$\{g_{UV}(x)\}$ can be a transition functions. and for $\mathbb{C}^k/\mathbb{C}^l$.

Given a C^∞ map $f: M \longrightarrow N$ of a differentiable manifolds M and N and a complex vector bundle $E \longrightarrow N$, we can define the pullback f^*E by setting

$$(f^*E)_x = E_{f(x)}.$$

If $\varphi: E_U \longrightarrow U \times \mathbb{C}^n$ is a trivialization of E in a nbd of $f(x)$, then the map

$$f^*\varphi : (f^*E)_{f^{-1}(U)} \longrightarrow f^{-1}(U) \times \mathbb{C}^n \text{ gives } f^*E$$

its manifold structure over the open set $f^{-1}(U)$.

$$\begin{tikzcd}
(f^*E)_{f^{-1}(U)} = (f^*E_U) \arrow[r, "f^*\varphi \,\to\, f^{-1}(U)\times\mathbb{C}^n"] \arrow[d] & E_U \arrow[r, "\varphi"] \arrow[d] & U \times \mathbb{C}^n \\
f^{-1}(U) \arrow[r, "f"] & U &
\end{tikzcd}$$

Transition functions for the pullback f^*E will, of course, be the pullback of the transition functions for E

$$\begin{tikzcd}
V\cap U \times \mathbb{C}^n & E_{U\cap V} \arrow[l, "\varphi_V^{-1}"'] \arrow[r, "\varphi_U"] & U\cap V \times \mathbb{C}^n \\
(f(x), v) \arrow[rr, mapsto] & & (f(x), g_{\alpha\beta}(f(x))(v)) \\
f^{-1}(V\cap U) \times \mathbb{C}^n \arrow[u] & f^*(E_{U\cap V}) \arrow[l] \arrow[r] \arrow[uu] & f^{-1}(U\cap V) \times \mathbb{C}^n \arrow[u] \\
(x, v) \arrow[rr, mapsto] & & (x, h_{\alpha\beta}(x)(v))
\end{tikzcd}$$

$$\Rightarrow \quad h_{\alpha\beta}(x) = g_{\alpha\beta}(f(x)) = (f^* g_{\alpha\beta})(x)$$

A map between vector bundles E and F on M is given by a C^∞ map $f: E \longrightarrow F$ such that $f(E_x) \subset F_x$. and