

the cone pC over a smooth conic curve C . Q then contains a single family of lines $\{L_q = \overline{pq} \mid q \in C\}$; since any two of these lines comprise a hyperplane section of Q , it follows that E_0 contains the vertex p of Q and every line $L_q \subset Q$ meets E_0 in two points other than p .

✓ To show that $p \in E_0$, note the following:

- ① $\dim \{ \text{hyperplanes containing } p \text{ in } \mathbb{P}^3 \} = \mathbb{P}^2$ ✓
 $\Rightarrow \exists$ three linearly independent sections τ_1, τ_2, τ_3 of $[H]$ over \mathbb{P}^3 s.t. $\tau_1(p) = \tau_2(p) = \tau_3(p) = 0$.

Suppose $p \notin E_0$. \Rightarrow For any hyperplane H passing p in \mathbb{P}^3 , $\#(H \cap E_p) = 5$. $H \cap E_p \subset H \cap Q = L_1 \cup L_2$, where L_1 & L_2 are lines in Q passing p . $\Rightarrow \#(L_1 \cdot E_p)$ and $\#(L_2 \cdot E_p)$ are 2 or 3. Suppose $\#(L_1 \cdot E_p) = 2$ and $\#(L_2 \cdot E_p) = 3$. Choose $L_3 \ni p$ and $L_3 \subset Q$, $L_1 \neq L_3$, $L_2 \neq L_3$. $\Rightarrow \#(L_3 \cdot E_p) = 2$ or 3 by the above

\Rightarrow (i) $\#(L_3 \cdot E_p) = 2$

$\Rightarrow \#(\overline{L_1, L_3} \cdot E_p) = 4$ Contradiction

(ii) $\#(L_3 \cdot E_p) = 3$

$\Rightarrow \#(\overline{L_2, L_3} \cdot E_p) = 6$ Contradiction since $\overline{L_2, L_3}$ can not lie in Q and E_p can not lie in $\overline{L_2, L_3}$. For, $Q = pC$, C smooth conic, and $g(E_p) = 2 + \frac{(5-1)(5-2)}{2}$.

Thus $p \in E_0$.

Since Q is the cone through p over the smooth cone C ,