$$= \psi' + \psi'' \wedge du_K - \frac{\partial \eta_I}{\partial u_K} du_K \wedge du_I - \frac{\partial \eta_I}{\partial u_j} du_j \wedge du_I$$

actually ±1. we can arrange

$$= \psi' + \psi'' \wedge du_K - \frac{\partial \eta_I}{\partial u_K} du_I \wedge du_K - \frac{\partial \eta_I}{\partial u_j} du_j \wedge du_I$$

$$= \psi' + \psi'' \wedge du_K - \left(\psi'' - \frac{\psi''(u')_{-1}}{u_K}\right) du_K - \frac{\partial \eta_I}{\partial u_j} du_j \wedge du_I$$

$$= \psi' - \frac{\partial \eta_I}{\partial u_j} du_j \wedge du_I + \frac{\psi''(u')_{-1}}{u_K} du_K .$$

$$= \left(\psi' - \frac{\partial \eta_I}{\partial u_j}\right) du_j \wedge du_I + \psi''(u')_{-1} \frac{du_K}{u_K}$$

$$\Rightarrow \quad \zeta' = \left(\psi' - \frac{\partial \eta_I}{\partial u_j}\right) du_j \wedge du_I \qquad \zeta'' = \psi''(u')_{-1}$$

$\Rightarrow \quad \zeta' \equiv 0\,(du') \qquad \zeta'' \equiv 0\,(u', du')$ since $\psi''(u')_{-1}$ is dependent of only u'. ⫽

Since $\tilde{\varphi}$ is closed, we deduce that $\zeta' \equiv 0\,(u', du')$ and

$$d\zeta' = 0 = d\zeta''.$$

$\ulcorner$

$$\tilde{\varphi} = \zeta' + \zeta'' \wedge \frac{du_K}{u_K}.$$

$$d\tilde{\varphi} = d\zeta' + d\zeta'' \wedge \frac{du_K}{u_K} = 0$$

Let $\zeta' = \zeta'_I du_I$.

$\Rightarrow \quad d\zeta' = d\zeta'_I \wedge du_I$, where $\zeta'_I(u_1, \cdots u_K)$. for each I.

$\zeta'' = \psi''(u')_{-1}$.

$\Rightarrow \quad d\zeta'_I \wedge du_I$ can not have a simple pole in u_K and ζ'' is a form in only $u_1, \cdots u_{K-1}$.

$\Rightarrow \quad d\zeta' = 0$ and $d\zeta'' \wedge \frac{du_K}{u_K} = 0$

$\Rightarrow \quad d\zeta' = 0$ and $d\zeta'' = 0$. $\Rightarrow$ ζ' can not be