By the exact sequence of cohomology,

$$H^q(M,\ \ker(\alpha)(k)) \longrightarrow H^q(M,\ \mathcal{E}_0(k)) \longrightarrow H^q(M,\ \mathcal{F}(k)) \longrightarrow$$

$$H^{q+1}(M,\ \ker(\alpha)(k)) \longrightarrow$$

$\Rightarrow$ For k large enough, $H^q(M, \ker\alpha(k)) = H^{q+1}(M, \ker\alpha(k))$
$= H^q(M, \mathcal{E}_0(k)) = 0$ for $q > 0$.
$\Rightarrow$ $H^q(M, \mathcal{F}(k)) = 0$. Refer to P 697. the proof.

Proof of 2. For each $x_0 \in M$, we have

$$0 \longrightarrow m_{x_0}\mathcal{F}(k) \longrightarrow \mathcal{F}(k) \longrightarrow \mathcal{F}(k)_{x_0}/m_{x_0}\mathcal{F}(k) \longrightarrow 0,$$

where $m_{x_0} \subset \mathcal{O}$ is the sheaf of ideals given by the maximal ideal m_{x_0} at x_0. The fiber $\mathcal{F}(k)_{x_0}/m_{x_0}\mathcal{F}(k)$ is an example of a coherent sheaf supported at a point – these are sometimes called skyscraper sheaves.

$$\Gamma\left(\mathcal{F}(k)_{x_0}/m_{x_0}\mathcal{F}(k)\right)_x = \begin{cases} \mathcal{F}(k)_{x_0}/m_{x_0}\mathcal{F}(k) & \text{if } x = x_0 \\ 0 & \text{otherwise} \end{cases}$$

Support $\left(\mathcal{F}(k)_{x_0}/m_{x_0}\mathcal{F}(k)\right) = \{x_0\}$.
See the def of support.

$$Z = \{z \in M : I_z \neq \mathcal{O}_z\} = \{z : M : (\mathcal{O}/I)_z \neq 0\}$$

$\mathcal{F}_{x_0}$ is a coherent sheaf, for

$$\begin{tikzcd} \mathcal{O}^{(q)} \arrow[r] & \mathcal{O}^{(p)} \arrow[r] \arrow[d, "\text{onto}"] & \mathcal{F}_{x_0} \arrow[r] & 0 \\ & \mathcal{F} \arrow[ur, "\text{onto}"'] & & \end{tikzcd}$$