$$\mathrm{Ext}^{n-p}(M; \mathcal{O}(E), \Omega^n) \cong H^{n-p}(M, \mathcal{O}(E)^* \otimes \Omega^n) = H^{n-p}(M, \Omega^n(E^*))$$

See P153.

Finally, there is an even more general duality theorem dealing with a map – cf. the reference to Hartshorne's notes at the end of this chapter.

Too abstract ! I don't get it.
Let's see how useful they are.

Global Ext and Vector Fields with Isolated Zeros

We shall prove a recent theorem, due to Carrell and Liebermann, which will illustrate several of the techniques developed above, and which also will tie in with several previous results in the book.

Let M be a compact Kähler manifold and v a holomorphic vector field having a set Z of isolated zeros.

Theorem. If Z is nonempty, then

$$H^{p,q}(M) = 0 \quad \text{for } p \neq q.$$

Actually, Carrell and Liebermann proved the more general statement

$$H^{p,q}(M) = 0 \quad \text{for } |p-q| > \dim_{\mathbb{C}} Z,$$

where Z is the zero set of any holomorphic vector field