$\sigma_2 = 0$, $\sigma_1 \in H^0(\mathbb{P}^2, \mathcal{O}(n))$. $\Rightarrow$ $(\sigma = 0) = ((\sigma_1, \sigma_2) = 0)$ is a curve. ⌟

We still have not found necessary and sufficient conditions that (**) may be solved. To do this, we assume for simplicity that

$$\begin{cases} \mathcal{L} = \mathcal{O}, \text{ and} \\ I_p = m_p \text{ is the maximal ideal for each point } p \in Z. \end{cases}$$

The exact sequence of global Ext applied to

$$0 \longrightarrow I \longrightarrow \mathcal{O} \longrightarrow \mathcal{O}_Z \longrightarrow 0$$

gives

$$\cdots \longrightarrow H^1(S, \mathcal{O}) \longrightarrow \mathrm{Ext}^1(S; I, \mathcal{O}) \longrightarrow \mathrm{Ext}^2(S; \mathcal{O}_Z, \mathcal{O}) \to H^2(S, \mathcal{O}) \longrightarrow \cdots.$$

⌜ $\mathrm{Ext}^1(S; \mathcal{O}, \mathcal{O}) \longrightarrow \mathrm{Ext}^1(S; I, \mathcal{O}) \to \mathrm{Ext}^2(S; \mathcal{O}_Z, \mathcal{O}) \to \mathrm{Ext}^2(S; \mathcal{O}, \mathcal{O}) \to \cdots$

By P706,
$$\mathrm{Ext}^1(S; \mathcal{O}, \mathcal{O}) = H^1(S, \mathcal{O}^* \otimes_{\mathcal{O}} \mathcal{O}) = H^1(S, \mathcal{O})$$
$$\mathrm{Ext}^2(S; \mathcal{O}, \mathcal{O}) = H^2(S, \mathcal{O}^* \otimes_{\mathcal{O}} \mathcal{O}) = H^2(S, \mathcal{O}).$$ ⌟

Since $\underline{\mathrm{Ext}}^q_{\mathcal{O}}(\mathcal{O}_Z, \mathcal{O}) = 0$ for $q \neq 2$ while $\underline{\mathrm{Ext}}^2_{\mathcal{O}}(\mathcal{O}_Z, \mathcal{O})$ is a skyscraper sheaf concentrated on Z with stalks

$$\underline{\mathrm{Ext}}^2_{\mathcal{O}}(\mathcal{O}_Z, \mathcal{O})_p \cong \Lambda^2 T_p'(S),$$