$\Rightarrow$ Since, when we compute $\int_\gamma \varphi$, we only need local representations, $\int_\gamma \varphi = 0$ as above.

$\Rightarrow$ By the equivalence on P455, φ is of the second kind.

"Comment"

I guess: $\Omega^p(U, alg) \xrightarrow{i} \Omega^p(U)$ induces an isomorphism on the cohomology groups. See P450

To give the interpretations of ρ_1 and ρ_2, we define the Picard number ρ to be the rank of the image

$$H^1(M, \mathcal{O}^*) \xrightarrow{c_1} H^{1+1}(M, \mathbb{Z}).$$

☞ ρ_1 = dimension of $\dfrac{(\text{1- forms of the second kind})}{d(\text{meromorphic 0- forms})}$

ρ_2 = dimension of $\dfrac{(\text{2-forms of the second kind})}{d(\text{meromorphic 1- forms})}$

Equivalently, according to the proof of the Lefschetz (1,1) theorem from Section 2 of Chapter 1, ρ is the rank of $H^{1,1}(M) \cap H^2(M, \mathbb{Z})$, which is the rank of the quotient group

$$\frac{\text{divisors on } M}{\text{homological equivalence}}$$

of all divisors on M modulo those homologous to zero.

☞ $$H^1(M, \mathcal{O}^*) \xrightarrow{c_1} H^2(M, \mathbb{Z}) \xrightarrow{\iota_*} H^2(M, \mathcal{O}) \cong H^{0,2}_{\bar\partial}(M)$$