$a_i - i \leq c_i - i \leq a_{i-1} - i \quad \Rightarrow \quad a_i \leq c_i \leq a_{i-1}$

Since $\sum c_i = a_d + (a_1 + a_2, \cdots a_{d-1})$ by Pieri's formula,

$\sum c_i = \sum c_i = \sum a_i$.

$\Rightarrow \quad c_i = a_i$ for all $i = 1, \cdots d. \quad \Rightarrow \quad c = a$ //

The Schubert cycle $\sigma_{a_1 \cdots a_d}$ thus appears once in (*), with coefficient $(-1)^d$.

⌈ $a_i - i \in [a_i - i, a_{i-1} - i]$ for all $i = 1, \cdots d$.

$\Rightarrow$ By the argument above, $\sigma_{a_1 \cdots a_d}$ can appear only in the last term of the sum (*).

$$(-1)^d \sum \sigma_{a_d} \cdot \sigma_{a_1 \cdots a_{d-1}}$$

$$= (-1)^d \sum_{\substack{a_i \leq c_i \leq a_{i-1} \\ \sum c_i = \sum a_i}} \sigma_c = (-1)^d \sigma_{a_1 \cdots a_d}, \text{ since}$$

$a_i \leq c_i$ and $\sum c_i = \sum a_i$, $c = a$ is the only possible choice.

//

2. If no integer $c_i - i$ appears in the interval $[a_k - k, a_{k-1} - k]$, then we have

$$c_1 - 1 \in [a_1 - 1, n-k]$$

$$\vdots$$

$$c_{k-1} - k+1 \in [a_{k-1} - k+1, a_{k-2} - k+1]$$

$$c_k - k \in [a_{k+1} - k - 1, a_k - k - 1]$$

$$\vdots$$

$$c_d - d \in [-d-1, a_d - d - 1],$$