$$\langle D'\eta, \tau\rangle = \langle \partial\eta_\alpha \otimes e_\alpha + \sum \eta_\beta \wedge \theta_{\beta\alpha} \otimes e_\alpha , \tau_\alpha \otimes e_\alpha \rangle$$

$$= \langle \partial\eta_\alpha , \tau_\alpha \rangle + \langle \sum_\beta \eta_\beta \wedge \theta_{\beta\alpha} , \tau_\alpha \rangle$$

$$\Rightarrow \langle D'\eta, \tau\rangle$$

$$= \int \langle \partial\eta_\alpha , \tau_\alpha \rangle + \langle \sum_\beta \eta_\beta \wedge \theta_{\beta\alpha} , \tau_\alpha \rangle$$

$$= \int \sum_\alpha \partial\eta_\alpha \wedge *\tau_\alpha + \sum_\beta \eta_\beta \wedge \theta_{\beta\alpha} \wedge *\tau_\alpha$$

$$\langle \eta, D'^*\tau\rangle = \langle \eta_\alpha \otimes e_\alpha , \partial^*\tau_\alpha \otimes e_\alpha + \theta^*\tau \rangle$$

$$= \int \sum_\alpha \langle \eta_\alpha , \partial^*\tau_\alpha \rangle + \langle \eta_\alpha , \theta^*\tau \rangle$$

$$= \int \sum_\alpha \eta_\alpha \wedge *\partial^*\tau_\alpha + \int \sum_\alpha \langle \eta_\alpha , \theta^*\tau \rangle$$

$$\Rightarrow \int \sum_\alpha \partial\eta_\alpha \wedge *\tau_\alpha - \int \sum_\alpha \eta_\alpha \wedge *\partial^*\tau_\alpha$$

$$= \int \sum_\alpha \partial(\eta_\alpha \wedge *\tau_\alpha) = \int_M d\left(\sum_\alpha \eta_\alpha \wedge *\tau_\alpha\right) = 0$$

Define $\theta^* : A^{p,q}(E)(U) \longrightarrow A^{p-1,q}(E)(U)$ by

$$\eta_\alpha \wedge \theta_{\alpha\beta} \wedge *\tau_\beta = \langle \eta_\alpha \otimes e_\alpha , \theta^*\tau \rangle \Phi,$$

where Φ is the volume form.