Since M is Kähler, $\exists$ a Kähler metric. $\Rightarrow$
Give the metric on $M - \{x\}$, and on $\tilde{U} = \pi^{-1}(U) = \{(z, l) \in U \times \mathbb{P}^{n-1} \mid z \in l\}$, give the product metric of the Kähler metric and the Fubini-Study metric on $\mathbb{P}^{n-1}$.
$\Rightarrow$ $\tilde{U}$ is Kähler. $\Rightarrow$ $M - \{x\} \cup_{\pi} \tilde{U}$ is Kähler since $\pi : \tilde{U} - E \longrightarrow U - \{x\}$ //
Up to this, I talked nonsense.

"The fact which we have to note is that if $L \to M$ is a positive line bundle, this implies that M is Kähler.
pf). By the assumption, (see p 154) $\exists$ a metric on L s.t $\frac{i}{2\pi} \Theta$ is a positive $(1,1)$-form on M, see the def of p148. $\Rightarrow$ Let $\omega = \frac{i}{2\pi}\Theta$. $\Rightarrow$ Since $d\Theta = 0$, ω is closed. $\Rightarrow$ By p29. ω induces a metric on M which is Kähler. Also, refer to P251~P252 Poor.

Thus since the blow-up $\tilde{M}$ of M has a positive line bundle $\pi^* L^k - E$ for $k \geq k_0$, $\tilde{M}$ is Kähler." //

Corollary. If $\tilde{M} \xrightarrow{\pi} M$ is a finite unbranched covering of compact complex manifolds, then M is algebraic $\Leftrightarrow$ $\tilde{M}$ is.
pf) Clearly, if $L \longrightarrow M$ is positive, then $c_1(\pi^* L) = \pi^* c_1(L)$ implies that $\pi^* L$ is positive.
$\ulcorner$ Since π_* is injective locally, $\pi^* c_1(L) = [\pi^* \omega]$ where ω is a positive form, and $\pi^*\omega$ is positive, see p148 Proposition. //
Conversely, say ω is an integral, positive $(1,1)$-