∵ $\gamma - \gamma_{D_i} = \partial(\Delta) - \partial\Delta_\epsilon = \partial(\Delta - \Delta_\epsilon)$.
Continue the procedure above, then $\gamma - \sum \alpha_i \gamma_{D_i} = \partial\sigma$, where σ is a 2-chain and $\sigma \cap D = \phi$.
$\Rightarrow \sigma \in M - D = U$. $\Rightarrow [\gamma - \sum \alpha_i \gamma_{D_i}] = 0$ in $H_1(U, \mathbb{Z})$.
$\Rightarrow [\gamma] = \sum \alpha_i [\gamma_{D_i}]$.

A consequence of this is:

For $p=1$, a closed meromorphic 1-form φ is of the second kind $\Leftrightarrow$ φ has no residues in any open set of the form $U = M - D$ where it is holomorphic.

∵ ($\Rightarrow$) Suppose φ is holomorphic in $U = M - D$.
By the equivalence on P455, $\exists$ a divisor D_0 s.t φ has no residues in $U_0 = M - D_0$. Let $\gamma \in H_1(U, \mathbb{Z})$ which is homologous to zero in M. Consider $D + D_0$.
$\Rightarrow \varphi$ has no residues in $M - (D + D_0) = U'$
Let $D = D_1 + \cdots + D_K$, & $D_0 = D_1^\circ + \cdots + D_l^\circ$.

$$\begin{array}{ccccc}
\gamma \in H_1(U, \mathbb{Z}) & \longrightarrow & H_1(U', \mathbb{Z}) & \longrightarrow & H_1(M, \mathbb{Z}) \\
\| & & \psi & & \\
\sum m_i \gamma_{D_i} & \longmapsto & \gamma = \sum m_i \gamma_{D_i} + \sum n_j \gamma_{D_j^\circ} & \longmapsto & 0
\end{array}$$

$\Rightarrow \gamma = \sum m_i \gamma_{D_i} + \sum n_j \gamma_{D_j^\circ}$, $\gamma_{D_i}, \gamma_{D_j^\circ} \in U'$
$\Rightarrow$ Since $D + D_0 \geq D_0$, $\int_\alpha \varphi = 0$ for all $\alpha \in H_1(U', \mathbb{Z})$.
$\Rightarrow 0 = \int_\gamma \varphi = \int_{\sum m_i \gamma_{D_i}} \varphi + \cancel{\int_{\sum n_j \gamma_{D_j^\circ}} \varphi}^{=0}$ ($\because \int_\beta \varphi = 0$ for all $\beta \in H_1(U_0, \mathbb{Z})$.)
$\Rightarrow \int_{\sum m_i \gamma_{D_i}} \varphi = 0$. $\Rightarrow \int_\gamma \varphi = 0$ for $\gamma \in H_1(U, \mathbb{Z})$.