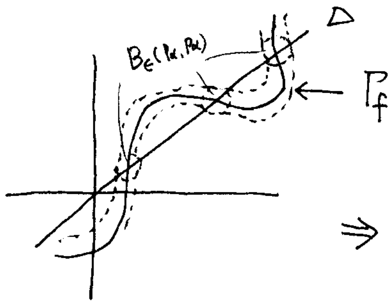
Moreover, $k$ is smooth on $M \times M - \Delta$, so that if we set

$$\varphi = T^{\circ}_{\Delta} - \bar{\partial} k\,;$$

$\varphi$ will be a $\bar{\partial}$-closed current representing $\eta^{\circ}_{\Delta}$, smooth in an open set containing $\Gamma_f$, and equal to $-\bar{\partial} k$ away from $\Delta$.

⌈ Clearly $\bar{\partial}\varphi = \bar{\partial} T^{\circ}_{\Delta} - \bar{\partial}\bar{\partial} k = 0 \Rightarrow \varphi$ is $\bar{\partial}$-closed current. & since $T^{\circ}_{\Delta}$ represents $\eta^{\circ}_{\Delta}$, and $\bar{\partial} k$ is an element of image of $\bar{\partial}$, $\varphi$ represents $\eta^{\circ}_{\Delta}$




Let $\mathcal{U}$ be an open set containing $\bigcup B_{\epsilon}(p_\alpha, p_\alpha)$ & $\Gamma_f$ as left.

$\Rightarrow$ Since $k$ is nonsingular in $B_{\epsilon}(p_\alpha, p_\alpha)$, and $\bar{\partial} k = T^{\circ}_{\Delta}$ (cancelled each other),

$\varphi = T^{\circ}_{\Delta} - \bar{\partial} k$ is smooth in $\mathcal{U}$. In $B_{\epsilon}(p_\alpha, p_\alpha)$, $\varphi = 0$. Away from $\Delta$, since $T^{\circ}_{\Delta}$ is equal to zero, $\varphi = -\bar{\partial} k$. ⌋

Then

$$\begin{aligned} \eta^{\circ}_{\Delta}(\Gamma_f) &= \int_{\Gamma_f} \varphi \\ &= -\int_{\Gamma_f - \bigcup B_{\epsilon}(p_\alpha, p_\alpha)} \bar{\partial} k \\ &= \sum_{\alpha} \int_{\partial(\Gamma_f \cap B_{\epsilon}(p_\alpha, p_\alpha))} k \end{aligned}$$