hyper surface, i.e for any point $p \in V \subset M$, V can be given in a nbd of p as the zeros of a single holomorphic function f. Moreover, any holomorphic function g defined at p and vanishing on V is divisible by f in a nbd of p.
f is called a local defining function for V near p, and is unique up to multiplication by a function non-zero at p.

Γ $g = 0$ on V. assume that f is not divisible by the square of any nonunit in $\mathcal{O}_p$. (See P47)
$\Rightarrow$ $f = f_1 \cdots f_m$. where each f_i is irreducible.
$\Rightarrow$ By P11. Corollary (Weak. Nullstellensatz), f_i divides. h in $\mathcal{O}_p$.
$\Rightarrow$ f divides h. in $\mathcal{O}_p$. $\lrcorner$

If V_1^* is a connected component of $V^* = V - V_s$, then $\overline{V_1^*}$ is an analytic subvariety in M. (See P21. Proposition *)
Thus V can be expressed uniquely as the union of irreducible analytic hypersufaces.

$$V = V_1 \cup \cdots \cup V_m,$$

where the V_i's are the closures of a the connected components of V^*. In particular, V is irreducible $\Leftrightarrow$ V^* is connected.

Γ Def: An analytic subvariety V of a complex manifold M is a subset given locally as the zeros of a finite collection of holomorphic functions. Given a point $p \in V$, $\exists$ open $U \subset M$. s.t $U \cap V = \{ f_1 = 0 = \cdots = f_n = 0 \}$.
A point $p \in V$ is called a smooth point of V if V is a submanifold of M near p, that is, if V is given in nbd of p by holomorphic functions $f_1, f_2 \cdots f_k$ with $\operatorname{rank} \mathcal{J}(f) = k$; the locus of smooth points of V is denoted V^*. A point $p \in V - V^*$ is called a singular point of V; the singular locus $V - V^*$ of V is denoted V_s.