This sequence will prove useful in a little while.

Cohomology of Coherent Sheaves. Suppose now that M is a compact complex manifold and $\mathcal{F}$ is a coherent sheaf on M. The fundamental global fact is: The cohomology $H^*(M, \mathcal{F})$ is a finite-dimensional vector space.

Again, this is proved in the references listed at the end of this chapter. We shall not prove it here but will show how the finite dimensionality may be used to draw consequences in case M is a smooth algebraic variety.

Suppose then that $L \to M$ is a positive line bundle, which we may as well take to be the hyperplane bundle relative to a smooth projective embedding $M \subset \mathbb{P}^N$. We let $\mathcal{L} = \mathcal{O}(L)$ and set $\mathcal{F}(k) = \mathcal{F} \otimes_{\mathcal{O}} \mathcal{L}^k$. The sections of $\mathcal{F}(k)$ may be thought of as sections of $\mathcal{F}$ having poles of order k along a hyperplane.

See P 139-P1.

The sections of $\mathcal{F}(k)$ may be thought of as sections of $\mathcal{F}$ having poles of order $\le k$ along a hyperplane.

Consider the following two assertions:

Theorem A. The global sections $H^0(M, \mathcal{F}(k))$ generate each