

$\Rightarrow \partial g / \partial w_0 \neq 0$  at  $(0, \frac{p_2}{p_1})$  since  $S$  is smooth.

$\Rightarrow \exists$  a holomorphic function  $h$  locally s.t.  $h(\frac{p_2}{p_1}) = 0$  and  $g(h(w_2), w_2) = 0$ , by the implicit function theorem.

$\Rightarrow \frac{\partial g}{\partial w_0} \frac{\partial h(w_2)}{\partial w_2} + \frac{\partial g}{\partial w_2} = 0$  by the chain rule

on some open set around  $(0, \frac{p_2}{p_1})$ .

$\Rightarrow$  Since  $\frac{\partial g}{\partial w_2}(0, \frac{p_2}{p_1}) = 0$ , and  $\frac{\partial g}{\partial w_0}(0, \frac{p_2}{p_1}) \neq 0$ ,

$$\frac{\partial h}{\partial w_2}(\frac{p_2}{p_1}) = 0.$$

$\Rightarrow$  Since  $w_2 \mapsto (h(w_2), w_2)$  gives an biholomorphic map between  $\mathbb{C}$  and  $S^{(U_1 \cong \mathbb{C}^*)}$  around  $(0, \frac{p_2}{p_1})$ ,

$\frac{\partial}{\partial w_2}$  corresponds to  $\frac{\partial h}{\partial w_2} \frac{\partial}{\partial u_1} + \frac{\partial}{\partial w_2}$ .

$\Rightarrow$  At  $\frac{p_2}{p_1}$ , it corresponds to  $\frac{\partial}{\partial w_2}$  which corresponds

to a tangent vector lying in  $(\mathbb{Z}_0 = 0)$ , since  $(\mathbb{Z}_0 = 0)$  corresponds to  $\{(0, w_2)\}$ .

Thus we can conclude that  $L = (\mathbb{Z}_0 = 0)$  is tangent to  $S$  at  $p$ , i.e.,  $(0, \frac{p_2}{p_1})$ .

This contradicts to the assumption above.

$\Rightarrow$  All points of  $S \cap (\partial f / \partial z_2 = 0)$  lie in the finite plane  $(\mathbb{Z}_0 \neq 0)$ .

$\Rightarrow$  We have only to consider  $\nabla$  on  $(\mathbb{Z}_0 = 0)$ . <sup>our problem</sup>

We know from the above  $\partial f / \partial z_2 \neq 0 \Rightarrow$  unramified.

②  $\partial f / \partial z_2 = 0 \Rightarrow v(q) - 1 = \text{multiplicity of } (S \cap (\partial f / \partial z_2 = 0)) \text{ at } q$ .

$\Rightarrow$  We get  $\sum (v(q) - 1) = \#(S \cdot (\partial f / \partial z_2 = 0)) = d(d-1)$ .