orem. We shall give several preliminary reductions before coming to the essential point.

1. Since $f(M)$ is a closed subset of N, the question is local around a point $p \in f(M)$ in N. So we may assume that N is a polycylinder $\Delta^N = \Delta$ in $\mathbb{C}^N$.

$y_0 \in \overline{f(M)} - f(M)$ ⇒ ∃ a sequence $\{y_n\}$ conversent to y_0 s.t $\{y_n\} \subset f(M)$. ⇒ Given any open set U s.t $\overline{U}$ compact and $U \ni y_0$, $U \subset N$, open $f^{-1}(U) \subset f^{-1}(\overline{U})$ which is compact. Choose $x_n \in M$ s.t $f(x_n) = y_n$ ⇒ $x_n \in f^{-1}(U)$ ⇒ Since $f^{-1}(\overline{U})$ is compact, ∃ a convergent subsequence $\{x_{n_k}\}$ to x_0 in $f^{-1}(\overline{U})$. ⇒ Since f is continuus, $\lim f(x_{n_k}) = \lim y_{n_k} = f(x_0) = y_0 \in f(M)$ ⇒ Contradiction ⇒ $\overline{f(M)} = f(M)$. ⇒ $f(M)$ is a closed subset of N.

I don't the reason why we need the closedness of $f(M)$ to assume that N is a polycylinder.

Also, we may assume that M is irreducible, since only a finite number of components of M will have images meeting a given compact set in N.

Given a compact set K in N, then $f^{-1}(K)$ is compact in M. Suppose $f^{-1}(K) \cap M_i \neq \emptyset$ and $x_i \in f^{-1}(K) \cap M_i$.