

$\Rightarrow$  Since  $\langle \cdot, \cdot \rangle$  is nondegenerate,  $\bar{V}$  is nonsingular.  
 $\Rightarrow \bar{V}a$  may assume any element in  $\mathbb{C}^n$ , i.e.,  
 given any  $c \in \mathbb{C}^n$ ,  $\exists a$  s.t.  $\bar{V}a = c$ . //  
 Clearly,  $W$  is a hyperplane in  $V$ .  
 $\{v \mid \langle v, w \rangle = 0\}$

□

In particular, if  $w_0 = v_1 \wedge v_2$  the hyperplane section  
 $H_{w_0} \cap G$  of  $G$  consists of the Schubert cycle  
 $\sigma_1(l_0)$  of lines in  $\mathbb{P}^3$  meeting the line  $l_0 = \overline{v_1, v_2}$   
 spanned by  $v_1$  and  $v_2$ . Thus

Every Schubert cycle  $\sigma_1(l_0) \subset G$  is a hyperplane section  
 of  $G$ .

$\Upsilon$  Let  $w = w_1 \wedge w_2 \in G$ , s.t.  $w \wedge v_1 \wedge v_2 = 0$ .

$\Rightarrow$  This implies that  $\{w_1, w_2, v_1, v_2\}$  is linearly  
 independent.  $\Rightarrow a_1 w_1 + a_2 w_2 + b_1 v_1 + b_2 v_2 = 0$ , not  
 all  $a_i$ 's  $b_i$ 's zero.  $\Rightarrow a_1 w_1 + a_2 w_2 = -b_1 v_1 - b_2 v_2$

$\Rightarrow$  The line  $\overline{w_1, w_2}$  meets with the line  $\overline{v_1, v_2}$ .

(Note: If  $a_1 = a_2 = 0$ , then since  $\{v_1, v_2\}$  is linearly  
 independent,  $b_1 = b_2 = 0$ , vice versa.)

Conversely, if  $\overline{w_1, w_2} \cap \overline{v_1, v_2} \neq \emptyset$ , then  $\{w_1, w_2, v_1, v_2\}$   
 is linearly independent.  $\Rightarrow w \wedge w_0 = 0$

□

Remember:  $\sigma_1(l_0) \stackrel{\text{in } \mathbb{P}^5}{\sim}$  is the image of the  $\sigma_1(l_0) \sim G(2, 4)$ .