

\mathbb{R}

Note: $\sqrt{-1} dz \wedge d\bar{z} = (\sqrt{-1}) \frac{dz}{z} \wedge (\bar{z} dz + z d\bar{z}) \frac{1}{2}$
 $= (-d\theta) \wedge dr = dr \wedge d\theta$
 $= (-d \arg z) \wedge (\bar{z} dz + z d\bar{z})$
 $\Rightarrow d \arg z \wedge dr = -\sqrt{-1} dz \wedge d\bar{z} //$

$P_{I \cup \{j\}} = \{z : |f_i(z)| = \epsilon, i \in I, |f_j(z)| = \epsilon, l \notin I \cup \{j\}, |f_l(z)| \leq \epsilon\}$
 \Rightarrow Its orientation is given by.

$$d \arg f_{i_1} \wedge \dots \wedge d \arg f_j \wedge \dots \wedge d \arg f_{i_p} \wedge \left(\bigwedge_{l \in I \cup \{j\}} \frac{\sqrt{-1}}{2} df_l \wedge d\bar{f}_l \right) \geq 0$$

$$\Rightarrow d \arg f_{i_1} \wedge \dots \wedge d \arg f_j \wedge \dots \wedge d \arg f_{i_p} \wedge \left(\right)$$

$$= (-1)^{(j, I \cup \{j\})-1} d \arg f_{i_1} \wedge \dots \wedge d \arg f_{i_p} \wedge d \arg f_j \wedge \left(\right)$$

\Rightarrow By the note above,

$$\epsilon (-1)^{(j, I \cup \{j\})-1} d \arg f_{i_1} \wedge \dots \wedge d \arg f_{i_p} \wedge d \arg f_j \wedge \left(\right)$$

\uparrow dr = the orientation of P_I

$$= d \arg f_{i_1} \wedge \dots \wedge d \arg f_{i_p} \wedge \left(\bigwedge_{l \in I} \frac{\sqrt{-1}}{2} df_l \wedge d\bar{f}_l \right)$$

normal derivative

since $\bigvee \epsilon P_{I \cup \{j\}}$ has to have the same orientation as P_I .

Thus $dr \wedge \epsilon (-1)^{(j, I \cup \{j\})-1} d \arg f_{i_1} \wedge \dots \wedge d \arg f_{i_p} \wedge d \arg f_j \wedge \left(\right)$

$$= \epsilon (-1)^{(j, I \cup \{j\})-1} d \arg f_{i_1} \wedge \dots \wedge d \arg f_{i_p} \wedge \left(\right) \wedge d \arg f_j \wedge d|f_j|$$

$$= \epsilon (-1)^{(j, I \cup \{j\})-1} d \arg f_{i_1} \wedge \dots \wedge d \arg f_{i_p} \wedge \left(\right) \wedge \left(d \arg f_j \wedge \left(\frac{1}{2} (f_j df_j + \bar{f}_j d\bar{f}_j) \right) \right)$$

$$= \epsilon (-1)^{(j, I \cup \{j\})-1} d \arg f_{i_1} \wedge \dots \wedge d \arg f_{i_p} \wedge \left(\right) \wedge (-\sqrt{-1}) \frac{df_j}{f_j} \wedge \left(\frac{1}{2} (f_j df_j + \bar{f}_j d\bar{f}_j) \right)$$

$$= \epsilon (-1)^{(j, I \cup \{j\})} d \arg f_{i_1} \wedge \dots \wedge d \arg f_{i_p} \wedge \left(\right) \wedge \left(\frac{\sqrt{-1}}{2} df_j \wedge d\bar{f}_j \right) =$$