ective, it would suffice to prove that

$$H^1(M, \mathcal{J}_x^2(L)) = H^1(M, \mathcal{J}_{x,y}(L)) = 0 ;$$

indeed, replacing L by L^k and using $H^1(M, \mathcal{O}(L^k)) = 0$ for $k \geq k_1$, the reader may check that our theorem is equivalent to this vanishing theorem for high powers of L.

$$H^0(M, \mathcal{J}_{x,y}(L)) \longrightarrow H^0(M, \mathcal{O}(L)) \xrightarrow{r_{x,y}} H^0(M, L_x \oplus L_y)$$

$\longrightarrow H^1(M, \mathcal{J}_{x,y}(L)) \Rightarrow$ If $H^1(M, \mathcal{J}_{x,y}(L)) = 0$, $r_{x,y}$ is surjective.

$H^0(M, L_x \oplus L_y)$ = Set of sections of $M \times (L_x \oplus L_y)$ over M.
$\Rightarrow$ Since $L_x \oplus L_y \cong \mathbb{C}^2$, $H^0(M, L_x \oplus L_y) = \{ f : M \to \mathbb{C}^2$ holomorphic $\}$ $\Rightarrow$ f must be constant
$\Rightarrow$ $H^0(M, L_x \oplus L_y) \cong L_x \oplus L_y$.

Similarly, $H^0(M, T_x^{*\prime} \otimes L_x) \cong T_x^{*\prime} \otimes L_x$.

By P 159, Th. B, let $E = M \times \mathbb{C}$.

$\Rightarrow \exists$ k_1 s.t. $H^1(M, \mathcal{O}(L^k \otimes (M \times \mathbb{C}))) = 0$ for $k \geq k_1$
$\Rightarrow$ $H^1(M, \mathcal{O}(L^k \otimes (M \times \mathbb{C}))) = H^1(M, \mathcal{O}(L^k))$

The reader may check that our theorem is equivalent to this vanishing theorem for high powers of L. This statement is going to be proved later.
The problem is that unless M is of dimension 1 neither of the sheaves $\mathcal{J}_{x,y}(L)$ and $\mathcal{J}_x^2(L)$ is the