over general rings. Taking cycles / boundaries gives respectively $H_*(K.) = \oplus H_n(K.)$ (homology) and $H^*(K^\cdot) = \oplus H^n(K^\cdot)$ (cohomology). We shall give the remaining discussion in homology, leaving the dual considerations to the reader

(b) A mapping or homomorphism of complexes $\varphi : K. \to L.$ is given by a commutative diagram

$$\begin{tikzcd}
\arrow[r] & K_n \arrow[r, "\partial"] \arrow[d, "\varphi"] \arrow[dr, phantom, "\circlearrowright"] & K_{n-1} \arrow[r] \arrow[d, "\varphi"] & \cdots \\
\arrow[r] & L_n \arrow[r, "\partial"] & L_{n-1} \arrow[r] & \cdots
\end{tikzcd}$$

It induces a map $\varphi_* : H_*(K.) \to H_*(L.)$ on homology. When necessary we shall write $\varphi_n : K_n \to L_n$ and ∂_K. ∂_L for the boundary maps. The set $\mathrm{Hom}(K., L.)$ of homomorphisms of complexes is a group with $(\varphi+\psi)_* = \varphi_* + \psi_*$.

(c) A homomorphism of complexes $\varphi : K. \to L.$ is homotopic to zero, denoted by $\varphi \sim 0$, if there is a chain homotopy

$$\varphi = \partial_L \alpha_n + \alpha_{n-1} \partial_K$$

as indicated by the diagram

$$\begin{tikzcd}
K_{n+1} \arrow[r] \arrow[d] & K_n \arrow[r, "\partial_K"] \arrow[d, "\varphi"] \arrow[dl, "\alpha_n"'] & K_{n-1} \arrow[d] \arrow[dl, "\alpha_{n-1}"'] \\
L_{n+1} \arrow[r, "\partial_L"] & L_n \arrow[r] & L_{n-1}
\end{tikzcd}$$

In this case $\varphi_* = 0$. Two maps φ and ψ are homoto