$\Rightarrow \iota_L^*(dw_1) = \frac{\partial(\frac{s_1}{s_0})}{\partial x_1} dx_1 + \frac{\partial(\frac{s_1}{s_0})}{\partial x_2} dx_2 = d\left(\frac{s_1}{s_0}\right)$

Similarly, we get $\iota_L^*(dw_i) = d\left(\frac{s_i}{s_0}\right)$.

$\Rightarrow$ Since ι_L^* is surjective, for any $v^* \in T_{x_0}^{*'}\mathcal{U}$, $\exists$ a_i's s.t $\iota_L^*(a_i\, dw_i) = v^*$.

$\Rightarrow \sum d\left(\frac{a_i s_i}{s_0}\right) = v^* \Rightarrow$ let $s = \sum a_i s_i$.

$\Rightarrow d\left(\frac{s}{s_0}\right) = v^*$ at x_0. $\Rightarrow \frac{s_0\, ds}{s_0^2} - \frac{s\, ds_0}{s_0^2}$

$= d\left(\frac{s}{s_0}\right)$. $\Rightarrow$ Consider the section $\sigma =$

$\frac{1}{s_0(x_0)} s - \frac{s(x_0)}{s_0^2(x_0)} s_0$. $\Rightarrow \sigma(x_0) = \frac{s(x_0)}{s_0(x_0)} - \frac{s(x_0)}{s_0(x_0)} = 0$.

$\Rightarrow \varphi_\alpha^{*-1}\sigma = \tau \in H^0(M, \mathcal{O}(L))$. $\Rightarrow \tau$ is the desired.

($\Leftarrow$). We have to show that ι_L^* is surjective i.e given $v^* \in T_{x_0}^*(M)$, $\exists$ s s.t $d\left(\frac{s_\alpha}{s_0}\right) = v^*$ at x_0.

By the assumption, $\exists$ $\sigma \in H^0(M, \mathcal{O}(L))$ s.t $\sigma_\alpha(x_0) = 0$ & $d\sigma_\alpha = v^*$.

Consider $\frac{\sigma_\alpha}{s_0}$. $\Rightarrow d\left(\frac{\sigma_\alpha}{s_0}\right) = \frac{1}{s_0^2}(s_0\, d\sigma_\alpha - \sigma_\alpha\, ds_0)$

$\Rightarrow$ At x_0, $\frac{1}{s_0(x_0)^2} s_0(x_0)\, d\sigma_\alpha - \cancel{\sigma_\alpha(x_0)}^{\,0}\, ds_0 = \frac{d\sigma_\alpha}{s_0(x_0)} = \frac{v^*}{s_0(x_0)}$

$\Rightarrow$ Let $\tau_\alpha = \frac{s_0^2(x_0)}{s_0(x_0)} \sigma_\alpha = s_0(x_0)\sigma_\alpha$.

$\Rightarrow d\left(\frac{\tau_\alpha}{s_0}\right) = v^*$. $\Rightarrow \frac{s_0^2(x_0)}{s_0(x_0)} \tau = s_0(x_0)\sigma$. is the desired section.