I think we had better read Plurisubharmonic functions and positive differential forms by P. Lelong.

P16.

1. Elementary Properties. (open and connected)

Let D be a domain in $\mathbb{C}^n$: we say that a function V of class (C^2) is plurisubharmonic in D, if at each point the hermitian form

$$\sum \frac{\partial^2 V}{\partial z_p \partial \bar{z}_q} dz_p d\bar{z}_q = L(V) \qquad (1)$$

is positive semi-definite.

$${}^t\bar{X}\left(\frac{\partial^2 V}{\partial z_p \partial \bar{z}_q}\right) X \geq 0 \qquad X \in \mathbb{C}^n.$$

There is an analogy between this condition and the condition

$$\sum_{p,q} \frac{\partial^2 V}{\partial x_p \partial x_q} dx_p dx_q \geq 0 \qquad (2)$$

which characterizes the convex (C^2) functions in $\mathbb{R}^m$. Nevertheless, condition (2) does not have an invariant interpretation on a variety if the x_p designate local coordinates, except in the case where the only changes of coordinates allowed are linear and the situation is then reduced to that where the domain of definition is a vector space. It is possible to give a generalization if the