$$= \alpha_{1n} - \alpha_{10}\frac{X_n}{X_0} = 0.$$

For simplicity, let $n = 2$.

$$\Rightarrow \; v_1 = \left(\alpha_{11} - \frac{X_1}{X_0}\alpha_{10}\right)\frac{X_1}{X_0}\frac{\partial}{\partial x_1} + \left(\alpha_{12} - \alpha_{10}\frac{X_2}{X_0}\right)\frac{X_1}{X_0}\frac{\partial}{\partial x_2}$$

$$\alpha_{11} = \frac{X_1}{X_0}\alpha_{10} \qquad \alpha_{12} = \alpha_{10}\frac{X_2}{X_0}$$

$\Rightarrow$ Wrong!

For simplicity, let $n = 2$.

$$v_1 = \left(-\frac{\alpha_{10}X_1}{X_0} + \frac{\alpha_{11}X_1}{X_0}\right)\frac{\partial}{\partial x_1} + \left(-\frac{\alpha_{10}X_2}{X_0} + \frac{\alpha_{12}X_2}{X_0}\right)\frac{\partial}{\partial x_2}$$

$$\Rightarrow \quad \alpha_{10}X_1 - \alpha_{11}X_1 \overset{0}{=} \;\&\; \alpha_{10}X_2 - \alpha_{12}X_2 = 0 \Rightarrow v_1 = 0$$

Since $\alpha_{10} \neq \alpha_{11}$ & $\alpha_{10} \neq \alpha_{12}$, $X_1 = X_2 = 0 \Rightarrow v_1 = 0$ at $[1, 0, 0]$. By symmetry, $v_1 = 0$ at $[0, 1, 0]$ & $[0, 0, 1]$.

$$v_2 = \pi_*\left(\sum_j \alpha_{2j}X_j\frac{\partial}{\partial X_j}\right) = \alpha_{2j}X_j\,\pi_*\left(\frac{\partial}{\partial X_j}\right)$$

$$= \alpha_{20}X_0\,\pi_*\left(\frac{\partial}{\partial X_0}\right) + \alpha_{21}X_1\,\pi_*\left(\frac{\partial}{\partial X_1}\right) + \alpha_{22}X_2\,\pi_*\left(\frac{\partial}{\partial X_2}\right)$$

$$= \alpha_{20}X_0\left(-\frac{X_1}{X_0^2}\frac{\partial}{\partial x_1} - \frac{X_2}{X_0^2}\frac{\partial}{\partial x_2}\right) + \alpha_{21}X_1\underbrace{\pi_*\left(\frac{\partial}{\partial X_1}\right)}_{=\frac{1}{X_0}\frac{\partial}{\partial x_1}} + \alpha_{22}X_2\underbrace{\pi_*\left(\frac{\partial}{\partial X_2}\right)}_{=\frac{1}{X_0}\frac{\partial}{\partial x_2}}$$

$$= (\alpha_{21} - \alpha_{20})\frac{X_1}{X_0}\frac{\partial}{\partial x_1} + (\alpha_{22} - \alpha_{20})\frac{X_2}{X_0}\frac{\partial}{\partial x_2}$$

let $n = 3$.

$$v_1 = \pi_*\left(\sum \alpha_{1j}X_j\frac{\partial}{\partial X_j}\right) = \alpha_{10}X_0\pi_*\frac{\partial}{\partial X_0} + \alpha_{11}X_1\pi_*\frac{\partial}{\partial X_1} + \alpha_{12}X_2\pi_*\frac{\partial}{\partial X_2} + \alpha_{13}X_3\pi_*\frac{\partial}{\partial X_3}$$

$$= \alpha_{10}X_0\left(-\frac{X_1}{X_0^2}\frac{\partial}{\partial x_1} - \frac{X_2}{X_0^2}\frac{\partial}{\partial x_2} - \frac{X_3}{X_0^2}\frac{\partial}{\partial x_3}\right) + \alpha_{11}X_1\frac{1}{X_0}\frac{\partial}{\partial x_1} + \alpha_{12}\frac{X_2}{X_0}\frac{\partial}{\partial x_2} + \alpha_{13}\frac{X_3}{X_0}\frac{\partial}{\partial x_3}$$

$$= (\alpha_{11} - \alpha_{10})\frac{X_1}{X_0}\frac{\partial}{\partial x_1} + (\alpha_{12} - \alpha_{10})\frac{X_2}{X_0}\frac{\partial}{\partial x_2} + (\alpha_{13} - \alpha_{10})x_3\frac{\partial}{\partial x_3}$$

$$= (\alpha_{11} - \alpha_{10})x_1\frac{\partial}{\partial x_1} + (\alpha_{12} - \alpha_{10})x_2\frac{\partial}{\partial x_2} + (\alpha_{13} - \alpha_{10})x_3\frac{\partial}{\partial x_3}$$

$$\Rightarrow \; v_2 = (\alpha_{21} - \alpha_{20})x_1\frac{\partial}{\partial x_1} + (\alpha_{22} - \alpha_{20})x_2\frac{\partial}{\partial x_2} + (\alpha_{23} - \alpha_{20})x_3\frac{\partial}{\partial x_3}$$