and satisfies

$$m_p(Z, W) \geq \quad mult_p(Z)\ mult_p(W).$$

Proof. We first argue that we may assume W to be smooth. For this we consider the product $M \times M$. By the formal properties of the Künneth formula and Poincare duality,

$$Z \cdot W = (Z \times W) \cdot \Delta,$$

where the right-hand side is the intersection number in $M \times M$ of $Z \times W$ with the diagonal Δ.

⌈ According to p59. $i_{(p,p)}(Z \times W \cdot \Delta) = (-1)^{2n-2p}\, i_p(Z \cdot W)$

$$= i_p(Z \cdot W).$$

$\Rightarrow \quad Z \cdot W = (Z \times W) \cdot \Delta$ We don't need anything famous ⌋

Set-theoretically,

$$(Z \times W) \cap \Delta = \{(p,p) ;\ p \in Z \cap W\}.$$

Also, it has been established in Section 1 of Chapter 0 that

$$mult_{p \times q}(Z \times W) = mult_p(Z)\ mult_q(W).$$

⌈ It follows from the definition of multiplicity on P22 ⌋

Since the diagonal is smooth, the general case is therefore reduced to the situation when W is smooth.