

$$= \{ \eta_{\alpha\omega} d\sigma_\alpha - (\tilde{\theta} \wedge \eta + t \eta \wedge \eta)_{\alpha\omega} \sigma_\alpha \} \otimes e_\omega$$

\Rightarrow

$$D_t(\eta(\sigma)) + \eta(D_t(\sigma))$$

$$= d(\eta_{\alpha\omega}^\sigma) - (\eta \wedge \tilde{\theta} + t \eta \wedge \eta)_{\alpha\omega} \sigma_\alpha \otimes e_\omega$$

$$+ \{ \eta_{\alpha\omega} d\sigma_\alpha - (\tilde{\theta} \wedge \eta + t \eta \wedge \eta)_{\alpha\omega} \sigma_\alpha \} \otimes e_\omega$$

$$= d(\eta_{\alpha\omega} \sigma_\alpha) \otimes e_\omega + \eta_{\alpha\omega} d\sigma_\alpha \otimes e_\omega$$

$$- (\eta \wedge \tilde{\theta} + \tilde{\theta} \wedge \eta)_{\alpha\omega} \sigma_\alpha \otimes e_\omega - 2t(\eta \wedge \eta)_{\alpha\omega} \sigma_\alpha \otimes e_\omega$$

$$\parallel$$

$$\{ (\eta \wedge \tilde{\theta} + \tilde{\theta} \wedge \eta)_{\alpha\omega} e_\alpha^* \otimes e_\omega \} (\sigma_\beta e_\beta)$$

\parallel

$$(\eta \wedge \tilde{\theta} + \tilde{\theta} \wedge \eta)(\sigma) \} - 2t(\eta \wedge \eta)_{\alpha\omega} e_\alpha^* \otimes e_\omega \} (\sigma_\beta e_\beta)$$

\parallel

$$(-2t \eta \wedge \eta)(\sigma)$$

$$d(\eta_{\alpha\omega} \sigma_\alpha) \otimes e_\omega + \eta_{\alpha\omega} d\sigma_\alpha \otimes e_\omega = (d\eta)_{\alpha\omega} \sigma_\alpha \otimes e_\omega$$

$$= (d\eta)_{\alpha\omega} e_\alpha^* \otimes e_\omega (\sigma_\beta e_\beta)$$

$$= (d\eta)(\sigma)$$

$$\text{Thus } D_t \eta = d\eta - (\tilde{\theta} \wedge \eta + \eta \wedge \tilde{\theta}) - 2t \eta \wedge \eta.$$

$$\Rightarrow D_t \eta = \left(\frac{\partial}{\partial t} \right) \Theta_t.$$

$$d\tilde{P}(\eta, \Theta_t, \dots, \Theta_t) = \tilde{P}(D_t \eta, \Theta_t, \dots, \Theta_t)$$

$$- \tilde{P}(\eta, D_t \Theta_t, \dots, \Theta_t) - \tilde{P}(\eta, \dots, D_t \Theta_t) - \dots$$

$$= \tilde{P}(D_t \eta, \Theta_t, \dots, \Theta_t) \text{ since } D_t \Theta_t = 0.$$

$$\Rightarrow d(\kappa \tilde{P}(\eta, \Theta_t, \dots, \Theta_t)) = \kappa \tilde{P}(D_t \eta, \Theta_t, \dots, \Theta_t) =$$