holomorphic section only if $c_1(L) \geq 0$, i.e.,

$$\deg L < 0 \Rightarrow H^0(S, \mathcal{O}(L)) = 0.$$

Γ Suppose $C_1(L) < 0$. i.e $\deg L < 0$. and L has a nontrivial global holomorphic section σ. $\Rightarrow [(\sigma=0)] = L$
$\Rightarrow (\sigma = 0) = \sum a_i p_i$ where $a_i \geq 0$ $\Rightarrow \deg L \geq 0$
$\Rightarrow$ Contradiction.

On the other hand, since the generator of $H^2(S, \mathbb{Z}) \cong \mathbb{Z}$ corresponding to $+1$ is represented by a positive form,

$$L \text{ positive} \Leftrightarrow \deg L > 0.$$

Γ The generator of $H^2(S, \mathbb{Z})$ corresponding to $+1$ is represented by $(1,1)$-form ω. $\Rightarrow$ By Proposition P148, if L is positive. $[\frac{i}{2\pi}\Theta] = C_1(L)$. where $\frac{i}{2\pi}\Theta$ is positive $(1,1)$-form. $\Rightarrow [\frac{i}{2\pi}\Theta] = n[\omega]$ where $n \in \mathbb{Z}_+$.
$\Rightarrow \underbrace{\langle C_1(L), [S] \rangle}_{\deg L} = \langle [\frac{i}{2\pi}\Theta], [S] \rangle = n \langle [\omega], [S] \rangle$
$= n > 0$

If $\deg L > 0$, $C_1(L) = [\frac{i}{2\pi}\Theta] \Rightarrow [\frac{i}{2\pi}\Theta] = n [\omega]$. $\Rightarrow \langle C_1(L), [S] \rangle = \langle n[\omega], [S] \rangle = n = \deg L \Rightarrow n > 0 \Rightarrow C_1(L) = [\frac{i}{2\pi}\Theta] = [n\omega]$
$\Rightarrow C_1(L)$ is represented by a positive $(1,1)$-form
$\Rightarrow$ Again by P148, Proposition, L is positive.

Thus, if $\deg L > \deg K_S$, then $L \otimes K_S^*$ is positive, and by Kodaira vanishing

$$H^1(S, \mathcal{O}(L)) = H^1(S, \Omega^1(L + K_S^*)) = 0.$$

Γ $K_S = \wedge T_S^{*\prime} = T_S^{*\prime}$ = cotangent bundle over S.