$$0 \longrightarrow E_1/\partial E_2 \xrightarrow{\eta} E_0 \longrightarrow M \longrightarrow 0$$

$$E_1/\partial E_2 \xrightarrow{\omega} N \ , \quad \text{since } \omega = 0 \text{ on } \partial E_2 \ (\because \delta\omega = 0)$$

By the arguments above,

$$0 \longrightarrow N \longrightarrow \frac{N \oplus E_0}{\mu(E_1/\partial E_2)} \longrightarrow M \longrightarrow 0 \qquad \text{exact}$$

$$n \longmapsto (n \oplus (0)) + \operatorname{im}\mu$$

$$n \oplus e_0 \longmapsto -\eta(e_0)$$

where $\mu : E_1/\partial E_2 \longrightarrow N \oplus E_0$

$$e_1 + \partial E_2 \longmapsto (\omega(e_1), \partial(e_1)) .$$

Define a map from $N \oplus E_0$ to E by

$$(n, e_0) \longmapsto \alpha(n) - \gamma(e_0) .$$

$$\Rightarrow \quad (\omega(e_1), \partial(e_1)) \longmapsto \alpha(\omega(e_1)) - \gamma(\partial(e_1)) = \delta(\gamma)(e_1) - \gamma(\partial(e_1))$$
$$= 0$$

$\Rightarrow$

$$\begin{tikzcd}
0 \arrow[r] & N \arrow[r] \arrow[d, equal] & \frac{N \oplus E_0}{\mu(E_1/\partial E_2)} \arrow[r] \arrow[d] & M \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & N \arrow[r, "\alpha"] & E \arrow[r, "\beta"] & M \arrow[r] & 0
\end{tikzcd}$$

$$n \longmapsto (n \oplus (0)) + \operatorname{im}\mu , \qquad (n \oplus e_0) + \operatorname{im}\mu \longmapsto -\eta(e_0)$$

$$n \longmapsto \alpha(n) , \qquad \alpha(n) - \gamma(e_0) \longmapsto -\beta \circ \gamma(e_0) = -\eta(e_0)$$