Let X be a n-dimensional complete intersection, and $Y_1, \dots, Y_{N-n}$ be hypersurfaces in $\mathbb{P}^N$ s.t $X = Y_1 \cap \dots \cap Y_{N-n}$.

$\Rightarrow$

$$Y_1 \cap \cdots \cap Y_{N-n} = X_1 = X \subset Y_2 \cap \cdots \cap Y_{N-n} \subset Y_3 \cap \cdots \cap Y_{N-n} \subset \cdots \subset Y_{N-n} \subset \mathbb{P}^N.$$

Suppose $\mathbb{P}^q$ lies in X, $q > n/2$.

By the Lefschetz theorem,
$$H^l(\mathbb{P}^N) \cong H^l(Y_{N-n}) \text{ if } l \le N-2$$
$$H^l(Y_{N-n}) \cong H^l(X_{N-n-1}) \text{ if } l \le N-3$$
$$\vdots$$
$$H^l(X_2) \cong H^l(X) \text{ if } l \le n-1.$$

$\Rightarrow$ $q > \frac{n}{2}$ $\Rightarrow$ $2q > n$ $\Rightarrow$ $n > 2n-2q$ $\Rightarrow$ $n-1 \ge 2n-2q$.

$\Rightarrow$ $H^{2n-2q}(X) \cong \cdots\cdots \cong H^{2n-2q}(Y_{N-n}) \cong H^{2n-2q}(\mathbb{P}^N)$

$\Rightarrow$ $H_{2q}(X) \overset{i_*}{\cong} H_{2q}(\mathbb{P}^N)$ $\quad i: X \longrightarrow \mathbb{P}^N$ inclusion.

$\Rightarrow$ A generator of $H_{2q}(X)$ is given by $[X \cap \mathbb{P}^{N+q-n}]$.

$\Rightarrow$ $[\mathbb{P}^q] = k\,[X \cap \mathbb{P}^{N-n+q}]$ $\quad$ $k = 1$

$\Rightarrow$ $X \cap \mathbb{P}^{N-n+q} = \mathbb{P}^q$ since $\deg(X \cap \mathbb{P}^{N-n+q})$ must be 1. But by the argument on P174, for generic $\mathbb{P}^{N-n+q}$ in $\mathbb{P}^N$, $X \cap \mathbb{P}^{N-n+q}$ is nondegenerate in $\mathbb{P}^{N-n+q}$. Thus the only possible choice is $q = N-n+q$. $\Rightarrow$ $N = n$ $\Rightarrow$ Impossible.

$\lrcorner$

Now, we deduce from the lemma that for each $p \in \mathbb{P}^3$ the set
$$X_p = X \cap \sigma(p)$$
of lines in the complex X passing through p forms a conic curve in $\sigma(p)$. There are three possible