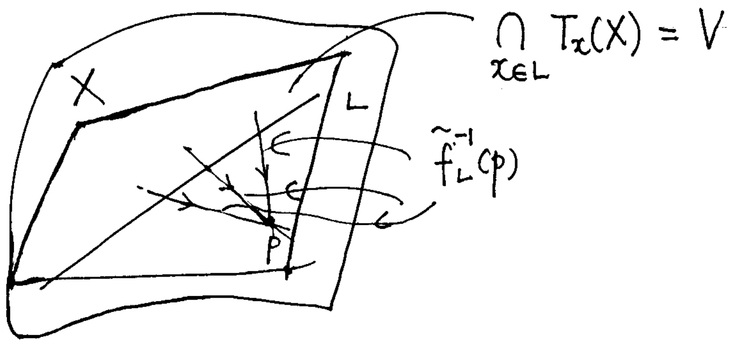
Thus $\tilde{f}_L : \tilde{X}_L \longrightarrow \mathbb{P}^3$ is again the blow-up of the curve E_L.

⌜ Since $\tilde{f}_L^{-1}(E_L) = \tilde{f}_L^{-1}(p) \cup (\bigcup_{L' \in B_L} L' - L)$, $\tilde{f}_L$ is one-to-one away from $\tilde{f}_L^{-1}(E_L)$.

⌟

(Recalling that a special line $L \subset X$ is to be counted among the lines meeting L, we may think of the line $\tilde{f}_L^{-1}(p)$ of points corresponding to normal vectors to $L \subset X$ in $\bigcap_{x \in L} T_x(X)$ as the "proper transform of L" itself in the blow-up $\tilde{X}_L$ of X).

⌜ $\tilde{f}_L^{-1}(p) = \{ [*, *, p] \mid$ All normal vectors to L passing p, and we may say that all lines in $\bigcap_{x \in L} T_x(X)$ passing $p \}$ is the line spanned by two points $[1, 0, p]$ and $[0, 1, p]$, since $\{ [(\alpha, 0, \alpha p) + (0, \beta, \beta p) \mid \alpha + \beta \neq 0 \} = \tilde{f}_L^{-1}(p)$.



⌟