$$= (-1)^{\frac{n(n+2)}{2}} \left(\frac{1}{2\pi}\right)^n (n-1)!$$

I can not belive that C_n is the constant in Bochner – Martinelli formula, anyway we can adjust the constant so that we get the desired answer.

$$\partial \log \|v\|^2 = \frac{\partial(v^i \bar{v}^i)}{\langle v, v\rangle} = \frac{\bar{v}^i \frac{\partial v^i}{\partial z_k} dz_k}{\langle v, v\rangle} = \frac{\bar{v}^i dv^i}{\langle v, v\rangle}$$

$$\bar{\partial} \log \|v\|^2 = \frac{\bar{\partial}\langle v^i \bar{v}^i\rangle}{\langle v, v\rangle} = \frac{v^i \frac{\partial \bar{v}^i}{\partial \bar{z}_l} d\bar{z}_l}{\langle v, v\rangle} = \frac{v^i d\bar{v}^i}{\langle v, v\rangle}$$

$$\partial\bar{\partial} \log \|v\|^2 = \partial\left(\frac{(v\ d\bar{v}^i)}{\langle v, v\rangle}\right) = \frac{dv^i \wedge d\bar{v}^i}{\langle v, v\rangle} - \frac{(v^i . d\bar{v}^i) \wedge \bar{v}^j dv^j}{\langle v, v\rangle^2}$$

Again since $\sum \bar{v}^i dv^i \wedge \sum \bar{v}^j dv^j = 0$.

$$\partial \log \|v\|^2 \wedge (\partial\bar{\partial} \log \|v\|^2)^{n-1}$$

$$= \frac{\bar{v}^i dv^i}{\langle v, v\rangle} \wedge \left(\frac{dv^i \wedge d\bar{v}^i}{\langle v, v\rangle} - \frac{v^i d\bar{v}^i \wedge \bar{v}^j dv^j}{\langle v, v\rangle^2}\right)^{n-1}$$

$$= \frac{\bar{v}^i dv^i}{\langle v, v\rangle} \wedge \left(\frac{dv^i \wedge d\bar{v}^i}{\langle v, v\rangle}\right)^{n-1}$$

$$= \frac{1}{\langle v, v\rangle^n} (\bar{v}^1 dv^1 + \cdots + \bar{v}^n dv^n)(dv^1 \wedge d\bar{v}^1 + dv^2 \wedge d\bar{v}^2 + \cdots + dv^n \wedge d\bar{v}^n)^{n-1}$$

$$= \frac{1}{\langle v, v\rangle^n} (-1)^{n-1} (\bar{v}^1 dv^1 + \cdots + \bar{v}^n dv^n)(d\bar{v}^1 \wedge dv^1 + d\bar{v}^2 \wedge dv^2 + \cdots + d\bar{v}^n \wedge dv^n)^{n-1}$$

$$= \frac{(-1)^{n-1}}{\langle v, v\rangle^n} (-1)^{\frac{n(n-1)}{2}} (n-1)! \sum_{i=0}^{n} (-1)^{i-1} \bar{v}^i\, d\bar{v}^1 \wedge \cdots \wedge \widehat{d\bar{v}^i} \wedge \cdots \wedge d\bar{v}^n \wedge dv^1 \wedge \cdots \wedge dv^n.$$