But we have seen that the locus of lines in G (resp. F) through any point x is just the intersection $T_x(G) \cap G$ (resp. $T_x(F) \cap F$), so the locus of lines in $X = F \cap G$ through x is

$$T_x(X) \cap X = T_x(F) \cap F \cap T_x(G) \cap G.$$

$\ulcorner$ By the results on P735 ~ P736 ⊛ (refer to P757), the locus of lines through x in G (F) $= T_x G \cap G$ ($T_x(F) \cap F$).

$\Rightarrow$ Clearly, $T_x(X) \cap X \subset T_x(F) \cap F$ $\Rightarrow$
$T_x(X) \cap X \subset T_x(G) \cap G$

$T_x(X) \cap X \subset T_x(F) \cap F \cap T_x(G) \cap G$.

Given any line $l_x \subset T_x(F) \cap F \cap T_x(G) \cap G$, then since $l_x \subset F \cap G$, $T_x(X) \supset l_x \Rightarrow l_x \subset T_x(X) \cap X$.

$\Rightarrow$ $T_x(X) \cap X = T_x(F) \cap F \cap T_x(G) \cap G$, as sets.

$\lrcorner$

$T_x(X) \cap X$ has degree 4, and – making the final assumption that it contains no multiple components – it must consist of four lines. We thus have

$$\deg S = \#(l_x \cap S) = 4.$$

$\ulcorner$ $T_x(X) \cap X = T_x(F) \cap T_x(G) \cap F \cap G = \mathbb{P}^3 \cap F \cap G$
$= \mathbb{P}^3 \cap F \cap \mathbb{P}^3 \cap G \sim 2H \cdot 2H \sim 4H^2$ in $\mathbb{P}^3$, where H is the hyperplane in $\mathbb{P}^3$. $\Rightarrow \deg(T_x(X) \cap X) = 4$.
$\dim(T_x(X) \cap X) = 3 + 3 - 5 = 1$ and $T_x(X) \cap X =$ Union of

CREART®