

$$= \begin{pmatrix} g_1 \cdot v^1, \dots, g_1 \cdot v^n \\ g_2 \cdot v^1, \dots, g_2 \cdot v^n \\ \vdots \\ g_k \cdot v^1, \dots, g_k \cdot v^n \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \vdots \\ \omega_k \end{pmatrix}$$

$$\Rightarrow \omega_1 = (g_1 \cdot v^1, \dots, g_1 \cdot v^n)$$

$$\omega_2 = (g_2 \cdot v^1, \dots, g_2 \cdot v^n)$$

\vdots

$$\omega_k = (g_k \cdot v^1, \dots, g_k \cdot v^n)$$

$$y_1 \omega_1 + \dots + y_k \omega_k = v = x_1 v_1 + \dots + x_n v_n$$

$$= (y_1 g_1 \cdot v^1, \dots, y_1 g_1 \cdot v^n) + (y_2 g_2 \cdot v^1, \dots, y_2 g_2 \cdot v^n)$$

$$+ \dots + (y_k g_k \cdot v^1, \dots, y_k g_k \cdot v^n)$$

$$= \{ (y_1 g_1 + y_2 g_2 + \dots + y_k g_k) \cdot v^1, (y_1 g_1 + y_2 g_2 + \dots + y_k g_k) \cdot v^2$$

$$\Rightarrow (y_1 g_1 + y_2 g_2 + \dots + y_k g_k) \cdot v^1 = x_1 v_1 + \dots + x_n v_n$$

$$= X \cdot v^1$$

$$(y_1 g_1 + \dots + y_k g_k) \cdot v^2 = X \cdot v^2$$

\vdots

$$\Rightarrow y_1 g_1 + \dots + y_k g_k = X \quad \text{since } v^1, \dots, v^n \text{ generates } \mathbb{C}^k$$

$$= (g_{11} y_1 + g_{21} y_2 + \dots + g_{k1} y_k, \dots, g_{1n} y_1 + \dots + g_{kn} y_k)$$

$$= (x_1, x_2, \dots, x_n)$$

$$\Rightarrow g_{ij} y_i = x_j$$

$$\Rightarrow {}^t g_{ji} y_i = x_j$$

$$= ({}^t g \ Y) \ j = X_j$$