Date

i.e.,

The points of D span exactly a $(d-r-1)$-plane.

Γ Suppose $v_1, v_2, \cdots v_d \in \mathbb{C}^g$.

$\Rightarrow$ If, l is the number of independent relations among $v_1, \cdots v_d$,

l is the dimension of the kernel of the following map:

Consider a map $\phi : \mathbb{C}^d \longrightarrow \mathbb{C}^g$ defined by

$$e_i \longmapsto v_i .$$

$\Rightarrow$ $\text{Rank}\,\phi = \dim \langle v_1 \cdots v_d \rangle = d - \dim \ker \phi$

$= d - l$.

$$x_1 v_1 + \cdots + x_d v_d = 0$$

$$\Leftrightarrow \quad x_1 (v_{11}, \cdots v_{1g}) + \cdots + x_d (v_{d1}, \cdots v_{dg}) = 0$$

$$\Leftrightarrow \quad \begin{array}{l} x_1 v_{11} + \cdots + x_d v_{d1} = 0 \\ x_1 v_{12} + \cdots + x_d v_{d2} = 0 \\ \quad \vdots \\ x_1 v_{1g} + \cdots + x_d v_{dg} = 0 \end{array} \quad \Leftrightarrow \quad \begin{pmatrix} v_{11}, \cdots & v_{d1} \\ \vdots & \\ v_{1g}, & \cdots v_{dg} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_d \end{pmatrix} = 0$$

$\omega = (\omega_1 \cdots \omega_g) \Rightarrow \omega \cdot v_j = 0$ for all j

$$\Rightarrow \quad \omega_1 v_{11} + \cdots + \omega_g v_{1g} = 0$$

$$\vdots$$

Exercise: $\omega_1 v_{d1} + \cdots + \omega_g v_{dg} = 0$.

$$\begin{pmatrix} v_{11}, \cdots & v_{1g} \\ \vdots & \\ v_{d1}, \cdots & v_{dg} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \vdots \\ \omega_g \end{pmatrix} = 0$$

$d - \text{rank}(v_{ij})$

$g - \text{rank}(v_{ij})$