$H_p \in I_p(F.G)$. $\Rightarrow$ By the Noether's theorem, $HL = AF + BG$, for some homogeneous polynomials $A.B$ with degree 1. $\lrcorner$

The linear form B vanishes at R_1 and R_2, and so $B = \beta L$.

Γ $HL = AF + BG \Rightarrow (HL)(R_1) = (AF)(R_1) + (BG)(R_1)$ $\Rightarrow 0 = 0 + B(R_1)\ G(R_1) \Rightarrow$ Since R_1 is not on a curve D, and $G(R_1) \neq 0$, $B(r_1) = 0$. Similarly, we get $B(R_2)$. $\Rightarrow$ B passes R_1 and R_2 and B is a line. $\Rightarrow$ $B = \beta L$ for some $\beta \in \mathbb{C}$. $\lrcorner$

It follows that L divides AF, and since the line $L=0$ is not a component of C, it follows that $A = \alpha L$. Then $H = \alpha F + \beta G$, and so $Ord_C(H)_q \geq Ord_C(G)_q$.

Γ $HL = AF + BG \Rightarrow HL = AF + \beta LG$ $\Rightarrow (H - \beta G)L = AF$. $L \cap C$ = Set of finite points $\Rightarrow$ L is not a component of C. $\Rightarrow$ L does not divide F. $\Rightarrow$ L must divide A. $\Rightarrow$ $A = \alpha L$ for some $\alpha \in \mathbb{C}$. $\Rightarrow (H - \beta G)L = \alpha L F \Rightarrow H - \beta G = \alpha F$ $\Rightarrow H = \alpha F + \beta G$.

Restrict H to $C (= (F=0))$, $H|_C = \beta G|_C \Rightarrow Ord_{C,q} H \geq Ord_{C,q}(G)$. $\lrcorner$