pf) ($\Leftarrow$) Suppose π_p is not an embedding. $\Rightarrow \exists\ q_1 \neq q_2 \in V$
s.t $\pi_p(q_1) = \pi_p(q_2)$.
Let $q_1 = [A_0, A_1, \cdots A_{n-1}, A_n]$, $q_2 = [B_0, B_1, \cdots B_{n-1}, B_n]$.
$\Rightarrow$ Since $[A_0, A_1, \cdots A_{n-1}, 0] = [B_0, B_1, \cdots B_{n-1}, 0]$, $\exists\ t_0$ s.t
$(A_0, A_1, \cdots A_n) + t_0(B_0, B_1, \cdots B_n) = (0, \cdots 0, A_n + t_0 B_n)$

$\Rightarrow A_n + t_0 B_n \neq 0$, otherwise $q_1 = q_2$.
$\Rightarrow [(A_0, \cdots A_n) + t_0(B_0, B_1 \cdots B_n)] = p \Rightarrow p \in C(V)$

($\Rightarrow$) Suppose $p \in C(V)$.
(i) Case 1 $q_1 \neq q_2 \in V$ s.t $[s(A_0, \cdots A_n) + t(B_0, \cdots B_n)] = [0, 0, \cdots 1]$, where $q_1 = [A_0, \cdots A_n]$, $q_2 = [B_0, B_1, \cdots B_n]$.
$\Rightarrow s A_i + t B_i = 0$, $i = 0, \cdots n-1$, $s \neq 0$. $s \neq 0 \neq t$.
$\Rightarrow [A_0, \cdots A_n] = [-\frac{t}{s} B_0, \cdots -\frac{t}{s} B_{n-1}, A_n]$
$\Rightarrow \pi_p(q_1) = [A_0, A_1, \cdots A_{n-1}, 0] = [-\frac{t}{s} B_0, \cdots -\frac{t}{s} B_{n-1}, 0] = \pi_p(q_2)$
$\Rightarrow \pi_p$ is not an imbedding.

(ii) Case 2. $\exists\ q \in C(V)$ s.t $\exists$ a line $\mathbb{P}^1$ tangent to V at q passing through p.
By the geometric description of tangent, $\exists\ q_1 \neq q_2 \in V$ (which is) near q s.t the line containing q_1 & q_2 passes p. $\Rightarrow \pi_p(q_1) = \pi_p(q_2)$. //

Suppose V can not be smoothly projected into a hyperplane even if $n > 2 \cdot \dim V + 1$.
$\Rightarrow \exists$ no point $p \notin V$ s.t $p \notin C(V)$, in other words,
For any point $p \in \mathbb{P}^n - V$, $p \in C(V)$
This implies $C(V) = \mathbb{P}^n$, since $C(V) \supset V$.
$\Rightarrow \dim C(V) = n$. But since $\dim C(V) \leq 2 \cdot \dim V + 1$,
$n \leq 2 \cdot \dim V + 1 \Rightarrow$ Contradiction to $n > 2 \cdot \dim V + 1$ //