$\Rightarrow$ By Kodaira vanishing theorem P155 (in a dualized form),

$$H^1(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}((1-d)H)) = 0$$

(ii) $d=1$. $\mathcal{O}_{\mathbb{P}^n}(H-H) = \mathcal{O}_{\mathbb{P}^n}(\mathbb{P}^n \times \mathbb{C}) = \mathcal{O}_{\mathbb{P}^n}$.
$\Rightarrow$ $H^1(\mathbb{P}^n, \mathcal{O}) = 0$ see P118 & P49.

(iii) $d > 1$ and assume $n=1$.
$\Rightarrow$ $V \subset \mathbb{P}^1$ hypersurface $\Rightarrow$ V is a set of discrete points. $\Rightarrow$ Obviously, $H^0(\mathbb{P}^n, \mathcal{O}(H)) \longrightarrow H^0(V, \mathcal{O}(H))$ is surjective.

By P139, $0 \longrightarrow \mathcal{O}_{\mathbb{P}^n}(H-V) \longrightarrow \mathcal{O}_{\mathbb{P}^n}(H) \xrightarrow{r} \mathcal{O}_V(H) \to 0$ is exact, in case V is smooth. $\lrcorner$

Note that two normal varieties $V, V' \subset \mathbb{P}^n$ will be projectively isomorphic — that is, V may be carried into V' by an automorphism of $\mathbb{P}^n$ — if V is biholomorphic to V' via a mapping carrying $H_{V'}$ to H_V.

$\ulcorner$

$$\begin{array}{ccc} H|_V & \xrightarrow[\text{bundle isomorphism}]{\tilde{f}} & H_{V'} \\ \downarrow & & \downarrow \\ V & \xrightarrow{f} & V' \end{array}$$

See P875 note, Vol 17. 1996, 4. 27

Let $\sigma_1, \sigma_2 \cdots \sigma_k$ be sections of H whose restrictions to V form a basis for $H^0(V, \mathcal{O}(H))$.
Consider $\tilde{f} \circ \sigma_i \circ f^{-1} : V' \longrightarrow H_{V'}$. $\Rightarrow$ $\{\tilde{f} \circ \sigma_i \circ f^{-1}\}$ are sections of $H|_{V'}$.