93 5 13

$$\begin{tikzcd} E \arrow[r] \arrow[d] & E' \arrow[d] \\ M \arrow[r, "f"] & f(M) \subset \mathbb{P}^N \end{tikzcd} \qquad \begin{tikzcd} E\otimes L^m \arrow[r, "\tilde{f}"] \arrow[d] & E'\otimes L'^m \arrow[d] \\ M \arrow[r, "f"] \arrow[u, bend left, dashed, "\sigma"] & f(M) \arrow[u, bend right, dashed, "\sigma'"'] \end{tikzcd}$$

$\Rightarrow \tilde{f}\circ\sigma = \sigma'\circ f$

Identify $V = H^0(M, \mathcal{O}(E\otimes L^m))$ with $V' = H^0(f(M), \mathcal{O}(E'\otimes L'^m))$ as follows:

$V = \langle \sigma_1, \sigma_2, \cdots \sigma_n \rangle \qquad V' = \langle \sigma_1', \cdots \sigma_n' \rangle$

$\sigma_i' = \tilde{f}\circ\sigma_i\circ f^{-1}$.

$\Rightarrow$ $\iota_{E'\otimes L'^m}$ is given locally as follows.

$$\iota'(x) = \begin{bmatrix} a_{11}, \cdots & a_{1n} \\ \vdots & \\ a_{k1}, \cdots & a_{kn} \end{bmatrix}$$

$\sigma_i = \sum a_{\alpha i} e_\alpha \quad \Rightarrow \quad \sigma_i' = \sum a_{\alpha i} e_\alpha'$

where $e_\alpha' = \tilde{f}\circ e_\alpha \circ f^{-1}$.

$\sigma_i^*(\sigma_j) = \delta_{ij} \qquad \sigma_i'^*(\sigma_j') = \delta_{ij}$

$$\Rightarrow \iota(x) = \iota_{E\otimes L^m}(x) = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{k1}, & \cdots & a_{kn} \end{bmatrix}$$

Thus if ι' is embedding, ι is embedding, too. $\lrcorner$

$\ulcorner$ Without loss of generality, assume that $E\otimes L^m$.

Suppose that $x \neq y \in U$. where $E|_U \cong U\times\mathbb{C}^k$.

If $\iota_E(x) = \iota_E(y)$, since $H^0(M, \mathcal{O}(E)) \rightarrow$