If we choose $e_{i_1} \wedge \cdots e_{i_{p-1}}$'s to be linearly independent,

$e_n \wedge e_{i} \wedge \cdots e_{i_{p-1}}$'s are linearly indepent.

$\Rightarrow$. $a_{i_1 \cdots i_{p-1}} = 0$. for all $i_1 \cdots i_{p-1}$.

$\Rightarrow$ f is one to one since $\ker f = 0$. ⌋⌋

But by the adjunction formula I, $N_V^* = [-V]|_V$; we can thus rewrite this last sequence as

$$(**) \qquad 0 \longrightarrow \Omega_V^{p-1}(-V) \longrightarrow \Omega_M^p|_V \longrightarrow \Omega_V^p \longrightarrow 0.$$

Now $[-V]$ is negative on M, and likewise $[-V]|_V$ is negative on V.

⌈⌈ By the assumption, $[V] = L$ is positive on M.

$[-V]$ is negative on M. ⌋⌋

The Kodaira vanishing theorem accordingly gives

$$H^q(M, \Omega_M^p(-V)) = 0 \qquad p+q < n.$$
$$H^q(V, \Omega_V^{p-1}(-V)) = 0 \qquad p+q < n.$$

⌈⌈ $\dim V = n-1$, $\quad p-1+q < n-1 \Leftrightarrow p+q < n$. ⌋⌋

By the exact cohomology sequences associated to the sheaf sequences $(*)$ and $(**)$, recalling that

$H^*(M, \Omega_M^p|_V) = H^*(V, \Omega_M^p|_V)$,

$$H^q(M, \Omega_M^p) \overset{r^*}{\cong} H^q(M, \Omega_M^p|_V) \overset{i^*}{\cong} H^q(V, \Omega_V^p)$$

for $p+q \leq n-2$, and with both maps injective for