In effect, $\tilde{\Delta}$ consists of all the lines through the origin in Δ made disjoint.

「 $z \neq 0 \Rightarrow \{(z, l)\}$ is the set of all lines through the origin in Δ. 」

$\tilde{\Delta}$, together with its projection map π to Δ, is called the blow-up of Δ at 0. The real points of the blow-up of $\Delta \subset \mathbb{C}^2$ are pictured in Fig. 1.

Note that we have encountered the manifold $\tilde{\Delta}$ before: together with the projection $\pi' : \tilde{\Delta} \longrightarrow \mathbb{P}^{n-1}$ on the second factor it is the universal bundle J on $\mathbb{P}^{n-1}$.

Now let M be a complex manifold of dimension n, $x \in M$ any point, and $z : U \longrightarrow \Delta$ a coordinate polydisc centered around $x \in M$. The restriction of the projection map

$$\pi : \tilde{\Delta} - E \longrightarrow U - \{x\} \subset M$$

gives an isomorphism between a neighborhood of $E = \pi^{-1}x$ in $\tilde{\Delta}$ and a nbd of x in M; we define the blow-up $\tilde{M}_x$ of M at x to be the complex manifold

$$\tilde{M}_x = M - \{x\} \cup_{\pi} \tilde{\Delta}$$

obtained by replacing $\Delta \subset M$ with $\tilde{\Delta}$, together with the natural projection map $\pi : \tilde{M}_x \longrightarrow M$.

「 $x \in M$, $x \in U \subset M$,

$$\begin{array}{ccc} & & U \ni x \\ & & \big\downarrow z \\ \tilde{\Delta} & \xrightarrow{\pi} & \Delta \end{array} \qquad \tilde{\Delta} - \pi^{-1}(0) \xrightarrow{\pi} \Delta \xrightarrow{z^{-1}} U - \{x\}$$

$\tilde{\Delta} - \pi^{-1}(0) \cong U - \{x\}$ $\quad \tilde{M}_x = M - \{x\} \cup_{\pi} \tilde{\Delta} \to M$ 」