$\Rightarrow \quad *_E : \; H^q(M, \Omega^p(E)) \longrightarrow H^{n-q}(M, \Omega^{n-p}(E^*))$ is isomorphic.

If $p=0$, $\quad *_E : H^q(M, \mathcal{O}(E)) \longrightarrow H^{n-q}(M, \Omega^n(E^*))$ is isomorphic, and $\Omega^n(E^*) = \mathcal{O}(\Lambda^n T^*M' \otimes E^*) = \mathcal{O}(K_M \otimes E^*)$.

As in the page 102. (see Kodaira-Serre Duality Theorem 2)

$$H^q(M, \Omega^p(E)) \otimes H^{n-q}(M, \Omega^{n-p}(E^*)) \xrightarrow{\ \wedge \ \leftarrow \text{see } p151.} H^n(M, \Omega^n) \cong \mathbb{C}$$

is non-degerate.

$$\Rightarrow \quad H^q(M, \Omega^p(E)) \cong (H^{n-q}(M, \Omega^{n-p}(E^*)))^*$$

$\Gamma \quad \varphi + \bar\partial A^{p,q-1}(E)$

$$\|\varphi + \bar\partial \eta\|^2 = \langle \varphi + \bar\partial\eta, \ \varphi + \bar\partial\eta \rangle = \|\varphi\|^2 + \|\partial\eta\|^2 \geq \|\varphi\|^2$$

in case $\bar\partial^* \varphi = 0$.

$$\|\varphi + t\bar\partial\eta\|^2 = f(t) = \|\varphi\|^2 + t^2\|\partial\eta\|^2 + t\langle \varphi, \bar\partial\eta\rangle + t\langle \bar\partial\eta, \varphi\rangle$$

$$\left.\frac{d f(t)}{d t}\right|_{t=0} = 0 = \langle \varphi, \bar\partial\eta\rangle + \langle \bar\partial\eta, \varphi\rangle = 0$$

$$= \langle \bar\partial^*\varphi, \eta\rangle + \langle \eta, \bar\partial^*\varphi\rangle = 0$$

$$= \langle \bar\partial^*\varphi, \eta\rangle + \overline{\langle \bar\partial^*\varphi, \eta\rangle}$$

$$= 2\,\mathrm{Re}\langle \bar\partial^*\varphi, \eta\rangle = 0$$

Plug. $i\bar\partial\eta$. in

$$\Rightarrow \quad -i\langle \varphi, \bar\partial\eta\rangle + i\langle \bar\partial\eta, \varphi\rangle = 0$$

$$\Rightarrow \quad \langle \varphi, \bar\partial\eta\rangle = \langle \bar\partial\eta, \varphi\rangle$$

$\Rightarrow \quad \langle \varphi, \bar\partial\eta\rangle = 0$ for all $\eta \quad \Rightarrow \bar\partial^*\varphi = 0$.