Date . .

The rational functions of V a priori form a subfield of the field $\mathcal{M}(V)$ of meromorphic functions; in fact,

Every meromorphic function on an algebraic variety $V \subset \mathbb{P}^n$ is rational.

The proof of this assertion is in two stages: first, we express V as a branched cover of a linear subspace $\mathbb{P}^k \subset \mathbb{P}^n$ by projection, and deduce from this representation that the pullback $\pi^* K(\mathbb{P}^k)$ to V of the field of rational functions on $\mathbb{P}^k$ has index at most $d = \deg(V)$ in the field $\mathcal{M}(V)$; we then show that the field $K(V)$ is an extension of degree at least d over $\pi^* K(\mathbb{P}^k)$.

If a field F is an extension field of E and E an extension field of K, then $[F:K] = [F:E][E:K]$, where $[F:K]$ is the dimension of K-vector space F.
See, for details, p 231 Hungerford.

$[\mathcal{M}(V);\ \pi^* K(\mathbb{P}^k)] \leq d$
$[K(V);\ \pi^* K(\mathbb{P}^k)] \geq d.$

$\Rightarrow$ Since $[\mathcal{M}(V);\ \pi^* K(\mathbb{P}^k)] = [\mathcal{M}(V);\ K(V)]\,[K(V);\ \pi^* K(\mathbb{P}^k)] \leq d$, $[\mathcal{M}(V);\ K(V)] = 1$ and $[\mathcal{M}(V);\ \pi^* K(\mathbb{P}^k)] = d.$

$\Rightarrow \quad \mathcal{M}(V) = K(V).$

$\pi : \mathbb{P}^n \longrightarrow \mathbb{P}^k$, $\frac{F}{G}$ on $\mathbb{P}^k$ is rational