

Γ Since $\overline{p, q}$ and L lie in V_2 , $\overline{p, q} \cap L \neq \emptyset$.
 $\Rightarrow \#(\overline{p, q} \cap H) \geq 3 \Rightarrow \overline{p, q} \subset H$. Similarly, $\overline{p, q} \subset G$.
 $\Rightarrow \overline{p, q} \subset H \cap G = X$.

□

Thus the map f_L is one-to-one away from the locus of lines in X meeting L ; this is sufficient to establish that f_L is birational.

Γ Suppose $p \neq q \in X - L$, and $f_L(p) = f_L(q)$.
 \Rightarrow For some a, b , $ap + bq \in L \Rightarrow \overline{p, q} \cap L \neq \emptyset$
 $\Rightarrow \langle L, \overline{p, q} \rangle$ is a α -plane in P^5 containing L .
 \Rightarrow By the argument above, $\overline{p, q} \subset X$ and $\overline{p, q} \cap L \neq \emptyset$.
 Thus we can say that f_L is one to one away from the locus of lines in X meeting L .
 $\Rightarrow f_L: X - B_L \rightarrow P^3$ is one to one.

By the remark on P495, 3., f_L may be extended to \tilde{X} by sending a point $r \in E$ in the exceptional divisor to the point of intersection of P^3 with the tangent line to X at L .

By the remark on P495, 3., since the exceptional divisor $E \rightarrow L$ is naturally identified with the projectivization $IP(N_{L/X})$ of the normal bundle $N_{L/X}$ of L in X . $\Rightarrow f_L$ may be extended to the blow-up of X along L by sending a point