

If $f(p) \notin \{p_i'\}$, $\text{ord}_{f(p)}(\omega) = 0$.

If $f(p) \in \{p_i'\}$, say $f(p) = p_i'$, $\Rightarrow \text{ord}_{f(p)}(\omega)$
 $= \text{ord}_{p_i'}(\omega) = a_i \Rightarrow v(p) a_i \cdot p$.

$$\Rightarrow \sum_{p \in f^{-1}(p_i')} v(p) \cdot a_i \cdot p = a_i f^*(p_i') = f^*(a_i p_i').$$

$$\text{Thus } \sum_{p \in f} v(p) \cdot \text{ord}_{f(p)}(\omega) \cdot p$$

$$= f^*(\sum a_i p_i') = f^*(\omega).$$

$$\Rightarrow (f^*\omega) = f^*(\omega) + \sum_{p \in f} (v(p)-1) \cdot p$$

$$\Rightarrow \underbrace{K_S}_{\Downarrow} = \underbrace{f^*K_{S'}}_{\Downarrow} + \underbrace{B}_{\Downarrow} \quad \Downarrow \quad [(\sum (v(p)-1) \cdot p)].$$

$$\Rightarrow \deg K_S = \deg(f^*K_{S'}) + \deg([B]).$$

$$\quad \quad \quad \parallel \quad \quad \quad n \cdot \deg K_{S'}$$

$$(\because \deg(f^*K_{S'}) = \int_S f^*(\frac{\bar{c}}{2\pi}(\mathbb{H})) = n \int_S \frac{\bar{c}}{2\pi}(\mathbb{H}))$$

$$= n \deg K_{S'})$$

$$\deg([B]) = \sum_{p \in f} (v(p)-1) \quad \text{by } P2.17, \& \text{ } P144.$$

$$\Rightarrow \deg K_S = \deg K_{S'} \cdot n + \sum_{p \in f} (v(p)-1). \quad \Downarrow$$

Now any compact Riemann surface S admits a holomorphic map to \mathbb{P}^1 : if $f \in M(S)$ is any global meromorphic function written locally as g/h with g, h