$$\begin{tikzcd} W \arrow[r, shift left, "G"] & V \subset R^n \arrow[l, shift left, "F"] \end{tikzcd} \qquad (a, 0 \cdots 0) \in W, \quad x_0 \in V$$

$$\Rightarrow \quad F \circ G(y_1 \cdots y_n) = (y_1 \cdots y_n)$$
$$G \circ F(x_1 \cdots x_n) = (x_1 \cdots x_n)$$

$F(x_1 \cdots x_n) = (f \circ \varphi^{-1}(x_1 \cdots x_n), x_2 \cdots x_n)$.

Consider $(a, 0, 0 \cdots 0)$. $\Rightarrow$ $f \circ \varphi^{-1}(x_0) = a$.

Restrict to $(f \circ \varphi^{-1})^{-1}(a)$

$\Rightarrow$ G is a diffeomorphism onto $(f \circ \varphi^{-1})^{-1}(a)$

$$\begin{tikzcd} W \cap \{a\} \times R^{n-1} \arrow[r, shift left, "G"] & (f \circ \varphi^{-1})^{-1}(a) \subset V \arrow[l, shift left, "F"] \end{tikzcd} \qquad x_0 \in (f \circ \varphi^{-1})^{-1}(a)$$

$$F((f \circ \varphi^{-1})^{-1}(a)) = \{(a, x_2 \cdots x_n) \mid (a, x_2, x_3 \cdots x_n) \in W\}$$

$\Rightarrow$ $(f \circ \varphi^{-1})^{-1}(a)$ is diffeomorphic to $W \cap (\{a\} \times R^{n-1})$

$V \cong W \longleftrightarrow$

By choosing open set U' smaller than U,

$$\begin{tikzcd} U' \supset f^{-1}(a) \arrow[r, "f"] \arrow[d, "\varphi"'] & R \ni a \\ V \supset (f \circ \varphi^{-1})^{-1}(a) = \varphi(f^{-1}(a)) \arrow[ur, dashed, "f \circ \varphi^{-1}"'] \arrow[d, "F"', "\cong"] & \\ W \supset W \cap \{a\} \times R^{n-1} & \end{tikzcd}$$

$W \subset R^n$

$\Rightarrow$ $U' \xrightarrow{\varphi} V \xrightarrow{F} W$

$\Rightarrow$ $F \circ \varphi : U' \longrightarrow W$

$F \circ \varphi \big|_{f^{-1}(a)} ; f^{-1}(a) \cong \{a\} \times R^{n-1}$.

If we let $F ; R^n \longrightarrow R \times R^{n-1}$

$(x_1 \cdots x_n) \longmapsto (f \circ \varphi^{-1}(x_1 \cdots x_n) - a, x_2 \cdots x_n)$