For $\sigma \in C^p(\underline{U}, \mathrm{Hom}(\mathcal{E}_q, \mathcal{G}))$, $D\sigma = \delta_p\sigma + \partial_q^*\sigma$.

$$\Rightarrow \phi^*(D\sigma) = \phi_q^*(\delta_p\sigma) + \phi_{q+1}^*(\partial_q^*\sigma)$$
$$= \delta_p'(\phi_q^*\sigma) + \partial_q'^*(\phi_q^*\sigma) \in C^{p+q+1}(\underline{U}, D)$$
$$= D(\phi_q^*\sigma)$$

For, $\phi_q^*(\delta_p\sigma)(x)$, $x \in \mathcal{E}_q'|U_{p+1}$

$$= (\delta_p\sigma)(\phi_q(x)) = \sigma|_{U_{p+1}}(\phi_q(x)|_{U_{p+1}}), \quad U_{p+1} = \bigcap_{i=1}^{p+2} U_{\beta_i}, \ \underline{U} = \{U_\beta\}$$

and

$$\delta_p'(\phi_q^*\sigma)(x) = \phi_q^*\sigma|_{U_{p+1}}(x) = \sigma|_{U_{p+1}}(\phi_q(x)) \Rightarrow \phi_q^* \circ \delta_p = \delta_p' \circ \phi_q^*.$$

$$\phi_{q+1}^*(\partial_q^*\sigma)(y) = (\partial_q^*\sigma)(\phi_{q+1}(y)) = \sigma(\partial_q \circ \phi_{q+1}(y)) = \sigma(\phi_q \circ \partial_q'(y))$$
$$= \phi_q^*(\sigma(\partial_q(y))) = (\partial_q^* \phi_q^* \sigma)(y)$$
$$\Rightarrow \phi_{q+1}^* \partial_q^* = \partial_q^* \phi_q^*.$$

Thus we have a map $\phi : H_\prime^n(\underline{U}, D) \longrightarrow H^n(\underline{U}, D)$. But here is a trouble! We can not find a chain homotopy to prove that if there is another map $\psi : H_\prime^n(\underline{U}, D) \longrightarrow H^n(\underline{U}, D)$, then $\phi = \psi$.

!!!. After I have been thinking about the statement that at least provided we allow ourselves to tensor with $\mathcal{L}^{-k}$, I guess (or hope) that the authors suggest the readers to accept that without reasoning. !!! Must be we need more machineries to prove the independence rigorously

In order to calculate global Ext, a main tool are the two spectral sequences of hypercohomology. The first of these is a spectral sequence $\{'E_r\}$ with

$$'E_2^{p,q} = H^p(M, \underline{Ext}_{\mathcal{O}}^q(\mathcal{F}, \mathcal{G})),$$