$\Rightarrow \quad b_{123} - b_{213} + b_{312} = 1 = a_{11} a_{22} a_{33} - a_{21} a_{12} a_{33} + a_{21} a_{32} a_{13}$
$a_{31} a_{12} a_{23} - a_{31} a_{22} a_{13} = \det(A)$.

$$x_1\, dx_2 \wedge dx_3 = \boxed{1}\; x_1' dx_2' \wedge dx_3' + \square\; x_2' dx_1' \wedge dx_3' + \square\; x_3' dx_1' \wedge dx_2'$$
$$x_2\, dx_1 \wedge dx_3 = \boxed{2}\; x_1' dx_2' \wedge dx_3'$$
$$x_3\, dx_1 \wedge dx_2 = \boxed{3}\; x_1' dx_2' \wedge dx_3'$$

$$\boxed{1} = a_{11} (a_{22} a_{33} - a_{32} a_{23}) = a_{11} \begin{vmatrix} a_{22}, & a_{23} \\ a_{32}, & a_{33} \end{vmatrix}$$
$$\boxed{2} = a_{12} (a_{21} a_{33} - a_{31} a_{23}) = a_{12} \begin{vmatrix} a_{21}, & a_{23} \\ a_{31}, & a_{33} \end{vmatrix}$$
$$\boxed{3} = a_{13} (a_{21} a_{32} - a_{22} a_{31}) = a_{13} \begin{vmatrix} a_{21}, & a_{22} \\ a_{31}, & a_{32} \end{vmatrix}$$

Think of choosing numbers.

$\Rightarrow \quad \boxed{1} - \boxed{2} + \boxed{3} = \det A$.

$\Rightarrow$ Thus, in general, $x_1\, dx_2 \wedge dx_3 \wedge \cdots \wedge dx_n - x_2 \wedge dx_1 \wedge dx_3 \cdots$
$= \sum_{i=1}^{n} \Phi_i(x) = \Phi_1(x') + \Phi_2(x') \cdots$

$\Rightarrow \quad \sum \Phi_i(x) = \sum \Phi_i(x')$. if $Ax = x'$. with $\det A = 1$

$\Rightarrow$ σ is invariant under the proper orthogonal group. i.e $A^tA = I$ and $\det A = 1$.

The unitary group $U(n) = U(n)$ is path-connected by P92, Husemoller Fiber Bundles, and so the corresponding orthogonal matrix is path-connected to the identity, which implies that its determinant is $+1$.

$$\begin{pmatrix} \alpha, & \beta \\ \gamma, & \omega \end{pmatrix} \longmapsto \begin{pmatrix} \alpha_1, & -\alpha_2, & \beta_1, & -\beta_2 \\ \alpha_2, & \alpha_1, & \beta_2, & \beta_1 \\ \gamma_1, & -\gamma_2 & \omega_1, & -\omega_2 \\ \gamma_2, & \gamma_1, & \omega_2, & \omega_1 \end{pmatrix}$$
$$\Big\downarrow$$
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \longmapsto \begin{pmatrix} 1. & 0 & 0. & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0. & 0. & 1 \end{pmatrix}$$

Thus when we decompose σ into types, each component of which is invariant under the unitary group, since a untitary