and the tautological bundle

$$T = N_{E|\tilde{\mathbb{P}}^5},$$

we obtain

$$e|_E = c_1(T) = \zeta.$$

⸢ $\omega|_{W_2} = 2\ell \Rightarrow \tilde{\omega}|_E = \pi^*\omega|_{\pi^* W_2} = \pi^*(\omega|_{W_2}) = \pi^*(2\ell) = 2\pi^*\ell = 2\tilde{\ell}$.

By the observation on P607, T = the normal bundle to $E = \mathbb{P}(N_{W_2|u})$ in $\tilde{W} = N_{E|\tilde{\mathbb{P}}^5}$.

$e|_E = e\cdot e = c_1(T) = \zeta$, since $e\cdot e$ is the self intersection number of E in $N_{E|\tilde{\mathbb{P}}^5} = T$, which is $c_1(T)$. ($\because$ $\dim E = 4$ and $\dim \tilde{W} = 5$. see P413. Gauss–Bonnet Formula II, $c_1(T)$ is the obstruction of constructing a nonvanishing global section of T. Refer to P393.

⸥

Also,

$$(\tilde{\omega}^5)_{\tilde{W}} = (\omega^5)_W = 1.$$

⸢ $(\pi^*\omega)^5|_{\pi^* W} = \omega^5|_W = 1$, since $\omega \in H^2(W, \mathbb{Z}) = \mathbb{Z}$ and ω^5 is a generator of $H^{10}(\mathbb{P}^5, \mathbb{Z}) = \mathbb{Z}$.

⸥