

Writing  $L = [D]$  for some divisor  $D = \sum n_i V_i$ ,

$$\gamma = c_1([D]) = \eta_D.$$

Q.E.D.

By Proposition (P161),  $\exists$  a divisor  $D$  s.t.

$$L = [D]. \Rightarrow \gamma = c_1([D]) = \eta_D. \text{ by P141, Proposition 2.}$$

### 3. Algebraic Varieties.

#### Analytic and Algebraic Varieties.

Let  $X_0, X_1, \dots, X_n$  denote Euclidean coordinates on  $\mathbb{C}^{n+1}$  and also the corresponding homogeneous coordinates on  $\mathbb{P}^n$ . Recall that the universal bundle  $J \rightarrow \mathbb{P}^n$  is the subbundle of the trivial bundle  $\mathbb{C}^{n+1} \times \mathbb{P}^n \rightarrow \mathbb{P}^n$  whose fibre over a point  $X \in \mathbb{P}^n$  is simply the line  $\{\lambda X\}_\lambda \subset \mathbb{C}^{n+1}$  corresponding to  $X$ .

The hyperplane bundle  $H \rightarrow \mathbb{P}^n$  is the dual of  $J$ , i.e., it is the bundle whose fibre over  $X \in \mathbb{P}^n$  corresponds to the space of linear functionals on the line  $\{\lambda X\}_\lambda$ . As we saw in Section 1 of this chapter, the Chern class of  $H$  is the fundamental class  $w$  of a hyperplane in  $\mathbb{P}^n$  — that is, a generator of  $H^2(\mathbb{P}^n, \mathbb{Z})$  — and it follows from  $H^1(\mathbb{P}^n, \mathcal{O}) = 0$  that every line bundle on  $\mathbb{P}^n$  is a multiple  $H^d$  of  $H$ .

Consider now the global sections of the bundle  $H$ .