itself has no base points, it follows that $|L+kC|$ has no base points for $k \le m$.

$\ulcorner$ If $k < m$, $(m-k)H$ is effective. Given any point $p \in C$, choose $p = p_1, p_2 \cdots p_{m-k} \in C$. $\Rightarrow \exists\ \sigma \in H^0(C, (m-k)H)$ s.t $\sigma(p_i) = 0$. We can choose different points $q_1, \cdots q_{m-k}$, s.t $q_i \neq p$, and $\exists\ \sigma'(q_i) = 0$. $\Rightarrow$ Since $\sigma'(p) \neq 0$, p is not a base point.

If $m = k$, then $[(0H)] = C \times \mathbb{C}$. $\Rightarrow \exists\ \sigma : C \to C \times \mathbb{C}$ s.t $\sigma(p) = 1$. $\Rightarrow \exists\ \tau \in H^0(M, \mathcal{O}(L+kC))$ s.t $\tau|_C = 1$. $\Rightarrow \tau(p) \neq 0 \Rightarrow p$ is not a base point. Thus the linear system $|L+kC|$ has no base point on C.

Since L is very ample, by the definition of "very ample" on P192, by the remark on P176, $|L|$ has no base points on M. For any point $p \in M - C$, $\exists$ a section $\sigma \in H^0(M, \mathcal{O}(L))$ s.t $\sigma(p) \neq 0$. Since $0 \le k \le m$, $\exists$ a section $\tau \in H^0(M, \mathcal{O}(kC))$ s.t $(\tau = 0) = kC$. Consider a section $\sigma \otimes \tau \in H^0(M, \mathcal{O}(L+kC))$. $\Rightarrow (\sigma \otimes \tau)(p) = \sigma(p) \otimes \tau(p) \neq 0$ since $\tau(p) \neq 0$ and $\sigma(p) \neq 0$. $\Rightarrow |L+kC|$ has no base points on $M - C$. $\Rightarrow |L+kC|$ has no base points on M, $0 \le k \le m$. $\lrcorner$

Consider now the map $\iota_{L'}$ given by the linear system $|L'| = |L + mC|$. Since $|L'|$ contains the subseries $|L| + mC$ and L is very ample, $\iota_{L'}$ embeds $M - C$ and seperates points of C from points of $M - C$.