NO.

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p is holomorphic in x. $\pi^{-1}(p) = X_p = \mathbb{P}^1$ for $p \in R$ and $\pi^{-1}(p)$ is a line in Σ. By the adjunction formula on P471,

$$0 = g(\pi^{-1}(p)) = \frac{K_\Sigma \cdot \pi^{-1}(p) + \pi^{-1}(p)\cdot\pi^{-1}(p)}{2} + 1$$
$$= \frac{\pi^{-1}(p)\cdot\pi^{-1}(p)}{2} + 1$$

$\Rightarrow$ $\pi^{-1}(p)\cdot\pi^{-1}(p) = -2$ $\Rightarrow$ By the arguments on P638 ~P639, p is a double point, if p is a singular point of multiplicity m, then $\pi^{-1}(p)\cdot\pi^{-1}(p) = -m$.

If the singular points are not discrete, then the set has dimension ≥ 1. $\Rightarrow$ $\pi^{-1}(p)\cdot\pi^{-1}(p) = 0$, since we can choose $\pi^{-1}(p')$ which is connected continuously with $\pi^{-1}(p)$, i.e. $0 \leq t \leq 1$, $\pi^{-1}(p_t)$, $p_0 = p$ $p_1 = p'$.

Note: I think; for the case of blow-ups, desingularization is unique. See P604

My guess might be correct, see P 500. According to the discussion of isolated singularities on P636 ~ P640, in particular, P640, if $p \in \hat{S}$ is nonoridinary singular, then $\pi^{-1}(p)$ is a union of several curves. Refer to Theorem on P621.

Question: Is a minimal desingularization unique?

It remains to determine the number $\# R$ of double