No.

Consider a map $\phi : V \longrightarrow E_x$ defined by

$\phi(\sigma) = \sigma(x)$. $\Rightarrow$ ϕ is linear $\Rightarrow$ $\ker \phi = \Lambda_x$ has the dimension $\dim V - \dim E_x = n-k$.

$$\begin{array}{ccc} \iota_V : M & \longrightarrow & G(n-k, V) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ x & \longmapsto & \Lambda_x \end{array}$$

We want to show that
$$\begin{tikzcd} E \arrow[r, "\tilde{\iota}_V"] \arrow[d] & Q \arrow[d] \\ M \arrow[r, "\iota_V"] & G(n-k, V) \end{tikzcd}$$

defined by $\tilde{\iota}_V(v) = \sigma + \Lambda_x$, where $\phi(\sigma) = v$, isomorphic, on each fiber. Then we can conclude that $\iota_V^* Q = E$, by p26 lemma 3.1 Milnor.

rank $Q = k =$ rank E. -- ⊛

If $\tilde{\iota}_V(v) = 0$, $\sigma \in \Lambda_x$. $\Rightarrow$ $\phi(\sigma) = v = 0$.

$\Rightarrow$ $\tilde{\iota}_V$ is injective $\Rightarrow$ By ⊛, $\tilde{\iota}_V$ is isomorphic.

Thus
$$\begin{tikzcd} E \arrow[r, "\tilde{\iota}_V"] \arrow[d] & Q \arrow[r] \arrow[d] & S^* \arrow[d] \\ M \arrow[r, "\iota_V"] & G(n-k, V) \arrow[r, "\cong"] & G(k, V^*) \end{tikzcd}$$

$\Rightarrow$ $\iota_V^* S^* = E$.

Given a section $\sigma : M \longrightarrow E$, define $\tau : G(n-k, V) \longrightarrow Q$ by $\tau(\Lambda) = \sigma + \Lambda$.

$\Rightarrow$ The following diagram is commutative.