$\ulcorner$ For each $p \in V$, a generic $(n-k-2)$-plane $\mathbb{P}^{n-k-2}$ satisfies $\#(\overline{\mathbb{P}^{n-k-2}\cdot p} \cap V) = 1$.

For, $\overline{\mathbb{P}^{n-k-1}\cdot p} \cap V$ is a set of a finite number of points. $\Rightarrow$ A generic $(n-k-2)$-plane in $\mathbb{P}^{n-k-1}$ misses the points except p, together with p.

If $\overline{\mathbb{P}^{n-k-2}\cdot p} \cap V$ has more than one point, then $\mathbb{P}^{n-k-2}$ must satisfy some relation; i.e. a holomorphic function.

$\Rightarrow$ $\exists$ an open set $\mathcal{U}$ in V s.t for each $q \in \mathcal{U}$, $\overline{\mathbb{P}^{n-k-2}, q} \cap V = \{q\}$. We can make sure by using the distance between two closed compact sets.

Thus π' is one to one over an open set $\mathcal{U}$ in its image. $\lrcorner$

Now, consider the rational function

$$x_{k+1} = \frac{X_{k+1}}{X_0} \quad \text{on } V.$$

$\ulcorner$ V does not contain the hyperplane $(X_0 = 0)$. $\Rightarrow$ x_{k+1} is a well-defined rational function. $\lrcorner$

Suppose that x_{k+1} satisfied an equation of the form

$$x_{k+1}^{d'} + \phi_1(x_1, \cdots x_k)\cdot x_{k+1}^{d'-1} + \cdots + \phi_{d'}(x_1 \cdots x_k) \equiv 0, \; d' < d.$$