n — i.e., by $H^0(\mathbb{P}^2, \mathcal{O}(nH))$.

$$\frac{n^2+3n}{2} = \binom{n+2}{2} - 1 = \frac{(n+2)(n+1)}{2} - 1$$

$$\mathbb{P}^2 \hookrightarrow \mathbb{P}^N \quad \text{see P178} \quad N = h^0(\mathbb{P}^2, \mathcal{O}(nH)) - 1$$

$$[z_0, z_1, z_2] \longmapsto [\cdots z^\alpha \cdots], \quad z^\alpha = z_0^{\alpha_0} z_1^{\alpha_1} z_2^{\alpha_2}, \quad \alpha_0 + \alpha_1 + \alpha_2 = n.$$

Since the hyperplane sections of $i_n(\mathbb{P}^2)$ are just the curves of degree n, we must show that:

(**) The points $i_n(p_\nu)$ lie on a $\mathbb{P}^{N-2}$ in $\mathbb{P}^N$.

This in turn will be the case if any N of the points $i_n(p_\nu)$ are linearly dependent in $\mathbb{P}^N$.

If any N of points $i_n(p_\nu)$ are linearly dependent in $\mathbb{P}^N$, $\{i_n(p_\nu)\}_{\nu=1}^{n^2}$ lie on a $\mathbb{P}^{N-2}$. For choose $\{i_n(p_\nu)\}$ $\nu = 1 \cdots N$. $\Rightarrow$ Since they are linearly dependent, $\exists$ a $\mathbb{P}^{N-1-1}$ in $\mathbb{P}^N$, and so if we consider $\{i_n(p_\nu)\}_{\nu=1}^{N} \cup \{i_n(p_\nu)\}_{\nu=N+1}$, then since they are lineal depent, they are on the $\mathbb{P}^{N-2}$ again ($\because$ $i_n(p_\nu)_{\nu=N+1}$ may be expressed as a linear combination of $\{i_n(p_\nu)\}_{\nu=1}^{N}$.)

$i_n : \mathbb{P}^2 \longrightarrow \mathbb{P}^N$ is smooth embedding as we saw on P178. Since $\{z^\alpha\}_{|\alpha|=n}$ is a basis, i.e lineal independt, $i_n(\mathbb{P}^2)$ does not lie on any hyperplane in $\mathbb{P}^N$. $\Rightarrow$ $i_n(\mathbb{P}^2) \cap H$ has dimension 1, since $2 + N-1 - N = 1$. more precisely.

for example $n=3$; $\mathbb{P}^2 \xrightarrow{i_3} \mathbb{P}^5$