since $\mathrm{Hom}_{\mathcal{O}}(\mathcal{F}(k), \mathcal{G}(k)) \cong \mathrm{Hom}_{\mathcal{O}}(\mathcal{F}, \mathcal{G})$, we may replace $\mathcal{F}$ by $\mathcal{F}(k)$ when convenient.

⌈ $\mathcal{F}(k) = \mathcal{F} \otimes_{\mathcal{O}} \mathcal{L}^k$

$$\Rightarrow \mathrm{Hom}_{\mathcal{O}}(\mathcal{F}(k)_x, \mathcal{G}(k)_x) = \mathrm{Hom}(\mathcal{F}_x \otimes_{\mathcal{O}} \mathcal{L}^k_x, \mathcal{G}_x \otimes \mathcal{L}^k_x)$$
$$\cong \mathcal{F}^*_x \otimes \mathcal{L}^{k*}_x \otimes \mathcal{G}_x \otimes \mathcal{L}^k_x \cong \mathcal{F}^*_x \otimes \mathcal{G}_x \otimes \mathcal{L}^{k*}_x \otimes \mathcal{L}_x \cong \mathcal{F}^*_x \otimes \mathcal{G}_x$$
$$\cong \mathrm{Hom}_{\mathcal{O}}(\mathcal{F}_x, \mathcal{G}_x) \quad \Rightarrow \quad \mathrm{Hom}_{\mathcal{O}}(\mathcal{F}(k), \mathcal{G}(k)) \cong \mathrm{Hom}_{\mathcal{O}}(\mathcal{F}, \mathcal{G})$$

Here we used $\mathcal{L}^{k*}_x \otimes \mathcal{L}_x \cong \mathrm{Hom}_{\mathcal{O}}(\mathcal{L}^k_x, \mathcal{L}^k_x) \cong \mathcal{O}_x \cong \mathcal{O}$ ⌋

Thus, suppose we are given a commutative diagram

$$\begin{tikzcd}
 & & \mathcal{E}' \arrow[d, dashed] \arrow[dr] & & \\
0 \arrow[r] & \mathcal{R} \arrow[r] & \mathcal{E} \arrow[r] & \mathcal{F} \arrow[r] & 0
\end{tikzcd}$$

of coherent sheaves on M where $\mathcal{E}$ and $\mathcal{E}'$ are locally free. It may not be possible to fill in the dotted arrow as it stands, but we can do the following: A section $s \in H^0(M, \mathcal{L}^k)$ gives an inclusion $\mathcal{E}'(-k) \subset \mathcal{E}'$, and we claim that, for $k \geq k_0$, the dotted arrow in the diagram

$$\begin{tikzcd}
 & & \mathcal{E}'(-k) \arrow[d, dashed] \arrow[dr] & & \\
0 \arrow[r] & \mathcal{R} \arrow[r] & \mathcal{E} \arrow[r] & \mathcal{F} \arrow[r] & 0
\end{tikzcd}$$

may be filled in.

⌈ $\mathcal{E}'(-k) \cong \mathcal{E}' \otimes \mathcal{L}^{-k} \xrightarrow{\otimes s} \mathcal{E}'$ ⌋