a frame for E in a nbd of z_0 for which $\theta(z_0)$ vanishes, we see that $[\Lambda, \bar\partial] + i D'^{*} = 0$.

Γ For simplicity, $\dim M = 2 = \dim E$.

Define $\exp : T_{z_0}M \longrightarrow M$ by $v \longmapsto \exp v$, where $\exp tv = \gamma(t)$. γ is geodesic s.t $\gamma'(0) = v$.

$\Rightarrow$ $\exp_*$ is invertible at z_0 $\Rightarrow$ By inverse function theorem, $\exists$ $\delta > 0$, open $U \subset M$ s.t

$\exp : \{ v \in T_{z_0}M \mid \|v\| < \delta \} \longrightarrow U \subset M$ is diffeomorphic and $\exp(0) = z_0$.

Let e_1, e_2 be an orthonormal basis for E_{z_0}, and consider extensions e_1, e_2 over some nbd of z_0.

$$\nabla e_1 = \theta_{11} e_1 + \theta_{21} e_2 \qquad \nabla e_2 = \theta_{12} e_1 + \theta_{22} e_2.$$

$e = f_1 e_1 + f_2 e_2$ where $f_1\ f_2$ are smooth over some nbd of z_0.
$(\, e(z) = f_1(z)\, e_1 + f_2(z)\, e_2$

$$\nabla e = (df_1 + \theta_{11} f_1 + \theta_{12} f_2) \otimes e_1 + (df_2 + \theta_{21} f_1 + \theta_{22} f_2) \otimes e_2$$

$$\Rightarrow \nabla_{\gamma'(t)} e = \gamma'(t)(f) + \theta(\gamma'(t))\, f(\gamma(t)) = 0.$$

$$\Rightarrow (df)(\gamma'(t)) + \theta(\gamma'(t))\, f(\gamma(t)) = 0$$

$\Rightarrow$ Let $f(\gamma(t)) = f(\exp tv) = \Psi(t, v)$.

$$\Rightarrow \frac{d\Psi(t,v)}{dt} + \theta(t,v)\,\Psi(t,v) = 0$$

$$\Psi(0, v) = f(z_0) = (1, 0).$$