P. Since the dimension of the linear system of curves of degree $2n-3$ is

$$n(2n-3) = 2n^2 - 3n,$$

there are plenty of such "test curves" E.

「 By P137, $\dim |D| = h^0(\mathbb{P}^2, \mathcal{O}(D)) - 1$

$D = H^{2n-3}$, where H is the hyperplane of $\mathbb{P}^2$.

$\Rightarrow$ By P166, $h^0(\mathbb{P}^2, \mathcal{O}(H^{2n-3})) = \binom{2n-3+2}{2} = \binom{2n-1}{2}$

$$= \frac{(2n-1)!}{(2n-3)!\,2!} = \frac{(2n-1)(2n-2)}{2} = (2n-1)(n-1) = 2n^2 - 3n + 1$$

$\Rightarrow$ $\dim |H^{2n-3}| = 2n^2 - 3n + 1 - 1 = 2n^2 - 3n$.

Here we used the fact that any curve in $\mathbb{P}^2$ of degree $2n-3$ is linearly equivalent to H^{2n-3}, since the curve represents a line bundle which is some multiple of $[H]$. See P145 」

The result is the following.

Proposition. Suppose that $\Gamma = p_1 + \cdots + p_{n^2}$ satisfies the Cayley-Bacharach property. Then Γ lies on a pencil of curves of degree n.

Proof. We consider the Veronese embedding

$$i_n : \mathbb{P}^2 \hookrightarrow \mathbb{P}^N, \quad N = \frac{n(n+3)}{2},$$

given by the complete linear system of curves of degree