has dimension given by

$$\dim |\mathcal{J}_{\Gamma_0}(4)| = 2 \text{ or } 3$$

where the second possibility holds exactly when the points of Γ are distinct and collinear.

Γ $h^0(\mathcal{O}_{\mathbb{P}^2}(4)) = \binom{2+4}{2} = {}_6C_2 = \frac{6\times5}{2} = 15$

$h^1(\mathcal{O}_{\mathbb{P}^2}(4)) = \dim H^1(\mathbb{P}^2, \mathcal{O}(4H)) = 0$ by p156.

From the exact sequence

$$0 \to \mathcal{J}_{\Gamma_0}(4) \to \mathcal{O}_{\mathbb{P}^2}(4) \to \mathcal{O}_{\Gamma_0}(4) \to 0,$$

we have

$$0 \to H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0}(4)) \to H^0(\mathbb{P}^2, \mathcal{O}(4)) \to H^0(\mathbb{P}^2, \mathcal{O}_{\Gamma_0}(4)) \to H^1(\mathbb{P}^2, \mathcal{J}_{\Gamma_0}(4))$$

$$\to \underset{\substack{\| \\ 0}}{H^1(\mathbb{P}^2, \mathcal{O}(4))} \to$$

$\Rightarrow$ $h^0(\mathcal{J}_{\Gamma_0}(4)) - h^0(\mathbb{P}^2, \mathcal{O}(4)) + h^0(\mathbb{P}^2, \mathcal{O}_{\Gamma_0}(4)) - h^1(\mathcal{J}_{\Gamma_0}(4)) = 0$

$h^0(\mathbb{P}^2, \mathcal{O}(4)) = {}_{n+2}C_2 = \frac{(n+1)(n+2)}{2}$

$H^0(\mathbb{P}^2, \mathcal{O}_{\Gamma_0}(4)) = \bigoplus_{p\in\Gamma_0} \mathcal{O}_{\Gamma_0,p}$. by p707

$\Rightarrow$ Since $\mathcal{O}_{\Gamma_0,p}$ is a complex vector space of dimension of 4 ($\because$ " $\mathcal{O}_{\Gamma_0,p} = \mathcal{O}_p / I_p$, $I_p = \{f_c, f_{c'}\}$,

where f_c & $f_{c'}$ are functions representing C & C' respectively,