To begin with, suppose that $V \subset U \subset R^n$ are open in R^n with relatively compact in the next. ($\bar{V} \subset U \subset \bar{U} \subset R^n$, $\bar{V}$. $\bar{U}$ compact.)
Functions with compact support in U may be considered as functions on a torus.

Suppose that $v_1(x), v_2(x) \dots v_n(x)$ are C^∞ vector fields in V that are every where linearly independent, and that $\rho(x)$ is positive function on V. For $\varphi \in C_c^\infty(U)$, the Sobolev 0- and 1- norms are equivalent to

$$\int_V \rho(x) \, |\varphi(x)|^2 dx \quad , \quad \int_V \rho(x) \{ |\varphi(x)|^2 + \sum_i |\underset{\substack{\uparrow \\ \text{directional derivative.}}}{v_i(x) \cdot \varphi(x)}|^2 \} dx.$$

respectively. More generally, note that the commutator

$$[v_i, v_j] \cdot \varphi = v_i(v_j \varphi) - v_j(v_i \varphi) \quad \text{is an}$$

operator of order 1, where an operator of order is one involving at most s-derivatives and denoted by a generic symbol $A^s \varphi$. An expression

$$v^\alpha(\varphi) = v_1^{\alpha_1}(v_2^{\alpha_2} \cdot \quad (v_n^{\alpha_n} \varphi) \cdots)$$

is independent of the ordering modulo operators of order $< [\alpha]$. It follows that the Sobolev s-norm of $\varphi \in C_c^\infty(U)$ is equivalent to

$$\sum_{[\alpha] \le s} \int |v^\alpha \varphi(x)|^2 dx .$$

Γ Since $0 < m \le \rho(x) \le M$ for all $x \in V$,

$$m \int_V |\varphi(x)|^2 dx \le \int_V \rho(x) \, |\varphi(x)|^2 dx \le M \int_V |\varphi(x)|^2 dx$$