$= K_V \otimes [-V]|_V \otimes [V]|_V = K_V.$ (*).

We can give the corresponding map on sections

$$\Omega^n_M(V) \xrightarrow{P.R} \Omega^{n-1}_V \quad \text{as follows:}$$

Considering a section ω of $\Omega^n_M(V)$ as a meromorphic n-form with a single pole along V and holomorphic elsewhere, we write

$$\omega = \frac{g(z)\, dz_1 \wedge \cdots \wedge dz_n}{f(z)},$$ (for the identification. See p138 ~p139)

where $z = (z_1, z_2, \cdots z_n)$ are local coordinates on M and V is given locally by $f(z)$.

Under the isomorphism (*), then, ω corresponds to the form ω' such that

$$\omega = \frac{df}{f} \wedge \omega'.$$

Explicitly, $df = \sum \frac{\partial f}{\partial z_i} \cdot dz_i$, and so we can take

$$\omega' = (-1)^{i-1} \frac{g(z)\, dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n}{\frac{\partial f}{\partial z_i}}$$

for any i such that $\partial f / \partial z_i \neq 0$.

fixed i

Γ $$\frac{df}{f} \wedge \omega' = \frac{1}{f}\Big(\sum_{i=1}^{n} \frac{\partial f}{\partial z_i} \cdot dz_i\Big) \wedge (-1)^{i-1} \frac{g(z)\, dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n}{\frac{\partial f}{\partial z_i}}$$

$$= \frac{1}{f} \frac{\partial f}{\partial z_i} dz_i \wedge (-1)^{i-1} \frac{g(z)\, dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n}{\partial f / \partial z_i} = \omega.$$ $\lrcorner$