Consider $\langle \omega\wedge\eta, \omega\wedge\eta\rangle$ at some fixed point.

$$\omega = \sum_{i_1<\cdots<i_p} a_I\, v_{i_1}\wedge\cdots\wedge v_{i_p}. \qquad \{v_i\} \text{ orthonormal}$$

$$\eta = \sum_{j_1<\cdots<j_q} b_J\, v_{j_1}\wedge\cdots\wedge v_{j_q}.$$

$$\Rightarrow \omega\wedge\eta = \sum a_I b_J\, v_{i_1}\wedge\cdots\wedge v_{i_p}\wedge v_{j_1}\wedge\cdots\wedge v_{j_q}.$$

$$\Rightarrow \langle \omega\wedge\eta, \omega\wedge\eta\rangle \leq \sum |a_I b_J|^2 \leq \left(\sum|a_I|^2\right)\left(\sum|b_J|^2\right)$$

$$= \|\omega\|^2\|\eta\|^2.$$

$$\Rightarrow \|L(\eta\otimes s)\|^2 = \int \langle\omega\wedge\eta, \omega\wedge\eta\rangle_x \langle s,s\rangle_x\, dx$$

$$\leq \int \|\omega\|_x^2\|\eta\|_x^2\langle s,s\rangle_x\, dx \leq C\|\omega\|^2\int \|\eta\|_x^2\langle s,s\rangle_x\, dx$$

$$= C\|\omega\|^2\int\langle\eta,\eta\rangle_x\langle s,s\rangle_x\, dx$$

$$= C\|\omega\|^2\langle\eta\otimes s, \eta\otimes s\rangle = C\|\omega\|^2\|\eta\otimes s\|^2$$

$$< C'\|\eta\otimes s\|^2 \Rightarrow L \text{ is bounded.}$$

$\Rightarrow$ By P93. Th 4.10. (Rudin. Functional Analysis), its adjoint Λ is bounded.

Let $D : \Gamma(E) \longrightarrow \Gamma(T^*M\otimes E)$ be a connection.

$s\in\Gamma(E)$ $\quad s = f_i e_i$. $\{e_i\}$ orthonormal frame on U, s.t $\overline{U}\subset V_\alpha$ where $\{V_\alpha\}$ locally