$\lambda : X \longrightarrow \mathbb{C}$. is uniformly continuous.
X^* : completion of X. $\langle\ \rangle$: inner product on X.

$\Rightarrow$ λ can be extended to X^* uniquely.

pf) Define $\lambda(a) = \lim_{n\to\infty} \lambda(a_n)$, where $\{a_n\}$ is a sequence converging to a, $a \in X^*$.

We have to check the well-definedness.
If we have another sequence $\{b_n\}$ converging to a,
we have to prove $\lim_{n\to\infty} \lambda(b_n) = \lambda(a)$.

$|\lambda(a) - \lambda(b)| \leq |\lambda(a) - \lambda(a_n)| + |\lambda(a_n) - \lambda(b_m)| + |\lambda(b_m) - \lambda(b)|$

If we choose n. m large enough, then we have
$|\lambda(a_n) - \lambda(b_m)| \leq C \, \|a_n - b_m\| \leq C(\|a_n - a\| + \|a - b_m\|) < \frac{\epsilon}{3}$.

Similarly, by using the (convergence) continuity of $\{\lambda(a_n)\}$, $\{\lambda(b_n)\}$,

$|\lambda(a) - \lambda(a_n)| < \frac{\epsilon}{3}$ & $|\lambda(b) - \lambda(b_m)| < \frac{\epsilon}{3}$.

$\Rightarrow$ $|\lambda(a) - \lambda(b)| < \epsilon$ $\Rightarrow$ Since ϵ is arbitrary,
$\lambda(a) = \lambda(b)$. Q.E.D.

The pairing
$$\langle u, v\rangle = \sum_{\xi} u_\xi v_\xi$$
identifies H_{-s} with the dual of H_s, so that $\lambda \in H_{-s}$ with its formal Fourier series given above.