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『 Without loss of generality, since L is a generic line through F, we may assume that
$F = (X_0^2 = 0)$ and G is a smooth quadric. $\Rightarrow$
$L = \{ F + \lambda G \}_{\lambda \in \mathbb{P}^1}$. $\Rightarrow$ Since L is a generic pencil, $(X_0 = 0) \cap G = \{p, p'\}$ is a set of two distinct points.

Let F' be a quadric in L. $\Rightarrow$ $F' = F + \lambda' G$.
$\Rightarrow$ F' passes p & p', and $^{\#}(F' \cap G) = 4$ since at p & p', $G = 0$ and $F = 0$ with multiplicity > 1.
In other words, solving $\{F=0\} \cap \{G=0\}$ is the same as solving $\{F=0\} \cap \{F + \lambda' G = 0\}$.
Let H be a conic passing through p & p' and H be tangent to G at those two points.
Suppose $p = [0, a_1, a_2]$ & $p' = [0, b_1, b_2]$.

$$\Rightarrow \left(\frac{X_2}{X_1} - \frac{a_2}{a_1} \right) \left(\frac{X_2}{X_1} - \frac{b_2}{b_1} \right) = 0$$

$\Rightarrow K = a_1 b_1 X_2^2 - (a_2 b_1 + a_1 b_2) X_1 X_2 + a_2 b_2 X_1^2 = 0$ passes p & p'.
$\Rightarrow$ $G = X_0 f(X_0, X_1, X_2) + K$ for some homogeneous polynomial f of deg 1.
Since H must pass through p & p', $H = X_0 g(X_0, X_1, X_2) + \lambda K$, for some g with $\deg g = 1$ and $\lambda \in \mathbb{C}$.

① $\lambda = 0$.
$H = X_0 g$ $\Rightarrow$ Since H is tangent to G at p & p', $g = \tau X_0$ for $\tau \in \mathbb{C}$.

② $\lambda \neq 0$