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$$\det \begin{bmatrix} (a_t)_{1,1} & (a_t)_{1,i} \cdot f_i(z) \\ (a_t)_{2,1} & (a_t)_{2,i} \cdot f_i(z) \\ \vdots & \vdots \\ (a_t)_{n,1} & (a_t)_{n,i} \cdot f_i(z) \end{bmatrix} = 0 .$$

Since $(a_t)_{ji}\, f_i(z) = (A_t f)_j = (g_t)_j$

$$\frac{1}{f_1(z)} \Big(\sum_{i,j} (-1)^j (A_t)_{j,1} (a_t)_{j,i} f_i(z) \Big) = \frac{1}{f_1(z)} \Big(\sum_j (-1)^j (A_t)_{j,1} (g_t)_j(z) \Big).$$

Thus $\det A_t(z)$ is in the ideal $\{g_{t,1} \cdots g_{t,n}\}_{p_t}$, and by the second elementary property of the residue integral

$$Res_{p_t} \left(\frac{h \det A_t \, dz_1 \wedge \cdots \wedge dz_n}{g_{t,1} \cdots g_{t,n}} \right) = 0 \quad \text{for } p_t \neq 0 .$$

Since $\det A_t(z) = \frac{1}{f_1(z)} \Big(\sum_j (-1)^j (A_t)_{j,1} (g_t)_j(z) \Big)$, $\det A_t$

$\in \{(g_t)_1, \cdots (g_t)_n\}_{p_t} \subset \mathcal{O}_{p_t}$.

$\Rightarrow$ By the final elementary property of the residual integral,

$$Res_{p_t} \left(\frac{\det A_t \, dz_1 \wedge \cdots \wedge dz_n}{(g_t)_1 \cdots (g_t)_n} \right) = 0 . \quad \text{and}$$

$Res_{p_t} \left(\frac{h \det A_t \, dz_1 \wedge \cdots \wedge dz_n}{(g_t)_1 \cdots (g_t)_n} \right) = 0$, since $h \det A_t \in \{(g_t)_1$

$\cdots (g_t)_n\}_{p_t}$.

Now then we use this together with case 2 to have: