2. Some Vanishing Theorems and Corollaries.

The Kodaira Vanishing Theorem

Let M compact Kähler manifold.

Def: A line bundle $L \longrightarrow M$ is positive if $\exists$ a metric on L with curvature form Θ s.t $\frac{i}{2\pi}\Theta$ is a positive

(1.1)-form; L is negative if L^* is positive.
A divisor D on M is positive if the line bundle $[D]$ is.

The positivity of a line bundle is a topological property, as we see from the

Proposition. If ω is any real, closed (1.1)-form with

$[\omega] = C_1(L) \in H^2_{DR}(M)$,
then there exists a metric connection on L with curvature form $\Theta = \left(\frac{i}{2\pi}\right)^{-1}\omega$. Thus L is positive $\Leftrightarrow$ its Chern class may be represented by a positive form in $H^2_{DR}(M)$. (This is the conclusion of the Proposition.)
<u>positive form</u> implies positive $(1,1)$-form

Pf ($\Rightarrow$) L positive $\Rightarrow$ $\exists$ a metric on L with curvature form Θ s.t $\frac{i}{2\pi}\Theta$ is a positive (1.1)-form.

$\Rightarrow$ By p141. Prop. 1. $C_1(L) = [\frac{i}{2\pi}\Theta]$.

($\Leftarrow$) If $C_1(L) = [\omega]$, ω is positive in $H^2_{DR}(M)$.
$\Rightarrow$ By Prop. $\exists$ a metric connection on L with curvature form $\Theta = \left(\frac{i}{2\pi}\right)^{-1}\omega$. $\Rightarrow \omega = \frac{i}{2\pi}\Theta \Rightarrow [\omega] = C_1(L)$.