p247. If $S \subset \mathbb{P}^{g-1}$ is any nondegenerate curve of genus g and degree $2g-2$, then S is a canonical curve.

pf). Let $\sigma_i \in H^0(S, \mathcal{O}(H))$ s.t $(\sigma_i = 0) = H_i \cap S$.

where $H_i = (Z_i = 0)$, $[Z_0, \cdots Z_{g-1}]$ coordinate for $\mathbb{P}^{g-1}$.

On $U_1 = (Z_1 \neq 0)$, $\sigma_1([Z_0, \cdots Z_{g-1}]) = 1$.

On $U_2 = (Z_2 \neq 0)$, $\sigma_1 = \frac{Z_1}{Z_2}$.

Similarly, we can describe σ_i on U_j. as $\begin{cases} \frac{Z_i}{Z_j} & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$

$\Rightarrow \quad S \xrightarrow{\iota} \mathbb{P}^{g-1}$

$[Z_0, \cdots Z_n] \longmapsto [Z_0, \cdots Z_n] = [\sigma_0(Z), \sigma_1(Z), \cdots \sigma_n(Z)]$.

$\Rightarrow$ Let E be the subspace of $H^0(S, \mathcal{O}(H))$ spanned by $\sigma_0, \sigma_1, \cdots \sigma_n$, $n = g-1$.

$\Rightarrow \quad \iota = \iota_E$. Let $D = (\sigma_0 = 0) = H_0 \cap S$.

$\Rightarrow$ By P247, if $D \neq K$, then $h^0(D) = g-1$. Here, we have to notice that $[H]|_S$ has degree $2g-2$, to use the result on P247. $\Rightarrow$ $h^0(D) = \dim H^0(S, \mathcal{O}(D))$ $= \dim H^0(S, \mathcal{O}(H_0 \cap S)) = g-1$. But we know already that $\dim E = g$. $\Rightarrow$ Contradiction.

Thus $D = K$ $\Rightarrow$ $[H]|_S = K_S = K$. and $\iota = \iota_K$.

$$4(d-1)^2 - 4(n-1)(d-1) + (n-1)^2 - 8g(n-1) \geq 0$$

Let $d-1 = a$, $n-1 = x$.

$\Rightarrow \quad x^2 - 4x(a + 2g) + 4a^2 \geq 0$

$\Rightarrow \quad x < 2(a+2g) - \sqrt{4(a+2g)^2 - 4a^2}$

$\Rightarrow \quad n < 2(d-1+2g) - \sqrt{4(a+2g)^2 - 4a^2} + 1$.