$\Rightarrow$ By syzygy theorem, $\mathcal{G}_1 \otimes \mathcal{L}^{-l} = 0$. But if $\mathcal{G}' \otimes \mathcal{L}^{-k}$ is locally free, we can stop there, i.e., we have an exact sequence

$$0 \to \underset{\mathcal{E}_1}{\underset{\parallel}{\mathcal{G}' \otimes \mathcal{L}^{-k}}} \to \mathcal{E}_0 \to \mathcal{F} \to 0$$

but $\mathcal{E}_1$ may not be a direct sum of $\mathcal{L}^{-k'}$, still locally free. So it depends on whether $\mathcal{G}' \otimes \mathcal{L}^{-k}$ is locally free or not. ┘

Returning to the proof of 1, we have proved in Section 4 of Chapter 1 that $H^q(M, \mathcal{E}(k)) = 0$ for $\mathcal{E}$ locally free, $q > 0$, and $k \geq k_0$.

┌ See P159. $H^q(M, \mathcal{O}(E \otimes L^k)) = 0$ for $q > 0$, $k \geq k_0$.
$\Rightarrow$ $H^q(M, \mathcal{E}(k)) = H^q(M, \mathcal{E} \otimes \mathcal{L}^k) = 0$ for $q > 0$, $k \geq k_0$ ┘

Applying this inductively on the length of a global syzygy gives Theorem B.

┌ Assume that Theorem B is true for the length $n-1$. Suppose we have a global syzygy of $\mathcal{F}$.

$$0 \to \mathcal{E}_n \to \mathcal{E}_{n-1} \to \cdots \to \mathcal{E}_0 \xrightarrow{\alpha} \mathcal{F} \to 0$$

$$\Rightarrow \quad 0 \to \mathcal{E}_n \to \mathcal{E}_{n-1} \to \cdots \mathcal{E}_1 \to \ker\alpha \to 0 \cdots$$

$$0 \to \ker\alpha \to \mathcal{E}_0 \to \mathcal{F} \to 0 \cdots$$

$\Rightarrow$ By the induction assumption, $H^q(M, \ker\alpha(k)) = 0$ for $q > 0$, $k \geq k_0$.