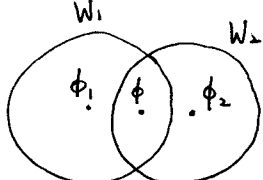


So that

$$(5) \quad \phi + \delta_i W_i \subset \phi_i + W_i \subset V_i \quad (i=1, 2).$$

Hence (1) holds with $W = (\delta_1 W_1) \cap (\delta_2 W_2)$, and (a) is proved.

$$\begin{aligned} \Gamma \quad \phi + W &\subset \phi + \delta_1 W_1 \quad \text{and} \quad \phi + W \subset \phi + \delta_2 W_2 \\ &\quad \cap \\ &\quad \phi_1 + W_1 \quad \cap \quad \phi_2 + W_2 \\ &\quad \cap \\ &\quad V_1 \quad \cap \quad V_2 \\ \Rightarrow \quad \phi + W &\subset V_1 \cap V_2. \end{aligned}$$


Suppose next that ϕ_1 and ϕ_2 are distinct elements of $\mathcal{D}(\Omega)$, and put

$$(6) \quad W = \{ \phi \in \mathcal{D}(\Omega) : \|\phi\|_0 < \|\phi_1 - \phi_2\|_0 \},$$

where $\|\phi\|_0$ is as in (1) in Section 6.2. Then $W \in \beta$ and ϕ_1 is not in $\phi_2 + W$.

$$\Gamma \quad \mathcal{D}_K \cap W \stackrel{?}{\in} \tau_K \quad W \text{ is convex and balanced clearly.}$$

To show $\mathcal{D}_K \cap W$ is open in \mathcal{D}_K , we want to find V_N s.t., for $f \in \mathcal{D}_K \cap W$, $f + V_N \subset \mathcal{D}_K \cap W$.

$$\|f\|_0 < M = \|\phi_1 - \phi_2\|_0 \Rightarrow \text{Let } M - \|f\|_0 = \ell. \quad \text{Choose } K_N \text{ s.t. } K_N > K \text{ and } \frac{1}{N} < \ell.$$

$$\begin{aligned} \Rightarrow \text{Since } V_N &= \{ \phi \in \mathcal{D}_K : \max \{ |D^\alpha \phi(x)| : x \in K, |\alpha| \leq N \} < \frac{1}{N} \}, \quad f + V_N \subset W \cap \mathcal{D}_K. \Rightarrow W \in \beta. \\ \|\phi_1 - \phi_2\|_0 &\Rightarrow \phi_1 - \phi_2 \in W \Rightarrow \phi_1 \notin \phi_2 + W. \end{aligned}$$