The quadric line complex X is determined up to isomorphism by the abstract variety A of lines lying on it.

$\ulcorner$ $B \hookrightarrow \mathbb{P}^3 \Rightarrow$ blow up the embedded curve $E_0 \Rightarrow$ blow down the family of proper transforms $\Rightarrow$ B determines X $\Rightarrow$ Since B is determined by A ($\because$ $B \cong B_L$, $L \in A$), X is determined by A up to isomorphism.

$\lrcorner$

Note, incidentally, that the preceding gives us another characterization of the special lines on X: for any line $L \subset X$ we have a C^∞ decomposition of vector bundles on L:

$$T(\mathbb{P}^5)|_L = N_{X/\mathbb{P}^5}|_L \oplus N_{L/X} \oplus T(L).$$

$\ulcorner$ $T(\mathbb{P}^5)|_L = (T(X) \oplus N_{X/\mathbb{P}^5})|_L = N_{X/\mathbb{P}^5}|_L \oplus T(X)|_L$
$= (T(L) \oplus N_{L/X})|_L \oplus N_{X/\mathbb{P}^5}|_L = T(L) \oplus N_{X/\mathbb{P}^5}|_L \oplus N_{L/X}|_L$
$= N_{X/\mathbb{P}^5}|_L \oplus N_{L/X} \oplus T(L)$.

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Now we have

$$c_1(T(\mathbb{P}^5))|_L = 6,$$
$$c_1(N_{X/\mathbb{P}^5}|_L) = c_1(N_{F/\mathbb{P}^5}|_L) + c_1(N_{G/\mathbb{P}^5}|_L)$$
$$= 2 + 2 = 4,$$