one A-line L_1 and one B-line L_2.

⌈ Obviously, $\bigcup_{[s_0,s_1]\in\mathbb{P}^1} \sigma([s_0,s_1]\times\mathbb{P}^1) = S$. $\Rightarrow$ Since $L \subset S$,

L must meet an A-line L_1. Similarly, L must meet one B-line L_2. ⌋

But L_1 and L_2 meet, and the plane they span in $\mathbb{P}^3$ can meet S in at most two lines, so either $L = L_1$ or $L = L_2$.

⌈ Since every A-line meets every B-line, L_1 and L_2 meet. Suppose $\underset{\langle L_1, L_2\rangle}{\underset{\parallel}{\mathbb{P}^2}} \cap S \supset L_1 \cup L_2 \cup L$, and $L \neq L_i$.

$\Rightarrow \mathbb{P}^2 = (a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 = 0) \qquad S = (X_1X_4 - X_2X_3 = 0)$

$\Rightarrow \mathbb{P}^2 \cap S$ is a curve of degree 2, since

$$\begin{aligned} Z_1 &= a_{11}X_1 + \cdots + a_{14}X_4 \\ &\vdots \\ Z_4 &= a_{41}X_1 + \cdots + a_{44}X_4 \end{aligned} \quad\Rightarrow\quad \begin{aligned} X_1 &= b_{11}Z_1 + \cdots + b_{14}Z_4 \\ &\vdots \\ X_4 &= b_{41}Z_1 + \cdots + b_{44}Z_4 \end{aligned}$$

$\Rightarrow$ S is given as the zero locus of quadratic polynomials in $Z_1, \cdots Z_4$. $\Rightarrow$ $\mathbb{P}^2 \cap S = \{Z_1 = 0\} \cap S$

$\Rightarrow$ Clearly $S \cap \mathbb{P}^2$ is the zero locus of quadratic polynomials in Z_2, Z_3, Z_4. ...... On the other hand, $L_1 + L_2$ is a curve of degree 2, and $L + L_1 + L_2$ is a curve of degree 3 in case $L \neq L_i$. Contradiction. ⌋