Clearly, $\delta \circ \rho_\varphi = \rho_\varphi \circ \delta$. (ρ_φ is a chain map).

$$\left(\begin{aligned} (\delta \circ \rho_\varphi(\sigma))_{\beta_0 \cdots \beta_{p+1}} &= \sum_{j=0}^{p+1} (-1)^j (\rho_\varphi(\sigma))_{\beta_0 \cdots \hat{\beta_j} \cdots \beta_{p+1}} \big| U'_{\beta_0} \cap \cdots \cap U'_{\beta_{p+1}} \\ &= \sum_{j=0}^{p+1} (-1)^j \sigma_{\varphi(\beta_0) \cdots \widehat{\varphi(\beta_j)} \cdots \varphi(\beta_{p+1})} \big| U'_{\beta_0} \cap \cdots \cap U'_{\beta_{p+1}} \\ &= (\delta\sigma)_{\varphi(\beta_0), \cdots \varphi(\beta_{p+1})} \big| U'_{\beta_0} \cap \cdots \cap U'_{\beta_{p+1}} = (\rho_\varphi(\delta\sigma))_{\beta_0 \cdots \beta_{p+1}}. \end{aligned} \right)$$

So ρ_φ induces a homomorphism

$\rho : H^p(\underline{U}, \mathcal{F}) \longrightarrow H^p(\underline{U}', \mathcal{F})$ which is independent of the choice of φ.

⌈ lemma 10.4.2 (Bott. Differential Forms in A.T. P 111).
If φ, ψ are two refinement maps from I' to I, then there is a homotopy operator between ρ_φ and ρ_ψ.

pf). Define $K : C^p(\underline{U}, \mathcal{F}) \longrightarrow C^{p-1}(\underline{U}', \mathcal{F})$ by

$$(K\sigma)_{\beta_0 \cdots \beta_{p-1}} = \sum_{i=0}^{p-1} (-1)^i \sigma_{\varphi(\beta_0) \cdots \varphi(\beta_i) \psi(\beta_i) \cdots \psi(\beta_{p-1})},$$ and

show that

$$\rho_\psi - \rho_\varphi = \delta K + K \delta.$$ ⌋

$\forall\ \sigma \in Z^p(\underline{U}, \mathcal{F})$

$$\Rightarrow \rho_\psi\sigma - \rho_\varphi\sigma = \delta K\sigma + K \cancel{\delta\sigma}^{\,0} \in \delta C^{p-1}(\underline{U}', \mathcal{F}).$$

$$\Rightarrow \rho_\psi\sigma = \rho_\varphi\sigma \text{ on } H^p(\underline{U}', \mathcal{F}).$$ //

We define the p-th Čech cohomology group of $\mathcal{F}$ on M to be the direct limit of the