less than n. By Lefschetz decomposition theorem,

$$H^n(M) = \bigoplus_{\substack{k=0 \\ 2k\le n}} L^k P^{n-2k}(M) = \underset{\substack{\uparrow \\ \text{primitive cohomology of } H^n(M)}}{P^n(M)} \oplus L P^{n-2}(M) \oplus L^2 P^{n-4}(M) \oplus \cdots$$

$$\begin{tikzcd} H^{n-2k}(M) \arrow[r, "L^k"] \arrow[d, leftrightarrow, "P.D"'] & H^n(M) \arrow[d, leftrightarrow, "P.D"] \\ H_{n+2k}(M) \arrow[r, "\cap \mathbb{P}^{N-k}"] & H_n(M) \end{tikzcd}$$

⇒ Thus any nonprimitive cycle in dimension n can be obtained by intersecting a cycle in dimension greater than n with hyperplanes. 」

Thus, the Lefschetz theorems together assert that the only "new" rational homology in varieties in each dimension is the primitive homology of the middle dimension.

「 Simply I think in this way.

From the Lefschetz hyperplane theorem, any cycle in $H_q(M.Q)$ comes from $H_q(V,Q)$ for $q \le n-1$.
For $q=n$, by the hard Lefschetz theorem, except the primitive homology, any nonprimitive cycle can be obtained by intersecting a cycle in dimension greater than n with hyperplanes.

$$\begin{tikzcd} H^{n-k}(M) \arrow[r, "L^k"] \arrow[d, leftrightarrow, "P.D"'] & H^{n+k}(M) \arrow[d, leftrightarrow] \\ H_{n+k}(M) \arrow[r, "\cap \mathbb{P}^{N-k}"] & H_{n-k}(M) \end{tikzcd}$$

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