$E \xrightarrow{\pi} M$ is a complex vector bundle together with the structure of a complex manifold on E, s.t for any $x \in M$, $\exists$ $U \ni x$ in M and a trivialization

$$\varphi_U : E_U \longrightarrow U \times \mathbb{C}^k$$ that is a biholomorphic map of complex manifolds. Such a trivialization is called a holomorphic trivialization. Note that if $\{\varphi_\alpha : E_{U_\alpha} \to U_\alpha \times \mathbb{C}^k\}$ are holomorphic trivializations, then the transition functions for E relative to $\{\varphi_\alpha\}$ are holomorphic maps, and that, conversely, given holomorphic maps

$$g_{\alpha\beta} : U_\alpha \cap U_\beta \longrightarrow GL_k \quad \text{satisfying the identies}$$

on p.66, we can construct a holomorphic vector bundle $E \longrightarrow M$ with transition functions $g_{\alpha\beta}$.

All the vector bundle phenomena discussed so far carry over directly to the category of holomorphic v.b. We can define the dual bundle and, direct, tensor, and alternating product bundles of holomorphic vector bundles to be holomorphic; likewise we observe that the pull back f^*E of a holomorphic vector bundle E under a holomorphic map $f: M \longrightarrow N$ of a complex manifolds has a natural holomorphic structure.
A holomorphic map of holomorphic vector bundles E, F and M is a <u>holomorphic map</u> $f: E \to F$ with $f: E_x \to F_x$ linear.;
a holomorphic subbundle of a holomorphic bundle E is a subbundle $F \subset E$ with F a complex submanifold of E, and the quotient bundle is again holomorphic.
A section σ of the holo bundle E over U is said to be holomorphic if $\sigma : U \to E$ is a holomorphic map,