that $[\omega] \in H^2_{DR}(\mathbb{P}^N)$ is Poincaré dual to some non-zero multiple of the homology class of a hyperplane $H \subset \mathbb{P}^N$. In fact, $[\omega]$ is Poincaré dual to (H), as the reader may verify by integrating ω over a line $\ell \cong \mathbb{P}^1$ to obtain

$$\int_\ell \omega = 1 = \#(H\cdot\ell).$$

Γ Let $\{w_i = Z_i/Z_0\}$ be coordinates on the open set $U_0 = (Z_0 \neq 0)$ in $\mathbb{P}^n$ and use the lifting $Z = (1, w_1, \cdots w_n)$ on U_0; we have

$$\omega = \frac{i}{2\pi}\partial\bar\partial \log(1+\Sigma w_i\bar w_i) = \frac{i}{2\pi}\left[\frac{\Sigma dw_i\wedge d\bar w_i}{1+\Sigma|w_i|^2} - \frac{\Sigma \bar w_i dw_i \wedge \Sigma w_i d\bar w_i}{(1+\Sigma|w_i|^2)^2}\right],$$

$$\begin{array}{ccc} U_0 & \longrightarrow & \mathbb{C}^n \\ [(Z_0, \cdots Z_n)] & \longmapsto & (w_1, w_2 \cdots w_n) \\ \cup & & \\ \ell & \longleftrightarrow & \{(w, w, \cdots w) \mid w \in \mathbb{C}\} \\ \shortparallel & & \end{array}$$

$[(1, w_1 \cdots w_n)]$ where $w_1 = \cdots = w_n$

$\shortparallel$

$[(1, w \cdots w)] \cong \mathbb{C} \equiv \mathbb{P}^1 - \{\infty\}$

Consider a map $f: \mathbb{C} \longrightarrow \mathbb{C}^n$ defined by $f(w) = (w, \cdots, w)$

We have only to show that $\displaystyle\int_{\mathbb{C}} f^*\omega = \int_\ell \omega = \int_{\ell-\{\infty\}} \omega = 1$.

$$\Rightarrow f^* dw_i = a\, dw$$

$$\Rightarrow f^* dw_i\left(\frac{\partial}{\partial w}\right) = a = dw_i\left(f_*\frac{\partial}{\partial w}\right) = dw_i\left(\frac{\partial}{\partial w_1} + \cdots + \frac{\partial}{\partial w_n}\right) = 1$$

$$\Rightarrow f^* dw_i = dw.$$

$$\Rightarrow f^*\omega = \frac{i}{2\pi}\left[\frac{\Sigma\, dw\wedge d\bar w}{1+n|w|^2} - \frac{\Sigma \bar w dw \wedge \Sigma w d\bar w}{(1+\Sigma|w|^2)^2}\right]$$