$\vdash$ By the Kodaira-Serre duality (P153. 4),

$$H^q(M, \Omega^p(L)) \cong H^{n-q}(M. \Omega^{n-p}(L^*))^* = 0 \quad \text{if } n-q+n-p > n \iff p+q < n$$

in case L^* is a positive line bundle (i.e L is a negative line bundle.) $\dashv$

The special case when $p = q = 0$ can be proved by elementary methods as follows: What we have to show is that

(*) $\quad H^0(M. \mathcal{O}(L)) = 0.$ in case $L \longrightarrow M$ has a metric with curvature form equal to $\frac{2\pi}{i}$ times a negative (1.1)-form. Suppose $s \neq 0 \in H^0(M. \mathcal{O}(L))$, and let $x_0 \in M$ be a point where $|s|^2$ attains a maximum. By hypothesis, if we write $z_i = x_i + i y_i$, the coefficient matrix for the curvature form

$$\left(\frac{\partial}{\partial z_i}\frac{\partial}{\partial \bar{z}_j} \log\left(\frac{1}{|s|^2}\right)\right) = \frac{1}{4}\left(\left(\frac{\partial^2}{\partial x_i \partial x_j} + \frac{\partial^2}{\partial y_i \partial y_j}\right)\right.$$

$$\left.+ i\left(\frac{\partial^2}{\partial x_j \partial y_i} - \frac{\partial^2}{\partial y_j \partial x_i}\right)\right)\left(\log\frac{1}{|s|^2}\right)$$

is negative definite hermitian, and in particular the real symmetric matrix

$$\left(\frac{\partial^2}{\partial x_i \partial x_j} + \frac{\partial^2}{\partial y_i \partial y_j}\right)\log\frac{1}{|s|^2} \quad \text{is negative definite.}$$

But $\log(1/|s|^2)$ attains a minimum at x_0, and by the maximum principle, the matrices

$$\left(\frac{\partial^2}{\partial x_i \partial x_j}\right)\log\frac{1}{|s|^2} \quad \text{and} \quad \left(\frac{\partial^2}{\partial y_i \partial y_j}\right)\log\frac{1}{|s|^2}$$

must both be positive semidefinite — a contradiction.