$$= \int_{\mathbb{R}^n} u(y)\, \Delta_x\, \chi_\epsilon(x-y)\, dy + \sum c_k(x) \int_{\mathbb{R}^n} u(y) \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, dy$$

$$+ b(x) \int_{\mathbb{R}^n} u(y)\, \chi_\epsilon(x-y)\, dy - \int_{\mathbb{R}^n} u(y)\, \Delta_y \chi_\epsilon(x-y)\, dy$$

$$- \int_{\mathbb{R}^n} u(y) \sum a_k(y) \frac{\partial}{\partial y} \chi_\epsilon(x-y)\, dy - \int_{\mathbb{R}^n} u(y)\, b(y)\, \chi_\epsilon(x-y)\, dy$$

$\Rightarrow$ If a_k's, b are constants, we have

$$\sum a_k \int_{\mathbb{R}^n} u(y) \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, dy - \int_{\mathbb{R}^n} \sum a_k\, u(y) \frac{\partial}{\partial y} \chi_\epsilon(x-y)\, dy \neq 0,$$

in general.

Anyway, we have

$$\sum a_k(x) \int_{\mathbb{R}^n} u(y) \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, dy - \int_{\mathbb{R}^n} u(y) \sum a_k(y) \frac{\partial}{\partial y_k} \chi_\epsilon(x-y)\, dy + b(x) \int_{\mathbb{R}^n} u(y)\, \chi_\epsilon(x-y)\, dy$$

$$- \int_{\mathbb{R}^n} u(y)\, b(y)\, \chi_\epsilon(x-y)\, dy.$$

$|b(x)| \leq M$. (on $x-y \in$ support of χ_ϵ and $y \in$ supp u.)

$$\Rightarrow \left\| b(x) \int_{\mathbb{R}^n} u(y)\, \chi_\epsilon(x-y)\, dy - \int_{\mathbb{R}^n} u(y)\, b(y)\, \chi_\epsilon(x-y)\, dy \right\|_0$$

$$\leq \quad \|u_\epsilon\|_0 \leq \|u\|_0.$$ Remember that we omit constants.

Thus it remains to bound

$$\sum a_k(x) \int_{\mathbb{R}^n} u(y) \frac{\partial}{\partial x_k} \chi_\epsilon(x-y)\, dy - \sum \int_{\mathbb{R}^n} u(y)\, a_k(y) \frac{\partial}{\partial y_k} \chi_\epsilon(x-y)\, dy.$$