We shall not give here the proof of this lemma, which is found in the references given at the end of this chapter.

Taking $q=1$, $F=(f_1, \cdots f_p)$ generats an ideal sheaf $I \subset \mathcal{O}$, and $\mathcal{R}$ is the sheaf of relations $\sum r_i f_i = 0$ among the generators. Oka's lemma is therefore a sort of Noetherian property of $\mathcal{O}$, not just in each stalk but, so to speak, spread out over sufficiently small open sets. We note the similarity to the following, which was proved in Section 1 of Chapter 0:

> If f and g are holomorphic in U and are relatively prime in the local ring $\mathcal{O}_{z_0}$, then they are relatively prime in $\mathcal{O}_z$ for $\|z-z_0\|<\epsilon$.

The proof of this result used the Weierstrass division theorem, and the same is true of the proof of Oka's lemma.

Here is the basic

Definition. Let M be a complex manifold with structure sheaf $\mathcal{O}$ and $\mathcal{F}$ a sheaf of $\mathcal{O}$-modules. Then $\mathcal{F}$ is coherent if locally it has a presentation

$$\mathcal{O}^{(p)} \longrightarrow \mathcal{O}^{(q)} \longrightarrow \mathcal{F} \longrightarrow 0.$$

In other words, $\mathcal{F}$ is coherent if, given any point $z_0 \in M$, there is a nbd U of z_0 and finitely many sections of $\mathcal{F}|U$