coordinates properly, we may assume that V is a linear subspace of $\mathbb{C}^n$.

$\ulcorner$ $p \in V$, $\Rightarrow$ since V is nonsingular, $\exists$ a coordinate chart $(z_1, \cdots z_n)$ on U s.t. U open in Δ^n.

$$\begin{array}{ccc} U & \xrightarrow{\varphi} & \mathbb{C}^n \\ \cup & & \cup \\ U \cap V & \xrightarrow{\varphi|} & \mathbb{C}^k \\ \Downarrow & & \\ p & \longmapsto & (*, \cdots *, 0, \cdots 0). \end{array}$$

$\varphi(D \cap U) \subset \mathbb{C}^n$ is an analytic subvariety.

If we prove $\overline{\varphi(D \cap U)}$ is an analytic subvariety in $\mathbb{C}^n$, then we can conclude that $\overline{D \cap U}$ is an analytic subvariety of U. $\Rightarrow$ Repeat the argument to all points $p \in V$. $\Rightarrow$ Then we are done for the nonsingular case. $\Rightarrow$ We may assume that V is a linear subspace of $\mathbb{C}^n$. $\lrcorner$

"Minor Concern"

$\ulcorner$ Suppose there are two open sets $U_1 \cup U_2 \supset V$ satisfying the above condition, i.e.,

$$\begin{array}{ccc} U_i \cap V & \longrightarrow & \mathbb{C}^k \\ \downarrow & & \downarrow \\ U_i & \xrightarrow{\varphi_i} & \mathbb{C}^k \end{array} \qquad i = 1, 2$$

Assume. $\overline{D \cap U_i}$ in U_i is analytic in U_i.

Q: ① $\bigcup \overline{D \cap U_i} = \overline{D}$?, where $D \subset U_1 \cup U_2$.

② $\overline{D}$ is analytic in Δ^n?