⌈ If E is line bundle, $g_{\alpha\beta}$ & Θ_α are complex valued. $\Rightarrow$ $g_{\alpha\beta}\,\Theta_\beta = \Theta_\beta\, g_{\alpha\beta}$ $\Rightarrow$ $\Theta_\alpha = \Theta_\beta$.
$\Rightarrow$ $\{\Theta_\alpha\}$ defines a global 2-form on M.
$\Rightarrow$ By P141, Proposition 1, $c_1(E) = [(\sqrt{-1}/2\pi\ \Theta)]$. ⌟

In order to define the general Chern classes of vector bundle, we digress for a moment to consider those functions of a variable metrix which are invariant under conjugation.

Let $M_n \cong \mathbb{C}^{n^2}$ denote the vector space of $n\times n$ matrices. A polynomial function $P: M_n \longrightarrow \mathbb{C}$, homogeneous of degree k in the entries, is said to be invariant if

$$P(A) = P(gAg^{-1})$$

for all $A \in M_n$, $g \in GL_n$. The basic examples of such polynomials $P(A)$ are the elementary symmetric polynomials of the eigenvalues of A, i.e., the polynomials $P^i(A)$ defined by the relation

$$\det(A + t\cdot I) = \sum_{k=0}^{n} P^{n-k}(A)\cdot t^k.$$

⌈ Let C be a metrix s.t $C^{-1}AC$ is the Jordan canonical form of A.

$$\Rightarrow \det(A + t\cdot I) = \det(C^{-1}(A+t\cdot I)C)$$

$$= \det\begin{pmatrix} t+\lambda_1 & * & 0 & \cdots & 0 \\ 0 & t+\lambda_2 & * & & \\ \vdots & & \ddots & & \\ 0 & & & & t+\lambda_n \end{pmatrix} = (t+\lambda_1)(t+\lambda_2)+\cdots+(t+\lambda_n)$$