$$\mathbb{H}^1(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}.(\mathcal{G}), \mathcal{F})) = H^1(C^*(\underline{\mathcal{U}}), D)$$

$\Rightarrow$ $e \in \mathrm{Ext}^1(M; \mathcal{G}, \mathcal{F})$ is given by a cocycle $\overset{\text{in}}{\in}$ $H^1(C^*(\underline{\mathcal{U}}), D)$.

In the diagram

$$\begin{tikzcd}
& C^0(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_1, \mathcal{F})) \arrow[dl, "\partial^*"'] \arrow[dr, "\delta"] & \oplus & C^1(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_0, \mathcal{F})) \arrow[dl, "\partial^*"'] \arrow[dr, "\delta"] & \\
C^0(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_2, \mathcal{F})) & \oplus & C^1(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_1, \mathcal{F})) & \oplus & C^2(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_0, \mathcal{F})).
\end{tikzcd}$$

the cocycle e is given by $\varphi \oplus \eta$, where

$\varphi = \{\varphi_\alpha\}$ with $\varphi_\alpha \in H^0(U_\alpha, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_1, \mathcal{F}))$,
$\eta = \{\eta_{\alpha\beta}\}$ with $\eta_{\alpha\beta} \in H^0(U_\alpha \cap U_\beta, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_0, \mathcal{F}))$.

$$\underset{\varphi \oplus \eta}{\overset{}{e}} \ \underset{\|}{} \in \underset{\overset{\Psi}{\varphi}}{C^0(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_1, \mathcal{F}))} \oplus \underset{\overset{\Psi}{\eta}}{C^1(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_0, \mathcal{F}))}$$

$\varphi \in C^0(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_1, \mathcal{F})) \Rightarrow \varphi = \{\varphi_\alpha\}$, $\varphi_\alpha \in H^0(U_\alpha, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_1, \mathcal{F})) \Rightarrow \varphi_\alpha \in \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_1, \mathcal{F})(U_\alpha)$

$$\underset{\{\eta_{\alpha\beta}\}}{\overset{\eta}{\|}} \in C^1(\underline{\mathcal{U}}, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_0, \mathcal{F})) = \prod_{\alpha \neq \beta} \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_0, \mathcal{F})(U_\alpha \cap U_\beta).$$

$$\eta_{\alpha\beta} \in \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_0, \mathcal{F})(U_\alpha \cap U_\beta) = H^0(U_\alpha \cap U_\beta, \underline{\mathcal{H}om}_{\mathcal{O}}(\mathcal{E}_0, \mathcal{F})).$$