

The only thing that remains to be proved is that  $\mu^{(g)}$  is generically one to one. But this is clear: by Abel's theorem the fiber of  $\mu^{(g)}$  over any point  $\lambda \in J(S)$  consists of the set  $|D|$  of effective divisors linearly equivalent to any divisor  $D \in \mu^{(g)-1}(\lambda)$ , which is a projective space.

¶ By Abel's theorem, if  $\mu^{(g)}(D) = \mu^{(g)}(D')$ , then  $\exists$  a meromorphic function  $f$  on  $S$  s.t.  $D = D' + (f)$ .  
 For,  $\mu^{(g)}(D) = \mu(D - gP_0) = \mu^{(g)}(D') = \mu(D' - gP_0)$ .  
 $\Rightarrow$  By Abel's theorem,  $D - gP_0 = D' - gP_0 + (f)$  for some meromorphic function  $f$  on  $S$ .  $\Rightarrow D = D' + (f)$ .  
 $D \in S^{(g)} \Rightarrow D$  is an effective divisor.  $\Rightarrow$  The fiber of  $\mu^{(g)}$  over any point  $\lambda \in J(S) = |D| \cong P(H^0(S, \mathcal{O}(D)))$ , see P.137.  $\square$

On the other hand, by dimension considerations the generic fiber of  $\mu^{(g)}$  is 0-dimensional; it follows that the generic fiber of  $\mu^{(g)}$  is one point. (The map  $\mu^{(g)}$  is an example of a birational map; we shall discuss these in detail in Chapter 4.) Q.E.D.

¶  $\mu^{(g)}: S^{(g)} \rightarrow J(S) = \mathbb{C}^g / \Lambda \Rightarrow \dim(J(S)) = g$   
 and  $\dim(S^{(g)}) = g \Rightarrow \mu^{(g)}$  is non-singular at generic points.  $\Rightarrow \dim \text{fiber} + \dim J(S) = \dim S^{(g)} \Rightarrow \dim \text{fiber} = 0$  at