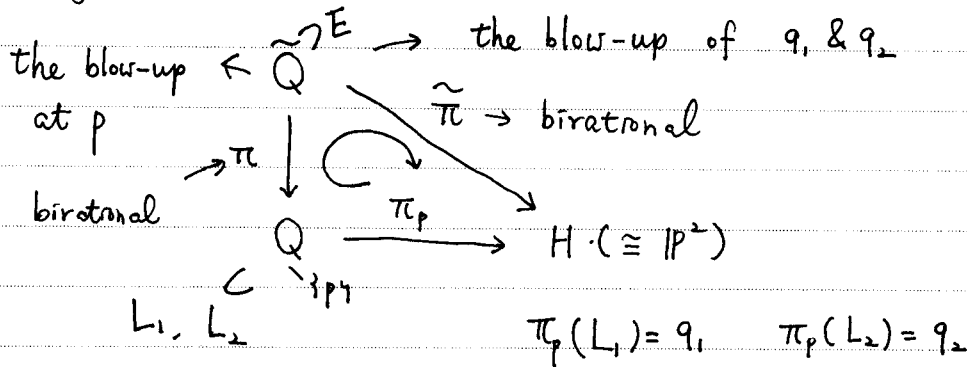


Note that in case V is a quadric hypersurface the map π_p is a birational isomorphism. We have already seen this in the case of Q a quadric surface in \mathbb{P}^3 , where the map π_p consists of the blow-up of the point p , followed by the blowing down of the two lines of Q through p .

By P499 & P480



$\Rightarrow \pi_p$ is birational, since

\exists an inverse rational map $\pi \circ \tilde{\pi}^{-1}: H \xrightarrow{\{q_1, q_2\}} \tilde{Q} \rightarrow Q$.

$$\pi_p \circ \pi = \tilde{\pi}$$

$$\pi_p \circ \pi \circ \tilde{\pi}^{-1} = \text{id}$$

For V a quadric hypersurface in \mathbb{P}^n , $p \in V$.

\Rightarrow for generic point $q \in V$, $\#(\overline{pq} \cap V) = 1$, since V is quadric. $\Rightarrow \pi_p(q) = \overline{pq} \cap H$, and π_p is generically one to one $\Rightarrow \pi_p$ is birational by P493.

More precisely, for any generic point $r \in H$, $\#(\overline{pr} \cap V) = 1$, since V is quadric. \square