$$\begin{aligned} M/mM &\xrightarrow{\phi} M \otimes_{\mathcal{O}} \mathbb{C} \\ x+mM &\longmapsto x \otimes_{\mathcal{O}} 1 \end{aligned}$$

$x + m'y \longmapsto x+m'y \otimes_{\mathcal{O}} 1 = x \otimes_{\mathcal{O}} 1 + m'y \otimes_{\mathcal{O}} 1 = x \otimes_{\mathcal{O}} 1 + y \otimes_{\mathcal{O}} m'1$

$\Rightarrow$ Since $m\mathbb{C} = 0$, $x+m'y \otimes_{\mathcal{O}} 1 = x \otimes 1$. $\Rightarrow$ ϕ is well-defined. $\phi \circ \psi(x \otimes \alpha) = \phi(\alpha x + mM) = x \otimes_{\mathcal{O}} \alpha = \alpha x \otimes_{\mathcal{O}} 1$ "$\alpha = \alpha + m$". $\psi \circ \phi(x + mM) = \psi(x \otimes_{\mathcal{O}} 1 + m) = x + mM$
$\Rightarrow$ ψ is isomorphic. $\Rightarrow$ $M \otimes_{\mathcal{O}} \mathbb{C} \cong M/mM$. ┘

By the Nakayama lemma, we may extend this isomorphism to a surjective map $E \to M \to 0$.

Γ $E \xrightarrow{\phi} M \to 0$. $E = \mathcal{O}^{(k)}$, k is the minimal number of generators of M.
$\Rightarrow$ $\exists$ the induced map $\phi : E/mE \longrightarrow M/mM \to 0$

$\phi(e + mE) = 0 \Rightarrow \phi(e) \in mM$, $e = x_1 f_1 + x_2 f_2$
$\Rightarrow$ $x_1 f_1 + x_2 f_2 \in mM \Rightarrow x_1 f_1 + x_2 f_2 = m_1 f_1 + m_2 f_2$
$\Rightarrow$ $(x_1 - m_1) f_1 + (x_2 - m_2) f_2 = 0$. If $x_1 \neq m_1$, and $x_1 \notin m$, then $x_1 - m_1$ is a unit. $\Rightarrow$ $f_1 = g f_2$. $\Rightarrow$ Contradiction to the minimality. Here we assumed $M = \{f_1, f_2\}$.
$\Rightarrow$ ϕ is isomorphic. $\Rightarrow$ $E_0 \cong M_0$.
The authors considered E s.t. $E_0 \cong M_0$ and used Nakayama lemma to get a surjective map from $E_0 \cong M_0$. $\Rightarrow$ This is stupid, because they have to explain how to get E, where they find a surjective map. Stupidly written. ┘