$\Rightarrow$ We have λ_1 and λ_{n+1} s.t $Q(\lambda_1, \lambda_{n+1}) = \delta_1$.

⌟

Then for every $\lambda \in \Lambda$, δ_1 divides $Q(\lambda, \lambda_1)$ and $Q(\lambda, \lambda_{n+1})$, and we can write

$$\lambda + \frac{Q(\lambda, \lambda_1)}{\delta_1} \cdot \lambda_{n+1} - \frac{Q(\lambda, \lambda_{n+1})}{\delta_1} \cdot \lambda_1 \in \mathbb{Z}\{\lambda_1, \lambda_{n+1}\}^{\perp},$$

i.e.,

$$\Lambda = \mathbb{Z}\{\lambda_1, \lambda_{n+1}\} \oplus \mathbb{Z}\{\lambda_1, \lambda_{n+1}\}^{\perp}.$$

⌜ $\{Q(\lambda_1, \lambda), \lambda \in \Lambda\} = d_{\lambda_1}\mathbb{Z} = \{\pm d_{\lambda_1}, \pm 2d_{\lambda_1}, \cdots\}$

$\Rightarrow$ $d_{\lambda_1} = \delta_1$. Similarly, we have $d_{\lambda_{n+1}} = \delta_1$.

$\Rightarrow$ δ_1 divides $Q(\lambda, \lambda_1)$ and $Q(\lambda, \lambda_{n+1})$ for all $\lambda \in \Lambda$.

$$Q\left(\lambda_1, \frac{Q(\lambda, \lambda_1)}{\delta_1} \lambda_{n+1} - \frac{Q(\lambda, \lambda_{n+1})}{\delta_1} \lambda_1 + \lambda\right)$$

$$= Q(\lambda_1, \lambda) + \frac{Q(\lambda, \lambda_1)}{\delta_1} Q(\lambda_1, \lambda_{n+1}) - \frac{Q(\lambda, \lambda_{n+1})}{\delta_1} \cancel{Q(\lambda_1, \lambda_1)}^{=0}$$

$$= Q(\lambda_1, \lambda) + \frac{Q(\lambda, \lambda_1)}{\delta_1} \delta_1 \quad \text{since } \delta_1 = Q(\lambda_1, \lambda_{n+1}).$$

$$= Q(\lambda_1, \lambda) + Q(\lambda, \lambda_1) = 0$$

Similarly, $Q\left(\lambda_{n+1}, \lambda + \frac{Q(\lambda, \lambda_1)}{\delta_1} \lambda_{n+1} - \frac{Q(\lambda, \lambda_{n+1})}{\delta_1} \lambda_1\right) = 0$

$\Rightarrow$ $\Lambda \subset \mathbb{Z}\{\lambda_1, \lambda_{n+1}\} \oplus \mathbb{Z}\{\lambda_1, \lambda_{n+1}\}^{\perp}$ $\Rightarrow$ Done. ⌟