

$x_2 \neq 0$, $x_3 \neq 0$, if we consider

$$\begin{pmatrix} \alpha_{11} - \alpha_{10}, & \alpha_{12} - \alpha_{10}, & \alpha_{13} - \alpha_{10} \\ \alpha_{21} - \alpha_{20}, & \alpha_{22} - \alpha_{20}, & \alpha_{23} - \alpha_{20} \\ \alpha_{31} - \alpha_{30}, & \alpha_{32} - \alpha_{30}, & \alpha_{33} - \alpha_{30} \end{pmatrix},$$

its determinant is zero, which contradicts to **(**)**.

Thus in general, we conclude that if v_1, \dots, v_q are linearly dependent at $X \in P^n$, then all but q of the homogeneous coordinates of X vanish, i.e.,

$$D_q = \bigcup_{\#I=q} P_{i_1, i_2, \dots, i_q}.$$

We are going to show that $\{v_1, v_2\}$ is generic.

$$v_1 = (\alpha_{11} - \alpha_{10}) x_1 \frac{\partial}{\partial x_1} + (\alpha_{12} - \alpha_{10}) x_2 \frac{\partial}{\partial x_2}$$

$$v_2 = (\alpha_{21} - \alpha_{20}) x_1 \frac{\partial}{\partial x_1} + (\alpha_{22} - \alpha_{20}) x_2 \frac{\partial}{\partial x_2}$$

$$\mathbb{C}^2 \xrightarrow{v_1} \mathbb{C}^2 \times \mathbb{C}^2$$

$$(x_1, x_2) \longmapsto (x_1, x_2, (\alpha_{11} - \alpha_{10}) x_1, (\alpha_{12} - \alpha_{10}) x_2)$$

$$\mathbb{C}^2 \xrightarrow{v_2} \mathbb{C}^2 \times \mathbb{C}^2$$

$$(x_1, x_2) \longmapsto (x_1, x_2, (\alpha_{21} - \alpha_{20}) x_1, (\alpha_{22} - \alpha_{20}) x_2)$$

$$(i) \quad (x_1, x_2) = (1, 0).$$

$\{(x_1, x_2, k(\alpha_{11} - \alpha_{10}) x_1, k(\alpha_{12} - \alpha_{10}) x_2)\}$ is the subspace E' of E spanned by v_1 .