$\Rightarrow \{f = g = 0\} \subset \mathbb{C}^3$ since we have z_1, z_2, z_4 variables. and $\pi : \mathbb{C}^3 \longrightarrow \mathbb{C}^2$ is onto locally.

Let z_4° be a point s.t $f(z_1^\circ, z_2^\circ, z_4^\circ) = g(z_1^\circ, z_2^\circ, z_4^\circ) = 0$.

Let h' be the greatest common divisor of f & g.

$\Rightarrow \{f = g = 0\} = \{f'h' = g'h' = 0\} = \{h' = 0\} \cup \{f' = g' = 0\}$

$\Rightarrow$ f' & g' are relatively prime. Assume that $f'(z_1^\circ, z_2^\circ, z_4^\circ) = g'(z_1^\circ, z_2^\circ, z_4^\circ) = 0$.

$\Rightarrow$ By the argument above (on P13), we have $\alpha' \beta' \gamma'$ s.t

$$\alpha' f' + \beta' g' = \gamma' \qquad \gamma' \in \mathcal{O}_2.$$

$\Rightarrow \pi(\{f' = g' = 0\}) = \{\gamma' = 0\}$ $\Rightarrow$ $\{f' = g' = 0\}$ is one-dimensional variety. Anyway, $\{\gamma' = 0\}$ is a set of measure zero in a nbd of (z_1°, z_2°).

$\{h' = 0\}$ is a finitely sheeted branched covering over a nbd of (z_1°, z_2°).

Now let's summarize the above result.

$W = \{f = g = 0\}$, f, g Weierstrass polynomials.

W does not contain a hypersurface.

$\alpha f + \beta g = \gamma$. $\deg \alpha < \deg g$ $\deg \beta < \deg f$.

γ is a Weierstrass polynomial in z_{n-1}.

Since $\{\gamma = f_1^{\alpha_1} f_2^{\alpha_2} \cdots f_k^{\alpha_k} = 0\} = \{f_1 \cdot f_2 \cdots f_k = 0\}$

we may assume γ has nonzero discriminent $D(\gamma)$.

$\Rightarrow$ For each $(z_1^\circ \cdots z_{n-2}^\circ) \in \mathbb{C}^{n-2} - \{D(\gamma) = 0\}$, by the inve