∵ $A + iB$ is negative definite hermitian $n \times n$ matrix.

$\Rightarrow \ {}^tX(A+iB)\bar{X} \leq 0$ for all $X \in \mathbb{C}^n$.

Let X = real.

$\Rightarrow \ {}^tXA\bar{X} + i\,{}^tXB\bar{X} \leq 0 \Rightarrow {}^tXB\bar{X} = 0$ & ${}^tXA\bar{X} \leq 0$.

(i) Suppose f has a minimum at $z_0 \in M$.

$\Rightarrow \exists$ open set $U \subset M$ s.t $U \ni z_0$ and $\varphi(z_0) = 0$

$U \xrightarrow[\cong]{\varphi} \mathbb{C}'$. $\Rightarrow f \circ \varphi^{-1}$ has a minimum at 0.

$\Rightarrow \ f \circ \varphi^{-1}(z) = f(z_0) + \left.\dfrac{\partial^2 f}{\partial x^2}\right|_{x=0}$

$$f \circ \varphi^{-1}(0 + tv) = f(z_0) + \frac{\partial^2 f}{\partial x^2} v_1^2 + 2\frac{\partial^2 f}{\partial x \partial y} v_1 v_2 + \frac{\partial^2 f}{\partial y^2} v_2^2$$

$+ \cdots$

$\Rightarrow$ If $f \circ \varphi^{-1}$ has a minim at 0,

$$\frac{\partial^2 f}{\partial x^2} v_1^2 + 2\frac{\partial^2 f}{\partial x \partial y} v_1 v_2 + \frac{\partial^2 f}{\partial y^2} v_2^2$$

$$= (v_1, v_2) \begin{pmatrix} \dfrac{\partial^2 f}{\partial x^2}, & \dfrac{\partial^2 f}{\partial x \partial y} \\ \dfrac{\partial^2 f}{\partial x \partial y}, & \dfrac{\partial^2 f}{\partial y^2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \geq 0$$

$$\Rightarrow \begin{pmatrix} \dfrac{\partial^2 f}{\partial x^2}, & \dfrac{\partial^2 f}{\partial x \partial y} \\ \dfrac{\partial^2 f}{\partial x \partial y}, & \dfrac{\partial^2 f}{\partial y^2} \end{pmatrix} \text{ is positive semidefinite.}$$

$\Rightarrow$ We pulled back f to $\mathbb{C}'$.

(ii) For $z_i = x_i + i y_i$, for $\forall v = (v_1, v_2, \cdots v_n, 0 \cdots 0)$,

$$\left(\begin{array}{c|c} \dfrac{\partial^2 f}{\partial x_i \partial x_j} & \\ \hline & \dfrac{\partial^2 f}{\partial y_i \partial y_j} \end{array}\right)$$