$\ulcorner$ Consider $\{(df_\alpha \otimes s_\alpha)\}$, where

$$\begin{tikzcd}
{[V]|_{U_\alpha}} \arrow[r, "\varphi_\alpha"] \arrow[dr] & U_\alpha \times \mathbb{C} \arrow[d] \\
 & U_\alpha \arrow[ul, bend left, "s_\alpha"] \arrow[u, "x \mapsto (x,1)"']
\end{tikzcd}$$

i.e $\varphi_\alpha^{-1}(x,1) = s_\alpha(x)$.

On $U_\alpha \cap U_\beta \cap V$, $df_\alpha \otimes s_\alpha \in N_V^*|_{U_\alpha} \otimes [V]|_{U_\alpha}$ actually $U_\alpha \cap V$.

$$df_\alpha \otimes s_\alpha = g_{\alpha\beta}\, df_\beta \otimes s_\alpha = df_\beta \otimes g_{\alpha\beta} s_\alpha. \text{ by } \circledast$$

$\circledast$

$$\varphi_\alpha^{-1}(x,1) = s_\alpha(x) \qquad \varphi_\beta^{-1}(x,1) = s_\beta(x)$$

$$\varphi_\beta(\varphi_\alpha^{-1}(x,1)) = \varphi_\beta(s_\alpha)$$

$$\parallel$$

$$(x, g_{\beta\alpha}(x)\, 1) = \varphi_\beta(s_\alpha)$$

$$\varphi_\beta^{-1}(x g_{\beta\alpha}(x)\, 1) = \varphi_\beta^{-1}(x, g_{\beta\alpha}(x)\, 1) = g_{\beta\alpha}(x)\, \varphi_\beta^{-1}(x, 1)$$

$$= g_{\beta\alpha}(x)\, s_\beta(x) = s_\alpha(x) \Rightarrow g_{\beta\alpha} s_\beta = s_\alpha. \Rightarrow s_\beta = g_{\alpha\beta} s_\alpha$$

Thus $df_\alpha \otimes s_\alpha = df_\beta \otimes g_{\alpha\beta} s_\alpha = df_\beta \otimes s_\beta$ on $V \cap U_\alpha \cap U_\beta$.

This means that $\{(df_\alpha \otimes s_\alpha)\}$ defines a global section of $N_V^* \otimes [V]|_V$.

Furthermore, since df_α is nonzero everywhere & s_α is nonzero everywhere, the global section is nonzero.

Since $N_V^* \otimes [V]$ is line bundle, it is the trivial bundle.

$$N_V^* \otimes [V]|_V = M|_V \times \mathbb{C}$$

$$\Rightarrow N_V^* \otimes [V]|_V \otimes [-V]|_V = (M|_V \times \mathbb{C}) \otimes [-V]|_V = [-V]|_V$$

$$\parallel$$

$$N_V^* = [-V]|_V$$

$\lrcorner$