Pf. If $L = 0$ on $\pi^{-1}D$, $\sigma_L(X) = L|_{\{\lambda X\}}$ vanishes on D only.

$\Rightarrow \dfrac{\sigma}{\sigma_L}$ holomorphic on all of $\mathbb{P}^n$. (?)

$\Rightarrow \dfrac{\sigma}{\sigma_L} = C$, constant.

To show $\dfrac{\sigma}{\sigma_L}$ holomorphic, we have to show σ_L has simple zeros. We will show this by example.

$$\mathbb{C}^3 \xrightarrow{L} \mathbb{C}$$
$$(z_0, z_1, z_2) \longmapsto a_0 z_0 + a_1 z_1 + a_2 z_2 .$$

Define $\mathbb{P}^2 \xrightarrow{\sigma_L} H$ by $\sigma_L([z_0, z_1, z_2])$

$= L|_{\{\lambda(z_0, z_1, z_2)\}}$.

Suppose $\{z_0 \neq 0\} = U \subset \mathbb{P}^2$

$$\begin{array}{ccccc} [z_0, z_1, z_2] \in U \subset \mathbb{P}^2 & \xrightarrow{\sigma_L} & H|_U \cong U \times \mathbb{C} \\ \downarrow \quad \swarrow & & \downarrow \\ \left(\frac{z_1}{z_0}, \frac{z_2}{z_0}\right) \in \mathbb{C}^2 & & \left\{ \left(\underset{W_1}{\overset{\shortparallel}{\frac{z_1}{z_0}}}, \underset{W_2}{\overset{\shortparallel}{\frac{z_2}{z_0}}}\right), \text{ value at } \left(1, \frac{z_1}{z_0}, \frac{z_2}{z_0}\right) \right\} \end{array}$$

$\Rightarrow \sigma_L : \mathbb{C}^2 \longrightarrow \mathbb{C}$

$(W_1, W_2) \longmapsto a_0 + W_1 a_1 + W_2 a_2$