$$\Rightarrow \quad \varphi_\Delta = \sum_{p,\mu,\nu} (-1)^{n-p} \delta_{\mu,\nu}\, \varphi_{\mu,\nu,p,n-p} = \sum_{p,\mu,\nu} (-1)^{n-p} \varphi_{\mu,\mu,p,n-p}. \qquad \lrcorner$$

Now let $f: M \longrightarrow M$ be a C^∞-map. We say that a fixed point $p \in M$ of f is nondegenerate if it is isolated and in terms of local coordinates $x_1, \cdots x_n$ on M centered around p, the Jacobian matrix

$$\mathcal{J}_f(p) : T_p(M) \longrightarrow T_p(M)$$

satisfies

$$\det(\mathcal{J}_f(p) - I) \neq 0;$$

under these circumstances, we define the index $\iota_f(p)$ of f at p to be

$$\iota_f(p) = \operatorname{sgn} \det(\mathcal{J}_f(p) - I).$$

We can give another interpretation of the nondegeneracy condition and the index as follows: let $\Gamma_f = \{(p, f(p))\} \subset M \times M$ be the graph of f. Γ_f is a submanifold of $M \times M$; we give it the orientation induced by the map

$$\tilde{f} : p \longmapsto (p, f(p)).$$

Pf

$$M \xrightarrow{\tilde{f}} \Gamma_f \hookrightarrow M \times M$$

For $(p, f(p))$, $U \times V \subset \underset{\cong \mathbb{R}^n}{M} \times \underset{\cong \mathbb{R}^n}{M}$, V, U open.

$$\begin{array}{ccc} U \times V & \xrightarrow{F} & U \times V \\ (p, q) & \longmapsto & (p, f(p) - q) \end{array} \Rightarrow J(F) = \begin{pmatrix} 1, & \frac{\partial f}{\partial p} \\ 0, & 1 \end{pmatrix}$$

$\Rightarrow$ By the inverse function theorem, we see that Γ_f is a submanifold of $\mathbb{R}^{2n}$.