positive, $d(|S|) \neq 0$ on V.

If $d(|S|) = 0$, $d\varphi = d\log|S|^2 = \frac{1}{|S|^2} d|S|^2 = \frac{1}{|S|^2} \cdot 2|S|$

$\cdot d|S| = \frac{2}{|S|} d(|S|) \Rightarrow$ We can not get any conclusion

I think $d\varphi = 0$ on V. is the important fact.

Suppose $d\varphi = 0$, $\Rightarrow \partial + \bar\partial (\log|S|^2) = 0$

$\Rightarrow \partial \log|S|^2 = 0 \quad \bar\partial \log|S|^2 = 0$

$\Rightarrow$

$\frac{i}{2\pi} \Theta = \frac{i}{2\pi} \partial\bar\partial \log|S|^{-2} = -\frac{i}{2\pi}\partial\bar\partial \log|S|^2 = 0$

is not positive.

Now for any critical point $x \in M$ of φ, the matrix

$$-\left(\frac{\partial}{\partial z_i}\frac{\partial}{\partial \bar z_j}\right)\log\frac{1}{|S|^2} = \left(\frac{1}{4}\left(\frac{\partial^2}{\partial x_i \partial x_j} + \frac{\partial^2}{\partial y_i \partial y_j}\right) + \frac{i}{4}\left(\frac{\partial^2}{\partial y_i \partial x_j} - \frac{\partial^2}{\partial x_j \partial y_i}\right)\right)\log|S|^2$$

is negative definite hermitian, and consequently the Hessian

$$H(\varphi) = \begin{bmatrix} \frac{\partial^2}{\partial x_i \partial x_j}, & \frac{\partial^2}{\partial x_i \partial y_j} \\ \frac{\partial^2}{\partial x_j \partial y_i}, & \frac{\partial^2}{\partial y_i \partial y_j} \end{bmatrix} \log|S|^2$$

of φ has at least n negative eigenvalues.

Γ $\frac{\partial}{\partial z_i}\frac{\partial}{\partial \bar z_j} = \frac{1}{2}\left(\frac{\partial}{\partial x_i} - i\frac{\partial}{\partial y_i}\right)\frac{1}{2}\left(\frac{\partial}{\partial x_j} + i\frac{\partial}{\partial y_j}\right) = \frac{1}{4}\left\{\left(\frac{\partial^2}{\partial x_i \partial x_j} + \frac{\partial^2}{\partial y_i \partial y_j}\right)\right.$

$\left. + i\left(\frac{\partial^2}{\partial x_i \partial y_j} - \frac{\partial^2}{\partial y_i \partial x_j}\right)\right\}$

$\Rightarrow$ Since $\partial\bar\partial\varphi = \partial\left(\frac{\partial\varphi}{\partial \bar z_j} d\bar z_j\right) = \frac{\partial^2\varphi}{\partial z_i \partial \bar z_j} dz_i \wedge d\bar z_j$, and

$-\frac{i}{2\pi}\partial\bar\partial\varphi$ is positive, & $\varphi = \log|S|^2$,