$$\omega = -g(z_1, z_2)\frac{dz_1}{(\partial f/\partial z_2)(z_1, z_2)}$$

for g a polynomial of degree $\leq d-3$. We have seen that the number of monomials of degree $\leq d$ in n variables is $\binom{n+d}{d}$, and so

$$g(S) = h^0(S, \Omega^1) = \binom{d-1}{2} = \frac{(d-1)(d-2)}{2}.$$

Γ We have only to count the number of linealy indepent polynomls g. See P166 for the number of monomials of degree $\leq d$ in n variable.
See P219. $g(S) = \dim H^0(S, \Omega^1) = h^0(S, \Omega^1)$

$$= \binom{2+d-3}{d-3} = \binom{d-1}{d-3} = {}_{d-1}C_{d-3} = {}_{d-1}C_2 = \frac{(d-1)(d-2)}{2}$$

By P124,
$$\begin{aligned} H^1(M, \mathbb{C}) &= H^{1,0}(M) \oplus H^{0,1}(M) \\ &= H^{1,0}(M) \oplus \overline{H^{1,0}(M)} \\ &= H^0(M, \Omega^1) \oplus \overline{H^0(M, \Omega^1)} \end{aligned}$$

$$\Rightarrow g(S) = \frac{1}{2}\dim H^1(M, \mathbb{C}) = \frac{1}{2}\dim(H^0(M, \Omega^1))$$

$$+ \frac{1}{2}\dim \overline{H^0(M, \Omega^1)} = \dim H^0(M, \Omega^1) = h^0(M, \Omega^1)$$

for M Riemann surface. ┘

Later on we will see how to extend this formula to certain singular curves.

Cases $g = 0, 1$

First, let S be any compact Riemann surface of