All these assertions are implicit in the following cohomological interpretation of the correspondence [ ]. The exact sheaf sequence

$$0 \longrightarrow \mathcal{O}^* \xrightarrow{i} \mathcal{M}^* \xrightarrow{j} \mathcal{M}^*/\mathcal{O}^* \longrightarrow 0 \qquad \text{on } M$$

gives us, in part, the exact sequence

$$H^0(M, \mathcal{M}^*) \longrightarrow H^0(M, \mathcal{M}^*/\mathcal{O}^*) \xrightarrow{\delta} H^1(M, \mathcal{O}^*) \longrightarrow H^1(M, \mathcal{M}^*)$$

of cohomology groups. The reader may easily verify that under the natural identifications

$$\mathrm{Div}(M) = H^0(M, \mathcal{M}^*/\mathcal{O}^*) \quad \text{and} \quad \mathrm{Pic}(M) = H^1(M, \mathcal{O}^*)$$

for any meromorphic function f on M, $\quad j_* f = (f)$,

and for any divisor D on M, $\quad \delta D = [D]$.

Indeed, we will generally violate the previous multiplicative notation and write $L + L'$ for the tensor product of two line bundles or mL for the mth tensor power $L^{\otimes m}$ of L.

① $f \in H^0(M, \mathcal{M}^*) \Rightarrow$ What is $j_* f \in H^0(M, \mathcal{M}^*/\mathcal{O}^*)$?

$\exists$ an open cover $\{U_\alpha\} = \underline{U}$, s.t

$$H^0(\underline{U}, \mathcal{M}^*) \xrightarrow{j_*} H^0(\underline{U}, \mathcal{M}^*/\mathcal{O}^*) \xrightarrow{\delta} H^1(\underline{U}, \mathcal{O}^*)$$

and $\delta \circ j_*(f) = 0$.

Let $f|_{U_\alpha} = f_\alpha$ for each α.

$$C^0(\underline{U}, \mathcal{M}^*) \xrightarrow{j} C^0(\underline{U}, \mathcal{M}^*/\mathcal{O}^*)$$

$$(f_\alpha) \longmapsto (j \circ f_\alpha)$$

$$f_\alpha \in \mathcal{M}^*(U_\alpha) \qquad j \circ f_\alpha \in \mathcal{M}^*/\mathcal{O}^*(U_\alpha)$$

$\Rightarrow$ If we go to refinement of $\underline{U}$, $\quad j \circ f_\alpha \in \dfrac{\mathcal{M}^*(U_\alpha)}{\mathcal{O}^*(U_\alpha)}$