To prove the Weitzenböck formular, we shall let $v_1, v_2 \cdots v_n$ be the vector field frame dual to $\varphi_1, \cdots \varphi_n$, and $v_{\bar{i}} = \overline{v_i}$.

For a function f, $\quad \bar{\partial} f = \sum_i (\bar{v}_i \cdot f)\, \bar{\varphi}_i$.

since $v_i = \sum a_\alpha \frac{\partial}{\partial z_\alpha}$ ($\because$ it is dual to φ_i).

and for a tensor $\tau = \{\tau_I\}$ the components of the $\bar{Z}$-covariant differential $\bar{\nabla}\tau$ are given by

$$(\bar{\nabla}\tau)_I = \bar{\partial}\tau_I + A^\circ(\tau).$$

[$\bar{Z}$-covariant differential, $\bar{\nabla}$ means ∇'', see P 73

$$\tau = \tau_I \widetilde{\varphi_I}$$

$$\nabla\tau = d\tau_I \otimes \widetilde{\varphi_I} + \tau_I \nabla\widetilde{\varphi_I} = \bar{\partial}\tau_I \otimes \widetilde{\varphi_I} + \partial\tau_I \otimes \widetilde{\varphi_I} + A^\circ(\tau)\varphi$$

$$\Rightarrow \quad \bar{\partial}\tau_I + A^\circ(\tau) = (\nabla''\tau)_I.$$]

It is convenient to use the symbol "$\equiv$" to denote "modulo lower-order terms," so that. e.g.

$$(\bar{\nabla}\tau)_I \equiv \bar{\partial}\tau_I.$$

We set $\quad \Phi' = \varphi_1 \wedge \cdots \wedge \varphi_n$.

It will suffice to prove (W) when $\psi = f\, \varphi_I \wedge \bar{\varphi}_J$ (no summation). Since the dz's act as, so to speak, vector bundle indices, we will assume $p = 0$. Finally, by the symmetry we may take $J = (1, 2, 3, \cdots q)$ so that

$$\psi = f\, \bar{\varphi}_1 \wedge \cdots \wedge \bar{\varphi}_q.$$