Let $\# \Gamma_0' = 5$. $\Rightarrow$ Choose a subset $\Lambda_0 \subset \Gamma_0'$ with $\# \Lambda_0 = 3$.

Then according to the application on P714, $|\mathcal{J}_{\Lambda_0}(2)|$ must satisfy case 1 or case 2.

① $|\mathcal{J}_{\Lambda_0}(2)|$ satisfies case 2.

By Reciprocity Formula I, Λ_0 will impose independent conditions on $|\mathcal{O}_{\mathbb{P}^2}(2)|$. and

$$\dim |\mathcal{J}_{\Lambda_0}(2)| = 2.$$

Since $\dim |\mathcal{J}_{\Gamma_0'}(2)| \leq \dim |\mathcal{J}_{\Lambda_0}(2)| = 2$,

by the correspondence between $H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0}(3H))$ and $H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0'}(2H))$,

$\dim |\mathcal{J}_{\Gamma_0'}(2)| = \dim |\mathcal{J}_{\Gamma_0}(3)| \leq 2$ which is impossible since $\dim |\mathcal{J}_{\Gamma_0}(3)| \geq 3$ is assumed.

Remark: The correspondence between $H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0}(3))$ and $H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0'}(2))$ is given as follows.

Since $|\mathcal{J}_{\Gamma_0}(3)|$ has a common fixed curve of line, if we let $H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0}(3)) = \langle \sigma_1, \sigma_2, \cdots \sigma_k \rangle$,

then $\sigma_1 = l\,\tau_1, \cdots \sigma_k = l\,\tau_k$, l common line

τ_i conic (homogeneous polynomial of deg 2)

σ_i cubic (" of deg 3)

l line (" of deg 1).

$$H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0}(3)) \longleftrightarrow H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0'}(2))$$

$$\overset{\psi}{\sigma} \longleftrightarrow \tau$$

where $\sigma = l\tau$.