

$f(t, x) = x + t K x + \text{matrix terms involving } t^2 \text{ or } t x_i x_j$.
where K is a $n \times n$ matrix.

$$\Rightarrow \frac{d f(t, x)}{dt} = B f(t, x) + [2] \dots (*)$$

where $B = \text{transpose of } A = (a_{ij})$
[2] is terms involving $x_i x_j$.

\Rightarrow The solution of $(*)$ is

$$f(t, x) = e^{tB} \cdot x + \text{terms involving } t^2 \text{ or } t x_i x_j.$$

by comparing $f(t, x) = x + t K x + \text{matrix terms involving } t^2$
or $t x_i x_j$. $\Rightarrow B = K$. furthermore.

$$\Rightarrow \left(\frac{\partial f_i}{\partial x_j} \right) = J_{f_t}(p) = e^{tB} + \text{terms involving } t^2.$$

\Rightarrow

$$\text{Thus } J_{f_t}(p) - I = t \left(A + \frac{tA^2}{2} + \frac{t^2A^3}{6} + \dots \right),$$

and for t positive and sufficiently small,

$$L_{f_t}(p) = \text{sgn det}(J_{f_t}(p) - I) = \text{sgn det } A = L_v(p).$$

$$\Uparrow J_{f_t}(p) - I = t \left(B + \frac{tB^2}{2} + \frac{t^2B^3}{6} + \dots \right) + t^2 \square$$

$\Rightarrow L_{f_t}(p) = \text{sgn det}(J_{f_t}(p) - I) =$
 $\text{sgn} \left(\det B \det \left(I + \frac{tB}{2} + \dots \right) \right)$ since $t^2 \square$ is so
small, for sufficiently small t . \Rightarrow