

is obtained from the given differential  $d$  by passing to the quotient. The cohomology of  $\{E_\bullet, d_\bullet\}$  is

$$\begin{aligned} E_1^{p,q} &= \frac{\ker d_\bullet}{\operatorname{im} d_\bullet} = \frac{\{a \in F^p K^{p+q} : da \in F^{p+1} K^{p+q+1}\}}{d(F^p K^{p+q-1}) + F^{p+1} K^{p+q}} \\ &= H^{p+q} \left( \frac{F^p K^*}{F^{p+1} K^*} \right) \\ &= H^{p+q} (Gr^p K^*) \end{aligned}$$

as specified.

$\square$

$$\begin{array}{ccccc} E_0^{p,q+1} & \xrightarrow{d_\bullet} & E_0^{p,q} & \xrightarrow{d_\bullet} & E_0^{p,q+1} \\ \parallel & & \parallel & & \parallel \\ \frac{F^p K^{p+q-1}}{F^{p+1} K^{p+q-1}} & \xrightarrow{d_\bullet} & \frac{F^p K^{p+q}}{F^{p+1} K^{p+q}} & \xrightarrow{d_\bullet} & \frac{F^p K^{p+q+1}}{F^{p+1} K^{p+q+1}} \end{array}$$

$$E_1^{p,q} = \frac{\ker d_\bullet}{\operatorname{im} d_\bullet} = H^{p+q} \left( \frac{F^p K^*}{F^{p+1} K^*} \right) \text{ as a definition}$$

Don't misunderstand this as

$$H^{p+q}(F^p K^*), \text{ since } \{a \in F^p K^{p+q} : da \in F^{p+1} K^{p+q+1}\}$$

$\neq \ker d + F^{p+1} K^{p+q}$  in general.

$$\text{Since } \frac{F^p K^r}{F^{p+1} K^r} = Gr^p K^r, \quad H^{p+q}(Gr^p K^*) = H^{p+q} \left( \frac{F^p K^*}{F^{p+1} K^*} \right). \quad \square$$