sending any smooth point of the image to its tangent plane in $\mathbb{P}^n$, is a rational map: explicitly, if φ is given locally by

$$\varphi(z) = [\varphi_0(z), \cdots \varphi_n(z)],$$

then the map $\mathcal{G}$ is given in terms of the Plücker embedding

$$G(k+1, n+1) \longrightarrow \mathbb{P}(\Lambda^{k+1}\mathbb{C}^{n+1})$$

by the minors of the Jacobian matrix $\partial\varphi_i/\partial z_\alpha$ of φ.

$$J(\varphi) = \begin{pmatrix} \frac{\partial\varphi_0}{\partial z_1}, & \cdots & \frac{\partial\varphi_0}{\partial z_k} \\ \vdots & & \\ \frac{\partial\varphi_n}{\partial z_1}, & \cdots & \frac{\partial\varphi_n}{\partial z_k} \end{pmatrix}$$

has rank $k+1$ if z is a smooth point.

$\operatorname{cod}\{z \in M : \operatorname{rank} J(\varphi) \leq k\} \geq 2$

$\Rightarrow$ Some $(k+1)\times(k+1)$ minor is nonzero.

By P193 & P209,

$$\begin{array}{ccc} J(\varphi) & \longmapsto & [\cdots |J(\varphi)|_{(k+1)\times(k+1)} \cdots] \\ \cap & & \cap \\ G(k+1, n+1) & \longrightarrow & \mathbb{P}(\Lambda^{k+1}\mathbb{C}^{n+1}) \end{array}$$

and see P21 proposition $(\operatorname{Im}\varphi)_s \neq \operatorname{Im}\varphi$

5. If $V \subset \mathbb{P}^n$ is any variety, we may define a rational map

$$V^k \longrightarrow G(k, n+1),$$

from the k-fold product of V with itself to the Grassmannian of $(k-1)$-planes in $\mathbb{P}^n$, by

$$(v_1, \cdots v_k) \longmapsto \overline{v_1, \cdots v_k}.$$