

$\eta \mapsto \eta \otimes s_0$  is well-defined, since  $(\eta \otimes s_0 = 0) = (\eta = 0) + (-q)$  and  $(\eta = 0) \geq q$ .

Suppose that all  $\sigma \in H^0(S, \Omega'(-p-q))$  are zero at  $p$ . Once again, by using the argument above,

$$\begin{array}{ccc} H^0(S, \Omega'(-q)) & \xleftarrow{\psi} & H^0(S, \Omega'(-p-q)) \\ \sigma \otimes s_1 & \xleftarrow{\quad} & \sigma \end{array}$$

$\psi$  is isomorphism  $\Rightarrow h^0(K-p) = h^0(K-p-q) = g+1 = h^0(K-q) \Rightarrow$  Contradicts to the fact that  $\dim H^0(K-p-q)$  is larger <sup>than</sup>  $\dim H^0(K-q)$ .

$\Rightarrow \exists \sigma \in H^0(S, \Omega'(-p-q))$  s.t.  $\sigma(p) \neq 0$ .

$\Rightarrow \eta' = \eta - \frac{\eta(p)}{\sigma(p)} \cdot \sigma \Rightarrow \eta'(q) \neq 0$  and  $\eta'(p) = 0$ .  
and  $\eta'/s_1 = \eta' \otimes s_1 \in H^0(S, \Omega')$  and  $\eta' \otimes s_1(p) \neq 0$  and  $\eta' \otimes s_1(q) = 0$

②  $p = q$ .

$$\begin{array}{ccc} H^0(S, \Omega'(-2p)) & \longrightarrow & H^0(S, \Omega'(-p)) \\ \psi & & \\ \sigma & \longmapsto & \sigma \otimes s_1 \end{array}$$

where  $(s_1 = 0) = p$ .

If all  $\eta \in H^0(S, \Omega'(-p))$  are zero at  $p$ ,

$\frac{\eta}{s_1} \in H^0(S, \Omega'(-2p)) \Rightarrow \dim H^0(S, \Omega'(-2p)) = h^0(K-p) \Rightarrow$  By the assumption, this can not happen.  $\Rightarrow \exists \eta \in H^0(S, \Omega'(-p))$  with  $\eta(p) \neq 0$ .  
 $\Rightarrow \eta \otimes s_1 \in H^0(S, \Omega') \Rightarrow \eta \otimes s_1(p) = 0$  and its order is exactly one.  $\square$

By Riemann-Roch,

$$h^0(K-p-q) = g-3 + h^0(p+q), \dots$$