「 Since $\dim D = \dim D' = 1$, and D intersects D' transversely at p, D & D' are smooth at p.
$\Rightarrow$ By the note on P475, $\pi^* D - E = \tilde{D}$ & $\pi^* D' - E = \tilde{D}'$ ($\because$ At a smooth point p, $mul_p(D) = 1$ see P22)」

This is as we would expect from our picture of the proper transforms of curves: for every point p' of intersection of D and D' other than p, $\tilde{D}$ and $\tilde{D}'$ will meet at $\pi^{-1}(p')$; since D and D' have distinct tangents at p, however, $\tilde{D}$ and $\tilde{D}'$ will not meet at any point of $E = \pi^{-1}(p)$.

「 If $p' \neq p$, since $\pi : \tilde{M} - E \longrightarrow M - \{p\}$ is isomorphic, $\tilde{D}$ and $\tilde{D}'$ will meet at $\pi^{-1}(p')$. Remember the definition of the blow-up. The blow-up counts the tangents at the point p. $\Rightarrow$ Since D & D' transverse at p, D & D' have distinct tangents at p. $\Rightarrow$ (p, v) & (p, v') are different points in $\tilde{M}$, where $v, v' \in \mathbb{P}(T_p'M)$. $\Rightarrow$ $\tilde{D}$ and $\tilde{D}'$ will not meet at any point of $E = \pi^{-1}(p)$.
」

One point that should be brought out here is that if $\{D_\lambda\}$ is a linear system of curves on the surface M, the proper transforms $\{\tilde{D}_\lambda\}$ of the curves D_λ on $\tilde{M}$ do not necessarily form a linear system on $\tilde{M}$. Indeed, since

$$\tilde{D}_\lambda = \pi^* D_\lambda - mult_p(D_\lambda) \cdot E$$