We can thus write

$$\tilde{V}_c \sim 6\tilde{\omega} - 2e \in H^2(\tilde{W}, \mathbb{Z}),$$

where $\tilde{\omega} = \pi^*\omega$ is the pullback to $\tilde{W}$ of the class ω of the hyperplane in W, and e the class of the exceptional divisor E.

⌈ By the results on P475 & P605, $\tilde{V}_c = \pi^* V_c - \mathrm{mult}_{W_2}(V_c)\cdot E$. Since $V_c \sim 6\omega$, $\tilde{V}_c \sim 6\pi^*\omega - 2\cdot e = 6\tilde{\omega} - 2e \in H^2(\tilde{W}, \mathbb{Z})$. ($\because$ $\tilde{\omega} \in H^2(\tilde{W}, \mathbb{Z})$ and, in general, $E \in H_{2n-2}(M, \mathbb{Z})$, $\dim_{\mathbb{C}} M = n$, here we are considering the Poincare dual, $\Rightarrow$ $e \in H^2(M, \mathbb{Z})$. Since $M = \tilde{W}$, $e \in H^2(\tilde{W}, \mathbb{Z})$. ⌋

Now, to determine the fivefold self-intersection

$$(6\tilde{\omega} - 2e)^5$$

of $\tilde{V}_c$ in $\tilde{W}$, recall from our discussion in Section 1 that the surface W_2 is the Veronese surface $\iota_{2H}(\mathbb{P}^2)$.

⌈ Remember the original question 'Given $C_1 \cdots C_5 \subset \mathbb{P}^2$, five smooth conics chosen generically, how many smooth conics in $\mathbb{P}^2$ are tangent to all five?'. We are not trying to calculate V_c^5.
To get the answer of the question, we have to calculate $\tilde{V}_c^5$, since $\tilde{\pi}$ is one to one, onto, isomorphism outside E. ⌋