$$\sum_{f(p)=p} \iota_f(p) = L(f).$$

Note that without computing signs the number of fixed points of f must be at least the absolute value of $L(f)$, i.e.,

$$\#\{p \in M : f(p) = p\} \geq |L(f)|,$$

and in particular,

$$L(f) \neq 0 \Rightarrow f \text{ has a fixed point.}$$

Γ $$\sum_{f(p)=p} \iota_f(p) = L(f)$$

$$\Rightarrow \quad |L(f)| \leq \sum_{f(p)=p} |\iota_f(p)| = \#\{p \in M : f(p) = p\}$$

$\Rightarrow$ If $L(f) \neq 0$, $\#\{p \in M : f(p) = p\} \geq 1$. $\Rightarrow$ f has a fixed point. ⌟

As an immediate corollary to the Lefschetz fixed-point formula we will prove the Hopf index theorem. Let M be as above, and let v be a global C^∞ vector field on M. We say a zero p of v is nondegenerate if it is isolated and, in terms of local coordinates $x_1, \cdots x_n$ centered around p,

$$v(x) = \sum a_{ij} x_i \frac{\partial}{\partial x_j} + [\partial].$$

with $\Delta = (a_{ij})$ nonsingular; in this case we define the index $\iota_v(p)$ of v at p to be the sgn of the determinant of Δ. Now, integrating the vector field v to time t gives a flow

$$f_t : M \longrightarrow M.$$