$[D] = [D']$ $\Leftrightarrow$ $D + (f) = D'$ for some meromorphic function f on S $\Rightarrow$ $\mu(D+(f)) = \mu(D) + \mu(f) = \mu(D) = \mu(D')$. $\Rightarrow$ Here we use the fact that, if $D = (f)$, then $\mu(f) = 0$. If $\tilde{\mu}(L) = 0 = \mu(D)$, where $L = [D]$, then $D = (f)$. $\Rightarrow$ $L = [(f=0)]$ $\Rightarrow$ L is trivial bundle see P134 to make sure. We use the converse.

Suppose $\exists$ a injective map $\tilde{\mu} : \mathrm{Pic}^0(S) \longrightarrow \mathcal{J}(S)$ s.t

$$\tilde{\mu}(L) = \mu(D) \text{ where } L = [D].$$

If $\mu(D) = 0$, then $\tilde{\mu}([D]) = 0$. $\Rightarrow$ By the injectiveness, $[D]$ is trivial. $\Rightarrow$ $\exists$ a meromorphic function f s.t $(f) = D$. see P134.
If $D = (f)$, then $[D]$ is trivial. $\Rightarrow \tilde{\mu}([D]) = 0 = \mu(D) = 0$

Jacobi Inversion

The second statement of Abel's theorem above suggests our next question: Is the map $\mu : \mathrm{Div}^0(S) \to \mathcal{J}(S)$ given by Abelian sums surjective, or, in other words, is the induced map $\tilde{\mu} : \mathrm{Pic}^0(S) \to \mathcal{J}(S)$ an isomorphism? The Jacobi inversion theorem asserts that the answer to this question is yes, and in fact tells us that we obtain what is suggested by counting dimensions.

Theorem (Jacobi Inversion). Given S a curve of genus