「 $e_{\lambda_{n+\beta}}(z+\lambda_{n+\alpha}) = e^{-2\pi i\,(z+\lambda_{n+\alpha})_\beta} = e^{-2\pi i(z_\beta + (\lambda_{n+\alpha})_\beta)}$

$\lambda_{n+\alpha} = \sum \omega_{\ell,\, n+\alpha}\, e_\ell . \quad \Rightarrow \quad (\lambda_{n+\alpha})_\beta = \omega_{\beta,\, n+\alpha}$

$$\Rightarrow \quad \Omega = \begin{pmatrix} \delta_1 & 0 & \cdots & 0 & Z_{11} & \cdots & Z_{1n} \\ 0 & \delta_2 & \cdots & 0 & \vdots & & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \delta_n & Z_{n1} & \cdots & Z_{nn} \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} \omega_{\beta,\, n+\alpha} &= \Omega_{\beta,\, n+\alpha} \\ &= Z_{\beta\alpha} \end{aligned}$$

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Now let $\varphi : \pi^* L \longrightarrow V \times \mathbb{C}$ be a trivialization of $\pi^* L$ inducing the multipliers given. Then for any section $\tilde\theta$ of L over $U \subset M$, $\theta = \varphi(\pi^*\tilde\theta)$ is an analytic function on $\pi^{-1}(U)$ satisfying

$$\theta(z+\lambda_\alpha) = \theta(z),$$
$$\theta(z+\lambda_{n+\alpha}) = e^{-2\pi i z_\alpha}\,\theta(z),$$

and conversely any such function defines a section of L.

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$$\begin{tikzcd} V \times \mathbb{C} \arrow[d] & \pi^* L \arrow[l, "\varphi"'] \arrow[d] & L \arrow[d] \\ V \arrow[r, "\cong"] & V \arrow[u, bend right, "\pi^*\tilde\theta"'] \arrow[r, "\pi"] & M \arrow[u, bend right, "\tilde\theta"'] \end{tikzcd}$$

$\theta = \varphi(\pi^*\tilde\theta)$ is an analytic function on $\pi^{-1}(U)$.

$$\Rightarrow \quad \begin{matrix} \{z\} \times \mathbb{C} \\ (z, \theta(z)) \end{matrix} \xleftarrow[\varphi_z]{} (\pi^* L)_z = (\pi^* L)_{z+\lambda} \xrightarrow[\varphi_{z+\lambda}]{} \begin{matrix} \{z+\lambda\} \times \mathbb{C} \\ (z+\lambda, \theta(z+\lambda)) \end{matrix}$$