Thus

$$d_1 \bar{\varphi} = d_x \Big(\sum_{\# I = p} \eta_I (x, y, dy) \wedge dx_I \Big),$$

and so $E_2^{p,q}$ is given by

$$E_2^{p,q} = H^p_{DR}(B, H^q_{DR}(F)),$$

where the right-hand side may be defined by first interpreting $H^q_{DR}(F) \longrightarrow B$ as a flat vector bundle – i.e., a vector bundle associated to a representation of the fundamental group – whose locally constant sections are just the sheaf $R^q_\pi(\mathbb{C})$, and then taking the de Rham cohomology of forms with values in this bundle. Granted that this interpretation needs some amplification, but once this is done we have derived the spectral sequence of a differentiable fibration.

Γ

$$E_1^{p,q} \xrightarrow{\ d_1\ } E_1^{p+1,q}$$

$$\frac{\{ \sum_{\# I = p} \eta_I \wedge dx_I \mid d_y \eta_I = 0 \} + F^{p+1} A^{p+q}}{\{ \sum_{\# I = p} d_y \eta_I \wedge dx_I \mid \eta_I \in A^{q-1} \} + F^{p+1} A^{p+q}} \longrightarrow \frac{\{ \sum_{\# I = p+1} \eta_I \wedge dx_I \mid d_y \eta_I = 0 \} + F^{p+2} A^{p+q+1}}{\{ \sum_{\# I = p+1} d_y \eta_I \wedge dx_I \} + F^{p+2} A^{p+q+1}}$$

$$\sum_{\# I = p} \eta_I \wedge dx_I + \square \longmapsto d \sum_{\# I = p} \eta_I \wedge dx_I + \square$$

$$\bar{\varphi} \qquad \text{well-defined} \qquad d_1 \bar{\varphi} = d_x \Big(\sum_{\# I = p} \eta_I \wedge dx_I \Big) + \square$$

since $d_y \eta_I = 0$, and $d \big(\sum_{\# I = p+1} \varphi_{IJ} \, dx_I \wedge dy_J \big)$

$$= \sum_{\# I = p+1} d_x \varphi_{IJ} \, dx_I \wedge dy_J + \sum_{\# I = p+1} d_y \varphi_{IJ} \, dx_I \wedge dy_J \in \{ \sum_{\# I = p+1} d_y \eta_I \wedge dx_I \}$$