

$\alpha, \beta, r$  s.t.  $\alpha + \beta + r = 2k + l$ ,  $1 \leq l \leq k$ .

$\Rightarrow$  For  $\Lambda \in \sigma_a(V) \cap \sigma_b(V') \cap \sigma_c(V'')$ ,  
 $\dim(\Lambda \cap V_{n-k+\alpha-a}) \geq \alpha$ ,  $\dim(\Lambda \cap V'_{n-k+\beta-b}) \geq \beta$   
 $\dim(\Lambda \cap V''_{n-k+r-c}) \geq r$ .

$\Rightarrow \dim(\Lambda \cap V_{n-k+\alpha-a} \cap V'_{n-k+\beta-b} \cap V''_{n-k+r-c}) \geq l$ . see note p511 ~ p512.

Thus  $n - k + \alpha - a_\alpha + n - k + \beta - b_\beta + n - k + r - c_r - 2n$   
 $= n - 3k + (\alpha + \beta + r) - (a_\alpha + b_\beta + c_r) = n - 3k + 2k + l$   
 $-(a_\alpha + b_\beta + c_r) = n - k + l - (a_\alpha + b_\beta + c_r)$

$\Rightarrow$  If  $a_\alpha + b_\beta + c_r > n - k$ ,  $d(V \cap V' \cap V'') \leq l - 1$ .

$\Rightarrow \exists$  no  $\Lambda \in \sigma_a(V) \cap \sigma_b(V') \cap \sigma_c(V'')$ .

$\Rightarrow \#(\sigma_a : \sigma_b \cdot \sigma_c) = 0$

If  $a_\alpha + b_\beta + c_r = n - k$ ,  $\#(\sigma_a \cdot \sigma_b \cdot \sigma_c)_{G(k,n)} = \#(\sigma_{a-\hat{a}_\alpha} \cdot \sigma_{b-\hat{b}_\beta} \cdot \sigma_{c-\hat{c}_r})_{G(k-1,n-1)}$ .

Suppose  $\Lambda \in \sigma_{a,0,\dots}(V)$ .

$$\bar{V}_1 = \pi(V_1)$$

$$\pi: \mathbb{C}^n \longrightarrow L^\circ$$

$$L^\circ \oplus L$$

$$\bar{V}_{n-k+1-a-1} = \pi(V_{n-k+1-a-1}) = \pi(V_{n-k+1-a})$$

$$\bar{V}_{n-1} = \pi(V_n) = L^\circ.$$

Claim:  $\bar{\Lambda}|_L = \Lambda \in \sigma_a(V) \stackrel{(?)}{\iff} \bar{\Lambda} \in G(k-1, n-1) = G(k-1, L^\circ)$

$\Gamma(\Rightarrow) \Lambda \in \sigma_a(V) \Rightarrow \dim(\Lambda \cap V_{n-k+1-a}) \geq 1$

$\Rightarrow$  Since  $\bar{\Lambda} \subset L^\circ$  and  $L \subset V_{n-k+1-a}$ ,

$\pi(\Lambda) = \bar{\Lambda}$  and  $d(\bar{\Lambda} \cap \bar{V}_{n-k+1-a-1}) \geq 0$ .

$\dim(\bar{\Lambda} \cap \bar{V}_{n-k+1-a-1}) \geq 0 \Rightarrow$  No problem