any two rows, so that the expression for $a_k(z)$ is independent of the order in which the points $A_j(z)$ are listed. Now as in the proof of 5. Theorem the functions $A_j(z)$ can be chosen so that they are holomorphic in an open nbd of any point of W_0; hence, the functions $a_k(z)$ are holomorphic in W_0. Furthermore, also as in the the proof of the preceding theorem, these functions a_k are continuous on W. It then follows from the extended form of Riemann's removable singularities theorem that a_k are holomorphic functions on all of W, so $Q_{f,g}(X) \in {}_n\mathcal{O}_W[X]$ as desired, and the proof is completed.

8. Corollary. If $\pi : V \to W$ is as above, then π^* exhibits ${}_V\mathcal{O}_V$ as an integral algebraic extension of ${}_n\mathcal{O}_W$ of degree at most ν. If, moreover, there is a function in ${}_V\mathcal{O}_V$ that seperates the sheets, then the degree of its extension is precisely ν, and as an ${}_n\mathcal{O}_W$-module ${}_V\mathcal{O}_V$ is isomorphic to a submodule of a free ${}_n\mathcal{O}_W$-module of rank ν.

Proof. In these circumstances Theorem 5 shows that for any $f \in {}_V\mathcal{O}_V$ there is a monic polynomial $P_f(X) \in {}_n\mathcal{O}_W[X] \subseteq {}_V\mathcal{O}_V[X]$ s.t $P_f(f) = 0$; that is just the condition that ${}_V\mathcal{O}_V$ be an integral algebraic extension of ${}_n\mathcal{O}_W$ of degree at most ν. If there is a function $f \in {}_V\mathcal{O}_V$ that seperates the sheets, then by Theorem 6 the polynomial $P_f(X)$ is of degree precisely ν and there is no nontrivial polynomial $P(X)$ of lower degree for which $P(f) = 0$, so the degree of this