$\ulcorner$ $f : M \longrightarrow N$ C^∞ - injective. one ~~to~~ one. (immersion)

Let $v_1 \cdots v_n$ be a unitary frame for TN.
" $\varphi_1 \cdots \varphi_n$ " its dual " T^*N.

Define a metric on M s.t $\langle\langle u_1, u_2 \rangle\rangle_M = \langle\langle f_* u_1, f_* u_2 \rangle\rangle_N$.
Choose $\{w_i\}$ s.t $f_* w_i = v_i$, $w_i \in TM$,

$\Rightarrow$ $\langle\langle w_i, w_j \rangle\rangle_M = \langle\langle v_i, v_j \rangle\rangle_N = \delta_{ij}$
$\Rightarrow$ $f^* \varphi_i (\ ) = \langle\langle \_ , a_j w_j \rangle\rangle_M$
$\Rightarrow$ $f^* \varphi_i (w_j) = a_j$
$\quad$ " $\varphi_i (f_* w_j) = \varphi_i (v_j) = \delta_{ij} = a_j$
$a_j = 1$ if $j = i$
$a_j = 0$ if $i \neq j$

$\Rightarrow$ $f^* \varphi_j \longleftrightarrow w_j$
$\Rightarrow$ $\langle\langle f^* \varphi_i , f^* \varphi_j \rangle\rangle_M = \langle\langle w_i, w_j \rangle\rangle_M = \delta_{ij}$
$\Rightarrow$ $\{ f^* \varphi_i \}$ orthonormal frame for T^*M.
$\Rightarrow$ $\wedge f^* \varphi_i$ is a volume form for M. $\lrcorner\lrcorner$

$\ulcorner$

$$\begin{array}{ccc} H^2(\mathbb{P}^N) & \xrightarrow{i^*} & H^2(M) \\ \omega & \longmapsto & i^*\omega . \end{array}$$

where $M \overset{i}{\hookrightarrow} \mathbb{P}^N$

$$\int_M i^*\omega = \int_M \omega$$

Let σ be a 2-cycle in M. i.e $\sigma \in H_2(M)$

$$\int_\sigma i^*\omega = \int_\sigma \omega = \#(H \cdot \sigma) = \#(H \cap M \cdot \sigma)$$

$\Rightarrow$ $i^*\omega = \omega|_M$ is Poincaré dual to $H \cap M = V \subset M$. $\lrcorner\lrcorner$

$\ulcorner$ $\sigma \in H_k(M)$, $\tau \in H_\ell(M)$ $\Rightarrow$ $\sigma \cap \tau \in H_{(k+\ell)-n}(M)$
$\eta_\sigma \in H^{n-k}_{DR}(M)$ $\quad \eta_\tau \in H^{n-\ell}_{DR}(M)$ Poincare-duals of σ, τ
respectively, $n = \dim M$.
$\Rightarrow$ $\eta_\sigma \wedge \eta_\tau \in H^{2n-(k+\ell)}_{DR}(M)$.