$$= \int_0^{2\pi} (\log \epsilon - i\theta)\, \psi(\epsilon e^{i\theta})\, \epsilon e^{i\theta} i\, d\theta$$

$$\Rightarrow \left| \int_0^{2\pi} (\qquad) d\theta \right| \leq \overset{\text{constant}}{\int_0^{2\pi}} |\log \epsilon|\, \epsilon + \theta \epsilon \, d\theta$$

$$= \epsilon |\log \epsilon| \cdot 2\pi + \frac{4\pi^2}{2} \epsilon \longrightarrow 0 \quad \text{as } \epsilon \to 0.$$

$$\Rightarrow (**) = 0.$$

The general case follows easily from the above since the essential point is to compute the integral on $\Delta^{\text{something}} \times (\Delta - \Delta(\epsilon))$. Thus we proved the claim.

By proving the claim, we can conclude that

$$\partial \varphi = \frac{1}{2} d \log h + \frac{1}{2} dj \quad \text{locally i.e on } U.$$

$$= \frac{1}{2} \partial \log h + \frac{1}{2} \partial j$$

$\Rightarrow$ $\partial\varphi$ is locally a meromorphic one-form (on U).

For each $q \in f(M-W) - f(W)$, we have an open set $U_q \subset \Delta^{n+1}$ s.t

$$\partial \varphi = \frac{1}{2} \partial \log h_q + \frac{1}{2} \partial j_q \quad \text{on } U_q.$$

$$\Rightarrow \left\{ \frac{1}{2} \partial \log h_q + \frac{1}{2} \partial j_q \right\}_{q \in f(M-W) - f(W)}$$

defines a meromorphic 1-form on an open set containing $f(M-W) - f(W)$, since on $U_q \cap U_{q'}$

$$\frac{1}{2} \partial \log h_q + \frac{1}{2} \partial j_q = \frac{1}{2} \partial \log h_{q'} + \frac{1}{2} \partial j_{q'}.$$

$\Rightarrow$ The open set obtained can be assumed $\Delta^{n+1} - f(W)$.

$\lrcorner$