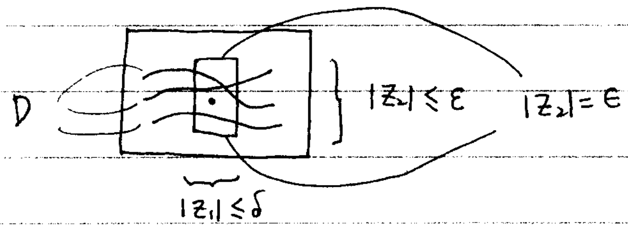
We may find a circle $|z_2| = \epsilon$ that does not meet these points. It follows by continuity that, for δ sufficiently small, the locus

$$\{\, |z_1| \leq \delta,\ |z_2| = \epsilon \,\}$$

will not meet D (Figure 2).



Since the finite set $D \cap \{z_1 = 0\}$ lie on the z_2-axis, we can find a circle $|z_2| = \epsilon$ ($z_1 = 0$). $\Rightarrow$ D is closed and $|z_2| = \epsilon$ ($z_1 = 0$) is compact $\Rightarrow$ By the tube lemma, $\exists\, \delta > 0$ s.t $\{\, |z_1| \leq \delta,\ |z_2| = \epsilon \,\} \cap D = \phi$.

For z with $0 < |z_1| \leq \delta$ the integral

$$\frac{1}{2\pi\sqrt{-1}} \int_{|z_2| = \epsilon} \frac{dh}{h}$$

is well-defined, continuous, and integer-valued.

$h : \Delta' = \{\, |z_1| < 1,\ |z_2| < 1 \,\} \longrightarrow \mathbb{C}$ s.t $(h = 0) = D \cap \Delta'$ $\Rightarrow$ Since $D \cap \{\, |z_1| \leq \delta,\ |z_2| = \epsilon \,\} = \phi$, $h \neq 0$ on $\{\, (z_1, z_2) \mid 0 \leq |z_1| \leq \delta,\ |z_2| = \epsilon \,\}$.

$\Rightarrow$ $\dfrac{1}{2\pi\sqrt{-1}} \displaystyle\int_{|z_2| = \epsilon} \frac{dh}{h}$ is well-defined.