Pf $V \xrightarrow{\pi_p} \mathbb{P}^{n-1}$ $\quad \pi_p(V) = \{ F_\alpha(X_0, X_1, \cdots X_{n-1}) = 0 \}$

$\overline{p,V} \ni [X_0, X_1, \cdots X_n] \Rightarrow \pi([X_0, X_1, \cdots X_n]) = [X_0, X_1, \cdots X_{n-1}] \Rightarrow$

Define $\tilde{F}$ on $\mathbb{P}^n$ by

$$\tilde{F}(X_0, X_1, \cdots X_n) = F(X_0, X_1, \cdots X_{n-1}) = F \circ \pi(X_0, \cdots X_n)$$

If $(X_0, \cdots X_{n-1})$ satisfies $F(X_0', \cdots X_{n-1}') = 0$,
$(X_0, \cdots X_n)$ satisfies $\tilde{F}(X_0', \cdots X_n') = 0$.
$\Rightarrow \overline{p,V} \subset \{ \tilde{F}_\alpha(X_0, X_1, \cdots X_n) = 0 \}$.

Suppose $[X_0, X_1, \cdots X_n]$ satisfies $\{ \tilde{F}_\alpha(X_0', \cdots X_n') = 0 \}$.
$\Rightarrow [X_0, \cdots X_{n-1}]$ satisfies $\{ F_\alpha(X_0', \cdots X_{n-1}') = 0 \}$. $\Rightarrow$
$[X_0, \cdots X_{n-1}] \in \pi_p(V) \Rightarrow$ For some q, $[X_0, X_1, \cdots X_{n-1}, q] \in V \subset \mathbb{P}^n$,
$\Rightarrow [X_0, \cdots X_n] \in \langle (X_0, \cdots X_{n-1}, q), (0, 0 \cdots 0, 1) \rangle = \mathbb{P}^1 \subset \overline{p,V}$.
$\Rightarrow \{ \tilde{F}_\alpha(X_0', \cdots X_n') = 0 \} \subset \overline{p,V}$.

Now if $H \subset \mathbb{P}^n$ is a generic hyperplane, not containing p, then the intersection of H with the cone $\overline{p,V}$ will be simply the projection $\pi_p(V)$ of V from p into H; so

$$\deg(\overline{p,V}) = \deg(H \cap \overline{p,V}) = \deg(\pi_p(V)) = \deg(V).$$

Pf $\mathbb{P}^{n-1} \cap \overline{p,V} = \pi_p(V)$. Take $H = \mathbb{P}^{n-1}$ and $p = [0, 0, \cdots, 1]$.
Clearly $\pi_p(V) \subset \mathbb{P}^{n-1}$.
If $[X_0, \cdots X_n] \in V$, $\pi_p([X_0, \cdots X_n]) = [X_0, X_1, \cdots X_{n-1}, 0] \in$
$\langle (X_0, \cdots X_n), (0, \cdots 0, 1) \rangle \subset \overline{p,V}$. $\Rightarrow \pi_p(V) \subset \mathbb{P}^{n-1} \cap \overline{p,V}$ -- ①

If $[Y_0, Y_1, \cdots Y_{n-1}, 0] \in \mathbb{P}^{n-1} \cap \overline{p,V}$, $[Y_0, Y_1, \cdots Y_{n-1}, 0] = [a(X_0, \cdots X_n) + b(0, 0 \cdots 0, 1)]$ where $[X_0, X_1, \cdots X_n] \in V$.
$\Rightarrow [Y_0, Y_1, \cdots Y_{n-1}] = [X_0, X_1, \cdots X_{n-1}, 0] = \pi([X_0, \cdots X_n]) \in \pi_p(V)$.
$\Rightarrow \mathbb{P}^{n-1} \cap \overline{p,V} \subset \pi_p(V)$ -- ②