② $h \in S^* - R^*$

$\Rightarrow \sigma(h) \cap X = L \cup L' = \sigma(p_L, h) \cup \sigma(p_{L'}, h)$. $p_L \neq p_{L'}$

$\Rightarrow$ By P768, $x = \overline{p_L, p_{L'}}$ lies in two confocal pencils.

$\Rightarrow \sigma(q) \cap X = \sigma(q, h_1) \cup \sigma(q, h_2)$, and $x \in \sigma(q, h_1) \cap \sigma(q, h_2)$, i.e., $x = h_1 \cap h_2$ is tangent to S at q.

$\Rightarrow T_x(X) \cap X = \sigma(q, h_1) \cup \sigma(q, h_2) \cup \sigma(p_L, h) \cup \sigma(p_{L'}, h)$.

(i) $\sigma(q, h_1) = \sigma(p_L, h)$

$\Rightarrow q = p_L \Rightarrow L = \sigma(p_L, h)$ is special $\Rightarrow$ By the argument on P792, h is tangent to S at $p_L = q$. //

(ii) $\sigma(q, h_1) = \sigma(q, h_2)$

$\Rightarrow q \in R \Rightarrow q$ is a double point of S

$\Rightarrow q \in \overline{p_L, p_{L'}} \subset h \Rightarrow q \in h \cap S \Rightarrow h$ is tangent to S at q ($\because$ q is a double point).

(iii) $\sigma(q, h_2) = \sigma(p_{L'}, h)$

$\Rightarrow q = p_{L'}$, $h_2 = h$.

$\Rightarrow L'$ is special $\Rightarrow$ By P792 & P793, h is tangent to S at $p_{L'} = q$. ~~* We can do the same thing to (ii)~~

~~(iv) $\sigma(q, h_1) = \sigma(q, h_2)$~~

~~$\Rightarrow \sigma(q, h')$ is special, $h_1 = h_2 = h'$~~

~~$\Rightarrow$ By P793, 3, h' is tangent to S at q.~~

~~By (ii), since~~

(iv) L, L' not special

Let q' be a singular point of $h \cap S$, s.t. $h = T_{q'}(S)$. (~~Claim q' is a smooth~~ point.)