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Conversely, if $C \subset \mathbb{P}^n$ is an irreducible, nondegenerate curve of degree n, let $p_1, p_2, \cdots p_{n-1}$ be any $(n-1)$ independent points of C, $V = \overline{p_1, p_2, \cdots p_{n-1}} \cong \mathbb{P}^{n-2}$ their linear span, and $\{H_\lambda\}_{\lambda \in \mathbb{P}^1}$ the pencil of hyperplanes in $\mathbb{P}^n$ containing V. Each hyperplane H_λ will then intersect C in n points: $p_1, p_2, \cdot p_{n-1}$ and an additional point we will call $q(\lambda)$. (In case H_λ is the hyperplane containing V and tangent to C at p_i, the point $q(\lambda) = p_i$.)

「 Since C is a curve of degree n, each hyperplane H_λ will intersect C in n points. 」

Every point of C will lie on a unique hyperplane H_λ, and so the map $q: \mathbb{P}^1 \longrightarrow C$ is an isomorphism.

「 For each H_λ, $C \cap H_\lambda = \{p_1, p_2 \cdots p_{n-1}, q(\lambda)\}$.

Suppose $C \cap H_\lambda \supset \{p_1, p_2 \cdots p_{n-1}, p.\}$ where $p \in \overline{p_1 \cdots p_{n-1}}$, $p \neq p_i$.
Since C is nondegenerate, $\exists\ q \in C$ s.t $q \notin H_\lambda$.

Consider $\overline{p_1, p_2, \cdots p_{n-1}, q} = \mathbb{P}^{n-1} = H_\lambda$ for some λ.
$\Rightarrow \mathbb{P}^{n-1} \cap C \supset \{p_1, p_2, \cdots p_{n-1}, p, q\}$ which is a set of $(n+1)$ points $\Rightarrow$ $\mathbb{P}^{n-1} \cap C$ contains a curve, see p64. $\Rightarrow$ Since C is irreducible, $C \subset \mathbb{P}^{n-1}$. $\Rightarrow$ Contradiction to the nondegeneracy of C.

(see

Thus we can conclude that $C \cap \overline{p_1 \cdots p_{n-1}}$ = exactly $\{p_1, p_2, \cdots p_{n-1}\}$.
Given $q' \in C - \overline{p_1 \cdots p_{n-1}}$, $\{q', p_1 \cdots p_{n-1}\}$ is a set of linearly independent points.

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