tangent vector $\partial / \partial X_i$ is given by

$$\pi_* \frac{\partial}{\partial X_i} = \frac{1}{X_0} \cdot \frac{\partial}{\partial x_i}, \quad i = 1, 2, \cdots n,$$

$$\pi_* \frac{\partial}{\partial X_0} = - \sum \frac{X_i}{X_0^2} \cdot \frac{\partial}{\partial x_i}.$$

$\ulcorner$ $\pi^* dx_i = d\, \pi^* x_i = d\, x_i \circ \pi = d\left(X_i / X_0 \right)$

$$= \frac{X_0\, dX_i - X_i\, dX_0}{X_0^2}.$$

$i \geq 1$, let $\pi_* \frac{\partial}{\partial X_j} = a_k \frac{\partial}{\partial x_k}$.

$$\pi^* dx_i \left(\frac{\partial}{\partial X_j} \right) = dx_i \left(\pi_* \frac{\partial}{\partial X_j} \right) = dx_i \left(a_k \frac{\partial}{\partial x_k} \right) = a_i$$

$$\frac{X_0\, dX_i - X_i\, dX_0}{X_0^2} \left(\frac{\partial}{\partial X_j} \right) = \frac{1}{X_0} \delta_{ij}$$

$$\Rightarrow a_i = \frac{1}{X_0} \delta_{ij}.$$

① $i \neq j$ $\quad a_i = 0$ $\qquad$ ② $i = j$ $\quad a_i = \frac{1}{X_0}$

$$\Rightarrow \pi_* \frac{\partial}{\partial X_j} = \frac{1}{X_0} \frac{\partial}{\partial X_j}.$$

let $\pi_* \frac{\partial}{\partial X_0} = b_k \frac{\partial}{\partial x_k}$.

$$\pi^* dx_i \left(\frac{\partial}{\partial X_0} \right) = dx_i \left(\pi_* \frac{\partial}{\partial X_0} \right) = dx_i \left(b_k \frac{\partial}{\partial x_k} \right) = b_i$$

$$\frac{X_0\, dX_i - X_i\, dX_0}{X_0^2} \left(\frac{\partial}{\partial X_0} \right) = - \frac{X_i}{X_0^2} \quad \Rightarrow \quad b_i = - \frac{X_i}{X_0^2}$$

$$\Rightarrow \pi_* \frac{\partial}{\partial X_0} = \sum - \frac{X_i}{X_0^2} \frac{\partial}{\partial x_i}$$

$\lrcorner$