$\Rightarrow$ If we let $\mathcal{O}_Z = {}_{m}\mathcal{O}$ and $\mathcal{O}_W = {}_{w}\mathcal{O}$. as we saw above, $f^*\mathcal{O}_W = \pi^*\mathcal{O}_W \subset \mathcal{O}_Z$, and if g is a function seperating the sheets, then $\{1, g, \cdots g^{d-1}\}$ generates $\mathcal{O}_Z$ over $f^*\mathcal{O}_W$ and is linearly independent.

Note that. if we consider $\mathcal{O}_Z/I$ as a module over $f^*\mathcal{O}_W / f^*m_W \cong \mathbb{C}$, $\mathcal{O}_Z/I$ is a complex vector space.

...

Suppose $\{1+I, g+I, \cdots g^{d-1}+I\}$ is linearly dependent.

$$\Rightarrow (a_0 + I) + (a_1 g + I) + \cdots (a_{d-1} g^{d-1} + I) = I, \quad a_i \in f^*\mathcal{O}_W.$$

$$\Rightarrow a_0 + a_1 g + \cdots + a_{d-1} g^{d-1} \in I$$

$$\Rightarrow a_0 + a_1 g + \cdots + a_{d-1} g^{d-1} = b_1 f_1 + \cdots + b_n f_n, \quad b_1, \cdots b_n \in \mathcal{O}_Z$$

$$= (b_{10} + b_{11} g + \cdots + b_{1,d-1} g^{d-1}) f_1, \quad b_{10}, b_{11}, \cdots b_{1,d-1} \in f^*\mathcal{O}_W$$

$$+ (b_{20} + b_{21} g + \cdots + b_{2,d-1} g^{d-1}) f_2, \quad b_{20}, b_{21}, \cdots b_{2,d-1} \in f^*\mathcal{O}_W$$

$$\vdots$$

$$+ (b_{n0} + b_{n1} g + \cdots + b_{n,d-1} g^{d-1}) f_n, \quad b_{n0}, \cdots \in f^*\mathcal{O}_W.$$

$$\Rightarrow a_0 = b_{10} f_1 + b_{20} f_2 + \cdots + b_{n0} f_n \in f^* m_W$$

$$a_1 = b_{11} f_1 + b_{21} f_2 + \cdots + b_{n1} f_n \in f^* m_W$$

$$\vdots$$

$$a_{d-1} = b_{1,d-1} f_1 + b_{2,d-1} f_2 + \cdots + b_{n,d-1} f_n \in f^* m_W$$

since $f_1 = f^*w_1, \cdots f_n = f^*w_n$. $\{w_1, \cdots w_n\} = m_W$.

$\Rightarrow a_i + f^*m_W = 0$. $\Rightarrow \{1+I, \cdots g^{d-1}+I\}$ is linearly independent.

As we saw above, $\mathcal{O}_Z \xrightarrow{d_g} d_g\mathcal{O}_Z$ is isomorphic as an $f^*\mathcal{O}_W$-module. $\Rightarrow d_g$ induces an isomorphism $d_g : \mathcal{O}_Z/I \longrightarrow d_g\mathcal{O}_Z / d_g I$