

the first reduction formula with  $\alpha=1$  and  $\beta=r=k$ , we have

$$\begin{aligned}\delta(a, b; c) &= \#(\sigma_a \cdot \sigma_b \cdot \sigma_{c_1-c_k}, \dots, c_1-c_2, 0) \text{ in } G(k, k+C_1) \\ &= \#(\sigma_b \cdot \sigma_{c_1-c_k}, \dots, c_1-c_2) \text{ in } G(k-1, k+C_1-1) \\ &= \#(\sigma_b \cdot \sigma_{c_1-b_{k-1}}, \dots, c_1-b_1) \\ &= 1\end{aligned}$$

By 1(case),  $C_i = b_{i-1}$  for all  $i$  since  $C_i \leq b_{i-1}$  for all  $i$ , if  $C_i < b_{i-1}$  for some  $i$ , contradiction to the assumption  $C_i \geq b_{i-1}$  for all  $i$ .

$\Rightarrow a = C_1, b_k = 0$  since  $\sum C_i = a + \sum b_i$ .

By the first reduction formula, with  $\alpha=1, \beta=r=k$ ,

$$\begin{aligned}\delta(a, b; c) &= \#(\sigma_a \cdot \sigma_b \cdot \sigma_{c_1-c_2}, \dots, c_1-c_2, 0) \text{ in } G(k, k+C_1) \\ &= \#(\sigma_{a-a_1} \cdot \sigma_{b-b_k} \cdot \sigma_{c_1-c_k}, \dots, c_1-c_2, 0-0) \text{ in } G(k-1, k+C_1-1) \\ &= \#(\sigma_0 \cdot \sigma_b \cdot \sigma_{c_1-c_k}, \dots, c_1-c_2)\end{aligned}$$

$$\begin{aligned}\Rightarrow \text{Since } \sigma_0 &= \{ \Lambda \in G(k, n) : \dim(\Lambda \cap V_{n-k+0-a_0}) \geq 0 \} \\ &= G(k, n), \quad \#(\sigma_0 \cdot \sigma_b \cdot \sigma_{c_1-c_k}, \dots, c_1-c_2) = \\ \#(\sigma_b \cdot \sigma_{c_1-c_k}, \dots, c_1-c_2) &= \#(\sigma_b \cdot \sigma_{c_1-b_{k-1}}, \dots, c_1-b_1) = 1\end{aligned}$$

by P198.

$$\text{in } G(k-1, k+C_1-1) \text{ since } c_1-b_{k-1} = k+C_1-1-(k-1)-b_{k-1}$$

$$c_1-b_{k-2} = k+C_1-1-(k-1)-b_{k-2} \dots c_1-b_1 = k+C_1-1-(k-1)-b_1$$

$\Rightarrow$

" Comment on Reduction Formulas I & II.

As in the page 201, consider a triple of indices