of M to be the Chern classes of its holomorphic tangent bundle $T'(M)$.

Note that the definition of $C_1(E)$ here agrees with our former definition of the Chern class of a holomorphic line bundle. In general — as will be clear by the end of this section — the Chern classes of a vector bundle are likewise purely topological invariants. The basic properties of the Chern classes are these:

1. First, if $f: M \longrightarrow N$ is any C^∞ map, $E \longrightarrow N$ a complex vector bundle, then

$$C_r(f^*E) = f^*C_r(E).$$

To see this, note that if D is a connection on E, $\underline{U} = \{U_\alpha\}$ an open cover of N with $e_{1,\alpha}, \cdots, e_{k,\alpha}$ a frame for E over U_α and θ_α the connection matrix for D relative to $\{e_{i,\alpha}\}$, then the matrices

$$f^*(\theta_\alpha) \quad \text{in} \quad f^{-1}(U_\alpha)$$

defines a connection D^* on $f^*E \longrightarrow M$ with curvature

$$\Theta(D^*) = f^*(\Theta(D)).$$

pf

$$\theta_\beta = dg \cdot g^{-1} + g \cdot \theta_\alpha \cdot g^{-1}, \quad g: U_\alpha \cap U_\beta \longrightarrow GL_n$$

$$\Rightarrow \quad f^*\theta_\beta = f^*(dg \cdot g^{-1}) + f^*(g \cdot \theta_\alpha \cdot g^{-1})$$
$$= d(f^*g) \cdot (f^*g)^{-1} + (f^*g) \cdot f^*\theta_\alpha \cdot (f^*g)^{-1}.$$

$$\Rightarrow \quad d\, f^*\theta_\alpha - f^*\theta_\alpha \wedge f^*\theta_\alpha = \Theta_{f^*D} = \Theta_{D^*}$$
$$= f^*(d\theta_\alpha - \theta_\alpha \wedge \theta_\alpha) = f^*(\Theta_D)$$