$$0 \to \overset{H^0(\mathcal{O}(K-L))}{\overset{\wr\!\!\shortparallel}{\mathrm{Ext}^0}}(\mathcal{S};\, \mathcal{J}(L),\, \Omega^2) \to \overset{H^0(\mathcal{O}(K))\oplus H^0(\mathcal{O}(K))}{\overset{\shortparallel}{\mathrm{Ext}^0}}(\mathcal{S};\, \mathcal{O}\oplus\mathcal{O},\, \Omega^2) \to \overset{H^0(\mathcal{O}(K+L))}{\overset{\shortparallel}{\mathrm{Ext}^0}}(\mathcal{S};\, \mathcal{O}(L^*),\, \Omega^2) \to \mathrm{Ext}^1(\mathcal{S};\, \mathcal{J}(L),\, \Omega^2) \to \underset{\underset{0}{\shortparallel}}{\mathrm{Ext}^2(\mathcal{S},\, \mathcal{O}\oplus\mathcal{O},\, \Omega^2)}$$

This gives the interpretation

$$(**) \qquad \mathrm{Ext}^1(\mathcal{S};\, \mathcal{J}(L),\, \Omega^2) \cong \left\{ \begin{array}{l} H^0(\mathcal{O}(K+L)) / \{ s\omega + s'\omega' \},\ \text{where} \\ \omega,\ \omega' \in H^0(\mathcal{O}(K)). \end{array} \right\}.$$

$$0 \to H^0(\mathcal{O}(K-L)) \to H^0(\mathcal{O}(K)) \oplus H^0(\mathcal{O}(K)) \to H^0(\mathcal{O}(K+L)) \to \mathrm{Ext}^1(\mathcal{S};\, \mathcal{J}(L),\, \Omega^2) \to 0$$

$\Rightarrow$ Since

$$\begin{array}{ccc} H^0(\mathcal{O}(K-L)) & \longrightarrow & H^0(\mathcal{O}(K)) \oplus H^0(\mathcal{O}(K)) \\ \sigma & \longmapsto & (-\sigma\otimes s',\quad \sigma\otimes s) \end{array}$$

and

$$\begin{array}{ccc} H^0(\mathcal{O}(K)) \oplus H^0(\mathcal{O}(K)) & \xrightarrow{\ \phi\ } & H^0(\mathcal{O}(K+L)) \\ (\omega,\quad \omega') & \longmapsto & s\otimes\omega + s'\otimes\omega', \end{array}$$

$$H^0(\mathcal{O}(K+L)) \big/ \operatorname{Im}\phi = \mathrm{Ext}^1(\mathcal{S};\, \mathcal{J}(L),\, \Omega^2)$$

$$= H^0(\mathcal{O}(K+L)) \big/ \{ s\omega + s'\omega' \}, \quad \omega, \omega' \in H^0(\mathcal{O}(K)).$$

Spectral sequences & sheaf exact sequence are natural.
$\Rightarrow$ Everything is functorial, i.e. natural.

Now since $\underline{\mathrm{Ext}}^0_{\mathcal{O}}(\mathcal{O}_p(L),\, \Omega^2) = \underline{\mathrm{Ext}}^1_{\mathcal{O}}(\mathcal{O}_p(L),\, \Omega^2) = 0$