

$\frac{\partial \Delta}{\partial y} \Big|_{(0,0)} = be$ , since we have only to consider the  $y^3$  term.

$$\Delta(0,0) = b^2$$

$$= \frac{p_y(0)}{f_y(0)^2} - \frac{p(0)e}{b^2} = \frac{p_y(0)}{f_y(0)^2} - \frac{p(0)f_{yy}(0)}{f_y(0)^3},$$

from  $f(x,y) = ax + by + \frac{cx^2}{2} + dxy + \frac{ey^2}{2}$ ,  $\frac{\partial^2 f}{\partial y^2} \Big|_{(0,0)} = e$   
 $f_y(0) = b$  □

On the basis of the lemma and elementary fiddling around, if we take

$$p = f_x^2 - f f_{xx},$$

then

$$\text{Res}_{1,0} \left( \frac{p(x,y) dx dy}{x f(x,y)^2} \right) = - \frac{(f_{xx} f_y^2 - 2 f_{xy} f_x f_y + f_{yy} f_x^2)}{f_y^3}.$$

Applying the residue theorem in the form (\*) gives the Reiss relation. Q.E.D.

$$\square \quad p_y = 2 f_x f_{xy} - f_y f_{xx} - f f_{xxy}$$

$$\Rightarrow p_y(0) = 2 f_x f_{xy} - f_y f_{xx} \quad \text{since } f(0) = 0.$$

$$\Rightarrow \text{Res}_{1,0} \left( \frac{p(x,y) dx dy}{x f(x,y)^2} \right) = \frac{p_y(0)}{f_y^2} - \frac{p(0) f_{yy}(0)}{f_y^3}$$

$$= \frac{f_y (2 f_x f_{xy} - f_y f_{xx}) - f_x^2 f_{yy}}{f_y^3} = - \frac{(f_{xx} f_y^2 - 2 f_{xy} f_x f_y + f_{yy} f_x^2)}{f_y^3}$$

Applying the residue theorem in the form (\*) (on P671) gives the Reiss relation. □