

Consider $V_\epsilon = \{ \phi \in \mathcal{D}(\Omega) : \|\phi\|_N < \epsilon \}$.
 $\Rightarrow V_\epsilon \in \tau_K$ since. if we choose ℓ sufficiently large so that $\frac{1}{\ell} < \epsilon$, and $\ell > N$, then $V_\ell = \{ \phi \in \mathcal{D}(\Omega) : \|\phi\|_\ell < \frac{1}{\ell} \} \subset V_\epsilon$. \Rightarrow The collection of V_ϵ 's forms a subcollection of the local base of τ_K . \Rightarrow We have a smaller topology \mathcal{V} on \mathcal{D}_K than τ_K . $\Rightarrow \Lambda$ is continuous on this topology \mathcal{V} on \mathcal{D}_K . The topology is called C^∞ -topology on $\mathcal{D}(\Omega)$.

If Λ is such that one N will do for all K , then since Λ is continuous on all \mathcal{D}_K . $\Rightarrow \Lambda$ is continuous on $\mathcal{D}(\Omega) \Rightarrow \Lambda$ is a distribution in Ω .

Even if Λ is of infinite order, for given compact set $K \subset \Omega$, $\exists N_K$ and C_K s.t. $|\Lambda\phi| \leq C_K \|\phi\|_{N_K}$.

$\Rightarrow N_K$ is the order of Λ locally.

6.9 Remark Each $x \in \Omega$ determines a linear functional δ_x on $\mathcal{D}(\Omega)$, by the formula

$$\delta_x(\phi) = \phi(x).$$

Theorem 6.8 shows that δ_x is a distribution, of order 0.

$$\|\delta_x(\phi)\| = |\phi(x)| \leq \|\phi\|_0 = \max \{ |\phi(x)| : x \in \Omega \} \quad \square$$

If $x=0$, the origin of \mathbb{R}^n , the functional $\delta = \delta_0$ is frequently called the Dirac measure on \mathbb{R}^n .

Since \mathcal{D}_K , for $K \subset \Omega$, is the intersection of the null spaces of these δ_x , as x ranges over the complement of K , it follows that each \mathcal{D}_K is a closed subspace of $\mathcal{D}(\Omega)$. [This follows also from Th. 1.27 and part (b) of Theorem 6.5, since each \mathcal{D}_K is complete.].