A manifold is called Kähler if it admits a Kähler metric; we now give some examples of Kähler manifolds.

Examples.

① Any metric on a compact Riemann surface is Kähler, since $d\omega$ is a 3-form and hence zero.

② If Λ is a lattice in $\mathbb{C}^n$, the complex torus $T = \mathbb{C}^n/\Lambda$ is Kähler with the Euclidean metric $ds^2 = \sum dz_i \otimes d\bar{z}_i$.

③ M and N are Kähler then. $M \times N$ is Kähler, with the product metric. since $ds^2_{M\times N} = ds_M^2 + ds_N^2$.

④ If $S \subset M$ is a submanifold, ω the associated (1.1)-form of a Kähler metric on M, we have al eady noted in Section 2 above that the associated (1.1)-form of the induced metric on S is just the pull-back to S of ω: thus if M is Kähler then S is Kähler. (See p29).

⑤ Recall that the Fubini-Study metric on $\mathbb{P}^n$ is given by its associated (1.1) form

$$\omega = \frac{i}{2\pi} \partial\bar{\partial} \log \|Z\|^2.$$

where Z is a local lifting of $U \subset \mathbb{P}^n$ to $\mathbb{C}^{n+1} - \{0\}$.

Since $\partial\bar{\partial} = -\bar{\partial}\partial$ $\quad (dz \wedge d\bar{z} = -d\bar{z} \wedge dz)$,

$$\begin{aligned} \omega &= \frac{i}{4\pi} (\partial + \bar{\partial})(\bar{\partial} - \partial) \log \|Z\|^2. \\ &= \frac{i}{4\pi} d\left((\bar{\partial} - \partial) \log \|Z\|^2\right), \end{aligned}$$

so we see that ω is closed, and the Fubini-Study metric is Kähler.