quadrics containing the singular point of F.

$\sqcap$ $F = X_1^2 + X_2^2 + X_3^2 = 0$

$$\frac{\partial F}{\partial X_0} = 0 \quad \frac{\partial F}{\partial X_1} = 2X_1, \quad \frac{\partial F}{\partial X_2} = 2X_2, \quad \frac{\partial F}{\partial X_3} = 2X_3$$

$\Rightarrow X_1 = X_2 = X_3 = 0 \quad \Rightarrow p = [1, 0, 0, 0] \in F$ is a singular point.

$|\lambda Q + Q'| = 0$ is of degree $2 \Leftrightarrow$ the quadric G corresponding to Q' contains p. $\Leftrightarrow \{\lambda Q + Q'\}$ is a tangent line (pencil) to W_1 at F, since $|\lambda Q + Q'| = 0$ is a polynomial of degree ≤ 2, and $m_F(L \cdot W_1) \geq 2$, $L = \{\lambda Q + Q'\}$.

Thus $T_F W_1 = \{$ all quadrics containing $p\}$. $\lrcorner$

Similarly, a quadric $F \in W_2 - W_3$ of rank two may be represented by the matrix

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

for generic Q', then, the polynomial $|\lambda Q + Q'|$ will have degree 2; that is, a generic pencil L in W containing F will meet W_1 in only two other points.

$\sqcap$ $\#(L \cap W - F) = 2$. $\lrcorner$