pic if $\varphi - \psi \sim 0$; then $\varphi_* = \psi_*$.

(d) Most importantly, an exact sequence of complexes

$$0 \to K. \longrightarrow L. \longrightarrow M. \longrightarrow 0$$

is defined in the obvious way. It gives rise to a long exact sequence homology sequence

$$\cdots \to H_n(K.) \to H_n(L.) \to H_n(M.) \xrightarrow{\partial_*} H_{n-1}(K.) \to \cdots.$$

The following definition and proposition are our primary technical tools:

Definition. A projective resolution $E.(M)$ of an $\mathcal{O}$-module M is given by an exact sequence

$$E.(M): \cdots \to E_m \xrightarrow{\partial} E_{m-1} \xrightarrow{\partial} \cdots \xrightarrow{\partial} E_0 \to M \to 0,$$

where the E_m are projective (= free) $\mathcal{O}$-modules.

Note that $H_n(E.(M)) = 0$ for $n > 0$ and $H_0(E.(M)) \cong M$

Pf At each $m > 0$, $E_{m+1} \xrightarrow{\partial} E_m \xrightarrow{\partial} E_{m-1}$ is exact &
$E_1 \xrightarrow{\partial} E_0 \xrightarrow{f} M$ is exact.
$\Rightarrow$ For $n > 0$, since $E_{n+1} \xrightarrow{\partial} E_n \xrightarrow{\partial} E_{n-1}$ is exact,

$$H_n(E.(M)) = \frac{\ker \partial}{\operatorname{im} \partial} = 0$$

For $n = 0$, $\quad E_1 \xrightarrow{\partial_1} E_0 \xrightarrow{\partial_0} 0$

$$H_0(E.(M)) = \frac{\ker \partial_0}{\operatorname{im} \partial_1} = \frac{E_0}{\partial_1 E_1}$$

since $E_1 \xrightarrow{\partial} E_0 \xrightarrow{f} M$ is exact, $\partial_1 E_1 = \ker f$.