where we write ω/dz for the function $h(z)$ such that $\omega = h(z)\,dz$.

$$\mu^{(g)}(D') = \mu^{(g)}(\sum z_i) = \left(\sum_\lambda \int_{p_0}^{z_\lambda} \omega_1, \ldots \sum_\lambda \int_{p_0}^{z_\lambda} \omega_g \right)$$

$$\Rightarrow \frac{\partial \mu^{(g)}(D')}{\partial z_i} = \left(\frac{\partial}{\partial z_i} \int_{p_0}^{z_i} \omega_j \right) = (\omega_j)/_{dz_i} = h_j(z).$$

$\omega_j = h_j(z)\,dz$

Thus, and this is a fundamental observation, the Jacobian matrix of the map $\mu^{(g)}$ is given by, near D,

$$J(\mu^{(g)}) = \begin{bmatrix} \omega_1/dz_1, & \cdots & \omega_1/dz_g \\ \vdots & & \vdots \\ \omega_g/dz_1, & \cdots & \omega_g/dz_g \end{bmatrix}.$$

We note that changing the local coordinate z_i multiplies the ith column by a nonzero factor but does not affect the rank of $J(\mu^{(g)})$.

$$z \longrightarrow z'$$

$$\frac{\partial}{\partial z} \int_{p_0}^{z} \omega = h(z), \quad \text{where } \omega = h(z) = f(z')\,dz'$$
$$= f(k(z))\,k'(z)\,dz$$
$$z' = k(z).$$

$$\frac{\partial}{\partial z'} \int_{p_0}^{z'} \omega = f(z') = f(k(z)) = h(z) \cdot \frac{1}{k'(z)} = h(z) \cdot \frac{1}{\frac{dk(z)}{dz}}.$$

$\Rightarrow$ Multiplies the ith column by a nonzero factor $1/k'(z)$.