these proper transforms onto the corresponding point of E_L, i.e., that

$$\tilde{f}_L : \tilde{X}_L \longrightarrow \mathbb{P}^3$$

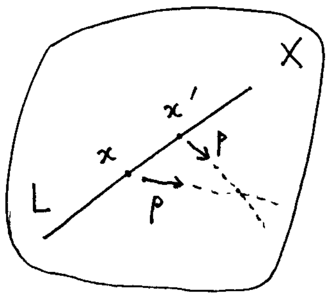
is the blow-up of the quintic curve $E_L \subset \mathbb{P}^3$.

⌈ By P796, since $f_L : X - L \longrightarrow \mathbb{P}^3$ is one-to-one away from the locus of lines in X meeting L, and if $\eta_1 \neq \eta_2 \in T_x(X)$, then $\overline{L, \eta_1} = \{[\ast\ \ast, \alpha\eta_1]\}$ and $\overline{L, \eta_2} = \{[\ast, \ast, \alpha\eta_2]\} \Rightarrow \eta_1 = \overline{L, \eta_1} \cap V_3 \neq \overline{L, \eta_2} = \eta_2$ where $V_3 = \{[0, 0, \ast, \ast \cdots]\}$, $\tilde{f}_L((x, \eta_1)) \neq \tilde{f}_L((x, \eta_2))$.

Thus, $\tilde{f}_L : \tilde{X}_L - \{\tilde{L}' \mid \tilde{L}'$ proper transform of L', s.t $L' \cap L \neq \phi\} \longrightarrow \mathbb{P}^3$ is one to one, and $\tilde{f}_L(\tilde{L}')$ is the one-point set $\overline{L, L'} \cap \mathbb{P}^3 \in E_L$.

$\Rightarrow \tilde{f}_L : \tilde{X}_L - \tilde{f}_L^{-1}(E_L) \longrightarrow \mathbb{P}^3 - E_L$ is one to one, and for each $p \in E_L$, $\tilde{f}_L^{-1}(p)$ is the set of normal vectors of L at x, where $\overline{x, p, L} \cap V_3 = p$.

Here x is uniquely determined for, if there is another x' s.t $\overline{x', p, L} \cap V_3 = p$.



$\Rightarrow \quad x + tp = x' + t'p$

$\Rightarrow \quad x - x' + (t - t')p = 0$

$\Rightarrow$ Since $x - x'$ and p are linearly independent, $x = x'$ and $t = t'$.

Note : $\tilde{f}_L^{-1}(p) \cong \mathbb{P}^1$.

⌟