We now illustrate how the singularities enter in a special case. Suppose that C and C' are two irreducible plane quartic curves having three ordinary double points p_i $(i = 1, 2, 3)$ in common.

⌜ By P281, $\deg C = 4$. and see the def of an ordinary double point on P228 ⌟

We assume that at each of these points the four tangent lines to the two curves are distinct. These curves then define an ideal $\mathcal{J}_{p_i} \subset \mathcal{O}_{p_i}$, which is contained in but not equal to the square m_i^2 of the maximal ideal and which we now shall describe:

⌜ $m_i = \{ f \in \mathcal{O}_p \mid f(p) = 0 \}$

$\mathcal{J}_{p_i} = \{ g_1 f_1 + g_2 f_2 \mid (f_1 = 0) = C, \ (f_2 = 0) = C' \}$
$g_1, g_2 \in \mathcal{O}_p$

$\Rightarrow \mathcal{J}_{p_i} \subset m_i^2$ since $f_i, \ i = 1, 2 \in m_i^2$.

$\mathcal{J}_{p_i} \neq m_i^2 = \{ z_1^2 h_1(z_1, z_2) + z_2^2 h_2(z_1, z_2) \mid h_1, h_2 \}$

for example

locally, $f_1 = z_1^2 (z_2^2 + z_1 z_2 + 2z_1^2)$, $f_2 = z_2^2 (z_1^2 + z_1 z_2 + z_2^2)$ ⌟

Choose local coordinates (x, y) relative to which C and C' have respective equations