

For example, $G = \mathbb{Z}^5 \oplus \text{Torsion group}$.

Let H be a subgr of G .

\Rightarrow The free part of H has rank less than 5 or equal to 5. Suppose not. $\Rightarrow \exists$ at least 6 vectors $v_1, \dots, v_6 \in \mathbb{Z}^5$, which are linearly indep.

\Rightarrow Consider $v_1, \dots, v_6 \in \mathbb{Q}^5$.

$\Rightarrow \exists \quad q_1 v_1 + \dots + q_6 v_6 = 0, \quad q_1, \dots, q_6 \in \mathbb{Q}$

$\Rightarrow \exists \quad n_1 v_1 + \dots + n_6 v_6 = 0, \quad \text{not all } n_i \text{ zero.}$

\Rightarrow Contradiction.

$\Rightarrow \frac{H}{F}$ is finitely generated, where F is the free part of H .

$\Rightarrow \quad 0 \rightarrow F \rightarrow H \rightarrow \frac{H}{F} \rightarrow 0$

$\Rightarrow H$ is finitely generated. \square

We have not dwelt on rational and algebraic equivalence of divisors or of general algebraic cycles, partly because we do not need these for our study of any specific varieties, and partly because the codimension-one theory is - at least as matters now stand - misleading as regards higher codimension cycles.

The Leray Spectral Sequence

This is in many ways the most useful general spectral sequence, and so we want at least to say what it is and give an illustration. Suppose we