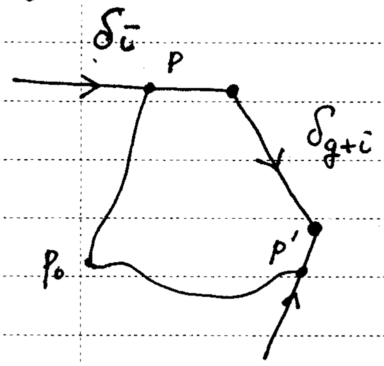
Now let ω be a holomorphic differential on S, η a meromorphic form whose only singularities are simple poles at points $s_\lambda \in S$. Assuming that η has no poles on the paths δ_i, let Π^i and N^i denote the periods of ω and η, respectively, along the path δ_i. Since the region Δ is simply connected and ω is holomorphic, we can set

$$\pi(s) = \int_{s_0}^{s} \omega$$

to obtain a holomorphic function π in $\bar{\Delta}$ with $\omega = d\pi$. (See Figure 3)



$\ulcorner$ Here π is holomorphic (of course, well-defined) in $\bar{\Delta}$ = (closure of Δ) which implies π is holomorphic on Δ. But we can define π on $\bar{\Delta}$

On Δ,

$$\lim_{h\to 0} \frac{\pi(s+h) - \pi(s)}{h} = \lim_{h\to 0} \frac{\int_{s_0}^{s+h}\omega - \int_{s_0}^{s}\omega}{h} = \lim_{h\to 0} \frac{\int_{s}^{s+h}\omega}{h}$$

$= f(s)$ where $\omega = f(z)\,dz$ locally. $\lrcorner$

Note that for any pair of points $p \in \delta_i$, $p' \in \delta_i^{-1}$ on $\partial\Delta$ that are identified on S.