$$\Rightarrow \quad c_1(N_{X/\mathbb{P}^5}|_L) = c_1(N_{F/\mathbb{P}^5}|_L) + c_1(N_{G/\mathbb{P}^5}|_L)$$
$$= -c_1(N^*_{F/\mathbb{P}^5}|_L) - c_1(N^*_{G/\mathbb{P}^5}|_L)$$
$$\overset{c_1}{=} -([-F]|_F)|_L - c_1([-G]|_G|_L)$$
$$= -c_1(-2H|_L) - c_1(-2H|_L)$$
$$= 2 + 2 = 4 \quad \text{since} \quad c_1(H|_L) = H\cdot L = 1.$$

$c_1(T(L)) = c_1(T(\mathbb{P}^1)) =$ coefficient of ω in $(1+\omega)^2 = 2$

$$\Rightarrow \quad c_1(T(\mathbb{P}^5)) = 6 = c_1(N_{X/\mathbb{P}^5}) + c_1(N_{L/X}) + c_1(T(L))$$
$$= 4 + c_1(N_{L/X}) + 2 \quad \Rightarrow \quad c_1(N_{L/X}) = 0$$

⏎

Thus, by our classification (Section 3, Chapter 4) of vector bundles on $\mathbb{P}^1$, we can write

$$N_{L/X} = H^n \oplus H^{-n}, \qquad n \geq 0,$$

where H is the hyperplane bundle on $L \cong \mathbb{P}^1$.

Pf $[H]$ is a line bundle of $\mathbb{P}^1$, and by the lemma on p516, any holomorphic bundle of rank 2 is of form $[H]^m \oplus [H]^n$. $\Rightarrow$ $N_{L/X} = [H]^n \oplus [H]^{-n}$ since $c_1(N_{L/X}) = 0$ and $c_1([H]^m \oplus [H]^{-n}) = c_1([H]^m) + c_1([H]^{-n}) = m\, c_1([H]) - n\, c_1([H])$. Here, $[H]^n = [H] \otimes \cdots \otimes [H]$.

⏎

If L is nonspecial, then we have seen that