The projection of $\{[(0,0,x,1)]\}$ near P_∞ is $[(0,0,x,0)]$, $|x|<\delta$. $\Rightarrow$ As $x \to 0$, $[(0,0,x,0)] \to [(0,0,1,0)]$.
In general, given a curve $[(ta_1, ta_2, ta_3, 1)]$, near P_∞, the projection of the curve is $[(ta_1, ta_2, ta_3, 0)]$
$[(ta_1, ta_2, ta_3, 1)] \cap \mathbb{P}^2 = [(ta_1, ta_2, ta_3, 0)] = [(a_1, a_2, a_3, 0)]$.
$\Rightarrow$ We have the continuous extension of π_p over E. $\lrcorner$

Now let L_1, L_2 be the two lines on S passing through p, and let q_1 and q_2 be their points of intersection with H. Then for any point $q \in H$ other than q_1 and q_2, the line $\overline{pq}$ will either meet S in one point other than p, or be simply tangent to S at p; in either case q will be the image under $\tilde{\pi}$ of a single point of $\tilde{S}$.

$\ulcorner$ Since $\deg S = 2$, $\#(\overline{pq} \cap S) = 2$.
$\Rightarrow$ $\overline{pq} \cap S = \{q'\}$, $q' \neq p$. or $q' = p$ ($\Rightarrow$ $\overline{pq}$ is tangent to S at p)

$$\begin{tikzcd} S - \{p\} \arrow[r, "\phi"] \arrow[d, "\pi_p"'] & \phi(S) - \underset{P_\infty}{[(0,0,0,1)]} \arrow[d, "\pi_{P_\infty}"'] \\ H \arrow[r] & \mathbb{P}^2 = \{[(*,*,*,0)]\} \end{tikzcd}$$

Let $[(a_1, a_2, a_3, a_4)] = x$
$\Rightarrow$ The line L spanned by P_∞ and x is given as