since $L: A^p(M) \longrightarrow A^{p+2}(M)$ and $\Lambda: A^p(M) \longrightarrow A^{p-2}(M)$, we obtain

$$(*) \qquad \begin{aligned} &[\Lambda . L] = h \\ &[h . L] = -2L \\ &[h . \Lambda] = 2\Lambda . \end{aligned}$$

Pf

(i) $[L . \Lambda] = p+q-n$, $\Rightarrow$ $[\Lambda . L] = n-p-q. = (n-p-q)$
$= n-(p+q)$.

$\Rightarrow$ On $A^r(M)$, $\quad [\Lambda . L] = n-r$.

$h = (n-r)$ on $A^r(M)$, $\Rightarrow$ $[\Lambda . L] = h$ //

(ii) $[h . L] = hL - Lh$

On $A^r(M)$, $\quad Lh = L\,(n-r)\,Id = (n-r)\,L$

$hL = (n-(r+2))L$

$\Rightarrow \quad hL - Lh = (n-r-2)L - (n-r)L = -2L$ //

(iii) $[h . \Lambda] = h\Lambda - \Lambda h$

On $A^r(M)$, $\quad \Lambda h = \Lambda\,(n-r)\,Id = (n-r)\,\Lambda$

$h\Lambda = (n-(r-2))\,\Lambda = (n-r+2)\,\Lambda$

$\Rightarrow \quad h\Lambda - \Lambda h = (n-r+2)\,\Lambda - (n-r)\,\Lambda = 2\Lambda$ //

The operators L, Λ and h all commute with Δ_d, and so act on the harmonic space $\mathcal{H}_d^*(M) \cong H^*(M)$ with relation $(*)$. We may therefore give a representation of sl_2 on $H^*(M)$ by sending

$$X = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \longrightarrow \Lambda,$$

$$Y = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \longrightarrow L,$$

$$H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \longrightarrow h;$$