$$d\left(\frac{*d\eta}{r^{n-2}}\right) + \frac{n-2}{r^n} \quad r\,dr \wedge *d\eta$$

$$= d\left(\frac{*d\eta}{r^{n-2}}\right) + \frac{n-2}{r^n} \quad \langle r\,dr, d\eta\rangle \,\Phi$$

$$= d\left(\frac{*d\eta}{r^{n-2}}\right) + \frac{n-2}{r^n} \quad \langle d\eta, r\,dr\rangle \,\Phi$$

$$= d\left(\frac{*d\eta}{r^{n-2}}\right) + \frac{n-2}{r^n} \quad d\eta \wedge *(r\,dr)$$

$$= d\left(\frac{*d\eta}{r^{n-2}}\right) + (n-2) \frac{d\eta \wedge *(r\,dr)}{r^n}$$

$$= d\left(\frac{*d\eta}{r^{n-2}}\right) + (n-2)\, d\eta \wedge \frac{\sigma}{C_n} = d\left(\frac{*d\eta}{r^{n-2}}\right) + \frac{(n-2)}{C_n} d(\eta\sigma)$$

$$\Rightarrow \frac{\Delta\eta}{\|x\|^{n-2}}\,dx = -d\left(\frac{*d\eta}{r^{n-2}}\right) - \frac{(n-2)}{C_n} d(\eta\sigma).$$

We apply Stokes' theorem to the region $\mathbb{R}^n - \{\|x\| \leq \epsilon\}$ and to each of the forms on the right-hand side. Thus

$$\int_{\mathbb{R}^n} \frac{\Delta\eta(x)\,dx}{\|x\|^{n-2}} = \lim_{\epsilon\to 0} \int_{\mathbb{R}^n - \{\|x\|\leq\epsilon\}} \frac{\Delta\eta(x)\,dx}{\|x\|^{n-2}}$$

$$= A_\epsilon + B_\epsilon,$$

where

$$A_\epsilon = \pm \int_{\|x\|=\epsilon} \frac{*d\eta}{\epsilon^{n-2}} = \frac{1}{\epsilon^{n-2}} \int_{\|x\|\leq\epsilon} \Delta\eta\,dx$$

$$\longrightarrow 0 \quad \text{as } \epsilon \to 0,$$