$$\dim|\mathcal{J}_{P_0}(n)| = \dim|L|$$
$$= \left\{ \frac{n(n+3)}{2} - d\right\} + \omega,$$

where $h^0(\mathcal{J}_P(K_S+L)) = \dim|\mathcal{J}_P(n-3)|+1$. Q.E.D.

⌈ By P712, $K_S = \pi^* H^{-3} + E \Rightarrow K_S + L = \pi^* H^{-3} + E + \pi^* H^n - E = \pi^* H^{n-3}$.

By P713, $\dim|\mathcal{J}_{P_0}(n)| = \dim|L| = \left\{\frac{n(n+3)}{2} - d\right\} + \omega$

$\omega = h^1(\mathcal{J}_{P_0}(n)) = \dim H^1(\mathbb{P}^2, \mathcal{O}(\pi^* H^n - E)) = \dim H^1(\mathbb{P}^2, \Omega(-nH+E)) = \dim H^0(S, I \otimes_{\mathcal{O}} \mathcal{O}(K_S)) - 2h^0(\mathcal{O}(K_S)) + h^0(\mathcal{O}(K_S - L)) = \dim H^0(S, I\otimes_{\mathcal{O}} \mathcal{O}(K_S))$ since $h^0(\mathcal{O}(K_S)) = P_g = \dim H^{2,0}(S)$ and $h^0(\mathcal{O}(K_S - L)) = \dim|K_S - L| + 1 = 0$.

$\Rightarrow \omega = h^0(S, I\otimes_{\mathcal{O}}\mathcal{O}(K_S))$ I feel strange. According to P716, the authors meant that C&C' intersects each other transversely by general curves. If it is correct, $I\otimes_{\mathcal{O}}\mathcal{O}(K_S)$ is simply $\mathcal{J}_P(K_S+L)$ = Sheaf of holomorphic sections of K_S+L vanishing on P.

My concern is this: Suppose $\dim|L| > 0$. $|L|$ has no base curves & $q=0$. Then, necessarily, $\exists$ general curves C, C $\in |L|$?

⊛ If $\exists$ no general curves. then that's O.K. Forget the Formula.

⊛ If $\exists$ general curves, we have the sound Formula, that's it. Make everything simpler & See P513 ~ P520.