Note that $\Lambda \in \Lambda^k V$ is decomposable $\Leftrightarrow$ Λ is in the image of $p : G(k, V) \longrightarrow \mathbb{P}(\Lambda^k V)$.

Thus $p(G(k, V)) = \{ \Lambda \in \Lambda^k V ; \ i(i(\bar{Z})\Lambda)\Lambda = 0 \} / \mathbb{C}^*$

$= \mathbb{P}\{ \Lambda \in \Lambda^k V : \ i(i(\bar{Z})\Lambda)\Lambda = 0 \}$.

Since i is linear, and $\dim(\Lambda^{k+1}V^*) = {}_nC_{k+1} = \binom{n}{k+1}$, if $\{ \bar{Z}_1, \bar{Z}_2, \cdots \bar{Z}_{\binom{n}{k+1}} \}$ is a set of basis elements of $\Lambda^{k+1}V^*$, we obtain $\binom{n}{k+1}$ equations.

It remains to show that each of ${}_nC_{k+1}$ equations is quadratic form.

For simplicity, $k = 2+1$, $n = 4$.

$\Rightarrow$ let $Z \in \Lambda^{3+1}V^* = \Lambda^4 V^*$.

$i(Z) : \Lambda^3 V \longrightarrow \Lambda^2 V$

$\Lambda = \Lambda_{123}\, e_1 \wedge e_2 \wedge e_3 + \Lambda_{124}\, e_1 \wedge e_2 \wedge e_4 + \Lambda_{234}\, e_2 \wedge e_3 \wedge e_4 + \Lambda_{134}\, e_1 \wedge e_3 \wedge e_4$

$\Rightarrow i(Z)\Lambda = \Lambda_{123}\big(a_{123,12}\, \underset{= e_1 \wedge e_2}{e_{12}} + a_{123,13}\, e_1 \wedge e_3 + a_{123,14}\, e_1 \wedge e_4 + \cdots + a_{123,34}\, e_3 \wedge e_4 \big) + \cdots$
$\cdots + \Lambda_{134}\big(a_{134,12}\, e_{12} + \cdots + a_{134,34}\, e_3 \wedge e_4 \big)$,

$\Rightarrow i(i(Z)\Lambda) :$

For simplicity, $k = 3$, $n = 4$.

$Z \in \Lambda^4 V^*$,

$$\Rightarrow \quad i(Z) : \Lambda^3 V \longrightarrow V^* \qquad i(i(Z)\Lambda) : \Lambda^3 V \longrightarrow \Lambda^2 V$$
$$\Lambda \longmapsto i(Z)\Lambda \qquad\qquad \Lambda' \longmapsto i(i(Z)\Lambda)\Lambda'$$