As before, follow the isomorphisms. $\lrcorner\!\!\rfloor$

Now we are about done: given $\gamma \in H^{1,1}(M) \cap H^{*}(M,\mathbb{Z})$, we have $\cancel{i_*(\gamma) = 0}$, and hence $\gamma = C_1(L)$ is the Chern class of ~~some~~ line bundle $L \in H^1(M, \mathcal{O}^*)$.

$\ulcorner$ $\gamma \in H^{1,1}(M) \cap H^2(M,\mathbb{Z}) \Rightarrow i_*(\gamma) = 0$ since $i_* = \pi^{0,2} \circ$ de Rham Iso.

I think, when we say $\gamma \in H^{1,1}(M) \cap H^2(M, \mathbb{Z})$

$H^2(M,\mathbb{Z})$ is the image of the following map

$$H^2(M,\mathbb{Z}) \hookrightarrow H^2(M,\mathbb{C}) \overset{\text{de Rham Iso.}}{\cong} H^2_{DR}(M,\mathbb{C})$$

$$\Rightarrow \quad \begin{tikzcd} \gamma \in H^2(M,\mathbb{Z}) \arrow[r, "i_*"] \arrow[d] & H^2(M,\mathcal{O}) \arrow[dd, "\cong \text{ Dolbeault}"] \\ H^2(M,\mathbb{C}) \arrow[d, "\cong"] & \\ \gamma \in H^2_{DR}(M,\mathbb{C}) \arrow[r, "\pi^{0,2}"] & H^{0,2}_{\bar\partial}(M) \end{tikzcd}$$

$\Rightarrow$ Since $\gamma \in H^{1,1}_{DR}(M,\mathbb{C})$, $\pi^{0,2}(\gamma) = 0$.

$\Rightarrow$ $i^*(\gamma) = 0$

$$\Rightarrow \quad H^1(M,\mathcal{O}^*) \xrightarrow{C_1} H^2(M,\mathbb{Z}) \xrightarrow{i_*} H^2(M,\mathcal{O})$$
$$\gamma \longmapsto 0$$

$\Rightarrow \exists\ L \in H^1(M,\mathcal{O}^*)$ s.t. $C_1(L) = \gamma$. $\lrcorner\!\!\rfloor$