beginning of this section, the global Sobolev norm induces a norm equivalent to the usual Sobolev norm on sections compactly supported in a nbd of a point, by using a partition of unity, we may conclude the

Global Sobolev Lemma. $\mathcal{H}_{[\frac{n}{2}]+1+s}(M.E) \subset C^s(M.E)$, the sections of differentiability class s on M, and

$$\bigcap_s \mathcal{H}_s(M.E) = C^\infty(M.E).$$

Global Rellich Lemma. For $s > r$, the inclusion

$$\mathcal{H}_s(M.E) \longrightarrow \mathcal{H}_r(M.E)$$

is a compact operator.

$\forall$ $p \in M$, $\exists$ $U \subset M$. s.t $E|_U \xrightarrow{\cong} U \times R^n$.

$$\begin{tikzcd}
\Gamma(E|_U) \arrow[r, "\nabla"] \arrow[d, leftrightarrow] & \Gamma(T^*M|_U \otimes E|_U) \arrow[d, leftrightarrow] \\
\Gamma(U \times R^n) \arrow[r, "\nabla"] & \Gamma(T^*M|_U \otimes U \times R^n)
\end{tikzcd}$$

1-form

$$\sum f_\alpha e_\alpha = f \longmapsto \sum df_\alpha \otimes e_\alpha + \theta_{\beta\alpha} f_\beta e_\alpha$$

$$(f_\alpha) \longmapsto (df_\alpha) + \left(\sum_\beta \theta_{\beta\alpha} f_\beta\right) = \left(df_\alpha + \sum_\beta \theta_{\beta\alpha} f_\beta\right)$$

$$\nabla gf = dg \otimes f + g \nabla f$$

$$\Rightarrow \quad dg\,(f_\alpha) + g\left(df_\alpha + \sum_\beta \theta_{\beta\alpha} f_\beta\right) = \left(d(gf_\alpha) + \sum_\beta \theta_{\beta\alpha}(gf_\beta)\right)$$

$\Rightarrow$ ∇ induces a connection on $U \times R^n$

Note that any map $f : U \longrightarrow R^n$ with compact support in U can be considered as a map on a torus to R^n.