Note that we can identify the Poincaré dual of the pull-back map on cohomology.
Explicitly, if $f: M \longrightarrow N$ is a C^∞ map of manifolds nonsingular over the cycle $A \subset N$, (f is transverse (to) over A) then with the proper orientation the cycle $f^{-1}(A)$ is ~~po~~ Poincare dual to the pull-back via f of the Poincare dual of A.
This is not hard to see: if $B \subset M$ any cycle on M meeting $f^{-1}(A)$ transversely, then $f(B)$ will meet A transversely at $f(B \cap f^{-1}(A))$.
If φ is a closed form on N Poincare dual to A, then

$$\int_B f^*\varphi = \int_{f(B)} \varphi = \#(A \cdot f(B)) = \#(f^{-1}(A) \cdot B)$$

so $f^{-1}(A)$ is Poincare dual to $f^*\varphi$.

$$\begin{array}{ccc} M & \xrightarrow{f} & N \\ \cup & & \cup \\ f^{-1}(A) & \longrightarrow & A \end{array}$$

If B intersects with $f^{-1}(A)$ transversely, let $p \in B \cap f^{-1}(A)$,
$\Rightarrow T_p(B) + T_p(f^{-1}(A)) = T_p M$.

$$f(B) \cap A \ni q.$$

Question: $T_q(f(B)) + T_q A = T_q N$?

$f(p) = q$, $\Rightarrow$ f is transverse to $A \Leftrightarrow$
$f_*(T_p M) + T_q A = T_q N$.

since $T_p M = T_p(B) + T_p(f^{-1}(A))$ and