$\Rightarrow \quad (l+m)d+1-\frac{m(m+1)}{2}(n-1)-m-1-ld \geq g.$

$\Downarrow$

$$(l+m)d-\frac{m(m+1)}{2}(n-1)-m-ld \geq g.$$

$$\Rightarrow \quad md-\frac{m(m+1)}{2}(n-1)-m \geq g.$$

$\Downarrow$

$$m(d-m(n-1)-1)+m^2(n-1)-\frac{m(m+1)}{2}(n-1) = m(d-m(n-1)-1) + \frac{m(n-1)}{2}(m-1).$$

Thus the genus of a nondegenerate curve of degree d in $\mathbb{P}^n$ is at most

$$\frac{m(m-1)}{2}(n-1)+m\epsilon, \quad \text{where } m=\left[\frac{d-1}{n-1}\right], \quad d-1=m(n-1)+\epsilon.$$

We will see in this section on ruled surfaces that in fact this bound is realized for each d and n, and an explicit description of these curves of maximal genus. For the time being, let us summarize what we know in general about nondegenerate curves in $\mathbb{P}^n$: if C has degree d, then

$d<n \Rightarrow C$ is degenerate,

$d=n \Rightarrow C$ is the rational curve,

$n<d<2n \Rightarrow g \leq d-n$, with equality if C is normal,

$d=2n \Rightarrow g \leq n+1$ with equality if and only if C is a canonical curve,

$d \geq 2n \Rightarrow g \leq \frac{m(m-1)}{2}(n-1)+m\epsilon$

where $m=\left[\frac{d-1}{n-1}\right]$, $d-1=m(n-1)+\epsilon$.

$\ulcorner$ ① $d<n$, by P173, if C is nondegenerate, then $d \geq n-1+1=n$. $\Rightarrow C$ must be degenerate.