ucible. $\lrcorner$

Of particular interest is the set of lines in $\mathbb{P}^3$ lying on a smooth quadric S. We see that under the Segré map σ, the curves $\{s\} \times \mathbb{P}^1$ and $\mathbb{P}^1 \times \{t\}$ on $\mathbb{P}^1 \times \mathbb{P}^1$ are sent into lines in $\mathbb{P}^3$.

Pf $s_0 = 1.\ s_1 = \alpha \quad \Rightarrow \quad [t_0, t_1, \alpha t_0, \alpha t_1] = (1, y, \alpha, \alpha y)$ $\lrcorner$

We will call these two families of lines on S the A-lines and the B-lines: clearly every A-line meets every B-line, and any two A-lines are disjoint, so are any two B-lines.

Pf $[a_0 t_0, a_0 t_1, a_1 t_0, a_1 t_1] = [s_0 b_0, s_0 b_1, s_1 b_0, s_1 b_1]$
for fixed points $[a_0, a_1]$ & $[b_0, b_1]$.
At $[a_0, a_1, b_0, b_1]$, these two lines meet.
$[a_0 t_0, a_0 t_1, a_1 t_0, a_1 t_1] = [a_0' t_0', a_0' t_1', a_1' t_0', a_1' t_1']$
$\Rightarrow$ If $a_0 \neq 0$, $a_0' \neq 0$, $[t_0, t_1, \frac{a_1}{a_0} t_0, \frac{a_1}{a_0} t_1] = [t_0', t_1', \frac{a_1'}{a_0'} t_0', \frac{a_1'}{a_0'} t_1']$
$\Rightarrow t_1/t_0 = t_1'/t_0'$ and $a_1/a_0 = a_1'/a_0'$ $\Rightarrow$ The lines are the same.
$a_0' = 0 \Rightarrow t_0 = 0. \Rightarrow [0, a_0 t_1, 0, a_1 t_1] = [0, 0, a_1' t_0', a_1' t_1']$
$\Rightarrow t_0' = 0 \Rightarrow$ Nonsense. $\Rightarrow$ Claims are true. $\lrcorner$

These are, moreover, all the lines on S: if $L \subset S$ is any line, then obviously L must meet at least