So, we have only to choose as follows.
Consider the increasing sequence of subspaces

$$0 \subset \Lambda \cap V_1 \subset \Lambda \cap V_2 \subset \cdots \qquad \subset \Lambda \cap V_{n-1} \subset \Lambda \cap V_n = \Lambda$$

Let $c_1$ be the number s.t $\dim(\Lambda \cap V_{c_1-1}) = 0$
and $\dim(\Lambda \cap V_{c_1}) = 1$
Let $c_2$ be the number s.t $\dim(\Lambda \cap V_{c_2-1}) = 1$
and $\dim(\Lambda \cap V_{c_2}) = 2$.

$\vdots \qquad \vdots \qquad \vdots$

Let $c_k$ be the number s.t $\dim(\Lambda \cap V_{c_k-1}) = k-1$
and $\dim(\Lambda \cap V_{c_k}) = k$. (Refer to P25. Milnor. Stasheff).
Here $V_{c_i} = \{e_1, \cdots e_i\}$.
Then $\Lambda \in W_{a_1,\cdots a_k}$ since $\Lambda \cap V_{\{c_1\ c_k\}}^{\circ} = (0)$.

$\Rightarrow$ Since $\Lambda$ is represented by the following matrix

$$\begin{array}{c} \begin{array}{cccccccccccccc} & & & & c_1 & & c_2 & & c_3 & & & & & \end{array} \\ \left( \begin{array}{cccccccccccccc} * & * & \cdots & * & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & & \\ * & * & \cdots & * & 0 & * & * & 1 & 0 & & 0 & 0 & & \\ & & & * & 0 & * & * & 0 & * & * & 1 & 0 & \cdots & \\ & & & & & & & & & & & & & \end{array} \right) \end{array}$$

$\therefore$ We can choose
$v_1 \in \Lambda \cap V_{c_1}$ s.t
$\langle v_1, e_{c_1} \rangle = 1$
Choose $v_2 \in \Lambda \cap V_{c_2}$ s.t
, $\langle v_2, e_{c_2} \rangle = 1$ & $\langle v_2, e_{c_1} \rangle = 0$

if $\Lambda \ni v$, $v = \cdots x_1 e_{c_1} + x_2 e_{c_2} + \cdots + x_k e_{c_k} + \cdots$
where, at least, one of $\{x_1, \cdots x_k\}$ is nonzero.
$\Rightarrow$ If $x_1 \neq 0$, $v \notin V_{\{c_1,\cdots c_k\}}^{\circ}$.

Thus $\Lambda \cap V_{\{c_1 \cdots c_k\}}^{\circ} = (0)$. $\Lambda \in U_I$, where
$I = \{c_1, \cdots c_k\}$. and if we let, $a_i = n-k+i-c_i$,