$$|f_\epsilon(\varphi)| = \left|\int_{\mathbb{R}^n} u_\epsilon(x)\,\varphi(x)\,dx\right| \leq \underset{\uparrow}{\|u_\epsilon\|_0}\ \underset{\nearrow}{\|\varphi\|_0}$$

L^2-norm

$\Rightarrow$ Since $s \geq 0$, $\|u_\epsilon\|_{s+1} \geq \|u_\epsilon\|_0$.

$$\Rightarrow \quad \frac{|f_\epsilon(\varphi)|}{\|\varphi\|_0} \leq \|u_\epsilon\|_0 \leq \|u_\epsilon\|_{s+1}.$$

If we show $\|u_\epsilon\|_{s+1}$'s are uniformly bounded, then $\|f_\epsilon\|$'s are uniformly bounded. - - - - - - — ⊛

$$\lim_{\epsilon\to 0} f_\epsilon(\varphi) = \lim_{\epsilon\to 0}\int_{\mathbb{R}^n} u_\epsilon(x)\,\varphi(x)\,dx$$

$$= u(\varphi)$$

For $u_\epsilon(\varphi) = \int_{\mathbb{R}^n} u_\epsilon(x)\,\varphi(x)\,dx = \int_{\mathbb{R}^n} u_y(\chi_\epsilon(x-y))\,\varphi(x)$ $dx = u_y\left(\int_{\mathbb{R}^n} \varphi(x)\,\chi_\epsilon(x-y)\,dx\right) = u_y(\varphi_\epsilon) = u(\varphi_\epsilon)$. and

$\varphi_\epsilon \longrightarrow \varphi$ in the topology of $C_c^\infty(\mathbb{R}^n) \Rightarrow u(\varphi_\epsilon) \to u(\varphi)$. Thus $\lim_{\epsilon\to 0} f_\epsilon(\varphi) = u(\varphi)$ for $\forall \varphi \in C_c^\infty(\mathbb{R}^n)$

which is dense in $H_0(\mathbb{R}^n) = L^2(\mathbb{R}^n)$ - - - ⊛⊛

$\Rightarrow$ By p125. Theorem 10. Yosida. $\{f_\epsilon\} \in L^2(\mathbb{R}^n)'$ converges weakly to the element u, which represents an element of $L^2(\mathbb{R}^n)'$ by $u(\varphi) = \int_{\mathbb{R}^n} u(x)\,\varphi(x)\,dx$.

By the Gårding inequality in our assumption, we can