$\ulcorner$ Given an open cover $\{U_\alpha\}$ of M and a set of meromorphic functions $\{f_\alpha\}$, s.t $f_\alpha \not\equiv 0$ on U_α. and

$$\frac{f_\alpha}{f_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta).$$

Consider $f_\alpha \mathcal{O}^*(U_\alpha)$. & $f_\beta \mathcal{O}^*(U_\beta) \in \dfrac{\mathcal{M}^*(U_\alpha)}{\mathcal{O}^*(U_\alpha)}$ & $\dfrac{\mathcal{M}^*(U_\beta)}{\mathcal{O}^*(U_\beta)}$

respectively.

Define $f: M \longrightarrow \bigcup_{x \in M} \left(\mathcal{M}^*/\mathcal{O}^*\right)_x$. by

$f(x) = \bar{f}_\alpha(x)$ where $x \in U_\alpha$. and $\bar{f}_\alpha(x)$ is the direct limit of $\bar{f}_\alpha$.

Well-definedness. If $x \in U_\beta$, i.e $x \in U_\alpha \cap U_\beta$.
$\Rightarrow \bar{f}_\beta . = f_\beta \mathcal{O}^*(U_\beta)$. $\Rightarrow \bar{f}_\beta(x) = \bar{f}_\alpha(x)$?

To show this, we need to find. some open set $V \ni x$. s.t
$\bar{f}_\beta|V = \bar{f}_\alpha|V$, $\Leftrightarrow f_\alpha \mathcal{O}^*(V) = f_\beta \mathcal{O}^*(V)$

But since $\dfrac{f_\alpha}{f_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta)$, $f_\alpha \mathcal{O}^*(U_\alpha \cap U_\beta) = f_\beta \mathcal{O}^*(U_\alpha \cap U_\beta)$.

Let $V = U_\alpha \cap U_\beta$. $\Rightarrow$ f is well-defined.

We have to check that given any point $x \in M$, $\exists$ U. s.t $f(x) = \bar{f}(x)$. $\Rightarrow$ This is done already when we defined f. $\lrcorner$

The f_α's are called local defining functions for D.
It follows immediately from the definitions that the identification

$$H^0(M, \mathcal{M}^*/\mathcal{O}^*) = \mathrm{Div}(M)$$