$\Rightarrow$ $K-L$ is a "some sort of" negative divisor. $\Rightarrow$ If $H^0(S, \mathcal{O}(K-L)) \neq 0$, $K-L$ is linearly equivalent to a "some sort of" positive divisor which is impossible. Thus $H^0(S, \mathcal{O}(K-L)) = 0 \Rightarrow$ $h^0(\mathcal{O}(K-L)) = 0$. ⏎

Proof. We consider the two exact sheaf sequences

$$\begin{cases} 0 \longrightarrow \mathcal{J}(L) \longrightarrow \mathcal{J}_{P_0}(L) \longrightarrow \mathcal{O}_P(L) \longrightarrow 0 \\ 0 \longrightarrow \mathcal{O}(L^*) \longrightarrow \mathcal{O} \oplus \mathcal{O} \longrightarrow \mathcal{J}(L) \longrightarrow 0, \end{cases}$$

where $\mathcal{O}_P = \mathcal{O}/\mathcal{J}_P$ in the first, and the second is the Kozul resolution.

Γ
$$0 \longrightarrow \mathcal{O}(L^*) \longrightarrow \mathcal{O} \oplus \mathcal{O} \longrightarrow \mathcal{J}(L) \longrightarrow 0$$
$$\eta \longmapsto (\eta \otimes s, \ -\eta \otimes s')$$
$$(f_1, \ f_2) \longmapsto f_1 s' + f_2 s$$

$\mathcal{J}(L)(U) = \{ f_1 s' + f_2 s \mid f_1, f_2 \in \mathcal{O}(U) \}$
see P703, P698, P713.
$\mathcal{J}(L)$ = an ideal sheaf generated by s, and s'.
$\mathcal{J}_{P_0}(L)$ = sheaf of holomorphic sections of L vanishing on P_0
$\Rightarrow \mathcal{J}(L)$ is a subsheaf of $\mathcal{J}_{P_0}(L)$