$\varphi_{\beta\gamma} \circ \varphi_{\alpha\beta} = \varphi_{\alpha\gamma}$ may not be true. The conditions are essential for gluing the local extensions $(\mathcal{E}_\alpha)$. ⌟

What is true is the

Lemma. The equivalence classes of global extensions $(\mathcal{E})$ are in bijective correspondence with $Ext^1(M; \mathcal{G}, \mathcal{F})$.

Proof. Given $(\mathcal{E})$, the exact sequence of global Ext's gives

$$\begin{array}{ccccc} Ext^0(M;\mathcal{G},\mathcal{E}) & \longrightarrow & Ext^0(M;\mathcal{G},\mathcal{G}) & \xrightarrow{\partial} & Ext^1(M;\mathcal{G},\mathcal{F}) \to \cdots \\ \| & & \| & & \\ H^0(M, \underline{Hom}_{\mathcal{O}}(\mathcal{G},\mathcal{E})) & \longrightarrow & H^0(M, \underline{Hom}_{\mathcal{O}}(\mathcal{G},\mathcal{G})) & & \end{array}$$

and the obstruction to splitting the sequence $(\mathcal{E})$ is just $\partial(1_{\mathcal{G}})$ as in the local case.

⌜ Since we have the two long exact sequences of global Ext, by P722,

$$Ext^0(M;\mathcal{G},\mathcal{E}) \longrightarrow Ext^0(M;\mathcal{G},\mathcal{G}) \xrightarrow{\partial} Ext^1(M;\mathcal{G},\mathcal{F}) \to \cdots .$$

$$H^0(M, \underline{Hom}_{\mathcal{O}}(\mathcal{G},\mathcal{E})) = Ext^0(M;\mathcal{G},\mathcal{E})$$
$$H^0(M, \underline{Hom}_{\mathcal{O}}(\mathcal{G},\mathcal{G})) = Ext^0(M;\mathcal{G},\mathcal{G})$$ by P706 & P700