

$$E_2 \rightarrow E_1 \rightarrow E_0 \rightarrow M \rightarrow 0$$

$$\downarrow \Phi_2 \quad \downarrow \Phi_1 \quad \downarrow \text{id} \quad \downarrow \Phi_0 \quad \downarrow \text{id}$$

$$E'_2 \rightarrow E'_1 \rightarrow E'_0 \rightarrow M \rightarrow 0$$

$$\downarrow P_2 \quad \downarrow P_1 \quad \downarrow P_0 \quad \downarrow \text{id}$$

$$E_2 \rightarrow E_1 \rightarrow E_0 \rightarrow M \rightarrow 0$$

$$\text{Ext}_0^n(M, N) \xrightarrow{\Phi^*} \text{Ext}_0^n(M, N) \xrightarrow{P^*} \text{Ext}_0^n(M, N)$$

id^*

By 3, $(P \circ \Phi)^* = \text{id}^*$, since clearly $\Phi^* \circ P^* = \text{id}^*$

$$E_2 \rightarrow E_1 \rightarrow E_0 \rightarrow M \rightarrow 0$$

$$\downarrow \text{id} \quad \downarrow \text{id} \quad \downarrow \text{id} \quad \downarrow \text{id}$$

$$E_2 \rightarrow E_1 \rightarrow E_0 \rightarrow M \rightarrow 0$$

\Rightarrow Similarly, $(\Phi \circ P)^* = \text{id}^* = P^* \circ \Phi^*$

$\Rightarrow \Phi^*$ is isomorphism $\Rightarrow \text{Ext}_0^n(M, N) \xrightarrow{\Phi^*} \text{Ext}_0^n(M, N)$

This proves that Ext is well-defined independently of the projective resolution $E(M)$. We can prove similarly $\text{Ext}_0^n(M, N') \rightarrow \text{Ext}_0^n(M, N') \rightarrow \text{Ext}_0^n(M, N)$

Next, we note that the definitions of Ext and Tor are not symmetric in M and N . For Tor this may be rectified as follows: Take projective resolutions $E(M)$