We had better explain this by giving an example. Suppose the last 3×3 minor as follows.

$$\begin{pmatrix} 0 & * = a_1 & * = a_2 \\ 1 & * = b_1 & * = b_2 \\ 0 & * = c_1 & * = c_2 \end{pmatrix} \text{ has rank } 2.$$

$\Rightarrow$ The rank depends on the top & bottom vector

$\Rightarrow$ We have only to consider $\begin{pmatrix} a_1, & a_2 \\ c_1, & c_2 \end{pmatrix}$.

Suppose $\operatorname{rank}\begin{pmatrix} a_1 & a_2 \\ c_1, & c_2 \end{pmatrix} = 1$. $\Rightarrow$ $\begin{pmatrix} 0 & a_1, & a_2 \\ 1 & b_1 & b_2 \\ 0 & c_1, & c_2 \end{pmatrix}$ has rank 2. Let $A = (a_1, a_2)$. $B = (c_1, c_2)$.

$\Rightarrow$ Let $v \perp B$ s.t $\|v\| < \delta$. ——— ①

$\Rightarrow$ $\begin{pmatrix} A+v \\ B \end{pmatrix}$ has rank 2.

In general, given a set of row vectors $A_1, A_2, \cdots A_n$ s.t $\dim \langle A_1 \cdots A_n \rangle = k < n$, then we can find $v_1, v_2 \ldots v_m \in \mathbb{C}^n$ s.t $\|v_i\| < \delta$. $\dim \langle A_1 + v_1, \cdots A_m + v_m, \cdots v_n \rangle = l$, $k \leq l \leq n$.

Suppose $A_1 \in \langle A_2, \cdots A_n \rangle$.

$\Rightarrow$ Choose v_1 s.t $v_1 \perp \langle A_2 \cdots A_n \rangle$ $\|v_1\| < \delta$

$\Rightarrow$ $A_1 + v_1 \notin \langle A_2, \cdots A_n \rangle$

$\Rightarrow$ $\operatorname{rank}\{A_1 + v_1, A_2, \cdots A_n\} = k+1$.

Continue this process, then we get the desired.