se of complex tori. In Section 7, we specialize to the case of the Jacobian of a curve. We see, by two lovely theorems of Riemann, how the geometry of the Jacobian is intimately connected to the special linear systems on the curve; following this we are finally able to prove some results on the Brill-Noether problem. The chapter concludes with Torelli's theorem, following Andreotti.

1. Preliminaries.

Embedding Riemann Surfaces

Let S be a compact Riemann surfaces. Througout this chapter we assume that S is connected. If ds^2 is any metric on S with associated $(1,1)$-form ω, then $d\omega$ has degree 3 and so is trivially 0; thus any metric on S is Kähler. Indeed, since the $\bar\partial$-Laplacian of any metric commutes with the decomposition into type, we see that a form φ, written in terms of a local coordinate $z = x+iy$ as

$$\varphi = p\,dx + q\,dy = \alpha\,dz + \beta\,d\bar z,$$

is harmonic if and only if $\varphi^{1,0} = \alpha\,dz$ is holomorphic and $\varphi^{0,1} = \beta\,d\bar z$ is antiholomorphic.

Γ $\Delta_{\bar\partial} : A^{p,q} \longrightarrow A^{p,q} \Rightarrow z = \sum z_{pq} \Rightarrow \Delta_{\bar\partial} z = \sum \Delta z_{pq} \Rightarrow (\Delta_{\bar\partial} z)_{p,q} = \Delta z_{p,q} \Rightarrow \Delta_{\bar\partial}$ commutes