

the tangent line of E_D at p is in Q . \Rightarrow Any line through p in Q other than the tangent line is not tangent to E_D at p . \Rightarrow For $L_{q_1} \neq L_{q_2}$ and L_{q_i} not tangent to E_D at p , $\overline{L_{q_1}, L_{q_2}} \cap E_D = (E_D \cap L_{q_1}) \cup (E_D \cap L_{q_2}) \Rightarrow \#(\overline{L_{q_1}, L_{q_2}} \cdot E_D) = \#(E_D \cdot L_{q_1}) + \#(E_D \cdot L_{q_2}) - 1 = 1 + \#(E_D \cdot (L_{q_1} - p)) + \#(E_D \cdot (L_{q_2} - p)) = 5$ since E_D is not tangent to $\overline{L_{q_1}, L_{q_2}}$, otherwise $\overline{L_{q_1}, L_{q_2}, L_{q'}} = \overline{L_{q_1}, L_{q_2}}$, where $L_{q'}$ is the tangent line $\Rightarrow \#(\overline{L_{q_1}, L_{q_2}} \cap C) \geq 3 \Rightarrow C$ is singular since C contains $\overline{L_{q_1}, L_{q_2}} \cap \mathbb{P}^2$, where $C \subset \mathbb{P}^2$. \Rightarrow Every line L_q , $q \neq q'$, $C \subset Q$ meets E_D in two points other than p , since $\#(E_D \cdot (L_{q_i} - p)) = 2$, refer to the previous argument. \equiv

But now the divisors

$$D_q = L_q \cdot E_D - p$$

form a linear system of degree 2, and so D_q must be the standard hyperelliptic divisor D_0 on B .

$$\text{① } \deg(D_q) = \deg(L_q \cdot E_D - p) = 2$$

② Since $\{H \mid H \text{ hyperplanes containing } L_{q_0} = \overline{q_0, p}, q_0 \in C\}$ is a linear system on \mathbb{P}^3 , for any $q_1, q_2 \in C$, $\overline{L_{q_0}, L_{q_1}} + \overline{L_{q_0}, L_{q_2}} = \overline{L_{q_0}, q_3}$ for some $q_3 \in C$.

$$\begin{aligned} \Rightarrow \overline{L_{q_0}, q_3} \cdot E_D &= L_{q_0} \cdot E_D + L_{q_3} \cdot E_D - p \\ &= (\overline{L_{q_0}, L_{q_1}} + \overline{L_{q_0}, L_{q_2}}) \cdot E_D = \overline{L_{q_0}, L_{q_1}} \cdot E_D + \overline{L_{q_0}, L_{q_2}} \cdot E_D \\ &= L_{q_0} \cdot E_D = L_{q_0} \cdot E_D + L_{q_1} \cdot E_D - p + L_{q_0} \cdot E_D + L_{q_2} \cdot E_D - p \end{aligned}$$