Now if φ is a 1-form, g a matrix of functions, and A_i a matrix of forms of degree d_i, by multilinearity

$$\sum_i (-1)^{d_1+\cdots+d_{i-1}} P(A_1, \cdots \varphi \wedge g A_i, \cdots A_k)$$

$$= \sum_i \varphi \wedge P(A_1, \cdots, g A_i, \cdots A_k)$$
$$= \sum_i \varphi \wedge P(A_1, \cdots, A_i g, \cdots A_k)$$
$$= \sum_i (-1)^{d_1+\cdots+d_i} P(A_1, \cdots, A_i \wedge \varphi g, \cdots A_k).$$

Pf Since $\sum_i P(A_1, \cdots g' A_i - A_i g', \cdots A_k) = 0$

$$= \sum_i P(A_1, \cdots g' A_i \cdots A_k) - \sum_i P(A_1, \cdots A_i g', \cdots A_k),$$

$$\sum_i P(A_1, \cdots, g' A_i, \cdots A_k) = \sum_i P(A_1, \cdots, A_i g', \cdots A_k)$$

$$\Rightarrow \quad \varphi \wedge \sum_i P(A_1, \cdots g' A_i, \cdots A_k) = \sum_i \varphi \wedge P(A_1, \cdots, g' A_i, \cdots A_k)$$

$$= \varphi \wedge \sum_i P(A_1, \cdots, A_i g', \cdots A_k) = \sum_i \varphi \wedge P(A_1, \cdots, A_i g', \cdots A_k)$$

If $\|g'\|$ is small enough, then $g' + I \in GL_n$.
At least $g' + I$ must be in GL_n.
Suppose $P : M_n \times M_n \longrightarrow \mathbb{C}$ is defined by

$$P(A, B) = (\text{trace } A) \times (\text{trace } B)$$

$\Rightarrow$ P is multilinear i.e

$$P(A_1 + A_2, B) = (tr(A_1 + A_2)) \times tr B = (tr A_1 + tr A_2) \cdot tr B$$
$$= tr A_1 \cdot tr B + tr A_2 \cdot tr B = P(A_1, B) + P(A_2, B)$$
$$P(\alpha A, B) = tr(\alpha A) \cdot tr B = \alpha \, tr A \cdot tr B$$
$$= \alpha P(A, B)$$

Note that $P(A, B) = P(B, A)$.

Thus for $\sum_i (-1)^{d_1+\cdots+d_{i-1}} P(A_1, \cdots, \varphi \wedge g A_i, \cdots A_k) =$