First, we prove the

Lemma. If $Q(\ ,\ )$ is an integral, skew-symmetric quadratic form on $\Lambda = \mathbb{Z}^{2n}$, then there exits a basis $\lambda_1, \cdots \lambda_{2n}$ for Λ in terms of which Q is given by the matrix

$$Q = \begin{pmatrix} 0 & \Delta_\delta \\ -\Delta_\delta & 0 \end{pmatrix}, \qquad \Delta_\delta = \begin{bmatrix} \delta_1 & & 0 \\ & \ddots & \\ 0 & & \delta_n \end{bmatrix}, \quad \delta_i \in \mathbb{Z}.$$

proof. For each $\lambda \in \Lambda$, the set of values $\{ Q(\lambda, \lambda'), \lambda' \in \Lambda \}$ forms a principal ideal $d_\lambda \mathbb{Z}$ in $\mathbb{Z}$, $d_\lambda \geq 0$.

$\ulcorner$ $\lambda \in \Lambda$, $\{ Q(\lambda, \lambda'), \lambda' \in \Lambda \} = K$
$Q(\lambda, \lambda_1') + Q(\lambda, \lambda_2') = Q(\lambda, \lambda_1' + \lambda_2')$
$\alpha Q(\lambda, \lambda') = Q(\lambda, \alpha\lambda')$, $\alpha \in \mathbb{Z}$
$\Rightarrow$ K is an ideal of $\mathbb{Z}$ $\Rightarrow$ $\mathbb{Z}$ is a principal ideal domain $\Rightarrow$ $K = d_\lambda \mathbb{Z}$ for $d_\lambda \geq 0$. $\lrcorner$

Let $\delta_1 = \min\ (d_\lambda : \lambda \in \Lambda,\ d_\lambda \neq 0)$, and take λ_1 and λ_{n+1} such that $Q(\lambda_1, \lambda_{n+1}) = \delta_1$.

$\ulcorner$ $\delta_1 = d_{\lambda'}$ for some $\lambda' \in \Lambda$ $\Rightarrow$ $\{ Q(\lambda', \lambda) \mid \lambda \in \Lambda \}$ $= d_{\lambda'} \mathbb{Z}$ $\Rightarrow$ $Q(\lambda', \lambda_{n+1}) = d_{\lambda'}$ for some $\lambda_{n+1} \in \mathbb{Z}$ $\Rightarrow$ Let $\lambda' = \lambda_1$.