morphic covering of pure order ν over an open subset $W \subset \mathbb{C}^n$, then to each holomorphic function $f \in {}_V\mathcal{O}_V$ there is canonically associated a monic polynomial $P_f(X) \in {}_n\mathcal{O}_W[X] \subseteq {}_V\mathcal{O}_V[X]$ of degree ν s.t $P_f(f) = 0$ in ${}_V\mathcal{O}_V$.

Proof. For any ν pt $z \in W$, let $\pi^{-1}(z) = \{A_1(z), \cdots A_\nu(z)\}$, where the ν points $A_j(z)$ are listed with repetitions according to the branching order. To any function $f \in {}_V\mathcal{O}_V$ associate the polynomial

$$P_f(X) = \prod_{j=1}^{\nu} [X - f(A_j(z))] = X^\nu + a_1(z) X^{\nu-1} + \cdots + a_\nu(z).$$

This is the unique monic polynomial of degree ν having as roots the ν values $f(A_j(z))$; the coefficients $a_i(z)$ are the elementary symmetric functions of these ν roots and are well-defined functions on W, independent of the particular ordering chosen for the points $A_j(z)$. $\Rightarrow$ Show the continuity of $a_i(z)$ and use the extended Riemann removable singularity theorem and the def of a covering to get the desired. See Gunning for details. ///

This theorem is in Griffiths & Harris. on P668. Lemma.

6. Theorem. If $\pi : V \to W$ is a finite branched $\cdots$ as above, and if $f \in {}_V\mathcal{O}_V$ seperates the sheets of this covering, then there is a unique monic polynomial $P_f(X) \in {}_n\mathcal{O}_W[X] \subseteq {}_V\mathcal{O}_V[X]$ of degree ν s.t $P_f(f) = 0$ in ${}_V\mathcal{O}_V$, and the discriminant of this polynomial is a function $d_f \in {}_n\mathcal{O}_W$ that does not vanish identically on any connected component of W.