We have only to prove that

$$i(u_\alpha^*)(\omega \otimes u) = \pm \omega \otimes i(u_\alpha^*)u.$$
$$(\text{on } \Lambda^{k-m}W \otimes \Lambda^m U) \qquad (\text{on } \Lambda^m U).$$

$i(u_\alpha^*)(\omega \otimes u)$ corresponds to $i(u_\alpha^*)(\omega \wedge u)$ (on $\Lambda^k V$) in $\Lambda^{k-1}V$.

$\Rightarrow \langle i(u_\alpha^*)(\omega \wedge u), \, Z \rangle = \langle \omega \wedge u, \; u_\alpha^* \wedge Z \rangle$

Let $Z = \sum \omega^* \wedge u^*$ since $\Lambda^{k-1}V^* = \Lambda^{k-1}(W^* \oplus U^*) = \bigoplus_{m=0}^{k} (\Lambda^{k-m}W^*) \otimes (\Lambda^{m-1}U^*)$

$\Rightarrow$ In the sum, we only need $\omega_\beta^* \wedge u_\beta^*$ with $\deg \omega^* = \deg \omega$, $\deg u_\beta^* = \deg u - 1$ since other terms become zero.

$\Rightarrow \langle \omega \wedge u, \; u_\alpha^* \wedge \omega_\beta^* \wedge u_\beta^* \rangle = \langle \omega \wedge u, \pm \omega_\beta^* \wedge u_\alpha^* \wedge u_\beta^* \rangle$
$= \pm \langle \omega, \omega_\beta^* \rangle \langle u, u_\alpha^* \wedge u_\beta^* \rangle$ $\cdots$ (proof later)
$= \pm \langle \omega \otimes u, \; \omega_\beta^* \otimes u_\alpha^* \wedge u_\beta^* \rangle$
$= \pm \langle \omega, \omega_\beta^* \rangle \langle i(u_\alpha^*)u, \; u_\beta^* \rangle$
$= \pm \langle \omega \wedge i(u_\alpha^*)u, \; \omega_\beta^* \wedge u_\beta^* \rangle = \pm \langle \omega \otimes i(u_\alpha^*)u, \; \omega_\beta^* \otimes u_\beta^* \rangle$

Thus we have
$\langle \omega \overset{(\otimes)}{\wedge} u, \; u_\alpha^* \wedge \omega_\beta^* \wedge u_\beta^* \rangle = \langle i(u_\alpha^*)(\omega \wedge u), \; \omega_\beta^* \wedge u_\beta^* \rangle$
$= \pm \langle \omega \underset{(\otimes)}{\wedge} i(u_\alpha^*)(u), \; \omega_\beta^* \wedge u_\beta^* \rangle$

$\Rightarrow$ We may have $i(u_\alpha^*)(\omega \otimes u) = \omega \otimes i(u_\alpha^*)u$ by making adjustments for sign.