

then the form Γ_ω is nondegenerate and for any $p = [v] \in \mathbb{P}^3$

$$X_p = \sigma(p, h),$$

where the hyperplane $h \subset \mathbb{P}^3$ is the kernel of the linear functional $\Gamma_\omega(v, \cdot)$ on \mathbb{C}^4 .

$$\forall p \in l = \overline{v, v'} \Rightarrow p = av + bv'.$$

$$\forall l' \ni p \Rightarrow l' = \overline{p, u} \Rightarrow \tilde{\phi}(l') = p \wedge u$$

$$\Rightarrow p \wedge u \wedge \omega = (av + bv') \wedge u \wedge v \wedge v' = 0$$

$\Rightarrow \tilde{\phi}(l') \in H_\omega \Rightarrow H_\omega \supset \tilde{\phi}(\sigma(p)) \Rightarrow$ Since H_ω is a hyperplane containing $\sigma(p)$, by the result on P759, H_ω is tangent to G at $l = \overline{v, v'}$, by P756, and $X = H_\omega \cap G = \sigma(l)$.

By changing the coordinates, the matrix of Γ_ω may be given as

$$\textcircled{1} \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{or} \quad \textcircled{2} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\textcircled{1}$ This implies $\Gamma_\omega(e_1, e_3) = -1 = \Gamma_\omega(e_2, e_4)$, otherwise 0. In case $\{e_1, \dots, e_4\}$ is a basis for \mathbb{C}^4 .

$$\Rightarrow e_1 \wedge e_3 + e_2 \wedge e_4 = \omega$$

$\textcircled{2}$ $\Gamma_\omega(e_1, e_2) = -1$ otherwise 0.

$$\Rightarrow e_1 \wedge e_2 + e_3 \wedge e_4 = \omega \text{ which is decomposable.}$$

$$\text{Thus } \Gamma_\omega(v, v') = (e_1 \wedge e_3 + e_2 \wedge e_4) \wedge v \wedge v'.$$

$$X_p = \sigma(p) \cap H_\omega.$$