

in homology  $f_*([S]) = n \cdot [S']$ ;

the integer  $n$  is called the sheet number, or degree, of the map. For any point  $p \in S'$ , let  $\Theta$  be a curvature form for the line bundle  $[(p)]$  associated to the divisor  $(p)$ . Then  $f^*\Theta$  is a curvature form for the line bundle  $f^*[(p)] = [f^*(p)]$  on  $S$ , and we see from the projection in Section 2 of Chapter 1 that

$$\deg f^*(p) = \int_S f^*\left(\frac{i}{2\pi} \Theta\right) = n \int_S \left(\frac{i}{2\pi} \Theta\right) = n$$

so the map  $f$  assumes all values  $p \in S'$  exactly  $n$  times, counting multiplicity in the sense of divisors.

⌈ See p144. "In the sense of divisors" means that multiplicities are considered.  $\cup$

For any  $p \in S$ , we can find local coordinates  $z$  around  $p$  in  $S$  and  $w$  near  $f(p)$  in  $S'$  such that the map  $f$  is given locally by

$$w = z^v.$$

⌈ 
$$\begin{array}{ccc} S & \xrightarrow{f} & S' \\ \downarrow & & \downarrow \\ p & \longmapsto & f(p) \end{array} \Rightarrow \begin{array}{l} \text{Choose local coordinates } z \text{ \& } w \\ \text{s.t. } p \longmapsto 0 \text{ in the } z \\ f(p) \longmapsto 0 \text{ in the } w. \end{array}$$

$\Rightarrow w = a_1 z + a_2 z^2 + \dots \Rightarrow$  But since  $f$  covers generically, constant number of times,  $w = z^v g(z)$

where  $g(z) \neq 0$  at  $z=0$ . By putting  $z' = z g(z)^{\frac{1}{v}}$ ,

we get  $w = z'^v$ .  $\cup$

Comment:  $z' = z g(z)^{\frac{1}{v}} \Rightarrow \frac{dz'}{dz} = z \frac{dg}{dz} + g(z)$  is not zero at  $z=0 \Rightarrow \exists$  an inverse function.  $\cup$