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$0 \to \mathrm{Hom}_{\mathcal{O}}(M, P) \longrightarrow \mathrm{Hom}_{\mathcal{O}}(M, Q) \longrightarrow \mathrm{Hom}_{\mathcal{O}}(M, R)$,

are exact.

『 See p200 ~ p201 & p209 ~ p210 Algebra by Hungerford 』

We express this by saying that $\otimes$ is right-exact and Hom is left-exact. Much of our discussion will be centered around the kernel of $P \otimes_{\mathcal{O}} M \longrightarrow Q \otimes_{\mathcal{O}} M$ and cokernel of $\mathrm{Hom}_{\mathcal{O}}(M, Q) \longrightarrow \mathrm{Hom}_{\mathcal{O}}(M, R)$.

Associated to an $\mathcal{O}$-module M is its fiber $M_0 = M/mM$ – the motivation for this terminology will emerge when we discuss coherent sheaves. This is a module over $\mathcal{O}/m = \mathbb{C}$ and is therefore a finit-dimensional vector space. Our main technical tool is the

Nakayama Lemma. If $M = mM$, then $M = (0)$.

Proof. We define the ideal $I = \{ f \in \mathcal{O} : f \cdot M = 0 \}$ and shall prove that $I = \mathcal{O}$.

『 If we prove the claim, then $M = (0)$, for $g \in M$, since for all $f \in \mathcal{O}$, $f \cdot g = 0 \Rightarrow f^{-1} f g = 0 \cdot f^{-1} = 0 \Leftrightarrow g = 0 \Rightarrow$ This implies $M = (0)$.
in case f unit 』

Suppose $m_1, \cdots, m_k$ generate M and write

$$m_i = \sum_j a_{ij} m_j \quad \text{or equivalently} \quad \underbrace{\sum_j (\delta_{ij} - a_{ij}) m_j}_{=0},$$

where $a_{ij} \in m$.

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