We assume that $\mathrm{ord}_{V_i}(f_\alpha) \neq 0$.

Thus given $f \in H^0(M, \mathcal{M}^*/\mathcal{O}^*)$, $\exists \{(f_\alpha, U_\alpha)\}$ s.t

① $\{U_\alpha\}$ open cover of M.

② If $V_i \cap U_\alpha \neq \phi$ and $\mathrm{ord}_{V_i}(f_\alpha) \neq 0$, V_i has a local defining function $g_{i\alpha} \in \mathcal{O}(U_\alpha)$. In case $\mathrm{ord}_{V_i}(f_\alpha) = 0$, $\exists$ wld.

$\Rightarrow$ Locally $f_\alpha = \frac{k}{h}$, i.e., for every point $p \in U_\alpha$,

$\exists$ open set $V_p \subset U_\alpha$ and $V_p \ni p$ s.t

$f_\alpha = \frac{k}{h}$ on V_p, k, h holomorphic on V_p.

$$\Rightarrow \quad k = \prod_i g_{i\alpha}^{a_i} \cdot l_1 \quad \text{where } l_1 \neq 0.$$

$$h = \prod_i g_{i\alpha}^{b_i}\, l_2 \quad \text{where } l_2 \neq 0.$$

and $a_i - b_i = \mathrm{ord}_{V_i}(f_\alpha)$

$$\Rightarrow \quad f_\alpha = \frac{k}{h} = \prod_i g_{i\alpha}^{a_i - b_i} \frac{l_1}{l_2} = \prod_i g_{i\alpha}^{\mathrm{ord}_{V_i}(f_\alpha)} \frac{l_1}{l_2}, \quad \frac{l_1}{l_2} \in \mathcal{O}^*(V_p) \text{ for each } i$$

(If we take k, h relatively prime, either $a_i = 0$ or $b_i = 0$)

If $V_p \cap V_q \neq \phi$, $f_\alpha = \prod_i g_{i\alpha}^{\mathrm{ord}_{V_i}(f_\alpha)}\, l$ on V_q, $l \in \mathcal{O}^*(V_q)$

$\Rightarrow$ Since $f_\alpha|_{V_p \cap V_q} = f_\alpha|_{V_q \cap V_p}$, $l = l_1/l_2 \in \mathcal{O}^*(V_p \cap V_q)$.

$\Rightarrow$ By the second property of sheaf,

$$f_\alpha = \prod_i g_{i\alpha}^{\mathrm{ord}_{V_i}(f_\alpha)}\, l, \quad l \in \mathcal{O}^*(U_\alpha).$$

$\Rightarrow \{(f_\alpha, U_\alpha)\}$ & $\{(\prod_i g_{i\alpha}^{\mathrm{ord}_{V_i}(f_\alpha)}, U_\alpha)\}$ define the same