$$\mathbb{C}^2 \times \mathbb{C} \xrightarrow{F} \Delta^3 \quad \Rightarrow \quad |J(F)| \neq 0 \text{ at } 0.$$

$$((z_1, z_2), w) \longmapsto (f_1, f_2, f_3 + w)$$

$\Rightarrow$ By the inverse function theorems $\exists$ locally invers G s.t $G(f_1, f_2, f_3 + w) = (z_1, z_2, w)$.

$\Rightarrow G(f_1, f_2, f_3) = (z_1, z_2, 0)$ if $w = 0$.

$\Rightarrow G$ gives a biholomorphic map between the image of f and $\mathbb{C}^2$ locally. $\Rightarrow$ The image of f is a hypersurface of Δ^3 around 0.

$x \in f(W) \Rightarrow \exists\ a_0 \in W$ s.t $f(a_0) = x$.

$\Rightarrow M - W$ is open and dense in $M \Rightarrow \exists \{a_n\} \subset M - W$ s.t $a_n \longrightarrow a_0$. Since f is continuous, $f(a_n) \to f(a_0) = x \Rightarrow x \in \overline{f(M-W)} \Rightarrow \overline{f(M-W)} \supset f(M-W) \cup f(W)$.

$x \in \overline{f(M-W)} - f(M-W) \Rightarrow \exists \{x_n\} \subset f(M-W)$ s.t $x_n \longrightarrow x$. $\Rightarrow \exists \{a_n\} \subset M - W$ s.t $f(a_n) = x_n$.

Choose a relatively compact open set $U \ni x$. $f^{-1}(U)$ is open in M and $f^{-1}(U) \supset a_n$'s infinitely many. $\Rightarrow \exists$ a limit point a_0 in $f^{-1}(U)$. $\Rightarrow f(a_0) = x$.

$\Rightarrow f(a_0) \in f(M) \subset f(M-W) \cup f(W) \Rightarrow f(a_0) = x \in f(W)$ since we assumed $x \notin f(M-W)$.

The problem is therefore to show that the two pieces $f(M-W)$ and $f(W)$ fit together nicely.

$\overline{f(M-W)}$ need not be analytic since we only saw that if $\cup V_i = V^*$, where V_i's components, then $\overline{V_i}$ is analytic

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