

The first assertion we can prove now: clearly the differential $d\varphi$ of the rational functions on V span the cotangent space to V at every point, and so a finite number of them do; any meromorphic form on V is then a linear combination of wedge products of these forms with meromorphic, hence rational, coefficient functions.

$$\Gamma \quad V \subset \mathbb{P}^n.$$

$$O_n \quad U_0 = (X_0 \neq 0), \quad U_0 \longrightarrow \mathbb{C}^n \quad \text{is}$$

$$[X_0, \dots, X_n] \longmapsto \left(\frac{X_1}{X_0}, \dots, \frac{X_n}{X_0} \right)$$

a holomorphic chart.

$$\Rightarrow T^*U_0 \text{ is generated by } \left\{ d\left(\frac{X_1}{X_0}\right), \dots, d\left(\frac{X_n}{X_0}\right) \right\}$$

$$\Rightarrow T^*\mathbb{P}^n \text{ is generated by } \left\{ d\left(\frac{X_1}{X_0}\right), \dots, d\left(\frac{X_n}{X_0}\right), d\left(\frac{X_0}{X_1}\right), \dots, d\left(\frac{X_0}{X_n}\right) \right\}$$

$$\text{Note that } \frac{X_i}{X_j} = \frac{X_i}{X_0} \frac{X_0}{X_j}.$$

$$\Rightarrow d\left(\frac{X_i}{X_j}\right) = d\left(\frac{X_i}{X_0}\right) \frac{X_0}{X_j} + \frac{X_i}{X_0} d\left(\frac{X_0}{X_j}\right)$$

$$= \frac{X_0}{X_j} d\left(\frac{X_i}{X_0}\right) - \frac{X_i}{X_0} \frac{X_0^2}{X_j^2} d\left(\frac{X_j}{X_0}\right).$$

$$\Rightarrow T^*V \text{ is generated by restrictions of } \left\{ d\left(\frac{X_i}{X_0}\right), \dots \right\} \text{ to } V.$$