Note that, implicitely, $\mathcal{F}(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p}) = 0$ if $\alpha_i = \alpha_j$ where $i \neq j$.

So any cocycle σ must satisfy the skew-symmetry condition.

$$\sigma_{i_0 \cdots i_p} = -\sigma_{i_0 \cdots i_{q-1}, i_{q+1}, i_q, i_{q+2} \cdots i_p}.$$

$$(\delta\sigma)_{i_0, i_1 \cdots i_{q-1}, \underline{i_q}, i_{q+1}, \underline{i_q}, i_{q+2} \cdots i_p} = 0 \in \mathcal{F}(U_{\alpha_0} \cap \cdots \cap U_{\alpha_{q-1}} \cap U_{\alpha_q} \cap U_{\alpha_{q+1}} \cap U_{\alpha_q} \cap \cdots \cap U_{\alpha_p}) = 0$$

$$\Rightarrow \quad 0 = \sigma_{i_0 i_1 \cdots i_{q-1}, i_{q+1}, i_q i_{q+2} \cdots i_p} + \sigma_{i_0 i_1 \cdots i_{q-1}, i_q, i_{q+1} \cdots i_p}$$

σ is called a coboundary if $\sigma = \delta\tau$ for some $\tau \in C^{p-1}(\underline{U}, \mathcal{F})$. It is easy to see that $\delta^2 = 0$.

We set

$$Z^p(\underline{U}, \mathcal{F}) = \ker\delta \subset C^p(\underline{U}, \mathcal{F}).$$

and $$H^p(\underline{U}, \mathcal{F}) = \frac{Z^p(\underline{U}, \mathcal{F})}{\delta C^{p-1}(\underline{U}, \mathcal{F})}.$$

Given two coverings $\underline{U} = \{U_\alpha\}_{\alpha \in I}$ & $\underline{U}' = \{U'_\beta\}_{\beta \in I'}$ of M,

we say that $\underline{U}'$ is a refinement of $\underline{U}$ if for every $\beta \in I'$, $\exists$ $\alpha \in I$ s.t $U'_\beta \subset U_\alpha$; we write $\underline{U}' < \underline{U}$.

If $\underline{U}' \subset \underline{U}$, we can choose a map $\varphi : I' \longrightarrow I$ s.t $U'_\beta \subset U_{\varphi(\beta)}$ for all β: then we have a map

$$\rho_\varphi : C^p(\underline{U}, \mathcal{F}) \longrightarrow C^p(\underline{U}', \mathcal{F}) \text{ given by}$$

$$(\rho_\varphi \sigma)_{\beta_0 \cdots \beta_p} = \sigma_{\varphi(\beta_0) \cdots \varphi(\beta_p)} \,|\, U_{\beta_0} \cap U_{\beta_1} \cap \cdots \cap U_{\beta_p}.$$