

....  $P_{n^2}$ .

$$\begin{aligned} \mathbb{F} \quad \dim H^0(\mathbb{P}^2, \mathcal{O}((n-3)H)) &= \binom{n-3+2}{n-3} = \binom{n-1}{n-3} = \frac{(n-1)!}{(n-3)! \cdot 2} \\ &= \frac{(n-1)(n-2)}{2} = \frac{n^2-3n+2}{2} = \frac{n(n-3)}{2} + 1 \end{aligned}$$

$$\dim H^0(\mathbb{P}^2, \mathcal{O}((n-3)H)) = \frac{n(n-3)}{2} + 1 = N' + 1$$

$\Rightarrow H^0(\mathbb{P}^2, \mathcal{O}((n-3)H)) = \langle \sigma_1, \sigma_2, \dots, \sigma_{N'+1} \rangle$  where  $\{\sigma_1, \dots, \sigma_{N'+1}\}$  is a set of linearly independent sections.

Let  $a_1\sigma_1 + a_2\sigma_2 + \dots + a_{N'+1}\sigma_{N'+1} \in H^0(\mathbb{P}^2, \mathcal{O}((n-3)H))$ .

$$(*) \quad \begin{cases} a_1\sigma_1(P_{N+1}) + a_2\sigma_2(P_{N+1}) + \dots + a_{N'+1}\sigma_{N'+1}(P_{N+1}) = 0 \\ \vdots \\ a_1\sigma_1(P_{n^2}) + a_2\sigma_2(P_{n^2}) + \dots + a_{N'+1}\sigma_{N'+1}(P_{n^2}) = 0 \end{cases}$$

$$\begin{aligned} \# \text{ of } a_i\text{'s} &= N' + 1 > \# \text{ of equations} = n^2 - N = \frac{n(n-3)}{2} \\ &= \frac{n(n-3)}{2} + 1 \end{aligned}$$

$\Rightarrow \exists$  nontrivial solutions for  $a_i$ 's i.e., not all  $a_i$ 's zero.  $\Rightarrow$  We can find a section  $\sigma$  s.t.  $\sigma(P_i) = 0$   $\forall i = N+1, \dots, n^2$ .  $\Rightarrow \{\sigma = 0\}$  is the curve passing through  $P_i$ 's.  $\forall i = N+1, \dots, n^2$ .  $\hookrightarrow$

Then  $A+B$  is a curve of degree  $2n-3$  passing through  $P_2, \dots, P_{n^2}$ , and consequently  $A+B$  contains  $P_1$ .

$\mathbb{F} \quad \deg \sigma = n-3, \quad \deg B = n-3, \quad \Rightarrow \deg(B+A) = n-3 + n$ .  
Since  $A \cap i(\mathbb{P}^2) = \{P_2, \dots, P_{n^2}\}$  and by the remark above, the hyperplane section of  $i_n(\mathbb{P}^2)$  is a curve of degree  $n$ .  $\Rightarrow$