(alternate discussion of intersections of analytic varieties will be given in Section 2 of Chapter 3 and in Section 2 of Chapter 5).

Let $V$ and $W$ be two analytic varieties of dimension $k$ and $n-k$ in the polycylinder $\Delta$ of radius 1 in $\mathbb{C}^n$ having the origin as their only point of intersection. Consider in the product $\Delta' \times \Delta'$ of the polycylinder of radius $\frac{1}{2}$ with itself the two varieties

$$\tilde{V} = \pi_1^{-1}(V) = \{(z, w) : z \in V\}$$

and $$\tilde{W} = \{(z, w) : z - w \in W\}.$$

For each $\epsilon$, of course, the varieties $\tilde{V}$ and $\tilde{W}$ meet the fiber $\pi_2^{-1}(\epsilon) = \Delta' \times \{\epsilon\} \cong \Delta'$ in the variety $V$ and the analytic variety $W + \epsilon$ – that is, $W$ translated by $\epsilon$ – respectively; moreover, $\pi_2^{-1}(\epsilon)$ will meet the intersection $\tilde{V} \cap \tilde{W}$ transversely at a point $(p, \epsilon)$ exactly when $V$ and $W+\epsilon$ meet transversely at $p$.

$\ulcorner$ $\pi_2^{-1}(\epsilon) \cap \tilde{W} = W+\epsilon \times \{\epsilon\}$

$\quad\Downarrow$

$(p, \epsilon) \in \tilde{W} \Rightarrow p - \epsilon \in W \Rightarrow p \in W + \epsilon$

$\Rightarrow (p, \epsilon) \in W + \epsilon \times \{\epsilon\}$

$(x, \epsilon) \in W+\epsilon \times \{\epsilon\} \Rightarrow x \in W + \epsilon \Rightarrow x - \epsilon \in W$

$\Rightarrow (x, \epsilon) \in \tilde{W} \Rightarrow (x, \epsilon) \in \tilde{W} \cap \pi_2^{-1}(\epsilon)$.

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Suppose $V$ and $W+\epsilon$ meet transversely at $p$.

$\Rightarrow T_pV + T_p(W+\epsilon) = T_pV + T_{p-\epsilon}W = T\Delta' = \mathbb{C}^n$.