The difficulties in proving these results are more technical than conceptual. For examples to prove assertion 3, note that if V is given near $0 \in \mathbb{C}^n$ by functions $f_1 \cdots f_k$, then $\pi(V)$ is defined in a nbd of $0 \in \mathbb{C}^{n-1}$ by the resultants of all pairs of relatively prime linear combinations of the f_i. The problem then is to show that the zero locus of an arbitrary collection of holomorphic functions in a polydisc is in fact given by a finite number of holomorphic functions in a slightly smaller polydisc.

$\ulcorner$ For simplicity, let $k = 3$.

$V = \{ f_1 = f_2 = f_3 = 0 \}$, where we may assume that f_1, f_2 & f_3 are Weierstrass polynomials in z_n, and they are relatively prime.

Let $z_0' \in \mathbb{C}^{n-1} - \pi(V)$. We are going to show that $\exists$ $a, b, c, a', b', c' \in \mathbb{C}$ s.t $af_1 + bf_2 + cf_3 = F(a,b,c)$ & $F(a',b',c)$ are relatively prime and on the line $\pi^{-1}(z_0')$ $F(a,b,c)$ & $F(a',b',c')$ have no common zeros. $\Rightarrow$ The "corrected" resultat of $F(a,b,c)$ & $F(a',b',c')$ does not have z_0' as zero. This proves that $\pi(V) = \bigcap_{\gamma} (\gamma = 0)$, since obviously $\pi(V) \subset \bigcap (\gamma = 0)$. γ resultant of $F(a,b,c)$ & $F(a',b',c')$

Now it remains to prove the claim!