

by Prop. P244, 8.2 Proposition), $B-B'$ is similar to

$$\begin{pmatrix} 0 & 1 & & 0 & & 0 \\ -1 & 0 & & & & \\ \hline & & 0 & 1 & & \\ 0 & & -1 & 0 & & \\ \hline & & & & 0 & 1 \\ & & & & -1 & 0 \\ & & & & & \ddots \\ & & & & & & 0 & 1 \\ & & & & & & -1 & 0 \end{pmatrix} \Rightarrow \text{rank}(B-B') \text{ is even.}$$

To prove the note, which implies P_0 is in one of the two families by Proposition 2 on P735, we will show the following lemma.

Lemma: Let Λ_B be the vector space generated by row vectors of $n \times 2n$ matrix (I, B) , B $n \times n$ matrix. Similarly, $\Lambda_{B'}$ is the v.s generated by row vectors of $n \times 2n$ matrix (I, B') B' $n \times n$. Then $\dim(\Lambda_B \cap \Lambda_{B'}) = n - \text{rank}(B-B')$.

proof). Let $e_i = (\overline{0 \cdots 0} \mid \overline{0 \cdots 0})$, $(b_{i1}, \dots, b_{in}) = B_i$ and $B'_i = (b'_{i1}, \dots, b'_{in})$ be row vectors of I, B, B' respectively. Let $\tilde{e}_i = (\overline{0 \cdots 1 \cdots 0} \mid \overline{0 \cdots 0})$, $\tilde{B}_i = (0 \cdots 0, b_{i1})$, $\tilde{B}'_i = (0 \cdots 0, b'_{i1})$, and $\tilde{e}_i + \tilde{B}_i = v_i$, $\tilde{e}_i + \tilde{B}'_i = w_i$.

Then since v_i 's are linearly independent and w_i 's are linearly independent, for any vector $u \in \Lambda_B \cap \Lambda_{B'}$,

$u = x_i v_i = x_i w_i$ (\because Each v_i can take care of only w_i).

$\Rightarrow \sum_{i=1}^n x_i (v_i - w_i) = 0 \Rightarrow \{(x_i) \mid \sum x_i (v_i - w_i) = 0\} = K$ has the dimension $n - \text{rank} \begin{pmatrix} v_1 - w_1 \\ \vdots \\ v_n - w_n \end{pmatrix}$, where