$$= -\frac{1}{2}\int_U dj \wedge \psi. \quad \Rightarrow \quad -(\partial f)(\psi) = -\frac{1}{2}(dj)(\psi)$$

$$\int_U \log|h|\,\partial\psi = \frac{1}{2}\int_U (\log|h|^2)\,\partial\psi$$

$$= \frac{1}{2}\int_U (\log h + \log \bar{h})\,\partial\psi = \frac{1}{2}\int_U (\log h)\,\partial\psi + \frac{1}{2}\int_U (\log \bar{h})\,\partial\psi.$$

$$\int_U (\log \bar{h})\,\partial\psi = \int_{\Delta^{n+1}} (\log \bar{z}_{n+1})\,\partial(\psi \circ \varphi^{-1}(z_1, \cdots z_{n+1})) \cdots (*)$$

Claim: $(*) = 0$.

To prove the claim, we have only to show that

$$\int_{\Delta^{n+1}} (\log \bar{z}_{n+1})\,\partial(\psi\, dz_1 \wedge \cdots \wedge d\hat{z}_i \wedge \cdots dz_{n+1} \wedge d\bar{z}), \quad \psi \in C_c^\infty(U) \underset{\text{identify}}{\overset{\shortparallel}{\to}} \Delta^{n+1}$$

$\Rightarrow \quad i = n+1,$

$$\int_{\Delta^{n+1}} \log \bar{z}_{n+1}\; \partial(\psi\, dz_1 \wedge \cdots \wedge dz_n \wedge d\bar{z})$$

$$= \int_{\Delta^n}\left(\int_{\Delta} \log \bar{z}_{n+1}\,\frac{\partial\psi}{\partial z_{n+1}}\,dz_{n+1} \wedge d\bar{z}_{n+1}\right) \wedge dz_1 \wedge \cdots \wedge dz_n \wedge d\bar{z}_1 \cdots d\bar{z}_n$$

$\Rightarrow$ We may write the integral simply

$$\Rightarrow \int_\Delta \log \bar{z}\,\frac{\partial\phi}{\partial z}\,dz \wedge d\bar{z} \qquad \phi \in C_c^\infty(\Delta)$$

$$= \int_\Delta \log\bar{z}\;\partial(\phi\, d\bar{z}) = \lim_{\epsilon \to 0}\int_{\Delta - \Delta(\epsilon)} \log\bar{z}\;\partial(\phi\, d\bar{z})$$