

Given any curve  $A_i$  of degree  $n$  passing through  $P_1, \dots, \hat{P}_i, \dots, P_k$ ,  $A_i + B$  will contain  $P$  and so  $P_i$  lies on  $A_i$ . Consequently, the points  $i_n(P_1), \dots, i_n(P_k)$  are linearly independent in  $\mathbb{P}^N$ .

This again implies (\*\*).

Q.E.D.

⌈ The exceptional case means that  $B$  contains more than  $\frac{n(n-3)}{2} + 1$  points. Let's label our points (points selected)  $P_1, \dots, P_{\frac{n(n+3)}{2}}$ , so that the curve  $B$  of degree  $n-3$  passes through exactly

$P_k, \dots, P_{\frac{n(n+3)}{2}}, k \leq \frac{n(n+3)}{2}$ .  
(In other words, in the exceptional case, we may have)  
 $P_i \in A \Rightarrow P'_{\frac{n(n+3)}{2}} = P_i$  In the argument above.)

Given any curve  $A_i$  of degree  $n$  passing through  $P_1, \dots, \hat{P}_i, \dots, P_k$ , then by the Cayley-Bacharach property,  $A_i + B$  contains  $P$ . Since  $B$  contains  $P_u, \forall u \geq k+1$ ,  $A_i$  must contain  $P_i \Rightarrow i_n(P_1), \dots, i_n(P_k)$  lie on a curve of degree  $n$ . "Remember that any degree  $n$  curve may be expressed as a linear combination of monomials  $Z^\alpha = Z_0^{\alpha_0} \dots Z_N^{\alpha_N}$ ." We don't need here.  
 $A_i \subset H \subset \mathbb{P}^N \Rightarrow \{i_n(P_1), \dots, i_n(P_k)\}$  is linearly dependent set.  $\Rightarrow$  In both cases,  $\{i_n(P_1), \dots, i_n(P_{\frac{n(n+3)}{2}})\}$  is linearly dependent.