that is, S^* is the locus of tangent planes to S and vice versa.

$\ulcorner$ $l_x^* \cdot S^* = l_x^* \cap S^*$, counting multiplicities.
For each x, $l_x^* \cdot S^*$ is singular since for each x, l_x^* is tangent to S^*. $\Rightarrow$ In the pencil $\{l_x^* \cdot S^*\}$, by Bertini, a generic element is smooth away the base locus $h^* = \cap\, l_x^*$. $\Rightarrow$ All elements of the pencil are singular at h^* since every element is singular. Since any line l_x^* in p^* is tangent to S^* at h^*, $p^* = T_{h^*}(S^*)$.

$*$ $h^* = \cap(l_x^* \cap S)$ for otherwise $\{l_x^* \cdot S^*\}$ has no base point $\Rightarrow$ By Bertini theorem, there exists a smooth $l_x^* \cdot S^*$ for some x, which is impossible since every $l_x^* \cdot S^*$ is singular. Thus we proved the following: Given a smooth point $p \in S$, then $T_p(S) \in S^*$. We can go over the argument above for S^* $\Rightarrow$ Given $h \in S^*$, $(T_h(S^*))^* \in S$
$\Rightarrow$ S and S^* are dual surfaces.

$\lrcorner$

Singular Lines of the Quadric Line Complex

The next step in our study of X is to introduce a subvariety $\Sigma \subset X$ closely related to the