

②  $|f_{\Lambda_0}(2)|$  satisfies case 1.

This means that any  $\tau \in |f_{\Lambda_0}(2)|$  is of  $\tau = l l'$ ,  
By the correspondence above,  $\dim |f_{\Lambda_0}(2)| \geq 3$ . for a fixed line  $l'$ .

$\exists \tau_1, \tau_2, \tau_3, \tau_4 \in H^0(\mathbb{P}^2, f_{\Lambda_0}(2))$ , which are linearly independent

$$\Rightarrow \tau_1 = l_1 l' \quad \tau_2 = l_2 l' \quad \tau_3 = l_3 l' \quad \tau_4 = l_4 l'$$

$\Rightarrow$  Since  $\{\tau_1, \tau_2, \tau_3, \tau_4\}$  is linearly independent,  
 $\{l_1, l_2, l_3, l_4\}$  is linearly independent.

But since  $\dim H^0(\mathbb{P}^2, \mathcal{O}(H)) = \binom{2+1}{2} = 3$ , we can not have such  $l_i$ 's.

In case  $\# P_0' = 6$ , we can apply the same arguments.

In case  $\# P_0' = 3$  or 4.

① Case 2

By the Reciprocity Formula I,  $P_0'$  will impose independent conditions on  $|O_{\mathbb{P}^2}(2)|$  and

$$\dim |f_{P_0'}(2)| = 5 - \# P_0' \leq 2$$

which is impossible, since  $\dim |f_{P_0'}(2)| = \dim |f_{P_0'}(3)| \geq 3$ .

② Case 1.

Again by the same argument above we can get a contradiction.

Point: Assumption  $\dim |f_{P_0'}(3)| \geq 3$  the correspondence, so that we can conclude  $\dim |f_{P_0'}(2)| \geq 3$ ,  
and  $\dim H^0(\mathbb{P}^2, \mathcal{O}(H)) = 3$