⌈ According to (P73.) lemma.

$\theta = \partial h' \cdot h'^{-1}$ $\Rightarrow$ Since h' is 1×1 matrix,

$$\theta = \frac{\partial h'}{h'} = \partial \log h'$$

$\Rightarrow$ $\Theta = d\theta - \cancel{\theta \wedge \theta} \overset{=0}{=} d\theta = d\,\partial \log h' = (\partial + \bar\partial)\,\partial \log h'$

$= \bar\partial \partial \log h' = -\partial\bar\partial \log h'$.

where $h' = \langle s, s \rangle$.

$\Rightarrow$ $-\partial\bar\partial \log \langle s, s\rangle = -\partial\bar\partial \log h \cdot |s_u|^2$

$= -\partial\bar\partial\left((\log h) + \log |s_u|^2\right) = -\partial\bar\partial \log h - \partial\bar\partial \log |s_u|^2$

$\Rightarrow$ $\partial\bar\partial \log |s_u|^2 = \partial\bar\partial(\log s_u \bar{s}_u) = \partial\bar\partial(\log s_u + \log \overline{s_u})$

$=0$ since s_u is holomorphic. ⌋

Now let $|s|'^2$ be another metric on L with curvature form Θ'. Then $\dfrac{|s|'^2}{|s|^2} = e^\rho$ for some real C^∞ function ρ on M, and from the local formular

$$h'(z) = e^{\rho(z)} h(z)$$

it follows that

$$\Theta = \partial\bar\partial \rho + \Theta'.$$

⌈ $\dfrac{|s|'^2}{|s|^2} = e^\rho = \dfrac{h' |s_u|^2}{h |s_u|^2} = \dfrac{h'}{h}$ $\Rightarrow$ $h' = e^\rho h$

$\Rightarrow$ $\Theta' = -\partial\bar\partial \log h' = -\partial\bar\partial \log(e^\rho h) = -\partial\bar\partial \log e^\rho - \partial\bar\partial \log h$ ⌋