process that every sequence in E contains a subsequence $\{f_i\}$ for which $\{D^\beta f_i\}$ converges. uniformly on compact subsets of Ω, for each multi-index β.

$\ulcorner$ For each multi-index β, we have a uniformly convergent subsequence by Ascoli's theorem. $\Rightarrow$ By Cantor's diagonal process, we get a uniform convergent subsequence on compact subsets for every multi-index β.

$\lrcorner$

Hence $\{f_i\}$ converges in the topology of $C^\infty(\Omega)$.

$\ulcorner$ Let $f(x) = \lim f_i(x)$, $x \in \Omega$. We have to show that $\forall \epsilon > 0$ $\exists N_\epsilon$ s.t $d(f_i, f) < \epsilon$ if $i \geq N_\epsilon$.

$$d(f_i, f) = \sum_{N=1}^{\infty} (2^{-N} P_N(f_i - f)) / (1 + P_N(f_i - f))$$

$$= \underbrace{\sum_{N=1}^{l-1} \frac{2^{-N} P_N(f_i - f)}{1 + P_N(f_i - f)}}_{①} + \underbrace{\sum_{N=l}^{\infty} \frac{2^{-N} P_N(f_i - f)}{1 + P_N(f_i - f)}}_{②}$$

Point 1 $\quad 2^{-N} P_N(f_i - f) / (1 + P_N(f_i - f)) < 2^{-N}$

$\Rightarrow$ For l sufficiently large, we make

$$② = \sum_{N=l}^{\infty} \frac{2^{-N} P_N(f_i - f)}{1 + P_N(f_i - f)} < \sum_{N=l}^{\infty} 2^{-N} = 2^{-l} \frac{1}{1 - \frac{1}{2}} = 2^{-(l-1)}$$

$$< \frac{\epsilon}{2}. \quad \cdots \quad (*)$$

Now, for a fixed l satisfying $(*)$, since N ranges from 1 to $l-1$, if we choose i sufficiently large, then we have $\sum_{N=1}^{l-1} \frac{2^{-N} P_N(f_i - f)}{1 + P_N(f_i - f)} < \frac{\epsilon}{2}$. $\Rightarrow$ Done. $\lrcorner$