In the same way, $\Phi_1 \wedge df_K \wedge d\bar{f}_K$ is the given orientation on M. $\Rightarrow$ $\Phi_1 = \Phi_K$. $\Rightarrow$ $c_1(\Lambda^k E) = c_1(E)$. $\lrcorner$

Finally, note that given generic sections $\sigma_1 \cdots \sigma_i$ we can define degeneracy cycles

$$D_i^{(j)} = \{ x : \dim \overline{\sigma_1(x), \cdots \sigma_i(x)} \leq i-j \}.$$

If, as before, we complete the sections $\sigma_1, \sigma_2, \cdots \sigma_i$ to a collection $\sigma_1 \cdots \sigma_n$ spanning each fiber, then the degeneracy cycle $D_i^{(j)}$ will be the inverse image, under the corresponding map $\iota : M \longrightarrow G(k,n)$, of the Schubert cycle

$$\underbrace{\sigma_{j \cdots j}}_{k-i+1} (V_{n-i-j+1}) = \{ \Lambda : \dim(\Lambda \cap V_{n-i-j+1}) \geq k-i+1 \}.$$

$\ulcorner$ $\overline{\sigma_1(x), \cdots \sigma_i(x)}$ = vector space spanned by $\sigma_1(x), \cdots \sigma_i(x)$.

Let $l = \dim \overline{\sigma_1(x), \cdots \sigma_i(x)}$. $\Rightarrow$ $l \leq i-j$. if $x \in D_i^{(j)}$.

For example, suppose $\sigma_1(x), \cdots \sigma_l(x)$ are linearly independent.

$$\begin{array}{lcl}
 & & \hphantom{0 + 0 + 0 + \cdots + {}} l \\
\sigma_1 &=& \sigma_1 + 0 + 0 \quad \cdots \quad 0 \quad 0 \quad \cdots \quad 0 \\
\sigma_2 &=& 0 + \sigma_2 + 0 \\
\vdots & & \\
\sigma_l &=& 0 + 0 + 0 + \cdots + \sigma_l + 0 + 0 \quad \cdots \quad 0 \\
\sigma_{l+1} &=& * \sigma_1 + * \sigma_2 + \cdots + * \sigma_l + 0 + 0 \cdots \quad 0 \\
\sigma_i &=& * \sigma_1 + * \sigma_2 + \cdots + * \sigma_l + 0 + 0 \cdots \quad 0 \\
\sigma_{i+1} &=& * \sigma_1 + \cdots + * + * \cdots \quad * \\
\vdots & & \\
\sigma_n &=& * \sigma_1 + \cdots + * \sigma_l + * + * \cdots \quad *
\end{array}$$