$$\sigma_1 \cdot \sigma_2 = \sum_{\Sigma c_i = 3} \sigma_c \qquad \begin{matrix} 1 \le c_1 \le 2 \\ 0 \le c_2 \le 1 \end{matrix} \qquad c = 2, 1$$

$$\Rightarrow \sigma_1 \cdot \sigma_2 = \sigma_{2,1}$$

$$\Rightarrow \sigma_1 \cdot \sigma_{2,1} = \sum_{\Sigma c_i = 4} \sigma_c \qquad \begin{matrix} 1 \le c_1 \le 2 \\ 0 \le c_2 \le 1 \end{matrix} \Rightarrow \text{No possible choice}$$

since $2+1 = 3 < 4 = \Sigma c_i$.

$$\sigma_1 \cdot \sigma_{4-2,\, 4-2-1} = \sigma_1 \cdot \sigma_{2,1} = 1 \quad \text{see } p198.$$

In general, the number of lines meeting four $(n+1)$-planes in general position in $\mathbb{P}^{2n+1}$ is given by the fourfold self-intersection of σ_n in $G(2, 2n+2)$; this is

$$(\sigma_n)^4 = (\sigma_n^2)^2 = \left(\sum_{i=0}^{n} \sigma_{2n-i,i}\right)^2 = n+1.$$

┌ We did this for $n=1$.

$$\sigma_n^2 = \sum_{\substack{\Sigma c_i = 2n \\ b_i \le c_i \le b_{i-1}}} \sigma_c = \sigma_{2n} + \sigma_{2n-1,1} \cdots + \sigma_{n,n} = \sum_{i=0}^{n} \sigma_{2n-i,i}$$

$$\begin{matrix} n \le c_1 \le 2n \\ 0 \le c_2 \le n \end{matrix}$$

$$\sigma_n \cdot \sigma_{2n-i,i} = \sum_{\substack{\Sigma c_i = 3n}} \sigma_c \quad \Rightarrow \quad \text{complicated} \Rightarrow$$

$$\begin{matrix} 2n-i \le c_1 \le 2n \\ i \le c_2 \le 2n-i \end{matrix}$$

Note.

$$\sigma_{2n-i,i} \cdot \sigma_{2n+2-2-i,\, 2n+2-2-(2n-i)} = \sigma_{2n-i,i} \cdot \sigma_{2n-i,i} = 1$$

$$\dim \sigma_{2n-i,i} = 2(2n) - 2n = 2n.$$