F Locally, $\frac{i}{2\pi}\Theta = i a\, dz \wedge d\bar{z}$, since $\dim M = 1$,

$a(z) < 0$ for all z, see P29.

$\Rightarrow \int_M i a\, dz \wedge d\bar{z} = i \int a\,(dx + i dy) \wedge (dx - i dy)$

$= i \int a \times (-2i)\, dx \wedge dy = 2 \int a\, dx \wedge dy < 0.$ 』

But if $s \neq 0 \in H^0(M, \mathcal{O}(L))$, then L is the line bundle associated to the effective divisor $D = (s)$, and we have

$$c_1(L) = \deg D \geq 0, \quad \text{a contradiction.}$$

As an immediate consequence of the vanishing theorem, we see that

$$H^q(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(kH)) = 0 \quad \text{for } 1 \leq q \leq n-1, \text{ all } k.$$

This follows directly from the dualized version of the vanishing theorem in case k is negative; if k is nonnegative,

$$H^q(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(kH)) = H^q(\mathbb{P}^n, \Omega^n_{\mathbb{P}^n}(kH - K_{\mathbb{P}^n}))$$
$$= H^q(\mathbb{P}^n, \Omega^n_{\mathbb{P}^n}((k+n+1)H)) = 0$$

by the original version of the theorem.

F (1) k is negative.

$[H]$ is positive line bundle (See P150)

$\Rightarrow [kH]$ is negative since k is negative.

$\Rightarrow$ By the dualized version of the vanishing theorem,

$$H^q(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(kH)) = 0 \quad \text{if } 0 + q \leq n-1 < n.$$