

Now we may follow our cocycle ω through this sequence of isomorphisms. Beginning with

$$\omega_{n-1} = \left(\frac{1}{2\pi\sqrt{-1}} \right)^n \omega \in H^{n-1}(U^*, Z_{\partial}^{n,0}),$$

we let $U_I = \bigcap_{i \in I} U_i$ and

$$\omega_p = \{ \omega_{p,I} \in Z_{\partial}^{n,n-p-1}(U_I) \}_{\#I=p+1}$$

denote a representative of

$$\bar{i}_{p+1} \circ \dots \circ \bar{i}_{n-1}(\omega_{n-1});$$

and then let

$$\xi_p = \{ \xi_{p,I} \in Q^{n,n-p-1}(U_I) \}_{\#I=p}$$

be cochains such that

$$\delta \xi_p = \omega_p, \quad \bar{\partial} \xi_p = \omega_{p+1}.$$

$$\Gamma \quad Z_{\partial}^{n,0} = \Omega^n, \quad \omega_{n-1} = \left(\frac{1}{2\pi\sqrt{-1}} \right)^n \omega \in H^{n-1}(U^*, Z_{\partial}^{n,0}) = H^{n-1}(U^*, \Omega^n)$$

$$H^{n-1}(U^*, Z_{\partial}^{n,0}) \xrightarrow{\bar{i}_{n-1}} H^{n-2}(U^*, Z_{\partial}^{n,1}) \rightarrow \dots \rightarrow H^{p+1}(U^*, Z_{\partial}^{n,n-p-2}) \xrightarrow{\bar{i}_{p+1}} H^p(U^*, Z_{\partial}^{n,n-p-1})$$

$$\parallel$$

$$H^{n-1}(U^*, \Omega^n) = H_{\partial}^{n,n-1}(U^*)$$

$$H^{n-1}(U^*, \Omega^n) = H^{n-1}(U^*, Z_{\partial}^{n,0}) \xrightarrow{\bar{i}_{p+1} \circ \dots \circ \bar{i}_{n-1}} H^p(U^*, Z_{\partial}^{n,n-p-1})$$

$$\downarrow \omega_{n-1} \longmapsto \bar{i}_{p+1} \circ \dots \circ \bar{i}_{n-1}(\omega_{n-1}) = \omega_p$$

$$\omega_p \in C^p(U^*, Z_{\partial}^{n,n-p-1})$$

$$\Rightarrow \omega_{p,I} \in Z_{\partial}^{n,n-p-1}(U_I), \quad U_I = \bigcap_{i \in I} U_i$$

$$H^{p+1}(U^*, Z_{\partial}^{n,n-p-2}) \xrightarrow{\bar{i}_{p+1}} H^p(U^*, Z_{\partial}^{n,n-p-1}) \rightarrow H^{p-1}(U^*, Z_{\partial}^{n,n-p})$$

$$\downarrow \omega_{p+1} \longmapsto \omega_p \longmapsto \omega_{p-1}$$