$H^p(\underline{\mathcal{U}}, \mathcal{F})$'s as $\underline{\mathcal{U}}$ becomes finer and finer:

$$H^p(M. \mathcal{F}) = \varinjlim_{\underline{\mathcal{U}}} H^p(\underline{\mathcal{U}}, \mathcal{F})$$

Clearly, for any covering $\underline{\mathcal{U}}$.

$$H^0(M. \mathcal{F}) = H^0(\underline{\mathcal{U}}, \mathcal{F}) = \mathcal{F}(M)$$

by the sheaf properties (i) & (ii).

Do we need this ?

Note that if $M \subset N$ closed subspace, $\mathcal{F}$ any sheaf on $M$, then extending $\mathcal{F}$ by zero to a sheaf on $N$,

we have $H^*(M. \mathcal{F}) = H^*(N, \mathcal{F})$

$$H^p(M. \mathcal{F}) = \varinjlim_{\underline{\mathcal{U}}} H^p(\underline{\mathcal{U}}, \mathcal{F}).$$

$$H^p(N. \mathcal{F}) = \varinjlim_{\underline{\mathcal{U}}} H^p(\underline{\mathcal{U}}, \mathcal{F}).$$

Given an open covering $\{V_\alpha\} = \underline{V}$ of $M$, for each $\alpha$,

$\exists$ $U_\alpha \subset N$ s.t $U_\alpha \cap M = V_\alpha$.

$\Rightarrow$ $\underbrace{\{U_\alpha\} \cup \{N - M\}}_{=\underline{\mathcal{U}}}$ is an open covering of $N$

$\Rightarrow$ $C^p(\underline{\mathcal{U}}, \mathcal{F}) = C^p(\underline{V}, \mathcal{F})$

X

$\Rightarrow$ Given an open covering $\{V_\alpha\}$ of $M$, we can have an open covering $\underline{\mathcal{U}}$ of $N$.

If $\underline{\mathcal{U}}' < \underline{\mathcal{U}}$, $\exists$ a map from $C^p(\underline{\mathcal{U}}, \mathcal{F})$ to $C^p(\underline{\mathcal{U}}', \mathcal{F})$.

$\underline{\mathcal{U}} \cap M = \underline{V}$, $\underline{\mathcal{U}}' \cap M = \underline{V}'$.

$\Rightarrow$ $C^p(\underline{\mathcal{U}}, \mathcal{F}) = C^p(\underline{V}, \mathcal{F})$ & $C^p(\underline{\mathcal{U}}', \mathcal{F}) = C^p(\underline{V}', \mathcal{F})$