$\alpha \in R^4 \Rightarrow \frac{1}{2}W \ni 0. \Rightarrow \exists\ \epsilon > 0$ s.t $(-\epsilon, \epsilon) \ni \forall r,$
$r\phi_0 \in \frac{1}{2}W$, since W is open. ⅃

Choose c so that $2c(|\alpha_0| + \delta) = 1$. Since W is convex and balanced, it follows that

(9) $\quad \alpha\phi - \alpha_0\phi_0 \in W$

whenever $|\alpha - \alpha_0| < \delta$ and $\phi - \phi_0 \in cW$.
This completes the proof. ///.

$\Gamma \quad \alpha\phi - \alpha_0\phi_0 = \alpha(\phi - \phi_0) + (\alpha - \alpha_0)\phi_0$

$(\alpha - \alpha_0)\phi_0 = \frac{\alpha - \alpha_0}{\delta}\delta\phi_0 \in \frac{\alpha - \alpha_0}{\delta}\frac{1}{2}W \subset \frac{1}{2}W$, since $\left|\frac{\alpha-\alpha_0}{\delta}\right| \leq 1$

and $\frac{1}{2}W$ is balanced.

$\phi - \phi_0 \in cW \Rightarrow \alpha(\phi - \phi_0) \in \alpha c W. \qquad c = \frac{1}{2(|\alpha_0| + \delta)}$

$|\alpha| < |\alpha - \alpha_0| + |\alpha_0| < \delta + |\alpha_0| \Rightarrow |\alpha c| < c(\delta + |\alpha_0|) \leq \frac{\delta + |\alpha_0|}{2(|\alpha_0| + \delta)}$

$= \frac{1}{2} \Rightarrow \alpha c W \subset \frac{1}{2}W$ since $2\alpha c W \subset W$ ($\because$ $|2\alpha c| \leq 1$).

$\Rightarrow \alpha\phi - \alpha_0\phi_0 \in \frac{1}{2}W + \frac{1}{2}W = W$ by convexity of W.

$$\begin{array}{ccc} \mathbb{C} \times \mathcal{D}(\Omega) & \xrightarrow{g} & \mathcal{D}(\Omega) \\ (\alpha, \phi) & \longmapsto & \alpha\phi. \end{array}$$

Pick $\alpha_0\phi_0$, then we can find δ & c s.t if $|\alpha - \alpha_0| < \delta$ and $\phi \in \phi_0 + cW$, $\alpha\phi \in \alpha_0\phi_0 + W$, which implies that