We ask accordingly whether we can evaluate the number $\eta^0_\Delta(\Gamma_f)$ in terms of the local behavior of f around its fixed points. What makes this possible is the fact that while the full decomposition of forms on $M\times M$ into bitype (cf. Section 2 in this chapter)

$$A^{p,q}(M\times M) = \bigoplus_{\substack{p_1+p_2=p\\ q_1+q_2=q}} A^{(p_1,q_1)(p_2,q_2)}(M\times M),$$

does not commute with the $\bar\partial$-operator, the coarser direct-sum decomposition

$$A^{p,q}(M\times M) = \bigoplus_{p_1} A^{(p_1,*),(p-p_1,q-*)}(M\times M)$$

does. Here $*$ represents an index running from zero to q. It follows that if T^0_Δ is the component of the current T_Δ of bitype $(0,*),(n,n-*)$ – i.e., the current defined by the linear function

$$T^0_\Delta(\varphi) = \int_\Delta \sum_q \varphi^{(n,n-q),(0,q)}$$

on test forms φ, then T^0_Δ is $\bar\partial$-closed and represents the Dolbeault cohomology class η^0_Δ.

$\ulcorner$ $\sigma\in A^{p,q}(M\times M) \Rightarrow \sigma = \bigoplus_{\substack{p_1+p_2=p\\ q_1+q_2=q}} \sigma^{(p_1,q_1),(p_2,q_2)}$

$\Rightarrow \bar\partial\sigma = \bigoplus_{\substack{p_1+p_2=p\\ q_1+q_2=q}} \bar\partial\sigma^{(p_1,q_1),(p_2,q_2)}$ $\quad$ $\bar\partial\sigma^{(p_1,q_1)(p_2,q_2)} \in A^{(p_1,q_1+1)(p_2,q_2)} \oplus A^{(p_1,q_1)(p_2,q_2+1)}$