

a set of distinct four points since the Veronese surface is of degree four. \Rightarrow Let $\mathbb{P}^3 \cap \tilde{\mathcal{C}}(\mathbb{P}^2) = \{ [\sigma_0(p_1), \dots, \sigma_5(p_1)], \dots, [\sigma_0(p_4), \dots, \sigma_5(p_4)] \}$.

Choose a generic point p_5 s.t. $[\sigma_0(p_5), \dots, \sigma_5(p_5)] \notin \mathbb{P}^3 \cap \tilde{\mathcal{C}}(\mathbb{P}^2)$. \Rightarrow The five points are linearly independent in $\mathbb{P}^5 \Rightarrow \exists$ 1-dimensional (a_0, \dots, a_5) s.t. $a_0 \sigma_0 + \dots + a_5 \sigma_5 = 0$ for all p_i 's.

□

Likewise, the conics through four generic points cut out on a generic line l a pencil of degree 2, which then has two branch points; so

$$\tilde{I}_p^4 \cdot \tilde{I}_l = 2.$$

$$\Gamma \quad \tilde{I}_p^4 \cdot \tilde{I}_l = \tilde{I}_{p_1} \cap \tilde{I}_{p_2} \cap \tilde{I}_{p_3} \cap \tilde{I}_{p_4} \cap \tilde{I}_l \ni C$$

$\Rightarrow C$ passes generic four points p_1, p_2, p_3, p_4 and is tangent to a generic line l .

For generic four points p_1, p_2, p_3, p_4 ,

$\{ [\sigma_0(p_1), \dots, \sigma_5(p_1)], \dots, [\sigma_0(p_4), \dots, \sigma_5(p_4)] \}$ is linearly independent, since $\tilde{\mathcal{C}}: \mathbb{P}^2 \rightarrow \mathbb{P}^5$ defined by

$p \mapsto [\sigma_0(p), \dots, \sigma_5(p)]$ is the Veronese embedding, and $\tilde{\mathcal{C}}(\mathbb{P}^2)$ is of degree 4.

$\Rightarrow \{ [a_0, \dots, a_5] \in \mathbb{P}^5 \mid a_0 \sigma_0 + \dots + a_5 \sigma_5 = 0 \text{ on } p_1, \dots, p_5 \}$
 $= \mathbb{P}^1$. Note: For generic four points p_1, p_2, p_3, p_4 ,

$\{ [\sigma_0(p_1), \dots, \sigma_5(p_1)], \dots, [\sigma_0(p_4), \dots, \sigma_5(p_4)] \}$ is a set of