NO.

DATE . . ( )

It follows from Bertini's theorem that h is tangent to S at every point $p \in S \cap h$, since otherwise the pencil cut out on C_h by the lines in h through p would be generically singular away from its base locus p.

「 I think the argument is wrong. We do not need Bertini's theorem. Given any point $p \in S \cap h$, since h contains 6 of the p_i's, choose two p_i's, say p_1, p_2 s.t p, p_1 & p_2 are not collinear ($\because \deg S = 2$ and p_i is a double point).
$\Rightarrow$ By the argument above, $\overline{pp_1}$ & $\overline{pp_2}$ are tangent to S at p. $\Rightarrow$ Since $h = \overline{p_0 p_1 p_2}$,
h is tangent to S. For example,
$F = 0$ on $\mathbb{P}^3$. $\Rightarrow$ F is a surface in $\mathbb{C}^2$.
$\Rightarrow$ If $\frac{\partial F}{\partial x} = 0$, and $\frac{\partial F}{\partial y} = 0$ at the origin,
$a\frac{\partial F}{\partial x} + b\frac{\partial F}{\partial y} = 0$, for all a, b, at the origin.
More precisely, in all directions at the origin, the directional derivatives are zero.
」

The curve C_h is thus a plane conic, counted with multiplicity 2 in the intersection $S \cdot h$.

「 Since h is tangent to S at every point $p \in$