

By $P672 - P1 = P671$, L will not have the line (the hyperplane $= \{ [0, x_1, x_2] \}$) at infinity as a component of its polar divisor provided that $\deg(p) \leq mn - 2 = \deg(f)^m + \deg(x) - (2+1) = mn + 1 - 3 = mn - 2$. \Rightarrow "For affine, see P166."

In case $m=2$, the restriction $\deg(p) \leq 2n-2$ suggests taking p to be of the form $p(x,y) = \alpha f_{xx} f + \beta f_{xy} f + \gamma f_{yy} f + \delta f_x^2 + \epsilon f_x f_y + \kappa f_y^2$.

$\deg f_{xx} = n-2$. $\deg f = n$. $\deg(f_{xy}) = \deg(f_{yx}) = \deg f_{yy} = n-2$ Jacobi relation.
 $\deg f_x = n-1 = \deg f_y \Rightarrow$ By looking at the relation $(*)$ on P671, we may guess $p(x,y) = \alpha f_{xx} f + \beta f_{xy} f + \gamma f_{yy} f + \delta f_x^2 + \epsilon f_x f_y + \kappa f_y^2$. \Rightarrow

To see what p to choose, we assume that the origin is one of the points of intersection and will prove the

Lemma.

$$\text{Res}_{x,y} \left(\frac{p(x,y) dx \wedge dy}{x f(x,y)^2} \right) = \frac{p_y}{f_y^2} - \frac{p f_{yy}}{f_y^3}.$$

Proof of Lemma. This is an application of the transformation formula from Section 2. We may assume that $f(x,y)$ has a Taylor series

$$f(x,y) = ax + by + \frac{cx^2}{2} + dxy + \frac{ey^2}{2} + \dots, \quad b \neq 0.$$

Since the n points of intersection of C with L