

⌈ As we saw above, since M is algebraic, if we consider a hyperplane section $M \cap H$, where $M \subset \mathbb{P}^n$ and H is a hyperplane of \mathbb{P}^n , $M \cap H \sim L^k$ for some $k > 0$.
 $\Rightarrow (M \cap H)$ is ^{effective} $\Rightarrow L^k$ is effective.

Since L is positive, $L^k \cdot L = \int_{L^k} c_1(L) = k(L \cdot L) > 0$.

($\because c_1(L)$ is a volume form on L^k which is represented by a curve in M). $\Rightarrow k > 0 \Rightarrow L \cdot L = 1$. \sqcup

Thus, if we write $K_M = L^m$, m must be negative since K_M is not positive, and so

$$9 = K_M \cdot K_M = m^2(L \cdot L) \Rightarrow m = -3,$$

i.e., $K_M = L^{-3}$.

⌈ Since we assumed that K_M is not positive, $c_1(K_M) \in H^2(M, \mathbb{Z}) = \text{Pic}(M)$, and $K_M = L^m$ for $m < 0$. \sqcup

Apply Riemann-Roch for L :

$$h^0(L) - h^1(L) + h^2(L) = 1 + \frac{L \cdot L - K \cdot L}{2} = 1 + \frac{1 - (-3)}{2} = 3.$$

⌈ By P472 & P246,

$$h^0(L) - h^1(L) + h^2(L) = \chi(L) = \chi(\mathcal{O}_M) + \frac{L \cdot L - K \cdot L}{2}$$

$$h^0(L) - h^1(L) + h^2(L)$$

$$= 1$$

$$K_M \cdot L = L^{-3} \cdot L = -3$$

\sqcup

But $h^2(L) = h^0(K - L) = h^0(L^{-4}) = 0$ since L^{-4} is negative.