

$$\Rightarrow g_{\alpha\beta}(x) \omega + \mathbb{C}^k = \tilde{g}_{\alpha\beta}(x) \omega_2 + \mathbb{C}^k$$

$$\Rightarrow K = \frac{\perp U_\alpha \times \mathbb{C}^n}{\mathbb{C}^k}$$

$$(x, \omega_2 + \mathbb{C}^k) \\ \sim (x, \tilde{g}_{\alpha\beta}(x)(\omega_2) + \mathbb{C}^k)$$

$$\text{Since } \omega_2 \in \mathbb{C}^{n-k} = \{ (0 \dots 0, \overset{\leftarrow n-k}{*} \dots *) \in \mathbb{C}^n \},$$

$$K \cong Q.$$

$$\text{Let } E = G(k, n) \times \mathbb{C}^n.$$

$$S = \{ (\Lambda, v) \in G(k, n) \times \mathbb{C}^n \mid v \in \Lambda \}.$$

$$\begin{array}{ccccc} U_{I'} \cap U_I \times \mathbb{C}^n & \xleftarrow{\varphi_{I'}'} & U_I \cap U_{I'} \times \mathbb{C}^n & \xrightarrow{\varphi_I} & U_I \cap U_{I'} \times \mathbb{C}^n \\ \cup & & \cup & & \cup \\ U_{I'} \cap U_I \times \mathbb{C}^k & \xleftarrow{\quad} & S|_{U_I \cap U_{I'}} & \xrightarrow{\quad} & U_I \cap U_{I'} \times \mathbb{C}^k \end{array}$$

$$\text{Let } g_{II'} = \varphi_I \circ \varphi_{I'}^{-1} (\overset{\vee}{\Lambda}, v) \longmapsto (\Lambda, w)$$

$$= \left(\begin{array}{c|c} h_{II'} & * \\ \hline 0 & \bar{j}_{II'} \end{array} \right)$$

$$\begin{aligned} w &= (x_1 \dots x_k) \\ \Lambda^I &= \begin{pmatrix} v_1 \\ \vdots \\ v_k \end{pmatrix} \quad v = x_1 v_1 + \dots + x_k v_k \end{aligned}$$

$$S = \{ (*\Lambda, l) \in G(n-k, n^*) \times \mathbb{C}^{n^*} \mid l \in *\Lambda \}$$

$$\downarrow$$

$$G(n-k, n^*)$$

$$\Rightarrow l \in *\Lambda \Leftrightarrow l(v) = 0 \text{ for all } v \in \Lambda.$$

$$\begin{array}{c} S \\ \downarrow \\ G(n-k, n^*) \end{array}$$

$$\longrightarrow$$

$$\begin{array}{c} S^* \\ \downarrow \\ G(n-k, n^*) \end{array}$$

$$\longrightarrow G(n-k, n^*)$$

$$S \longrightarrow S^*$$

$$(*\Lambda, l) \longmapsto (*\Lambda, w)$$