Suppose $(0, \cdots 1)$ is a linear combination of $\nabla F_1 \cdots \nabla F_{n-k}$.

$\Rightarrow (0, \cdots 1) = a_1 \nabla F_1 + \cdots + a_{n-k} \nabla F_{n-k}$

Since V meets H_λ transversely,

$(\nabla F_1)^\perp \cap (\nabla F_2)^\perp \cap \cdots \cap (\nabla F_{n-k})^\perp + (0, \cdots, 1)^\perp = \mathbb{C}^n$ at p -- ⊛

Here $(\nabla F_1)^\perp \cap \cdots (\nabla F_{n-k})^\perp = T_p V$.

By the relation $(0, 0, \cdots 1) = a_1 \nabla F_1 + \cdots + a_{n-k} \nabla F_{n-k}$ at p,

for any $v \in (\nabla F_1)^\perp \cap \cdots \cap (\nabla F_{n-k})^\perp$, $v \perp (0, 0, \cdots 1)$.

$\Rightarrow v \in (0, 0, \cdots 1)^\perp \Rightarrow (\nabla F_1)^\perp \cap \cdots \cap (\nabla F_{n-k})^\perp \subset (0, 0, \cdots 1)^\perp$.

$\Rightarrow$ Contradiction to ⊛.

Since $H_\lambda \cap V = V_1 \cup \cdots \cup V_\ell$, where each V_i is irreducible, if $p \in V_1 \cap V_2$, p can not be smooth.

Let V' be the union of the irreducible components of the sections $H_\lambda \cap V$ that contain Z. Then V' is an open k-dimensional analytic variety contained in V, and hence its closure $\overline{V'}$ must be all of V; thus $H_\lambda \cap V = H_\lambda \cap V'$ is irreducible for generic λ.

「 I think (i) V' is an open k-dimensional set contained in V. See p438 back. $\Rightarrow \overline{V'}$ is an algebraic variety in V. $\Rightarrow \dim \overline{V'} = \dim V \Rightarrow \overline{V'} = V$.

(ii) V' is k-dimensional variety contained in V. $\Rightarrow V = V'$

Accept this fact.

Since $H_\lambda \cap V'$ is closed, $H_\lambda \cap \overline{V'} = H_\lambda \cap V'$.

Remember A closed in X, U open in X

$\Rightarrow A \cap \overline{U} = A \cap U$ if $A \cap U$ closed in A.