NO.
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It follows that $\omega$ does not have the hyperplane at infinity as a component of its polar divisor exactly when the degree restriction

$$\deg(g) \leq (d_1+\cdots+d_n)-(n+1)$$

is satisfied.

Pf For the expression of the most general meromorphic $n$-form on $\mathbb{P}^n$, see P170 & refer to P148.

By the expression above, at infinity

$$\omega = -\frac{1}{(x_1')^{n+1}}\, dx_1'\wedge\cdots\wedge dx_n'\Big(\prod_{i=1}^{n}\frac{1}{(x_1')^{d_i}} f_i'(x_1',x_2',\cdots x_n')\Big)^{-1}\cdot g\Big(\frac{1}{x_1'}\cdots\frac{x_n'}{x_1'}\Big) = -\frac{dx_1'\wedge\cdots\wedge dx_n'}{(x_1')^{n+1-\Sigma d_i}}\,\frac{1}{f_1'\cdots f_n'}\,g\Big(\frac{1}{x_1'},\cdots\frac{x_n'}{x_1'}\Big)$$

To check whether $\omega$ has a component of its polar divisor at infinity, we have to see whether $\omega=\infty$ at $x_1'=0$.

$\Rightarrow$ If we let $\deg(g)=\ell$,

$$\frac{1}{(x_1')^{n+1-\Sigma d_i}}\,\frac{1}{(x_1')^{\ell}} = (x_1')^{\Sigma d_i-(n+1)-\ell}$$ is what we count on. $\Rightarrow$ If $\Sigma d_i-(n+1)-\ell \geq 0$, then $\omega$ does not have the hyperplane at infinity $\underset{(X_0=0)}{\parallel}$ as a component of its polar divisor.

⊛ Note that $f_i'(x_1',\cdots x_n')$ is not divided by $x_1'$. ⊛

$\Rightarrow$ We don't need to worry the residue at the infinity. $\Rightarrow \left(0=\int_{\mathbb{P}^n\setminus\mathbb{C}^n}\right) \int_{\mathbb{C}^n}\omega = 0 = \Sigma\cdots = \int_{\mathbb{P}^n}\omega$