

$(\varphi)_\infty$ of φ are expressible as the loci of homogeneous polynomials $F(X)$ and $G(X)$.

According to P131,

$$(\varphi) = (\varphi)_0 - (\varphi)_\infty$$

$\Rightarrow (\varphi)_0$ and $(\varphi)_\infty$ are analytic subvarieties of projective space, actually hypersurfaces.

$\Rightarrow [(\varphi)_0]$ is of the form H^d for some d , and $[(\varphi)_\infty]$ is the zero locus of some section σ of $[(\varphi)_0]$. But all sections σ of H^d are of the form σ_F , and so

$$(\varphi)_0 = (\sigma_F) = (F(X_0, \dots, X_n) = 0).$$

In the exactly same way,

$$\text{we have } (\varphi)_\infty = (G(X_0, X_1, \dots, X_n) = 0)$$

$\Rightarrow (\varphi)_0$ and $(\varphi)_\infty$ are expressible as the loci of homogeneous polynomials $F(X)$ and $G(X)$. \Downarrow

Since moreover the divisor (φ) is homologous to zero, F and G have the same degree, so F/G is a well-defined rational function on \mathbb{P}^n ; then from

$$(F/G) = (\varphi)$$

it follows that $\varphi = \lambda F/G$ for some $\lambda \in \mathbb{C}$.