If $\Lambda \in \iota(M) \cap \sigma_{1,\dots,1}(V)$, then $\iota(x) = \Lambda$, $x \in M$.

$\Lambda \in \sigma_{\underbrace{1,\dots,1}_{r}}(V) \Rightarrow \dim(\Lambda \cap V_{n-k+r-1}) \geq r \Rightarrow$ By the previous result, $\sigma_1 \cdots \sigma_{k-r+1}$ are linearly dependent at x.

$\Rightarrow x \in D_{k-r+1}$.

Conversely, if $x \in D_{k-r+1}$, consider $\iota(x) = \Lambda$. What we have to check is if $\Lambda \in \sigma_{1,1,\dots,1}(V)$. Again, by the previous result, $\dim(\Lambda \cap V_{n-k+r-1}) \geq r$

$x \in D_{k-r+1} \subset D_{k-(r-1)+1} \subset D_{k-(r-2)+1} \subset \cdots \subset D_k$.

$\Rightarrow$ By the previous result,

$$\left.\begin{aligned} &\dim(\Lambda \cap V_{n-k+(r-1)-1}) \geq r-1 \\ &\dim(\Lambda \cap V_{n-k+(r-2)-1}) \geq r-2 \\ &\qquad\vdots \\ &\dim(\Lambda \cap V_{n-k+(r-r)-1}) \geq r-r. \end{aligned}\right\} ①$$

Suppose $x \notin D_{k-(r+1)+1}$.

$$\begin{aligned} \Rightarrow \dim(\Lambda \cap V_{n-k+r+1-0}) &= \dim \Lambda + \dim V_{n-k+r+1} - \dim(\Lambda + V_{n-k+r+1}) \\ &= k + n-k+r+1 - n \\ &= r+1 \quad \text{--------} ② \end{aligned}$$

$\Rightarrow$ Clearly $x \notin D_{k-(r+i)+1}$ $i \geq 2$.

By the same argument, $\dim(\Lambda \cap V_{n-k+r+i}) = r+i \cdots ③$

$\Rightarrow$ By ①, ② & ③, $\Lambda \in \sigma_{1,1,\dots,1}(V)$.

Moreover the condition that $\sigma_1, \sigma_2, \cdots \sigma_k$ be generic assures that $\iota(M)$ meets $\sigma_{1,1,\dots,1}(V)$ transversely.

Let M_1 be a submanifold of M_2 and let M_2 be a submanifold of M_3. Then M_1 is a submanifold of M_3.

pf) $M_1 \subset M_2 \subset M_3$. Given $x \in M_1$, we have an open set $U_2 \ni x$ s.t U_2 is diffeomorphic to an open disk and