$\dim(\Lambda \cap V_{x_2}) \geq 2$ and $\dim(\Lambda \cap V'_{y_2}) \geq k-1$

$\Rightarrow \exists\ v_1, \cdots v_{k-1}$ linear indept vectors in $\Lambda \cap V'_{y_2}$

s.t $v_i = (0 \cdots 0 \overset{x_2}{*} * \cdots *)$ $i = 1, 2, \cdots k-1$.

$\Rightarrow \{e_{x_1}, v_1 \cdots v_{k-1}\}$ is a basis for Λ.

$\Rightarrow \dim(\Lambda \cap V'_{y_2}) = k-1$, otherwise $\dim \Lambda > k$.

Since $\dim(\Lambda \cap V_{x_2}) \geq 2$, $\exists\ v \in \Lambda \cap V'_{y_2}$ s.t

$v \in V_{x_2}$. $\Rightarrow v = (0 \cdots 0 \overset{x_2}{*}, 0 \cdots 0)$

($\because$ v is a linear combination of $e_{x_1}, v_1 \cdots v_{k-1}$.)

$\Rightarrow \dim(\Lambda \cap V_{x_2}) = 2$. and $\Lambda \cap V_{x_2} = \langle e_{x_1}, e_{x_2} \rangle$.

Continue this procedure, then we have

$\Lambda \cap V_{x_i} = \langle e_{x_1}, e_{x_2}, \cdots e_{x_i} \rangle$.

$\Rightarrow \Lambda = \langle e_{x_1} \cdots e_{x_k} \rangle = \{(0, \cdots 0 \overset{x_1}{*} 0 \cdots 0 \overset{x_2}{*} \cdots \overset{x_3}{*} 0 \cdots \overset{x_k}{*} 0 \cdots 0) \in \mathbb{C}^n\}$

Thus we proved the claim, in other words, $\sigma_a \cap \sigma_b$ is just one-point set.

Summing up, then, we have the formula

$$\#(\sigma_a \cdot \sigma_b) = \delta_{(a_1, a_2 \cdots a_k)}^{(n-k-b_k, \cdots n-k-b_1)}.$$

Pf $\#(\sigma_a \cdot \sigma_b) = \begin{cases} 1 & \text{if } b_{k-i+1} = n-k-a_i \text{ for all } i \\ 0 & \text{otherwise.} \end{cases}$

$b_1 = n-k-a_k$, $b_2 = n-k-a_{k-1}$ $\cdots$ $b_k = n-k-a_{k-k+1}$

$\Updownarrow$ $\Updownarrow$

$a_k = n-k-b_1$ $a_1 = n-k-b_k$

$a_1 = n-k-b_k$. $a_2 = n-k-b_{k-1}$ $\cdots$ $a_k = n-k-b_1$.

$\Rightarrow \#(\sigma_a \cdot \sigma_b) = 1 = \delta_{(a_1 \cdots a_k)}^{(n-b_k-k, \cdots n-k-b_1)}$.

This enables us to express an arbitrary cycle γ on