the commutative diagram

$$\begin{tikzcd}
H^p(B, P^{n-k}) \arrow[r, "d_2"] \arrow[d, "L^{k+1}=0"] & H^{p+2}(B, R^{n-k-1}) \arrow[d, "L^{k+1}\cong"] \\
H^p(B, R^{n+k+2}) \arrow[r, "d_2"] & H^{p+2}(B, R^{n+k+1})
\end{tikzcd}$$

⌜ $H^p(B, P^{n-k}) \subset H^p(B, R^{n-k}) = E_2^{p,n-k}$

$H^{p+2}(B, R^{n-k-1}) = E_2^{p+2,n-k-1}$, $H^p(B, R^{n+k+2}) = E_2^{p,n+k+2}$

$H^{p+2}(B, R^{n+k+1}) = E_2^{p+2,n+k+1}$ ⌟

The right-hand vertical arrow is an isomophism by hard Lefschetz, and the left-hand one is zero by definition of primitive. Thus $d_2=0$. Q.E.D.

⌜ $R^{n-k} \cong \bigoplus L^l P^{n-k-2l}$

Consider $LP^{n-k-2} \subset R^{n-k}$. $P^{n-k-2} \subset R^{n-k-2}$.

$\Rightarrow$ By the argument above,

$$H^p(B, P^{n-k-2}) \xrightarrow{d_2} H^{p+2}(B, R^{n-k-3})$$

is zero.

$\Rightarrow$

$$\begin{tikzcd}
H^p(B, LP^{n-k-2}) \arrow[r, "d_2"] & H^{p+2}(B, R^{n-k-1}) \\
H^p(B, P^{n-k-2}) \arrow[r, "d_2", "\circledast"'] \arrow[u, "L"] \arrow[ur, phantom, "\circlearrowright"] & H^{p+2}(B, R^{n-k-3}) \arrow[u, "L"']
\end{tikzcd}$$

Since L is onto, and $\circledast$ is zero, d_2 above