$$\left\| \int_{\mathbb{R}^n} \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, u(y)\, dy \right\|_0^2$$

$$= \int_{\mathbb{R}^n} \left| \int_{\mathbb{R}^n} \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, u(y)\, dy \right|^2 dx$$

$$\leq \int_{\mathbb{R}^n} \left(\int_{\mathbb{R}^n} \left| \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, u(y) \right| dy \right)^2 dx$$

$$= \int_{x \in \mathrm{supp}\, u + \epsilon K} \Big\} \left(\int_{\mathbb{R}^n} \left| \frac{\partial}{\partial x_k} \chi_\epsilon(x-y) \right|^2 dy \right) \cdot \int_{\mathbb{R}^n} |u(y)|^2 dy \Big\} dx$$

$\leq \quad \|u\|_0^2$, where $K = \mathrm{supp}\, \chi$ and we use here the Hölder's inequality.

Similarly, for

$$\left\| \int_{\mathbb{R}^n} \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, u(y)\, a(y)\, dy \right\|_0$$

$\leq \|u \cdot a\|_0^2 \leq \|u\|_0^2$, since $|a| \leq$ some constat for supp u.

Q.E.D.

"Comments: ① We omitted constants throughout the proo-
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② Regularity Lemma is messed up completely but the idea is there, using the Gårding inequality.