is projectively isomorphic to the Veronese surface.

Comment: To show that the Veronese surface is the only nondegenerate surface in $\mathbb{P}^5$ whose variety of chords $C(S) = \bigcup_{p,q \in S} \overline{pq}$ is a subvariety of $\mathbb{P}^5$,

we need to show that, given any surface $V \subset \mathbb{P}^5$ whose degree is ≥ 5, $C(V) = \mathbb{P}^5$. and if $\deg V = 4$, V is a rational normal scroll with $C(V) = \mathbb{P}^5$. (see P525. Proposition.) ⌟

⌜ For references, see Hartshorne P316 Ex. 3.11 (a) & (b). and see P262. Yang. Complex Algebraic Geometry. ⌟

⌜ One more comment: $f : \mathbb{P}^2 \longrightarrow \mathbb{P}^5$ is an embedding s.t $f(\mathbb{P}^2) = S$ is nondegenerate.

$\Rightarrow$ Since $S \not\subset D = H$, f^*H is well-defined, see P132. By P134, $f^*([H]) = [f^*H]$.

$\Rightarrow$ $f^*([H]) = n[H]$ for some n, since $\mathbb{P}^2$ has a bundle of the unique type nH.

$\Rightarrow$ $\deg S = (c_1(f^*H))^2 = k^2$, for $k \in \mathbb{Z}$. ⌟

We may state the original question of this section as: Given $L \longrightarrow M$ a holomorphic line bundle, when is $i_L : M \longrightarrow \mathbb{P}^N$ an embedding?