$$\begin{pmatrix} \alpha & \beta \\ \gamma & \omega \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} \Rightarrow \begin{pmatrix} \alpha_1 & -\alpha_2 & \beta_1 & -\beta_2 \\ \alpha_2 & \alpha_1 & \beta_2 & \beta_1 \\ \gamma_1 & -\gamma_2 & \omega_1 & -\omega_2 \\ \gamma_2 & \gamma_1 & \omega_2 & \omega_1 \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \end{pmatrix}$$

$\Rightarrow \quad |\alpha|^2 + |\beta|^2 = 1 = |\gamma|^2 + |\omega|^2$

$\alpha\bar{\gamma} + \beta\bar{\omega} = 0 \iff 0 = \alpha_1\gamma_1 + \alpha_2\gamma_2 + \beta_1\omega_1 + \beta_2\omega_2 = \alpha_2\gamma_1 - \alpha_1\gamma_2 + \beta_2\omega_1 - \beta_1\omega_2$

$\Rightarrow \quad \alpha_1^2 + \alpha_2^2 + \beta_1^2 + \beta_2^2 = \gamma_1^2 + \gamma_2^2 + \omega_1^2 + \omega_2^2 = 1$

$\Rightarrow \quad x_1'\, dx_2' \wedge dx_3' \wedge dx_4' - x_2'\, dx_1' \wedge dx_3' \wedge dx_4' + x_3'\, dx_1' \wedge dx_2' \wedge dx_4'$
$- x_4'\, dx_1' \wedge dx_2' \wedge dx_3'$
$= x_1'\, dy_1' \wedge dx_2' \wedge dy_2' - y_1'\, dx_1' \wedge dx_2' \wedge dy_2' + x_2'\, dx_1' \wedge d\, y_1' \wedge dy_2'$
$- y_2'\, dx_1' \wedge dy_1' \wedge dx_2'$

$dx_2' \wedge dy_2' = d(\gamma_1 x_1 - \gamma_2 y_1 + \omega_1 x_2 - \omega_2 y_2) \wedge d(\gamma_2 x_1 + \gamma_1 y_1 + \omega_2 x_2 + \omega_1 y_2) = (\gamma_1\, dx_1 - \gamma_2\, dy_1 + \omega_1\, dx_2 - \omega_2\, dy_2) \wedge (\gamma_2\, dx_1 + \gamma_1\, dy_1 + \omega_2\, dx_2 + \omega_1\, dy_2) = -\gamma_2^2\, dy_1 \wedge dx_1 + \omega_1\gamma_2\, dx_2 \wedge dx_1 - \omega_2\gamma_2\, dy_2 \wedge dx_1 + \gamma_1^2\, dx_1 \wedge dy_1 + \gamma_1\omega_1\, dx_2 \wedge dy_1 - \gamma_1\omega_2\, dy_2 \wedge dy_1 + \gamma_1\omega_2\, dx_1 \wedge dx_2 - \gamma_2\omega_2\, dy_1 \wedge dx_2 - \omega_2^2\, dy_2 \wedge dx_2 + \gamma_1\omega_1\, dx_1 \wedge dy_2 - \gamma_2\omega_1\, dy_1 \wedge dy_2 + \omega_1^2\, dx_2 \wedge dy_2 = +(\gamma_1^2 + \gamma_2^2)\, dx_1 \wedge dy_1 + (-\omega_1\gamma_2 + \gamma_1\omega_2)\, dx_1 \wedge dx_2 + (\gamma_2\omega_2 + \gamma_1\omega_1)\, dx_1 \wedge dy_2 + (\gamma_1\omega_1 + \gamma_2\omega_2)\, dx_2 \wedge dy_1 + (\gamma_1\omega_2 - \gamma_2\omega_1)\, dy_1 \wedge dy_2 + (\omega_1^2 + \omega_2^2)\, dx_2 \wedge dy_2$

So complicated !

Note the following: $n = 3$.

$$x_1 = a_{11}x_1' + a_{21}x_2' + a_{31}x_3'$$
$$x_2 = a_{12}x_1' + a_{22}x_2' + a_{32}x_3' \quad \text{assume } \det(a_{ij}) = 1.$$
$$x_3 = a_{13}x_1' + a_{23}x_2' + a_{33}x_3'$$

$\Rightarrow \quad d(x_1\, dx_2 \wedge dx_3) = dx_1 \wedge dx_2 \wedge dx_3 = dx_1' \wedge dx_2' \wedge dx_3'$

Suppose $x_1\, dx_2 \wedge dx_3 = b_{123}\, x_1'\, dx_2' \wedge dx_3' + b_{213}\, x_2'\, dx_1' \wedge dx_3' + b_{312}\, x_3'\, dx_1' \wedge dx_2'$