

$\dim \{f_1 = \dots = f_n = 0\} = 0 \Rightarrow \mathbb{Z}$ is a finite set of points.

Thus locally $I = \{f_1, \dots, f_n\}$, where $n = \dim_{\mathbb{C}} M$. Setting $\mathcal{O}_{\mathbb{Z}} = \mathcal{O}/I$ the ringed space $(\mathbb{Z}, \mathcal{O}_{\mathbb{Z}})$ consists of the points $p \in \mathbb{Z}$ together with a finite-dimensional \mathbb{C} -algebra $\mathcal{O}_{\mathbb{Z}, p} = \mathcal{O}_p/I_p$. The associated zero-cycle is $\sum_{p \in \mathbb{Z}} \dim_{\mathbb{C}}(\mathcal{O}_{\mathbb{Z}, p}) \cdot p$. The space $(\mathbb{Z}, \mathcal{O}_{\mathbb{Z}})$ contains more information than just the set of points $p \in \mathbb{Z}$, even if we include the multiplicities $\dim_{\mathbb{C}}(\mathcal{O}_{\mathbb{Z}, p})$.

□ If $p \in \mathbb{Z}$, $\mathcal{O}_p \neq I_p \Rightarrow \dim_{\mathbb{C}} \mathcal{O}_p/I_p \neq 0$. If $p \notin \mathbb{Z}$, $\mathcal{O}_p = I_p \Rightarrow \dim_{\mathbb{C}} \mathcal{O}_p/I_p = 0$. \Rightarrow The associated zero cycle is $\sum_{p \in \mathbb{Z}} \dim_{\mathbb{C}}(\mathcal{O}_{\mathbb{Z}, p}) \cdot p$, where $\dim_{\mathbb{C}}(\mathcal{O}_{\mathbb{Z}, p}) = \text{degree of}$ the map $f = (f_1, \dots, f_n) : \bigcup_{p \in \mathbb{Z}} U_p \rightarrow \mathbb{C}^n$, see P667 ~ P669 \square

Now ^{we} come to the question of sheafifying Ext and Tor. Recall from the section on homological algebra that we proved a proposition giving four properties of projective resolutions of $\mathcal{O}_{\mathbb{Z}}$ -modules. The definition and basic properties of Ext and Tor for modules over local rings were formal consequences of this proposition. The point we wish to make here is this: By the propagation principle, the same four properties in that proposition are valid locally for coherent sheaves. For example, the first one, that projective resolutions exist, becomes that local syzygies exist, which we have already