Comment: Suppose f is homogeneous polynomial, & g is irreducible homogeneous polynomial, and $\{g=0\} \subset \{f=0\}$. $\Rightarrow$ f is divisible by g.

pf) Let $f(1, z_1, z_2, \cdots z_n) = \tilde{f}(z_1, z_2 \cdots z_n)$,

$g(1, z_1, z_2, \cdots z_n) = \tilde{g}(z_1, z_2, \cdots z_n)$.

$\Rightarrow$ By Corollary on P11, $\tilde{f}/\tilde{g}$ is holomorphic, and is polynomial in $z_1, z_2, \cdots z_n$.

$\Rightarrow$ $\tilde{f} = \tilde{g}\tilde{h}$

$\Rightarrow$ $f = gh$.

$*$ $y^2 = x^2(x+1)$. $\Rightarrow$ $y = \pm x\sqrt{x+1}$

$y' = \pm 1$ $\Rightarrow$ $y = \pm x$ are the tangent lines, at the origin, to $y^2 = x^2(x+1)$.

$\Rightarrow$ $y^2 = x^2(x+1)$ is irreducible globally, but reducible locally at the origin. ⌟

If, conversely, $\Delta \in |\mathcal{O}_{\mathbb{P}^2}(3)|$, then the curves $\Delta + L$, L a line in $\mathbb{P}^2$, give a $\mathbb{P}^2$ in the projective space $|\mathcal{O}_{\mathbb{P}^2}(4)|$.

⌜ Let Δ be represented by a section $\sigma_0 \in \mathcal{O}_{\mathbb{P}^2}(3)$. Let s_1, s_2, s_3 be sections of $\mathcal{O}(H)$ and be linearly independent. $\Rightarrow$ $\langle s_1, s_2, s_3 \rangle = \mathbb{P}^2$.

$\Rightarrow$ $\sigma_0 s_1$, $\sigma_0 s_2$, & $\sigma_0 s_3$ give a $\mathbb{P}^2$ in the projective space $|\mathcal{O}_{\mathbb{P}^2}(4)|$. ⌟

Since not all curves in this linear system are