where s is the section of H vanishing exactly on H and r is the restriction map.

Γ First of all, we want to show that for any $s \in H^0(\mathbb{P}^N, \mathcal{O}(H))$, $(s) = H'$ where $H' \subset \mathbb{P}^N$ is a hyperplane.

Let $(s) = \sum a_i V_i$, where V_i's irreducible analytic hypersurfaces of $\mathbb{P}^N$, and $a_i \geq 0$. since s is holomorphic.

$\Rightarrow$ Since each V_i is linearly equivalent to kH and $[(s)] = [H]$, $(s) = V$. and $[V] = [H]$,

$\Rightarrow$ $V + (f) = H$ for some meromorphic function f on $\mathbb{P}^N$.

$\Rightarrow$ By P144, since (f) is homologous to zero, V is homologous to H. (See also P61)

$\Rightarrow$ By P64, (any analytic subvariety of $\mathbb{P}^n$ homologous to a hyperplane is a hyperplane), V is a hyperplane.

$\Rightarrow$ $\mathbb{P}(H^0(\mathbb{P}^N, \mathcal{O}(H)))$ corresponds to the set of all hyperplanes in $\mathbb{P}^N$.

$\Rightarrow$ Since $\{ s|_M \in H^0(M, \mathcal{O}(H|_M)) \}$ a linear subspace of $\mathbb{P}(H^0(\mathbb{P}^N, \mathcal{O}(H)))$, the corresponding subset $\{ H \cap M \mid H$ hyperplane in $\mathbb{P}^N \}$ of