

all. It follows that the Reiss relation is both necessary and sufficient.

⌈ $f(x, y) = f_1(y) + f_2(y)x + f_3(y)x^2 + x^3g(x, y)$ is the unique representation. Since we have only to consider points of $\{f=0\} \cap \{x=0\}$, $L \cap C = \{(0, q_u) = p_u\}$.

$$f_{xx} = 2f_3(y)$$

$$f_{xy} = (f_2)_y + 2x(f_3)_y$$

$$f_{yy} = (f_1)_{yy} + (f_2)_{yy}x + x^2(f_3)_{yy}$$

$$f_{xx}(p_u) = 2f_3(q_u) \quad f_{xy}(p_u) = (f_2)_y(q_u) \quad f_{yy}(p_u) = (f_1)_{yy}(q_u)$$

Given $a_u, b_u, c_u \in \mathbb{C}$, we have to find $f(x, y)$ s.t.
 $2f_3(q_u) = a_u, (f_2)_y(q_u) = b_u, (f_1)_{yy}(q_u) = c_u$

If we let $U = \{f(x, y) \mid \deg f = n\}$, $W = \{x^3g(x, y) \mid \deg(g) = n-3\}$,

$$f \longmapsto f+W \in V = \frac{U}{W}$$

$$\downarrow$$

$$[f+W] \in \mathbb{P}(V) \cong \mathbb{P}^{3n-1}$$

\Rightarrow If we have a curve f with prescribed second-order behavior at points p_u , we get a point in $\mathbb{P}(V)$.

Conversely, given a point \bar{z} in $\mathbb{P}(V)$, $\exists f \in U$ s.t. $[f+W] = \bar{z}$. \Rightarrow We have a curve f .

At each point $p_u \in C \cap L$, f must satisfy the following:

$$\textcircled{1} \quad f(p_u) = 0 = y(p_u)$$

$$\textcircled{2} \quad f_x(p_u) + f_y(p_u) y'(p_u) = 0 \quad y'(p_u) \text{ tangent}$$

$$\textcircled{3} \quad f_{xx}(p_u) + 2f_{xy}(p_u) y'(p_u) + f_{yy}(p_u) y'(p_u)^2 + f_y(p_u) y''(p_u) = 0$$

$y''(p_u)$ second order of the curve.