$\operatorname{sgn}\det(J_{f_t}(p) - I) = \operatorname{sgn}\det B = \operatorname{sgn}\det A = \iota_v(p)$.

Different Way.

We know that $f(t,x) = x + tKx +$ terms involving t^2 or $t x_i x_j$.

$v(f(t,x)) = {}^tAx + t\,{}^tAKx +$ terms involving $x_i x_j$, $t x_i x_j$.

where $v(x) = {}^tAx +$ terms involving $x_i x_j$.

By $\frac{d}{dt} f(t,x) = v(f(t,x))$,

$Kx +$ terms involving t or $x_i x_j$

$= {}^tAx + t\,{}^tAKx +$ terms involving $x_i x_j$ or $t x_i x_j$

$\Rightarrow$ By comparing, $K = {}^tA$.

$\Rightarrow$ $f(t,x) = x + tKx +$ terms involving t^2 or $t x_i x_j$

$\Rightarrow$

$J_{f_t}(p) = tK + I +$ terms involving t^2.

$\Rightarrow \operatorname{sgn}\det(J_{f_t}(p) - I) = \operatorname{sgn}\det K = \operatorname{sgn}\det A$. for positive sufficiently small t. ⊔

Since f_t is homotopic to the identity, f_t^* acts as the identity on the cohomology of M, so that

$$\operatorname{trace} f_t^*|_{H^p_{DR}(M)} = \dim H^p_{DR}(M),$$

i.e., $L(f_t) = \chi(M)$;