$$\iota(v)\,\partial\,\Phi_r = (\bar\partial\omega)^{n-r}\wedge\iota(v)P_r(E,\Theta) + (-1)^{2(n-r)+r+1}(\bar\partial\omega)^{n-r-1}\wedge\iota(v)P_{r+1}(E,\Theta)$$

since $\iota(v)(\iota(v)P_{r+1}(E,\Theta)) = 0$ & $\iota(v)(\omega) = 1$. ⌟

" Comment on $\iota(\bar\partial v)\omega + \iota(v)\bar\partial\omega = \bar\partial(\omega(v))$."

Not natural, because we did not define $\iota_{\bar\partial v}$, when $\bar\partial v$ is of type (1,1).

So we are going to show $\iota(v)\bar\partial\omega = 0$ in a different way. Let $\omega = a_i\, dz_i$, $v = v_i \frac{\partial}{\partial z_i}$.

$\Rightarrow$ $\omega(v) = 1$ $\Rightarrow$ $a_i v_i = 1$.

$\bar\partial\omega = \frac{\partial a_i}{\partial \bar z_l} d\bar z_l \wedge dz_i$.

$$(\iota(v)\bar\partial\omega)(b_k \tfrac{\partial}{\partial \bar z_k}) = \bar\partial\omega(v, b_k\tfrac{\partial}{\partial\bar z_k}) = \frac{\partial a_i}{\partial\bar z_l} d\bar z_l\wedge dz_i(v_\alpha\tfrac{\partial}{\partial z_\alpha}, b_k\tfrac{\partial}{\partial\bar z_k}) = \frac{\partial a_i}{\partial \bar z_l} v_\alpha b_k\; d\bar z_l\wedge dz_i(\tfrac{\partial}{\partial z_\alpha}, \tfrac{\partial}{\partial \bar z_k})$$

$$= \frac{\partial a_i}{\partial\bar z_l} v_\alpha b_k\; \delta_{kl}\,\delta_{\alpha i} = -\frac{\partial a_i}{\partial \bar z_l} v_i b_l \cdots (*)$$

But since $\frac{\partial a_i v_i}{\partial \bar z_l} = 0 = \frac{\partial a_i}{\partial\bar z_l} v_i + a_i \cancel{\frac{\partial v_i}{\partial \bar z_l}}^{=0}$ (since v_i's are holomorphic), $\frac{\partial a_i}{\partial\bar z_l} v_i = 0$.

$\Rightarrow$ $(*) = 0$. $\Rightarrow$ $\iota(v)\bar\partial\omega = 0$

$(\iota(v_1)(\varphi\wedge\eta))(v_2,\cdots v_{n+m})$ ↓

$= \varphi_1\wedge\cdots\wedge\varphi_n\wedge\eta_1\wedge\cdots\wedge\eta_m(v_1,\cdots v_{n+m})$

$= \sum_\sigma \epsilon(\sigma)\,\varphi_1(v_{\sigma(1)})\cdots\varphi_n(v_{\sigma(n)})\,\eta_1(v_{\sigma(n+1)})\cdots\eta_m(v_{\sigma(n+m)})$

Fix σ, then we have $v_{\sigma(1)}, \cdots v_{\sigma(n+m)}$.