Continuing in this way, we obtain a basis $(\lambda_1 \cdots \lambda_{2n})$ for Λ having the desired properties.

Note that the integers $\{\delta_i\}$ obtained satisfy $\delta_1 | \delta_2$, $\delta_2 | \delta_3$, and so on: if, for example, $\delta_1 \nmid \delta_2$, then for some k we would have

$$0 < Q(k\lambda_1 + \lambda_2, \lambda_{n+1} + \lambda_{n+2}) < \delta_1 .$$

「 If δ_1 does not divide δ_2, since $0 < \delta_1 \leq \delta_2$.
$\exists$ k' s.t $0 < \delta_2 - k'\delta_1 < \delta_1$.
$\Rightarrow$ $Q(\lambda_2, \lambda_{n+2}) - k' Q(\lambda_1, \lambda_{n+1}) = Q(-k'\lambda_1 + \lambda_2, \lambda_{n+1} + \lambda_{n+2})$, since $Q(\lambda_2, \lambda_{n+1}) = Q(\lambda_1, \lambda_{n+2}) = 0$.
$\Rightarrow$ Put $k = -k'$.
」

We observe that with the additional condition $\delta_i | \delta_{i+1}$ the integers δ_i are invariants of the quadratic form Q. Q.E.D.

「 First of all, Q is given by

$$\left(\begin{array}{c|c} 0 & \begin{matrix} \delta_1 & & 0 \\ & \ddots & \\ 0 & & \delta_n \end{matrix} \\ \hline \begin{matrix} -\delta_1 & & 0 \\ & \ddots & \\ 0 & & -\delta_n \end{matrix} & 0 \end{array}\right).$$

$$\Rightarrow |Q - \lambda I| = \left|\begin{array}{c|c|c|c} \begin{matrix} -\lambda & \delta_1 \\ -\delta_1 & -\lambda \end{matrix} & 0 & & 0 \\ \hline 0 & \ddots & \vdots & 0 \\ \hline & & & \begin{matrix} -\lambda & \delta_n \\ -\delta_n & -\lambda \end{matrix} \end{array}\right| = (\lambda^2 + \delta_1^2) \cdots (\lambda^2 + \delta_n^2)$$