

represented by  $Z$ , i.e.

$$J_Z = \{ \lambda(z_0, \dots, z_n), \lambda \in \mathbb{C}^* \}.$$

To see that in fact  $J = [-H]$ , consider the section  $e_0$  of  $J$  over  $U_0 = (z_0 \neq 0) \subset \mathbb{P}^n$  given by

$$e_0(z) = (1, \frac{z_1}{z_0}, \dots, \frac{z_n}{z_0}).$$

$e_0$  is clearly holomorphic and nonzero in  $U_0$  and extends to a global meromorphic section of  $J$  with a pole of order 1 along the hyperplane  $(z_0 = 0) \subset \mathbb{P}^n$ . Thus  $J = [(e_0)] = [-H]$ .

$$\Gamma \quad \text{On } U_0 = (z_0 \neq 0), \quad J|_{U_0} \cong U_0 \times \mathbb{C}.$$

$$\begin{array}{ccc} J|_{U_0} & \longrightarrow & U_0 \times \mathbb{C} \\ \downarrow \psi & & \\ ([z_0, z_1, \dots, z_n], & \longmapsto & ([z_0, z_1, \dots, z_n], \lambda) \\ [\lambda, \lambda \frac{z_1}{z_0}, \dots, \lambda \frac{z_n}{z_0}]) & & \\ & & U_0 \times \mathbb{C}^n \longrightarrow J|_{U_0} \subset U_0 \times \mathbb{C}^{n+1} \\ & & (1, \frac{z_1}{z_0}, \dots, \frac{z_n}{z_0}) \in \mathbb{C}^n. \quad ([z_0, \dots, z_n], (\lambda_1, \dots, \lambda_{n+1})) \longmapsto \begin{pmatrix} [z_0, \dots, z_n] \\ \lambda_1 (1, \frac{z_1}{z_0}, \dots, \frac{z_n}{z_0}) \\ \lambda_2 (0, 1, 0, \dots, 0) \\ \vdots \\ \lambda_n (0, 0, \dots, 1, 0) \\ \lambda_{n+1} (0, 0, \dots, 0, 1) \end{pmatrix} \\ \Rightarrow \lambda (1, \frac{z_1}{z_0}, \dots, \frac{z_n}{z_0}) \in \mathbb{C}^n. & & \text{biholomorphic.} \\ & & (p19). \\ U_0 & \xrightarrow{e_0} & J|_{U_0} \\ & & \Downarrow \\ & & J \text{ is subbundle of } \mathbb{P}^n \times \mathbb{C}^{n+1}. \end{array}$$

$$[z_0, \dots, z_n] \longmapsto (1, \frac{z_1}{z_0}, \dots, \frac{z_n}{z_0})$$

Extend  $e_0$  to  $\mathbb{P}^n$ . as follows:

For simplicity,  $n=2$ .

$$\text{On } U_0 = (z_0 \neq 0), \quad e_0 : [z_0, z_1, z_2] \longmapsto (1, \frac{z_1}{z_0}, \frac{z_2}{z_0}).$$

On  $U_1 = (z_1 \neq 0)$ , to extend, we have to find transition functions.