

$$(D_1, \dots, D_n)_{\text{tot}} = \dim_{\mathbb{C}}(\mathcal{O}/I).$$

In general, if  $D_i$  are divisors on a complex manifold  $M$  meeting at a finite number of isolated points  $P_U$ , we define the effective zero cycle

$$D_1 \cdots D_n = \sum_U m_U P_U,$$

where  $m_U = (D_1 \cdots D_n)_{P_U}$ .

The degree

$$\deg(D_1 \cdots D_n) = \sum_U m_U$$

is the total intersection number of the  $D_i$ .

### Applications to Plane Projective Geometry

We shall apply the residue theorem to the simplest global case  $M = \mathbb{P}^n$ , with special attention to the case  $n=2$ . Suppose then that  $D_1, \dots, D_n$  are hypersurfaces of <sup>respective</sup> degrees  $d_1, \dots, d_n$  and meeting in isolated points  $P_U$ . According to the discussion in the preceding section, we may write the intersection as a zero cycle

$$D_1 \cdots D_n = \sum_U m_U P_U,$$

where the local intersection numbers  $m_U = (D_1 \cdots D_n)_{P_U}$  are given by a residue, and the global Bezout theorem

$$\deg(D_1 \cdots D_n) = \sum_U m_U = d_1 \cdots d_n$$

is valid. In a suitable Euclidean coordinate system  $(x_1, \dots, x_n)$ , we may assume that all  $P_U$  lie in  $\mathbb{C}^n \subset \mathbb{P}^n$  and that  $D_i$  is the divisor of a polynomial  $f_i(x_1, \dots, x_n)$  of de-