Applying Stokes' theorem once again, we may express the integral on the left of $(*)$ as a difference of integrals

$$C_n \int_{\|x\|=\rho} \psi * d\varphi .$$

$$\pm C_n \int_{B[\delta,\epsilon]} * d\varphi \wedge \frac{dr}{r^{n-1}} = \pm C_n \int_{B[\delta,\epsilon]} * d\varphi \wedge d\psi$$

$$= \pm C_n \int_{B[\delta,\epsilon]} d(\psi * d\varphi) \overset{\text{Stokes' theorem}}{=} \text{difference of the integrals } C_n \int_{\|x\|=\rho} \psi * d\varphi .$$

For fixed $\rho$, this integral is a constant times

$$\int_{\|x\|=\rho} * d\varphi = \int_{\|x\|\le\rho} d * d\varphi = 0 .$$

Since $\psi = \begin{cases} \log r & \text{in case } n=2 \\ \left(-\frac{1}{n-2}\right) \frac{1}{r^{n-2}} & \text{in case } n \ge 3, \end{cases}$

for fixed $\rho$,

$$\psi = \begin{cases} \log \rho & \text{in case } n=2 \\ \left(-\frac{1}{n-2}\right) \frac{1}{\rho^{n-2}} & \text{in case } n \ge 3. \end{cases}$$

$$\Rightarrow \int_{\|x\|=\rho} \psi * d\varphi = \psi \int_{\|x\|=\rho} * d\varphi = \psi \int_{\|x\|\le\rho} d * d\varphi = 0$$