

a hexagon such that the opposite sides meet in three collinear points, then the vertices of H are on a conic.

Proof. Set $P_{ij} = L_i \cap L_j$ and let L be the line through P_{14} , P_{25} and P_{36} . Then if Q is a conic passing through the five vertices P_{12} , P_{23} , P_{34} , P_{45} , P_{56} of H , we may take $C = L_1 + L_3 + L_5$, $D = L_2 + L_4 + L_6$, and $E = Q + L$ to conclude that Q passes through P_{61} . Q.E.D.

\square L is the line passing through the three collinear points. According to Corollary 4.2, P397, Algebraic Geometry by Hartshorne, \exists a conic passing P_{12} , P_{23} , P_{34} , P_{45} , P_{56} . By the classical statement above, $E = Q + L$ must pass through P_{61} . Since L can not pass through P_{61} (\because If so, L contains L_6), Q must pass through P_{61} . \Rightarrow All P_{12}, \dots, P_{61} of H are on Q . "More on."

$L = L_6 \ni P_{14}, (P_{25}), P_{36}, (P_{26}) \Rightarrow L = L_2$ contradiction since $L = L_2 = L_6$. \square

Along similar lines but at a deeper level we shall prove a converse to the Cayley-Bacharach theorem.

Suppose that

$$P = P_1 + \dots + P_{n^2}$$

is a zero-cycle consisting of n^2 distinct points. We say that P satisfies the Cayley-Bacharach property if every curve E of degree $\geq n-3$ that passes through all but one point of P necessarily contains