Then $\alpha_1^3 \alpha_2^3 \alpha_3^3 \alpha_4^3 = \left(\frac{a_5(\lambda)}{a_1(\lambda)}\right)^3$,

$$\alpha_1^6 + \alpha_2^6 + \alpha_3^6 + \alpha_4^6 = (\alpha_1+\alpha_2+\alpha_3+\alpha_4)^6 - \cdots$$
$$= \left(\frac{a_2(\lambda)}{a_1(\lambda)}\right)^6 + \cdots$$

$\Rightarrow$ Δ is, at most, degree 24. //

This argument is not useful at this moment.

From ① & ②, for a generic pencil L of conics, $\#(L \cap V_c) \leq 6$, for, we can find a generic pencil L s.t L meets V_c transversely.

Suppose $\#(L \cap V_c) > 6$.

$\Rightarrow$ By ① & ②, since L represents a 4-sheeted cover of $\mathbb{P}^1$ with branch points ≥ 7. ($\because$ an element in $L \cap V_c$ is tangent to C.) This is impossible by the Riemann-Hurwitz formula

$$b = \#\text{ of branch points (counting multiplicity)}$$
$$= 2 \cdot 4 + 2 \cdot g - 2 = 6.$$

Thus $\#(L \cdot V_c) = 6 \Rightarrow \deg V_c = 6$.

It remains to show that, given a branched covering $\pi : C \longrightarrow \mathbb{P}^1$, then $\exists$ a pencil $\{\mathfrak{D}\}$ on C.

For each $\lambda \in \mathbb{P}^1$, $\exists$ a divisor D_λ on C s.t $\pi(D_\lambda) = \lambda$.

$$\begin{tikzcd}
\pi^*[H] \arrow[r] \arrow[d] & {[H]} \arrow[d] \\
C \arrow[r, "\pi"] & \mathbb{P}^1 \arrow[u, bend right, "\sigma"']
\end{tikzcd}$$

$\sigma \to$ section $\Rightarrow$ $\pi^*\sigma$ is a section of $\pi^*[H]$.