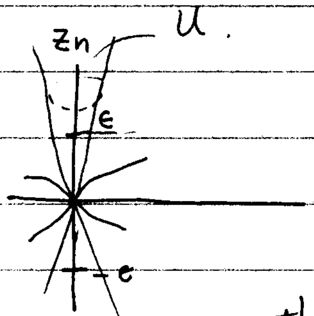

Notice that $(f=0)$ does not intersect with $\{|z_n|=\epsilon,\ \|z'\|<\delta\}$ for small enough δ and for a proper $\epsilon>0$.
$\Rightarrow$ The set of all lines through the origin which do not lie in $(f=0)$ are open set in $G(1,n)$.

$\Rightarrow$ Given $V=\{f_1=\cdots=f_k=0\}$ since f_i's are finite, we may assume that $(f_1=0),\cdots$ & $(f_k=0)$ do not contain a common line through the origin.

$k=4$. $V=\{f_1=f_2=f_3=f_4\}$.

Suppose $f_1,\cdots$ & f_4 are relatively prime and any three of them are not relatively prime (otherwise, say f_1,f_2,f_3, since $V=\{f_1=\cdots=f_4\}\subset\{f_1=f_2=f_3=0\}\subset$ the common locus of relatively prime functions, by the previous argument.) $\Rightarrow$ $f_1=hg_1$, $f_2=hg_2$, $hg_3=f_3$ where g_1, g_2 & g_3 are relatively prime. $\Rightarrow$ By the previous argument, we have two relatively prime functions g_1' & g_2' s.t $\{g_1'=0=g_2'\}\supset\{g_1=g_2=g_3=0\}$.

$\Rightarrow$ $\{f_1=f_2=f_3=0\}=\{h=0\}\cup\{g_1=g_2=g_3=0\}$
$\subset\{h=0\}\cup\{g_1'=g_2'=0\}=\{hg_1'=hg_2'=0\}$
where $g_2'=g_3$, $g_1'\in\mathcal{O}_{n-1}$.

$\Rightarrow$ Again g_1', g_3 & f_3 are relatively prime.
$\Rightarrow$ We have two relatively prime functions s.t the common locus of the two contains the V.