Correction : Let $\sigma_1(p_i(l)) = p_1(l) + p_2(l) + p_3(l)$, $\sigma_2(p_i(l)) = p_1(l)\,p_2(l) + p_2(l)\,p_3(l) + p_1(l)\,p_3(l)$
$\sigma_3(p_i(l)) = p_1(l)\,p_2(l)\,p_3(l)$ are expressed as a holomorphic function of a_0, a_1, a_2
and $\sum \int_{p_0}^{p_i(l)} \frac{dx}{y}$ is expressed as a holomorphic function of $\sigma_1, \sigma_2, \sigma_3$. $\Rightarrow$

$l \overset{\text{denote}}{=} L = \{ a_0 Z_0 + a_1 Z_1 + a_2 Z_2 = 0 \}$

$\Rightarrow \{ p_1(L), \ p_2(L), \ p_3(L) \} = L \cap C = (y^2 = x^3 + ax^2 + bx + c)$. $\Rightarrow \{ a_0 + a_1 x + a_2 y = 0 \} = L$

$\Rightarrow \ a_2 y = -a_1 x - a_0 \ \Rightarrow \ y = -\frac{a_1}{a_2} x - \frac{a_0}{a_2}$ assuming $a_2 \neq 0$

$\Rightarrow \ \left(-\frac{a_1}{a_2} x - \frac{a_0}{a_2} \right)^2 = x^3 + ax^2 + bx + c$.

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$\Rightarrow$ $x(p_1)$, $x(p_2)$, $x(p_3)$ are expressed as functions of a, b, c, a_0, a_1, a_2 $\Rightarrow$ $y(p_1), \cdots y(p_3)$ are so if they are distinct

$\Rightarrow$ Clearly. $\sum \int_{p_0}^{p_i} \omega$ is holomorphic.

In general, $g(z, x_1, \cdots x_5) = z^5 + x_1 z^4 + x_2 z^3 + x_3 z^2 + x_4 z + x_5$ and α is a simple zero of $g(z) = 0$ at $x_1 = a_1$, $x_2 = a_2$, $x_3 = a_3$, $x_4 = a_4$, $x_5 = a_5$.

Then locally, the zeros around α are expressed as a holo. function of $a_1, a_2, \cdots a_5$.

Consider the following map

$$F : \mathbb{C}^6 \longrightarrow \mathbb{C}^6$$

$$(z, x_1, x_2, \cdots x_5) \longmapsto (g(z, x_1 \cdots x_5), \ x_1, \cdots, x_5)$$

$$\Rightarrow J(F) = \begin{pmatrix} \frac{\partial g}{\partial z}, & \frac{\partial g}{\partial x_1}, & \cdots & \frac{\partial g}{\partial x_5} \\ 0, & 1, & 0 \cdots & 0 \\ 0 & 0, & 1 \cdots & 0 \\ 0 & & & \vdots \\ \vdots & & & \\ 0 & & & 1 \end{pmatrix}$$

$\Rightarrow \det J(F) \neq 0$ at $(z, x_1, \cdots x_5) = (\alpha, a_1, a_2 \cdots a_5)$.

$\Rightarrow$ By the inverse function theorem, $\exists$ G s.t $G \circ F = id$. $\Rightarrow G(g(z, x_1 \cdots x_5), x_1 \cdots x_5) = (z, x_1, x_2 \cdots x_5)$

$\Rightarrow$ On $g(z, x_1 \cdots x_5) = 0$, $G(0, x_1, \cdots x_5) = (z, x_1, \cdots x_5)$.