quely. ⌋

$\tilde{\mathbb{C}}^n$ is the total space of the universal line bundle over projective space, and $\pi^*\Omega$ is smooth up on $\tilde{\mathbb{C}}^n$. Thus, on $\tilde{\mathbb{C}}^n$

$$\pi^*\beta = C_n\,\theta \wedge (\pi^*\Omega)^{n-1},$$

where $\theta = \partial \log \|z\|^2$ is a $(1,0)$-form that on each fiber $\{\lambda z\}_{\lambda\in\mathbb{C}}$ of $\tilde{\mathbb{C}}^n \longrightarrow \mathbb{P}^{n-1}$ reduces to $d\lambda/\lambda$.

⌜ $\pi : \tilde{\mathbb{C}}^n \longrightarrow \mathbb{P}^{n-1}$. For $l \in \mathbb{P}^{n-1}$, $\exists$ a line $\{z\}$, s.t $z \in l$.

$$\begin{array}{ccl} \tilde{\mathbb{C}}^n & \xrightarrow{\phi} & J = \{(l, z') \in \mathbb{P}^{n-1}\times\mathbb{C}^n \mid z' \in l\} \\ \Psi\ z & \longmapsto & (l, z) \quad z \in l. \end{array}$$

ϕ is isomorphic outside $\pi'^{-1}(0)$ and around $\pi'^{-1}(0)$. $\tilde{\mathbb{C}}^n$ is $\Delta \times \mathbb{P}^{n-1}$. $\Rightarrow$ ϕ is isomorphic.

$$\pi : \tilde{\mathbb{C}}^n \longrightarrow \mathbb{P}^{n-1} \quad \Rightarrow \quad \pi^* : H^2_{DR}(\mathbb{P}^{n-1}, \mathbb{C}) \longrightarrow H^2_{DR}(\tilde{\mathbb{C}}^n, \mathbb{C}).$$

$$\beta = C_n\,(\partial \log\|z\|^2) \wedge (\partial\bar\partial \log\|z\|^2)^{n-1}$$
$$= C_n\,\theta' \wedge \left(\frac{4\pi}{\sqrt{-1}}\Omega\right)^{n-1} = C_n'\,\theta' \wedge \Omega^{n-1}.$$

$$\Rightarrow \pi^*\beta = C_n'\,\pi^*(\theta' \wedge \Omega^{n-1}) = C_n'\,\pi^*\theta' \wedge \pi^*\Omega^{n-1}) = C_n'\,\pi^*\theta' \wedge (\pi^*\Omega)^{n-1}$$

β is $(n, n-1)$ type on $\mathbb{C}^n - \{0\}$. (on $\mathbb{C}^n$ with a singularity)

$$\tilde{\mathbb{C}}^n \setminus \pi'(0) \xrightarrow{\pi'} \mathbb{C}^n - \{0\} \xrightarrow{p} \mathbb{P}^{n-1} \quad \text{where } \pi' : \tilde{\mathbb{C}}^n \longrightarrow \mathbb{C}^n.$$

$\Rightarrow$ $\pi^*\beta$ means the form on $\tilde{\mathbb{C}}^n$, which is $\pi'^*\beta$.
To get $\pi^*\beta = C_n\,\theta \wedge (\pi^*\Omega)^{n-1}$, we had better check that the Kähler form of the Fubini-Study metric pulls back to Ω under the projection p.