

$$\mathbb{H}_\beta^\alpha = \sum_\mu A_\mu^\alpha \wedge \bar{A}_\mu^\beta.$$

where $A_\mu^\alpha = \sum A_{\mu j}^\alpha dz_j$ is a matrix of (1,0) forms.

According to P19, we have a surjective holomorphic bundle map

$$\ker f \longrightarrow M \times \mathbb{C}^n \xrightarrow{f} E \longrightarrow 0.$$

$$\Rightarrow E = \frac{M \times \mathbb{C}^n}{\ker f} \Rightarrow \text{By the computation on P18,}$$

$$\mathbb{H}_E = \mathbb{H}_{M \times \mathbb{C}^n}|_E + A \wedge {}^t \bar{A}.$$

$$\text{Since } \mathbb{H}_{M \times \mathbb{C}^n} = 0, \quad \mathbb{H}_E = A \wedge {}^t \bar{A}.$$

$$\text{On P19, } A = \sum a_{\lambda j}^\alpha dz_\alpha \otimes e_\lambda \otimes e_j^*.$$

$$\text{Let } A_{j\alpha}^\lambda = a_{\lambda j}^\alpha \Rightarrow A = \sum A_{j\alpha}^\lambda dz_\alpha \otimes e_\lambda \otimes e_j^*$$

$$\begin{aligned} \Rightarrow {}^t \bar{A} &= \sum \bar{a}_{\lambda j}^\alpha d\bar{z}_\alpha \otimes e_\lambda^* \otimes e_j = \sum \bar{A}_{j\alpha}^\lambda d\bar{z}_\alpha \otimes e_\lambda^* \otimes e_j \\ &= \sum \bar{A}_{i\ell}^\beta d\bar{z}_\ell \otimes e_\beta^* \otimes e_i \end{aligned}$$

$$A \wedge {}^t \bar{A} (e_k) = A \wedge (\sum \bar{A}_{i\ell}^k d\bar{z}_\ell \otimes e_i)$$

$$\begin{aligned} &= \sum A_{i\alpha}^\lambda dz_\alpha \otimes e_\lambda \wedge \bar{A}_{i\ell}^k d\bar{z}_\ell = \sum A_{i\alpha}^\lambda dz_\alpha \wedge \bar{A}_{i\ell}^k d\bar{z}_\ell \\ &\otimes e_\lambda \Rightarrow A \wedge {}^t \bar{A} = \sum A_{i\alpha}^\lambda dz_\alpha \wedge \bar{A}_{i\ell}^k d\bar{z}_\ell \otimes e_\lambda = \mathbb{H}^\lambda_k \end{aligned}$$