$\ulcorner$ $\tilde{\tau} = \sum_i (d\varphi_i' - \sum \theta_{ij}'^{*} \wedge \varphi_j') \otimes v_i'$

In our case, $\varphi_i' = dz_i$ $\quad v_i' = \frac{\partial}{\partial z_i}$.

$$\Rightarrow \tau = \sum_i (0 - \sum \theta_{ij}'^{*} \wedge dz_j) \otimes \frac{\partial}{\partial z_i}$$

$$= -\sum_{i,j} \theta_{ij}'^{*} \wedge dz_j \otimes \frac{\partial}{\partial z_i}$$

$$= -\sum_{i,j} \theta_{ji}'^{*} \wedge dz_i \otimes \frac{\partial}{\partial z_j} = \sum_{i,j} \theta_{ij} \wedge dz_i \otimes \frac{\partial}{\partial z_j}$$

$\Uparrow$ (by P76 $\theta_{ji}'^{*} = -\theta_{ij}$)

$$= \sum_{i,j,k} \Gamma_{ik}^{j} dz_k \wedge dz_i \otimes \frac{\partial}{\partial z_j}$$

$$= \frac{1}{2}\left(\sum_{i,j,k} \Gamma_{ik}^{j} dz_k \wedge dz_i \otimes \frac{\partial}{\partial z_j} + \sum_{i,j,k} \Gamma_{ik}^{j} dz_k \wedge dz_i \otimes \frac{\partial}{\partial z_j}\right)$$

$$= \frac{1}{2}\left(\sum_{i,j,k} \Gamma_{ki}^{j} dz_i \wedge dz_k \otimes \frac{\partial}{\partial z_j}\right) + \frac{1}{2}\sum_{i,j,k}(-1)\Gamma_{ik}^{j} dz_i \wedge dz_k \otimes \frac{\partial}{\partial z_j}$$

$$= \frac{1}{2}\sum_{i,j,k} (\Gamma_{ki}^{j} - \Gamma_{ik}^{j}) \frac{\partial}{\partial z_j} \otimes (dz_i \wedge dz_k)$$

$$\iota(v)\cdot\tilde{\tau} = \iota(v)\left(\frac{1}{2}\sum_{i,j,k} \Gamma_{ki}^{j} - \Gamma_{ik}^{j}) \frac{\partial}{\partial z_j} \otimes dz_i \wedge dz_k\right)$$

$$= \frac{1}{2}\sum_{i,j,k} (\Gamma_{ki}^{j} - \Gamma_{ik}^{j}) \frac{\partial}{\partial z_j} \otimes \iota(v)(dz_i \wedge dz_k)$$

$$\iota(v)(dz_i \wedge dz_k) = -v^k dz_i + v^i dz_k.$$

$$\Rightarrow \iota(v)\tau = \frac{1}{2}\sum_{i,j,k} (\Gamma_{ki}^{j} - \Gamma_{ik}^{j}) \frac{\partial}{\partial z_j} \otimes (-v^k dz_i + v^i dz_k)$$

$$\Rightarrow \frac{1}{2}\sum_{i,j,k} (\Gamma_{ki}^{j} - \Gamma_{ik}^{j}) \frac{\partial}{\partial z_j} (-v^k) dz_i$$

$$= -\frac{1}{2}\sum_{i,j,k} (\Gamma_{ik}^{j} - \Gamma_{ki}^{j}) \frac{\partial}{\partial z_j} v^i dz_k \quad \cdots ①$$

$$\frac{1}{2}\sum_{i,j,k} (\Gamma_{ki}^{j} - \Gamma_{ik}^{j}) \frac{\partial}{\partial z_j} \otimes v^i dz_k \quad \cdots ② \quad \Longrightarrow \quad ① + ②$$