Actually, it is obvious that a Hilbert space whose unit ball is compact is finite dimensional, and we may make $H^0(M, \Omega^q)$ into a Hilbert space by defining

$$\langle \varphi, \psi \rangle = \sum_{i,j} \int_{V_i} \varphi_{i,j}(z)\, \overline{\psi_{i,j}(z)}\; \Phi(z_i).$$

where $\Phi(z_i)$ is the Euclidean volume form in the coordinates z_i. Since (1) a sequence $\psi_\nu \in \mathcal{O}(U_i)$ that is Cauchy in $L^2(V_i)$ has a subsequence converging uniformly on compact subsets of V_i to $\psi \in \mathcal{O}(V_i)$, and (2) a sequence $\psi_\nu \in \mathcal{O}(U_i)$ that is bounded in $L^2(V_i)$ has a similarly convergent subsequence, we may adopt the previous argument to this Hilbert-space setting.

「 See Gunning Lecturs on Riemann Surfaces. On p59 ~ p61, he showed that sup norm is bounded by $\| \|_2$ norm (L^2-norm). 」

「 From this result, we can show that $\exists$ a uniformly convergent subsequence on a compact subset of V_i and by Montel theorem, a sequence which is bounded has a convergent subsequence uniformly. 」

This argument may be modified to prove the finite dimensionality of all $H^q(M, \Omega^p)$, and indeed the finite dimensionality of $H^q(M, \mathcal{F})$ for any coherent analytic sheaf $\mathcal{F}$ — these matters will be discussed further in Section 3 of Chapter 5, where it will emerge that the finite dimensionality is the central fact