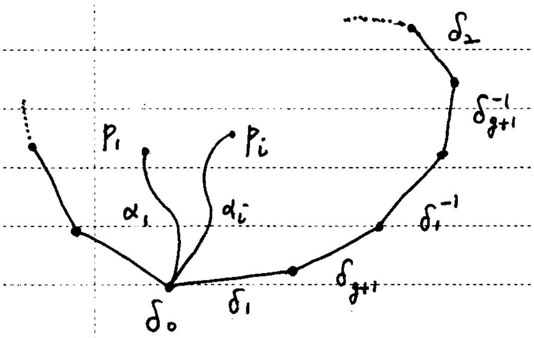
on one rather pretty result that is similarly obtained.

Theorem (Weil). Let f, g be meromorphic functions on the compact Riemann surface S, with (f) disjoint from (g). Then

$$\prod_{p\in S} f(p)^{\mathrm{ord}_p(g)} = \prod_{p\in S} g(p)^{\mathrm{ord}_p(f)} .$$

Proof. Let $\delta_1, \delta_2, \cdots \delta_{2g}$ and Δ be as above. Let $\{p_i\}$ denote the support of (f), $\{q_i\}$ the support of (g), and draw smooth arcs α_i from s_0 to p_i disjoint except for their common base point s_0 and not containing any of the points $\{q_i\}$. Let Δ' be the complement of the arcs α_i in Δ; Δ' can again be considered as a polygon with sides $\cdots \delta_i, \delta_{g+i}, \delta_i^{-1}, \delta_{g+i}^{-1} \cdots \alpha_i, \alpha_i^{-1} \cdots$ as drawn in Figure 5.



Since Δ' is simply connected and f is nonzero holomorphic in Δ', we can choose a single branch of the function $\log f$ in Δ'; we consider the meromorphic differential

$$\varphi = \log f \cdot d\log g = \log f \cdot \frac{dg}{g} \quad \text{in } \Delta'.$$

⌜ By P85, Silverman, a branch of $\log z$ is defined to