Date

3. Linear Systems on Curves.

Reciprocity Law II

Let S be a compact Riemann surface of genus g, ω a global holomorphic 1-form on S, and η a differential of the second kind, i.e., a global meromorphic 1-form with no residues. We want, as in the first reciprocity law, to relate the periods of ω and η to the singularities of η. Since these singularities are not described by the intrinsically defined residue, we choose a local coordinate z around each singular point p of η, and write

$$\eta(z) = (a_{-n}^{p} z^{-n} + \cdots + a_0^p + a_1^p z + \cdots)\, dz,$$
$$\omega(z) = (b_0^p + b_1^p z + \cdots)\, dz.$$

Note that $a_{-1}^p = \operatorname{Res}_p(\eta) = 0$, and that $b_0^p(p) = (\omega/dz)(p)$ as defined earlier.

☞ By the def of η (differential of the 2nd kind), $a_{-1}^p = 0$. $b_0^p(p) = (\omega/dz)(p)$, see P236.

Now let $\delta_1, \delta_2, \cdots \delta_{2g}$ be cycles on S representing a canonical basis for $H_1(S, \mathbb{Z})$, disjoint except for a common base point $s_0 \in S$ and not containing any singular points of η; let Π^i and N^i denote the periods of ω and η along δ_i. As before, $\Delta = S - \bigcup \delta_i$ is simply

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