must appear two times. But as we saw above, they are found just one time.
$\Rightarrow$ The multiplicity is 1. $\Rightarrow$ $\#(\Sigma_{k,n} \cdot \sigma_{n-k,\cdots})$ $= 2^{k+1}$. By the result on p.740 that $\Sigma_{k,n}$ has intersection number 0 with all Schubert cycles of complementary dimension except $\sigma_{n-k,n-k-1,\cdots}$ and the result on p739

$$\#(\sigma_{a_1 \cdots a_{k+1}} \cdot \sigma_{b_1 \cdots b_{k+1}}) = \begin{cases} 1 & \text{if } a_i + b_{k+2-i} = n-k+1 \ \forall \text{ for all } i, \\ 0, & \text{otherwise} \end{cases},$$

$\Sigma_{k,n} = a\, \sigma_{k+1,k,k-1,\cdots 1}$ and $\#(\sigma_{k+1,\cdots 1} \cdot \sigma_{n-k,n-k-1,\cdots}) = 1$,
$\Rightarrow a = 2^{k+1}$. $\Rightarrow$ $\Sigma_{k,n}$ is homologous to $2^{k+1}\sigma_{k+1,k,\cdots 1}$

Note that since the two families of n-planes on a $2n$-dimensional quadric in $\mathbb{P}^{2n+1}$ may be taken into one another by an automorphism of $\mathbb{P}^{2n+1}$, they represent the same class on $G(n+1, 2n+1)$, and hence each represents the class $2^n \cdot \sigma_{n+1, n \cdots 1}$.

Γ By the automorphism φ_I on p738, $\#I = $ odd, the two families of n-planes on a $2n$-dimensional quadric in $\mathbb{P}^{2n+1}$ may be taken into one another. $\mathrm{Aut}(\mathbb{P}^{2n+1})$ is path-connected, since GL_{2n+1} is path-connected, see p65 & p196.