(Note that the row vectors of such a matrix representative for $\Lambda \in U_I$ are just the points of intersection of Λ with the affine $(n-k)$-planes $\{ V_{I^\circ} + e_j : j \in I \}$.)

Suppose $I = \{1, 2, \cdots k\}$. Λ is spanned by $\{ e_i + A_i \mid i = 1 \cdots k \}$, where $A_i = (0, \cdots 0, a_{i1}, \cdots a_{i\,n-k})$. $\Rightarrow$ Each $e_i + A_i \in e_i + V_{I^\circ}$ where $I^\circ = \{ k+1, k+2, \cdots n \}$.

Conversely, any $k \times n$ matrix of the form above represents a k-plane $\Lambda \in U_I$; thus the $k(n-k)$ entries of the I° th $k \times (n-k)$ minor $\Lambda^{I}_{I^\circ}$ of Λ^I give a bijection of sets

$$\varphi_I : U_I \longrightarrow \mathbb{C}^{k(n-k)}$$

for each I. Note that $\varphi_I(U_I \cap U_{I'})$ is open in $\mathbb{C}^{k(n-k)}$ for all I, I'; we claim that in fact the map $\varphi_I \circ \varphi_{I'}^{-1}$ is holomorphic on this open set and hence that the map φ_I give $G(k,n)$ the structure of a complex manifold.

To show that $\varphi_I(U_I \cap U_{I'})$ is open in $\mathbb{C}^{k(n-k)}$, we need to show that given $A \in \varphi_I(U_I \cap U_{I'})$, $\exists\ \epsilon > 0$ s.t if $\| A - A' \| < \epsilon$, then $\varphi_I^{-1}(A') \in U_I \cap U_{I'}$.

Without loss of generality.

Let $A = \begin{pmatrix} a_{11}, & a_{12}, & \cdots & a_{1,n-k} \\ a_{21}, & a_{22}, & \cdots & a_{2,n-k} \\ \vdots & & & \\ a_{k1}, & a_{k2}, & \cdots & a_{k,n-k} \end{pmatrix}$. $\Rightarrow$ $\varphi_I^{-1}(A) = \Lambda$ is spanned by $\{ e_i + A_i \}$ where $A_i = (0, 0, \cdot 0, a_{i1}, \cdots a_{i,n-k})$, $e_i = (\overline{0, 0 \cdots 0}, \overset{i}{1}, 0, 0 \cdots 0)$

$\Rightarrow \langle e_i + A_i \rangle + V_{I'^\circ} = \mathbb{C}^n$ since $\Lambda \cap V_{I'^\circ} = \{0\}$ and $\dim \Lambda = k$, $\dim V_{I'^\circ} = n-k$

This implies that $\det \begin{bmatrix} e_1 + A_1 \\ \vdots \\ e_k + A_k \\ e_{j_1} \\ \vdots \\ e_{j_{n-k}} \end{bmatrix} \neq 0$ where $e_{j_l} \in V_{I'^\circ}$.