$\underline{Ext}^m_{\mathcal{O}}(\mathcal{F}, \mathcal{G}) = H^m(E.(\mathcal{F}), \mathcal{G})$.

$$0 \to \mathcal{O}^{(k_n)} \to \cdots \quad \mathcal{O}^{(k_1)} \to \mathcal{O}^{(k_0)} \to \mathcal{F} \to 0$$

Take $Hom_{\mathcal{O}}(\cdot, \mathcal{G})$,

$$0 \to Hom(\mathcal{F}, \mathcal{G}) \to Hom(\mathcal{O}^{(k_0)}, \mathcal{G}) \to \cdots \xrightarrow{\delta} Hom(\mathcal{O}^{(k_m)}, \mathcal{G}) \xrightarrow{\delta}$$

$\Rightarrow$ we have the following exact sequence

$$0 \to \ker\delta \to Hom(\mathcal{O}^{(k_m)}, \mathcal{G}) \to \operatorname{Im}\delta \to 0.$$

$\Rightarrow$ By Th. 14 above, $\ker\delta$, & $\operatorname{Im}\delta$ are coherent.

$\Rightarrow$ From the following exact sequence,

$$0 \to \operatorname{Im}\delta \to \ker\delta \to \underline{Ext}^m_{\mathcal{O}}(\mathcal{F}, \mathcal{G}) \to 0.$$

by the property ⊛ P697, since $\operatorname{Im}\delta$ & $\ker\delta$ are coherent, $\underline{Ext}^m_{\mathcal{O}}(\mathcal{F}, \mathcal{G})$ is coherent.

For the property 2, P683 ~ P684.

$$\begin{tikzcd}
\mathcal{F}_1 \arrow[r, "\alpha_1"] \arrow[d, dashed, "\phi_1"] & \mathcal{F}_0 \arrow[r, "\alpha_0"] \arrow[d, dashed, "\phi_0"] & \mathcal{F} \arrow[r] \arrow[d, "\phi"] & 0 \\
\mathcal{G}_1 \arrow[r, "\beta_1"] & \mathcal{G}_0 \arrow[r, "\beta_0"] & \mathcal{G} \arrow[r] & 0
\end{tikzcd}$$

to show that ϕ_0 & ϕ_1 exist locally is easy application of the proof on P684. We don't need worry about the size of nbds, since we only care things near a point. See P642 note. ■

As an illustration of property 3, given an exact sequence $0 \to \mathcal{G}' \to \mathcal{G} \to \mathcal{G}'' \to 0$ of coherent sheaves, application of $\otimes_{\mathcal{O}} \mathcal{F}$ gives

$$\cdots \to \underline{Tor}^{\mathcal{O}}_1(\mathcal{G}, \mathcal{F}) \to \underline{Tor}^{\mathcal{O}}_1(\mathcal{G}'', \mathcal{F}) \to \mathcal{G}'\otimes_{\mathcal{O}}\mathcal{F} \to \mathcal{G}\otimes_{\mathcal{O}}\mathcal{F} \to \mathcal{G}''\otimes_{\mathcal{O}}\mathcal{F} \to 0.$$