coordinates in $\Delta$ and $l = [l_1, \cdots l_n]$ corresponding homogeneous coordinates on $\mathbb{P}^{n-1}$. Let $\tilde{\Delta} \subset \Delta \times \mathbb{P}^{n-1}$ be the submanifold of $\Delta \times \mathbb{P}^{n-1}$ given by the quadratic relations

$$\tilde{\Delta} = \{ (z, l) ; z_i l_j = z_j l_i \text{ for all } i, j \}.$$

If we consider points $l \in \mathbb{P}^{n-1}$ as lines in $\mathbb{C}^n$, then writing these equations as $z \wedge l = 0$ we see that this is just the incidence correspondence defined as $\{ (z, l) : z \in l \}$.

$\Gamma$ $\tilde{\Delta} = \{ (z, l) \in \Delta \times \mathbb{P}^{n-1} : z_i l_j = z_j l_i \text{ for all } i, j \}$

$$\frac{z_i}{z_j} = \frac{l_i}{l_j} = k \quad \Rightarrow \quad z_i = k z_j \quad l_i = k l_j$$

$$\frac{z_1}{z_2} = \frac{l_1}{l_2} \qquad \frac{z_i}{z_j} = \frac{l_i}{l_j} \quad \Rightarrow \quad \frac{z_i}{l_i} = \frac{z_j}{l_j} = k'$$

$$\Rightarrow z \in l. \quad \Leftrightarrow \quad z \wedge l = 0$$

//

Now $\tilde{\Delta}$ maps onto $\Delta$ via projection on the first factor $\pi : (z, l) \longmapsto z$; from the geometric interpretation it follows that the map is an isomorphism away from the origin in $\Delta$, and $\pi^{-1}(0)$ is just the projective space of lines in $\Delta$.

$\Gamma$

$$\begin{tikzcd} \tilde{\Delta} = \{ (z, l) \in \Delta \times \mathbb{P}^{n-1} \mid z \in l. \} \arrow[d, "\pi"] \arrow[r, phantom, "\ni"] & (z, l) \arrow[d, mapsto] \\ \Delta & z \end{tikzcd}$$

$$\pi^{-1}(0) = \{ (0, l) \} = \mathbb{P}^{n-1}. \qquad z = z' \neq 0$$

$$\Rightarrow [z] = l = [z'] = l' \Rightarrow \pi \text{ is } 1\text{-}1. \; //$$