iety of U. And it is proper, since D is proper in an open set of C^n, and π_I is open map. ⏌

Thus, for $H \in U - \bigcup_I \pi_I^{-1}(D)$, the points of $H \cap C$ are in general position. Q.E.D.

Now, we can characterize the dimension of a linear system $|D|$ as follows: $\dim |D| \geq t$ if and only if for every t points $p_1, \cdots p_t \in S$ there exists a divisor $E \in |D|$ containing $p_1, p_2, \cdots p_t$.

⎾ $(\Rightarrow)$ $H^0(S, \mathcal{O}(D)) \ni \sigma_1, \sigma_2, \cdots \sigma_{t+1}$ which are linearly independent.

$x_1\sigma_1 + \cdots + x_{t+1}\sigma_{t+1}$ is a nontrivial section of $\mathcal{O}(D)$ if not all x_i's zero.

Consider $\{x_1\sigma_1(p_i) + x_2\sigma_2(p_i) + \cdots + x_{t+1}\sigma_{t+1}(p_i) = 0$ for $i = 1, \cdots t\}$ $\Rightarrow$ Always $\exists$ nontrivial solutions $x_1 \cdots x_{t+1}$ satisfying the equations. If $p_1 = p_2$, take derivative as P.633 note.

$(\Leftarrow)$ Consider a map $\phi : \underbrace{S \times S \times \cdots \times S}_{t} \longrightarrow |D|$ defined by $\phi(p_1 \cdots p_t) = D_{p_1 \cdots p_t}$ which is an effective divisor containing $p_1 \cdots p_t$. We can choose such D by the assumption.

$\Rightarrow$ There are a finite number of possibility of existence of $(p_1 \cdots p_t)$'s s.t $\phi(p_1 \cdots p_t) = D_{p_1 \cdots p_t}$ i.e if $\deg D = d$, at most ${}_dC_t$ number of $(p_1 \cdots p_t)$'s.

$\Rightarrow$ $\dim |D| \geq t$, for $S \times \cdots \times S \longrightarrow \phi(S^t)$ has fiber dimension ${}_dC_t$. ⏌

Thus, if D and D' are two effective divisors on S, we can find a divisor, $E \sim D + D'$ containing any $h^0(D) - 1 + h^0(D') - 1$ points of S, and so we have

$$h^0(D + D') \geq h^0(D) + h^0(D') - 1.$$