and we have the

Hopf Index Theorem. $\sum_{v(p)=0} \iota_v(p) = \chi(M)$.

$\ulcorner$ $\operatorname{trace} f_t^*|_{H^p_{DR}(M)} = \operatorname{tr}(id|_{H^p_{DR}(M)}) = \operatorname{tr}\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = \dim H^p_{DR}(M)$

$$\Rightarrow \quad L(f_t) = \sum_{f_t(p)=p} \iota_{f_t}(p) = \sum (-1)^p \operatorname{trace}(f_t^*|_{H^p_{DR}(M)})$$

$$= \sum_p (-1)^p \dim H^p_{DR}(M) = \chi(M)$$

$$\sum_{v(p)=0} \iota_v(p) = \sum_{f_t(p)=p} \iota_v(p) = \sum_{f_t(p)=p} \iota_{f_t}(p) = L(f_t) = \chi(M).$$

$\lrcorner$

The Holomorphic Lefschetz Fixed-Point Formula

Suppose now that M is a compact complex manifold of dimension n and $f : M \longrightarrow M$ a holomorphic map. Then f act not only on the deRham cohomology of M but on the Dolbeault cohomology groups as well, and we may hope, by analogy with the Lefschetz fixed-point formula, that the action of f on $H^{*,*}_{\bar\partial}(M)$ will be reflected in the local behavior of f around its fixed points. This is in fact the case, and we will spend the remainder of this section deriving the corresponding formula.

Our starting point, as before, is a computation of the Dolbeault cohomology class of the diagonal $\Delta \subset M \times M$. To