$$\Rightarrow (\Lambda^I_{I'})^{-1} \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} v_1' \\ \vdots \\ v_n' \end{pmatrix} = \Lambda^{I'}$$

where $\Lambda^I_{I'}$ is the I' th $k \times k$ minor of Λ^I.

$$\Rightarrow (\Lambda^I_{I'})^{-1} \Lambda^I = \Lambda^{I'} \Rightarrow \Lambda^I = \Lambda^I_{I'} \Lambda^{I'}$$

$$(x_1 \cdots x_k) \begin{pmatrix} v_{11}, & \cdots & v_{1n} \\ & & \\ v_{k1}, & \cdots & v_{kn} \end{pmatrix} = (x_1 v_{11} + x_2 v_{12} + \cdots + x_k v_{k1},$$

$$x_1 v_{12} + \cdots + x_k v_{k2}, \quad \cdots, x_1 v_{1n} + x_2 v_{2n} + \cdots + x_k v_{kn})$$

$$\Rightarrow {}^t({}^tX \Lambda^I) = {}^t\Lambda^I X = v = {}^t\Lambda^{I'} Y$$
$$= {}^t\Lambda^I \; {}^t(\Lambda^I_{I'})^{-1} Y.$$

$$\Rightarrow {}^t\Lambda^I (X - {}^t(\Lambda^I_{I'})^{-1} Y) = 0.$$

Since $\operatorname{rank}({}^t\Lambda^I) = k$, $\quad X - {}^t(\Lambda^I_{I'})^{-1} Y = 0$

$$\Rightarrow X = {}^t(\Lambda^I_{I'})^{-1} Y \quad \Rightarrow \quad Y = {}^t\Lambda^I_{I'} X.$$

$\Rightarrow$ The transition function $g_{II'} (= {}^t\Lambda^I_{I'})$ is given by $({}^t\Lambda^I_{I'})^{-1}$.

$$\begin{array}{ccccc} U_{I'} \times \mathbb{C}^k & \longleftarrow & S|_{U_I \cap U_{I'}} & \longrightarrow & U_I \times \mathbb{C}^k \\ & & \rotatebox{90}{$\in$} & & \\ (\Lambda, \underset{\substack{\shortparallel \\ g_{I'I}(\Lambda) X)}}{Y}) & \longleftarrow\!\shortmid & (\Lambda, v) & \longmapsto & (\Lambda, X) \end{array}$$