The essential step in our construction of $\Lambda$ is to express the tensor

$$\iota(v)(\Theta) \in A^{0,1}(T' \otimes T'^{*})$$

as $\bar\partial$ of a global section of $T' \otimes T'^{*}$. To do this, we recall from Section 5 of Chapter 0 the definition of the torsion associated to a metric on $T'(M)$.

See P77 & P107

As we saw then, if $v_1, \cdots v_n$ is a unitary frame for $T'(M)$, $\varphi_1, \cdots \varphi_n$ the dual coframe for $T'^{*}$, $\theta$ the connection matrix of $D$ in terms of $\{v_i\}$ and $\theta^{*} = -{}^t\theta$ the connection matrix of the dual connection $D^{*}$ on $T'^{*}$ in terms of $\{\varphi_i\}$, then

$$d\varphi_i = \sum \theta^{*}_{ij} \wedge \varphi_j + \tau_i$$

with $\tau_i$ of type $(2,0)$; the vector $\tau = (\tau_1, \cdots \tau_n)$ of 2-forms is called the torsion of the metric in terms of $\{v_i\}$.

According to P76. $\theta = -{}^t\theta^{*} \Rightarrow \theta^{*} = -{}^t\theta. \Rightarrow \theta^{*} = \bar\theta$.
See Lemma on P76.

Now if

$$\{v_i' = \sum g_{ij} \cdot v_j\}$$

is another frame, $\{\varphi_i'\}$ the dual coframe, and $\theta'$, and $\theta'^{*}$ the connection matrices of $D$ and $D^{*}$ in terms of $\{v_i'\}$ and $\{\varphi_i'\}$, then in matrix notation and setting $g^{*} = {}^t g^{-1}$

$$\varphi' = g^{*} \cdot \varphi,$$
$$\theta'^{*} = g^{*} \cdot \theta^{*} \cdot {}^t g + d(g^{*}) \cdot {}^t g.$$

and