If g is the genus of the curve C, it follows that

$$g = \frac{1}{2}\deg K_C + 1 = \frac{1}{2}\deg (K_M + C)|_C + 1 = \frac{K_M \cdot C + C \cdot C}{2} + 1 .$$

By p216, $2g-2 = \deg K_C$.

$$\Rightarrow \quad g = \frac{1}{2}\deg K_C + 1 = \frac{1}{2}\deg (K_M + C)|_C + 1$$

Since $\deg (K_M+C)|_C = \frac{\sqrt{-1}}{2\pi}\int_C c_1((K_M+C)|_C) = \frac{\sqrt{-1}}{2\pi}\int_C c_1(K_M|_C) + \frac{\sqrt{-1}}{2\pi}\int_C c_1(C|_C) = K_M \cdot C + C \cdot C$,

$$g = \frac{K_M \cdot C + C \cdot C}{2} + 1$$

More on degree

$$\begin{array}{ccc} L|_C & \longrightarrow & L \\ \downarrow & & \downarrow \\ C & \longrightarrow & M \end{array}$$

$\boxed{\int_C c_1(L|_C) = \int_M c_1(L) \wedge c_1(C) = C \cdot L \text{ by the Poincare duality theorem. P59}}$

$\Rightarrow \deg(L|_C) = \#$ of points of $L \cap C$ since $L|_C$ is the restriction of L to C, and, if $L = (s=0)$, $(s|_C = 0) = L|_C$.

This formula is also referred to as the adjunction formula. In general, we define the virtual genus $\pi(C)$ of an arbitrary curve C on M by