

since $\langle dz, z \rangle \wedge (z, dz) = -\langle dz, z \rangle \wedge (dz, z) = 0$.

$$\begin{aligned} \Gamma \quad 0 &= d r = d \|z\|^2 = d z_i \bar{z}_i = d z_i \bar{z}_i + z_i d \bar{z}_i \dots \textcircled{*} \\ &= \langle dz, z \rangle + \langle z, dz \rangle. \end{aligned}$$

$$\begin{aligned} \partial \bar{\partial} \log \|z\|^2 &= \partial \left(\frac{\bar{\partial} \|z\|^2}{\|z\|^2} \right) = \partial \left(\frac{z_i d \bar{z}_i}{\|z\|^2} \right) = \partial \left(\frac{\langle z, dz \rangle}{\|z\|^2} \right) \\ &= \frac{(\partial \langle z, dz \rangle) \|z\|^2 - \langle z, dz \rangle \wedge \partial \|z\|^2}{\|z\|^4} \end{aligned}$$

$$= \frac{1}{\|z\|^2} \langle dz, dz \rangle - \frac{\langle z, dz \rangle \wedge \langle dz, z \rangle}{\|z\|^4} = \frac{\langle dz, dz \rangle}{\langle z, z \rangle}$$

$$\begin{aligned} \langle dz, z \rangle \wedge \langle z, dz \rangle &= -\langle z, dz \rangle \wedge \langle dz, z \rangle \\ &= \bar{z}_i dz_i \wedge z_j d \bar{z}_j = 0 \quad \text{since } \langle dz, z \rangle \wedge \langle z, dz \rangle \\ &= -\langle z, dz \rangle \wedge \langle z, dz \rangle = 0, \quad \text{on } \|z\|=r. \end{aligned}$$

The last integral is therefore equal to

$$\left(\frac{\sqrt{-1}}{2} \right)^{n-p} \int_{\|z\|=r} \psi \wedge \bar{\partial} \log \|z\|^2 \wedge (\partial \bar{\partial} \log \|z\|^2)^{n-p-1}.$$

$$\Gamma \quad \frac{\bar{\partial} \|z\|^2}{r^2} = \frac{\bar{\partial} \|z\|^2}{\|z\|^2} = \bar{\partial} \log \|z\|^2.$$

$$\frac{(\partial \bar{\partial} \|z\|^2)^{n-p-1}}{r^{2n-2p-2}} = \left(\frac{\partial \bar{\partial} \|z\|^2}{r^2} \right)^{n-p-1} = \left(\frac{\partial \bar{\partial} \|z\|^2}{\|z\|^2} \right)^{n-p-1}.$$

$$\begin{aligned} \frac{\partial \bar{\partial} \|z\|^2}{\|z\|^2} &\stackrel{?}{=} \partial \bar{\partial} \log \|z\|^2 \\ &\quad \parallel \frac{\langle dz, dz \rangle}{\|z\|^2} \parallel \end{aligned}$$