

assumption. $\Rightarrow m_{(p,p)}(Z \times W, \Delta) = m_p(Z, W)$ by the formal properties of Künneth formula again (?), which we had better make clearer.

$$\Rightarrow \text{Thus } Z \cdot W = \sum_p m_p(Z, W).$$

So the key points are $m_{(p,p)}(Z \times W, \Delta) = m_p(Z, W)$, and $Z \cdot W = (Z \times W) \cdot \Delta$.

$Z \cdot W = [Z] \cdot [W] = [Z \times W] \cdot [\Delta]$ by P59 since $(-1)^{2n-2p} = 1$, where $[Z]$ & $[W]$ are homology classes represented by Z & W respectively.

Now it remains to show that

$$m_{(p,p)}(Z \times W, \Delta) = m_p(Z, W).$$

To show this, let's return to the definition of the intersection multiplicity m . P62.

Let V and W be two analytic varieties of dimension k and $n-k$ in the polycylinder Δ of radius 1 in \mathbb{C}^n having the origin as their only point of intersection. Consider in the product $\Delta' \times \Delta'$ of the polycylinder of radius $\frac{1}{2}$ with itself the two varieties

$$\tilde{V} = \pi_1^{-1}(V) = \{(z, w) : z \in V\} \quad \text{and}$$

$$\tilde{W} = \{(z, w) : z - w \in W\}.$$

For each e , of course, the varieties \tilde{V} and \tilde{W} meet the fiber $\pi_2^{-1}(e) = \Delta' \times \{e\} \cong \Delta'$ in the analytic variety V