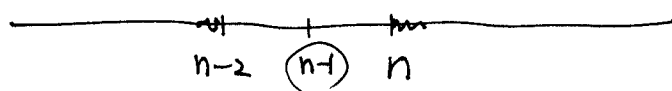


By (ii),  $H^{2k-1}(V, \mathbb{Q}) \cong H^{2k-1}(\mathbb{P}^n, \mathbb{Q}) = 0$  if  $2k-1 \leq n-2$  388  
 $\Leftrightarrow 2k+1 \leq n$



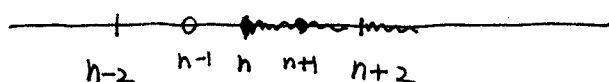
$$\Rightarrow 2k-1 \neq n-1 \Rightarrow 2k \neq n \quad k \neq \frac{n}{2}$$

$$\Rightarrow \text{If } k \neq \frac{n}{2}, \quad H^{2k-1}(V) = 0.$$

By (i),  $H^{2k+2}(\mathbb{P}^n, \mathbb{Z}) \cong H^{2k}(V, \mathbb{Z})$  if  $n \leq 2k$

$$\Leftrightarrow n \leq 2k \Leftrightarrow k \geq \frac{n}{2} \quad \dots (*)$$

By (ii),  $H^{2k}(\mathbb{P}^n, \mathbb{Z}) \cong H^{2k}(V, \mathbb{Q})$  if  $2k \leq n-2$  \*\*



$$\Rightarrow \text{If } 2k \neq n-1, \quad H^{2k}(V, \mathbb{Q}) \cong \mathbb{Q}.$$

By (\*),  $n = 2l-1, \Rightarrow k \geq \frac{2l-1}{2} = l - \frac{1}{2} \quad k = l.$

$$H^{2l}(V, \mathbb{Z}) \cong \mathbb{Z} \quad \text{if } 2l = 2k = n+1$$

By (\*\*),  $H^{2k}(V, \mathbb{Q}) \cong \mathbb{Q}$  if  $2k \leq n-2$ .

$$\Rightarrow H^{2k}(\mathbb{P}^n, \mathbb{Q}) \xrightarrow{\cong} H^{2k}(V, \mathbb{Q}) \quad \text{if } 2k \leq n-2.$$

$\Rightarrow H^{2k}(V, \mathbb{Q})$  is generated by the class of a  $k$ -plane section of  $V$  for  $k \leq \frac{n}{2}-1$

$$\begin{array}{ccc} H^{2k}(\mathbb{P}^n, \mathbb{Q}) & \longrightarrow & H^{2k}(V, \mathbb{Q}) \\ \parallel & \updownarrow & \parallel \\ \text{Hom}(H_{2k}(\mathbb{P}^n, \mathbb{Q}), \mathbb{Q}) & \longrightarrow & \text{Hom}(H_{2k}(V, \mathbb{Q}), \mathbb{Q}) \\ \Leftrightarrow & & H_{2k}(\mathbb{P}^n, \mathbb{Q}) \xleftarrow{\cong} H_{2k}(V, \mathbb{Q}) \quad \text{if } 2k \leq n-2 \end{array}$$