it applies if $a_i + b_{k+1-i} = n-k$ for any i.

$\ulcorner$ $\beta = \gamma = k \Rightarrow \alpha = 1. \Rightarrow a_1 + b_k + c_k = n-k.$

Suppose $b_k \neq 0$ or $c_k \neq 0$. $\Rightarrow a_1 \neq n-k$.

I can not see the reason why $b_k = c_k = 0$ should be true. See P517 for this explanation.

As suggested, we can apply this first reduction to the intersection of cycles $\#(\sigma_{a^*} \cdot \sigma_{b^*} \cdot \sigma_{c^*})$ in $G(n-k, n)$; we obtain

Reduction Formula II. For any three coeffients $a_\alpha, b_\beta, c_\gamma$ with $a_\alpha + b_\beta + c_\gamma \geq 2(n-k)+1$,

$$\#(\sigma_a \cdot \sigma_b \cdot \sigma_c)_{G(k,n)}$$

$$= \begin{cases} 0 & \text{if } \alpha+\beta+\gamma > k \\ \#(\sigma_{a_1-1, \cdots a_\alpha - 1, a_{\alpha+1}, \cdots a_k} \cdot \sigma_{b_1-1, \cdots b_\beta - 1, b_{\beta+1}, \cdots b_k} \cdot \sigma_{c_1-1, \cdots c_\gamma-1, c_{\gamma+1}, \cdots c_k})_{G(k, n-1)} & \text{if } \alpha+\beta+\gamma = k. \end{cases}$$

$\ulcorner$ If $a_\alpha = a_{\alpha+1}$, $a_\alpha - 1 < a_{\alpha+1}$. $\Rightarrow$

$\sigma_{a_1-1, \cdots a_\alpha-1, a_{\alpha+1} \cdots a_k} = 0$

$\Rightarrow$ I think that Reduction Formula II is not correct.

For example, $k = 6$. $n = 12$.

$a_1 = 6 > a_2 = 5$. $a_3 = a_4 = a_5 = 4 > a_6 = 2$.

$\Rightarrow a^*_{a_1} = a^*_6 = 1 \quad a^*_4 = 5 \quad a^*_5 = 2 \quad a^*_2 = 6$