φ_M $(p.q)$ type. φ_N (p',q') type $\Rightarrow$ (p,q) (p',q')
ψ_M $(a.b)$ " ψ_N (a',b') type. (a,b), (a',b') types.

$$\langle \bar{\partial}_M \varphi, \psi \rangle = \langle \varphi, \bar{\partial}_M^* \psi \rangle.$$

$$\bar{\partial}_M^* : \overset{M}{(a,b)}. \overset{N}{(a',b')} \longmapsto (a, b-1)\ (a',b')$$

since, if $p=a$. $q=b-1$. $p'=a'$. $q'=b'$.
the equation holds. In the similar way.

$$\bar{\partial}_N^* ; \overset{M}{(p.q)}\ \overset{N}{(p',q')} \longmapsto (p.q)\ (p', q'-1).$$

$$*_{M\times N} ; (p.q)\ (p',q') \longmapsto (m-p.\ m-q).\ (n-p'\ n-q')$$

where $m.n$ complex dimension of complex manifolds M, N respectively.

$$\Rightarrow *_{M\times N} \bar{\partial}_M *_{M\times N} ; (p.q)\ (p',q') \longmapsto (m-p, n-q)\ (n-p', n-q')$$
$$\longmapsto (m-p.\ n-q+1)\ (n-p', n-q') \longmapsto (p.\ q-1)\ (p'.q')$$

$$\Rightarrow *_{M\times N} \bar{\partial}_N *_{M\times N} ; (p.q)\ (p',q') \longmapsto (p, q)\ (p', q'-1).$$

From ⊛, we know that

$$-*_{M\times N} \bar{\partial}_{M\times N} *_{M\times N} \psi = \bar{\partial}_M^* \psi + \bar{\partial}_N^* \psi$$
$$/\!/$$
$$-*_{M\times N} \bar{\partial}_M *_{M\times N} \psi + (-*_{M\times N} \bar{\partial}_N *_{M\times N})\psi$$

By comparing the types, we get

$$-*_{M\times N} \bar{\partial}_M *_{M\times N} \psi = \bar{\partial}_M^* \psi. \quad \& \quad -*_{M\times N} \bar{\partial}_N *_{M\times N} \psi = \bar{\partial}_N^* \psi.$$

It remains to prove that $\bar{\partial}_M \bar{\partial}_N^* + \bar{\partial}_N^* \bar{\partial}_M = 0 = \bar{\partial}_M^* \bar{\partial}_N + \bar{\partial}_N \bar{\partial}_M^*$.