

Specially, $T_{p_1}W' \cap W$ is in \mathbb{P}^2 .

Recall that if $l \in \tau(W) \cap \sigma_{n-2, n-3}$, & l passes p_1 ,

$l \subset T_{p_1}W' \cap W \Rightarrow T_{p_1}W' \cap W = \{ \text{two lines} \}$

for, l is an irreducible component of $T_{p_1}W' \cap W$ so to have deg 2. the rest should be deg 1 subvariety of \mathbb{P}^2 which is isomorphic to \mathbb{P}^1 , see P 174. $\Rightarrow T_{p_1}W' \cap W = \{ \text{two lines} \}$.

Suppose $l \not\subset T_{p_1}W' \cap W$ $l \in \tau(W) \cap \sigma_{n-2, n-3}$

Suppose $T_{p_1}W' \cap W$ is a curve of deg 2.

Let C be an irreducible nondegenerate curve of deg 2

$\Rightarrow C$ must be the rational normal curve which is smooth. (See P 179).

But because of p_1 , $T_{p_1}W' \cap W$ is singular.

Proof of $T_{p_1}W' \cap W = \{ \text{two lines} \}$.

Let $T_{p_1}W' \cap W = V$. $V^* = V_1 \cup \dots \cup V_r$ where V_i 's are connected components. \Rightarrow Each $\overline{V_i}$ is analytic subvariety in $S \cong \mathbb{P}^3$, in particular, in $\mathbb{P}^2 \cong T_{p_1}W'$.

If $\overline{V_i}$ is degenerate, $\overline{V_i} \subset \mathbb{P}^1 \Rightarrow \overline{V_i} = \mathbb{P}^1$.

If $\overline{V_i}$ is nondegenerate, of deg 1, then $\overline{V_i} = \mathbb{P}^1$ by 174.

If $\overline{V_i}$ is nondegenerate, irreducible curve of deg 2, then $\overline{V_i}$ is the rational normal curve by 179.

But this is impossible, since $V = \overline{V_i}$ and the rational normal curve is smooth, while V is singular because of p_1 .

Thus, each $\overline{V_i}$ must be \mathbb{P}^1 , and so, because of degree count, $V = \mathbb{P}^1 \cup_{p_1} \mathbb{P}^1$, since V must be singular, $\mathbb{P}^1 \cap \mathbb{P}^1 \ni p_1$. \square