rightarrow$$

$$\Lambda \cap V_{n+1-k-a_i+i} = \Lambda \cap V_{n-k+i-a_i} \Rightarrow \dim(\Lambda \cap V_{n+1-k-a_i+i}) = \dim(\Lambda \cap V_{n-k+i-a_i}) \Rightarrow \Lambda \in \sigma_a(V) \Rightarrow \bar{\iota}_1^{-1}(\sigma_a(V')) \subset \sigma_a(V).$$

$$\bar{\iota}_2: G(k, n) \longrightarrow G(k+1, n+1)$$

$$\Lambda \longmapsto \Lambda \oplus \langle e_{n+1} \rangle.$$

$$\sigma_a(V') = \{ \Lambda \in G(k+1, n+1) \mid \dim(\Lambda \cap V'_{n+1-k+i-a_i}) \geq i \}.$$