Proper Mapping Theorem. If M, N are complex manifolds, $f: M \longrightarrow N$ a holomorphic map, and $V \subset M$ an analytic variety such that $f|_V$ is proper, then $f(V)$ is an analytic subvariety of N.

The proof will be given in Section 2 of Chapter 3.

P 60

Intersection of Analytic Cycles

Suppose now that M is a compact complex manifold of dimension n, $V \subset M$ a possibly singular analytic subvariety of dimension k. As we have seen, Stokes' theorem

$$\int_V d\psi = 0$$

holds for any $(2k-1)$-form ψ on M. We may thus define a linear functional on $H^{2k}_{DR}(M)$ by

$$[\varphi] \longmapsto \int_V \varphi,$$

where V is given the natural orientation.

$\ulcorner$ $\varphi - \varphi' = d\psi$

$$\Rightarrow \int_V \varphi = \int_V \varphi' + d\psi = \int_V \varphi' + \overbrace{\int_V d\psi}^{=0 \text{ by the above}} = \int_V \varphi'. \lrcorner$$