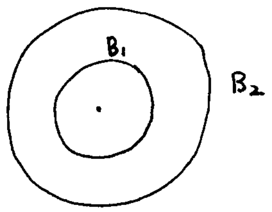
This prove that E is compact.

Hence $C^\infty(\Omega)$ has the Heine-Borel property. It follows from Theorem 1.23, (P 17), that $C^\infty(\Omega)$ is not locally bounded, hence not normable. The same conclusion holds for $\mathcal{D}_K$ whenever K has nonempty interior (otherwise $\mathcal{D}_K = \{0\}$.) because $\dim \mathcal{D}_K = \infty$ in that case. This last statement is a consequence of the following proposition:

If B_1 and B_2 are concentric closed balls in $\mathbb{R}^n$, with B_1 in the interior of B_2, then there exists $\phi \in C^\infty(\mathbb{R}^n)$ such that $\phi(x) = 1$ for every $x \in B_1$ and $\phi(x) = 0$ for every x outside B_2.

Pf $\qquad \phi = 1$ on B_1 $\quad \phi = 0$ on B_2^c.



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Now! Return to P136.

The union of the spaces $\mathcal{D}_K$, as K ranges over all compact subsets of Ω, is the test function space $\mathcal{D}(\Omega)$. It is clear that $\mathcal{D}(\Omega)$ is a vector space, with respect to the usual definitions of addition and scalar multiplication of complex functions. Explicitly, $\phi \in \mathcal{D}(\Omega) \Leftrightarrow \phi \in C^\infty(\Omega)$ and the support of ϕ is a compact subset of Ω.

Let us introduce the norms

(1) $$\|\phi\|_N = \max\{|D^\alpha \phi(x)| : x \in \Omega,\ |\alpha| \le N\},$$