$w_2, w_1 \cdots w_{k_2}\ v_2 v_1 \cdots v_{n-k_1-k_2}, u_2, u_1 \cdots u_{k_1}$ is negatively oriented for $T_p(M)$.

C, with this orientation, is called the intersection cycle $A \cdot B$ of A & B. Much Harder.

Poincare Duality

The fundamental result on intersection of cycles is the

Theorem (Poincaré Duality). M. compact, oriented n-manifold $\Rightarrow$ The intersection pairing

$$H_k(M.\mathbb{Z}) \times H_{n-k}(M.\mathbb{Z}) \longrightarrow \mathbb{Z} \text{ is unimodular .i.e.}$$

any linear functional $H_{n-k}(M.\mathbb{Z}) \longrightarrow \mathbb{Z}$ is expressible as intersection with some class $\alpha \in H_k(M.\mathbb{Z})$, and any class $\alpha \in H_k(M.\mathbb{Z})$ having intersection number 0 with all classes in $H_{n-k}(M.\mathbb{Z})$ is a torsion class.

pf) We may assume that M is the underlying manifold of a simplicial complex $K = \{\sigma_\alpha^k, \partial\}_{\alpha,k}$.
The essential step in the proof is the construction of the dual cell decomposition of M, as follows.

First, let $\{\tau_\alpha^k, \partial\}$ be the first barycentric subdivision of the complex K. For each vertex σ_α^0 in the original triangulation,

let $*\sigma_\alpha^0 = \bigcup_{\tau_\beta^n \ni \sigma_\alpha^0} \tau_\beta^n$ be the n-cell given as