⇒ By the result above, $a_1'\omega_1 + \cdots + a_g'\omega_g = 0$. ⇒ $a_1' = \cdots = a_g' = 0$ since $\omega_1, \cdots, \omega_g$ are linearly independent over $\mathbb{C}$.

"Comment" One forms on Riemann Surfaces.

ω is a 1-form on S (Riemann surface)

We can define a function $\int_{z_0}^{z} \omega$ modulo the periods.

⇒ $d \int_{z_0}^{z} \omega = \omega$. This is the property which the only Riemann surface has, since we can define the line integral. That's because the Riemann surface is of dim 1.

Once we know this, we can take our basis $\omega_1, \cdots, \omega_g$ for $H^0(S, \Omega^1)$ so that

$$\int_{\delta_i} \omega_j = \delta_{ij} \quad \text{for} \quad 1 \le i, j \le g,$$

* $\int_{\delta_{g+i}} \omega_j = 0$ need not be true. *

i.e., so that the period matrix has the form

$$(I_g , Z).$$

Such a basis for $H^0(S, \Omega^1)$ is called normalized.

$$\begin{pmatrix} \int_{\delta_1} a_{11}\omega_1 + a_{12}\omega_2 , & \int_{\delta_2} a_{11}\omega_1 + a_{12}\omega_2 \\ \int_{\delta_1} a_{21}\omega_1 + a_{22}\omega_2 , & \int_{\delta_2} a_{21}\omega_1 + a_{22}\omega_2 \end{pmatrix} = \begin{pmatrix} a_{11} , & a_{12} \\ a_{21} , & a_{22} \end{pmatrix} \begin{pmatrix} \int_{\delta_1} \omega_1 , & \int_{\delta_2} \omega_1 \\ \int_{\delta_1} \omega_2 & \int_{\delta_2} \omega_2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \text{Take} \begin{pmatrix} a_{11} , & a_{12} \\ a_{21} , & a_{22} \end{pmatrix} = \begin{pmatrix} \int_{\delta_1} \omega_1 , & \int_{\delta_2} \omega_1 \\ \int_{\delta_1} \omega_2 , & \int_{\delta_2} \omega_2 \end{pmatrix}^{-1}$$