$\Rightarrow \exists$ a finite # of open sets U_i's, $i=1\cdots n$, covering X'.
$\Rightarrow \exists$ a partition of unity φ_i's s.t. $\mathrm{supp}\,\varphi_i \subset U_i$ and $\sum \varphi_i = 1$.
Let X_i be the lifted vector field on U_i. $\Rightarrow \sum \varphi_i X_i$ is a global vector field v on X'. Note that $v \neq 0$ on X', everywhere

since $X_i \neq 0$ everywhere on U_i and $(\gamma^{-1}\circ\pi)_*(\sum \varphi_i X_i)$
$= \sum \varphi_i(l)(\gamma^{-1}\circ\pi)_*(X_i) = \sum \varphi_i(l)(-\partial/\partial t) = -\frac{\partial}{\partial t}$ for each $l \in X'$.

Consider the following map

$$\mathbb{R} \longrightarrow \mathbb{R}.$$
$$t \longmapsto \phi(\varphi_t(p)) \qquad \phi = \gamma^{-1}\circ\pi : X' \longrightarrow I$$

φ_t is the flow of v.

We claim that $\phi(\varphi_t(p)) = -t + \phi(p)$.
To show this,

$$\frac{d\phi(\varphi_t(p))}{dt}\,\frac{\partial}{\partial t}\bigg|_{t=\varphi_t(p)} = \phi_*\left(\frac{d}{dt}\varphi_t(p)\right)$$

$= \phi_*\left(\frac{d}{ds}\Big|_{s=0}\varphi_s(\varphi_t(p))\right) = \phi_*\left(v(\varphi_t(p))\right)$ (by the definition of a flow) $= -\frac{\partial}{\partial t}\Big|_{t=\varphi_t(p)}$ since $v(\varphi_t(p))$

is a lifted vector field of $-\frac{\partial}{\partial t}\Big|_{t=\varphi_t(p)}$.

$\Rightarrow \dfrac{d\phi(\varphi_t(p))}{dt} = -1 \Rightarrow \phi(\varphi_t(p)) = -t + \text{constant}$
$t=0 \Rightarrow \phi(p) = \text{constat}$

Thus if $p \in \phi^{-1}(t)$, $\phi(\varphi_t(p)) = -t + \phi(p) = -t+t = 0$
$\Rightarrow \varphi_t(p) \in \phi^{-1}(0) \Rightarrow \varphi_t : \phi^{-1}(t) \longrightarrow \phi^{-1}(0)$ is