As the authors pointed out on P705, there is subtlety on a sheaf map from $\mathcal{O}(L)$ to $\mathcal{J}_{P_0}(L)$.

Locally, we can define a map as follows:

① $p \in P_0$.

Given $\eta \in \mathcal{O}(L)_p$,

$\Rightarrow \left(\dfrac{\mathcal{O}(L)}{\mathcal{J}_{P_0}(L)}\right)_p$ is a complex vector space by P669

$\Rightarrow \left(\dfrac{\mathcal{O}(L)}{\mathcal{J}_{P_0}(L)}\right)_p$ has a basis $\{1 + \mathcal{J}_{P_0}(L)_p,\ f + \mathcal{J}_{P_0}(L)_p, \ldots + f^{n-1} + \mathcal{J}_{P_0}(L)_p\}$ by note P633 ~ P640.

$$\Rightarrow \eta + \mathcal{J}_{P_0}(L)_p = (\alpha_1 + \alpha_2 f + \cdots + \alpha_n f^{n-1}) + \mathcal{J}_{P_0}(L)_p$$
$$\Rightarrow \eta - (\alpha_1 + \alpha_2 f + \cdots + \alpha_n f^{n-1}) = \sigma \in \mathcal{J}_{P_0}(L)_p.$$
$$\Rightarrow \eta = \alpha_1 + \alpha_2 f + \cdots + \alpha_n f^{n-1} + g_1 s + g_2 s'$$

Define a map from $\mathcal{O}(L)_p$ to $\mathcal{J}_{P_0}(L)_p$ by

$$\eta \longmapsto g_1 s + g_2 s'$$

② $p \notin P_0$

$\Rightarrow$ Define a map from $\mathcal{O}(L)_p$ to $\mathcal{J}_{P_0}(L)_p$ by

$$\eta \longmapsto \eta.$$

But the problem is that we can not define a sheaf map from $\mathcal{O}(L)(\mathcal{U})$ to $\mathcal{J}_{P_0}(L)(\mathcal{U})$.