quadratic form on $\mathbb{C}^n$ is its rank, there are all in all only three quadratic surfaces without multiple components in $\mathbb{P}^3$: (1) those given as the locus of a nondegenerate form

$$X_0^2 + X_1^2 + X_2^2 + X_3^2$$

on $\mathbb{C}^4$ — these are the smooth quadrics; (2) those given as the locus of a form

$$X_0^2 + X_1^2 + X_2^2;$$

such a quadric is the cone over a plane conic curve and singular at the vertex $[0,0,0,1]$ of the cone; and (3) those given as the locus of a form

$$X_0^2 + X_1^2;$$

these consist of the union of two planes.

$\ulcorner$ Rank of Q = rank of $(Q(e_i, e_j))$ = $\dim \mathbb{C}^4 - \dim V$, $Q = 0$ on V, V is maximal.

If Q_1 & Q_2 have the same rank, then $Q_1 = {}^tA Q_2 A$, by the same argument on P732 note.

$$\left(\frac{\partial\, X_0^2 + \cdots + X_3^2}{\partial X_i}\right) \neq 0. \Rightarrow X_0^2 + X_1^2 + X_2^2 + X_3^2 \text{ is nonsingular.}$$

$$\left(\frac{\partial X_0^2 + X_1^2 + X_2^2}{\partial X_i}\right) = (2X_0, 2X_1, 2X_2, 0) = 0$$

at $[0,0,0,1]$.

If $X_3 = 0$, in $\{[(X_0, X_1, X_2, 0)]\}$, $X_0^2 + X_1^2 + X_2^2 = 0$ is a plane curve.

For example, $X_0^2 + X_1^2 = 0 \Rightarrow X_0 = \pm i X_1$