$\geq k-r-i$, since $\Lambda \cap V_{k-r-i} \supset V_{k-r} \cap V_{k-r-i} = V_{k-r-i}$, for $i=0,\cdots$

$\dim(\Lambda \cap V_{n-k+k-r+i-(n-k-1)}) = \dim(\Lambda \cap V_{k-r+i+1})$

$= -\dim(\Lambda + V_{k-r+i+1}) + \dim(\Lambda) + \dim(V_{k-r+i+1})$

$\geq -(k+1) + k + (k-r+i+1) = k-r+i$.

$\Rightarrow \quad \Lambda \in \sigma_{n-k,\cdots,n-k,\, n-k-1,\cdots n-k-1}(V) = Z_r$. ⅃

$Z_k \cong \mathbb{P}(V_{k+1})^*$ is just the dual projective space of hyperplanes in V_{k+1}, and $Z_r \subset Z_k$ the linear subspace of hyperplanes containing V_{k-r}.

Γ See P15 ⅃

The bundle $S|Z_k$, moreover, is just the subbundle of the trivial bundle $V_{k+1} \times Z_k$ whose fiber over any $\Lambda \in Z_k$ is the hyperplane $\Lambda \subset V_{k+1}$. The quotient Q of $V_{k+1} \times Z_k$ by S is thus the universal quotient bundle on $Z_k \cong \mathbb{P}^k$; that is, the hyperplane line bundle.

Γ $\dim Q = \dim V_{k+1} \times Z_k / S = \dim(V_{k+1} \times Z_k) - \dim S$

$= (k+1) - k = 1$.

According to P207,

$$\begin{tikzcd} Q \arrow[rr] \arrow[d] & & S^* = H. \arrow[d] \\ G(k,k+1) \arrow[rr, "* = f"] & & G(1,k+1) \end{tikzcd}$$

let $f = *$.

$\Rightarrow$ The quotient Q of $V_{k+1} \times Z_k$ by S is the pull-back of the hyperplane line bundle over $G(1,k+1) = Z_k^*$.