Date

Note that away from the branch locus the map π is a covering map and we can take coordinates $(z_1(p_1), \cdots z_d(p_d))$ on $S^{(d)}$. At the other extreme, around a point d.p local coordinates are

$$(z_1 + \cdots + z_d, \cdots\cdots, z_1 \cdots z_d).$$

For $d = 3$, $(z_1, z_2, z_3) \xmapsto{\phi} (z_1+z_2+z_3, z_1z_2+z_2z_3+z_1z_3, z_1z_2z_3)$.

$$\Rightarrow J(\phi) = \begin{pmatrix} 1, & 1, & 1 \\ z_2+z_3, & z_1+z_3, & z_1+z_2 \\ z_2z_3, & z_1z_3, & z_2z_1 \end{pmatrix}$$

$\Rightarrow |J(\phi)| = a(z_1-z_2)(z_2-z_3)(z_3-z_1)$ since we note that, if $z_1 = z_2$, $z_2 = z_3$ or $z_1 = z_3$, $|J(\phi)| = 0$, and deg $|J(\phi)| = 3$. We can see the coefficient of $z_2^2 z_1$ is equal to 1. $\Rightarrow$ $a = 1$.

$\Rightarrow |J(\phi)| = (z_1-z_2)(z_2-z_3)(z_3-z_1)$.

In general d, $|J(\phi)| = a(z_1-z_2)(z_2-z_3)(z_3-z_4)\cdots(z_d-z_1)$ where $a = +1$ or -1.

The compact complex manifold $S^{(d)}$ is called the dth symmetric product of S. (It is interesting to verify that $\mathbb{P}^{1(d)} = \mathbb{P}^d$.)

Again for $d=2$, consider a map ϕ defined by

$$\mathbb{P}^1 \times \mathbb{P}^1 \longrightarrow \mathbb{P}^2$$

$$([z_0, z_1], [w_0, w_1]) \longmapsto [z_0w_0, z_1w_0 + z_0w_1, z_1w_1]$$

$\Rightarrow$ Obviously, ϕ is holomorphic, and $\phi([w_0, w_1], [z_0,$