$\Rightarrow \partial \Gamma_1 = \pm \Gamma = \pm \{ z \mid |f_i(z)| = \epsilon_i \}$

$\Rightarrow \int_\Gamma \omega = \pm \int_{\partial \Gamma_1} \omega = \pm \int_{\Gamma_1} d\omega = 0$ by Stokes' theorem.

We now give a sheaf-cohomological interpretation of the residue. To motivate this we note that, even though the meromorphic form ω has polar divisor $D_1 + \cdots + D_n$, it is the origin $\{0\} = D_1 \cap \cdots \cap D_n$ with which we are concerned most. To express this, we consider $\omega \in H^0(U_1 \cap U_2 \cap \cdots \cap U_n, \Omega^n)$ as a Čech $(n-1)$-cochain for the sheaf Ω^n and covering $\underline{U} = \{U_i\}$ of U^*.

$U_i = U - D_i$, $\phi \neq \cap U_i = \cap U - D_i = \cap D_i^c = (\cup D_i)^c$
$= U - (D_1 + \cdots + D_n) = \cap U_i \not\ni \{0\}$. since $0 \in D_i$.
Recall that $\cap U_i$ is open and dense in U.

Thus $\omega \in C^{n-1}(\underline{U}, \Omega^n)$, and since trivially $\delta\omega = 0$, we obtain a class in $H^{n-1}(U^*, \Omega^n)$.

Since $(\delta\omega)_{U_{\alpha_0} \cap \cdots \cap U_{\alpha_{n+1}}} = \omega|_{U_{\alpha_1} \cap \cdots \cap U_{\alpha_{n+1}}} - \omega|_{U_{\alpha_0} \cap \hat{U}_{\alpha_1} \cap U_{\alpha_n}}$
$\cdots\cdots = 0$ ($\because$ $C^n(\underline{U}, \Omega^n) = 0$),
$[\omega] \in H^{n-1}(U^*, \Omega^n) = \lim_{\underline{U}} H^{n-1}(\underline{U}, \Omega^n)$.

Denote by η_ω the image of $(1/2\pi\sqrt{-1})^n \omega$ under the Dolbeault isomorphism
$$H^{n-1}(U^*, \Omega^n) \cong H_{\bar\partial}^{n,n-1}(U^*).$$