

We now are in a position to globalize the local duality theorem. Suppose that $I \subset \mathcal{O}$ is a sheaf of regular ideals s.t. the support $\mathbb{Z} = \text{supp}(\mathcal{O}/I)$ has dimension zero. Equivalently, locally $I = \{f_1, \dots, f_n\}$, where the f_i form a regular sequence and $n = \dim_{\mathbb{C}} M$.

⌈ See p 660. ⌋

We consider \mathbb{Z} as a ringed space with structure sheaf $\mathcal{O}_{\mathbb{Z}} = \mathcal{O}/I$.

We now refer to the intrinsic form of the local duality theorem in Section 3 above. According to the proposition in that section, the sheaves $\underline{\text{Ext}}_{\mathcal{O}}^q(\mathcal{O}_{\mathbb{Z}}, \Omega^n) = 0$ for $q < n$.

$$\begin{aligned} \lceil \mathcal{O}_{\mathbb{Z}} &= \mathcal{O}/I \Rightarrow (\mathcal{O}_{\mathbb{Z}})_x \cong \mathcal{O}_x/I_x \\ \Omega_x^n &\cong \mathcal{O}_x \\ \Rightarrow \underline{\text{Ext}}_{\mathcal{O}}^q(\mathcal{O}_{\mathbb{Z}}, \Omega^n)_x &\cong \underline{\text{Ext}}_{\mathcal{O}_x}^q(\mathcal{O}_x/I_x, \mathcal{O}_x) = 0 \text{ for } q < n \\ n &= \# \{f_1, \dots, f_n\}. \text{ See p 690. } \rceil \end{aligned}$$

Moreover, since the sheaves $\mathcal{O}_{\mathbb{Z}}$ and $\underline{\text{Ext}}_{\mathcal{O}}^n(\mathcal{O}_{\mathbb{Z}}, \Omega^n)$ are skyscraper sheaves,

$$\left\{ \begin{aligned} H^0(M, \mathcal{O}_{\mathbb{Z}}) &\cong \bigoplus_{p \in \mathbb{Z}} \mathcal{O}_{\mathbb{Z}, p} \\ H^0(M, \underline{\text{Ext}}_{\mathcal{O}}^n(\mathcal{O}_{\mathbb{Z}}, \Omega^n)) &\cong \bigoplus_{p \in \mathbb{Z}} \underline{\text{Ext}}_{\mathcal{O}_p}^n(\mathcal{O}_{\mathbb{Z}, p}, \Omega_p^n) \\ H^q(M, \mathcal{O}_{\mathbb{Z}}) &= H^q(M, \underline{\text{Ext}}_{\mathcal{O}}^n(\mathcal{O}_{\mathbb{Z}}, \Omega^n)) = 0 \text{ for } q > 0. \end{aligned} \right.$$