$p \in D. \Rightarrow p \in V_i \Rightarrow U_\alpha \ni p$ & a local defing fnt f_α.
$\Rightarrow$ σ is divided by f. $\Rightarrow \sigma \in \mathcal{E}(-D)$ locally.
$\Rightarrow I_n \otimes S_0 = \ker \gamma.$ ⅃

Chern Classes of Line Bundles

Let M now be a complex compact manifold of dim n. The exact sequence of sheaves

$$0 \longrightarrow \mathbb{Z} \longrightarrow \mathcal{O} \xrightarrow{\exp} \mathcal{O}^* \longrightarrow 0$$

gives a boundary map in cohomology

$$H^1(M, \mathcal{O}^*) \xrightarrow{\delta} H^2(M, \mathbb{Z}).$$

For a line bundle $L \in Pic(M) = H^1(M, \mathcal{O}^*)$, we define the first Chern class $c_1(L)$ of L (or simply Chern class) to be $\delta(L) \in H^2(M, \mathbb{Z})$; for D a divisor on M, we define the Chern class of D to be $c_1([D])$. By a slight abuse of language, we will sometimes write $c_1(L) \in H^2_{DR}(M)$ for the image of $c_1(L)$ under the natural map $H^2(M, \mathbb{Z}) \longrightarrow H^2_{DR}(M)$.

As an immediate consequence of the definition, note that

$$c_1(L \otimes L') = c_1(L) + c_1(L')$$

and $$c_1(L^*) = -c_1(L).$$

$\ulcorner$ $\delta(L \otimes L') = c_1(L \otimes L') = \delta(L) + \delta(L')$

$$0 \longrightarrow C^1(\underline{U}, \mathbb{Z}) \longrightarrow C^1(\underline{U}, \mathcal{O}) \longrightarrow C^1(\underline{U}, \mathcal{O}^*) \longrightarrow 0$$

$$h = \{(h_{\alpha\beta}, U_\alpha \cap U_\beta)\} \longmapsto \{(g_{\alpha\beta}, U_\alpha \cap U_\beta)\} = g$$
$$h' = \{(h'_{\alpha\beta}, U_\alpha \cap U_\beta)\} \longmapsto \{(g'_{\alpha\beta}, U_\alpha \cap U_\beta)\} = g'$$