We will sometimes refer to the Riemann-Hurwitz formula as being any of the following:

$$\deg K_S = 2g-2,$$

$$\chi(S) = n\,\chi(S') - \sum_{q\in S} (v(q)-1),$$

$$K_S = f^* K_{S'} + B.$$

Note two things about maps $f: S \longrightarrow S'$ between compact Riemann surfaces: first, that the number of branch points of f, counting multiplicity, is always even; and second, that unless f is constant, $g(S) \geq g(S')$.

$\ulcorner$ If f is not constant, $g(S) = n\cdot(g(S')-1)+1 + \frac{1}{2}\sum_{p\in S}(v(p)-1)$.

$$\Rightarrow \quad g(S) - g(S') = (n-1)(g(S')-1) + \frac{1}{2}\sum_{p\in S}(v(p)-1)$$

① $n=0$

Since $\sum_{q\in f^{-1}(p)} v(q) = n$, for any $p \in S'$,

$v(q)=0$ for all $q\in f^{-1}(p)$. $\Rightarrow$ locally, $f(z)=w=1$ $\Rightarrow$ f is constant. $\Rightarrow$ $*$

② $n\geq 1$ & $g(S')=0$

$\Rightarrow$ $S' = \mathbb{P}^1$. $\Rightarrow$ Since $g(S)\geq 0$, $g(S)\geq g(S') = 0$

③ $n\geq 1$ & $g(S')>0$ $\Rightarrow$ Since $(n-1)(g(S')-1)\geq 0$ and $\frac{1}{2}\sum_{p\in S}(v(p)-1)\geq 0$, $g(S)-g(S')\geq 0$.

I think that the # of branch points, counting mul-