we see that ψ_i extends to a meromorphic function in Δ, and hence in all of $\mathbb{P}^k$.

$\ulcorner$

$$\varphi' = \varphi \cdot \pi^* f^m$$

$$\psi_1' = \sum \varphi' = \sum \varphi \cdot \pi^* f^m = \pi^* f^m \psi_1$$

$$\Rightarrow \quad \psi_1 = \frac{\psi_1'}{\pi^* f^m}$$

$$\psi_2' = \sum \varphi' \cdot \varphi' = \sum \varphi \cdot \pi^* f^m \cdot \varphi \cdot \pi^* f^m$$

$$= \sum \varphi \cdot \varphi \cdot (\pi^* f^m)^2 = (\pi^* f^m)^2 \psi_2$$

$$= ((\pi^* f)^m)^2 \psi_2 = (\pi^* f)^{2m} = \pi^* f^{2m} \psi_2$$

$$\Rightarrow \quad \psi_2 = \frac{\psi_2'}{\pi^* f^{2m}}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\psi_i = \frac{\psi_i'}{\pi^* f^{im}}$$

Since $\mathbb{P}^k$ is compact and the (by identity theorem) Riemann extension is unique, we can patch all extensions together and get a meromorphic function on $\mathbb{P}^k$. $\lrcorner$

Thus the functions ψ_i are rational functions.

$\ulcorner$ By P168, any meromorphic function on $\mathbb{P}^n$ is rational. $\lrcorner$