i.e any holomorphic bundle with a finite number of global sections that generate each fiber has a metric ~~a~~ with non-negative curvature.

$$0 \longrightarrow \ker f \longrightarrow M \times \mathbb{C}^n \xrightarrow{f} E \longrightarrow 0$$

$\uparrow$ holomorphic subbundle.

$$\Rightarrow \quad \frac{M \times \mathbb{C}^n}{\ker f} \overset{C^\infty\text{-bundle isomorphic}}{\cong} E$$

$$\Rightarrow \quad \Theta\left(\frac{M \times \mathbb{C}^n}{\ker f}\right) \geq \Theta(M \times \mathbb{C}^n) = 0$$

The connection between the sign of the curvature of a vector bundle and the existence of global sections is fundamental in the theory of complex manifolds.

6. Harmonic Theory on Compact Complex Manifolds.

The Hodge Theorem

M connected, compact complex manifold of complex dim n. with a hermitian metric ds^2 which has an associated $(1,1)$ form

$$\omega = \frac{i}{2} \sum \varphi_i \wedge \overline{\varphi_i} \qquad \text{in terms of a unitary coframe } \{\varphi_1, \varphi_2, \cdots \varphi_n\}.$$

The metric ds^2 induces a hermitian metric on all tensor bundles $T^{*(p,q)}(M)$; the inner product in $T_z^{*(p,q)}(M)$ is given by taking the basis $\{\varphi_I(z) \wedge \overline{\varphi_J}(z)\}_{\#I=p,\ \#J=q}$ to be orthogonal and of length $\|\varphi_I \wedge \overline{\varphi_J}\|^2 = \cancel{2^{p+q}}\ (1)$