Fix $z_0' \in \mathbb{C}^{n-1} - \pi(V)$. Let $z_n^1, \cdots z_n^{\ell}$ be roots of $\underset{f_3(z_0', z_n)}{\overset{\shortparallel}{f_3(z_n)}}$ $=0$. Choose $x \in \mathbb{C}$ s.t

$$x \neq -\frac{f_1(z_n^i)}{f_2(z_n^i)} \quad \text{in case } f_2(z_n^i) \neq 0, \ i = 1 \cdots \ell.$$

Let $F(1, x, 1) = f_1(z_0', z_n) + x f_2(z_0', z_n) + f_3(z_0', z_n)$

$F(1, x, 2) = f_1(z_0', z_n) + x f_2(z_0', z_n) + 2 f_3(z_0', z_n)$.

We may choose x so that either $F(1, x, 1)$ or $F(1, x, 2)$ is of type $\overset{\text{nonzero}}{\square} z_n^p + a_1(z_0') z_n^{p-1} + \cdots$.

$\Rightarrow$ $F(1, x, 1)(\alpha) = 0$ & $F(1, x, 2)(\alpha) = 0$ $\Rightarrow$

$f_3(z_0', \alpha) = 0 \Rightarrow \alpha = z_n^i$ for some i.

$\Rightarrow f_1(z_0', z_n^i) + x f_2(z_0', z_n^i) = 0$

① $f_2(z_0', z_n^i) = 0$

$\Rightarrow f_1(z_0', z_n^i) = 0 \Rightarrow (z_0', z_n^i) \in V \Rightarrow *$

② $f_2(z_0', z_n^i) \neq 0$

$f_1(z_0', z_n^i) + x f_2(z_0', z_n^i) \neq 0$ by the choice of x.

$\Rightarrow F(1, x, 1)$ & $F(1, x, 2)$ have no common zeros along the line $\pi^{-1}(z_0')$. Furthermore, if $f_1(z', z_n) + x f_2(z', z_n) + f_3(z', z_n) = F$ & $F' = f_1(z', z_n) + x f_2(z', z_n) + 2 f_3(z', z_n)$ have a common factor $h(z', z_n)$, i.e.,

$$F = h \cdot g$$
$$F' = h \cdot g',$$

plug $z' = z_0'$. $\Rightarrow$ $F(1, x, 1) = h(z_0', z_n)\, g(z_0', z_n)$

$F(1, x, 2) = h(z_0', z_n)\, g'(z_0', z_n)$

Since $F(1, x, 1)$ & $F(1, x, 2)$ have no common zeros