Date . .

Now $[\Lambda, \Theta_E]$ is bounded on $A^{0,*}(L^{-\mu}\otimes E)$, so we can write

$$|\langle[\Lambda, \Theta_E]\eta, \eta\rangle| \leq C\|\eta\|^2, \quad \text{and}$$

consequently for $p<n$,

$$\mu > \frac{C}{2\pi} \Rightarrow \eta = 0$$

i.e $\mathcal{H}^{0,p}(L^{-\mu}\otimes E) = 0$ for $\mu > \frac{C}{2\pi}$, $p<n$. Q.E.D.

$\sqcap\!\sqcap$ $[\Lambda, 1\otimes\Theta_E]$ is an operator on $A^{0,p}(L^{-\mu}\otimes E)$

For,

first of all $L : A^{p,q}(E) \longrightarrow A^{p+1,q+1}(E)$ defined by

$L(\eta\otimes s) = \omega\wedge\eta\otimes s$. is bounded.

Since all forms of type $\eta\otimes s$ are dense in $A^{p,q}(E)$, we have only to show L is bounded on the forms of type $\eta\otimes s$.

$$\|L(\eta\otimes s)\|^2 = \langle L(\eta\otimes s), L(\eta\otimes s)\rangle$$

$$= \langle \omega\wedge\eta\otimes s, \omega\wedge\eta\otimes s\rangle$$

$$= \int \langle \omega\wedge\eta\otimes s, \omega\wedge\eta\otimes s\rangle_x \, dx$$

$$= \int \langle \omega\wedge\eta, \omega\wedge\eta\rangle_x \langle s, s\rangle_x \, dx$$