

The form $dz_1 \wedge dz_2$ extends to a meromorphic ω -form on \mathbb{P}^2 ; since $K_{\mathbb{P}^2} = -3H$ and $dz_1 \wedge dz_2$ is nonzero holomorphic on $\mathbb{P}^2 - L$, it follows that $dz_1 \wedge dz_2$ must have a pole of order 3 along the line L .

$$\Gamma \quad \mathbb{P}^2, \quad U_0 = (z_0 \neq 0) \quad U_1 = (z_1 \neq 0) \quad U_2 = (z_2 \neq 0).$$

$$\sigma_0 = dz_1 \wedge dz_2$$

$$= d\frac{1}{x_0} \wedge d\frac{x_2}{x_0}, \quad \text{where } \frac{1}{x_0} = \left(\frac{z_0}{z_1}\right)^{-1} = z_1,$$

$$\frac{x_2}{x_0} = \frac{z_2}{z_0} = \left(\frac{z_2}{z_1}\right) / \left(\frac{z_0}{z_1}\right)$$

$$\Rightarrow dz_1 \wedge dz_2 = d\left(\frac{1}{x_0}\right) \wedge d\left(\frac{x_2}{x_0}\right) = \left(-\frac{1}{x_0^2} dx_0\right) \wedge \left(-\frac{x_2}{x_0^2} dx_0 + \frac{1}{x_0} dx_2\right) = -\frac{1}{x_0^3} dx_0 \wedge dx_2.$$

$\Rightarrow \sigma_0$ extends to U_1 , and eventually to \mathbb{P}^2 . The rest follows easily. \square

Similarly, f extends to a meromorphic function on \mathbb{P}^2 , and since f is a polynomial with a single zero along a curve of degree d in $\mathbb{P}^2 - L$, it must have a pole of order d along L .

$$\Gamma \quad f(z_1, z_2) = f\left(\frac{z_1}{z_0}, \frac{z_2}{z_0}\right) = f\left(\frac{1}{x_0}, \frac{x_2}{x_0}\right)$$

$\Rightarrow f$ can be extended to \mathbb{P}^2 .

Note that, when we write w locally as follows

$$w = \frac{g(z_1, z_2)}{f(z_1, z_2)} dz_1 \wedge dz_2, \quad f \text{ is the function}$$

mentioned before, i.e. $f\left(\frac{z_1}{z_0}, \frac{z_2}{z_0}\right) = F\left(1, \frac{z_1}{z_0}, \frac{z_2}{z_0}\right).$