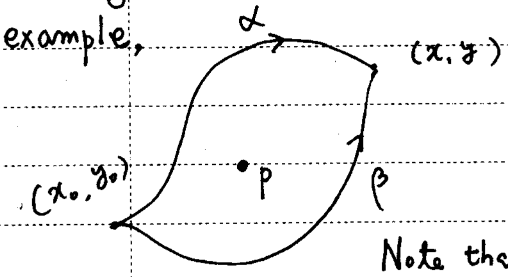
Note moreover that since $\mathbb{P}^1$ is simply connected and dx/y is closed, the only dependence of the integral on the choice of path arises from the residues of dx/y, which are readily calculated.

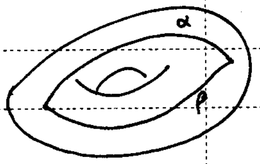
Γ $d\left(\frac{dx}{y}\right) = d\left(\frac{1}{\sqrt{x^2+ax+b}}\right) \wedge dx = 0.$

For example,



$$\int_\alpha \frac{dx}{y} - \int_\beta \frac{dx}{y} = \mathrm{Res}_p\left(\frac{dx}{y}\right)$$

Note that $\frac{dx}{y}$ is meromphic in general unless C is smooth.



$\int_\alpha \omega \neq \int_\beta \omega$ in general.

since the torus is not simply connected.

Remember that $\int_C \omega = 0$ if C bounds a simply connected domain. 95. 2. 22. $\frac{dx}{y}$ is holomorphic on $\mathbb{P}^1$ if C is smooth, by Jell and since $\mathbb{P}^1$ is simply connected, $\frac{dx}{y}$ is closed. $\Rightarrow$ $\frac{dx}{y}$ is exact. $\Rightarrow$ The line integral is indepent of the choice of paths.

The integral (**), on the other hand, is the integral of the form dx/y on the cubic curve $C = (y^2 = x^3 + ax^2 + bx + c)$. Now, if C is smooth then by the genus formula it has genus 1, and hence can not be parametrized by a single meromorphic function; thus no such simple expression as the one given above for (*) is possible for (**). Moreover, C is topologically a torus and therefore not simply connected; so the integral

$$\int_p^q \frac{dx}{y}$$

is well-defined only modulo the periods of dx/y, that