a curve of degree d. $\Rightarrow$ By p172, $\#\left(\left(\partial f/\partial z_2 = 0\right) \cdot S\right)$
$= d(d-1)$ (counting multiplicity).

I think we had better understand $(f=0)$ as $(F=0)$, and $(\partial f/\partial z_2 = 0)$ as $(\partial F/\partial Z_2 = 0)$.
The condition that $(Z_0 = 0)$ ~~is~~ be not tangent to S can be described as follows:
By p 175, for any $p \in S \cap (Z_0 = 0)$,
since the tangent space at p is represented by $\frac{\partial F}{\partial Z_0}(p) Z_0 + \frac{\partial F}{\partial Z_1}(p) Z_1 + \frac{\partial F}{\partial Z_2}(p) Z_2 = 0$,

both $\frac{\partial F}{\partial Z_1}(p)$ & $\frac{\partial F}{\partial Z_2}(p)$ can not be zero at the same time.

Suppose $p \in S \cap (\partial F/\partial Z_2 = 0) \cap (Z_0 = 0)$.

$\Rightarrow \frac{\partial F}{\partial Z_1}(p) \neq 0$

Let $p = [(0, p_1, p_2)]$ where $p_1 \neq 0$.

$$U_1 = (Z_1 \neq 0) \longrightarrow \mathbb{C}^2$$
$$[(Z_0, Z_1, Z_2)] \longmapsto \Big(\underset{\overset{\shortparallel}{\omega_0}}{\frac{Z_0}{Z_1}}, \underset{\overset{\shortparallel}{\omega_2}}{\frac{Z_2}{Z_1}}\Big).$$

$\Rightarrow S = \{ g(\omega_0, \omega_2) = 0 \mid g(\omega_0, \omega_2) = F(\frac{Z_0}{Z_1}, 1, \frac{Z_2}{Z_1}) \}$

$\left(\frac{\partial F}{\partial Z_2} = 0\right) = \left(\frac{\partial g}{\partial \omega_2} = 0\right)$ on $U_1 \cong \mathbb{C}^2$.

$(Z_0 = 0) = (\omega_0 = 0) = L$

If $\omega_0 = 0$, $\omega_2 = \frac{p_2}{p_1}$, $\frac{\partial g}{\partial \omega_2}(0, \frac{p_2}{p_1}) = 0$ & $g(0, \frac{p_2}{p_1}) = 0$.