Thus since, once we fix z_λ's, φ_a is expressed as $(a_1, a_2 \cdots a_d)$,

$$\psi\begin{pmatrix} a_1 \\ \vdots \\ a_d \end{pmatrix} = \begin{pmatrix} \frac{\omega_1}{dz_1}(p_1) & \cdots & \frac{\omega_1}{dz_d}(p_d) \\ \vdots & & \vdots \\ \frac{\omega_g}{dz_1}(p_1), & \cdots & \frac{\omega_g}{dz_d}(p_d) \end{pmatrix}\begin{pmatrix} a_1 \\ \vdots \\ a_d \end{pmatrix} . \Rightarrow \psi \text{ is given}$$

by the matrix $\left(\frac{\partial \omega_i}{\partial z_j}(p_j)\right)$.

Now the number of independent relations among the row vectors of this matrix is just the number of linearly independent holomorphic differentials vanishing at p_λ for all λ, that is, the dimension of $H^0(S, \Omega^1(-D))$.

Thus

$$\begin{aligned} h^0(D) &= \dim(\ker \psi) + 1 \\ &= d - \operatorname{rank}\psi + 1 . \\ &= d - g + h^0(K-D) + 1 . \end{aligned}$$

This is the classical Riemann–Roch formula.

$\ulcorner$ For $g=3$, $d=2$,

$$\begin{bmatrix} \frac{\omega_1}{dz_1}(p_1), & \frac{\omega_1}{dz_2}(p_2) \\ \frac{\omega_2}{dz_1}(p_1), & \frac{\omega_2}{dz_2}(p_2) \\ \frac{\omega_3}{dz_1}(p_1), & \frac{\omega_3}{dz_2}(p_2) \end{bmatrix}$$

Assume that the first two rows are the linearly independent vectors.

$\Rightarrow \exists\ c_1, c_2$ s.t

$$\left(\frac{\omega_3}{dz_1}(p_1), \frac{\omega_3}{dz_2}(p_2)\right) = c_1\left(\frac{\omega_1}{dz_1}(p_1), \frac{\omega_1}{dz_2}(p_2)\right)$$

$$+ c_2\left(\frac{\omega_2}{dz_1}(p_1), \frac{\omega_2}{dz_2}(p_2)\right) \Rightarrow \frac{\omega_3}{dz_1}(p_1) = \frac{c_1\omega_1}{dz_1}(p_1) + \frac{c_2\omega_2}{dz_1}(p_1)$$