⇒ Thus the tangent space to W_1 at a smooth point $F = l_1 + l_2$ is just the plane $H \subset W$ of conics passing through the point $p \in l_1 \cap l_2 \in \mathbb{P}^2$.

Again we may assume $F = 2l = X_0^2$.

To find the tangent cone to W_1 at F,

$$\frac{\partial^2 W_1}{\partial X_2 \partial X_1} = X_0 \qquad \frac{\partial^2 W_1}{\partial X_1 \partial X_2} = X_0 \qquad \frac{\partial^2 W_1}{\partial X_5^2} = -2X_0.$$

otherwise 0 at $F = [1, 0 \cdots 0]$.

⇒ The tangent cone is $X_1 X_2 - X_5^2 = 0$.

Let G be an element in the tangent cone.

⇒

$$(X_0, X_1, X_2)\begin{pmatrix} a_0 & a_3 & a_4 \\ a_3 & a_1 & a_5 \\ a_4 & a_5 & a_2 \end{pmatrix}\begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} = 0 \quad : G.$$

$a_1 a_2 - a_5^2 = 0$

⇒ $X_0 = 0$ ⇒

$$(0, X_1, X_2)\begin{pmatrix} a_0 & a_3 & a_4 \\ a_3 & a_1 & a_5 \\ a_4 & a_5 & a_2 \end{pmatrix}\begin{pmatrix} 0 \\ X_1 \\ X_2 \end{pmatrix} = 0$$

⇒ $a_1 X_1^2 + 2a_5 X_1 X_2 + a_2 X_2^2 = 0$

⇒ $a_1 a_2 - a_5^2 = 0 \iff G$ is a conic tangent to X_0.

⇒ Thus the tangent cone to W_1 at a double point $F = 2l$ is the locus of conics tangent to l.

⅃