$s = z_1^m f_1 \quad s' = z_2^n f_2$ or $s = z_2^m f_1,\ s' = z_1^n f_2.$

We may assume that $s = z_1^m \quad s' = z_2^n$ since $f_1, f_2 \in \mathcal{O}$ are unit. For example, $s = z_1^2,\ s' = z_2^3$.
Since $\sigma(0) = 0$ with 6 multiplicity,
$\sigma = z_1^2 g_1 + z_2^3 g_2 + g_3$, g_3 has 6 multiplicity at 0. $\Rightarrow$ $g_3 = z_1 z_2^6 g_3'$ or $z_1^6 z_2 g_3' \cdots$
$\Rightarrow g_3 = z_1^2 g_1 + z_2^3 g_2 \Rightarrow \sigma \in \mathcal{J}(L)_p$.

② $p \notin P$ $\qquad \mathcal{O}(L)_p / \mathcal{J}_P(L)_p = 0$. trivially done.

Remark: $\mathcal{J}_{P_0}(L)$ is the sheaf of holomorphic sections of L vanishing on P_0. If $\{p, p\} \subset P_0$, the vanishing order at p is ≥ 2. $\lrcorner$

By the duality theorem, $H^1(\mathcal{J}_{P_0}(L))$ and $\mathrm{Ext}^1(S; \mathcal{J}_{P_0}(L), \Omega^2)$ are canonically dual vector spaces.

$\ulcorner$ By Global Duality Theorem II on p 708,

$H^p(M, \mathcal{F}) \otimes \mathrm{Ext}^{n-p}(M; \mathcal{F}, \Omega^n) \longrightarrow \mathbb{C}$ is non-degenerate.
$\Rightarrow M = S.\ \mathcal{F} = \mathcal{J}_{P_0}(L), \Rightarrow n = 2.$
$\Rightarrow H^1(S, \mathcal{J}_{P_0}(L)) \otimes \mathrm{Ext}^1(S; \mathcal{J}_{P_0}(L), \Omega^2) \longrightarrow \mathbb{C}$ is nondegenerate $\Rightarrow$ $H^1(S, \mathcal{J}_{P_0}(L))$ and $\mathrm{Ext}^1(S; \mathcal{J}_{P_0}(L), \Omega^2)$ are canonically dual vector spaces. $\lrcorner$