U_α is the union of U'_β's. ⌟

We can give the set of line bundles on M the structure of a group, multiplication being given by tensor product and inverses by dual bundles.
If L is given by data $\{g_{\alpha\beta}\}$, L' by $\{g'_{\alpha\beta}\}$, we have seen that

$$L \otimes L' \sim \{g_{\alpha\beta}\, g'_{\alpha\beta}\}, \qquad L^* \sim \{g_{\alpha\beta}^{-1}\},$$

and so the group structure on the set of line bundles is the same as the group structure on $H^1(M, \mathcal{O}^*)$. The group $H^1(M, \mathcal{O}^*)$ is called the Picard group of M, denoted $\mathrm{Pic}(M)$.

We now describe the basic correspondence between divisors and line bundles. Let D be a divisor on M, with local defining functions $f_\alpha \in \mathcal{M}^*(U_\alpha)$ over some open cover $\{U_\alpha\}$ of M. Then the functions

$$g_{\alpha\beta} = \frac{f_\alpha}{f_\beta} \quad \text{holomorphic and non-zero in } U_\alpha \cap U_\beta.$$

and in $U_\alpha \cap U_\beta \cap U_\gamma$, we have

$$g_{\alpha\beta}\, g_{\beta\gamma} \cdot g_{\gamma\alpha} = \frac{f_\alpha}{f_\beta}\,\frac{f_\beta}{f_\gamma}\cdot\frac{f_\gamma}{f_\alpha} = 1.$$

The line bundle given by the transition functions $\{g_{\alpha\beta} = f_\alpha/f_\beta\}$ is called the associated line bundle of D, and written $[D]$. We check that it is well-defined: if $\{f'_\alpha\}$ are the alternate local data for D, then $h_\alpha = f_\alpha/f'_\alpha \in \mathcal{O}^*(U_\alpha)$,

and $$g'_{\alpha\beta} = \frac{f'_\alpha}{f'_\beta} = g_{\alpha\beta}\cdot\frac{h_\beta}{h_\alpha} \quad \text{for each } \alpha, \beta.$$

⌜ $\Rightarrow$ By (**) on p. 133, they define the same line bundle. ⌟