$$\underbrace{(* \cdots * \;\; 0\,0 \cdots)}_{= {}^t v} \left(\begin{array}{ccc|c} * & K & * & \\ & \ddots & & \\ * & & * & \\ \hline & & & * \end{array}\right) \underbrace{\begin{pmatrix} * \\ \vdots \\ * \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{= v} \geq 0$$

$${}^t v K v \geq 0, \quad \Rightarrow \quad K \text{ is positive definite.}$$

Since the hyperplane bundle on $\mathbb{P}^n$ is positive, by the note, the hyperplane bundle on any complex submanifold of $\mathbb{P}^n$ is positive.

Our aim in this section is to prove that certain Čech cohomology groups $H^q(M, \Omega^p(L))$ associated to a positive line bundle $L \longrightarrow M$ are zero. To begin with, we transpose the problem into one involving $\bar{\partial}$-cohomology and harmonic forms by a technique that will be familiar from the previous discussion.

Recall that for any holomorphic vector bundle $E \to M$, the $\bar{\partial}$-operator

$$\bar{\partial} : A^{p,q}(E) \longrightarrow A^{p,q+1}(E)$$

is defined for global C^∞ E-valued differential forms, and satisfies $\bar{\partial}^2 = 0$. We let $Z_{\bar{\partial}}^{p,q}(E)$ denote the space of $\bar{\partial}$-closed E-valued differential forms of type (p,q), and we define the Dolbeault cohomology groups $H_{\bar{\partial}}^{p,q}(E)$ of E to be

$$H_{\bar{\partial}}^{p,q}(E) = \frac{Z_{\bar{\partial}}^{p,q}(E)}{\bar{\partial} A^{p,q-1}(E)} .$$

Let $\mathcal{Z}_{\bar{\partial}}^{p,q}(E)$ denote the sheaf of $\bar{\partial}$-closed E-valued (p,q)-forms.