alizations in higher dimensions. The principal fact that holds in general is this: the holomorphic Euler characteristic of a vector bundle $E \longrightarrow M$ on a compact complex manifold is a topological invariant of E and M. In these terms, the essential point of the classical Riemann - Roch formula is the duality $h^1(D) = h^0(K-D)$.

Pf $h^0(K-D) = h^0(D) - d + g - 1 = \dim H^0(S, \mathcal{O}(D)) - d + \dim H^0(S, \Omega^1(D)) - 1$

$h^1(D) = \dim H^1(S, \mathcal{O}(D)) = \dim H^0(S, \Omega^1(-D)) = \dim H^0(S, \mathcal{O}(K-D)) = h^0(K-D) = h^0(D) - d + g - 1.$

Canonical Curves

Let S be a compact Riemann surface of genus $g \geq 2$, K the canonical bundle on S. We note immediately that the complete linear system $|K|$ has no base points: if $p \in S$ were in the base locus of $|K|$, we would have

$$h^0(K-p) = h^0(K) = g,$$

and hence by Riemann - Roch

$$h^0(p) = \deg(p) - g + 1 + h^0(K-p) = 1 - g + 1 + g = 2,$$

i.e., there would exist a nonconstant meromorphic function on S holomorphic on $S - \{p\}$ and having only a single pole at p, so S would be biholomorphic to $\mathbb{P}^1$.