either all of $\sigma(p)$, or a line in $\sigma(p)$.

$\ulcorner$ Obviously, since $\sigma(p)$ is a 2-plane and $H = \mathbb{P}^4$, $\sigma(p) \cap \mathbb{P}^4 = \mathbb{P}^2$ or $\mathbb{P}^1$. $\Rightarrow$ X_p is either $\sigma(p)$ or a line in $\sigma(p)$. $\lrcorner$

But the set of tangent planes

$$\{ T_x(G) \}_{x \in \sigma(p)}$$

to G at points of $\sigma(p)$ form the linear system of all hyperplanes containing $\sigma(p)$, i.e., any hyperplane containing $\sigma(p)$ is tangent to G.

$\ulcorner$ $\tilde{\Phi}(\sigma(p))$ is a 2-plane in $\mathbb{P}^5$.
$K = \{ H \supset \tilde{\Phi}(\sigma(p)) \mid H \text{ is a hyperplane in } \mathbb{P}^5 \}$
We may assume that $\tilde{\Phi}(\sigma(p)) = \{ [*,*,*,0,0,0] \} \subset \mathbb{P}^5$. Suppose $H = (a_0X_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_4X_5 + a_5X_5 = 0)$ contains $\tilde{\Phi}(\sigma(p))$. $\Rightarrow$ $a_0 = a_1 = a_2 = 0$
$\Rightarrow$ $K = \{ [0,0,0,*,*,*] \in \mathbb{P}^5 \} = \mathbb{P}^2$.
But $\{ T_x(G) \}_{x \in \sigma(p)}$ is 2-dimensional subvariety of $\mathbb{P}^{5*}$ $\Rightarrow$ Since K is irreducible, and $\{ T_x(G) \} \subset K$ $\Rightarrow$ $\{ T_x(G) \}_{x \in \sigma(p)} = K$.
To show that $\{ T_x(G) \}_{x \in \sigma(p)}$ is a 2-dim subvariety of $\mathbb{P}^{5*}$, we need to show that

$$T_x(G) \cap G \neq T_y(G) \cap G \quad \text{if} \quad x \neq y.$$

Assume $T_x(G) \cap G = T_y(G) \cap G$.