$= k\int_D c_1(E) > 0 \Rightarrow \int_D c_1(E) > 0$ since $k>0$, even if $D = kE$

$$\int_{kE} c_1(E) = k\int_E c_1(E) > 0 \Rightarrow \int_E c_1(E) = E\cdot E > 0.$$

If $E = mD$, $\Rightarrow \int_{kmD} c_1(E) = km\int_D c_1(E) = k\int_{mD} c_1(E)$

$= k\int_E c_1(E) > 0$. If $E = -mD$, then $\int_E c_1(E) < 0$.

$\Rightarrow$ 무엇 하고 있는 거냐. 정신 차려.! (What the hell am I doing?)

Suppose mD is effective.

$\Rightarrow \int_{mD} c_1(E) = m\int_D c_1(E) > 0 \Rightarrow mE\cdot D > 0 \Rightarrow E\cdot D \neq 0$.

Contradiction. The same works for $-mD$. ⌟

Applying Riemann-Roch, we find

$$h^0(mD) - h^1(mD) + h^2(mD) = \frac{1}{2}m^2 d - \frac{m}{2}K\cdot D + \chi(\mathcal{O}_S);$$

i.e.

$$h^0(K-mD) = h^2(mD) \geq \frac{1}{2}m^2 d - \frac{m}{2}K\cdot D + \chi(\mathcal{O}_S) + h^1(mD)$$

becomes arbitrarily large as m goes to either $-\infty$ or $+\infty$.

Pf $h^0(mD) - h^1(mD) + h^2(mD) = \chi([mD]) = \chi(\mathcal{O}_S) + \frac{mD\cdot mD - mD\cdot K}{2} = \chi(\mathcal{O}_S) + \frac{m^2 D\cdot D - mK\cdot D}{2}$ by Riemann-Roch on P472.

$$D\cdot D = \int_S c_1(D)\wedge c_1(D) = \int_D c_1(D) = \deg D = d.$$

in case D is smooth.

$= \chi(\mathcal{O}_S) + (m^2 d - mK\cdot D)/2$