subsection entitled " Proof of the Hodge Theorem II: Global Theory " we want to prove that if φ lies in the Sobolev space $\mathcal{H}_s^{p,q}(M)$ and $\psi \in \mathcal{H}_0^{p,q}(M)$ is a weak solution of the equation

$$\Delta \psi = \varphi,$$

then $\psi \in \mathcal{H}_{s+2}^{p,q}(M)$.

「 P 94 」

Writing $P = \bar\partial + \bar\partial^*$, $P^2 = \Delta$.

「 $P^2 = (\bar\partial + \bar\partial^*)(\bar\partial + \bar\partial^*) = \bar\partial^*\bar\partial + \bar\partial\bar\partial^* = \Delta_{\bar\partial}$ 」

Therefore, we consider the weak solution

$$(*) \qquad P\theta = \eta$$

and show that if $\eta \in \mathcal{H}_s^{p,q}(M)$, then $\theta \in \mathcal{H}_{s+1}^{p,q}(M)$.

「 Once we show the above, then $PP\psi = \varphi \Rightarrow$ $P\psi \in \mathcal{H}_{s+1}^{p,q}(M) \Rightarrow$ Again $\psi \in \mathcal{H}_{s+2}^{p,q}(M)$, done. 」

If $\rho \in C^\infty(M)$, then

$$P(\rho\theta) = P(\rho) \wedge \theta + \rho P(\theta)$$
$$= P(\rho) \wedge \theta + \rho\eta.$$

「 I think $P\theta = \eta$ does not make sense since $P = \bar\partial + \bar\partial^*$, $\bar\partial : A^{p,q} \longrightarrow A^{p,q+1}$ and $\bar\partial^* : A^{p,q} \longrightarrow A^{p,q-1}$. Their whole plan for proving Regularity lemma I is disastrous. Anyway it is worth to reading, and