$$\Rightarrow H_0(E.(M)) = \frac{E_0}{\partial_1 E_1} = \frac{E_0}{\ker f}$$

$$0 \to \ker f \to E_0 \xrightarrow{f} M \to 0 \quad \Rightarrow \quad E_0/\ker f \cong M.$$

$$\Rightarrow H_0(E.(M)) \cong M.$$

Given a projective resolution $E.(M)$ and an exact sequence

$$\to F_m \to F_{m-1} \to \cdots \to F_0 \to 0,$$

where the F_m are free, we obtain a new projective resolution $E'.(M)$ by setting $E'_m = E_m \oplus F_m$. We shall prove later that any projective resolution may be modified as to have $E'_m = 0$ for $m > n$ (Syzygy theorem).

Proposition. 1. Projective resolutions exist;
2. Given $\varphi : M \to N$ and projective resolutions $E.(M)$ and $E.(N)$, we may find a mapping of complexes $\Phi : E.(M) \to E.(N)$ inducing φ in the sense of the commutative diagram

$$\begin{array}{ccc} H_0(E.(M)) & \xrightarrow{\Phi_*} & H_0(E.(N)) \\ \wr\| & & \wr\| \\ M & \xrightarrow[\varphi]{} & N \end{array}$$

3. Φ is unique up to homotopy; and
4. If $0 \to M' \to M \to M'' \to 0$ is exact, then we may choose projective resolutions and mapping of complexes so that $0 \to E.(M') \to E.(M) \to E.(M'') \to 0$ is an exact