$$d : C^p(\underline{U}, K^q) \longrightarrow C^p(\underline{U}, K^{q+1}),$$

satisfy $\delta^2 = d^2 = 0$, $\quad d\delta + \delta d = 0$; and hence gives rise to a double complex

$$\{ C^{p,q} = C^p(\underline{U}, K^q) ; \delta, d \}.$$

$$\begin{tikzcd}
\{\sigma_I\}_{\#I=p} \in C^p(\underline{U}, K^q) \arrow[r, "\delta"] \arrow[d, "d"'] & C^{p+1}(\underline{U}, K^q) \arrow[d, "d"] \\
C^p(\underline{U}, K^{q+1}) \arrow[r, "\delta"] & C^{p+1}(\underline{U}, K^{q+1})
\end{tikzcd}$$

$$(\delta\sigma)_{\alpha_0 \cdots \alpha_{p+1}} = \sum (-1)^i \sigma_{\alpha_0, \cdots \hat{\alpha}_i \cdots \alpha_{p+1}}$$

$$\Rightarrow \quad \underset{(d(\delta\sigma))_{\alpha_0 \cdots \alpha_{p+1}}}{d((\delta\sigma)_{\alpha_0 \cdots \alpha_{p+1}})} = \sum (-1)^i d\sigma_{\alpha_0, \cdots \hat{\alpha}_i \cdots \alpha_{p+1}} = \sum (-1)^i (d\sigma)_{\alpha_0, \cdots \hat{\alpha}_i \cdots \alpha_{p+1}}$$

$$= (\delta(d\sigma))_{\alpha_0, \cdots \alpha_{p+1}}$$

$\Rightarrow \quad d\delta = \delta d. \quad \Rightarrow \quad d\delta - \delta d = 0$ plays the same role as $d\delta + \delta d = 0$. See P443 ⊛.

Let $(C^*(\underline{U}), D)$ be the associated single complex. A refinement $\underline{U}' < \underline{U}$ of coverings induces mappings

$$C^p(\underline{U}, K^q) \longrightarrow C^p(\underline{U}', K^q),$$
$$H^*(C^*(\underline{U})) \longrightarrow H^*(C^*(\underline{U}')),$$

and we define the hypercohomology

$$\mathbb{H}^*(X, K^*) = \varinjlim_{\underline{U}} H^*(C^*(\underline{U}), D).$$