by the result on P456, φ is of the second kind.

$$\Rightarrow \quad \mathrm{Ker}\, R = \frac{\{\text{1-forms of the second kind}\}}{\{\text{exact forms}\}}.$$

$$\Rightarrow \quad \rho_1 = \dim H^1(M, \mathbb{C}) = b_1$$

Next, we have

$$\frac{H^2(M, \mathbb{C})}{\left\{\begin{array}{l}\text{Chern classes} \\ \text{of holomorphic} \\ \text{line bundles}\end{array}\right\}} = \frac{H^2(M, \mathbb{C})}{i\, H^0(\oplus\, \mathbb{C}_D)}$$

$$\cong \text{image } \{H^2(M, \mathbb{C}) \longrightarrow \mathbb{H}^2(\Omega(*))\}$$

$$= \frac{\{\text{2-forms of the second kind}\}}{\{\text{exact forms}\}},$$

and so $\rho_2 = b_2 - \rho$.

Q.E.D.

Γ $\left\{\begin{array}{l}\text{Chern classes} \\ \text{of holomorphic} \\ \text{line bundles}\end{array}\right\}$ means the set of all linear combination of Chern classes of holomorphic line bundles.

From $H^0(\oplus\, \mathbb{C}_D) \xrightarrow{i} H^2(M, \mathbb{C}) \xrightarrow{f} \mathbb{H}^2(\Omega(*))$,

$\mathrm{Ker}\, f = \mathrm{Im}\, i$

$\Rightarrow \quad \dfrac{H^2(M, \mathbb{C})}{\mathrm{Im}\, i} \xrightarrow{f} \mathrm{Im}\, f$ is isomorphic.