$\Rightarrow$ W_1 = image of f. ⌟

Now the cohomology ring of $\mathbb{P}^{2*} \times \mathbb{P}^{2*}$ is generated by the classes ω_1 and ω_2, where ω_1 and ω_2 are the pullbacks of the hyperplane class in $\mathbb{P}^{2*}$ via the two projection maps. with the relations $\omega_1^3 = \omega_2^3 = 0$, $\omega_1^2 \omega_2^2 = 1$.

Γ $\mathbb{P}^{2*} \cong \mathbb{P}^2$ $\qquad$ $\mathbb{P}^{2*} \times \mathbb{P}^{2*} \xrightarrow{\pi_1} \mathbb{P}^{2*}$

$$\Rightarrow \begin{array}{ccc} H^2(\mathbb{P}^{2*}) & \xrightarrow{\pi_1^*} & H^2(\mathbb{P}^{2*} \times \mathbb{P}^{2*}) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ [\ell] & \longmapsto & \pi_1^*[\ell] = \omega_1 \end{array} \qquad \begin{array}{l} \ell \text{ line in } \mathbb{P}^{2*} \\ \;\,\| \\ \;\,\mathbb{P}^1 \end{array}$$

Similarly, $\pi_2^*[\ell] = \omega_2$. $\pi_1^*([\ell]^3) = (\pi_1^*[\ell])^3 = \omega_1^3$.

$\Rightarrow$ Since $[\ell]^3 \in H^6(\mathbb{P}^{2*}) = 0$, $\omega_1^3 = 0$. Similarly, $\omega_2^3 = 0$. Note the following commutative diagram

$$\begin{array}{ccc} H^2(\mathbb{P}^{2*}) \otimes H^2(\mathbb{P}^{2*}) & \xrightarrow{\cap} & H^4(\mathbb{P}^{2*}) \to \text{generator} \\ [\ell] \qquad\quad [\ell] & & [\ell]^2 \\ \Big\downarrow \pi_1^* \otimes \pi_1^* & & \Big\downarrow \pi_1^* \\ H^2(\mathbb{P}^{2*} \times \mathbb{P}^{2*}) \otimes H^2(\mathbb{P}^{2*} \times \mathbb{P}^{2*}) & \xrightarrow{\cap} & H^4(\mathbb{P}^{2*} \times \mathbb{P}^{2*}) \\ \rotatebox{90}{$\in$} \qquad\qquad \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ \omega_1 \quad \otimes \quad \omega_1 & \longmapsto & \omega_1^2 \end{array}$$

$\Rightarrow$ $\pi_1^*([\ell]^2) = \omega_1^2$. But again, $H^4(\mathbb{P}^{2*} \times \mathbb{P}^{2*})$ is generated by ω_1^2 and ω_2^2, i.e.
$H^4(\mathbb{P}^{2*} \times \mathbb{P}^{2*}) = \langle \omega_1^2 \rangle \oplus \langle \omega_2^2 \rangle$.

Since ω_1^2 & ω_2^2 are represented by $\mathbb{P}^{2*} \times \{*\}$ & $\{*\} \times \mathbb{P}^{2*}$ respectively, $\omega_1^{1+1} \cdot \omega_2^2 = \#((\mathbb{P}^{2*} \times \{*\}) \cdot (\{*\} \times \mathbb{P}^2)) = 1$ ⌟

$\pi_1 : \mathbb{P}^2 \times \mathbb{P}^2 \longrightarrow \mathbb{P}^2$ $\qquad$ $\ni \ell \otimes 1$

$\pi_1^* : H^2(\mathbb{P}^2) \longrightarrow H^2(\mathbb{P}^2 \times \mathbb{P}^2) = H^2(\mathbb{P}^2) \otimes H^0(\mathbb{P}^2)$
$\qquad\qquad [\ell] \qquad\qquad\qquad\qquad\qquad \oplus\, H^1(\mathbb{P}^2) \otimes H^1(\mathbb{P}^2)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \oplus\, H^0(\mathbb{P}^2) \otimes H^2(\mathbb{P}^2)$