In general, the family of effective divisors on M corresponding to a linear subspace of $\mathbb{P}(H^0(M, \mathcal{O}(L)))$ for some $L \to M$ is called a linear system of divisors; a linear system is called complete if it is of the form $|D|$, i.e if it contains every effective divisor linearly equivalent to any of its members.

Γ Suppose that K is a complete linear system.
$\Rightarrow \exists\ L \longrightarrow M.$ s.t

① $K \subset \{$effective divisors on $M\}$.
② K corresponds to a linear subspace of $\mathbb{P}(H^0(M, \mathcal{O}(L)))$
③ $K = |D|$ for some D (divisor on M).

$H^0(M, \mathcal{O}(L)) = 1 \Rightarrow$ Trivial. nothing to say.
$H^0(M, \mathcal{O}(L)) \neq 1$. and L is not trivial.
$\Rightarrow \exists$ a global holomorphic section s of L.
$\Rightarrow [(s)] = L \Rightarrow$ Any holomorphic section s' of L is fs. for, $(s') = (fs) = (f) + (s) \ \cancel{\Rightarrow} \ (f) = 0 \Rightarrow f$ is holomorphic function. $\Rightarrow f$ is constant $(\neq 0)$.

Here is a big mistake. $(s) = (s')$ need not be true. Different Divisors can make the same line bundle.

Let $(s) = D. \Rightarrow L = [D]$.

$$H^0(M, \mathcal{O}(L)) = H^0(M, \mathcal{O}([D]))$$

$$\Rightarrow \mathbb{P}(H^0(M, \mathcal{O}([D]))) \cong |D|$$

$$\mathbb{P}(H^0(M, \mathcal{O}([D]))) = \mathbb{P}(H^0(M, \mathcal{O}(L)))$$

Note: $\dim H^0(M, \mathcal{O}(D)) < \infty$ by 1 & 3 on P152