

ctive over $S-R$, with $\pi^{-1}(p) = X_p \cong P^1$ for $p \in R$.

$$\Gamma \quad \pi : \Sigma \longrightarrow S$$

For $\forall p \in S-R$, \exists a unique singular line through p . $\Rightarrow \exists$ a unique $x \in \Sigma$ s.t. $\pi(x) = p$. This implies that π is one to one and onto $S-R$. For $p \in R$, any line l_x of X through p is singular. $\Rightarrow \sigma(p) \cap X = X_p = \pi^{-1}(p) = \sigma(p, h) \cong P^1$.

\Rightarrow

Σ is easy to describe, once we have the following characterization.

Lemma. For $x \in X$,
 $x \in \Sigma \Leftrightarrow T_x(F)$ is tangent to G .

Proof. Say $T_x(F)$ is tangent to G at x' . Then $x \in T_{x'}(G)$, and so l_x meets $l_{x'}$ at a point $p \in P^3$.

$$\Gamma \quad T_x(F) \subset T_{x'}(G) \Rightarrow x \in T_{x'}(G) \Rightarrow x \in T_{x'}(G) \cap G = \sigma(l_{x'}) \Rightarrow l_x \cap l_{x'} \neq \emptyset. \text{ See p 157}$$

\Rightarrow

The plane $\sigma(p)$ is then contained in $T_{x'}(G) = T_x(F)$, i.e., is tangent to F at x ; thus $x \in \Sigma$.