exists a rank-two holomorphic vector bundle $\mathbb{C}^2 \to E \to S$ with $\det E = L$ and having a section $s \in H^0(S, \mathcal{O}(E))$ with $(s) = Z$ if and only if, Z has the Cayley-Bacharach property relative to the linear system $|K \otimes L|$.

Pf Let $H^0(S, \Omega^2(L)) = \langle \psi_1, \psi_2, \cdots \psi_m \rangle$ and $Z = \{p_1, p_2, \cdots p_n\}$.
Assume $m \geq n$.

$$\begin{pmatrix} \psi_1(\tau_1), & \cdots & , \psi_1(\tau_{n-1}), & \psi_1(\tau_n) \\ \psi_2(\tau_1), & \cdots & , \psi_2(\tau_{n-1}), & \psi_2(\tau_n) \\ \vdots & & & \\ \psi_m(\tau_1), & \cdots & , \psi_m(\tau_{n-1}), & \psi_m(\tau_n) \end{pmatrix}$$

$\Rightarrow \exists$ $m-(n-1)$ linearly independent vectors
$(a_{11}, a_{12}, \cdots a_{1m})$, $\cdots$ $(a_{m-n+1,1} \cdots a_{m-n+1,m})$ in $\mathbb{C}^m$
s.t

$$\begin{aligned} a_{11}\psi_1 + &\cdots + a_{1m}\psi_m = 0 \\ &\vdots \\ a_{m-n+1,1}\psi_1 + &\cdots + a_{m-n+1,m}\psi_m = 0 \quad \text{at } \tau_1, \tau_2, \cdots \tau_{n-1}. \end{aligned}$$

$\Rightarrow$ Since Z has the Cayley-Bacharach property,

$$\begin{aligned} a_{11}\psi_1 + &\cdots + a_{1m}\psi_m = 0 \\ &\vdots \\ a_{m-n+1,1}\psi_1 + &\cdots + a_{m-n+1,m}\psi_m = 0 \quad \text{at } \tau_n, \text{ too.} \end{aligned}$$