In the same way, a meromorphic section s of L over U – defined to be a section of the sheaf $\mathcal{O}(L) \otimes_{\mathcal{O}} \mathcal{M}$ – is given by a collection of meromorphic functions $s_\alpha \in \mathcal{M}(U \cap U_\alpha)$ satisfying $s_\alpha = g_{\alpha\beta} \cdot s_\beta$ in $U \cap U_\alpha \cap U_\beta$. Note that the quotient of two meromorphic sections $s, s' \not\equiv 0$ of L is a well-defined meromorphic function.

Γ $$\mathcal{O}(L)(U_\alpha) \otimes_{\mathcal{O}(U_\alpha)} \mathcal{M}(U_\alpha) = \left(\mathcal{O}(L) \otimes_{\mathcal{O}} \mathcal{M}\right)(U_\alpha)$$

$$\Rightarrow \begin{tikzcd}
U_\alpha \times \mathbb{C} \arrow[dr] & L_{U_\alpha} \arrow[l, "\varphi_\alpha"'] \arrow[d] \arrow[r, "\varphi_\beta"] & U_\beta \times \mathbb{C} \\
& U_\alpha \arrow[ul, bend left] \arrow[u, bend right, "\bar{s}_\alpha"'] \arrow[ur, bend right] &
\end{tikzcd}$$

$(x, s_\alpha(x))$ $\qquad U_\alpha = U_\beta \qquad (x, s_\beta(x))$

$$\Rightarrow \quad s_\beta(x) = \varphi_\beta \circ \varphi_\alpha^{-1}|_x (s_\alpha(x)) = g_{\alpha\beta}(x)\, s_\alpha(x)$$

Given any section s of $\mathcal{O}(L)(U_\alpha) \otimes_{\mathcal{O}(U_\alpha)} \mathcal{M}(U_\alpha)$ on U_α, s_α can be expressed as $\sum_i s_{\alpha,i} \otimes_{\mathcal{O}} f_{\alpha,i}$ where $s_{\alpha,i} \in \mathcal{O}(L)(U_\alpha)$ and $f_{\alpha,i} \in \mathcal{M}(U_\alpha)$.

$\Rightarrow$ Since $s_{\alpha,i}$ can be identified with $\overline{s_{\alpha,i}} \in \mathcal{O}(U_\alpha)$, s.t

$g_{\beta\alpha}\, \overline{s_{\alpha,i}}(x) = \overline{s_{\beta,i}}(x)$ for each i,

$\sum_i s_{\alpha,i} \otimes_{\mathcal{O}} f_{\alpha,i}$ can be expressed as $\sum_i \overline{s_{\alpha,i}} \otimes_{\mathcal{O}} f_{\alpha,i}$

$$= \sum_i 1 \otimes \overline{s_{\alpha,i}} f_{\alpha,i} \in \mathcal{M}(U_\alpha).$$

On $U_\alpha \cap U_\beta$, $\sum_i \overline{s_{\alpha,i}} \otimes_{\mathcal{O}} f_{\alpha,i} = \sum g_{\alpha\beta}\, \overline{s_{\beta,i}} \otimes_{\mathcal{O}} f_{\beta,i}$

$$= g_{\alpha\beta} \otimes 1 \left(\sum \overline{s_{\beta,i}} \otimes f_{\beta,i}\right) \Rightarrow \sum_i \overline{s_{\alpha,i}} \otimes f_{\alpha,i} = g_{\alpha\beta} \sum \overline{s_{\beta,i}} \otimes f_{\beta,i}$$

$$\sum_i 1 \otimes \overline{s_{\alpha,i}} f_{\alpha,i} = g_{\alpha\beta} \sum 1 \otimes \overline{s_{\beta,i}} f_{\beta,i}.$$