$$= (-1)^{n-k} \left(\begin{array}{c|c} \sigma & \\ \tau & 0 \\ \hline I & I \end{array} \right) =$$

in even dimension, all boundary maps are zero, and so the homology of $\mathbb{P}^n$ is freely generated by the classes of the closures $\mathbb{P}^k$ of the cells, i.e by the homology classes of its linear subspaces given the natural orientation.

Inasmuch as a k-plane $\mathbb{P}^k$ and an $(n-k)$-plane $\mathbb{P}^{n-k}$ in $\mathbb{P}^n$ will generically intersect transversely in one point, Poincare duality is clear in this case. Indeed, since an $(n-k_1)$-plane will generically intersect an $(n-k_2)$-plane transversely in an $(n-k_1-k_2)$-plane

$$((\mathbb{P}^{n-k_1}) \cdot (\mathbb{P}^{n-k_2})) = \pm (\mathbb{P}^{n-k_1-k_2}).$$

$$H_k(\mathbb{P}^n) \times H_{n-k}(\mathbb{P}^n) \longrightarrow \mathbb{Z}$$

5. Vector Bundles, Connections. and Curvature.

Complex and Holomorphic Vector Bundles.

Def: M differentiable manifold. A C^∞ complex vector bundle on M consists of a family $\{E_x\}_{x \in M}$ of complex vector spaces parametrized by M, together with a C^∞ manifold structure on $E = \bigcup_{x \in M} E_x$ s.t

1. The projection map $\pi : E \longrightarrow M$ taking E_x to x is C^∞ and
2. For every $x_0 \in M$, $\exists$ an open $U \subset M$, $U \ni x_0$ and