by P. For example, for $k=2$ we have

$$\tilde{P}(A,B) = \frac{1}{2}\left(P(A+B) - P(A) - P(B)\right).$$

$\tilde{P}: V \times V \longrightarrow \mathbb{C}$ See note P410.

$P(A) = \tilde{P}(A,A)$

$$\tilde{P}(A+B, A+B) = \tilde{P}(A+B, A) + \tilde{P}(A+B, B) = \tilde{P}(A,A)$$
$$+ 2\tilde{P}(A,B) + \tilde{P}(B,B) = P(A+B)$$
$$\Rightarrow 2\tilde{P}(A,B) = P(A+B) - P(A) - P(B).$$

In general, to polarize P^k, if for $(A^1, \cdots A^k) \in (M_n)^k$, $\tau \in S_k$ a permutation and $I \subset \{1, 2, \cdots n\}$ a multiindex of order k, we let A_I^τ be the $k \times k$ matrix whose ith column is the ith column of $A_{I,I}^{\tau(i)}$, then

$$\tilde{P}^k(A_1, \cdots A_k) = \frac{1}{k!}\sum_{\tau \in S^k}\sum_{\#I=k} \det(A_I^\tau);$$

and the polarization of a general invariant polynomial—expressed as a polynomial in the elementary invariant polynomials P^i — can be written out in a similarly unenlightening way.

Clearly, $\tilde{P}^k(A, A, \cdots A) = \frac{1}{k!}\sum_{\tau \in S^k}\sum_{\#I=k} \det(A_I)$

$$\overset{\frac{1}{k!}\cdot k!}{=} \sum_{\#I=k} \det(A_{I,I}) = P^k(A).$$

$$\tilde{P}^k(gA_1g^{-1}, \cdots gA_kg^{-1}) = \frac{1}{k!}\sum_{\tau \in S^k}\sum_{\#I=k} \det\left((gAg^{-1})_I^\tau\right)$$