Note that since each factor on the right has length $< d$, this already implies that σ_a is expressible as a polynomial in the special Schubert cycles $\sigma_{b,0,\cdots}$, i.e., that

The cohomology ring of the Grassmannian $G(k,n)$ is generated by the classes of the special Schubert cycles.

「 By induction, the length decreases to 1.」

Now, we will use the relation (**) to prove Giambelli's formula

$$\sigma_{a_1,\cdots a_d} = \begin{vmatrix} \sigma_{a_1}, & \sigma_{a_1+1}, & \sigma_{a_1+2}, & \cdots & \sigma_{a_1+d-1} \\ \sigma_{a_2-1}, & \sigma_{a_2} & \sigma_{a_2+1}, & \cdots & \sigma_{a_2+d-2} \\ \sigma_{a_3-2}, & \sigma_{a_3-1} & \sigma_{a_3} & & \vdots \\ \vdots & & & & \\ \sigma_{a_d-d+1}, & \cdots & & & \sigma_{a_d} \end{vmatrix}.$$

We will prove this by induction; clearly it is true for $d=1$. Assume that it holds for $d-1$; expanding by cofactors along the left-hand row, the determinant is given by

$$\sum (-1)^j \sigma_{a_j+d-j} \cdot \begin{vmatrix} \sigma_{a_1} & \cdots & \sigma_{a_1+d-2} \\ \vdots & & \\ \sigma_{a_{j+1}-j} & \cdots & \sigma_{a_{j+1}+d-j} \\ \vdots & & \\ \sigma_{a_d-d+1} & \cdots & \sigma_{a_d-1} \end{vmatrix}$$