$$\Rightarrow \quad H^r(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q}) \cong H^{r+1}(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q-1}) \quad \text{for } r>0.$$

$$\Rightarrow \quad H^r(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q}) \cong H^{r-1}(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q+1}) \cong H^{r-2}(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q+2}) \cong \cdots$$
$$\cong H^1(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q+r-1})$$

$$0 \longrightarrow Z_{\bar\partial}^{p,q+r-1} \longrightarrow \mathcal{A}^{p,q+r-1} \longrightarrow Z_{\bar\partial}^{p,q+r} \longrightarrow 0$$

$$\Rightarrow \quad H^0(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q+r-1}) \longrightarrow H^0(\underline{\mathcal{U}}, \mathcal{A}^{p,q+r-1}) \xrightarrow{\bar\partial^*} H^0(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q+r})$$
$$\longrightarrow H^1(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q+r-1}) \longrightarrow \underset{0}{\underset{\parallel}{H^1(\underline{\mathcal{U}}, \mathcal{A}^{p,q+r-1})}} \longrightarrow$$

$$\Rightarrow \quad H^1(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q+r-1}) \cong \qquad \cong \frac{H^0(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q+r})}{\bar\partial^* H^0(\underline{\mathcal{U}}, \mathcal{A}^{p,q+r-1})}$$
$$= \frac{Z_{\bar\partial}^{p,q+r}(M)}{\bar\partial\, \mathcal{A}^{p,q+r-1}(M)} = \frac{Z_{\bar\partial}^{p,q+r}(M)}{\bar\partial\, A^{p,q+r-1}(M)} = H_{\bar\partial}^{p,q+r}(M) \cong H^{q+r}(M, \Omega^p)$$

$\Rightarrow$ *Finally, we get* $H^r(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,q}) \cong H^{q+r}(M, \Omega^p)$, *and put* $q=0$, *then* $H^r(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,0}) \cong H^r(M, \Omega^p)$

$\Rightarrow$ *Since* $Z_{\bar\partial}^{p,0} = \Omega^p$,

$$H^r(\underline{\mathcal{U}}, Z_{\bar\partial}^{p,0}) = H^r(\underline{\mathcal{U}}, \Omega^p) \cong H^r(M, \Omega^p).$$

Computations.

1. M n-dim complex manifold.

$H^q(M, \mathcal{O}) \cong H_{\bar\partial}^{0,q}(M) = 0$ for $q > n$, since $d\bar z_I = 0$ if $|I| = q > n$, in general $dz_I \wedge d\bar z_J = 0$ $|I| + |J| > n$.