

This implies $d_1 \bar{\varphi} = \overline{d\varphi}$.

$$\text{Thus } E_2^{p,q} = \frac{\ker d_1}{\text{im } d_1} = H_{\text{DR}}^p(B, H_{\text{DR}}^q(\bar{F}))$$

$H_{\text{DR}}^q(\bar{F})_x = (\tilde{B} \times_p H_{\text{DR}}^q(\bar{F}))_x = [(x, g, v)]$, where $g \in \pi_1(B)$ and $v \in H_{\text{DR}}^q(\bar{F})$. And as stated on P463,

$$H_{\text{DR}}^q(\bar{F})(U) = R_{\pi}^q(\mathcal{O})(U)$$

\uparrow
locally constant sheaf over U . \Rightarrow

These spectral sequences are generally nontrivial — i.e., $E_2 \neq E_{\infty}$ — and may be extremely complicated. Even the simplest nontrivial fibration, the Hopf fibration,

$$\pi: S^{2n+1} \longrightarrow \mathbb{P}^n,$$

has an interesting spectral sequence: The fiber is the circle S^1 , and since \mathbb{P}^n is simply connected,

$$E_2^{p,q} \cong H^q(S^1) \otimes H^p(\mathbb{P}^n).$$

$$\mathbb{F} \quad E_2^{p,q} = H_{\text{DR}}^p(\mathbb{P}^n, H_{\text{DR}}^q(S^1)) = H_{\text{DR}}^p(\mathbb{P}^n, H_{\text{DR}}^q(S^1))$$

$$H_{\text{DR}}^q(S^1) = \mathbb{P}^n \times_p H_{\text{DR}}^q(S^1), \text{ where}$$

$$p: \pi_1(\mathbb{P}^n) \longrightarrow \text{Aut}(H_{\text{DR}}^q(S^1)), \text{ but since } \pi_1(\mathbb{P}^n) = 0, \quad p(\pi_1(\mathbb{P}^n)) = \text{identity of } H_{\text{DR}}^q(S^1) \Rightarrow H_{\text{DR}}^q(S^1) = \mathbb{P}^n \times H_{\text{DR}}^q(S^1) \Rightarrow E_2^{p,q} = H_{\text{DR}}^p(\mathbb{P}^n, H_{\text{DR}}^q(S^1)) = H_{\text{DR}}^p(\mathbb{P}^n) \otimes H_{\text{DR}}^q(S^1)$$

by the universal coefficient theorem, since $H_{\text{DR}}^p(\mathbb{P}^n)$ & $H_{\text{DR}}^q(S^1)$ are torsion-free. \Rightarrow

Figure 6 pictures the E_2 term. If $\eta \in E_2^{0,1} \cong H^1(S^1)$ is a generator, then $d_2 \eta \neq 0$, since $H^q(S^{2n+1}) = 0$ for $q \neq 0, 2n+1$.