「 We know that the topology τ_K of $\mathcal{D}_K$ is induced from $\mathcal{D}(\Omega)$ by Th. 6.5 (b), and $\mathcal{D}_K$ is Fréchet space by P33. $\Rightarrow$ By Th 1.27, $\mathcal{D}_K$ is a closed subspace of $\mathcal{D}(\Omega)$. But I think that this proof is nonsense, because, to prove $\mathcal{D}_K$ is F-space, they used the closedness of $\mathcal{D}_K(\Omega)$ in $C^\infty(\Omega)$, which was made of use the fact that δ_x is continuous in the topology of $C^\infty(\Omega)$. It is "kind of" redundant.

Second thought, maybe it is not nonsense since they use a different topology on $C^\infty(\Omega)$ to prove δ_x is continuous. 」

It is obvious that each $\mathcal{D}_K$ has empty interior, relative to $\mathcal{D}(\Omega)$.

「 $$\mathcal{D}_K = \bigcap_{x \in K^c} \delta_x^{-1}(0) \Rightarrow \mathcal{D}_K^c = \bigcup_{x \in K^c} \delta_x^{-1}(0)^c$$

$\delta_x^{-1}(0)^c$ is open set in $\mathcal{D}(\Omega)$.

If $\mathcal{D}_K$ has an interior point, then $\exists\ \phi + V_N \subset \mathcal{D}_K$.
$V_N = \{ \psi \in \mathcal{D}(\Omega) : \|\psi\|_N < \frac{1}{N} \}$ $\Rightarrow$ This implies that if $\psi \in \mathcal{D}(\Omega)$ s.t $\|\psi\|_N < \frac{1}{N}$, $\operatorname{supp}\psi \subset K$.

Consider $d_K(x) = d(x, K)$

Consider $\eta(x) = \psi(x-\alpha)$. $\Rightarrow$ $\operatorname{supp}\eta \subset \alpha + K \subset \Omega$ but $\|\eta\|_N = \|\psi\|_N$ $\Rightarrow$ Since $\alpha + K \not\supset K$, we have a contradiction. 」

Since there is a countable collection of sets $K_i \subset \Omega$ s.t $\mathcal{D}(\Omega) = \bigcup \mathcal{D}_{K_i}$, $\mathcal{D}(\Omega)$ is of the first category in itself. Since Cauchy sequences converge in $\mathcal{D}(\Omega)$ (Theorem 6.5), Baire's theorem implies that $\mathcal{D}(\Omega)$ is not metrizable.