Applying the Riemann-Roch

$$\chi(L_i) = \frac{L_i \cdot L_i - L_i \cdot K_S}{2} + \chi(\mathcal{O}_S) = \chi(\mathcal{O}_S).$$

Γ

By p472, $\chi(L_i) = \frac{L_i \cdot L_i - L_i \cdot K_S}{2} + \chi(\mathcal{O}_S) = \frac{-1-(-1)}{2} + \chi(\mathcal{O}_S)$
$= \chi(\mathcal{O}_S)$. $\lrcorner$

Moreover, by Kodaira-Serre duality $h^2(L_i) = h^0(K_S - L_i)$;
but
$\deg(K_S - L_i)|_{S\cap H} = K_S \cdot H_S - L_i \cdot H_S = K_{S_0} \cdot H_{S_0} - E_i \cdot H_{S_0} = -3-1$
$= -4$,
so $K_S - L_i$ can not have any global sections.

Γ $H^2(S, \mathcal{O}(L_i)) \cong H^0(S, \Omega^2(L_i^*)) = H^0(S, \mathcal{O}(K_S - L_i))$
$\Rightarrow \dim H^2(S, \mathcal{O}(L_i)) = h^0(L_i) = \dim H^0(S, \mathcal{O}(K_S - L_i)) = h^0(K_S - L_i)$.

First of all, note that since $\#((H\cap S)\cap \ell) = \#(\ell \cap S)$
$= 3$, $H \cap S$ is a cubic curve $\overset{\text{in } H}{,}$ here $\ell \subset H$.
$K_S = -H'|_S$. $\Rightarrow \deg(K_S|_{S\cap H}) = -\#((S\cap H)\cap(S\cap H'))$
$= -\#(S_0 \cap H \cap H') = -\#(S_0 \cap \ell) = -3$.
$\deg(L_i|_{S\cap H}) = \#([L_i]\cdot[S\cap H]) = \#(\ell \cap (S_0 \cap H)) \overset{\#(S\cap *)}{=}$
$\#(S_0 \cap (\ell \cap H)) = 1$

since $\ell \subset S$, ($\because$ $[L_i]$ is represented by a line in S).
$\Rightarrow \deg(K_S - L_i)|_{S\cap H} = \deg(K_S|_{S\cap H}) - \deg(L_i|_{S\cap H}) = -3-1 = -4$
$\Rightarrow K_S - L_i \longrightarrow S$ has no global section. Otherwise,