Clearly, any such matrix represents an element of $G(k,n)$ and any two such matrices A, A' the same element of $G(k,n) \Leftrightarrow A = gA'$ for some $g \in GL_k$.

⊢ $\Lambda = \langle v_1, v_2 \cdots v_k \rangle$, $v_i \in \mathbb{C}^n$,
$\Lambda' = \langle w_1, w_2, \quad w_k \rangle$, $w_i \in \mathbb{C}^n$,

$$\Rightarrow \begin{aligned} v_1 &= g_{11} w_1 + g_{21} w_2 + \cdots + g_{k1} w_k \\ v_2 &= g_{12} w_1 + g_{22} w_2 + \cdots + g_{k2} w_k \\ v_i &= g_{1i} w_1 + g_{2i} w_2 + \cdots + g_{ki} w_k \\ v_k &= g_{1k} w_1 + g_{2k} w_2 + \cdots + g_{kk} w_k \end{aligned} \quad \Rightarrow A = {}^t g A', \text{ where } g = (g_{ij}).$$

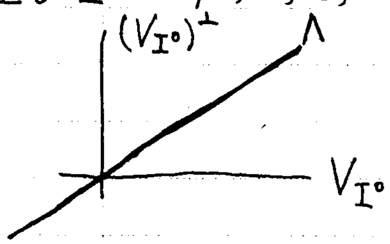
⫯

For every multindex $I = \{i_1, \cdots i_k\} \subset \{1, 2, \cdots n\}$ of cardinality k, let $V_{I^\circ} \subset \mathbb{C}^n$ be the $(n-k)$-plane in $\mathbb{C}^n$ spanned by the vectors $\{e_j : j \notin I\}$ and let

$$U_I = \{ \Lambda \in G(k,n) : \quad \Lambda \cap V_{I^\circ} = \{0\} \};$$

U_I is just the set of $\Lambda \in G(k,n)$ such that the Ith $k\times k$ minor of one, and hence for any, matrix representation for Λ is nonsingular.

⊢ $I \cup I^\circ = \{1, 2, 3, \cdots n\}$. Since $A = gA'$, $g \in GL_k$, if for some g,



$$A = \begin{pmatrix} 0 & \cdots & 0, & v_{1k+1}, & \cdots & v \\ v_{21}, & \cdots & v_{2k}, & v_{2k+1} & \cdots & v_{2n} \\ \vdots \\ v_{k1} & \cdots & v_{kk} \end{pmatrix},$$

$$\Lambda \cap V_{I^\circ} \ni (0, 0, \cdots 0, v_{1k+1}, \cdots v_{1n})$$

$v_1 = (v_{11}, \cdots \quad v_{1n})$ $\qquad$ $w_1 = (w_{11}, \cdots \quad w_{1n})$
$\vdots$ $\qquad$ $\vdots$
$v_k = (v_{k1}, \cdots \quad v_{kn})$ $\qquad$ $w_k = (w_{k1}, \cdots \quad w_{kn})$

$$\Rightarrow \begin{aligned} v_{11} &= g_{11} w_{11} + g_{21} w_{21} + \cdots + g_{k1} w_{k1} \\ v_{21} &= g_{12} w_{11} + g_{22} w_{21} + \cdots + g_{k2} w_{k1} \\ v_{ij} &= g_{1i} w_{1j} + g_{2i} w_{2j} + \cdots + g_{ki} w_{kj} \end{aligned}$$