$U_{\{i\}}) \Rightarrow \omega_{\{i\}} = \eta_{\omega.} = n!\,(-1)^i\,(-1)^{\frac{(n+1)(n+2)}{2}} \dfrac{\bar\partial\rho_1\wedge\cdots\widehat{\bar\partial\rho_i}\wedge\cdots\bar\partial\rho_n\wedge dz_1\wedge\cdots\wedge dz_n}{f_1\cdots f_n}$

But, setting $f=(f_1,\cdots f_n)$ and $\|f\|^2=\sum_i |f_i|^2$,

$$\bar\partial\rho_i = \frac{f_i\,d\bar f_i}{\|f\|^2} - \frac{|f_i|^2\sum f_j d\bar f_j}{\|f\|^4},$$

and so the wedge product

$$\bigwedge_{j\neq i}\bar\partial\rho_j = \frac{\left(\bigwedge_{j\neq i} f_j\,d\bar f_j\right)}{\|f\|^{2n-2}} - \frac{\sum_{k\neq i}(-1)^{(k,\{i\}^c)}\left(\bigwedge_{j\neq i,k} f_j d\bar f_j\right)\overbrace{\textstyle\sum f_\ell\,d\bar f_\ell}^{|f_k|^2}}{\|f\|^{2n}}$$

$$= \frac{1}{\|f\|^{2n}}\left(\|f\|^2\bigwedge_{j\neq i} f_j\,d\bar f_j - \sum_{k\neq i}|f_k|^2\left(\bigwedge_{j\neq i} f_j d\bar f_j\right)\right.$$

$$\left. - |f_k|^2\sum_{k\neq i}(-1)^{k-i-1}\bigwedge_{j\neq k} f_j\,d\bar f_j\right)$$

$$= \frac{1}{\|f\|^{2n}}\left(|f_i|^2\bigwedge_{j\neq i} f_j\,d\bar f_j + \sum_{k\neq i}|f_k|^2(-1)^{k-i}\bigwedge_{j\neq k} f_j\,d\bar f_j\right)$$

$$= \frac{f_1\cdots f_n\,(-1)^i\left(\sum(-1)^k\bar f_k\bigwedge_{j\neq k} d\bar f_j\right)}{\|f\|^{2n}}.$$

$$\bigwedge_{j\neq i}\bar\partial\rho_j = \bigwedge_{j\neq i}\left(\frac{f_j\,d\bar f_j}{\|f\|^2} - \frac{|f_j|^2\sum f_\ell d\bar f_\ell}{\|f\|^4}\right) = \text{expand \& rearrange}$$

$$= \frac{1}{\|f\|^{2n}}\|f\|^2\bigwedge_{j\neq i} f_j d\bar f_j - \sum_{k<i}\frac{|f_k|^2}{\|f\|^{2n}}(-1)^{n-k-1}\left((-1)^{n-k-1}\bigwedge_{j\neq i} f_j d\bar f_j + (-1)^{n-i}\bigwedge_{j\neq k} f_j d\bar f_j\right)$$

$$- \sum_{k>i}(-1)^{n-k}\frac{|f_k|^2}{\|f\|^{2n}}\left((-1)^{n-k}\bigwedge_{j\neq i} f_j d\bar f_j + (-1)^{n-i-1}\bigwedge_{j\neq k} f_j d\bar f_j\right),$$ since $\sum f_\ell d\bar f_\ell\wedge\sum f_\ell d\bar f_\ell = 0$,