Section 2 of the geometry of the Grassmannian $G(2,4)$ to see in detail the behavior of 2-planes on a quadric in $\mathbb{P}^5$.

The first statement of the proposition is readily verified: since the Gauss map $\mathcal{G}$ on a smooth quadric $F \subset \mathbb{P}^{m+1}$ is the restriction of a linear isomorphism $\mathbb{P}^{m+1} \longrightarrow \mathbb{P}^{m+1*}$, the family of tangent planes to F along a linear subspace $\Lambda_k \subset F$ forms a k-dimensional linear subspace of $\mathbb{P}^{m*}$.

Pf We have the following commutative diagram:

$$\begin{tikzcd} F \arrow[r, "\mathcal{G}"] \arrow[d, hook] & \mathbb{P}^{m+1*} \arrow[d, equal] \\ \mathbb{P}^{m+1} \arrow[r, "Q", "\cong"'] & \mathbb{P}^{m+1*} \end{tikzcd}$$

$$\begin{array}{ccc} \mathbb{P}^{m+1} & & \mathbb{P}^{m+1*} \\ \Downarrow & & \Downarrow \\ [a_0, \cdots a_{m+1}] & \longmapsto & \{{}^tX Q a = 0\} = [Qa] \end{array}$$

$\Rightarrow$ If $\Lambda_k \subset F$, $Q(\Lambda_k)$ is a k-dimensional linear subspace of $\mathbb{P}^{m*}$, since $k \leq m$. ⌟

Since the tangent space to F at any point of Λ contains Λ, moreover, the image $\mathcal{G}(\Lambda)$ lies entirely in the $(m-k)$-dimensional subspace of $\mathbb{P}^{m+1*}$ of planes through Λ; thus

$$k \leq m-k$$