Lemma. Given an open set $U \subset \mathbb{R}^n$ and $T \in \mathcal{D}(U)$. with $\Delta T = 0$, then $T = T_\psi$ for a function ψ harmonic in U.

Proof. Given $V \subset U$ a relatively compact open subset, then for $\varphi \in C_c^\infty(V)$ and ϵ sufficiently small, $\operatorname{supp} \varphi_\epsilon \subset U$.

⌈ $\overline{V} \subset U$ $\overline{V}$ compact. 」

We can then define T_ϵ by

$$T_\epsilon(\varphi) = T(\varphi_\epsilon),$$

and repeat the previous argument to conclude that $T_\epsilon = T_{\psi_V}$ for some ψ_V harmonic in V.

⌈ $T_\epsilon(x) = T_y(\chi_\epsilon(x-y))$ $\chi_\epsilon(x-y)$ has supp $= x - \epsilon K$

In general, $\Rightarrow T_\epsilon(x)$ is not defined on V, but, for $x \in V$, if we choose ϵ small enough, then $x - \epsilon K \subset U$. Thus $T_\epsilon(x)$ is well-defined on V. So we can apply the properties of T_ϵ (P374) & the arguments on P376 ~ P378 to this local case. $\Rightarrow$ We can conclude that $T_\epsilon = T_{\psi_V}$ for some ψ_V harmonic in V. 」

Since ψ_V is the same for all ϵ, if $V \subset W \subset U$ we have $\psi_W|_V = \psi_V$.

⌈ $\epsilon_1 < \epsilon_2$. If $T_{\epsilon_1} = T_{\psi_V}$ and $T_{\epsilon_2} = T_{\phi_V}$.

$\psi_V = \phi_V = T_\delta(x)$, $\delta > 0$ in the proof of the lemma P376~7.