

acc.

$$\begin{aligned}
 \Gamma \quad \pi^* D' \cdot \pi^* D &= \int_{\tilde{M}} \eta_{\pi^* D'} \cdot \pi^* D = \langle \eta_{\pi^* D'} \cdot \pi^* D, [\tilde{M}] \rangle \\
 &= \langle \eta_{\pi^* D'} \wedge \eta_{\pi^* D}, [\tilde{M}] \rangle = \langle \pi^* \eta_{D'} \wedge \pi^* \eta_D, [\tilde{M}] \rangle \\
 &= \langle \pi^* (\eta_{D'} \wedge \eta_D), [\tilde{M}] \rangle \\
 &= \langle \eta_{D'} \wedge \eta_D, \pi_* [\tilde{M}] \rangle \\
 &= \langle \eta_{D'} \wedge \eta_D, [M] \rangle, \text{ since } \pi_* \text{ has degree 1} \\
 &= \int_M \eta_{D'} \wedge \eta_D = D' \cdot D. \quad (\because \pi \text{ is one to one on } \tilde{M} - \pi^{-1}(p).)
 \end{aligned}$$

$$\pi^* D \cdot E = \pi^* D \cdot \pi^* E = \pi^* (D \cdot E) = 0, \text{ since } \pi_* E = * \text{ is homologous to zero in } H_1(M).$$

$$\tilde{D}_1, \tilde{D}_2 \in \text{Div}(\tilde{M}).$$

$$\Rightarrow \tilde{D}_1 = \pi^* D_1 + n_1 E \quad \& \quad \tilde{D}_2 = \pi^* D_2 + n_2 E$$

$$\Rightarrow \tilde{D}_1 \cdot \tilde{D}_2 = (\pi^* D_1 + n_1 E) \cdot (\pi^* D_2 + n_2 E) = \pi^* D_1 \cdot \pi^* D_2 + n_1 n_2 E \cdot E$$

\Rightarrow When we do a product on $\text{Div}(\tilde{M})$, we have only to product separately on $\pi^* \text{Div}(M)$ and $\mathbb{Z}\langle E \rangle$.

$$\tilde{D}_1 = \pi^* D_1 + n_1 E = (\pi^* D_1, n_1 E) \quad \tilde{D}_2 = (\pi^* D_2, n_2 E)$$

$$\tilde{D}_1 \cdot \tilde{D}_2 = (\pi^* D_1 \cdot \pi^* D_2, n_1 E \cdot n_2 E) \quad \square$$

Note in particular that if D, D' are two divisors on M intersecting transversely at p , \tilde{D} & \tilde{D}' their proper transforms in \tilde{M} , we have

$$\begin{aligned}
 \tilde{D} \cdot \tilde{D}' &= (\pi^* D - E) \cdot (\pi^* D' - E) \\
 &= \pi^* D \cdot \pi^* D' + E \cdot E \\
 &= D \cdot D' - 1.
 \end{aligned}$$