$= \lambda^d \sum X_{i_1} X_{i_2} \cdots X_{i_d} F(e_{i_1}, e_{i_2}, \cdots e_{i_d})$

homogeneous polynomial of deg d in $X_0, \cdots X_n$

If $\sigma_F \equiv 0$, $\sigma_F(X) = 0$ for all X.

$Q \equiv 0$ on $\mathbb{C}^{n+1}$ $\Rightarrow$ Since the map $F \longmapsto Q$

where $Q(v) = F(v, \cdots v)$, is injective, $F \equiv 0$.

Suppose $[X]$ is a solution of $\sigma_F([X]) = 0$.

$\Rightarrow \sigma_F([X]) = Q(\lambda X_0 \cdots \lambda X_n) = 0. = \sum X_{i_1} \cdots X_{i_d} F(e_i \cdots e_{i_d})$
$= 0$.

$\Rightarrow$ The zero divisor of the section σ_F is the image in $\mathbb{P}^n$ of the zero locus of $F(X_0 \cdots X_n)$ in $\mathbb{C}^{n+1}$.

Actually, $F(X_0 \cdots X_n) = Q(\lambda X_0, \cdots, \lambda X_n)$

We claim now that these are all the global sections of H^d. To show this, let σ be any global section of H^d, and denote by σ_F be the section of H^d corresponding to an arbitrary homogeneous polynomial $F(X_0, X_1, \cdots X_n)$. The quotient σ/σ_F is then a meromorphic function on $\mathbb{P}^n$; let

$$G' = \pi^*\left(\frac{\sigma}{\sigma_F}\right)$$

be its pullback to $\mathbb{C}^{n+1} - \{0\}$.

$$\frac{\sigma_\alpha}{\sigma_\beta} = g_{\alpha\beta} = \frac{\sigma_{F\alpha}}{\sigma_{F\beta}} \Rightarrow \frac{\sigma_\alpha}{\sigma_{F\alpha}} = \frac{\sigma_\beta}{\sigma_{F\beta}}$$

$$\Rightarrow \frac{\frac{\sigma_\alpha}{\sigma_{F\alpha}}}{\frac{\sigma_\beta}{\sigma_{F\beta}}} = 1. \Rightarrow \frac{\sigma_\alpha}{\sigma_{F\alpha}} \text{ is a meromorphic function } \mathbb{P}^n.$$