Let α be a curve s.t $\alpha : I \longrightarrow \delta_i$
$$0 \longmapsto \delta_i(0)$$
$$1 \longmapsto \delta_i(1).$$

$$\Rightarrow \int_{\delta_i} \pi \cdot \eta = \int_0^1 \pi(\alpha(t)) \cdot \eta(\alpha(t))$$
$$= \int_0^1 \pi(\beta(1-t))\, \eta(\beta(1-t))$$

$$\int_{\delta_i^{-1}} \pi \cdot \eta = -\int_0^1 \pi(\beta(t)) \cdot \eta(\beta(t)).$$ (accept this as a definition)

where β is a curve s.t $\beta : I \longrightarrow \delta_i^{-1}$
$$0 \longmapsto \delta_i^{-1}(0)$$
$$1 \longmapsto \delta_i^{-1}(1).$$

and $\alpha(t) = \beta(t)$ on S, $\forall t \in I$.

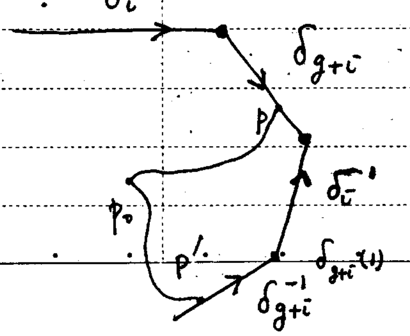
$$\Rightarrow \int_{\delta_i + \delta_i^{-1}} \pi \cdot \eta = \int_0^1 \pi(\alpha(t)) \cdot \eta(\alpha(t)) - \int_0^1 \pi(\beta(t))\, \eta(\alpha(t))$$

(since $\eta(\alpha(t)) = \eta(\beta(t))$.)

$$= \int_0^1 \pi(\alpha(t)) \cdot \eta(\alpha(t)) - \int_0^1 \pi(\beta(t))\, \eta(\alpha(t))$$
$$= \int_0^1 \{ \pi(\underset{\delta_i}{\alpha(t)}) - \pi(\underset{\delta'}{\beta(t)}) \}\, \eta(\alpha(t))$$
$$= \int_0^1 -\pi^{g+i}\, \eta(\alpha(t)) = -\pi^{g+i} \int_0^1 \eta(\alpha(t)) = -\pi^{g+i} \int_{\delta_i} \eta$$
$$= -\pi^{g+i} N^i$$

Similarly $$\int_{\delta_{g+i} + \delta_{g+i}^{-1}} \pi \cdot \eta = \pi^i N^{g+i}.$$

Pf.



$$\pi(p') - \pi(p)$$
$$= \int_{p_0}^{p'} \pi - \int_{p_0}^{p} \pi = \int_{p}^{\delta_{g+i}(1)} \pi + \int_{\delta_i^{-1}} \pi + \int_{\delta_{g+i}(1)}^{p'} \pi$$
$$= \int_{\delta_i^{-1}} \pi = -\pi^i \quad \text{since} \int_p^{p'} \omega = 0$$