with $\quad \Pi_2 = \dfrac{1}{2\sqrt{-1}} (\mathrm{Im}\, Z)^{-1}$,

$$\Pi_2 = \frac{\sqrt{-1}}{2} \Delta_\delta^{-1} \bar{Z} (\mathrm{Im}\, Z)^{-1}.$$

$\sqcap$

$$I = \begin{pmatrix} \Pi_1 & \bar{\Pi}_1 \\ \Pi_2 & \bar{\Pi}_2 \end{pmatrix} \begin{pmatrix} \Delta_\delta & Z \\ \Delta_\delta & \bar{Z} \end{pmatrix} = \begin{pmatrix} \Pi_1 \Delta_\delta + \bar{\Pi}_1 \Delta_\delta & \Pi_1 Z + \bar{\Pi}_1 \bar{Z} \\ \Pi_2 \Delta_\delta + \bar{\Pi}_2 \Delta_\delta & \Pi_2 Z + \bar{\Pi}_2 \bar{Z} \end{pmatrix}$$

$$\Rightarrow \quad \Pi_1 \Delta_\delta + \bar{\Pi}_1 \Delta_\delta = I \qquad \Pi_2 \Delta_\delta + \bar{\Pi}_2 \Delta_\delta = 0$$
$$\Pi_1 Z + \bar{\Pi}_1 \bar{Z} = 0 \qquad \Pi_2 Z + \bar{\Pi}_2 \bar{Z} = I$$

$\Rightarrow$ From $(\Pi_2 + \bar{\Pi}_2)\Delta_\delta = 0$, since $\delta_\alpha \neq 0$ for all α,
$\Pi_2 + \bar{\Pi}_2 = 0. \quad \Rightarrow \quad \Pi_2 Z + \bar{\Pi}_2 \bar{Z} = I = \Pi_2 (Z - \bar{Z}) = I = \Pi_2 (2\, \mathrm{Im}\, Z)$
$\sqrt{-1} = I \quad \Rightarrow \quad \Pi_2 = \dfrac{1}{2\sqrt{-1}} (\mathrm{Im}\, Z)^{-1}.$

$(\Pi_1 + \bar{\Pi}_1)\Delta_\delta = I \quad \Rightarrow \quad \Pi_1 + \bar{\Pi}_1 = \Delta_\delta^{-1}$
$\Rightarrow \quad \Pi_1 Z + (\Delta_\delta^{-1} - \Pi_1)\bar{Z} = 0 \quad \Rightarrow \quad \Pi_1 (Z - \bar{Z}) = -\Delta_\delta^{-1} \bar{Z}$
$\Rightarrow \quad 2\sqrt{-1}\, \Pi_1\, \mathrm{Im}\, Z = -\Delta_\delta^{-1} \bar{Z} \quad \Rightarrow \quad \Pi_1 = -\dfrac{1}{2\sqrt{-1}} \Delta_\delta^{-1} \bar{Z} (\mathrm{Im}\, Z)^{-1}.$

$\lrcorner$

The cohomology class $[\omega]$ of a Hodge form ω on an Abelian variety $M = V/\Lambda$ is called a polarization of M. The integers δ_α appearing in the expression
$$\omega = \sum \delta_\alpha\, dx_\alpha \wedge dx_{n+\alpha}, \qquad \delta_\alpha | \delta_{\alpha+1}$$