

Def: A curvature operator $\Theta \in A^2(\text{Hom}(E, E))$ is positive at $x \in M$ if for $\lambda \neq 0 \in E_x$, the multivector

$-\bar{i}(\lambda, \Theta \lambda) \in \Lambda^2 T_x^* M$ is positive of type (1,1), or equivalently if for any holomorphic tangent vector $v \in T_x(M)$, the hermitian matrix

$\langle \Theta(x): v, \bar{v} \rangle \in \text{Hom}(E_x, E_x)$ is positive definite.

We write $\Theta > 0$ if Θ is positive everywhere,
 $\Theta \geq 0$ if Θ is positive semidefinite, and
 $\Theta > \Theta'$ if $\Theta - \Theta' > 0$.

Let A the second fundamental form of the subbundle SCE above, and write.

$$A = \sum_{1 \leq j \leq s} a_{\lambda j}^\alpha dz_\alpha \otimes e_\lambda \otimes e_j^* \in \Lambda^{1,0} T^* M \otimes Q \otimes S^*$$

$$\Rightarrow \overset{\text{on } S, Q}{\oplus} \overset{\text{on } T^* M}{\ominus} A = \sum_{s < \lambda < r} \bar{a}_{\lambda j}^\alpha d\bar{z}_\alpha \otimes e_\lambda^* \otimes e_j \in \Lambda^{0,1} (T^* M) \otimes Q^* \otimes S$$

$$\Rightarrow A \wedge {}^t \bar{A} = \sum \underbrace{a_{i\kappa}^\alpha \bar{a}_{j\kappa}^\beta}_{\text{wavy line}} dz_\alpha \wedge d\bar{z}_\beta \otimes e_i \otimes e_j^*.$$

$$\Uparrow \sum (A \wedge {}^t \bar{A})_{ij} e_i \otimes e_j^*$$

Applying e_i^* and e_j , $(A \wedge {}^t \bar{A})_{ij} = \sum_\kappa A_{i\kappa} \wedge ({}^t \bar{A})_{\kappa j}$

$$\begin{aligned} A_{i\kappa} &= \sum a_{i\kappa}^\alpha dz_\alpha \\ ({}^t \bar{A})_{\kappa j} &= \sum \bar{a}_{j\kappa}^\beta d\bar{z}_\beta \Rightarrow A_{i\kappa} \wedge ({}^t \bar{A})_{\kappa j} \\ &= \sum a_{i\kappa}^\alpha dz_\alpha \wedge \bar{a}_{j\kappa}^\beta d\bar{z}_\beta \end{aligned}$$