

There are a few points to be made about the intersection of divisors on an algebraic surface:

1. If L is a positive line bundle, then for any effective divisor D

$$L \cdot D = \int_D C_1(L) > 0.$$

Since L is positive, $C_1(L) = [\frac{\sqrt{-1}}{2\pi} \Theta]$, where $\frac{\sqrt{-1}}{2\pi} \Theta$ is positive definite.

$$\Rightarrow \text{Locally, } \frac{\sqrt{-1}}{2\pi} \Theta = -\frac{\sqrt{-1}}{2\pi} (\sqrt{-1} h_{11} dz_1 \wedge d\bar{z}_1 + \sqrt{-1} h_{12} dz_1 \wedge d\bar{z}_2$$

$$+ \sqrt{-1} h_{21} dz_2 \wedge d\bar{z}_1 + \sqrt{-1} h_{22} dz_2 \wedge d\bar{z}_2) \Rightarrow h_{11} > 0, h_{22} > 0$$

$$h_{12} = \overline{h_{21}}$$

$$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

positive definite, hermitian
see p29 & p148.

$$L \cdot D = \int_D C_1(L) = \int_{D^*} C_1(L) \quad \text{by the def. of integral}$$

by p32.

For a point $p \in D^* \subset M$, \exists a holomorphic function on $f \sim$ s.t. $(f=0) \cap U = U \cap D^*$, U open in M

\Rightarrow Since p is smooth, we may assume that locally, on U , $(z_2=0) = D^*$. see p20.

$$\Rightarrow \int_U C_1(L) = \int_U \frac{\sqrt{-1}}{2\pi} \Theta = \int_U \frac{h_{11}}{2\pi} dz > 0, \quad U \text{ open in } \mathbb{C}.$$