

Since I_p is a hyperplane in W , see P150,
 $I_p = H_p$, by P151, $I_p \sim \tilde{w}$.

Let $\{C_\lambda\}$ be a generic pencil. $\Rightarrow \{C_\lambda\}$ cuts out
 on l a linear system of degree 2 without base
 points, refer to P150. \Rightarrow The corresponding map
 expresses l as a 2-sheeted cover of P^1 ; and
 by the Riemann-Hurwitz formula, the # of branch
 points of this map is

$$b = 2g(P^1) - 2 + 2\chi(P^1) = 2 \cdot 0 - 2 + 2 \cdot 2 = 2.$$

\Rightarrow The pencil $\{C_\lambda\}$ contains 2 conics tangent to l ,
 and $\deg I_e = 2 \Rightarrow I_e \sim 2\tilde{w}$ ①

$2L$ a generic double line, and $\{C_\lambda\}$ a generic
 pencil of conics containing $2L$, \Rightarrow Again $\{C_\lambda\}$
 cuts out on l a pencil of degree 2 without base
 points. \Rightarrow The corresponding map has 2 branch points
 as before - but one of these two is just the point
 of intersection of L with l . $\Rightarrow \{C_\lambda\}$ has
 one point of intersection with I_e other than $2L$;
 it follows that

$$\text{mult}_{2L}(\{C_\lambda\}, I_e) = 1,$$

and so,

$$\text{mult}_W(I_e) = 1. \dots \dots \dots \textcircled{2}$$

\Rightarrow By the result on P605 together with ① & ②,
 $\tilde{I}_e \sim 2\tilde{w} - e$.

By P151, $\tilde{V}_c \sim 6\tilde{w} - 2e$ and $2\tilde{I}_p + 2\tilde{I}_e \sim$
 $2\tilde{w} + 4\tilde{w} - 2e = 6\tilde{w} - 2e \Rightarrow \tilde{V}_c \sim 2\tilde{I}_p + 2\tilde{I}_e. \quad \square$