to compute the ordinary cohomology $H^*(M, \mathbb{C})$ of a complex manifold M purely in terms of the holomorphic differentials. First, note that the Poincare lemma holds for these forms: If φ is a closed holomorphic p-form ($p>0$), then locally $\varphi = d\eta$ for a holomorphic $(p-1)$-form η. The proof may be done by the same method as the $\bar\partial$-Poincare lemma — a much more sophisticated lemma will be proved when we discuss the log complex in the next example.

「 $H^{0,p}_{\bar\partial}(\Delta)=0$, $p>0$ by the $\bar\partial$-Poincare lemma on P25. Suppose φ is a closed p-form.
$\Rightarrow \exists\ \eta \in A^{0,p-1}(\Delta)$ s.t $\bar\partial\eta = \varphi$. We don't know that η is holomorphic. Need some work. 」

Now the holomorphic de Rham complex

$$\Omega^0 \xrightarrow{d} \Omega^1 \xrightarrow{d} \Omega^2 \to \cdots$$

and trivial

$$\mathbb{C} \to 0 \to 0 \to$$

are such that the inclusion

$$\mathbb{C}^* \longrightarrow (\Omega^*, d)$$

is a quasi-isomorphism, and repeating the previous argument gives

$$(*) \qquad H^*(M, \mathbb{C}) \cong \check{\mathbb{H}}^*(M, \Omega^*),$$

expressing the complex Čech cohomology in terms of the holomorphic forms.

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$$\begin{array}{ccccccc}
\mathbb{C} & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow \\
\downarrow f_0 & & \downarrow f_1 & & \downarrow f_2 & \\
\Omega^0 & \xrightarrow{d} & \Omega^1 & \xrightarrow{d} & \Omega^2 & \longrightarrow
\end{array}$$