$\bar{\omega}(a_{24})e_2 \quad \Rightarrow \quad \iota_V^* \tau = \sigma$ on an open set.
$\Rightarrow \iota_V^* \tau = \sigma$ on M. See below

As in the case of line bundles, we have an imbedding theorem:

Theorem: For M any compact complex manifold, $L \to M$ a positive line bundle and $E \to M$ any holomorphic vector bundle, then for m sufficiently large, the map $\iota_{E \otimes L^m}$ is an embedding.

proof. Most of the work has been done for us already by the Kodaira embedding theorem: since M has a positive line bundle, we may take $M \subset \mathbb{P}^N$ an algebraic variety and $L \to M$ the hyperplane bundle.

Additional comments on $\iota_V^* \tau = \sigma$.

In general, we get (easily guess)

$$\begin{aligned}
f_1 &= x_1 + y_1 z_{11} + y_2 z_{12} + \cdots + y_{n-k} z_{1\,n-k} \\
f_2 &= x_2 + y_1 z_{21} + y_2 z_{22} + \cdots + y_{n-k} z_{2\,n-k} \\
&\;\vdots \\
f_k &= x_k + y_1 z_{k1} + y_2 z_{k2} + \cdots + y_{n-k} z_{k\,n-k}
\end{aligned}$$

$$\Rightarrow \sigma = x_1 \sigma_1 + \cdots + x_k \sigma_k + y_1 \sigma_{k+1} + \cdots + y_{n-k} \sigma_n$$

where $\begin{pmatrix} 1 & & 0 & z_{11} & \cdots & z_{1\,n-k} \\ & \ddots & & \vdots & & \\ 0 & & 1 & z_{k1} & \cdots & z_{k\,n-k} \end{pmatrix}$. //