Since differentiability is local, $\varphi_1\sigma$, $\varphi_2\sigma$ are diff'ble
$\Rightarrow$ σ is in $C^s(M, E)$.
Point : $\rho_1\sigma = f_\alpha e_\alpha \Rightarrow (f_\alpha) \in C^s(U, R^n)$ //

(Rellich lemma).
Given a bounded sequence $\{u_\ell\}$ in $\mathcal{H}_s(M, E)$,
consider $\{\rho_1 u_\ell\}$.

$\Rightarrow$ ρ_1 C^∞ and compact support $\Rightarrow$ ρ_1 bounded
, i.e $|D^\alpha \varphi_1| \leq M'$ for all $[\alpha] \leq s$.

$\Rightarrow$ $\{\rho_1 u_\ell\}$ is a bounded sequence in $\mathcal{H}_s(M, E)$.

$\Rightarrow$ $\rho_1 u_\ell = \Sigma (f_\alpha)_\ell e_\alpha$ $\Rightarrow$ $(f_\alpha)_\ell$ has a compact support in U_1 $\Rightarrow$ $\{(f_\alpha)_\ell\} : U_1 \longrightarrow R^n$. in $\mathcal{H}_s(U, R^n)$.
which can be considered as a map from a torus to R^n.
$\Rightarrow$ By local Rellich lemma, $\exists$ a convergent subsequence $\{(f_\alpha)_{\ell_k}\}$ in $\mathcal{H}_r(U, R^n)$.

$\Rightarrow$ $\exists$ a corresponding subsequence $\{u_{\ell_k}\}$.

$\Rightarrow$ Now consider $\{\rho_2 u_{\ell_k}\}$ $\Rightarrow$ In the same way, we get a subsequence of $\{u_{\ell_k}\}$, say $\{u_m\}$.

$\Rightarrow$ $\{\rho_1 u_m\}$ converges to $\rho_1 u_0 = v_1$ and v_i can be
$\{\rho_2 u_m\}$ converges to $\rho_2 v_0 = v_2$ extended to M by defining zero outside U_i.

Question ? u_m converges to one element.

$$\|u_m - (v_1+v_2)\|_r = \|\rho_1 u_m + \rho_2 u_m - (v_1 + v_2)\|_r \leq \|\rho_1 u_m - v_1\|_r + \|\rho_2 u_m - v_2\|_r$$

$$< \epsilon + \epsilon = 2\epsilon \Rightarrow u_m \longrightarrow v_1 + v_2$$

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