Grassmannian of the cycle of k-planes lying on a smooth quadrics in $\mathbb{P}^n$.

☞ P739, see.

It is interesting to try and answer the same question for the cycle of k-planes lying on a linear system of quadrics; we will discuss here the case of lines on quadrics in $\mathbb{P}^3$.

To begin with, recall that the integral homology of the Grassmannian $G(2,4)$ is generated by the Schubert cycles

$$\sigma_1(l_0) = \{x \in G : l_x \cap l_0 \neq \phi\},$$
$$\sigma_2(p_0) = \{x \in G : l_x \ni p_0\},$$
$$\sigma_{1,1}(h_0) = \{x \in G : l_x \subset h_0\},$$

and

$$\sigma_{2,1}(p_0, h_0) = \{x \in G : p_0 \in l_x \subset h_0\}$$

for $p_0 \in l_0 \subset h_0$ any choice of point, line, and hyperplane in $\mathbb{P}^3$. The intersections of these Schubert cycles are

$$\sigma_1 \cdot \sigma_1 = \sigma_2 + \sigma_{1,1},$$
$$\sigma_1 \cdot \sigma_2 = \sigma_1 \cdot \sigma_{1,1} = \sigma_{2,1}$$
$$\sigma_2 \cdot \sigma_2 = \sigma_{1,1} \cdot \sigma_{1,1} = \sigma_1 \cdot \sigma_{2,1} = 1,$$
$$\sigma_2 \cdot \sigma_{1,1} = 0.$$