The Levi Extension and Proper Mapping Theorems

We first recall the statement of Remmert's

Proper Mapping Theorem. Let U and N be complex manifolds, $M \subset U$ an analytic subvariety, and $f: U \longrightarrow N$ a holomorphic mappings whose restriction to M is proper. Then the image $f(M)$ is an analytic subvariety of N.

We shall give a proof of this result under one additional technical assumption, which will be trivially satisfied in all of our applications. This is:

For each smooth point $p \in M$ and each k-plane Λ_p in the tangent space to M at p ($k \leq n = \dim M$), there is a k-dimensional analytic subvariety Z of M having Λ_p as tangent plane at p.

⌈ To each smooth point $p \in M$, choose a k-plane Λ_p in the tangent space to M at p. $\Rightarrow \exists$ a k-dimensional analytic subvariety Z of M having Λ_p as tangent plane at p. This is not the case. ⌋

In practice U will be an open subset of an algebraic variety V, and we may take Z to be a linear