

In some sense, <sup>we may consider</sup>  $a=0$  and  $L_{bc} = \infty \Rightarrow \varphi_{abc}(a) = \infty$  and  $\varphi_{abc}''(L_{bc}) = 0$

$$\varphi_{edf}(e) = \infty \Rightarrow \varphi_{edf} \circ \varphi_{abc}(\infty) = \infty \quad \varphi_{edf} \circ \varphi_{abc}(a) = 0$$

$\Rightarrow$  "Some sort of reciprocity"

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$\Rightarrow \varphi_{edf} \circ \varphi_{abc}$  is holomorphic 1995.12.27

$$\mathbb{P} [a'(1, 0, 0) + b'(X'_0, X'_1, X'_2)] = L_a$$

$\Rightarrow$  For any  $X'_0 \neq 0$ ,  $a' + b'X'_0 = 0$  has nontrivial solutions  $(a', b')$ , in case  $X'_1 \neq 0$  or  $X'_2 \neq 0$ .

$$\varphi [(a' + b'X'_0, b'X'_1, b'X'_2)]$$

$$= [(b'^2 X'_1 X'_2, (a' + b'X'_0) b'X'_2, (a' + b'X'_0) b'X'_1)]$$

$\Rightarrow$  Consider the line  $X'_1 X_1 - X'_2 X_2 = 0$ ,  $X'_1 \neq 0$  or  $X'_2 \neq 0$ .

$\Rightarrow$  Clearly  $\varphi(L_a) \subset \{X'_1 X_1 - X'_2 X_2 = 0\}$ .

Suppose  $L = \{a_0 X_0 + a_1 X_1 + a_2 X_2 = 0\} \nsubseteq a, b, c$ .

$\Rightarrow a_0 \neq 0, a_1 \neq 0, a_2 \neq 0$ .

$$Z_0 = X_1 X_2, \quad Z_1 = X_0 X_2, \quad Z_2 = X_0 X_1$$

$$\Rightarrow a_0 + a_1 \frac{X_1}{X_0} + a_2 \frac{X_2}{X_0} = 0 \Rightarrow \frac{X_1}{X_0} = \frac{Z_0}{Z_1}, \quad \frac{X_2}{X_0} = \frac{Z_0}{Z_2}$$

$$\Rightarrow a_0 + a_1 \frac{Z_0}{Z_1} + a_2 \frac{Z_0}{Z_2} = 0 \Rightarrow a_0 Z_1 Z_2 + a_1 Z_0 Z_2 + a_2 Z_0 Z_1 = 0$$

Clearly,  $\{a_0 Z_1 Z_2 + a_1 Z_0 Z_2 + a_2 Z_0 Z_1\} \ni d, e, f$ .

$$d = c, \quad e = a, \quad f = b.$$

Note that  $\varphi_{abc}$  is described locally as

$$(x, y) \mapsto \left(\frac{1}{x}, \frac{1}{y}\right).$$

$$\varphi_{edf} \circ \varphi_{abc} : [X_0, X_1, X_2] \mapsto [X_1 X_2, X_0 X_2, X_1 X_0] \mapsto$$

$$[X_0 X_2 X_1 X_0, X_1 X_2 X_1 X_2, X_1 X_2 X_0 X_1]$$

$$= [X_0, X_2, X_1]$$

is holomorphic  $\Rightarrow$

Another Cremona transformation has been implicitly mentioned in the last section. Let  $a_1, a_2, \dots$

$a_6$  be six points in  $\mathbb{P}^2$  in general position with respect to lines and conics.  $\mathbb{P}$  See P420  $\perp$