$$\langle A^1_\mu A^1_\lambda \bar{A}^1_\mu \bar{A}^1_\lambda,\ \tau_1\wedge\bar{\tau}_1\wedge\tau_2\wedge\bar{\tau}_2\rangle$$
$$= -\left| A^1_\mu(\tau_1) A^1_\lambda(\tau_2) - A^1_\lambda(\tau_1) A^1_\mu(\tau_2)\right|^2 = -\left|\det\begin{pmatrix} A^1_\mu(\tau_1), & A^1_\mu(\tau_2) \\ A^1_\lambda(\tau_1), & A^1_\lambda(\tau_2)\end{pmatrix}\right|^2$$

$$\langle A^2_\mu A^2_\lambda \bar{A}^2_\mu \bar{A}^2_\lambda,\ \tau_1\wedge\bar{\tau}_1\wedge\tau_2\wedge\bar{\tau}_2\rangle$$
$$= -\left| A^2_\mu(\tau_1) A^2_\lambda(\tau_2) - A^2_\lambda(\tau_1) A^2_\mu(\tau_2)\right|^2.$$
$$= -\left|\det\begin{pmatrix} A^2_\mu(\tau_1), & A^2_\mu(\tau_2) \\ A^2_\lambda(\tau_1), & A^2_\lambda(\tau_2)\end{pmatrix}\right|^2$$

Thus

$$\langle C_1(\Theta)^2 - 2C_2(\Theta),\ \left(\frac{2}{\sqrt{-1}}\right)^2 \tau_1\wedge\bar{\tau}_1\wedge\tau_2\wedge\bar{\tau}_2\rangle$$

$$= \sum_{\mu,\lambda} \left|\det\begin{pmatrix} A^1_\mu(\tau_1), & A^1_\mu(\tau_2) \\ A^1_\lambda(\tau_1), & A^1_\lambda(\tau_2)\end{pmatrix}\right|^2 + \left|\det\begin{pmatrix} A^2_\mu(\tau_1), & A^2_\mu(\tau_2) \\ A^2_\lambda(\tau_1), & A^2_\lambda(\tau_2)\end{pmatrix}\right|^2$$

$$+ 2\det\begin{pmatrix} A^1_\mu(\tau_1), & A^1_\mu(\tau_2) \\ A^2_\lambda(\tau_1), & A^2_\lambda(\tau_2)\end{pmatrix} \overline{\det\begin{pmatrix} A^2_\mu(\tau_1), & A^2_\mu(\tau_2) \\ A^1_\lambda(\tau_1), & A^1_\lambda(\tau_2)\end{pmatrix}} \quad \cdots\cdots \circledast$$

In case $\dim M = 2$, $1 \le \mu, \lambda \le 2$.

$\circledast$ becomes

$$2\left|\det\begin{pmatrix} A^1_1(\tau_1), & A^1_1(\tau_2) \\ A^1_2(\tau_1), & A^1_2(\tau_2)\end{pmatrix}\right|^2 + 2\left|\det\begin{pmatrix} A^2_1(\tau_1), & A^2_1(\tau_2) \\ A^2_2(\tau_1), & A^2_2(\tau_2)\end{pmatrix}\right|^2$$

$$+ 2\left|\det\begin{pmatrix} A^1_1(\tau_1) & A^1_1(\tau_2) \\ A^2_1(\tau_1), & A^2_1(\tau_2)\end{pmatrix}\right|^2(-1) - 2\left|\det\begin{pmatrix} A^1_2(\tau_1), & A^1_2(\tau_2) \\ A^2_2(\tau_1), & A^2_2(\tau_2)\end{pmatrix}\right|^2$$

$$+ 2\det\begin{pmatrix} A^1_1(\tau_1), & A^1_1(\tau_2) \\ A^2_2(\tau_1), & A^2_2(\tau_2)\end{pmatrix} \overline{\det\begin{pmatrix} A^2_1(\tau_1), & A^2_1(\tau_2) \\ A^1_2(\tau_1), & A^1_2(\tau_2)\end{pmatrix}}$$

$$+ 2\det\begin{pmatrix} A^1_2(\tau_1), & A^1_2(\tau_2) \\ A^2_1(\tau_1), & A^2_1(\tau_2)\end{pmatrix} \overline{\det\begin{pmatrix} A^2_2(\tau_1), & A^2_2(\tau_2) \\ A^1_1(\tau_1), & A^1_1(\tau_2)\end{pmatrix}}$$

If we plug in $(A^1_1(\tau_1), A^1_1(\tau_2)) = (1, 1)$, $(A^1_2(\tau_1), A^1_2(\tau_2)) = (1,1)$