First, we note that any linear functional L on $\mathbb{C}^{n+1}$ induces a section σ_L of H by restriction, i.e. by setting

$$\sigma_L(X) = L|_{\{\lambda X\}} .$$

Clearly σ_L is identically zero only if L is, so we have an injection

$$\mathbb{C}^{n+1\,*} \longrightarrow H^0(\mathbb{P}^n, \mathcal{O}(H)).$$

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$$\begin{array}{ccc} (\mathbb{C}^{n+1})^* & \xrightarrow{\ \phi\ } & H^0(\mathbb{P}^n, \mathcal{O}(H)) \\ \Downarrow & & \Downarrow \\ L & \longmapsto & \sigma_L \end{array}$$

If $\sigma_L \equiv 0$. $L|_{\{\lambda X\}} = 0$. $\Rightarrow$ $L \equiv 0$. $\Rightarrow$ ϕ is one to one. 」

In fact, all of $H^0(\mathbb{P}^n, \mathcal{O}(H))$ is obtained in this way: if σ is any section of H, $D = (\sigma)$ its zero divisor, then the fundamental class η_D is given by

$\eta_D = c_1(H) = \omega$. and by the argument

of Section. 4, Chapter 0, it follows that D is a hyperplane in $\mathbb{P}^n$. 「 See P64 」.

If we let $L \in (\mathbb{C}^{n+1})^*$ be any linear functional vanishing on the hyperplane $\pi^{-1}D \subset \mathbb{C}^{n+1}$, then, the meromorphic function σ/σ_L will be holomorphic on all of $\mathbb{P}^n$, hence constant.