Since everything is natural, and $\tilde{\rho} : \mathcal{O} \to \mathcal{O}_Z$ is natural projection, ρ is a restriction. ⌟

Now what we are seeking is
$e \in \mathrm{Ext}^1(S; I, \mathcal{O})$ with $e_p \neq 0$ in each $\Lambda^2 T'_p(S)$ $(p \in Z)$.

⌜
$$\underset{\ni e_p \neq 0}{\underline{\mathcal{E}xt^1_{\mathcal{O}}(I, \mathcal{O})_p}} \cong \underline{\mathcal{E}xt^2_{\mathcal{O}}(\mathcal{O}_Z, \mathcal{O})_p} \cong \Lambda^2 T'_p(S) \cong \mathbb{C}$$
⌟

Applying the duality in (***), we have the following result:

Given a set of points $Z \subset S$, there is a rank-two holomorphic vector bundle $E \to S$ with $\Lambda^2 \mathcal{E} \cong \mathcal{O}$ and section $s \in H^0(S, \mathcal{O}(\mathcal{E}))$ that defines Z
$\Leftrightarrow$ there are bivectors $0 \neq \tau_p \in \Lambda^2 T'_p(S)$ $(p \in Z)$ such that
$$\sum_{p \in Z} \langle \psi, \tau_p \rangle = 0$$
for all $\psi \in H^0(S, \Omega^2)$. In particular, if $\deg Z > p_g(S)$, then (E, s) always exists.

⌜
$$\begin{tikzcd}
\mathrm{Ext}^1(S; I, \mathcal{O}) \arrow[r, "\alpha"] \arrow[d, leftrightarrow] & \mathrm{Ext}^2(S; \mathcal{O}_Z, \mathcal{O}) \arrow[r] \arrow[d, leftrightarrow] & \mathrm{Ext}^2(S, \mathcal{O}, \mathcal{O}) \arrow[d, leftrightarrow] \\
\mathrm{Ext}^1(S; I, \mathcal{O})^* & \mathrm{Ext}^2(S; \mathcal{O}_Z, \mathcal{O})^* \arrow[l] & \mathrm{Ext}^2(S; \mathcal{O}, \mathcal{O})^* \arrow[l]
\end{tikzcd}$$

$$e \mapsto \bigoplus_{p \in Z} \tau_p, \ \tau_p \neq 0 \mapsto \ ?$$

$$\bigoplus \psi|_p \longleftarrow \psi$$