-ar transformation of $\mathbb{C}^{n+1}$.

Let $X_0, X_1, \cdots X_n$ be homogeneous coordinates on $\mathbb{P}^n$, $x_i = X_i/X_0$ the corresponding Euclidean coordinates on the complement of the hyperplane $H = (X_0 = 0)$. Since the fundamental class of a hyperplane in $\mathbb{P}^n$ generates $H^2(\mathbb{P}^n, \mathbb{Z}) \cong \mathbb{Z}$, any holomorphic automorphism φ of $\mathbb{P}^n$ must take a hyperplane into a complex submanifold of $\mathbb{P}^n$ homologous to a hyperplane, hence to a hyperplane.

$\ulcorner$ $H^2(\mathbb{P}^n, Z) \cong H_{2n-2}(\mathbb{P}^n, Z) \cong Z$.

$\varphi : \mathbb{P}^n \longrightarrow \mathbb{P}^n$ automorphism and H hyperplane in $\mathbb{P}^n$ $\Rightarrow$ $\varphi(H)$ is homologous to a hyperplane in $\mathbb{P}^n$.
$\Rightarrow$ By the argument above, since $\varphi(H)$ is an analytic subvariety of $\mathbb{P}^n$ homologous to a hyperplane, $\varphi(H)$ is a hyperplane. $\lrcorner$

Consequently, after composing φ with a linear transformation of $\mathbb{P}^n$, we may assume that $\varphi(H) = H$.

$\ulcorner$ $\varphi(H) = H' \Rightarrow \exists$ a linear transformation L s.t $L(H') = H$.
$\Rightarrow L \circ \varphi(H) = H$. $\lrcorner$

Similarly, φ must carry the coordinate hyperplanes $H_i = (x_i = 0)$ into hyperplanes other than

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