Before we can apply the reciprocity law, we need to prove a similar result, which will enable us to normalize our basis for $H^0(S, \Omega^1)$. Let ω, ω' be two holomorphic 1-forms on S, Π^i and Π'^i their respective periods around δ_i. Let Δ and π be as above, and consider the integral around $\partial\Delta$ of the form $\pi \cdot \overline{\omega'}$. The exterior derivative $d(\pi \cdot \overline{\omega'}) = d\pi \wedge \overline{\omega}' = \omega \wedge \overline{\omega}'$, and so by Stokes' theorem

$$\int_{\partial\Delta} \pi \cdot \overline{\omega}' = \int_S \omega \wedge \overline{\omega}',$$

and, evaluating the line integral just as in the proof of the reciprocity law, we obtain

$$\int_S \omega \wedge \overline{\omega}' = \sum \left(\Pi^i \overline{\Pi'^{i+g}} - \Pi^{i+g} \overline{\Pi'^i} \right).$$

$\ulcorner$ Since $\overline{\omega}'$ is $d\overline{\pi}'$, $d\overline{\omega}' = 0 \Rightarrow d(\pi \cdot \overline{\omega}') = d\pi \wedge \overline{\omega}' = \omega \wedge \overline{\omega}'$.

$$\int_{\delta_i + \delta_i^{-1}} \pi \cdot \overline{\omega}' = -\Pi^{g+i} \int_{\delta_i} \overline{\omega}' = -\Pi^{g+i} \overline{\Pi^{i\prime}}$$

$$\int_{\delta_{g+i} + \delta_{g+i}^{-1}} \pi \cdot \overline{\omega}' = +\Pi^i \int_{\delta_{g+i}} \overline{\omega}' = \Pi^i \overline{\Pi'^{g+i}}$$

$$\Rightarrow \int_{\partial\Delta} \pi \cdot \overline{\omega}' = \sum \int_{\delta_i + \delta_i^{-1}} \pi \cdot \overline{\omega}' + \sum \int_{\delta_{g+i} + \delta_{g+i}^{-1}} \pi \cdot \overline{\omega}'$$

$$= -\sum \Pi^{g+i} \overline{\Pi^{i\prime}} + \sum \Pi^i \overline{\Pi'^{g+i}} = \sum \left(\Pi^i \overline{\Pi'^{g+i}} - \Pi^{g+i} \overline{\Pi'^i} \right). \lrcorner$$

"Comment" We can integrate $\pi \cdot \overline{\omega}'$ or $\pi \cdot \eta$ just inside $\overline{\Delta}$. $\Rightarrow$ Those evaluations are the same since the domain enclosed by the two curves are holomorphic.

$$\Rightarrow \int_{\partial\Delta} \pi \cdot \overline{\omega}' = \int_C \pi \cdot \overline{\omega}' = \int_S \omega \wedge \overline{\omega}'.$$