$$\Rightarrow \quad \pi^*\beta = \pi'^*\beta = C_n \, \pi'^*\left((\partial \log \|z\|^2) \wedge (\partial\bar{\partial} \log \|z\|^2)^{n-1}\right)$$

$$= C_n \, \pi'^*(\partial \log \|z\|^2) \wedge \left(\pi'^*(\partial\bar{\partial} \log \|z\|^2)\right)^{n-1}$$

$$= C_n \, \partial \log \|z\|^2 \wedge \left(\pi^*(\text{Kähler form of the Fubini-Study metric})\right)^{n-1}$$

$$= C_n \, \partial \log \|z\|^2 \wedge \pi^*\left((\text{Kächer form of the Fubini-Study metric})^{n-1}\right)$$

$$= C_n \, \partial \log \|z\|^2 \wedge \pi^*(\text{volume form on } \mathbb{P}^{n-1})$$

where π' is the identity. $\widetilde{\mathbb{C}}^n - \pi'(0)$.

$$\partial \log \|z\|^2 = \frac{1}{\|z\|^2} \sum \bar{z}_i \, dz_i .$$

$$\lambda\,(z_{01}, z_{02}, \cdots z_{0n}) = (z_1, \cdots z_n). \quad \Rightarrow \quad dz_i = z_{0i}\, d\lambda .$$

$$\|z\|^2 = |\lambda|^2 \left(|z_{01}|^2 + \cdots + |z_{0n}|^2\right).$$

$$\Rightarrow \quad \sum \bar{z}_i \, dz_i = \sum \bar{\lambda}\, \bar{z}_{0i} \, z_{0i} \, d\lambda = \bar{\lambda}\, d\lambda \sum |z_{0i}|^2 .$$

$$\Rightarrow \quad \partial \log \|z\|^2 = \frac{\bar{\lambda}\, d\lambda}{|\lambda|^2} \quad \text{on } \{\lambda\,(z_{01}, \cdots z_{0n})\}$$

$$= \frac{d\lambda}{\lambda} .$$

$\lrcorner$

Summarizing, $\pi^*\beta$ on $\widetilde{\mathbb{C}}^n$ is the pull-back of the standard volume form on $\mathbb{P}^{n-1}$ times a form θ that reduces to the Cauchy kernel in each fiber of $\widetilde{\mathbb{C}}^n \to \mathbb{P}^{n-1}$. Using this interpretation, the n-variable Bochner – Martinelli formulas may, by pulling forms back to $\widetilde{\mathbb{C}}^n$ and making an obvious iteration of the integrals, be reduced to the one-variable Cauchy formula.

~~$\ulcorner$ I think: For $n = 2$, $\varphi \in C^\infty(\mathbb{C}^2)$~~

~~Suppose $\pi' : \widetilde{\mathbb{C}}^2 \to \mathbb{C}^2$ is the natural projection.~~

~~$\pi'^*\varphi(z_1, z_2)$ on $\widetilde{\mathbb{C}}^2$. $\quad \beta = C_2 \,(\partial \log \|z\|^2) \wedge (\partial\bar{\partial} \log \|z\|^2)$.~~

~~$\pi'^*\varphi(0, z_2) = -\int_{\mathbb{C}} \bar{\partial}_{z_1} \pi'^*\varphi(z_1, z_2) \wedge \pi'^*\beta$ up to a constant.~~