2. The second map assigns to a closed meromorphic 1-form its residue as in the proof of the lemma above.

⊢

$$\mathbb{H}^1(M, \Omega(*)) = \lim_{\underline{\mathcal{U}}} H^1(C^*(\underline{\mathcal{U}}), D)$$

$$C^n(\underline{\mathcal{U}}) = \bigoplus_{p+q=n} C^p(\underline{\mathcal{U}}, \Omega^q(*))$$

See note P 470 back.

$$\mathbb{H}^1(M, \Omega(*)) = {}''E_2^{1,0} = \frac{\{\text{closed meromorphic 1-forms}\}}{\{\text{exact forms}\}}$$

$${}'E_2^{0,1} = H^0(M, \mathcal{H}^1(\Omega(*))) = H^0(\bigoplus_{\substack{D \in \mathrm{Div}\, M \\ \text{irreducible}}} \mathbb{C}_D)$$

$${}''E_2^{1,0} \longrightarrow {}'E_2^{0,1}$$

$$\| $$

$$\frac{\{a \in {}''F^0 C^1 \; ; \; (d+\delta)(a) \in {}''F^3 C^2\}}{d({}''F^{-\text{III}} C^0) + {}''F^2 C^1}$$

$${}'E_2^{0,1} = \frac{\{a \in {}'F^0 C^1 \; ; \; (d+\delta)(a) \in {}'F^2 C^2\}}{d({}'F^{-1} C^0) + {}'F^1 C^1}$$

$$= \frac{\{a \in C^0(\underline{\mathcal{U}}, \Omega^1(*)) + C^1(\underline{\mathcal{U}}, \Omega^0(*)) : (d+\delta)(a) \in C^2(\underline{\mathcal{U}}, \Omega^0(*))\}}{C^1(\underline{\mathcal{U}}, \Omega^0(*))}.$$

$$= \frac{\{a \in C^0(\underline{\mathcal{U}}, \Omega^1(*)) + C^1(\underline{\mathcal{U}}, \Omega^0(*)) : (d+\delta)(a) \in C^2(\underline{\mathcal{U}}, \Omega^0(*))\}}{C^1(\underline{\mathcal{U}}, \Omega^0(*)) \cap \{a \in C^0(\underline{\mathcal{U}}, \Omega^1(*)) + C^{1,0} : (d+\delta)(a) \in C^{2,0}\}}$$

by P 441