— is holomorphic in ω.

⌈ Since any symmetric function can be expressed as polynomials in the power sums, the remark above is proved. See P8. ⌋

One application is a proof of the proper mapping theorem for $f: U \longrightarrow W$, and hence for general finite surjective mappings: If $V \subset U$ is an analytic variety defined by equations $\{h_\alpha(z)=0\}$, then $f(V) \subset W$ is defined by $\{H_\alpha(w)=0\}$, where $H_\alpha(\omega) = h_\alpha(z_1(\omega)) \cdots h_\alpha(z_\alpha(\omega))$.

⌈ $U = \{ (z_1, z_2) \in \mathbb{C}^{n^2} \mid |z_1|<1,\ |z_2|<1 \}$ $\quad W = U$

$$\begin{array}{ccc} f: U & \longrightarrow & W \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ (z_1, z_2) & \longmapsto & (z_1^2, z_2) \end{array} \Rightarrow f \text{ is surjective, open and finite.}$$

$$\begin{array}{ccc} h_1: U & \longrightarrow & \mathbb{C} \\ (z_1, z_2) & \longmapsto & z_1 - \frac{1}{2} \end{array} \qquad \begin{array}{ccc} h_2: U & \longrightarrow & \mathbb{C} \\ (z_1, z_2) & \longmapsto & z_2 + \frac{1}{3} \end{array}$$

$\Rightarrow \{ (z_1, z_2) \mid h_1 = 0 = h_2 \} = \{ (\frac{1}{2}, -\frac{1}{3}) \}$.

$H_1(\omega) = h_1(z^1(\omega))\, h_1(z^2(\omega)) = h_1(\sqrt{\omega_1}, \omega_2)\, h_1(-\sqrt{\omega_1}, \omega_2) = (\sqrt{\omega_1} - \frac{1}{2})(-\sqrt{\omega_1} - \frac{1}{2})$

$H_2(\omega) = h_2(\sqrt{\omega_1}, \omega_2)\, h_2(-\sqrt{\omega_1}, \omega_2) = (\omega_2 + \frac{1}{3})(\omega_2 + \frac{1}{3})$
$= (\omega_2 + \frac{1}{3})^2$.

$\Rightarrow \{ H_1 = H_2 = 0 \} = \{ (\frac{1}{4}, -\frac{1}{3}) \} \frown f\{(\frac{1}{2}, -\frac{1}{3})\}$

If $h_1(z_1, z_2) = z_1 - \frac{1}{2}$, $h_2(z_1, z_2) = z_1 + \frac{1}{2}$, then
$\{ h_1 = h_2 = 0 \} = \phi$, $H_1(\omega) = (\sqrt{\omega_1} - \frac{1}{2})(-\sqrt{\omega_1} - \frac{1}{2})$ $H_2(\omega) = (-\sqrt{\omega_1} + \frac{1}{2})(\sqrt{\omega_1} + \frac{1}{2})$