To show h is invertible, we have only to show that $J(h)$ has nonzero determinant.

$$J(h) = \begin{pmatrix} \frac{\partial \sigma_1}{\partial z_1'}, & \frac{\partial \sigma_1}{\partial z_2'} \\ \frac{\partial \sigma_2}{\partial z_1'}, & \frac{\partial \sigma_2}{\partial z_2'} \end{pmatrix} = \begin{pmatrix} 1, & 1 \\ z_2', & z_1' \end{pmatrix}$$

$$\Rightarrow \quad |J(h)| = z_1' - z_2' \neq 0 \quad \Rightarrow \quad \begin{array}{l} z_1'(q_1) = f_1(\sigma_1, \sigma_2) \\ z_2'(q_2) = f_2(\sigma_1, \sigma_2) \end{array}$$

$\Rightarrow \phi \circ \psi^{-1}$ is holomorphic in $\sigma_1(z_1', z_2')$ and $\sigma_2(z_1', z_2')$.

If $q_1 + q_2 \in \pi(U_1 \times U_2) \cap \pi(U_1' \times U_2')$, $\exists$ open sets $V_1 \ni q_1$, $q_2 \in V_2$ s.t. $V_1 \subset U_1 \cap U_1'$, $V_2 \subset U_2 \cap U_2'$ with loss of generality.

$$\begin{array}{c} \mathbb{C} \times \mathbb{C} \\ z_1 \uparrow z_2 \\ V_1 \times V_2 \\ z_1' \downarrow z_2' \\ \mathbb{C} \times \mathbb{C} \end{array}$$

$$\pi(V_1 \times V_2) \subset \pi(U_1 \times U_2) \cap \pi(U_1' \times U_2').$$

Aha! Here are two points we have to notice.

① $z_1' + z_2'$ & $z_1' z_2'$ are functions of z_1 & z_2

② $z_1' + z_2'$ & $z_1' z_2'$ are unchanged even if we exchange q_1' with q_2'. Remember that $z_1' = z_1'(q_1')$, $q_1' \in V_1$
$z_2' = z_2'(q_2')$, $q_2' \in V_2$.

But since exchanging q_1' with q_2' means exchanging z_1 with z_2, $z_1' + z_2'$ & $z_1' z_2'$ are symmetric functions in z_1, z_2 $\Rightarrow$ $z_1' + z_2'$ & $z_1' z_2'$ may be expressed in terms of $\sigma_1(z_1, z_2)$ & $\sigma_2(z_1, z_2)$.

Similarly for the other way.