$\ulcorner$ $$\frac{d}{dt}\alpha(t,x)\Big|_{t=0} = v(x). \quad \alpha(0,x) = x.$$

$\Rightarrow \alpha(t,x) = f_t(x)$. by the theory of the ordinary differential equations. $\lrcorner$

For t small, the fixed points of f_t will be exactly the zeros of v, and if v is given as above near a zero p, then in terms of the coordinates x,

$$J_{f_t}(p) = e^{tA} + \text{higher-order terms}.$$

$\ulcorner$ We may assume that $f_t : (-\delta,\delta) \times B(x_0,\epsilon) \longrightarrow U$ where $B(x_0,\epsilon) \subset \mathbb{R}^n$, $U \subset \mathbb{R}^n$ open.

$\Rightarrow \frac{df_t}{dt}\Big|_{t=0} = v \qquad \Rightarrow f_t(x) = tv + x$

For example, if $v(x) = x^2$. $n=1$.

$f_t(x) = tx^2 + x = 0 \qquad \Rightarrow \quad x = 0, -\frac{1}{t}$ for small t.

$\Rightarrow$ f_t is not a diffeomph. $\Rightarrow$ Wrong conclusion!

$f_t(x) = tx^2 + x$

$\frac{d}{dt}f_t(x) = x^2 \neq (tx^2+x)^2$

Given a vector field v, does there exist a curve $f(t)$ s.t. $\frac{d}{dt}f_t(x) = v(f_t(x))$

$f_0(x) = x$.

$\Rightarrow f_t(x) = tv + x$ is not a solution!