$$\begin{tikzcd}
0 \arrow[r] & C^{p-1}(\underline{\mathcal{U}}, Z_{\bar\partial}^{n,n-p-1}) \arrow[r, hook] \arrow[d, "\delta"] & C^{p-1}(\underline{\mathcal{U}}, \mathcal{A}^{n,n-p-1}) \arrow[r, "\bar\partial"] \arrow[d, "\delta"] & C^{p-1}(\underline{\mathcal{U}}, Z_{\bar\partial}^{n,n-p}) \arrow[r] \arrow[d, "\delta"] & 0 \\
0 \arrow[r] & C^{p}(\underline{\mathcal{U}}, Z_{\bar\partial}^{n,n-p-1}) \arrow[r, hook] & C^{p}(\underline{\mathcal{U}}, \mathcal{A}^{n,n-p-1}) \arrow[r, "\bar\partial"] & C^{p}(\underline{\mathcal{U}}, Z_{\bar\partial}^{n,n-p}) \arrow[r] & 0
\end{tikzcd}$$

$$\xi_p \longmapsto \omega_{p-1} = \bar\partial \xi_p, \qquad \omega_p \longmapsto \delta\xi_p \longmapsto 0$$

$\xi_p \in C^{p-1}(\underline{\mathcal{U}}, \mathcal{A}^{n,n-p-1})$ s.t. $\bar\partial \xi_p = \omega_{p-1}$, $\omega_p = \delta \xi_p$.

If necessary, we have to choose a refinement of $\underline{\mathcal{U}} = \{U_i\}$. See p40. (We don't need a refinement. see p527 note ⌟

Next, let Γ_I be the chain defined by

$$\Gamma_I = \{ z : |f_i(z)| = \varepsilon \text{ for } i \in I,\ |f_j(z)| \le \epsilon \text{ for } j \notin I \}$$

and with orientation

$$d(\arg f_{i_1}) \wedge \cdots \wedge d(\arg f_{i_p}) \wedge \Big(\bigwedge_{j \notin I} \frac{\sqrt{-1}}{2}\, df_j \wedge d\bar f_j \Big) \ge 0,$$

where $I = \{ i_1 < \cdots < i_p \}$.

⌜ Refer to p650. For example, $\underbrace{\{ |z_1| = \epsilon_1,\ |z_2| \le \epsilon_2 \}}_{= \Gamma_1}$

$\Rightarrow$ $-i \dfrac{dz_1}{z_1} \wedge i \dfrac{dz_2 \wedge d\bar z_2}{2}$ is an orientaion on $\Gamma_{I=\{1\}}$. ⌟

Then the boundary

$$\partial \Gamma_I = \sum_{j \notin I} (-1)^{(j,\, I - \{j\})} \Gamma_{I \cup \{j\}},$$

where $(j, I-\{j\})$ is the position from the rear of the