the points of Γ_0 must lie on C_0.

Γ Suppose all the points of Γ_0 do not lie on C_0.
$\Rightarrow$ $C_0 = (\tau = 0)$ and $\exists$ linearly independent $\sigma_1, \sigma_2, \sigma_3$
$\in H^0(\mathcal{J}_{\Gamma_0}(3))$ s.t $\sigma_1 = \tau l_1$, $\sigma_2 = \tau l_2$ $\sigma_3 = \tau l_3$.
$\{l_1, l_2, l_3\}$ is linearly independent. $\Rightarrow$ $\Gamma_0'' = \Gamma_0 - \Gamma_0' \neq \phi$ where $\Gamma_0' = \Gamma_0 \cap C_0$, and
$\dim H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0''}(H)) \geq 3$, this impossible. For
$\# \Gamma_0'' = 1$ $\Rightarrow$ $\exists$ only 2 linearly independent lines passing Γ_0''.
$\# \Gamma_0'' = 2$. $\Rightarrow$ $\exists$ only one line.
Thus $\Gamma_0 \subset C_0$. $\lrcorner$

If C_0 is a line, then the previous argument shows that at most three points of Γ_0 can fail to lie on C_0. Q.E.D.

Γ Suppose $\#(C_0 \cap \Gamma_0) \leq 4$.
Let $\Gamma_0' = C_0 \cap \Gamma_0$. and $\Gamma_0'' = \Gamma_0 - C_0 \cap \Gamma_0 = \Gamma_0 - \Gamma_0'$.
Let l be the homogeneous polnomial of deg 1 representing C_0. Note that $\# \Gamma_0'' \geq 4$.
Choose a subset $\Lambda_0 \subset \Gamma_0''$ s.t $\# \Lambda_0 = 4$.

① $\dim |\mathcal{J}_{\Lambda_0}(2)| \geq 3$.
$\Rightarrow$ $\exists$ $\tau_1, \tau_2, \tau_3, \tau_4$ linearly independent conics
(i) $|\mathcal{J}_{\Lambda_0}(2)|$ contains a fixed curve l'
$\Rightarrow$ $\tau_1 = l' l_1$, $\tau_2 = l' l_2$. $\tau_3 = l' l_3$ $\tau_4 = l' l_4$