Date

For, $S_0 = z^2 g(z)$, $g(0) \neq 0$ around the center p

$\sigma = h(z)\,dz$, $h(z) \neq 0$.

$\Rightarrow \frac{S_0}{\sigma} = \frac{h(z)}{z^2}\,dz$. $\Rightarrow$ It has only a double pole at $z=0$ and has no residue.

It follows that if we let z_λ be a local coordinate around the point p_λ, for any sequence $a_1, a_2, \cdots a_d$ of complex numbers there exists a meromorphic 1-form on S, holomorphic on $S - \{p_\lambda\}$ and having principal part

$$\eta_a(z) = (a_\lambda z_\lambda^{-2} + [0])\,dz_\lambda \quad \text{at } p_\lambda.$$

$\sigma_1(p_1) \neq 0$, $\sigma_2(p_2) \neq 0$, $\cdots$ $\sigma_d(p_d) \neq 0$.

$(S_{01} = 0) = 2p_1$, $\cdots$ $(S_{0d} = 0) = 2p_d$.

$\Rightarrow \frac{\sigma_1}{S_{01}} + \frac{\sigma_2}{S_{02}} + \cdots + \frac{\sigma_d}{S_{0d}}$ is the meromorphic we want.

At p_λ. $h_1(z_\lambda)\,dz_\lambda + \cdots + \frac{h_\lambda(z_\lambda)}{z_\lambda^2}\,dz_\lambda + \cdots + h_d(z_\lambda)\,dz_\lambda$

$$= \left(\frac{h_\lambda(z_\lambda)}{z_\lambda^2} + f(z_\lambda)\right)dz_\lambda = (a'_\lambda z_\lambda^{-2} + [0])\,dz_\lambda$$

$\Rightarrow$ Take $b z_\lambda = z'_\lambda$. $\Rightarrow (a'_\lambda b^2 z_\lambda'^{-2} + [0])\, b^{-1} dz'_\lambda$

$= (a'_\lambda b + [0])\,dz'_\lambda \Rightarrow b = \frac{a_\lambda}{a'_\lambda}$. Fix z_λ, and make adjustment of σ_i.

*(Here fix z_λ's, and determine σ_i's to get a_λ's)

Since any two such forms differ by a holomorphic 1-form on S, we see moreover that there exists a unique such differential φ_a with all A-periods zero.