1. Suppose $L = \{F_\lambda\}$ is a generic line in W, that is, a generic pencil of quadrics in $\mathbb{P}^2$. The conics F_λ of the pencil L may be given as the zero loci of the quadratic forms

$$Q^\lambda(X) = \Sigma\, q_{ij}^\lambda X_i X_j,$$

where $Q^\lambda = (q_{ij}^\lambda) = Q^0 + \lambda Q^\infty$
for suitable choice of nonsingular symmetric matrices Q^0 and Q^∞. F_λ will then be singular exactly when the determinant $|Q^0 + \lambda Q^\infty|$ vanishes; since this determinant is a cubic polynomial in λ, this will occur for three values of λ.

$\ulcorner$ We may choose Q^0, Q^∞ nonsingular since L is a generic pencil. $|Q^0 + \lambda Q^\infty| = |Q^\infty||Q^{\infty-1}Q^0 + \lambda I|$
$= |Q^\infty| (\lambda^3 + \square\cdot\lambda^2 + \square\lambda + \square) \Rightarrow \exists$ three values of λ
s.t $|Q^0 + \lambda Q^\infty| = 0$ $\lrcorner$

L thus intersects W_1 three times, so $\deg(W_1) = 3$.

$\ulcorner \#(L \cap W_1) = 3$. $\Rightarrow$ L is a line in W and W_1 is a hypersurface. $\Rightarrow$ $\deg W_1 = 3$ $\lrcorner$

Note that in general by this argument the singular quadrics in $\mathbb{P}^n$ form a hypersurface of degree $n+1$ in the system of all quadrics.