Thus given $\sigma \in H^{\oplus(n+1)}$ with $\mathcal{E}(\sigma)=0$, at each pt $[X_0,\cdots X_n]$, some number is obtained uniquely.

$\Rightarrow$

$$
\begin{tikzcd}
\mathbb{P}^n \times \mathbb{C} \arrow[rr, "\cong"] \arrow[dr] & & \ker \mathcal{E} \arrow[dl] \\
& \mathbb{P}^n &
\end{tikzcd}
$$

$$(X, \lambda) \longmapsto \sigma, \quad \sigma(X) = (\lambda X_0, \cdots \lambda X_n)$$

」

Thus we have an exact sequence of bundles on $\mathbb{P}^n$:

$$0 \longrightarrow \mathbb{C} \longrightarrow H^{\oplus(n+1)} \xrightarrow{\mathcal{E}} T'(\mathbb{P}^n) \longrightarrow 0,$$

called the Euler sequence. Now, from the C^∞ decomposition

$$H^{\oplus(n+1)} = T'(\mathbb{P}^n) \oplus \mathbb{C}$$ 「 See. Milnor P 35 Problem 3-B」

and the Whitney product formula, we find

$$c(T'(\mathbb{P}^n)) = c(H^{\oplus(n+1)}) = c(H)^{n+1} = (1+\omega)^{n+1},$$

where $\omega = \eta_H \in H^2(\mathbb{P}^n, \mathbb{Z})$ is the class of a hyperplane.

The Gauss - Bonnet Formulas

As we have seen, the first Chern class of a holomorphic line bundle is Poincare dual to the cycle represented by the zero-locus of a global holomorphic section.

「 See P141. Proposition 2 」

A similar geometric description of the general Chern class-