Now v_1 and v_2 will be linearly dependent at $X \in \mathbb{P}^n$ when there exist $(\lambda_1, \lambda_2) \neq 0$ such that

$$[\lambda_1 \alpha_{10} X_0 + \lambda_2 \alpha_{20} X_0, \cdots \lambda_1 \alpha_{1n} X_n + \lambda_2 \alpha_{2n} X_n] = [X_0, \cdots X_n],$$

and by the assumption that all 2×2 minor determinants of A are distinct and nonzero, this will be the case only when all but two of the homogeneous coordinates of X are zero, i.e., when X lies on a line $\overline{P_i P_j}$ for some $0 \leq i \neq j \leq r$. D_2 thus consists of the union of the $\binom{n+1}{2}$ lines $\overline{P_i, P_j}$; or in other words, if

ω is the hyperplan class on $\mathbb{P}^n$,

$$C_{n-1}(\mathbb{P}^n) = \binom{n+1}{2} \cdot \omega^{n-1}.$$

$\ulcorner$ We already compute D_2 on P356 back.
Again by Gauss - Bonnet Formula II, $C_{n-1}(\mathbb{P}^n)$ is Poincare dual to the degeneracy cycle D_{n-n+2}.
$\omega \in H^2(\mathbb{P}^n)$, $H^{2n-2}(\mathbb{P}^n) = \langle \omega^{n-1} \rangle$.
$\omega^{n-1} \in H^{2n-2}(\mathbb{P}^n) \longleftrightarrow [\mathbb{P}^2] \in H_2(\mathbb{P}^n)$.
$\Rightarrow$ Since $D_2 = {}_{n+1}C_2 [\mathbb{P}^2]$, $C_{n-1}(\mathbb{P}^n) = {}_{n+1}C_2 \omega^{n-1}$. $\lrcorner$

In general, $v_1, v_2, \cdots v_q$ will be linearly dependent at X exactly when all but q of the homogeneous coordinates of X vanish, i.e.,

$$D_q = \bigcup_{\#I=q} \overline{P_{i_1}, \cdots \cdots P_{i_q}}$$

consists of the union of the coordinate $(q-1)$-planes span