

$C, \mathcal{O}(\mathcal{L}H) = H^0(C, \mathcal{O}(G)),$
 $\Rightarrow K_C = -2H + \mathcal{L}H|_C = 0|_C \Rightarrow K_C$ is a trivial
 bundle over C . \Rightarrow By the Gauss-Bonnet theorem,
 $\chi(C) = 0 \Rightarrow$ In case C is connected,
 then C is a torus, and the genus of C is 1.

(3) By Bezout's theorem on P^3 , since F meets G
 transversely,

$$\mathcal{L} = \sum_i \text{mult}_{Z_i}(F \cdot G) \cdot \text{degree}(Z_i).$$

where $\text{mult}_{Z_i}(F \cdot G) = 1$ for all i .

\Rightarrow (i) $\mathcal{L} = 1 + 1 + 1 + 1 \Rightarrow$ impossible since $F \cap G$
 does not contain a line. (see ① & $\deg Z_i = 1$
 $\Rightarrow Z_i = \mathbb{P}^1$, see P^3).

(ii) $\mathcal{L} = 2 + 1 + 1 \Rightarrow$ impossible by the same
 reason above

(iii) $\mathcal{L} = 2 + 2$.

$\Rightarrow F \cap G = C_1 \cup C_2$.

$\deg C_1 = 2 \Rightarrow C_1 \subset \mathbb{P}^3 \Rightarrow$ By the result on P^3 ,
 C_1 is degenerate. $\Rightarrow C_1 \subset H_1 = \mathbb{P}^2$.

\Rightarrow Since G & F are irreducible, $F \cap H_1 = C_1 = G \cap H_1$.

Similarly, $C_2 \subset H_2 = \mathbb{P}^2$ and $F \cap H_2 = C_2 = G \cap H_2$.

(a) Case $H_1 = H_2 = H$.

$\Rightarrow F \cap H = C_1 \cup C_2$

\Rightarrow By Bezout's theorem, since $\deg F = 2$ & $\deg H = 1$,
 $2 = \deg C_1 + \deg C_2 = 4$, which is absurd.