$\ulcorner$ $D = \sum a_i V_i$.

$D + (f) \geq 0$ effective.

$(f) = \sum \mathrm{ord}_V(f) V$.

On $M - \bigcup V_i$, $\mathrm{ord}_V(f) \geq 0 \Leftrightarrow$ f is holomorphic.
and $\mathrm{ord}_{V_i}(f) + a_i \geq 0. \Leftrightarrow \mathrm{ord}_{V_i}(f) \geq -a_i$. $\lrcorner$

We denote by $|D| \subset \mathrm{Div}(M)$ the set of all effective divisors linearly equivalent to D; if $L = [D]$, we write $|L|$ for $|D|$. Let s_0 be a global meromorphic section of $[D]$ with $(s_0) = D$. Then for any global holomorphic section s of $[D]$, the quotient

$f_s = \dfrac{s}{s_0}$ is a meromorphic function on M with

$$(f_s) = (s) - (s_0) \geq -D, \qquad \text{i.e} \quad f_s \in \mathcal{L}(D)$$

and $(s) = D + (f_s) \in |D|$.

$\ulcorner$ the quotient $f_s = \dfrac{s}{s_0}$ is a meromorphic function on M with

$(f_s) = (s) - (s_0) \geq -D \Leftrightarrow (s) + D - (s_0) \geq 0$
since $D = (s_0)$ $\Rightarrow (f_s) + D \geq 0 \Rightarrow f_s \in \mathcal{L}(D)$.
and $(s) = D + (f_s) \in |D|$ since $D + (f_s)$ is equivalent to (s) linearly as an effective divisor. $\lrcorner$

On the other hand, for any $f \in \mathcal{L}(D)$, the section $s = f \cdot s_0$ of $[D]$ is holomorphic. Thus multiplication by s_0 gives an identification

$$\begin{array}{ccc} \mathcal{L}(D) & \xrightarrow{\otimes s_0} & H^0(M, \mathcal{O}([D])) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ f & \longmapsto & f \cdot s_0 \end{array}.$$