DATE . . PAGE

$\Rightarrow$ By the composition of ψ and φ,

$$\begin{array}{ccccc} \mathbb{P}^2 & \longrightarrow & \mathbb{P}^2 & \longrightarrow & \mathbb{P}^2 \\ [X_0, X_1, X_2] & \longmapsto & [G_2 G_3 l_1, \ G_1 G_3 l_2, \ G_1 G_2 l_3] & & \end{array}$$

$$[G_1 G_2 G_3^2 l_1 l_2, \ G_1 G_2^2 G_3 l_1 l_3, \ G_1^2 G_2 G_3 l_2 l_3]$$
$$\parallel$$
$$[G_3 l_1 l_2, \ G_2 l_1 l_3, \ G_1 l_2 l_3]$$

$l_1 = \overline{a_2 a_3} \qquad l_2 = \overline{a_1 a_3} \qquad l_3 = \overline{a_1 a_2}.$

By changing the coordinates, we may assume that

$a_1 = [1.0.0] \quad a_2 = [0.1.0]. \quad a_3 = [0.0.1].$

$\Rightarrow \ l_1 = \{X_0 = 0\} \qquad l_2 = \{X_1 = 0\}. \qquad l_3 = \{X_2 = 0\}.$

$\Rightarrow$

$\tilde{\psi} = \varphi \circ \psi : \mathbb{P}^2 \longrightarrow \mathbb{P}^2$

$[X_0, X_1, X_2] \longmapsto [\tilde{G}_3 X_0 X_1, \ \tilde{G}_2 X_0 X_2, \ \tilde{G}_1 X_1 X_2]$

Since $\tilde{G}_3$ passes a_1 & a_2, not a_3,

$\tilde{G}_3 = \underset{b_0}{\wedge} X_2^2 + b_1 X_0 X_1 + b_2 X_0 X_2 + b_3 X_1 X_2, \quad b_0 \neq 0.$

Similarly, we get

$\tilde{G}_2 = c_0 X_1^2 + c_1 X_0 X_1 + c_2 X_0 X_2 + c_3 X_1 X_2, \quad c_0 \neq 0$

$\tilde{G}_1 = d_0 X_0^2 + d_1 X_0 X_1 + d_2 X_0 X_2 + d_3 X_1 X_2, \quad d_0 \neq 0.$

We have only to show that $\tilde{\psi}$ is a composition of two quadratic transformations.