The common intersection of the divisors in a linear system is called the base locus of the system; in particular, a divisor F in the base locus — that is, such that $D_\lambda - F \geq 0$ for all λ — is called a fixed component of E.

⟦ Since each D_λ is effective, $D_\lambda - F \geq 0$. ⟧

The second property is more remarkable: like the first, it is peculiar to linear system and is not the case for general families of divisors, even general families of linearly divisors. This is

Bertini's Theorem. The generic element of a linear system is smooth away from the base locus of the system.

pf) If the generic element of a linear system is singular away from the base locus of the system, then the same will be true for a generic pencil contained in the system; thus it suffices to prove Bertini for a pencil.

Suppose $\{D_\lambda\}_{\lambda \in \mathbb{P}^1}$ is a pencil, given a polydisc Δ contained in M by

$$D_\lambda = (f(z_1, \cdots z_n) + \lambda \cdot g(z_1, \cdots z_n) = 0)$$

and suppose P_λ is a singular point of the divisor D_λ ($\lambda \neq 0, \infty$) but not in the base locus B of the pencil.

⟦ $\mathbb{P}(H^0(M, \mathcal{O}(L))) = \mathbb{P}^1 \Rightarrow$ This implies $\exists$ two nonzero linearly independent global holomorphic sections s_1, s_2 of L.
$\Rightarrow$ Given $\lambda \in \mathbb{P}^1$, we can identify λ with $s_1 + \lambda s_2$ in $\mathbb{P}(H^0(M, \mathcal{O}(L)))$.