$\Rightarrow$ The pencil $\{a\sigma_1 + b\sigma_2\}$ contains p, and is contained in the net of quadrics from W containing p, i.e $\{a\sigma_1 + b\sigma_2\} \subset N_p$.
(Here a generic net N_p means that its base locus consists of 8 distinct points.)
Thus we have only to count the lines in N_p.

There is another way to make the count, based on the fact that a smooth point p on a quadric F will lie on on two lines of F if F is smooth, but only one if F is singular.

By the result ⊛ on P479, in case F is smooth, $T_p(F) \cap F$ = distinct two lines.

(i) $F = X_0^2 + X_1^2$

$\Rightarrow$ $p = [1, \pm i, *, *]$ is a smooth point of F.

Assume $p = [1, i, *, *] \in X_0 + iX_1 = 0 \subset F$ and $* X_2 - * X_3 = 0$ $\Rightarrow$ $\{X_0 + iX_1 = 0\} \cap \{* X_2 - * X_3 = 0\} \ni p$. But $\exists$ infinitely many lines in $X_0 + iX_1 = 0$ passing $p = [1, i, *, *]$.

If $p = [0, 0, 1, 1]$, p is a singular point.

$p \in X_0 + iX_1 = 0$ Again $\exists$ infinitely many lines in $X_0+iX_1 = 0$ passing through p. Strange. !!!

See back for an explanation.

Actually, the arguments continues.