the union of the n-simplices τ_β^n in the subdivision having σ_α^0 as a vertex. Then for each k-simplex σ_α^k in the original decomposition,

let $\displaystyle *\sigma_\alpha^k = \bigcap_{\sigma_\beta^0 \in \sigma_\alpha^k} *\sigma_\beta^0$ be the intersection of the

n-cells $*\sigma_\beta^0$ associated to the $(k+1)$ vertices of σ_α^k. The cells $\{\Delta_\alpha^{n-k} = *\sigma_\alpha^k\}$ then give a decomposition of M, called the dual cell decomposition to $\{\sigma_\alpha\}$.

Note that since the only point of a k-simplex σ_α^k of our original complex held in common by $(k+1)$ cells of the dual decomposition is its barycenter, the dual cell $\Delta_\alpha^{n-k} = *\sigma_\alpha^k$ of σ_α is the only $(n-k)$-cell of the dual decomposition meeting σ_α^k; Δ_α^{n-k} will ~~meet~~ intersect σ_α^k transversely.

Given an orientation on σ_α^k, we may take the dual orientation on Δ_α^{n-k} to be the one s.t at $p \in \sigma_\alpha^k \cap *\sigma_\alpha^k$,

$$i_p(\sigma_\alpha, \Delta_\alpha) = +1.$$

Hereafter, if σ_α is considered an oriented simplex, $*\sigma_\alpha$ will denote the oriented cell Δ_α with the dual orientation; we will also write $*\Delta_\alpha$ to denote the original oriented simplex σ_α.

We now relate the boundary operator ∂ on the complex $\{\sigma_\alpha^k\}$ to the coboundary operator δ on $\{\Delta_\alpha^{n-k}\}$.
Note first that if σ_α^k has vertices $\sigma_0^0, \cdots \sigma_k^0$, then the dual cell $\Delta_\alpha^{n-k} = *\sigma_\alpha^k$ is given as the $(k+1)$-fold intersection $\bigcap_i \Delta_i^n = \bigcap_i *\sigma_i^0$ of the dual