⊤ $X_h = \sigma(h) \cap H \Rightarrow X_h$ is a line in $\sigma(h)$ or $\sigma(h)$. $\Rightarrow$ The set of tangent planes

$$\{T_x(G)\}_{x\in\sigma(h)} = \{H \supset \sigma(h) \mid H \text{ hyperplane in } P^5\}$$

$\Rightarrow$ Again $X_h$ is a line. $\Rightarrow$ The line $X_h$ is a pencil $\sigma(p, h)$.

⊥

Here is another way to view this: any element $\omega$ of $\Lambda^2 \mathbb{C}^4$ corresponds to a skew-symmetric quadratic form

$$\Gamma_\omega(v, v') = \omega \wedge v \wedge v' \in \Lambda^4 \mathbb{C}^4 \cong \mathbb{C};$$

the corresponding linear line complex $X = H_\omega \cap G$ is then given by

$$X = \{ l = \overline{v, v'} : \Gamma_\omega(v, v') = 0 \}.$$

⊤ $$\Gamma_\omega : \mathbb{C}^4 \times \mathbb{C}^4 \longrightarrow \Lambda^4\mathbb{C}^4 \cong \mathbb{C}$$
$$(v, v') \longmapsto \Gamma_\omega(v, v') = \omega \wedge v \wedge v'$$

By the results on P156.

$$H_\omega \cap G = \{ v \wedge v' \in G \mid v \wedge v' \wedge \omega = \Gamma_\omega(v, v') = \omega \wedge v \wedge v' = 0 \}. = \{ l = \overline{v, v'} \mid \Gamma_\omega(v, v') = 0 \}.$$

⊥

If $\omega = v \wedge v'$ is decomposable, then $H_\omega$ is tangent to $G$ at $l = \overline{v, v'}$, and $X = H_\omega \cap G$ is the Schubert cycle $\sigma(l)$; if, on the other hand, $\omega$ is indecomposable,