Date . .

We may illustrate the correspondence between maps to projective space and base-point-free linear systems with a classical example: the Veronese map associated to the line bundle dH on $\mathbb{P}^n$. We have seen that the global sections of dH correspond to homogeneous polynomials of degree d in $Z = [Z_0, Z_1, \cdots Z_n]$, so that if $\{Z^\alpha = Z_0^{\alpha_0} \cdots Z_n^{\alpha_n}\}$ denotes the set of monomials of degree d in Z, then the Veronese map is given by

$$[Z_0, \cdots Z_n] \longmapsto [, \ldots, Z^\alpha, \cdots].$$

Γ $\alpha_0 + \alpha_1 + \cdots + \alpha_n = d$.

$$[Z_0, \cdots Z_n] \longmapsto [\cdots, Z^\alpha, \cdots] \in \mathbb{P}^{{}_{d+n}C_n}$$ see p 166.

It is easily verified that the Veronese map is a smooth embedding, with the property that every hypersurface of degree d in $\mathbb{P}^n$ becomes a hyperplane section of $i_{dH}(\mathbb{P}^n) \subset \mathbb{P}^N$.

Γ For simplicity, $n = 1$.

$$[Z_0, Z_1] \xmapsto{\phi} [Z_0^2, Z_0 Z_1, Z_1^2].$$

Suppose $\phi([Z_0, Z_1]) = \phi([Z_0', Z_1'])$

$\Rightarrow [Z_0^2, Z_0 Z_1, Z_1^2] = [Z_0'^2, Z_0' Z_1', Z_1'^2]$

$\Rightarrow Z_0^2 = \lambda Z_0'^2 \quad Z_0 Z_1 = \lambda Z_0' Z_1' \quad Z_1^2 = \lambda Z_1'^2$

$\Rightarrow \dfrac{Z_0^2}{Z_0 Z_1} = \dfrac{\lambda Z_0'^2}{\lambda Z_0' Z_1'} \Rightarrow \dfrac{Z_0}{Z_1} = \dfrac{Z_0'}{Z_1'}$ $\Rightarrow \phi$ is one to one. Clearly ϕ is smooth. $\dfrac{\partial \phi}{\partial Z_0} = (2Z_0, Z_1, 0)$