Λ'' only in the point p. ⫽

Thus, by our first argument, Λ and Λ'' belong to the same family on F if and only if $n \equiv 0\ (2)$; it follows that

$$\Lambda \text{ and } \Lambda' \text{ belong to the same family}$$
$$\Leftrightarrow \quad n \equiv -1 = \dim(\Lambda \cap \Lambda')\ (2).$$

Γ Our first argument means the proof of the case $\Lambda \cap \Lambda'' \neq \phi$. $\Rightarrow$ Λ and Λ'' belong to the same family on F if and only if $n \equiv 0\ (2)$.

Λ and Λ' belong to the same family $\Leftrightarrow$

Λ and Λ'' belong to opposite families $\Leftrightarrow$

$n \not\equiv 0\ (2) \Leftrightarrow n \equiv -1\ (2) \Leftrightarrow n \equiv -1 = \dim(\Lambda \cap \Lambda')\ (2)$

since $\dim(\Lambda \cap \Lambda') = \dim \phi = -1$. ⫽

This completes the proof of the proposition.

We can write down explicitly the two families of n-planes on the smooth $2n$-dimensional quadric $F \subset \mathbb{P}^{2n+1}$ given by

$$Q(X) = \sum_{i=0}^{n} X_i X_{n+i+1}.$$

In this case for B any $(n+1) \times (n+1)$ matrix the n-plane Λ_B spanned by the row vectors $e_i = (0, \cdots 1 \cdots 0, b_{i,0} \cdots b_{i,n})$ of the $2(n+1) \times (n+1)$ matrix (I, B) lies in F if and