ن.ز

is of order 0.

2. The δ-function is the distribution defined by

$$\delta(\varphi) = \varphi(0).$$

☞ As we saw on P141, Rudin's F.A., δ is a distribution of order 0. ن.ز

Next we extend the operators D_i to the space of distributions by setting

$$(D_i T)(\varphi) = -T(D_i \varphi).$$

☞ See P140, F.A. $P_4 = P136$. (2) ن.ز

If $T = T_\psi$ is the distribution associated to a function ψ of class C^1, then for $\varphi \in C_c^\infty(\mathbb{R}^n)$

$$\begin{aligned}(D_i T_\psi)(\varphi) &= -T_\psi(D_i \varphi) \\ &= -\int_{\mathbb{R}^n} \psi(x)\left[\frac{\partial \varphi}{\partial x_i}(x)\right] dx \\ &= \int_{\mathbb{R}^n} \frac{\partial \psi(x)}{\partial x_i}\, \varphi(x)\, dx \quad (\text{by Stokes' theorem}) \\ &= (T_{D_i \psi})(\varphi),\end{aligned}$$

so that our extended notion of differentiating distributions makes sense.

An example that illustrates the principle underlying the various residue theorems we shall discuss is obtained by considering the locally L^1 function $\psi(x)$ on $\mathbb{R}$ defined by

$$\begin{cases} \psi(x) = 0, & x < 0, \\ \psi(x) = 1, & x \geq 1. \end{cases}$$

☞ See. F.A P164. Ex 24. Heaviside function ن.ز