

$$\dim T_{2L}(W_2) = 2 \quad \text{and} \quad \dim \{X_0, L\} = 2,$$

$$T_{2L}(W_2) \subset \{X_0, L\}, \quad \text{and so } T_{2L}(W_2) = \{X_0 + L\}_{L \in \mathbb{P}^k}$$

If $\{C_\lambda\}$ is any pencil through $2L$ but not in the tangent space $T_{2L}(W_2)$, then, it can have only finitely many base points.

⌈ Otherwise, $\{C_\lambda\}$ has a fixed line. Since $\{C_\lambda\}$ passes $2L$, $\{C_\lambda\} = L(L + \lambda L_0) \Rightarrow \{C_\lambda\}$ is a tangent line of $T_{2L}(W_2)$, by the argument above. \Rightarrow Contradiction.

Choosing the conic C to miss these base points, the same argument as before shows that $\{C_\lambda\}$ meets V_C with multiplicity ≥ 2 at $2L$.

⌈ Given a set of finite points, $\{p_1, \dots, p_k\}$, then $\bigcup H_{p_i}$ is a set of measure zero set in W , where H_{p_i} is a hyperplane in W as in the page 250.
 \Rightarrow We can find a smooth conic C to miss the finite-point set.

For a fixed pencil $\{C_\lambda\}$, a generic conic C misses the base points of $\{C_\lambda\}$.
 (\Rightarrow) By fixing a smooth conic C , a generic pencil $\{C_\lambda\}$ not tangent to W_2 at $2L$