

When we speak of the dimension of a linear system, we will refer to the dimension of the projective space parametrizing it; thus when we write $\dim |D|$ for the dimension of the complete linear system associated to a divisor D , we have

$$\dim |D| = h^0(M, \mathcal{O}(D)) - 1.$$

A linear system of dimension 1 is called a pencil, of dimension 2 a net, and of dimension 3 a web.

We will mention here two special properties of linear systems. The first is elementary: if $E = \{D_\lambda \mid \lambda \in \mathbb{P}^n\}$ is a linear system, then for any $\lambda_0, \dots, \lambda_n$ linearly independent in \mathbb{P}^n ,

$$D_{\lambda_0} \cap \dots \cap D_{\lambda_n} = \bigcap_{\lambda \in \mathbb{P}^n} D_\lambda.$$

$$\overline{\Gamma} \quad D_{\lambda_0} \cap \dots \cap D_{\lambda_n} \supset \bigcap_{\lambda \in \mathbb{P}^n} D_\lambda \quad \text{obviously.}$$

Let $H^0(M, \mathcal{O}(L)) = (n+1)$ -dimensional complex space.

Each λ_i corresponds to a global section $S_i \in H^0(M, \mathcal{O}(L))$.

Let $D_i = (S_i) \Rightarrow$ Since S_i is a global holomorphic function on M , D_i is effective.

Given any global holomorphic section $S \in H^0(M, \mathcal{O}(L))$,

S may be expressed as a linear combination of S_0, S_1, \dots, S_n .

$$\Rightarrow S = 0 \text{ on } \bigcap \{S_i = 0\} \Rightarrow (S) \supset \bigcap_{i=1}^n (S_i).$$

$$\Rightarrow D_S \supset \bigcap_{i=1}^n D_i = \bigcap_{i=1}^n D_{\lambda_i} \Rightarrow \bigcap D_\lambda \supset \bigcap D_{\lambda_i} = D_{\lambda_0} \cap \dots \cap D_{\lambda_n}$$