$V_1(W) \sim 7\sigma_2 + 3\sigma_{1,1}$.

$\ulcorner$ $V_1(W) \cap \sigma_{1,1} \ni l \Rightarrow l \subset H$ (a generic hyperplane chosen above). and l lies on a pencil of quadrics from W, $\Rightarrow$ Let $\{F_\lambda\}$ be a pencil of quadrics in W. $\Rightarrow$ The pencil of conics $\{F_\lambda \cap H\}$ are all singular, since $\{F_\lambda \cap H\}$ has l as a fixed line.
$\Rightarrow$ To count the number of all lines in $V_1(W) \cap \sigma_{1,1}$, we have only to ask for the number of pencils of singular conics in $W|H = X$ having a fixed line, since if L_1 & L_2 have the same fixed line, then $\langle L_1, L_2\rangle$ has the fixed line, which is absurd ($\because$ Singular conics pencils are discrete).

$\Rightarrow$ $\#(V_1(W)\cdot\sigma_{1,1}) = 3$

$\Rightarrow$ $V_1(W) \sim a\sigma_{1,1} + b\sigma_2$

$\Rightarrow$ $a = 3$, $b = 7$ since $\sigma_{1,1}\cdot\sigma_{1,1} = 1$ & $\sigma_2\cdot\sigma_2 = 1$ $\lrcorner$

We may check this calculation as follows: let N_1, N_2 be two generic nets in the web W, $L = N_1 \cap N_2$ their common pencil, and consider the intersection $V_0(N_1) \cap V_0(N_2)$. If a line l lies on a quadric $F_1 \in N_1$ and a quadric $F_2 \in N_2$, then either

(1) $F_1 \neq F_2$, so l lies on the pencil $\overline{F_1F_2} \subset W$,