pf $n=2$. $f: \mathbb{C}^2 \longrightarrow \mathbb{C}$ holomorphic, $f(0,0)=0$.

Consider $g = |f|^2 = u^2+v^2$, $f = u+iv$.

Claim: Critical points of g are isolated. (on $\{u^2+v^2 \neq 0\}$)

At $(z_1^0, z_2^0) \in \{u^2+v^2\neq 0\}$, $Dg=0$.

$$\Rightarrow \left.\begin{aligned} \frac{\partial g}{\partial x_1} &= 2u\frac{\partial u}{\partial x_1} + 2v\frac{\partial v}{\partial x_1} = 0, \\ \frac{\partial g}{\partial y_1} &= 2u\frac{\partial u}{\partial y_1} + 2v\frac{\partial v}{\partial y_1} = 0 \\ \frac{\partial g}{\partial x_2} &= 2u\frac{\partial u}{\partial x_2} + 2v\frac{\partial v}{\partial x_2} = 0 \\ \frac{\partial g}{\partial y_2} &= 2u\frac{\partial u}{\partial y_2} + 2v\frac{\partial v}{\partial y_2} = 0 \end{aligned}\right\} \circledast$$

$z_1 = x_1+iy_1$, $z_2 = x_2+iy_2$

Since f is holomorphic,

$$\frac{\partial f}{\partial z_1} = \frac{\partial u}{\partial x_1} + i\frac{\partial v}{\partial x_1} = -i\left(\frac{\partial u}{\partial y_1} + i\frac{\partial v}{\partial y_1}\right)$$

$$\frac{\partial f}{\partial z_2} = \frac{\partial u}{\partial x_2} + i\frac{\partial v}{\partial x_2} = -i\left(\frac{\partial u}{\partial y_2} + i\frac{\partial v}{\partial y_2}\right)$$

$$\Rightarrow \frac{\partial u}{\partial x_1} = \frac{\partial v}{\partial y_1}, \quad \frac{\partial v}{\partial x_1} = -\frac{\partial u}{\partial y_1}, \quad \frac{\partial u}{\partial x_2} = \frac{\partial v}{\partial y_2}, \quad \frac{\partial v}{\partial x_2} = -\frac{\partial u}{\partial y_2}$$

$\Rightarrow \circledast$ becomes

$$\left.\begin{aligned} u\frac{\partial v}{\partial y_1} + v\frac{\partial v}{\partial x_1} &= 0 \\ -u\frac{\partial v}{\partial x_1} + v\frac{\partial v}{\partial y_1} &= 0 \end{aligned}\right\} \Rightarrow (u^2+v^2)\frac{\partial v}{\partial x_1} = 0 \Rightarrow \frac{\partial v}{\partial x_1} = 0 = \frac{\partial u}{\partial y_1}$$

$$\left.\begin{aligned} u\frac{\partial v}{\partial y_2} + v\frac{\partial v}{\partial x_2} &= 0 \\ -u\frac{\partial v}{\partial x_2} + v\frac{\partial v}{\partial y_2} &= 0 \end{aligned}\right\} \Rightarrow (u^2+v^2)\frac{\partial v}{\partial x_2} = 0 \Rightarrow \frac{\partial v}{\partial x_2} = 0 = \frac{\partial u}{\partial y_2}$$