sequence

$$0 \longrightarrow \mathcal{O}_M \longrightarrow \mathcal{O}_M(L) \longrightarrow \mathcal{O}_{\mathbb{P}^1}(H_{\mathbb{P}^1}) \longrightarrow 0$$

and the fact that $H^1(M, \mathcal{O}_M) = 0$, it follows that

$$H^0(M, \mathcal{O}_M(L)) \longrightarrow H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(H_{\mathbb{P}^1})) \longrightarrow 0.$$

$\ulcorner$ Since $L \cdot L = L \cdot D = 1$, $L|_D$ is the hyperplane bundle, i.e., the point bundle.

By P139,

$$0 \longrightarrow \mathcal{O}_M(L \otimes [-D]) \longrightarrow \mathcal{O}_M(L) \longrightarrow \mathcal{O}_D(L|_D) \longrightarrow 0$$

$$\mathcal{O}_M(L \otimes [-D]) = \mathcal{O}_M(L - L) = \mathcal{O}_M, \qquad \mathcal{O}_D(L|_D) = \mathcal{O}_{\mathbb{P}^1}(H|_{\mathbb{P}^1})$$

$$\Rightarrow \quad H^0(M, \mathcal{O}_M(L)) \longrightarrow H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(H_{\mathbb{P}^1})) \longrightarrow H^1(M, \mathcal{O}) \longrightarrow$$

$$H^1(M, \mathcal{O}) = H^{0,1}(M) = 0, \text{ since } H^1(M) = 0.$$

$\lrcorner$

$H_{\mathbb{P}^1}$ is very ample on $\mathbb{P}^1$ so the linear system $|L|$ seperates points on each curve $D \in |L|$.

$\ulcorner$ Given any two points $P_1 \neq P_2 \in D$,

$$\begin{array}{ccc} L|_D = \iota^*(L) & \longrightarrow & L \\ \downarrow & & \downarrow \\ D & \xrightarrow{\iota} & M \end{array}$$

$\Rightarrow$ Since $H_{\mathbb{P}^1}$ is very ample, $\exists$ σ s.t σ is a section of