This is the place we have to adjust.
By the Kodaira, $H^1(S, \Omega^1(2p)) = 0 \Rightarrow$
$0 \to \Omega^1(2p) \to \Omega^1(3p) \to \mathbb{C}_p \to 0$.
$\Rightarrow \exists$ a meromorphic form on S, holomorphic on $S - \{p\}$, and having a triple pole at p; this can not have any residues $\Rightarrow$ Let z_1, z_2 be local coordinates at p & q. $\Rightarrow$ For any a_1, a_2, $\exists$ a meromorphic 1-form η_a on S, holomorphic on $S - \{p, q\}$ and having principal part

$$\eta_a(z) = (a_1 z_1^{-3} + [0])\, dz_1 + (a_2 z_2^{-2} + [0])\, dz_2$$

Then by Kodaira vanishing theorem, for $p \in S$, $q \in S$

$$H^1(S, \Omega^1(2p)) = 0 = H^1(S, \Omega^1(q)) = H^1(S, \Omega^1(p)) = 0$$

and so from the exact sequences $0 \to \Omega^1(p) \to \Omega^1(2p) \to \mathbb{C}_p \to 0$

$$0 \to \Omega^1(2p) \to \Omega^1(3p) \to \mathbb{C}_p \to 0$$
$$0 \to \Omega^1(q) \to \Omega^1(2q) \to \mathbb{C}_q \to 0.$$

we see that $\exists$ a meromorphic on S, holomorphic on $S - \{p\}$ and having a triple pole (and a double pole) at p, & $\exists$ a meromorphic form on S, holomorphic on $S - \{q\}$ and having a double pole at q, clearly these forms can not have any residues. since σ/s_0 is the form of those forms, where $(s_0 = 0) = 3p$ or $2q$. $\sigma(p) \neq 0$ or $\sigma(q) \neq 0$.
It follows that if we let z_1 be a local coordinate around the point p and let z_2 be a local coordinate around q, for any sequence a_1, a_2, a_3 of complex numbers, $\exists$ a meromorphic 1-form η_a on S, holomorphic on $S - \{p, q\}$. and having principal parts

$$\eta_a(z) = (a_1 z_1^{-3} + [0])\, dz_1$$
$$= (a_3 z_2^{-2} + [0])\, dz_2$$
$$= (a_2 z_1^{-2} + [0])\, dz_1$$