

$$\begin{aligned}
 \int_U (\varphi - \log|h|) \partial\psi &= \int_U \varphi \partial\psi - \int_U (\log|h|) \partial\psi \\
 &= (\partial\varphi)(\psi) - \int_U (\log|h|) d\psi = (\partial\varphi)(\psi) - (d\log|h|)(\psi) \\
 &= (dj)(\psi) \Rightarrow \text{In the distributional sense.}
 \end{aligned}$$

$$\partial\varphi = d\log|h| + dj \quad (\text{on } U) \leftarrow \text{locally.}$$

$\Rightarrow \partial\varphi$ is closed, and meromorphic 1-form on $U \subset \Delta^{n+1} - f(W)$. $\Rightarrow \partial\varphi$ is closed and meromorphic 1-form on $\Delta^{n+1} - f(W)$.

Wrong!

$$\begin{aligned}
 \varphi - \log|h| &= f, \quad \bar{j} = f + \sqrt{-1}g \Rightarrow \partial\bar{j} = \partial f + \sqrt{-1}\partial g \\
 \Rightarrow \partial\bar{j} &= \partial f - \sqrt{-1}\partial g = 0 \quad \Rightarrow \partial f = \sqrt{-1}\partial g.
 \end{aligned}$$

For $\psi \in A_c^{n,n+1}(U)$,

$$\begin{aligned}
 (\partial\varphi)(\psi) &= - \int_U \varphi \partial\psi = - \int_U (\log|h|) \partial\psi \\
 &\quad - \int_U f \partial\psi.
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \int_U f \partial\psi &= \int_U \partial(f\psi) - \int_U \partial f \wedge \psi \\
 &= \int_U d(f\psi) - \int_U \partial f \wedge \psi
 \end{aligned}$$

$$\stackrel{\text{by Stokes' theorem and } \text{supp } \psi \subset U}{=} 0 - \int_U \partial f \wedge \psi = -\frac{1}{2} \int_U \partial\bar{j} \wedge \psi$$