From our original discussion of Ext, it follows from the vanishing of $\mathrm{Ext}_{\mathcal{O}}^{n}(E, N)$ for all N and $n \geq 1$ that E is projective (= free); this happens if e is a unit.

See P 686.

If e is not a unit, then $\mathrm{Ext}_{\mathcal{O}}^{1}(E, \mathcal{O}) \neq 0$ and E is not projective. Q. E. D.

Ext^1 and Extensions - Global Case.

Let M be an algebraic variety, and $\mathcal{F}$, $\mathcal{G}$ coherent sheaves on M. We may speak of global extensions

$$(\mathcal{E}) \qquad 0 \longrightarrow \mathcal{F} \longrightarrow \mathcal{E} \longrightarrow \mathcal{G} \longrightarrow 0,$$

by which we mean an exact sequence of sheaves of $\mathcal{O}$- modules - then $\mathcal{E}$ is necessarily coherent - and with the equivalence relation and notion of trivial extension as in the local case.

One's first guess might be that such $(\mathcal{E})$'s are in bijective correspondence with $H^0(M, \underline{\mathrm{Ext}}_{\mathcal{O}}^{1}(\mathcal{F}, \mathcal{G}))$. This is not quite correct