

The conclusion that the Chern class $C_1([D])$ represents, on the one hand, the Poincaré dual of the fundamental homology cycle carried by a divisor D , and on the other hand is given in de Rham cohomology by $\frac{i}{2\pi}$ times the curvature of any connection in the line

bundle $[D]$, is of fundamental importance for what follows. The method of proof of this proposition, i.e. applying Stoke's theorem to a differential form with singularities - is likewise ubiquitous, and will be systematized in Chapter 3.

The simplest consequence of this proposition is the fact that the divisor (f) of a meromorphic function is homologous to zero. This is intuitively clear:

$$\begin{aligned} \sqcap \quad (f) = D. \quad \Rightarrow \quad L = [D] = M \times \mathbb{C} \text{ since } f \text{ is a meromorphic section of the trivial bundle. al } (f) = L. \\ \text{See P136.} \quad \Rightarrow \quad C_1(L) = 0 = \eta_D \Rightarrow V \sim 0. \quad \sqcup \end{aligned}$$

Drawing an arc γ from $\lambda_0 = 0$ to $\lambda_1 = \infty$ on the Riemann sphere P'_λ ,

the divisors $\{(\lambda_0 f + \lambda_1) \mid [\lambda_0, \lambda_1] \in \gamma\}$ trace out a chain with boundary $(f)_0 - (f)_\infty$.

$$\begin{aligned} \sqcap \quad IP'_\lambda &= [(\lambda_0, \lambda_1)] \quad \lambda_0, \lambda_1 \in \mathbb{C}. \\ \bar{\lambda}_0 &= [(0, 1)] \quad \bar{\lambda}_1 = [(1, 0)] \end{aligned}$$

$$\gamma(t) = [(\cos t, \sin t)] \quad 0 \leq t \leq \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\gamma(t) = [(\sin 2\pi t, \cos 2\pi t)] \quad 0 \leq t \leq \frac{1}{4}$$

$$\Rightarrow \{(\sin 2\pi t)f + \cos 2\pi t\} \mid 0 \leq t \leq \frac{1}{4}.$$