ian differential geometry. Let M be a complex manifold, $z = (z_1, \cdots z_n)$ local coordinates on M, and

$$ds^2 = \sum \varphi_i \otimes \overline{\varphi_i}$$

a hermitian metric on M with associated $(1,1)$-form ω. Write $\varphi_i = \alpha_i + \sqrt{-1}\beta_i$; then the associated Riemannian metric on M is

$$Re(ds^2) = \sum_i \alpha_i \otimes \alpha_i + \beta_i \otimes \beta_i,$$

and the volume element associated to $Re(ds^2)$ is given by

$$d\mu = \alpha_1 \wedge \beta_1 \wedge \cdots \wedge \alpha_n \wedge \beta_n.$$

$\ulcorner$ $ds^2 = \sum \varphi_i \otimes \overline{\varphi_i} = \sum (\alpha_i + \sqrt{-1}\beta_i) \otimes (\alpha_i - \sqrt{-1}\beta_i)$

$= \sum \alpha_i \otimes \alpha_i + \beta_i \otimes \beta_i - \sqrt{-1} \sum (\alpha_i \otimes \beta_i - \beta_i \otimes \alpha_i)$

$\Rightarrow Re(ds^2) = \sum \alpha_i \otimes \alpha_i + \beta_i \otimes \beta_i.$

Consider the case $n = 1$

$ds^2 = \varphi \otimes \overline{\varphi}$, where $\varphi = \alpha + i\beta$.

$\Rightarrow ds^2 = (\alpha + i\beta) \otimes (\alpha - i\beta) = \alpha \otimes \alpha + \beta \otimes \beta - i(\alpha \otimes \beta - \beta \otimes \alpha)$. Suppose v & ω satisfy the following

$ds^2(\ , v) = \alpha(\ )$ and $ds^2(\ , \omega) = \beta(\ )$.

$\Rightarrow \alpha(x) = \alpha(x)\alpha(v) + \beta(x)\beta(v)$ for all x

$\Rightarrow \alpha(x)(1 - \alpha(v)) - \beta(x)\beta(v) = 0$

$\Rightarrow$ Since α & β are linearly independent, $1 - \alpha(v) = 0$ and $\beta(v) = 0$. Similarly, $\beta(\omega) = 1$ and $\alpha(\omega) = 0$.

$\Rightarrow \langle \alpha, \alpha \rangle = 1 = \langle \beta, \beta \rangle$, $\langle \alpha, \beta \rangle = 0$.

We can generalize this result to n. More precisely, we have v_i, ω_i s.t $\alpha_i(x) = ds^2(x, v_i)$ and $\beta_i(x) = ds^2(x, \omega_i)$.