

$\Rightarrow m_F(L, W_1) = 1$. Assume, at F , W_1 is singular.

\Rightarrow By choosing a proper coordinate, we may assume that $F = [1, 0 \dots 0]$. $L = [X_0, X_1, 0 \dots 0]$.

$W_1 = G(X_0, \dots, X_5) = 0$. G deg 3 homogeneous polynomial. \Rightarrow Consider the homogeneous forms corresponding to F , L and $G|L$.

$\Rightarrow F$ the origin. $L = \mathbb{C}$, $G|L =$ a polynomial of degree 3. $G|L$ has a zero with multiplicity at the origin

Since $\frac{\partial G}{\partial X_i}(F) = 0$ for all i . (Think of a Taylor series)

$\Rightarrow L \cap W_1$ is not a set of three distinct points. \Rightarrow Thus F is a smooth point of W_1 .

\Rightarrow

On the other hand, if F is a double line and L a generic line through F , we see (Figure 2) that the pencil L will consist of all conics passing through the points p, p' of intersection of F with a second conic G of L , and tangent to (i.e., having intersection multiplicity > 1 with) G at those points.

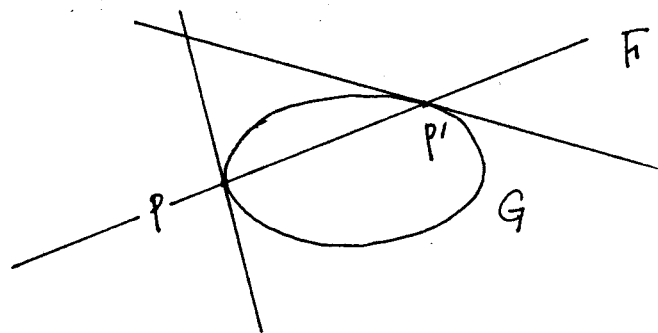


Figure 2