$$\begin{cases} xy = 0, \\ (x-y)(x-ry) = 0. \end{cases}$$

This is possible by identifying the directions through p_i with $\mathbb{P}^1$ and noting that any four points of $\mathbb{P}^1$ may be projectively transformed to $\{0, 1, r, \infty\}$.

$\ulcorner$ $p = [(x_0, y_0, z_0)] \in \mathbb{P}^2$.

A direction through p may be expressed as a line through p, i.e. $[(a,b,c)] \in \mathbb{P}^1$ where $H = \{[(a,b,c)] \mid ax_0 + by_0 + cz_0 = 0\} \cong \mathbb{P}^1$

$\Rightarrow$ The directions through p may be identified with $\mathbb{P}^1$. Here we use the fact that any hyperplane in $\mathbb{P}^2$ may be transformed to $\mathbb{P}^1$ in $\mathbb{P}^2$ by using a proper projective isomorphism.

Given any four distinct points in $\mathbb{P}^1$, say, $[(a_0, a_1, 0)]$, $[(b_0, b_1, 0)]$, $[(c_0, c_1, 0)]$, $[(d_0, d_1, 0)]$, assume that $\{[(a_0, a_1, 0)], [(b_0, b_1, 0)]\}$ is linearly independent.

$$\Rightarrow \begin{pmatrix} a_0 & b_0 & 0 \\ a_1 & b_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : \mathbb{C}^3 \longrightarrow \mathbb{C}^3$$

sends $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} a_0 \\ a_1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} b_0 \\ b_1 \\ 0 \end{pmatrix}$

respectively. Note that $\begin{pmatrix} \alpha a_0 & \beta b_0 & 0 \\ \alpha a_1 & \beta b_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} =$