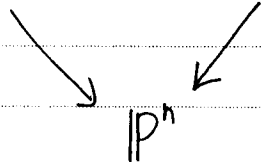


Thus given  $\sigma \in H^{\oplus(n+1)}$  with  $\mathcal{E}(\sigma) = 0$ , at each pt  $[X_0, \dots, X_n]$ , some number is obtained uniquely.

$\Rightarrow$

$$\mathbb{P}^n \times \mathbb{C} \xrightarrow{\cong} \ker \mathcal{E}$$

$$(X, \lambda) \longmapsto \sigma, \quad \sigma(X) = (\lambda X_0, \dots, \lambda X_n)$$



$\Rightarrow$

Thus we have an exact sequence of bundles on  $\mathbb{P}^n$ :

$$0 \rightarrow \mathbb{C} \rightarrow H^{\oplus(n+1)} \xrightarrow{\mathcal{E}} T'(\mathbb{P}^n) \rightarrow 0,$$

called the Euler sequence. Now, from the  $C^\infty$  decomposition

$$H^{\oplus(n+1)} = T'(\mathbb{P}^n) \oplus \mathbb{C} \quad \text{[See Milnor P35 Problem 3-B]}$$

and the Whitney product formula, we find

$$C(T'(\mathbb{P}^n)) = C(H^{\oplus(n+1)}) = C(H)^{n+1} = (1 + \omega)^{n+1},$$

where  $\omega = \eta_H \in H^2(\mathbb{P}^n, \mathbb{Z})$  is the class of a hyperplane.

## The Gauss - Bonnet Formulas

As we have seen, the first Chern class of a holomorphic line bundle is Poincaré dual to the cycle represented by the zero-locus of a global holomorphic section.

[See P141. Proposition 2]

$\Rightarrow$

A similar geometric description of the general Chern class