$\bar{Z}$ - covariant differential of the tensor ψ, and $A'(\psi)$ is a first-order operator involving $\bar{Z}$- derivatives of ψ.
Using (*), we obtain.

$$\alpha \,|\,(A'\psi, \psi)| \leq \epsilon \|\bar{\nabla}\psi\|^2 + \frac{1}{\epsilon}\|\psi\|^2,$$

which implies

$$\|\bar{\nabla}\psi\|^2 \leq C'\{(\Delta\psi, \psi) + \|\psi\|^2\}, \quad C' > 0.$$

$$\ulcorner \; \alpha \,|\langle A'\psi, \psi\rangle| \leq \epsilon \|A'(\psi)\|^2 + \frac{1}{\epsilon}\|\psi\|^2 \text{ since } \alpha\,|\langle A'\psi, \psi\rangle|$$

$$\leq \alpha \|A'\psi\|\,\|\psi\| .$$

$$\|A'(\psi)\|^2 \leq K(\|\bar{\nabla}\psi\|^2 + \|\psi\|^2)$$

$$\Rightarrow \alpha\,|\langle A'\psi, \psi\rangle| \leq \epsilon K \|\bar{\nabla}\psi\|^2 + \left(K + \frac{1}{\epsilon}\right)\|\psi\|^2$$

$$\alpha\left|(\Delta\psi, \psi) - \|\bar{\nabla}\psi\|^2\right| \leq \epsilon K \|\bar{\nabla}\psi\|^2 + \left(K + \frac{1}{\epsilon}\right)\|\psi\|^2$$

$$\Downarrow$$

$$\alpha\left(\|\bar{\nabla}\psi\|^2 - \langle\Delta\psi, \psi\rangle\right) \leq \epsilon K \|\bar{\nabla}\psi\|^2 + \left(K + \frac{1}{\epsilon}\right)\|\psi\|^2$$

$$\alpha\|\bar{\nabla}\psi\|^2 - \epsilon K \|\bar{\nabla}\psi\|^2 \leq \alpha\langle\Delta\psi, \psi\rangle + \left(K + \frac{1}{\epsilon}\right)\|\psi\|^2$$

$$(\alpha - \epsilon K)\|\bar{\nabla}\psi\|^2 \leq \alpha \langle\nabla\psi, \psi\rangle + \left(K + \frac{1}{\epsilon}\right)\|\psi\|^2$$

Choose ϵ small enough so that $\alpha - \epsilon K > 0$,

$$\|\bar{\nabla}\psi\|^2 \leq \frac{\alpha}{\alpha - \epsilon K}\langle\nabla\psi, \psi\rangle + \left(\frac{K + \frac{1}{\epsilon}}{\alpha - \epsilon K}\right)\|\psi\|^2$$

$$\leq C'\{\langle\Delta\psi, \psi\rangle + \|\psi\|^2\}.$$

We now repeat the argument applied this time to