If we consider a_{ij}'s and b_{ij}'s as variables and g_{ij}'s some constants, in computing $\sum_{\#I=2} \det(gAg^{-1})_I$, for each $a_{i_1j_1} a_{i_2j_2} \cdots a_{i_\ell j_\ell}$ term, summation of g_{ij}'s turn out to be some constant.
In other words, g_{ij}'s do not affect on a_{ij}'s or b_{ij}'s. They play a role as coefficients of a_{ij}'s and b_{ij}'s. Therefore,

$$\sum_{\#I=2} \det(A_I^{id}) = \sum_{\#I=2} \det(gAg^{-1})_I^{id}.$$

Given a symmetric homogeneous polynomial f of deg k, its polarization is expressed as follows:

$$\frac{\partial^k}{\partial t_1 \partial t_2 \cdots \partial t_k} f(t_1 v_1 + t_2 v_2 + \cdots t_k v_k) = Q(v_1, v_2, \cdots v_k)\, k!.$$

For example. $k=2$.

$$\begin{aligned} f(t_1 v_1 + t_2 v_2) &= Q(t_1 v_1 + t_2 v_2,\ t_1 v_1 + t_2 v_2) \\ &= t_1^2 Q(v_1, v_1) + 2 t_1 t_2 Q(v_1, v_2) \\ &\quad + t_2^2 Q(v_2, v_2) \end{aligned}$$

$$\Rightarrow \frac{\partial^2}{\partial t_1 \partial t_2} f(t_1 v_1 + t_2 v_2) = 2\, Q(v_1, v_2)$$

The polarizations of a general invariant polynomial is expressed as a polynomial in the elementary invariant polynomials p^i by P402 $f(A) = G(P^1(A), \cdots P^n(A))$. ↵