$\Rightarrow$ ⊛ has no nontrivial solution, i.e., not all zero a, b, c satisfying ⊛.

$\Rightarrow$ Thus $\nexists$ no quadric in N_p which has p as a vertex. ///

From the above result, the base point p lies on two lines of F if F is smooth, but only one if F is singular, i.e rank $F = 3$. So the projection of I onto N_p expresses I as a double cover of $N_p \cong \mathbb{P}^2$, branched over the locus of singular quadrics in N_p.

⌟

But by our analysis of the linear system of quadrics in $\mathbb{P}^3$, the locus of singular quadrics in N_p is a smooth quartic curve.

⌜ Since N_p is a generic $\mathbb{P}^2$ in $W = \mathbb{P}^9$, by Bertini's theorem, $N_p \cap W_1$ is a smooth curve in $\mathbb{P}^2 = N_p$. $\Rightarrow$ Since it is a plane curve, it is irreducible, otherwise, it is not smooth for, if $N_p \cap W_1 = C_1 \cup C_2$, $C_1 \cap C_2 \neq \phi$ and at the intersection, $C_1 \cup C_2$ is not smooth. see P144 and $\deg W_1 = 4$

$\Rightarrow$ $\deg N_p \cap W_1 = 4$ by Bezout's theorem on P172. $\Rightarrow$ $N_p \cap W_1$, the locus of singular quadrics in N_p is a smooth quartic curve.

⌟