We shall now define another locally convex topology τ on $\mathcal{D}(\Omega)$ in which Cauchy sequences do converge. The fact that this τ is not metrizable is only a minor inconvenience, as we shall see.

6.3 Definitions Let Ω be a nonempty open set in R^n.

(a) For every compact $K \subset \Omega$, τ_K denotes the Frechet space topology of $\mathcal{D}_K$, as described in Sections 1.46 and 6.2.
(b) β is the collection of all convex balanced sets $W \subset \mathcal{D}(\Omega)$ such that $\mathcal{D}_K \cap W \in \tau_K$ for every compact $K \subset \Omega$.

⌈ A set $B \subset X$ is said to be balanced if $\alpha B \subset B$ for every $\alpha \in \Phi$ with $|\alpha| \leq 1$, where $\Phi = \mathbb{C}$ or $\mathbb{R}$.

$\mathcal{D}_K$ is an element of τ_K, where τ_K is a topology on $\mathcal{D}_K$. ⌋

(c) τ is the collection of all unions of sets of the form $\phi + W$, with $\phi \in \mathcal{D}(\Omega)$ and $W \in \beta$.

Through-out this chapter, K will always denote a compact subset of Ω.

6.4 Theorem
(a) τ is a topology on $\mathcal{D}(\Omega)$, and β is a local base for τ.
(b) τ makes $\mathcal{D}(\Omega)$ into a locally convex topological vector space.

proof. Suppose $V_1 \in \tau$, $V_2 \in \tau$, $\phi \in V_1 \cap V_2$. To prove (a),