$$\begin{cases} R_0 = M \\ R_k = \text{image } E_k \to E_{k-1} & (0 < k < n), \\ R_n = F. \end{cases}$$

Then we have short exact sequences

$$\begin{cases} 0 \to R_1 \longrightarrow E_0 \longrightarrow R_0 \longrightarrow 0 \\ 0 \longrightarrow R_{k+1} \longrightarrow E_k \longrightarrow R_k \longrightarrow 0 & (0 < k < n), \\ 0 \longrightarrow R_n \longrightarrow E_{n-1} \longrightarrow R_{n-1} \longrightarrow 0. \end{cases}$$

Since higher Tor's are zero if one of the modules is free, the long exact sequence in the second factor gives

$$Tor^{\mathcal{O}}_{q+1}(\mathbb{C}, R_{k-1}) \cong Tor^{\mathcal{O}}_{q}(\mathbb{C}, R_k), \quad q \geq 1.$$

$$\longrightarrow Tor^{\mathcal{O}}_{q}(\mathbb{C}, R_{k+1}) \longrightarrow \underset{\underset{0}{\|}}{Tor^{\mathcal{O}}_{q}(\mathbb{C}, E_k)} \longrightarrow Tor^{\mathcal{O}}_{q}(\mathbb{C}, R_k) \to Tor^{\mathcal{O}}_{q-1}(\mathbb{C}, R_{k+1}) \longrightarrow Tor^{\mathcal{O}}_{q-1}(\mathbb{C}, E_k) = 0 \quad \text{if } q-1 \geq 1$$

$$\Rightarrow \quad Tor^{\mathcal{O}}_{q+1}(\mathbb{C}, R_k) \cong Tor^{\mathcal{O}}_{q}(\mathbb{C}, R_{k+1}) \quad q \geq 1$$

In particular,

$$Tor^{\mathcal{O}}_{1}(\mathbb{C}, R_n) \cong Tor^{\mathcal{O}}_{n+1}(\mathbb{C}, M).$$

$$Tor^{\mathcal{O}}_{1}(\mathbb{C}, R_n) \cong Tor^{\mathcal{O}}_{2}(\mathbb{C}, R_{n-1}) \cong Tor^{\mathcal{O}}_{3}(\mathbb{C}, R_{n-2}) \cong \cdots \cong Tor^{\mathcal{O}}_{n+1}(\mathbb{C}, R_0) = Tor^{\mathcal{O}}_{n+1}(\mathbb{C}, M)$$

To show that the right-hand side is zero, we let

$$K. : 0 \longrightarrow K_n \longrightarrow K_{n-1} \longrightarrow \cdots \to K_0 \longrightarrow \mathcal{O}/m = \mathbb{C} \longrightarrow 0$$

be the Koszul complex associated to the maximal ideal $m = \{z_1, \cdots z_n\}$. Since

$$Tor^{\mathcal{O}}_{*}(\mathbb{C}, M) = H_*(K. \otimes_{\mathcal{O}} M),$$