λ_1, λ_2 satisfy ⊛.

One more example

$$A = \begin{pmatrix} 10, & 100, & 30 \\ 4, & 7, & -11 \\ 3, & 2, & 8 \end{pmatrix}$$

$\begin{vmatrix} 4, 7 \\ 3 & 2 \end{vmatrix} = 8-21=-13$ $\quad\begin{vmatrix} 7, -11 \\ 2 & 8 \end{vmatrix} = 56+22=78$ $\quad\begin{vmatrix} 4, -11 \\ 3, 8 \end{vmatrix} = 32+33=65$

$\begin{vmatrix} 10, 100 \\ 4 \cdot 7 \end{vmatrix} = -400+70 = -330.$ $\quad\begin{vmatrix} 100 & 30 \\ 7. & -11 \end{vmatrix} = -1100-210 = -1310$ $\quad\begin{vmatrix} 10, 30 \\ 4, -11 \end{vmatrix} = -110-120 = -230$

$\begin{vmatrix} 10, 100 \\ 3, 2 \end{vmatrix} = 20-300 = -280$ $\quad\begin{vmatrix} 10, 30 \\ 3, 8 \end{vmatrix} = 80-90 = -10.$ $\quad\begin{vmatrix} 100, 30 \\ 2, 8 \end{vmatrix} = 800-60 = 740.$

$|A| = 10 \cdot 78 - 100 \cdot 65 + 30 \cdot (-13) = 780 - 6500 - 390$

$= -6110.$

All minors are distinct and nonzero.

$v_1 = (7-4)\, x_1 \frac{\partial}{\partial x_1} + (-11-4)\, x_2 \frac{\partial}{\partial x_2} = 3x_1 \frac{\partial}{\partial x_1} - 15 x_2 \frac{\partial}{\partial x_2}$

$v_2 = (2-3)\, x_1 \frac{\partial}{\partial x_1} + (8-3)\, x_2 \frac{\partial}{\partial x_2} = -x_1 \frac{\partial}{\partial x_1} + 5 x_2 \frac{\partial}{\partial x_2}$

$\Rightarrow$ v_1 & v_2 are linearly dependent on $\mathbb{P}^2$.

Suppose v_1, v_2 & v_3 are linearly dependent at $X \in \mathbb{P}^n$.

$\Rightarrow \quad \lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = \pi_* (\lambda_1 \alpha_{1j} X_j \frac{\partial}{\partial X_j} + \lambda_2 \alpha_{2j} X_j \frac{\partial}{\partial X_j}$
$+ \lambda_3 \alpha_{3j} X_j \frac{\partial}{\partial X_j}) = 0 \quad$ for $(\lambda_1, \lambda_2, \lambda_3) \neq 0.$

$\Rightarrow \quad \lambda_1 \alpha_{1j} X_j \frac{\partial}{\partial X_j} + \lambda_2 \alpha_{2j} X_j \frac{\partial}{\partial X_j} + \lambda_3 \alpha_{3j} X_j \frac{\partial}{\partial X_j} = k X_j \frac{\partial}{\partial X_j}$