$\Rightarrow \dim V = \dim H^{1,1}(M) - 1 = \#$ of negative eigenvalues of a matrix associated to Q, which is invariant under a change of bases. $\Rightarrow$ Since $\dim P^{1,1} = \dim H^{1,1}(M)-1$, Q must be negative definite on V. ⅃

In particular, if $D \cdot D > 0$, then for any divisor D' on M such that $D' \cdot D = 0$, either $D' \cdot D' < 0$ or $(D') = 0$; this is commonly called the index theorem.

☞ For any divisor D' s.t $D' \cdot D = 0$, $D' \cdot D = \int_M \eta_{D'} \wedge \eta_D = 0$, and $\eta_{D'} \in V$. $\Rightarrow$ $\eta_{D'} \cdot \eta_{D'} < 0$ if not $\eta_{D'} = 0$. In case $\eta_{D'} = 0$, $(D') = 0$. ⅃

By way of terminology, we define a curve C on the surface M to be any effective divisor on M; a curve C is called smooth if it is the locus of a submanifold of M taken with multiplicity 1 and irreducible if it is not the sum of two nontrivial effective divisors. ☞ Compare with the definition on P20 & P22. ⅃

Let C be a smooth, irreducible curve on M. By the adjunction formula from Section 1 of Chapter 1

$$K_C = (K_M + C)|_C,$$

where K_C and K_M denote, as usual, the canonical line bundles of C and M.

☞ See P147, Adjunction Formula II

$$K_V = (K_M \otimes [V])|_V, \quad V = C$$ ⅃