limit we obtain two spectral sequences abutting to $\mathbb{H}^*(X, \mathcal{K}^*)$ with

$$'E_2^{p,q} = H^p(X, \mathcal{H}^q(\mathcal{K}^*)),$$
$$''E_2^{p,q} = H_d^q(H^p(X, \mathcal{K}^*)).$$

Explanations. $H^*(X, \mathcal{H}^*(\mathcal{K}^*))$ is the Cech cohomology of the cohomology sheaves $\mathcal{H}^*(\mathcal{K}^*)$, and $H_d^*(H^*(X, \mathcal{K}^*))$ is the cohomology of the complex

$$H^*(X, \mathcal{K}^\circ) \xrightarrow{d} H^*(X, \mathcal{K}') \xrightarrow{d} \cdots .$$

Γ
$$K^n = \bigoplus_{p+q=n} C^p(\underline{U}, \mathcal{K}^q)$$

$$'F^p K^n = \bigoplus_{\substack{p'+q=n \\ p' \geq p}} C^{p'}(\underline{U}, \mathcal{K}^q)$$

$$'E_0^{p,q}(\underline{U}) = \frac{'F^p K^{p+q}}{'F^{p+1} K^{p+q}} \cong C^p(\underline{U}, \mathcal{K}^q)$$

$\underline{U}' < \underline{U}$,

$$\begin{tikzcd}
'E_0^{p,q}(\underline{U}) \arrow[rr, "\cong"] \arrow[dr, "D"] \arrow[dd, "\varphi"'] & & C^p(\underline{U}, \mathcal{K}^q) \arrow[dr, "d"] \arrow[dd, "\varphi"] & \\
& 'E_0^{p,q+1}(\underline{U}) \arrow[rr, "\cong"] \arrow[dd, "\varphi"] & & C^p(\underline{U}, \mathcal{K}^{q+1}) \arrow[dd, "\varphi"] \\
'E_0^{p,q}(\underline{U}') \arrow[rr, "\cong"] \arrow[dr, "D=\delta+d"'] & & C^p(\underline{U}', \mathcal{K}^q) \arrow[dr, "d"] & \\
& 'E_0^{p,q+1}(\underline{U}') \arrow[rr, "\cong"] & & C^p(\underline{U}', \mathcal{K}^{q+1})
\end{tikzcd}$$