

Let $V = \{V_1 \subset V_2 \subset \dots \subset V_n = W\}$ be a flag in W , and let $V^* = \{V_1^* \subset V_2^* \subset \dots \subset V_n^* = W^*\}$ be the dual flag in W^* given by

$$V_i^* = \text{Ann}(V_{n-i}).$$

$$\begin{array}{ccc} \mathbb{F} & \text{Ann}(V_{n-i}) \subset \text{Ann}(V_{n-(i+1)}) & \\ & \text{" } V_i^* & \text{" } V_{i+1}^* \end{array} \quad \supset$$

By linear algebra, for any k -plane in W ,

$$\dim(\Lambda \cap V_{n-k+i-a_i}) \geq i \iff \dim(*\Lambda \cap V_{k-i+a_i}^*) \geq a_i.$$

\mathbb{F} 93. 3.20. Suppose $\sigma_a', \sigma_b', \sigma_c' \in G(k, n+1)$ and $(\sigma_a' \cdot \sigma_b') = \sum n_c' \cdot \sigma_c'$. ^{Now} we want to show that

$$\text{, if } \sigma_a \cdot \sigma_b = \sum n_c \cdot \sigma_c, \text{ then } n_c = n_c',$$

$$\sigma_a, \sigma_b, \sigma_c \in G(k, n), \sigma_a = \bar{i}_1^{-1}(\sigma_a') \text{ for all } a.$$

Let $\tilde{\sigma}_a'$ be the Poincaré dual of σ_a' .
 " $\tilde{\sigma}_a$ " of σ_a .

$$\Rightarrow \sigma_a' \cdot \sigma_b' = \sum n_c' \cdot \sigma_c' \iff \tilde{\sigma}_a' \wedge \tilde{\sigma}_b' = \sum n_c' \cdot \tilde{\sigma}_c'$$

Applying \bar{i}_1^* , we get

$$\begin{aligned} \bar{i}_1^* (\tilde{\sigma}_a' \wedge \tilde{\sigma}_b') &= \sum n_c' \cdot \bar{i}_1^* \tilde{\sigma}_c' \\ &= \bar{i}_1^* (\tilde{\sigma}_a') \wedge \bar{i}_1^* (\tilde{\sigma}_b') = \sum n_c' \bar{i}_1^* \tilde{\sigma}_c' \\ &= \tilde{\sigma}_a \wedge \tilde{\sigma}_b = \sum n_c' \tilde{\sigma}_c \iff \sigma_a \cdot \sigma_b = \sum n_c' \sigma_c \\ &\Rightarrow n_c' = n_c. \end{aligned}$$