$$\int_U (\varphi - \log|h|)\,\partial\psi = \int_U \varphi\,\partial\psi - \int_U (\log|h|)\,\partial\psi$$

$$= (\partial\varphi)(\psi) - \int_U (\log|h|)\,d\psi = (\partial\varphi)(\psi) - (d\log|h|)(\psi)$$

$= (dj)(\psi)$ $\Rightarrow$ In the distributional sense.

$\partial\varphi = d\log|h| + dj$ (on U) $\leftarrow$ locally.

$\Rightarrow$ $\partial\varphi$ is closed, and meromorphic 1-form on $U \subset \Delta^{n+1} - f(W)$. $\Rightarrow$ $\partial\varphi$ is closed and meromorphic 1-form on $\Delta^{n+1} - f(W)$.

Wrong!

$\varphi - \log|h| = f$, $\quad j = f + \sqrt{-1}\,g \Rightarrow \partial j = \partial f + \sqrt{-1}\,\partial g$

$\Rightarrow \partial\bar{j} = \partial f - \sqrt{-1}\,\partial g = 0 \quad \Rightarrow \partial f = \sqrt{-1}\,\partial g$.

For $\psi \in A_c^{n,n+1}(U)$,

$$(\partial\varphi)(\psi) = -\int_U \varphi\,\partial\psi = -\int_U (\log|h|)\,\partial\psi$$

$$- \int_U f\,\partial\psi.$$

$$\Rightarrow \int_U f\,\partial\psi = \int_U \partial(f\psi) - \int_U \partial f \wedge \psi$$

$$= \int_U d(f\psi) - \int_U \partial f \wedge \psi$$

$$= 0 - \int_U \partial f \wedge \psi = -\frac{1}{2}\int_U \partial j \wedge \psi$$

by Stokes' theorem and $\operatorname{supp}\psi \subset U$.