$= \sum_{I,j} \frac{\partial T_I}{\partial \bar{z}_j} d\bar{z}_j \wedge dz_I = 0 \qquad \frac{\partial T_I}{\partial \bar{z}_j} = 0$ for all I, j.

$\Rightarrow \bar{\partial} T_I = 0$ for all I $\Rightarrow \exists\ f_I \in \mathcal{O}(U)$ s.t

$T_{f_I} = T_I$, by P380. regularity for the $\bar{\partial}$-operator.

$\Rightarrow T = \sum_{\#I=p} T_{f_I}\, dz_I \in \Omega^p(U) = \{ T_f : f \in \mathcal{O}(U) \}$

For the full exterior derivative d, we have a complex

$$0 \longrightarrow \mathbb{C} \longrightarrow C^\infty \xrightarrow{d} \mathcal{A}^1 \xrightarrow{d} \mathcal{A}^2 \longrightarrow \cdots .$$

Given $T \in C^\infty(U)$, $dT = 0 \Rightarrow$

$(dT)(\varphi) = 0$. for all $\varphi \in A_c^{n-1}(U)$.

$\Rightarrow \varphi = \sum_i f_i\, dx_1 \wedge \cdots \wedge \widehat{dx_i} \wedge \cdots \wedge dx_n \Rightarrow \varphi = f\, dx_2 \wedge \cdots \wedge dx_n$

, specially. $\Rightarrow dT(\varphi) = -T(d\varphi) = -T(\frac{\partial f}{\partial x_1} dx_1 \wedge \cdots \wedge dx_n)$

$\Rightarrow$ I can not go further. Let's try a different way. $dT = 0 \Rightarrow \Delta T = 0 \Rightarrow \exists\ \psi$ s.t

$T = T_\psi$. $\Rightarrow dT = d(T_\psi) = T_{d\psi} = 0 \Rightarrow d\psi = 0$

$\Rightarrow \psi$ is constant

To prove the $\bar{\partial}$-Poincaré lemma for higher-degree currents, we shall give another proof for the C^∞ case that can be adapted to currents. This proof will be based on finding a homotopy operator

$$K : A_c^{0,q}(\mathbb{C}^n) \longrightarrow A^{0,q-1}(\mathbb{C}^n).$$

The construction of K is based on the Bochner-Martinelli formula above, and the explicit expression will turn