ain C'. $\Rightarrow$ This is a wrong interpretation. see below. ⌟

Having fixed the tangent directions to C to correspond to the points $0, \infty \in \mathbb{P}^1$, the tangent directions to the curve C_f defined by f will be μ, λ and the condition $\mu\lambda = r$ has intrinsic meaning and defines the ideal $\mathcal{J}_{p_i} \subset \mathcal{O}_{p_i}$.

⌜

$$\frac{f}{\beta} = \frac{\alpha}{\beta}\, xy + (x-y)(x-ry)$$

Let $f/\beta = f$. $\alpha/\beta = \alpha$.

$$\Rightarrow \quad f(x,y) = \alpha(x,y)\, xy + (x-y)(x-ry)$$

$$\alpha(x,y) = \underset{\overset{\shortparallel}{\alpha_{00}}}{\alpha} + \alpha_{10} x + \alpha_{01} y + \alpha_{11} xy + \cdots$$

$$\Rightarrow \quad f(x,y) = \alpha\, xy + (x-y)(x-ry) + (\text{higher-order terms})$$

$$= (x-\mu y)(x-\lambda y) + (\text{higher-order terms})$$

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\, y' = 0$$

$$\frac{\partial f}{\partial x} = 2x - (\mu+\lambda) y + \text{higher-order terms}$$

$$\frac{\partial f}{\partial y} = 2y\mu\lambda - (\mu+\lambda) x + \quad ''$$

$$y'_{x=0} = - \left.\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}\right|_{x=0} = -\lim_{x\to 0} \frac{2x - (\mu+\lambda) y + \text{higher-order terms}}{2y\mu\lambda - (\mu+\lambda)x + \text{higher-order terms}}$$