For $g=4$, $d=2$, if we do this, we can easily guess the general case. Suppose $\omega(P_i)=0$. $\Rightarrow \omega = a\omega_1+b\omega_2+c\omega_3+d\omega_4$.
Assume the first two rows are the linearly indepent set.

$$\Rightarrow \left(\frac{\omega_3}{dz_1}(P_1),\ \frac{\omega_3}{dz_2}(P_2)\right) = x_1\left(\frac{\omega_1}{dz_1},\ \frac{\omega_1}{dz_2}\right) + y_1\left(\frac{\omega_2}{dz_1},\ \frac{\omega_2}{dz_2}\right) \quad \text{--} ①$$

$$\left(\frac{\omega_4}{dz_1}(P_1),\ \frac{\omega_4}{dz_2}(P_2)\right) = x_2\left(\frac{\omega_1}{dz_1},\ \frac{\omega_1}{dz_2}\right) + y_2\left(\frac{\omega_2}{dz_1},\ \frac{\omega_2}{dz_2}\right) \quad \text{---} ②$$

$$\Rightarrow \frac{\omega}{dz_1}(P_1) = \left(a\frac{\omega_1}{dz_1} + b\frac{\omega_2}{dz_1} + c\frac{\omega_3}{dz_1} + d\frac{\omega_4}{dz_1}\right)(P_1) = 0 \quad \text{--} ③$$

$$\frac{\omega}{dz_2}(P_2) = \left(a\frac{\omega_1}{dz_2} + b\frac{\omega_2}{dz_2} + c\frac{\omega_3}{dz_2} + d\frac{\omega_4}{dz_2}\right)(P_2) = 0 \quad \text{--} ④$$

Plug ① in ③ & ② in ④.

$$\Rightarrow \text{We get} \quad (a+cx_1+dx_2)\left(\frac{\omega_1}{dz_1}\right) + (b+cy_1+dy_2)\left(\frac{\omega_2}{dz_1}\right) = 0$$
$$(a+cx_1+dx_2)\left(\frac{\omega_1}{dz_2}\right) + (b+cy_1+dy_2)\left(\frac{\omega_2}{dz_2}\right) = 0$$

$\Rightarrow$ Since $\left(\frac{\omega_1}{dz_1}, \frac{\omega_1}{dz_2}\right)$ & $\left(\frac{\omega_2}{dz_1}, \frac{\omega_2}{dz_2}\right)$ are linearly independent,
$a+cx_1+dx_2 = b+cy_1+dy_2 = 0$

$$\Rightarrow \omega = c(\omega_3 - x_1\omega_1 - y_1\omega_2) + d(\omega_4 - x_2\omega_1 - y_2\omega_2)$$

$H^0(S, \Omega^1(-D)) \longleftrightarrow$ Set of holomorphic differentials vanishing at P_λ for all λ.

$\ni$

$\sigma \longleftrightarrow \frac{\sigma}{s_0}$

where $(s_0=0) = -D$.

$\Rightarrow$ We got what we wanted to prove.

$h^0(D) = \dim H^0(S, \mathcal{O}(D)) = \dim V + 1 = \dim(\ker\Psi)+1$
$= d - \operatorname{rank}\Psi + 1 = d - g + h^0(K-D) + 1$ since