$$= \sum a_k(x) \int_{\mathbb{R}^n} u(y) \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, dy - \sum_k \int_{\mathbb{R}^n} u(y) a_k(y) \frac{\partial}{\partial y_k} \chi_\epsilon(x-y)\, dy$$

$$= \sum a_k(x) \int_{\mathbb{R}^n} u(y) \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, dy - (-1) \sum_k \int_{\mathbb{R}^n} u(y)\, a_k(y) \frac{\partial \chi_\epsilon}{\partial x_k}(x-y)\, dy$$

$$= \sum \int_{\mathbb{R}^n} \frac{\partial \chi_\epsilon}{\partial x_k}(x-y)\; u(y)\; (a_k(x) + a_k(y))\; dy$$

Again $x-y \in \operatorname{supp} \chi_\epsilon$ $\quad y \in \operatorname{supp} u$.

$\Rightarrow |\, a_k(x) + a_k(y) \,| <$ some constant.

By the Minkowski inequality

$$\|f+g\|_p \le \|f\|_p + \|g\|_p .$$

$$\left\| \sum_k \int_{\mathbb{R}^n} \frac{\partial \chi_\epsilon}{\partial x_k}(x-y)\; u(y)\; (a_k(x) + a_k(y))\, dy \right\|_0$$

$$\le \sum_k \left\| \int_{\mathbb{R}^n} \frac{\partial \chi_\epsilon}{\partial x_k}(x-y)\; u(y)\; (a_k(x) + a_k(y))\, dy \right\|_0$$

$$\le \sum_k \left(\left\| \int_{\mathbb{R}^n} \frac{\partial \chi_\epsilon(x-y)}{\partial x_k}\; u(y)\; a_k(x)\, dy \right\|_0 + \left\| \int_{\mathbb{R}^n} \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, u(y)\, dy\; a_k(y) \right\|_0 \right) < \sum_k \left\| \int_{\mathbb{R}^n} \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\;\; u(y)\, dy \right\|_0$$

$$+ \sum_k \left\| \int_{\mathbb{R}^n} \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\;\; u(y)\;\, a(y)\;\, dy \right\|$$