

$$E' = \{ (x_1, x_2, K(\alpha_{11} - \alpha_{10})x_1, K(\alpha_{12} - \alpha_{10})x_2) - 1(0, 0, \begin{smallmatrix} * \\ * \end{smallmatrix}) \}$$

V_2 intersect with E' at $(0, 0, 0, 0)$ $(*, *) \neq 0$

$$\begin{aligned} \textcircled{1} \quad \lim_{h \rightarrow 0} \frac{(0, 0, 0, 0) - (0, h, 0, k_2 h)}{h} \\ = -(0, 1, 0, -k_2). \end{aligned}$$

$$\textcircled{2} \quad \lim_{h \rightarrow 0} \frac{(0, 0, 0, 0) - (h, 0, k_1 h, 0)}{h} = -(1, 0, k_1, 0)$$

$$\textcircled{3} \quad \text{im } V_{2*} = \left\langle \begin{pmatrix} 1, 0, \alpha_{21} - \alpha_{20}, 0 \\ 0, 1, 0, \alpha_{22} - \alpha_{20} \end{pmatrix} \right\rangle$$

$$\det \begin{pmatrix} 1, 0, k_1, 0 \\ 0, 1, 0, k_2 \\ 1, 0, \alpha_{21} - \alpha_{20}, 0 \\ 0, 1, 0, \alpha_{22} - \alpha_{20} \end{pmatrix} = \begin{vmatrix} 1, 0, k_1, 0 \\ 0, 1, 0, k_2 \\ 0, 0, \alpha_{21} - \alpha_{20} - k_1, 0 \\ 0, 0, 0, \alpha_{22} - \alpha_{20} - k_2 \end{vmatrix}$$

$$= (\alpha_{21} - \alpha_{20} - k_1)(\alpha_{22} - \alpha_{20} - k_2) \neq 0 \text{ for sufficiently small } k_1, k_2$$

$$\Rightarrow T_0 E' + \text{im } V_{2*} = T_0 \mathbb{C}^4$$

$\Rightarrow V_2$ meets with E' at $(0, 0, 0, 0)$ transversely.

In general, observe the following:

$$V_1 = (\alpha_{11} - \alpha_{10})x_1 \frac{\partial}{\partial x_1} + \dots + (\alpha_{1n} - \alpha_{10})x_n \frac{\partial}{\partial x_n}$$

$$V_2 = (\alpha_{21} - \alpha_{20})x_1 \frac{\partial}{\partial x_1} + \dots + (\alpha_{2n} - \alpha_{20})x_n \frac{\partial}{\partial x_n}$$

$$\vdots$$

$$V_q = (\alpha_{q1} - \alpha_{q0})x_1 \frac{\partial}{\partial x_1} + \dots + (\alpha_{qn} - \alpha_{q0})x_n \frac{\partial}{\partial x_n}$$

$$\Rightarrow E' = \langle V_1, \dots, V_q \rangle \text{ is locally } \mathbb{C}^n \times \mathbb{C}^q.$$