NO.

DATE . . ( )

has rank 1. $\Rightarrow$ The tangent vector of $(\alpha_{ij}(t))$ is of rank 1.

Thus $\beta\omega_1 - \alpha\omega_2 = 0$. Suppose $\omega_1 \neq 0$. ($\because$ Both ω_1 & ω_2 can not be zero)

$$\Rightarrow \begin{pmatrix} 1 & 0 & r'\omega_1 \\ 0 & 1 & r'\omega_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & r'\omega_1 \\ 0 & 1 & \frac{\beta}{\alpha} r'\omega_1 \end{pmatrix} = M(r)$$

$$\Rightarrow M(r) \cap L = \frac{\beta}{\alpha} e_1 - e_2 = [\beta, -\alpha] \in \mathbb{P}^1 = L$$

As $r \to 0$, $M \cap L \longrightarrow [\beta, -\alpha] \in L$. More generally:

Suppose $\begin{pmatrix} \alpha_{11}(t) & \cdots & \alpha_{14}(t) \\ \alpha_{21}(t) & \cdots & \alpha_{24}(t) \end{pmatrix}$ is a curve passing L in B_L.

$$\begin{pmatrix} \alpha_{11}(t) & \cdots & \alpha_{14}(t) \\ \alpha_{21}(t) & \cdots & \alpha_{24}(t) \end{pmatrix} = t \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} + t^2 \begin{pmatrix} \\ \end{pmatrix} + \cdots$$

$$\Rightarrow \text{Let } M(t) = \begin{pmatrix} 1 & 0 & \alpha_{11}(t) & \cdots & \alpha_{14}(t) \\ 0 & 1 & \alpha_{21}(t) & \cdots & \alpha_{24}(t) \end{pmatrix}.$$

$$M(t) \cap L = \left[\frac{\alpha_{21}}{\alpha_{11}}, -1 \right] = [\alpha_{21}, -\alpha_{11}]$$

$$= [t\,\omega_{21} + t^2 \cdots, -t\,\omega_{11} - t^2 * \cdots]$$

$$\Rightarrow M(t) \cap L \longrightarrow [\omega_{21}, \omega_{11}] \text{ as } t \longrightarrow 0.$$

$\Rightarrow$ Since B_L is a curve, r has an extension.

$\blacksquare$

Note that r can be realized as the composition of $j|_{B_L}$ with the projection map of C_L from the point P_L.

i.e.

$$B_L \xrightarrow{j|_{B_L}} C_L \xrightarrow{\pi} L$$