$x_0 = p_0, \cdots x_{k+1} = p_{k+1}.$

$$\Rightarrow \quad x = [p_0, \cdots p_{k+1}, x_{k+2}, \cdots x_n].$$

$$\Rightarrow \quad x - p = [(0, \cdots 0, \underset{\substack{\uparrow \\ \mathbb{P}^{n-k-2}}}{x_{k+2} \cdots x_n})] - [(0, 0 \cdots 0, \underset{\substack{\uparrow \\ \mathbb{P}^{n-k-2}}}{p_{k+2}, \cdots p_n})]$$

$$\Rightarrow \quad x \in \overline{\mathbb{P}^{n-k-2}, p}$$

$$\Rightarrow \quad \overline{\mathbb{P}^{n-k-2}, p} \cap V \ni x \quad \Rightarrow \text{ Contradiction to}$$

$\overline{\mathbb{P}^{n-k-2}, p}$ misses V. ⟫

By what we have seen, we can find a homogeneous polynomial $F(X_0, X_1, \cdots X_{k+1})$ vanishing along $\pi(V)$ but not at $\pi(p)$; correspondingly, the polynomial

$$\widetilde{F}(X_0, X_1, \cdots X_n) = F(X_0, X_1, \cdots X_{k+1}) \text{ vanishes}$$

on V but not at p.

Pf We proved that $\exists$ a homogeneous polynoml $F(X_0, \cdots X_{k+1})$ vanishing along $\pi(V)$ but not outside $\pi(V)$.

$\Rightarrow \; F \neq 0$ at p.

$$\Rightarrow \quad \widetilde{F}(X_0, X_1, \cdots X_n) = F(X_0, \cdots X_{k+1}) = 0 \quad \text{on } V$$

since $\widetilde{F}(X_0, X_1 \cdots X_n) = F \circ \pi(X_0, \cdots X_n)$. ⟫

We can thus find, for any point $p \in V$, a polynomial vanishing identically on V but not at p,