

A divisor $D = \sum a_i V_i$ is called effective if $a_i \geq 0$ for all i ; we write $D \geq 0$ for D effective.

An analytic hypersurface V will usually be identified with the divisor $\sum V_i$, where the V_i 's are the irreducible components of V .

Let $V \subset M$ be an irreducible analytic hypersurface, $p \in V$ any point, and f a local defining function for V near p . For any holomorphic function g defined near p , we define the order $\text{ord}_{V,p}(g)$ of g along V at p to be the largest integer a s.t. $f^a \mid g$ in the local ring $\mathcal{O}_{M,p}$,

$$g = f^a \cdot h, \quad h \text{ holomorphic function.}$$

By the result from p. 10 that relatively prime elements of $\mathcal{O}_{M,p}$ stay relatively prime in nearby local rings,

we see that for g a holomorphic function on M , $\text{ord}_{V,p}(g)$ is independent of p . Thus we can define the order $\text{ord}_V(g)$ of g along V to be simply the order of g along V at any point $p \in V$.

Since V is irreducible, f is irreducible by p. 12.

Suppose g is a holomorphic function on M .

At \forall point $p \in M$, let $\text{ord}_{V,p}(g) = a$.

Let $\{x \in V \mid \text{ord}_{V,x}(g) = a\} = K$.

$U' \subset U$

In $g = f^a h$ for some nbd U' of p , since a is the largest number, f can not divide h in any nbd U' of p .

In other words, $[f]$ and $[h]$ are relatively prime in $\mathcal{O}_{M,p}$.

\Rightarrow By p. 10, proposition and irreducibility of f , $[f], [h]$ are relatively prime again in $\mathcal{O}_{M,q}$ for $q \in$ some nbd V' of p , where