$$
\begin{aligned}
g^{-1}_{im} v'_m &= \sum g^{*-1}_{il}\, g^{-1}_{im} \,(d\varphi'_l - (\theta'^{*}\varphi')_l) \otimes v'_m \\
&= \sum ({}^t g_{mi}\, g^{*}_{il})^{-1} \,(d\varphi'_l - (\theta'^{*}\varphi')_l) \otimes v'_m \\
&= \sum \delta_{ml} \,(d\varphi'_l - (\theta'^{*}\varphi')_l) \otimes v'_m \\
&= \sum (d\varphi'_l - (\theta'^{*}\varphi')_l) \otimes v'_l \\
&= \sum (d\varphi'_i - \Sigma\theta'^{*}_{ij}\wedge\varphi'_j) \otimes v'_i .
\end{aligned}
$$

Now let $(z_1, \cdots z_n)$ be local coordinates on M and $\theta = (\theta_{ij})$ the connection matrix of D in terms of the frame $\{\partial/\partial z_i\}$ for $T(M)$. Write

$$
\theta_{ij} = \sum_k \Gamma^j_{ik}\, dz_k ,
$$

so that

$$
Dv = D\Big(\sum_i v^i \frac{\partial}{\partial z_i}\Big) = \sum_{j,k} \Big(\frac{\partial v^j}{\partial z_k} + \sum_i \Gamma^j_{ik} v^i \Big) \frac{\partial}{\partial z_j} \otimes dz_k .
$$

「 Here I think the authors assumed v^j's are holomorphic. See P429. middle line, & P427
According to P73. θ is of (1.0) type. 」

The torsion tensor $\tilde{\tau}$ is given by

$$
\begin{aligned}
\tilde{\tau} &= - \sum_{i,j} \theta^{*}_{ji} \wedge dz_i \otimes \frac{\partial}{\partial z_j} \\
&= \sum_{i,j} \theta_{ij} \wedge dz_i \otimes \frac{\partial}{\partial z_j} \\
&= \frac{1}{2} \sum_{i,j,k} (\Gamma^j_{ki} - \Gamma^j_{ik}) \frac{\partial}{\partial z_j} \otimes (dz_i \wedge dz_k),
\end{aligned}
$$

and so the contraction $\iota(v)\cdot\tilde{\tau} \in C^\infty(T'\otimes T'^{*})$ of $\tilde{\tau}$ by v is given by

$$
\iota(v)\cdot\tilde{\tau} = \sum_{i,j,k} (\Gamma^j_{ik} - \Gamma^j_{ki})\, v^i \frac{\partial}{\partial z_j} \otimes dz_k .
$$