of the exterior algebra $E^{2n}(dz_k, d\bar{z}_k)$ i.e., it satisfies condition (5) for each system L^{n-p} of purely linear forms $\alpha_1, \cdots \alpha_{n-p}$.

Remarks.

(1) Assume that on W^n a metric is given by a hermitian positive form expressed in the local coordinates $dz_k, d\bar{z}_k$:

$$\sum g_{p\bar{q}}\, dz_p\, d\bar{z}_q = ds^2$$

Set

$$\Omega = \frac{i}{2} \sum g_{p\bar{q}}\, dz_p \wedge d\bar{z}_q .$$

Then we can replace τ_n in condition (5) by the "element of volume" $\frac{1}{n!}\Omega^n$.

Pf

$$\phi \wedge (i\alpha_1 \wedge \bar{\alpha}_1) \wedge \cdots (i\alpha_{n-p} \wedge \bar{\alpha}_{n-p}) = l(\phi, L^{n-p})\, \tau_n$$

$$\Omega \wedge (i\, dz_1 \wedge d\bar{z}_1) \wedge \cdots (i\, dz_{n-1} \wedge d\bar{z}_{n-1})$$
$$= \frac{(i)^n}{2} (dz_1 \wedge d\bar{z}_1 \wedge \cdots \wedge dz_n \wedge d\bar{z}_n)\, g_{n\bar{n}}$$

$$\Omega^n = \left(\frac{i}{2}\right)^n \sum g_{1\overline{\sigma(1)}}\, g_{2\overline{\sigma(2)}} \cdots g_{n\overline{\sigma(n)}} \quad \epsilon(\sigma)\, dz_1 \wedge d\bar{z}_1 \wedge \cdots \wedge dz_n \wedge d\bar{z}_n$$
$$= \left(\frac{i}{2}\right)^n \det(g_{i\bar{j}})\, dz_1 \wedge d\bar{z}_1 \wedge \cdots \wedge dz_n \wedge d\bar{z}_n$$

$$\Rightarrow \frac{\Omega^n}{n!} = \frac{1}{n!} \left(\frac{i}{2}\right)^n \det(g_{i\bar{j}})\, \Phi(z) \wedge \overline{\Phi(z)}\, (-1)^{\frac{(n-1)(3n-4)}{2}}$$

$\det(g_{i\bar{j}}) \geq 0$ since every eigenvalue of $(g_{i\bar{j}})$ is