special cases.

Before we go on to consider general intersections, we want to offer two general observations.

First, we will alter our formalism slightly, as follows: for any sequence $a = a_1, a_2 \cdots$ of nonnegative integers, we let $\sigma_a(V)$ denote the cycle

$$\sigma_a(V) = \{\Lambda : \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i\} \subset G(k,n)$$

so that the symbol σ_a can be used to refer to a Schubert cycle in any Grassmannian. Of course, σ_a will be null in $G(k,n)$ unless $a_i \leq n-k$ for all i, $a_i = 0$ for all $i > k$, and a is nonincreasing.

Now, the inclusion $\mathbb{C}^n \longrightarrow \mathbb{C}^{n+1}$ induces inclusions

$$i_1 : G(k,n) \longrightarrow G(k,n+1)$$

and $\quad i_2 : G(k,n) \longrightarrow G(k+1,n+1)$

obtained by sending $\Lambda \subset \mathbb{C}^n$ to $\Lambda \subset \mathbb{C}^{n+1}$ and $\Lambda \oplus \{e_{n+1}\} \subset \mathbb{C}^{n+1}$, respectively. Under these inclusions, it is not hard to see that for appropriate choices of flags V in $\mathbb{C}^n$ and V' in $\mathbb{C}^{n+1}$,

$$\sigma_a(V) = i_1^{-1}(\sigma_a(V')) = i_2^{-1}(\sigma_a(V')),$$

i.e., if we denote the Poincare dual of σ_a by $\tilde{\sigma}_a$, $i_1^* \tilde{\sigma}_a = i_2^* \tilde{\sigma}_a = \tilde{\sigma}_a$.

Pf

$$V = (0 \subset \overset{V_1}{\overset{\shortparallel}{\mathbb{C}^1}} \subset \overset{V_2}{\overset{\shortparallel}{\mathbb{C}^2}} \subset \cdots \subset \overset{V_{n-1}}{\overset{\shortparallel}{\mathbb{C}^{n-1}}} \subset \overset{V_n}{\overset{\shortparallel}{\mathbb{C}^n}})$$

$$V' = (0 \subset \underset{V_1'}{\underset{\shortparallel}{\mathbb{C}^1}} \subset \underset{V_2'}{\underset{\shortparallel}{\mathbb{C}^2}} \subset \cdots \subset \underset{V_n'}{\underset{\shortparallel}{\mathbb{C}^n}} \subset \mathbb{C}^{n+1})$$

$$\Rightarrow \sigma_a(V) = \{\Lambda \in G(k,n) \mid \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i\}$$