

we have lines tangent to  $V$ , and so  $\pi_3(\bar{I}) = C(V) \cup$

$\bar{I}$  is an algebraic subvariety of  $\mathbb{P}^n \times \mathbb{P}^n \times \mathbb{P}^n$ , and so  $C(V)$  is an analytic subvariety in  $\mathbb{P}^n$ .

$$\Gamma \quad I = \{ (p, q, r) : p \neq q \in V, \quad p \wedge q \wedge r = 0 \}$$

$\Rightarrow$  Let  $f : \mathbb{P}^n \times \mathbb{P}^n \times \mathbb{P}^n \longrightarrow \mathbb{C}^n$  be a map defined by  $f(p, q, r) = p \wedge q \wedge r$  locally.

More precisely, given  $(x_1, x_2, x_3) \in \mathbb{P}^n \times \mathbb{P}^n \times \mathbb{P}^n$ , since  $x_1 = [Z_{10}, Z_{11}, \dots, Z_{1n}]$ ,  $x_2 = [Z_{20}, Z_{21}, \dots, Z_{2n}]$ ,  $x_3 = [Z_{30}, Z_{31}, \dots, Z_{3n}]$ ,

define  $f : U_0 \times U_0 \times U_0 \longrightarrow \mathbb{C}^n$  by

$$f(x_1, x_2, x_3) = \left( \frac{Z_{11}}{Z_{10}}, \dots, \frac{Z_{1n}}{Z_{10}} \right) \wedge \left( \frac{Z_{21}}{Z_{20}}, \dots, \frac{Z_{2n}}{Z_{20}} \right) \wedge \left( \frac{Z_{31}}{Z_{30}}, \dots, \frac{Z_{3n}}{Z_{30}} \right)$$

where we assumed here  $Z_{10} \neq 0 \neq Z_{20} \neq 0 \neq Z_{30}$ .

For simplicity,  $n = 2$ .

$$f(x_1, x_2, x_3) = \left( \frac{Z_{11}}{Z_{10}} e_1 + \frac{Z_{12}}{Z_{10}} e_2 \right) \wedge \left( \frac{Z_{21}}{Z_{20}} e_1 + \frac{Z_{22}}{Z_{20}} e_2 \right) \wedge \left( \frac{Z_{31}}{Z_{30}} e_1 + \frac{Z_{32}}{Z_{30}} e_2 \right) =$$

$$\left( \frac{Z_{11}}{Z_{10}} \frac{Z_{22}}{Z_{20}} - \frac{Z_{12}}{Z_{10}} \frac{Z_{21}}{Z_{20}} \right) e_1 \wedge e_2 \wedge \left( \frac{Z_{31}}{Z_{30}} e_1 + \frac{Z_{32}}{Z_{30}} e_2 \right) = 0 \Rightarrow \text{not good example.}$$

$$\text{For } n = 3, \quad f(x_1, x_2, x_3) = \left( \frac{Z_{11}}{Z_{10}} e_1 + \frac{Z_{12}}{Z_{10}} e_2 + \frac{Z_{13}}{Z_{10}} e_3 \right) \wedge \left( \frac{Z_{21}}{Z_{20}} e_1 + \frac{Z_{22}}{Z_{20}} e_2 + \frac{Z_{23}}{Z_{20}} e_3 \right) \wedge \left( \frac{Z_{31}}{Z_{30}} e_1 + \frac{Z_{32}}{Z_{30}} e_2 + \frac{Z_{33}}{Z_{30}} e_3 \right) = \left( \frac{Z_{11}}{Z_{10}} \frac{Z_{22}}{Z_{20}} \frac{Z_{33}}{Z_{30}} - \frac{Z_{11}}{Z_{10}} \frac{Z_{23}}{Z_{20}} \frac{Z_{32}}{Z_{30}} + \dots \right) e_1 \wedge e_2 \wedge e_3$$