$H_1 + H_2 \in \mathbb{P}(H^0(\mathbb{P}^3, \mathcal{O}(2H))) = W$ ⌟

The cohomology of $\mathbb{P}^{3*} \times \mathbb{P}^{3*}$ is generated by the pullbacks ω_1 and ω_2 of the hyperplane class in $\mathbb{P}^{3*}$ via the two projection maps; as before, since f is 2-sheeted, we obtain

$$\begin{aligned} \deg W_2 &= \frac{1}{2}(\omega_1 + \omega_2)^6 \\ &= \frac{1}{2} \cdot 20\, \omega_1^3 \omega_2^3 = 10. \end{aligned}$$

⌜ By the argument on P830 note, $f^*H = \omega_1 + \omega_2$.
$\deg W_2 = \#(H^6 \cap W_2) = \#((H \cap W_2) \cap (H \cap W_2) \cap \cdots)$ since W_2 is 6-dimensional ($\because$ $\mathbb{P}^{3*} \times \mathbb{P}^{3*}$ is 6-dimensional and $f(\mathbb{P}^{3*} \times \mathbb{P}^{3*}) = W_2$). Note that $H^6 = \mathbb{P}^3$.
$\Rightarrow$ $\#(H^6 \cap W_2) = \frac{1}{2}(f^*H)^6 = \frac{1}{\deg f}(f^*H)^6 = \frac{1}{2}(\omega_1 + \omega_2)^6$
$= \frac{1}{2}\, {}_6C_3\, \omega_1^3 \omega_2^3 = \frac{1}{2}\, 6 \cdot 5 \cdot 4/6 = 10$ since $\omega_1^i \omega_2^j = 0$ if $(i, j) \neq (3, 3)$. ⌟

“ By the argument on P831 note, $\omega_1 = \widetilde{\mathbb{P}^2 \times \mathbb{P}^3}$
$\Rightarrow$ $\omega_1^3 = * \times \mathbb{P}^3$. Similarly $\omega_2^3 = \widetilde{\mathbb{P}^3 \times *}$.
$\Rightarrow$ $\omega_1^3 \cap \omega_2^3 = 1 = \#((* \times \mathbb{P}^3) \cdot (\mathbb{P}^3 \times *))$.
Note $\omega_1^2 = \widetilde{\mathbb{P}^1 \times \mathbb{P}^3}$ ···· etc. ·· ”

Lines on Linear Systems of Quadrics

Earlier in this section, we found the class on the