$$\begin{tikzcd} Q_2 = E_2/F_2 \arrow[r, "\partial"] \arrow[d, phantom, "\cong"] & Q_1 = E_1/F_1 \arrow[d, phantom, "\cong"] \\ \mathcal{O}(e_r \otimes \mathbb{C}^{r-1}) \arrow[r, "\partial"] & \mathcal{O}\cdot e_r \end{tikzcd}$$

by P597 note

This made us use the expressions above.
We assumed $Q.$ is exact sequence and examined the lower right-hand corner, which is O.K. And $F.$ is a Koszul complex of $r-1$. From the commutative and exact diagram, we get a long exact sequence

$$\underset{0}{\underset{\parallel}{H_n(F.)}} \to H_n(E.) \to \underset{0}{\underset{\parallel}{H_n(Q.)}} \to H_{n-1}(F.) \to H_{n-1}(E.) \to H_{n-1}(Q.) \to .$$

$$\Rightarrow \quad H_n(E.) = 0 \quad n > 0$$

Q.E.D.

Intrinsic Form of Local Duality. We shall use the Koszul complex to compute $\mathrm{Ext}^*_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O})$, and interpret the result as an intrinsic form of the duality theorem from Section 2 above. In fact, we shall reprove the duality theorem in this new form.

The following plays an analogouous role to the $*$-operator in Hodge theory:

Lemma. There are isomorphisms $\mathrm{Hom}_{\mathcal{O}}(E_k, \mathcal{O}) \cong E_{r-k}$ such that the diagram

$$\begin{array}{ccc} \mathrm{Hom}_{\mathcal{O}}(E_k, \mathcal{O}) & \xrightarrow{\sim} & E_{r-k} \\ \downarrow \partial^* & & \downarrow \partial \\ \mathrm{Hom}_{\mathcal{O}}(E_{k+1}, \mathcal{O}) & \xrightarrow{\sim} & E_{r-k-1} \end{array}$$