

\Rightarrow the map ψ is given by the matrix

$$\begin{bmatrix} \frac{1}{2} \left(\frac{w_1}{dz_1} \right)'(p), & \left(\frac{w_1}{dz_1} \right)(p), & \left(\frac{w_1}{dz_2} \right)(q) \\ \frac{1}{2} \left(\frac{w_2}{dz_1} \right)'(p), & \left(\frac{w_2}{dz_1} \right)(p), & \left(\frac{w_2}{dz_2} \right)(q) \\ \vdots & \vdots & \vdots \\ \frac{1}{2} \left(\frac{w_g}{dz_1} \right)'(p), & \left(\frac{w_g}{dz_1} \right)(p), & \left(\frac{w_g}{dz_2} \right)(q) \end{bmatrix}$$

Now the number of independent relations among the row vectors of this matrix is just the number of linearly independent holomorphic differentials vanishing up to order 2 at p and vanishing at q , that is, the dimension of $H^0(S, \Omega^1(-2p-q))$.

This explains that "Here, of course, we take the "linear span" of a point P_i with multiplicities a_i in D to be the span of P_i together with the first a_i-1 derivatives of the canonical map. \hookrightarrow

Finally, since the dimension of the linear span of d points on C is just $d-1$ less the number of independent linear relations on the points, we have the geometric version of the Riemann-Roch:

The dimension r of the complete linear system containing a divisor $D = \sum P_i$ is equal to the number of independent linear relations on the points P_i on the canonical curve,