along the line $\pi^{-1}(z_0')$, $h(z_0', z_n)$ is independent of z_n.
By the choice of x, $h(z_0', z_n)$ is a constant independent of z' & z_n. $\Rightarrow$ F & F' are relatively prime.
For the general case,

$$f_1 + x_2 f_2 + f_3 + x_4 f_4 + \cdots = 0$$ will work

$$f_1 + x_2 f_2 + 2f_3 + x_4 f_4 + \cdots = 0.$$

in the same way. ⌟

Granted assertions 3 and 1, 2 is not hard to prove by a sequence of projections.

⌜ By assertion 1, we may assume V is irreducible at 0. $\Rightarrow V = \{f_1 = \cdots = f_K = 0\}$.
$\Rightarrow$ By the irreducibility of V at 0, $f_1, f_2, \cdots$ & f_K are irreducible in $\mathcal{O}_n$, and assume that $f_1, f_2 \cdots f_K$ are Weierstrass polynomials in z_n.

For simplicity, $k = 2$.
$V = \{f_1 = f_2 = 0\}$. We want to show that $V \xrightarrow{\pi} \pi(V)$ is a finite-sheeted branched covering.
Let $z \in \mathbb{C}^{n-1}$ s.t $f_1(z, z_n)$ & $f_2(z, z_n)$ have distinct roots. $\Rightarrow$ By the inverse function theorem, $\exists$ locally $z_n = h_1(z)$, $z_n = h_2(z)$, $z_n = g_1(z)$, $z_n = g_2(z)$, $z_n = g_3(z)$ s.t $f_1(h_i(z), z_n) = 0$ and $f_2(g_i(z), z_n) = 0$.
$\Rightarrow$ If $h_1(z) = g_1(z)$ for some z, $h_2(z) \neq g_2(z)$ locally.