Chern classes of projective space. Let $X_0, X_1, \cdots X_n$ be linear coordinates on $\mathbb{C}^{n+1}$, and let $\mathcal{E}$ and π_* be as in the Euler sequence above. Let $A = (\alpha_{ij})$ be an $(n+1) \times (n+1)$ matrix all of whose minors are distinct and nonzero, and consider the vector fields

$$v_i = \mathcal{E}(\alpha_{i,0} X_0, \cdots \alpha_{i,n} X_n)$$
$$= \pi_* \sum_j \alpha_{ij} X_i \frac{\partial}{\partial X_j}.$$

$$\begin{pmatrix} \alpha_{0,0}, & \alpha_{01}, & \cdots & \alpha_{in} \\ \alpha_{1,0}, & \alpha_{1,1} & \cdots & \alpha_{in} \\ \vdots & \vdots & & \\ \alpha_{n,0}, & \alpha_{n,1} & \cdots & \alpha_{nn} \end{pmatrix} = A \qquad v_i = \mathcal{E}(\alpha_{i0} X_0, \cdots \alpha_{in} X_n)$$
$$= \pi_* \left(\sum \sigma_j(X) \frac{\partial}{\partial X_j} \right)$$

$\alpha_{i0} X_0$ may be considered the linear functional L: $\mathbb{C}\{X\} \longrightarrow \mathbb{C}$ defined by $(X_0' \cdots X_n') \mapsto \alpha_{i0} X_0'$. $= \pi_* \left(\sum_j \alpha_{ij} X_j \frac{\partial}{\partial X_j} \right)$

We will leave it as an excercise to verify that, under the assumptions made about A, $v_1, \cdots v_n$ are generic sections of $T'(\mathbb{P}^n)$ (this is simply a matter of writing v_i out in terms of Euclidean coordinates on $\mathbb{P}^n$), and compute the degeneracy cycles D_i of $v_1 \cdots v_i$.

$$v_1 = \pi_* \left(\alpha_{1j} X_j \frac{\partial}{\partial X_j} \right) = \alpha_{1j} X_j \pi_* \left(\frac{\partial}{\partial X_j} \right) = \alpha_{10} X_0 \pi_* \left(\frac{\partial}{\partial X_0} \right) + \alpha_{10} X_1 \pi_* \left(\frac{\partial}{\partial X_1} \right) + \cdots + \alpha_{1n} X_n \pi_* \left(\frac{\partial}{\partial X_n} \right) = \alpha_{10} X_0 \left(- \sum \frac{X_i}{X_0^2} \frac{\partial}{\partial x_i} \right) + \alpha_{11} X_1 \frac{1}{X_0} \frac{\partial}{\partial x_1} + \cdots + \alpha_{1n} X_n \frac{1}{X_0} \frac{\partial}{\partial x_n}$$

$$= \left(\alpha_{11} - \frac{X_1}{X_0} \alpha_{10} \right) \frac{X_1}{X_0} \frac{\partial}{\partial x_1} + \left(\alpha_{12} - \alpha_{10} \frac{X_2}{X_0} \right) \frac{X_1}{X_0} \frac{\partial}{\partial x_2} + \left(\alpha_{13} - \alpha_{10} \frac{X_3}{X_0} \right) \frac{X_1}{X_0} \frac{\partial}{\partial x_3} + \cdots + \left(\alpha_{1n} - \alpha_{10} \frac{X_n}{X_0} \right) \frac{X_1}{X_0} \frac{\partial}{\partial x_n}.$$

$$\Rightarrow D_1 = \{ x \in \mathbb{P}^n : v_1 = 0 \} \Rightarrow X_1 = 0 \text{ or } \alpha_{11} - \frac{X_1}{X_0} \alpha_{10} = \cdots$$