$-(E_1 + \cdots + E_d)$. $\Rightarrow$ Since $E_i \cap E_j = \phi$, $i \neq j$ and $E_i \cdot E_i = -1$,
$(n_1+1)E_1 + \cdots + (n_d+1)E_d \sim 0$, and so $-(n_1+1)^2 - (n_2+1)^2 - \cdots = 0$.
which implies $n_1 = n_2 = \cdots = n_d = -1$.

One more thing to be clear here. $C' = \pi^* C + n_1 E_1 + \cdots + n_d E_d \sim \pi^* H^m + n_1 E_1 + \cdots + n_d E_d$. since $C \sim H^n$ for some m. $\Rightarrow$ $m = n$ for

$$\pi^* H^m + n_1 E_1 + \cdots + n_d E_d \sim \pi^* H^n - E \sim C'$$

$\Rightarrow$ $\pi_* : S \longrightarrow \mathbb{P}^2$ is a map degree one.

$\Rightarrow$ $\pi_* [\pi^* H^n + n_1 E_1 + \cdots + n_d E_d] = \pi_* [\pi^* H^n - E] = [H^n]$
$\quad \overset{\shortparallel}{[H^m]} \qquad \Longrightarrow \quad m = n$.

Thus $C' = \pi^* C - E$ which proves $\{\pi^* C - E\} = |\pi^* H^n - E|$ where C is a curve of degree n in $\mathbb{P}^2$, since $\{\pi^* C - E\} \subset |\pi^* H^n - E|$. It remains to show that for a generic curve C of degree n passing P_0, $\tilde{C} = \pi^* C - E_1 - n_2 E_2 - \cdots - n_d E_d$.

For example, consider a curve in $\mathbb{P}^2$ which is of degree 2 and passes $P_1 = [(1.0.0)]$, $P_2 = [(0.0,1)]$ $P_3 = [(0.1.0)]$ $P_4 = [(1.1.0)]$, $P_5 = [(1.0.1)]$.

$\Rightarrow$ Any curve of degree 2 in $\mathbb{P}^2$ is expressed as a linear combination of homogeneous polynomials of degree 2 in z_0, z_1, z_2. See P165 & P166.

$\Rightarrow$ $a_0 z_0^2 + a_1 z_1^2 + a_2 z_2^2 + b_0 z_1 z_2 + b_1 z_0 z_2 + b_2 z_0 z_1 = 0$

$\Rightarrow$ Plug in it all the five points. $\Rightarrow$ $a_0 = a_1 = a_2 = 0$. by pluging P_1, P_2, P_3. $\Rightarrow$ $b_2 = 0 = b_1$. $\Rightarrow$ $C = (z_0 z_1 = 0)$.

$\Rightarrow$ There is only one effective divisor in $|\mathcal{O}_{\mathbb{P}_2}(2)|$, $P_0 = \{P_1 \cdots P_5\}$. At $P_2 = [(0.0.1)]$, $(z_0 = 0)$ meets with $(z_1 = 0)$ transversely. $\Rightarrow$ P_2 is a double point of C.

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