

Thus. let s_1/Δ & s_2/Δ be expressed as f & g respectively. 28/

$$\Rightarrow D_\lambda \text{ can be identified with } (s_1 + \lambda s_2 = 0) \\ = (f + \lambda g = 0). \quad \lambda \in P' = \mathbb{C} \cup \{\infty\}. \quad \text{..}$$

We have then

$$f(P_\lambda) + \lambda g(P_\lambda) = 0$$

and

$$\frac{\partial f}{\partial z_i}(P_\lambda) + \lambda \frac{\partial g}{\partial z_i}(P_\lambda) = 0, \quad i = 1, \dots, n.$$

Since P_λ is not a base point of $\{D_\lambda\}$, f and g can not both vanish at P_λ and so neither one can: thus

$$\lambda = \frac{-f(P_\lambda)}{g(P_\lambda)} \quad (\because f(P_\lambda) + \lambda g(P_\lambda) = 0) \\ \lambda \neq 0, \infty$$

and

$$\frac{\partial f}{\partial z_i}(P_\lambda) - \frac{f(P_\lambda)}{g(P_\lambda)} \frac{\partial g}{\partial z_i}(P_\lambda) = 0.$$

Then

$$\frac{\partial}{\partial z_i} \left(\frac{f}{g} \right) (P_\lambda) = \frac{\left(\frac{\partial f}{\partial z_i} \right) (P_\lambda) - \left[\frac{f(P_\lambda)}{g(P_\lambda)} \right] \cdot \left(\frac{\partial g}{\partial z_i} \right) (P_\lambda)}{g(P_\lambda)} = 0$$

Now the locus V of singular points of the divisors D_λ , being locally the image in Δ of the variety $S \subset \Delta \times P'$ cut out by the equations $\{\lambda g + f = 0, \quad \frac{\partial f}{\partial z_i} + \lambda \frac{\partial g}{\partial z_i} = 0\}$, is an analytic subvariety of Δ .

Define functions as follows:

$$F_0: \Delta \times P' \longrightarrow \mathbb{C}$$

$$((z_1, \dots, z_n), w) \longmapsto f(z_1, \dots, z_n) + w g(z_1, \dots, z_n)$$

$$F_i: \Delta \times P' \longrightarrow \mathbb{C} \quad \text{for } i = 1, 2, \dots, n.$$

$$((z_1, \dots, z_n), w) \longmapsto \frac{\partial f}{\partial z_i}(z_1, \dots, z_n) + w \frac{\partial g}{\partial z_i}(z_1, \dots, z_n)$$