or, commonly, the "number of handles."

$\ulcorner$ $\chi(S) = \underset{\text{dim } H_0(S)}{\underset{\parallel}{b_0(S)}} - \underset{\text{dim } H_1(S)}{\underset{\parallel}{b_1(S)}} + \underset{\text{dim } H_2(S)}{\underset{\parallel}{b_2(S)}} = 2 - b_1(S)$

$\Rightarrow$ $b_1(S) = 2 - \chi(S)$

$\text{dim } H_1(S) = 2 \cdot \#$ of handles. $\lrcorner$

We saw in Section 2 of Chapter 1 that the curvature form of a metric on the holomorphic tangent bundle $T'(S) = K_S^*$ is just the Gaussian curvature of the metric times the volume form divided by $\sqrt{-1}$.

$\ulcorner$ See P77. $i \underset{\text{curvature form}}{\Theta} = \underset{\text{Gaussian curvature}}{K} \cdot \underset{\text{volume form}}{\Phi}$

$\Rightarrow$ $\Theta = K \cdot \Phi / i$. $\lrcorner$

By the classical Gauss-Bonnet theorem, then

$$\deg K_S = -\chi(S) = 2g - 2.$$

$\ulcorner$ $\deg K_S^* = \int_S \frac{i}{2\pi} \Theta$ by P144

$= \frac{1}{2\pi} \int_S K \Phi \underset{\text{P144}}{=} \chi(S)$ $\Rightarrow$ $\chi(S) = -\deg K_S = 2g-2$ $\lrcorner$

This is a form of the Riemann-Hurwitz formula and can be proved directly as follows: Let $f : S \longrightarrow S'$ be a holomorphic map between compact Riemann surfaces S and S'. For the induced map $f_* : H_2(S, \mathbb{Z}) \longrightarrow H_2(S', \mathbb{Z})$,