Then in the blow-up $\tilde{\mathbb{P}}^2$ of $\mathbb{P}^2$ at the points a_i the proper transforms of the conics $G_i \ni \{a_j\}_{j\neq i}$ in $\mathbb{P}^2$ passing through five of the six points a_i are disjoint rational curves of self-intersection -1, and so in turn may be blown down.

See P 484

The resulting surface is $\mathbb{P}^2$; thus we have another Cremona transformation ψ.

$$\begin{tikzcd} & \tilde{\mathbb{P}}^2_{a_1\cdots a_6} \supset G_1,\cdots G_6 \arrow[dl, "\tilde{\pi}"'] \arrow[dr, "\pi"] & \\ \mathbb{P}^2 \arrow[rr, "\pi\circ\tilde{\pi}^{-1}"] & & \mathbb{P}^2 \end{tikzcd}$$

($\because$ If we let M be the surface obtained by blowing down $G_1,\cdots G_6$ in $\tilde{\mathbb{P}}^2$, then

$$b^0(M) = b^0(\tilde{\mathbb{P}}^2) = b^4(M) = b^4(\tilde{\mathbb{P}}^2) = 1.$$

$$H_i(M) \oplus H_i(G_1) \oplus \cdots \oplus H_i(G_6) = H_i(\tilde{\mathbb{P}}^2)$$

$$\Rightarrow \quad b^1(M) = b^1(\tilde{\mathbb{P}}^2) = b^3(\tilde{\mathbb{P}}^2) = b^3(M) = 1 = b^1(\mathbb{P}^2) = b^3(\mathbb{P}^2)$$

$$H_2(M) \oplus \mathbb{C}^6 = H_2(\tilde{\mathbb{P}}^2) = H_2(\mathbb{P}^2) \oplus \mathbb{C}^6$$

$$\Rightarrow \quad H_2(M) = H_2(\mathbb{P}^2) \quad \Rightarrow \quad b^2(M) = 1$$

Since $K_{\tilde{\mathbb{P}}^2}$ is negative, $D \cdot K_M = \pi^* D \cdot (K_{\tilde{\mathbb{P}}^2} - G_1 - \cdots - G_6)$ $\leq \pi^* D \cdot K_{\tilde{\mathbb{P}}^2} < 0 \quad \Rightarrow \quad K_M$ is not positive.

Here D is a divisor in M. $\Rightarrow$ Again by lemma on P487, $M \cong \mathbb{P}^2$)