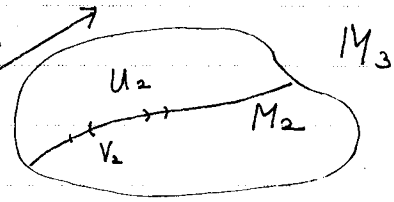
$U_3 \overset{\psi}{\cong} U_2 \times \mathbb{R}^{n_3-n_2}$. U_3 open set in M_3, where $\dim M_i = n_i$. → tubular nbd of U_2 (normal bundle of U_2)

Since M_1 is a submanifold of M_2, $\exists$ an open set V_2 s.t

$$\begin{array}{ccc} V_2 & \xrightarrow{\varphi_2} & \mathbb{R}^{n_2} \\ \cup & & \\ V_2 \cap M_1 & \xrightarrow{\varphi_2|} & \mathbb{R}^{n_1}. \end{array}$$

$\Rightarrow$ Consider $\psi(V_2 \times \mathbb{R}^{n_3-n_2})$ in M_3.

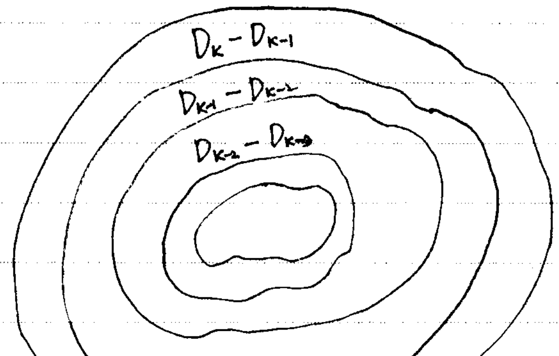
$$\begin{tikzcd} V_2 \times \mathbb{R}^{n_3-n_2} \arrow[r, "\psi"] & M_3 \\ V_2 \cap M_1 \times \mathbb{R}^{n_3-n_2} \arrow[u, phantom, "\bigcup"] \arrow[ur, "\psi|"'] & \end{tikzcd}$$



$\Rightarrow V_2 \cap M_1 = \psi(V_2 \times \mathbb{R}^{n_3-n_2}) \cap M_1$

$\Rightarrow$ This proves that M_1 is a submanifold of M_3. //

$r = 1$ $\qquad D_K \supset D_{K-1} \supset D_{K-2} \supset \cdots$



$$M \xrightarrow{\iota} G(k,n)$$

$$\iota_*(T_x M) + T_{\iota(x)} \sigma_1(V) \overset{?}{=} T_{\iota(x)} G(k,n).$$

(i) $x \in D_K - D_{K-1}$.

$\Rightarrow \operatorname{codim}_{\mathbb{R}} (D_K - D_{K-1}) = 2(k-(k-1)) = 2$

$\sigma_1, \sigma_2, \cdots \sigma_{K-1}$ are linearly independent at x. & σ_K is a linear combination of $\sigma_1, \cdots \sigma_{K-1}$.