$\Rightarrow$ Since $e^{\int_1^z \frac{1}{w} dw}$ is bounded around $z=0$, by Riemann extension theorem, $e^{\int_1^z \frac{1}{w} dw}$ is extendable to $\mathbb{C}$ and is holomorphic. Furthermore, it has zero at $z=0$.

In general, $h(z)$ is holomorphic on Δ^{n+1}.

Again, for simplicity, $n=1$ $h(z_1, z_2)$ holomorphic in Δ^2. Choose a point $\eta_0 = (\eta_{01}, \eta_{02})$ s.t. $h(\eta_0) \neq 0$.

$$\int_{\eta_0}^{z} \partial \log h$$

$$\alpha : [0,1] \longrightarrow \Delta^2 \quad \text{s.t} \quad \alpha(0) = \eta_0, \ \alpha(1) = z$$
$$t \longmapsto (\alpha_1(t), \alpha_2(t))$$

$$= \int_{\eta_0}^{z} \frac{\partial h}{h} = \int_{\eta_0}^{z} \frac{1}{h}\left(\frac{\partial h}{\partial w_1} dw_1 + \frac{\partial h}{\partial w_2} dw_2\right)$$

$$= \int_0^1 \frac{1}{h(\alpha_1(t), \alpha_2(t))}\left(\frac{\partial h}{\partial w_1}(\alpha_1, \alpha_2) \cdot \alpha_1'(t)\, dt + \frac{\partial h}{\partial w_2} \alpha_2'(t)\, dt\right)$$

$$= \int_0^1 \frac{1}{h(\alpha_1(t), \alpha_2(t))}\left(\frac{\partial h}{\partial w_1}\alpha_1'(t) + \frac{\partial h}{\partial w_2}\alpha_2'(t)\right) dt$$

$$= \int_0^1 \frac{1}{h(\alpha_1(t), \alpha_2(t))} \frac{d\, h(\alpha_1(t), \alpha_2(t))}{dt}\, dt.$$

$\exp \int_{\eta_0}^{\eta} \partial \log h$ is well-defined on $\Delta^2 - (h=0)$, by the same reason as above, and the following note.

Note : $\operatorname{codim}_{\mathbb{R}}(\Delta^2 - (h=0)) = 2$. $\Rightarrow$ $\Delta^2 - (h=0)$ is path-connected.