$\{C_\lambda\}$ thus has four points of intersection with V_C other than $2L$: it follows that

$$\mathrm{mult}_{2L}(\{C_\lambda\}, V_C) = 2,$$

and so, for generic C,

$$\mathrm{mult}_{W_2}(V_C) = 2.$$

Γ $W_2 \subset V_C$

If the multiplicity at $2L$ is ≥ 3, then, again by Bertini's theorem, for a generic pencil $\{C_\lambda\}$ through $2L$, $[\{C_\lambda\} \cap V_C - 2L]$ is smooth, and is a set of distinct points ≤ 3. ($\because$ By the argument above, the # of branch points is 6.) $\Rightarrow$ And we have a branched 4-sheeted cover of $\mathbb{P}^1$ from $\{C_\lambda\}$, and the number of branch points of that cover is $\leq 2 + 3 = 5$, since clearly, counting multiplicity the # of branch points $\underline{\text{at}\quad 2L}$ is 2 and
(in fact, for some λ, $C_\lambda = 2L$)

the # of branch points at points (conics) other than $2L$ is ≤ 3. Contradiction.

$\Rightarrow$ $\mathrm{mult}_{2L}(\{C_\lambda\}, V_C) = 2$ .. $\circledast$

$\Rightarrow$ Since $\circledast$ is true for smooth C, for generic C $\mathrm{mult}_{W_2}(V_C) = 2$.

Point: Use Bertini's theorem.