$f_3(z_1) = a_{31} z_1^{-1} + \cdots \qquad f_3(z_2) = a_{32} z_2^{-1} + \cdots$

$$\Rightarrow \begin{bmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \\ a_{31}, & a_{32} \end{bmatrix} \text{ has rank } 2 < 3 = r$$

Thus r must be $\le d$.

Now, if ω is any holomorphic 1-form on S, then by the residue theorem, for each ν,

$$0 = \sum_i Res_{p_i}(f_\nu \omega)$$
$$= \sum_i a_{\nu i} \cdot \left(\frac{\omega(p_i)}{dz_i} \right).$$

~~$D + (f_0) \ge 0 \quad \cdots \quad D + (f_r) \ge 0$~~

~~Let $D' = D + (f_0) \Rightarrow D' + (f_1/f_0) \ge 0 \ \cdots \ D' + (f_r/f_0) \ge 0$.~~

~~$\Rightarrow f_1 = f_1/f_0$ has poles at points in D'.~~

Since we assume that $f_0 = 1$, and $D + (f_\lambda) \ge 0$, poles of f_λ's are in D, i.e., f_λ's have poles at p_i's.

$\Rightarrow 0 = \sum_i Res_{p_i}(f_\nu \omega)$ by P222, Residue Theorem.

$= \sum_i a_{\nu i} \frac{\omega(p_i)}{dz_i}$, since $f_\nu \omega = (a_{\nu i} z_i^{-1} + \cdots) \cdot \left(\frac{\omega(p_i)}{dz_i} + [1] \right)$.

This gives r independent relations on the points p_i on the canonical curve, establishing the inequality (*).