If $\bar{\Lambda} \in G(k-1, L^\circ)$, consider $\Lambda = \bar{\Lambda}.L \in G(k,n)$. We have to check if $\Lambda \in \sigma_{a,0 \dots}(V)$.

$\dim(\Lambda \cap V_{n-k+1-a}) \geq \dim(L) = 1$ since $\Lambda \cap V_{n-k+1-a} \supset L$.

Thus we proved the claim.

We have the revised reduction formula I. as follows:

For any three indices $0 \leq \alpha, \beta, \gamma \leq k$ with $\alpha+\beta+\gamma = 2k+l$, $1 \leq l \leq k$,

$$\#(\sigma_a \cdot \sigma_b \cdot \sigma_c) = \begin{cases} 0 & \text{if } a_\alpha + b_\beta + c_\gamma > n-k \\ \#(\sigma_{a-\hat{a}_\alpha} \cdot \sigma_{b-\hat{b}_\beta} \cdot \sigma_{c-\hat{c}_\gamma})_{G(k-1,n-1)} & \text{if } a_\alpha + b_\beta + c_\gamma = n-k. \end{cases}$$

$\Rightarrow$

Reduction Formula II. For any three coefficients a_α, b_β, c_γ with $a_\alpha + b_\beta + c_\gamma \geq 2(n-k)+1$.

$$\#(\sigma_a \cdot \sigma_b \cdot \sigma_c)_{G(k,n)} = \#(\sigma_{a^*} \cdot \sigma_{b^*} \cdot \sigma_{c^*})_{G(n-k,n)}$$

$$= \begin{cases} 0 & \text{if } a^*_{a_\alpha} + b^*_{b_\beta} + c^*_{c_\gamma} > n-(n-k) = k \\ \#(\sigma_{a^*-\hat{a}^*_{a_\alpha}} \cdot \sigma_{b^*-\hat{b}^*_{b_\beta}} \cdot \sigma_{c^*-\hat{c}^*_{c_\gamma}})_{G(n-k-1,n-1)} & \text{if } a^*_{a_\alpha} + b^*_{b_\beta} + c^*_{c_\gamma} = k. \end{cases}$$

$$= \begin{cases} 0 & \text{if } \alpha+\beta+\gamma > k \\ \#(\sigma_{a_1-1,\dots,a_\alpha-1,a_{\alpha+1},\dots,a_k} \cdot \sigma_{b_1-1,\dots,b_\beta-1,b_{\beta+1},\dots,b_k} \cdot \sigma_{c_1-1,\dots,c_\gamma-1,c_{\gamma+1},\dots,c_k})_{G(k,n-1)} & \alpha+\beta+\gamma = k \end{cases}$$