Let $g(z_1) = z_2(z_1) = z_2$. $\Rightarrow$ If we let $v(q)$ = the ramification index of π_p at q, and $q = (a, b) \in \mathbb{C}^2$,

$$g(z_1) - b = (z_1 - a)^{v(q)} h(z_1), \quad h(a) \neq 0$$

$\Rightarrow$ $f(z_1, g(z_1)) = 0$ and f is a polynomial in z_1, z_2. $\Rightarrow$ $h(z_1)$ is polynomial in z_1.

$$\Rightarrow \frac{\partial g}{\partial z_1} = v\,(z_1 - a)^{v-1} h(z_1) + (z_1 - a)^{v} \frac{\partial h}{\partial z_1}$$

Plug in $z_1 = a$. (i.e evaluate at q).

$$\frac{\partial g}{\partial z_1}(q) = 0 .$$

$$\frac{\partial g}{\partial z_1} = (z_1 - a)^{v-1}\left(v\, h(z_1) + (z_1 - a) \frac{\partial h}{\partial z_1} \right).$$

$\Rightarrow$ $\frac{\partial g}{\partial z_1}$ has $v-1$ as the order of vanishing at q.

From $\frac{\partial f}{\partial z_2} + \frac{\partial f}{\partial z_1}\frac{\partial z_1}{\partial z_2} \equiv 0$ on S,

$$\frac{\partial f}{\partial z_2} \equiv - \frac{\partial f}{\partial z_1}\frac{\partial z_1}{\partial z_2}.$$ $\Rightarrow$ The order of vanishing of $\frac{\partial z_1}{\partial z_2}$ at q = The order of zero of $\frac{\partial f}{\partial z_2}$ at q.
= The multiplicity of intersection of S with the curve $(\partial f/\partial z_2 = 0)$ at q.

$(\partial f/\partial z_2 = 0)$ is a curve of degree $d-1$ in $\mathbb{P}^2$, and so its intersection number with S is $d(d-1)$; since all points of $S \cap (\partial f/\partial z_2 = 0)$ lie in the finite plane $(Z_0 \neq 0)$,

$$\Sigma\,(v(q) - 1) = d(d-1).$$

Since $(f=0)$ is a curve of deg d, $(\frac{\partial f}{\partial z_2} = 0)$ is