is quite visible. We also note that

$$d_2(\eta \wedge \omega^q) = \omega^{q+1} \qquad (0 \leq q \leq n).$$

╔ $d_2(\eta \wedge \omega^q) = d_2\eta \wedge \omega^q - \eta \wedge d_2\omega^q = \omega \wedge \omega^q - 0 = \omega^{q+1}$

since $d_2\omega = 0$ $(\because d\omega = 0)$. ┘

By way of contrast, we suppose that E, B are compact Kähler manifolds and

$$\pi : E \longrightarrow B$$

is surjective, holomorphic mapping of maximal rank. This is a differentiable fiber bundle whose fiber F is a compact manifold, and we shall prove:

The Leray spectral sequence degenerates at E_2; i.e.,

$$E_2 \cong E_\infty$$

so that

$$H^*(E, Q) \cong H^*(B, R^*_\pi(Q)).$$

╔ $\pi : E \longrightarrow B$, $\dim E = n$ $\dim B = k$.

π is of maximal rank.

$\Rightarrow$ Since π is surjective and $\dim \pi(E) \leq n$ by the proper mapping theorem, see P398, $n \geq k$.

That π is of maximal rank means that the rank of the Jacobian of π at every point is k.

$$\begin{array}{ccc} E & \xrightarrow{\pi} & B \\ \cup & & \cup \\ U & \longrightarrow & V \\ \varphi\downarrow & & \downarrow \\ \mathbb{C}^n & \longrightarrow & \mathbb{C}^k \end{array} \qquad \begin{array}{ccc} \mathbb{C}^n & \xrightarrow{F} & \mathbb{C}^k \times \mathbb{C}^{n-k} \\ (x_1, \cdots x_n) & \longmapsto & (\pi(x), x_{k+1}, \cdots, x_n) \end{array}$$