

Then by the sheaf property (ii), \exists a section $s \in \mathcal{F}(U)$ s.t. $s|_{W_p} = s(p)|_{W_p}$ for each $p \in U$.

It remains to prove that $\varphi(U)(s) = t$.

For each $p \in U$, $\varphi(U)(s)|_{W_p} = t|_{W_p}$. since

$$\varphi(U)(s)|_{W_p} = \varphi_{W_p}(s(p)|_{W_p}) = t|_{W_p}.$$

\Rightarrow Again by the sheaf property (i) applied to $\varphi(U)(s) - t$, $\varphi(U)(s) = t$.

Def: Let $\varphi: \mathcal{F} \rightarrow \mathcal{G}$ be a morphism of presheaves. We define the presheaf kernel of φ , presheaf cokernel of φ , and presheaf image of φ to be the presheaves given by $U \mapsto \ker(\varphi(U))$, $U \mapsto \operatorname{coker}(\varphi(U))$ and $U \mapsto \operatorname{im}(\varphi(U))$ respectively.

Note that if $\varphi: \mathcal{F} \rightarrow \mathcal{G}$ is a morphism of sheaves, then the presheaf kernel of φ is a sheaf, but the presheaf cokernel and presheaf image of φ are in general not sheaves. This leads us to the notion of a sheaf associated to a presheaf.

Proposition - Definition.

Given a presheaf \mathcal{F} , there is a sheaf \mathcal{F}^+ , and a morphism $\theta: \mathcal{F} \rightarrow \mathcal{F}^+$ with the property that for any sheaf \mathcal{G} , and any morphism $\varphi: \mathcal{F} \rightarrow \mathcal{G}$, \exists a unique morphism $\psi: \mathcal{F}^+ \rightarrow \mathcal{G}$ s.t. $\varphi = \psi \circ \theta$.