$|H|$ is a linear system.

$\Rightarrow$ By Bertini's theorem, $\exists$ a generic hyperplane H s.t. $H \cap M$ is smooth,

since $\bigcap_{H \subset \mathbb{P}^N} H = \phi$ i.e. base locus is empty. //

By P139.

$$0 \to \mathcal{O}_M(E \otimes [-D]) \xrightarrow{\otimes s_0} \mathcal{O}_M(E) \xrightarrow{r} \mathcal{O}_D(E|_D) \to 0.$$

$(S_0) = D.$

Let $E = L \otimes [H]^{\otimes \mu}$, $M \cap H = D$. H

$$\qquad = L \otimes [H|_M]^{\otimes \mu}.$$

$\Rightarrow$ We get

$$0 \to \mathcal{O}_M(L \otimes [H]^{\otimes \mu-1}) \xrightarrow{\otimes s} \mathcal{O}_M(L \otimes [H]^{\otimes \mu}) \xrightarrow{r} \mathcal{O}_V(L \otimes [H]^{\otimes \mu}) \to 0$$

For $\mu \gg 0$, we have both

$$H^0(V, \mathcal{O}(L + \mu H)) \neq 0.$$

by induction and

$$H^0(M, \mathcal{O}(L+\mu H)) \to H^0(V, \mathcal{O}(L+\mu H)) \to 0,$$

since $H^1(M, \mathcal{O}(L + (\mu-1)H)) = 0$. by Th.B.