$\Rightarrow$ We have $\mathbb{C}^n = \oplus \langle v_{ij} \rangle$, ${}^t v_{ij} v_{kl} = \begin{cases} \neq 0 & \text{if } i=k, j=l \\ 0 & \text{if } i\neq k \text{ or } j\neq l. \end{cases}$

$\Rightarrow (Q(v_{ij}, v_{kl}))$ is a diagonal matrix, and $({}^t v_{ij} v_{kl})$ is a diagonal matrix, too.

In sum, given two nonsingular symmetric Q, Q', we can make Q, Q' into I, D by changing bases, where D is diagonal.

$\sqcup\!\sqcup$

The singular elements of the pencil

$$F_\lambda = \left(\sum_{i=0}^{5} (\lambda - \lambda_i) X_i^2 = 0 \right)$$

are then the six quadrics $F_{\lambda_0}, \cdots F_{\lambda_5}$. The map π is thus branched at the six points $\lambda_0, \cdots \lambda_5$; and consequently

The variety A of lines on the quadric line complex X given as the intersection of the two quadrics

$$G = (\Sigma X_i^2 = 0) \quad \text{and} \quad F = (\Sigma \lambda_i X_i^2 = 0)$$

is the Jacobian of the curve expressible as a double cover of $\mathbb{P}^1$ branched at the six points $\lambda_0, \cdots \lambda_5$.