

For $b = (b_1, b_2) \in (h=0)$, Since $\Delta^2 - (h=0)$ is open and $(h=0)$ is nowhere dense, $\exists b_n \rightarrow b$ s.t. $b_n \in \Delta^2 - (h=0)$.

$$\int_0^1 \frac{1}{h(\gamma_t)} \frac{dh}{dt} dt = \ln h(b_n) + i \theta_n^{\leftarrow \text{argument}} - \ln h(\gamma_0) - i \theta_0.$$

$$\Rightarrow e^{\int_0^1 \frac{1}{h(\gamma_t)} \frac{dh}{dt} dt} = e^{\ln h(b_n)} e^{-i \theta_n} C^{\leftarrow \text{constant}}$$

$$\Rightarrow \text{As } b_n \rightarrow b, |h(b_n)| \rightarrow 0 \Rightarrow e^{\ln h(b_n)} \rightarrow 0$$

$$\Rightarrow e^{\int_0^1 \frac{1}{h(\gamma_t)} \frac{dh}{dt} dt} = e^{\int_{\gamma_0}^{b_n} \partial \log h} \rightarrow 0 \text{ as } b_n \rightarrow b.$$

\Rightarrow Locally, $e^{\int_{\gamma_0}^z \partial \log h}$ is bounded. \Rightarrow it is extended to Δ^2 by Riemann extension theorem.

Note that $(e^{\int_{\gamma_0}^z \partial \log h} = 0) = (h=0)$.

This proves that $(e^{\int_{\gamma_0}^z \partial \log h} = 0) = f(M)$. \square

Comment on the path-connectedness of $\Delta^2 - (h=0)$.

In \mathbb{C}^2 , let $(z_1=0)$, which is the z_2 -plane.

Given two points (a_1, b_1) & $(a_2, b_2) \in \mathbb{C}^2 - (z_1=0)$, $a_1 \neq 0 \neq a_2$.

$\Rightarrow \exists$ a path $\alpha: [0, 1] \rightarrow \mathbb{C}$ s.t. $\alpha(t) \neq 0$, $\alpha(0) = a_1$, $\alpha(1) = a_2$ since $\mathbb{C} - \{0\}$ is path-connected. \Rightarrow Consider a path $\beta: [0, 1] \rightarrow \mathbb{C}^2$