In general, then, the tangent cone to an analytic variety $V \subset M$ at $p \in V$ is taken to be the intersection of the tangent cones at p to all local analytic hypersurfaces in M containing V. In case V is smooth at p, of course, this is just the tangent space to V at p.

Γ In case $V = \{f_1 = f_2 = 0\} \subset M^3$ smooth at p, locally

$$(z_1, z_2, z_3) \longmapsto (f_1, f_2, z_3).$$

where we assume $\left|\left(\frac{\partial f_i}{\partial z_j}\right)\right| \neq 0$ at p, $1 \le i, j \le 2$.

$\Rightarrow$ $T_pV = T_p\{f_1 = 0\} \cap T_p\{f_2 = 0\} = \mathbb{C}$, since every tangent cone of any local hypersurface containg V contains $\mathbb{C}$. For, given a local hypersurface V' containing V, $\Rightarrow$ $V' = \{f = 0\}$.

Since V is smooth, $V \overset{\text{biholomph}}{\cong} \mathbb{C}$ locally at p.

$\Rightarrow$ For $(\alpha_1, \alpha_2, \alpha_3) \in T_pV$, $\exists$ $\alpha : I \to V$ s.t $\alpha'(0) = (\alpha_1, \alpha_2, \alpha_3)$. $\Rightarrow$ Since $V \subset \{f = 0\}$, $f(\alpha(t)) = 0$. $\Rightarrow$ $\frac{\partial f}{\partial z_1} \cdot \alpha_1 + \frac{\partial f}{\partial z_2} \cdot \alpha_2 + \frac{\partial f}{\partial z_3} \cdot \alpha_3 = 0$

$\Rightarrow$ $(\alpha_1, \alpha_2, \alpha_3) \in T_pV'$.

Thus in case V is smooth at p, T_pV is just