Pf. ($\Rightarrow$) $\exists\ f_1 \cdots f_k$ s.t $V = \{f_1 = \cdots = f_k = 0\}$ &

$\det\left(\frac{\partial f_i}{\partial z_j}\right) \neq 0 \quad 1 \le i, j \le k.$

$\Rightarrow \quad \pi : V \longrightarrow \mathbb{C}^{n-k}$

$(z_1, \cdots z_n) \longmapsto (z_{k+1}, \cdots z_n)$

$\Rightarrow$ π is one to one. Covering map $\Rightarrow$ $\mathrm{mult}_p(V) = 1$.

($\Leftarrow$) Suppose $\pi : V \cap U \longrightarrow O$ is a one-sheeted branched covering, where O is an open set in $\mathbb{C}^k$. By the assertion 2 on P14. π is branched over an analytic hypersurface of O.

$\Rightarrow$ As in the proof of Proposition (P21 ~ P22), we have an analytic function f s.t

$z' \longmapsto (z', \ z_n)$. $\quad z_n = f(z')$.

since π is one to one generically. □

In general, if $W \subset M$ is an irreducible subvariety, we define the multiplicity $\mathrm{mult}_W(V)$ of V along W to be simply the multiplicity of V at a generic point of W.

P31

The Wirtinger Theorem. The interplay between the real and imaginary parts of a hermitian metric now gives us the Wirtinger theorem, which expresses another fundamental difference between Riemannian and herm-