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$f^*\sigma = f^*\nu \wedge f^*\beta$. $\Rightarrow$ $\nu = dr$, where $r = \|W\|$.
$\Rightarrow f^*\nu = f^* dr = f^* d\|W\| = d\|W \circ f\| = \alpha\, d\|z\|$, for $\alpha \geq 0$ $\Rightarrow$ $f^*\beta \geq 0$ on $\|z\| = \varepsilon$. If $df_1 \wedge \cdots \wedge df_n \neq 0$ at z_0, then f is locally biholomorphic. $\Rightarrow$ $f^*\beta > 0$ at z_0.

$\Rightarrow \int_{\|z\|=\|z_0\|} f^*\beta > 0$. $\Rightarrow$ $\int_{\|z\|=\|z_0\|} f^*\beta = \int_{\|z\|=\|z_0\|} \eta(f_1, \cdots f_n)$

$= \mathrm{Res}_{\{0\}}\left(\frac{df_1}{f_1} \wedge \cdots \wedge \frac{df_n}{f_n}\right) = (D_1, \cdots D_n)_{\{0\}} > 0$ ∎

"Comment on σ P370~P371"

"Orthogonal to dr" means that $\sigma = 0$ on the vertical vectors of spheres in $\mathbb{R}^n$. Consider σ at $(1, 0 \cdots 0)$. $\Rightarrow$
$dr = dx_1$ & $\sigma_0 = c\; dx_2 \wedge dx_3 \wedge \cdots \wedge dx_n$.

$$\Rightarrow R_g^*\sigma_0 = c\; dx_2 \circ R_g \wedge dx_3 \circ R_g \wedge \cdots \wedge dx_n \circ R_g$$
$$= c\; d(g_{21}x_1 + \cdots + g_{2n}x_n) \wedge d(g_{31}x_1 + \cdots + g_{3n}x_n) \wedge \cdots \wedge d(g_{n1}x_1 + \cdots + g_{nn}x_n)$$
$$= c\;(g_{21}\, g_{32} \cdots g_{n\,n-1})\, dx_1 \wedge dx_2 \wedge \cdots \wedge dx_{n-1} + (\quad) \cdots$$

For $n = 3$,
$$R_g^*\sigma_0 = c\; dx_2 \circ R_g \wedge dx_3 \circ R_g$$
$$= c\; d(g_{21}x_1 + g_{22}x_2 + g_{23}x_3) \wedge d(g_{31}x_1 + g_{32}x_2 + g_{33}x_3)$$
$$= c\,(g_{21}\, g_{32} - g_{22}\, g_{31})\, dx_1 \wedge dx_2 + c\,(g_{22}\, g_{33} - g_{23}\, g_{32})\, dx_2 \wedge dx_3 + c\,(g_{21}\, g_{33} - g_{23}\, g_{31})\, dx_1 \wedge dx_3$$
$$= c\,(x_3\, dx_1 \wedge dx_2 - x_1\, dx_2 \wedge dx_3 + x_2\, dx_1 \wedge dx_3),$$
since $R_g^*\sigma_0$ & σ are $SO(n)$ invariant.