$\ulcorner$ $\omega^n \in H^{2n}$ $\quad \varphi \in H^1 \quad \Rightarrow \omega^n \wedge \varphi \in H^{2n+1} = 0$.

$\iota(v)(\omega^n \wedge \varphi) = \iota(v)\omega \wedge \omega^{n-1} \wedge \varphi + (-1)^2 \omega \wedge \iota(v)(\omega^{n-1}\wedge\varphi)$

$= \omega^{n-1} \wedge \iota(v)\omega \wedge \varphi + \omega \wedge (\iota(v)\omega \wedge \omega^{n-2} \wedge \varphi + \omega \wedge \iota(v)(\omega^{n-2}\wedge\varphi))$

$\cdots\cdots = n\,\omega^{n-1} \wedge \iota(v)\omega \wedge \varphi + \omega^n \wedge \overset{=0}{\iota(v)\varphi} = n\,\omega^{n-1}\wedge \iota(v)\omega \wedge \varphi$

$\Rightarrow$ Obviously $\int_M \omega^{n-1} \wedge \iota(v)\omega \wedge \varphi = 0$.

$\Rightarrow$ Since φ is arbitrary, $\iota(v)\omega = 0$ in $H^{0,1}(M)$ $\lrcorner$

In particular, $d_1' L = 0$, *so that* L *defines a class in* ${}'E_2^{n-1,1}$.

$\ulcorner$

$$\begin{tikzcd}
L \in {}'E_1^{n-1,1} \arrow[r, "d_1'"] \arrow[d] & {}'E_1^{n,1} \arrow[d] \\
H^{1,1}(M) \arrow[r, "\iota(v)"] & H^{0,1}(M)
\end{tikzcd}$$

$L \mapsto \omega \in H^{1,1}(M)$

Since $\omega \in H^{1,1}(M)$, $\quad d_1' L = 0 \Leftrightarrow \iota(v)\omega = 0$. $\lrcorner$

Since

$$d_r' : {}'E_r^{n-1,1} \longrightarrow {}'E_r^{n-1+r,\,2-r} = 0$$

for $r \geq 2$, it follows that L defines a class in ${}'E_r^{n-1,1}$ for all r.

$\ulcorner$ ${}'E_r^{n-1+r,\,2-r} = 0$, since

for $r \geq 3$, ${}'E_3^{n-1+r,\,2-r} = 0$

$(\because 2-r<0)$ for $r=2$, ${}'E_2^{n+1,0} = 0$ (since ${}'E_1^{n+1,0} = H^0(M, \Omega^{n+1}) = 0$)

$\Rightarrow$ ${}'E_3^{n-1,1} = \dfrac{{}'E_2^{n-1,1}}{\sim}$ $\quad {}'E_4^{n-1,1} = \dfrac{{}'E_3^{n-1,1}}{\sim}$ $\quad \cdots \Rightarrow L$ defines a class in ${}'E_r^{n-1,1}$ for all r $\lrcorner$