Given $\sigma \in Z^p(\underline{U}, \mathcal{a}^{r,s})$, define $\tau \in C^{p-1}(\underline{U}, \mathcal{a}^{r,s})$ by

$$\tau_{\alpha_0,\alpha_1 \cdots \alpha_{p-1}} = \sum \rho_\beta \, \sigma_{\beta, \alpha_0, \alpha_1 \cdots \alpha_{p-1}}.$$

where the section $\rho_\beta \sigma_{\beta,\alpha_0\alpha_1\cdots\alpha_{p-1}}$ extends to $U_{\alpha_0} \cap \cdots \cap U_{\alpha_{p-1}}$ by zero. In the case $p=1$, explicitly,

$$\sigma = \{ \sigma_{U,V} \in \mathcal{a}^{r,s}(U \cap V) \}.$$

$$\sigma_{UV} + \sigma_{VW} + \sigma_{WU} = 0 \quad \text{in } U \cap V \cap W.$$

Set $\tau_U = \sum_V \rho_V \sigma_{V,U}$. $\Rightarrow$ $(\delta\tau)_{U,V} = -\tau_U + \tau_V$

$$= -\sum_V \rho_V \sigma_{V,U} + \sum_U \rho_U \sigma_{U,V} = -\sum_W \rho_W . \sigma_{W,U} + \sum_W \rho_W \sigma_{W,V}$$

$$= \sum \rho_W (\sigma_{W,V} - \sigma_{W,U}) = \sum \rho_W (\rho_{W,V} + \sigma_{U,W}) = \sum \rho_W (-\sigma_{VU})$$

$$= \sigma_{U,V} \qquad = -\sigma_{VU} \sum \rho_W = -\sigma_{VU}$$

Def: Sheaves that admit partitions of unity, more precisely, for any $\underline{U} = \{U_\alpha\}$, maps $\eta_\alpha : \mathcal{F}(U_\alpha) \to \mathcal{F}(U = \bigcup U_\alpha)$ s.t the support of $(\eta_\alpha \sigma)$ is contained in U_α and $\sum \eta_\alpha (\sigma|U_\alpha) = \sigma$ for $\sigma \in \mathcal{F}(U)$, are called fine.

The same argument shows that their higher cohomology groups vanish.

2. For K, a simplicial complex with underlying topological space M,

$$H^*(K, \mathbb{Z}) \cong \check{H}^*(M, \mathbb{Z}).$$

i.e. the Cech cohomology of the constant sheaf $\mathbb{Z}$ on M is isomorphic to the simplicial cohomology of the complex K.