and the subspace $E \subset H^0(M, \mathcal{O}(L))$, and we have a basic dictionary

$$\left\{ \begin{array}{l} \text{nondegenerate maps} \\ f: M \longrightarrow \mathbb{P}^N, \text{ modulo} \\ \text{projective} \\ \text{transformations} \end{array} \right\} \rightleftarrows \left\{ \begin{array}{l} \text{line bundles } L \longrightarrow M \\ \text{with } E \subset H^0(M, \mathcal{O}(L)) \\ \text{such that } |E| \text{ has no} \\ \text{base points} \end{array} \right\}$$

where the choice of homogeneous coordinates on $\mathbb{P}^N$ corresponds to the choice of basis $s_0, \cdots s_N$ for $E$.

We will often write $\iota_L$ for $\iota_{H^0(M, \mathcal{O}(L))}$ and $\iota_D$ for $\iota_{[D]}$.

$\ulcorner$ Given a line bundle $L \longrightarrow M$ with $E \subset H^0(M, \mathcal{O}(L))$ s.t $|E|$ has no base points, then we can get a map

$$\begin{aligned} \iota_E : M &\longrightarrow \mathbb{P}^N \\ p &\longmapsto [s_0(p), \cdots s_N(p)]. \end{aligned} \qquad \begin{array}{ll} \{s_0, \cdots s_N\} & \text{basis for } E \\ \{t_0, \cdots t_N\} & \quad \text{"} \quad E \end{array}$$

$$\Rightarrow \begin{aligned} t_0 &= a_{00} s_0 + \cdots + a_{N0} s_N \\ t_1 &= a_{01} s_0 + \cdots + a_{N1} s_N \\ &\vdots \\ t_N &= a_{0N} s_0 + \cdots + a_{NN} s_N \end{aligned} \qquad (a_{ij}) \in GL_{N+1}.$$

$\Rightarrow$ By using $\{t_0, \cdots t_N\}$, we have a different map $\iota_E' : M \longrightarrow \mathbb{P}^N$ defined by $\iota_E'(p) = [t_0(p), \cdots t_N(p)]$.

$\Rightarrow$ $\iota_E \equiv \iota_E'$ ( modulo projective transformation induced by $(a_{ij})$ ) //