$$\frac{\partial W_1}{\partial X_0}(F) = X_1 X_2 - X_5^2 \Big|_F = 0$$

$$\frac{\partial W_1}{\partial X_1} = 0 \quad \frac{\partial W_1}{\partial X_2} = 1, \quad \frac{\partial W_1}{\partial X_3} = 0 = \frac{\partial W_1}{\partial X_4} = \frac{\partial W_1}{\partial X_5} \text{ at } F.$$

$\Rightarrow$ By P175, the tangent space is $X_2 = 0$.

$$\Rightarrow T_F W_1 = \left\{ \begin{pmatrix} X_0 & X_3 & X_4 \\ X_3 & X_1 & X_5 \\ X_4 & X_5 & 0 \end{pmatrix} \right\} \Big/ \mathbb{C}^* = \{ [X_0, X_1, 0, X_3, X_4, X_5] \} = \mathbb{P}^5$$

Given any element $\left[\begin{pmatrix} a_0 & a_3 & a_4 \\ a_3 & a_1 & a_5 \\ a_4 & a_5 & 0 \end{pmatrix} \right]$ in $T_F W_1$,

$$(X_0, X_1, X_2) \begin{pmatrix} a_0 & a_3 & a_4 \\ a_3 & a_1 & a_5 \\ a_4 & a_5 & 0 \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} = 0 \text{ passes through}$$

the point $[0, 0, 1]$. Conversely, if a conic

$$(X_0, X_1, X_2) \begin{pmatrix} a_0 & a_3 & a_4 \\ a_3 & a_1 & a_5 \\ a_4 & a_5 & a_2 \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} = 0 \text{ passes through}$$

the point $[0, 0, 1]$, then

$$(0, 0, 1) \begin{pmatrix} a_0 & a_3 & a_4 \\ a_3 & a_1 & a_5 \\ a_4 & a_5 & a_2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (a_4, a_5, a_2) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = a_2 = 0$$