to the fact that $h^0(K_{S'}) = g$. Here we used the fact that $\iota_L(S') = S$ has the degree $2g-2$, which is equal to $c_1(L)$, see p 177. ⊛. ⌟

The canonical curve of a Riemann surface S derives much of its importance from the fact that it is intrinsically defined by S, and so as a general rule, any projective invariant of the canonical curve reflects the intrinsic structure of S. We will see this principle applied when we discuss Weierstra's's points, and again in discussing the Torelli theorem.

We can rephrase the Riemann-Roch formula in terms of the geometry of the canonical curve: for any divisor $D = \Sigma p_i$ on the Riemann surface S, $h^0(K-D)$ is just the number of hyperplanes in $\mathbb{P}^{g-1}$ containing the points $\iota_K(p_i)$, and so $h^0(D)$ is equal to the degree of D minus the dimension of the linear space $\overline{D}$ spanned by the points p_i on the canonical curve.

⌜

$$S \xrightarrow{\iota_K} \mathbb{P}^{g-1}$$

$$\cup$$

$$D = \Sigma p_i.$$

$$p \longmapsto [\omega_1(p), \omega_2(p)]$$

$$p_1 \longmapsto [\tfrac{\omega_1}{dz_1}(p_1), \tfrac{\omega_2}{dz}(p_1)]$$

$$p_2 \longmapsto [\tfrac{\omega_1}{dz_2}(p_2), \tfrac{\omega_2}{dz_2}(p_2)]$$

$$p_3 \longmapsto [\tfrac{\omega_1}{dz_3}(p_3), \tfrac{\omega_2}{dz_3}(p_3)].$$

$\Rightarrow$ $h^0(K-D)$ is the number of independent relations among column vectors