⌜ By Kodaira–Serre duality, $h^2(L) = h^0(K-L) = h^0(L^{-3}-L)$ $= h^0(L^{-4}) = 0$ since L^{-4} is negative ($\because$ L is positive). ⌟

Also, by the Kodaira vanishing theorem,

$$h^1(L) = h^1(K+4L) = 0,$$

since L^4 is positive.

⌜ $H^1(M, \mathcal{O}(L)) = H^1(M, \mathcal{O}(-3L+4L)) = H^1(M, \mathcal{O}(K_M+4L))$ $= H^1(M, \Omega^2(4L)) = 0$ by the Kodaira vanishing theorem, since $1+2 > \dim M = 2$, and L^k is positive. ⌟

Consequently, $h^0(L) = 3$.

Now if $D \in |L|$ is any divisor in the linear system $|L|$, D must be irreducible: if $D = D_1 + D_2$ where $D_1, D_2 > 0$, we would obtain

$$1 = L \cdot L = L \cdot D_1 + L \cdot D_2.$$

⌜ Suppose D is not irreducible. $D = D_1 + D_2$, where D_1, D_2 are effective. $1 = L \cdot L = L \cdot D_1 + L \cdot D_2$

$\Rightarrow$ $L \cdot D_1 = \int_{D_1} c_1(L) \geq 0$ $\quad L \cdot D_2 = \int_{D_2} c_1(L) \geq 0$

$D_1 = L^{k_1}$, $D_2 = L^{k_2}$, $k_1, k_2 > 0$

$\Rightarrow$ $L \cdot D_1 + L \cdot D_2 \geq 2$ $\Rightarrow$ Contradiction. ⌟

Moreover, D must be a smooth curve: if $p \in D$ is a singular point, since $\dim |L| = 2$ we can find $D' \neq D \in |L|$ such that $p \in D'$; we would then have