(*) $$0 \longrightarrow \Omega^p \longrightarrow \mathcal{D}^{p,0} \xrightarrow{\bar\partial} \mathcal{D}^{p,1} \longrightarrow \cdots \xrightarrow{\bar\partial} \mathcal{D}^{p,n} \longrightarrow 0.$$

Since distributions may be multiplied by C^∞ functions, the sheaves $\mathcal{D}^{p,q}$ admit partitions of unity. Consequently, $H^k(M, \mathcal{D}^{p,q}) = 0$ for $k>0$, and the sheaf-theoretic proof of the deRham and Dolbeault theorems from Section 3 of Chapter 0 will apply verbatim if we can prove that (*) is exact. In other words, we must establish the $\bar\partial$-Poincaré lemma for currents.

$\ulcorner$ $\underline{U} = \{U_\alpha\}_{\alpha\in I}$ locally finite cover of M

Given $\sigma \in Z^k(\underline{U}, \mathcal{D}^{p,q})$, define $\tau \in C^{p-1}(\underline{U}, \mathcal{D}^{p,q})$ by setting

$$\tau_{\alpha_0,\cdots\alpha_{k-1}} = \sum_{\beta\in I} \rho_\beta\, \sigma_{\beta,\alpha_0,\cdots\alpha_{k-1}}.$$

$\Rightarrow$ $\delta\tau = \sigma$. $\Rightarrow$ $H^k(M, \mathcal{D}^{p,q}) = 0$.

See p 42, & p44 ~ p45. $\lrcorner$

The first step is just the regularity theorem for the $\bar\partial$-operator. Note that this step is trivial for the full exterior derivative d.

$\ulcorner$ Given $T \in \mathcal{D}^{p,0}(U)$ s.t $\bar\partial T = 0$, then by p380, regularity for the $\bar\partial$-operator, $\exists$ $f \in \mathcal{O}(U)$ s.t $T = T_f$, in case $p=0$. $\Rightarrow$ In general, $p\neq 0$.

$T = \sum_{\# I} T_I\, dz_I$. $\Rightarrow$ $\bar\partial T = \sum_{\# I} \bar\partial T \wedge dz_I$.