

Moreover, to any function  $g \in \mathcal{O}_V$  there corresponds a unique polynomial  $Q_{f,g}(X) \in {}_n\mathcal{O}_W[X] \subseteq {}_V\mathcal{O}_V[X]$  of degree  $\nu-1$  s.t.  $d_f \cdot g = Q_{f,g}(f)$  in  ${}_V\mathcal{O}_V$ .

Def: A holomorphic function  $f \in \mathcal{O}_V$  is said to separate the sheets of a finite branched holomorphic covering  $\pi: V \rightarrow W$  with a regular part  $\pi|_{V_0}: V_0 \rightarrow W_0$  provided that for each connected component of  $W_0$  there is a point  $B$  in that component s.t. if  $\pi^{-1}(B) = \{A_1, \dots, A_\nu\}$  where  $A_j$  are distinct points, then  $f(A_i) \neq f(A_k)$  whenever  $i \neq k$ .

Proof. Suppose  $P(X) \in \pi^*({}_n\mathcal{O}_W)[X]$  s.t.  $P(f) = 0$ ,  $\deg P = \nu$   
 $\Rightarrow P(f(A_j(z))) = 0 \Rightarrow P(X) = 0$  has  $\nu$  distinct roots.  
 $\Rightarrow P(X) = P_f(X)$  on an open set  $\subset W \Rightarrow$  Since  $P$  &  $P_f$  have holomorphic coefficients,  $P = P_f$  on  $W$ .

With the same notation as in the proof of Theorem 5, the discriminant of the polynomial  $P_f(X)$  is the function

$$d_f(z) = \left( \prod_{j > k} [f(A_j(z)) - f(A_k(z))] \right)^2$$

and is a well-defined complex-valued function on  $W$ . Since  $d_f(z)$  is evidently a symmetric polynomial in the values  $f(A_j(z))$ , it is necessarily a polynomial in the elementary symmetric functions of the values  $f(A_j(z))$ , the coefficients of the polynomial  $P_f(X)$ , and hence is actually a holomorphic function on  $W$ ; and if  $B$  is a point of  $W$  for which  $A_j(B)$  are distinct points and  $f(A_j(B)) \neq f(A_k(B))$ ,  $j \neq k$ ,