$$= -C_2 \int_{B(r)} \frac{dz_1}{z_1} \wedge \int_{B((\frac{r}{|z_1|^2}-1)^{\frac{1}{2}})} \phi_1 \circ \psi_1^{-1}(\omega) \left(\frac{\partial \varphi}{\partial \bar{z}_1} d\bar{z}_1 + \frac{\partial \varphi}{\partial \bar{\omega}} d\bar{\omega} \right) \wedge \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega}$$

simply

$$= -C_2 \int_{B(r)} \frac{dz_1}{z_1} \wedge \int_{B(\cdot\ )} \phi_1(\omega) \frac{\partial \varphi}{\partial \bar{z}_1} d\bar{z}_1 \wedge \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega}$$

$\Rightarrow$ To use the Cauchy integral formula, we have to have the following form

$$? \Rightarrow -C_2 \int_{B(r)} \frac{dz_1}{z_1} \wedge \bar{\partial} \int_{B((\frac{r}{|z_1|^2}-1)^{\frac{1}{2}})} \phi_1(\omega)\, \varphi(z_1, \omega) \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega}.$$

But the problem is in $B((\frac{r}{|z_1|^2}-1)^{\frac{1}{2}})$ which depends on z_1. So, in general we can not expect

$$\bar{\partial} \int_{B((\frac{r}{|z_1|^2}-1)^{\frac{1}{2}})} \phi_1(\omega)\, \varphi(z_1,\omega) \frac{1}{1+|\omega|^2)^2} d\omega \wedge d\bar{\omega}$$

$$= \int_{B((\frac{r}{|z_1|^2}-1)^{\frac{1}{2}})} \phi_1(\omega) \frac{\partial \varphi}{\partial \bar{z}_1} d\bar{z}_1 \wedge \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega}.$$

So we need to change the ball $B(r)$ into the polar disk. $\Delta = \{ (z_1, z_2) \in \mathbb{C}^2 \mid \quad |z_i| < r$

Wrong, here

$$\Rightarrow -C_2 \int_{|z_1|<r} \frac{dz_1}{z_1} \wedge \int_{\mathbb{C}} \phi_1 \circ \psi_1^{-1}(\omega) \frac{\partial \varphi}{\partial \bar{z}_1} d\bar{z}_1 \wedge \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega}$$

$$= -C_2 \int_{|z_1|<r} \frac{dz_1}{z_1} \wedge \bar{\partial} \int_{\mathbb{C}} \phi_1 \circ \psi_1^{-1}(\omega)\, \varphi(z_1, \omega) \frac{1}{(1+|\omega|^2)^2} d\omega \wedge d\bar{\omega} \quad (**)$$

$\Rightarrow$ By the Cauchy integral formula. $-\int_{|z|<r} \frac{\partial f}{\partial \bar{z}} d\bar{z} \wedge dz$