NO.

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$\ulcorner$ F_i is a line in $\mathbb{P}^2$ $\Rightarrow$ F has ${}_6C_2$ double points $\Rightarrow$ Since $\#\{r(p_i)\} = 15$, the set of singular points of F is $\{r(p_i)\}$.

$\lrcorner$

It follows that each of the lines L_i contains exactly five of the points $r(p_i)$, and correspondingly that the plane $\overline{p_0 L_i} \subset \mathbb{P}^3$ contains exactly six of the double points p_i of S.

$\ulcorner$ L_1 intersects $L_2, \cdots L_6$ $\Rightarrow$ L_1 contains five of $r(p_i)$'s. $\Rightarrow$ $\overline{p_0 L_i}$ contains six of double points, since $\overline{p_0 p_i'}$ does not contain any other double point ($\because$ $\deg S = 4$), where p_i' is a double point on L_i.

$\lrcorner$

Our first observation, then, is that

Through each double point p of S there pass six hyperplanes, each containing six of the points p_i.

$\ulcorner$ $\overline{p_0 L_i}$ is a hyperplane in $\mathbb{P}^3$.

$\lrcorner$

Let us consider in more detail one of the hyperplanes $h = \overline{p_0, L}$ found in the last argument.