In case $n < m$, we can assume that $\begin{pmatrix} a_{11} & & a_{1n} \\ \vdots & & \vdots \\ a_{k1} & & a_{kn} \end{pmatrix}$ is a linearly

in $\begin{pmatrix} a_{11}(x) & \cdots & a_{1n}(x) \\ \vdots & & \\ a_{k1}(x) & \cdots & a_{kn}(x) \\ \vdots & & \\ a_{m1}(x) & \cdots & a_{mn}(x) \end{pmatrix}$ k

independent set. $\Rightarrow$

$$\begin{tikzcd} f_u ; U \times \mathbb{C}^n \arrow[r] \arrow[dr] & U \times \mathbb{C}^m \\ & U \times \mathbb{C}^{l+l+k-2l} \arrow[u] \end{tikzcd}$$

For $\ker f$, $U \times \mathbb{C}^n \xrightarrow{f_u} U \times \mathbb{C}^{k}$ can be used instead of $f_u ; U \times \mathbb{C}^n \longrightarrow U \times \mathbb{C}^m$.

For $\operatorname{im} f$, construct G on $V \times \mathbb{C}^m$ $\overset{V \times \mathbb{C}^m}{\|}$
$V \times \mathbb{C}^n \times \mathbb{C}^{m-n}$ as follows; $V \times \mathbb{C}^n \times \mathbb{C}^{m-n}$

$$\left(x, \begin{pmatrix} a_{11} & \cdots & a_{1n} & 0 & \\ a_{21} & \cdots & a_{2n} & \vdots & \\ \vdots & & & & \\ a_{n1} & \cdots & a_{nn} & \vdots & \\ a_{m1} & \cdots & a_{mn} & 0 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \\ w_{n+1} \\ \vdots \\ w_m \end{pmatrix} \right) \overset{G}{\longmapsto} \left(x, \begin{pmatrix} a_{11} & \cdots & a_{1k} & 0 & 0 \\ \vdots & & & \vdots & 0 \\ a_{k1} & \cdots & a_{kk} & 0 & \\ 0 & \cdots & 0 & 1 & 0 \\ 0 & & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ \vdots \\ z_k \\ \vdots \\ z_n \\ w_{n+1} \\ \vdots \\ w_m \end{pmatrix} \right)$$

$$\begin{tikzcd} \cup & \uparrow \\ \operatorname{Im}(f_V) \arrow[r, "G"] & G(\operatorname{Im}(f_V)) \end{tikzcd}$$

$$\begin{tikzcd} \operatorname{Im}(f_V) \subset V \times \mathbb{C}^m \arrow[d] \\ \wr\wr \quad V \times \mathbb{C}^k \subset V \times \mathbb{C}^m \end{tikzcd}$$

Two bundles E and F on M are isomorphic if $\exists$ a map f: $E \to F$ with $f_x : E_x \longrightarrow F_x$ an isomorphism for all $x \in M$; a vector bundle on M is called trivial if it is isomorphic to the product bundle $M \times \mathbb{C}^k$. Finally, a section σ of the vector bundle $E \xrightarrow{\pi} M$ over $U \subset M$ is a C^∞-map

$$\sigma ; U \longrightarrow E \quad \text{s.t} \quad \sigma(x) \in E_x \quad \text{for all } x \in U.$$

A frame for E over $U \subset M$ is a collection $\sigma_1, \sigma_2 \cdots \sigma_k$