

Then for a generic point $p = [\alpha_0, \alpha_1, \dots, \alpha_k]$ in \mathbb{P}^k , the inverse image of p in $\pi'(V) \subset \mathbb{P}^{k+1}$ would consist of at most of the d' points $[\alpha_0, \dots, \alpha_k, \beta]$,

where

$$\beta^{d'} + \psi_1\left(\frac{\alpha_1}{\alpha_0}, \dots, \frac{\alpha_k}{\alpha_0}\right) \beta^{d'-1} + \dots + \psi_{d'}\left(\frac{\alpha_1}{\alpha_0}, \dots, \frac{\alpha_k}{\alpha_0}\right) = 0.$$

⌈ We don't need terms of x_{k+2}, \dots, x_n , since x_1, x_2, \dots, x_n are independent variables. $x_{k+1} \in K(V)$.

For a generic point $p = [\alpha_0, \alpha_1, \dots, \alpha_k]$ where $\alpha_0 \neq 0$, since

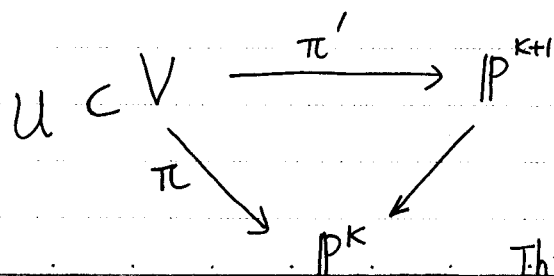
$$x_{k+1}^{d'} + \psi_1(x_1, \dots, x_k) \cdot x_{k+1}^{d'-1} + \dots + \psi_{d'}(x_1, \dots, x_k) \equiv 0, \\ d' < d,$$

$$\left(\frac{\beta}{\alpha_0}\right)^{d'} + \psi_1\left(\frac{\alpha_1}{\alpha_0}, \dots, \frac{\alpha_k}{\alpha_0}\right) \left(\frac{\beta}{\alpha_0}\right)^{d'-1} + \dots + \psi_{d'}\left(\frac{\alpha_1}{\alpha_0}, \dots, \frac{\alpha_k}{\alpha_0}\right) = 0 \quad (*)$$

$\Rightarrow \exists$ at most of d' points $[\alpha_0, \dots, \alpha_k, \beta]$ satisfying the equation $(*)$. \Downarrow

But since the projection $\pi': V \rightarrow \mathbb{P}^{k+1}$ is generically one to one onto its image and the fibers of π generically consist of $d > d'$ points, this is impossible. Q.E.D.

⌈



Choose a generic point $p = [\alpha_0, \alpha_1, \dots, \alpha_k]$ so that $\pi^{-1}(p) \subset U$, where U is the open set above.

This is possible.

$$[\alpha_0, \alpha_1, \dots, \alpha_k] = p \in \mathbb{P}^k$$