Consider A and B as points. $\Rightarrow$ $\pi_1 : I^* \longrightarrow F$ is an unbranched 2-sheeted cover of F.

$I^* = I/_{A,B} = \{$ connected components of the fibers $\}$

By Theorem 4.5, the fundamental group of F is one to one & onto $\pi_1^{-1}(p)$, if I^* is path-connected. But since $\# \pi_1^{-1}(p) = 2$, I^* must be disconnected.

It follows that I has two connected components, each mapping via π_1 onto F with fibers isomorphic to one irreducible component of Σ_{n-1}; as in the last argument, each of the connected components of I is irreducible of dimension $n(n-1)/2 + 2n$. Since the fibers of the projection map $\pi_2 : I \longrightarrow \Sigma_n$ are irreducible of dimension n, we see that Σ_n has two connected components Σ_n^1 and Σ_n^2, each irreducible of dimension

$$\frac{n(n-1)}{2} + 2n - n = \frac{n(n+1)}{2}.$$

$\pi_2 : I \longrightarrow \Sigma_n$ is a smooth fibration, and since $\pi_2^{-1}(\Lambda) = \mathbb{P}^n$ is irreducible, Σ_n has two connected components, Σ_n^1, Σ_n^2. Since each of the connected components of I is of dim $n(n-1)/2 + 2n$,

$$\dim \Sigma_n^i + n = n(n-1)/2 + 2n.$$

$$\Rightarrow \quad \dim \Sigma_n^i = n(n-1)/2 + 2n - n = n(n+1)/2$$