The pencil $\{C_\lambda\}$ therefore contains six conics tangent to C, and consequently

$$\deg V_C = 6.$$

「 Note: ① By the result on P255, $\exists$ a 4-sheeted covering $\mathbb{P}^1$ of $\mathbb{P}^1$ with 6 distinct branch points.
$\Rightarrow$ Since C is biholomorphic to $\mathbb{P}^1$, we have a 4-sheeted covering C of $\mathbb{P}^1$ with 6 distinct branch points.

② By the basic correspondence on P283, $\exists$ a pencil $\{D_\lambda\}$ on C without base points which corresponds to the branched covering above. (Later, we will prove the correspondence.)

Then, by the exact sequence on P139,

$$0 \longrightarrow \mathcal{O}_{\mathbb{P}^2}([2H] \otimes [-C]) \longrightarrow \mathcal{O}_{\mathbb{P}^2}(2H) \longrightarrow \mathcal{O}_C(2H|_C) \longrightarrow 0$$

we have the following exact sequence

$$0 \longrightarrow \underset{\mathbb{C}}{\underset{\|}{H^0(\mathbb{P}^2, \mathcal{O})}} \longrightarrow \underset{\mathbb{C}^6}{\underset{\|}{H^0(\mathbb{P}^2, \mathcal{O}(2H))}} \longrightarrow \underset{\underset{\mathbb{C}^5.}{\underset{\|}{H^0(\mathbb{P}^1, \mathcal{O}(4H))}}}{\underset{\|}{H^0(C, \mathcal{O}_C(2H|_C))}} \longrightarrow \underset{0}{\underset{\|}{H^1(\mathbb{P}^2, \mathcal{O})}}$$

$\Rightarrow \exists$ a pencil of conics $\{C_\lambda\}$ in $\mathbb{P}^2$ s.t

$$C_\lambda \cap C = D_\lambda.$$

Thus we proved that $\exists$ a pencil $\{C_\lambda\}$ in $H^0(\mathbb{P}^2, \mathcal{O}(2H))$ so that the corresponding map $\pi : C \longrightarrow \mathbb{P}^1$ is a branched, 4-sheeted covering with