

By the result on P735, $T_p(F) \cap F$ is the cone through p over a smooth quadric in $\mathbb{P}^{2n+2-2} = \mathbb{P}^{2n}$.

By the construction of a smooth quadric \tilde{F}_{2n-1} on P734, $\tilde{F}_{2n-1} = T_p(F) \cap F \cap \mathbb{P}^{2n}$, \mathbb{P}^{2n} is a generic $2n$ -plane complementary to p in \mathbb{P}^{2n+2} ($\supset T_p(F) \cap F$).

Thus

$$\begin{aligned} \tilde{F}_{2n-1} \cap \Lambda &= T_p(F) \cap F \cap \mathbb{P}^{2n} \cap \Lambda = (T_p(F) \cap F \cap \Lambda) \cap \mathbb{P}^{2n} \\ &= \Lambda \cap \mathbb{P}^{2n} \text{ since } T_p(F) \cap F \supset \Lambda. \end{aligned}$$

$\Rightarrow \tilde{F}_{2n-1} \cap \Lambda$ is a linear space.

It remains to show that $\tilde{F} \cap \Lambda^{\neq K}$ is a $(n-1)$ -plane.

Clearly $\langle p, K \rangle \subset \Lambda$.

Let $q \in \Lambda$. $\Rightarrow q \in F \cap T_p(F)$ and since $F \cap T_p(F)$ is the cone through p over \tilde{F} , $\exists p' \in \tilde{F}$ s.t.

$$\overline{pp'} \ni q \Rightarrow \overline{pq} \ni p' \Rightarrow \overline{pq} \subset \Lambda.$$

$$\Rightarrow p' \in \Lambda \Rightarrow p' \in \tilde{F} \cap \Lambda = K$$

$$\Rightarrow \overline{pp'} \subset \langle p, K \rangle \Rightarrow q \in \langle p, K \rangle \Rightarrow \Lambda \subset \langle p, K \rangle$$

$\Rightarrow \Lambda = \langle p, K \rangle$. This proves that K is a $(n-1)$ -plane. \square

The fibers of π_1 are therefore isomorphic to Σ'_{n-1} , which by hypothesis is irreducible of dimension $n(n+1)/2$; it follows that I itself is irreducible of dimension $n(n+1)/2 + 2n+1$.

By the argument above, $\pi_1^{-1}(p)$ is isomorphic to the set of $(n-1)$ -planes lying on a smooth quadric \tilde{F}_{2n-1} of