$$\begin{pmatrix} v_{11}, v_{12} \cdots v_{1n} \\ v_{21}, v_{22}, \cdots v_{2n} \\ \vdots \quad \vdots \quad v_{ij} \quad \vdots \\ v_{k1}, v_{k2}, \cdots v_{kn} \end{pmatrix} = {}^t\begin{pmatrix} g_{11}, g_{12} \cdots g_{1k} \\ g_{21}, g_{22} \cdots g_{2k} \\ g_{i1} \; g_{i2} \cdots g_{ik} \\ g_{k1}, g_{k2} \cdots g_{kk} \end{pmatrix} \begin{pmatrix} w_{11}, w_{12} \cdots w_{1n} \\ w_{21}, w_{22} \cdots w_{2n} \\ \vdots \\ w_{k1}, w_{k2} \cdots w_{kn} \end{pmatrix}$$ ⅃

Any $\Lambda \in U_I$ has a unique matrix representation Λ^I whose Ith $k\times k$ minor is the identity matrix, e.g., any $\Lambda \in U_{\{1,2,\cdots k\}}$ can be represented uniquely by a matrix of the form

$$\begin{bmatrix} 1 & 0 & \cdots & 0 & * & \cdots & * \\ 0 & 1 & & 0 & * & \cdots & * \\ \vdots & & \ddots & 0 & \vdots & & \vdots \\ 0 & 0 & & 1 & * & \cdots & * \end{bmatrix}.$$

Γ By choosing ${}^t g = \begin{pmatrix} w_{11} & \cdots & w_{1k} \\ \vdots & & \\ w_{k1}, & \cdots & w_{kk} \end{pmatrix}^{-1}$, we have a matrix representation Λ^I whose Ith $k\times k$ minor is the identity matrix.

Suppose Λ has two matrix representations.

$$\begin{pmatrix} I, & A \end{pmatrix} \quad \begin{pmatrix} I, & B \end{pmatrix}, \quad A=(a_{ij}), \; B=(b_{ij}).$$

$\Rightarrow \{(e_1, A_1), (e_2, A_2) \cdots\}$ & $\{(e_i, B_i)\}$ span Λ

where $(e_i, A_i) = (0 \cdots 0 \overset{i}{1} 0 \cdots 0, a_{i1} \cdots a_{in-k})$

$(e_j, B_j) = (0 \cdots \overset{j}{1} 0 \cdots 0, b_{j1} \cdots b_{j,n-k})$.

$\Rightarrow$ To span the same Λ, $(e_i, A_i) = (e_i, B_i)$.

$\Rightarrow A_i = B_i \Rightarrow \Lambda$ has a unique matrix representation. ⅃