$$v_1(x) = a_{11}(x)\, D_1 + a_{21}(x)\, D_2 + \cdots + a_{n1}(x)\, D_n$$
$$v_2(x) = a_{12}(x)\, D_1 + a_{22}(x)\, D_2 + \cdots + a_{n2}(x)\, D_n$$
$$\vdots \qquad\qquad \vdots$$
$$v_n(x) = a_{1n}(x)\, D_1 + a_{2n}(x)\, D_2 + \cdots + a_{nn}(x)\, D_n.$$

For simplicity, $n=2$

$$v_1(x) = a_{11}(x) D_1 + a_{21}(x) D_2 \qquad a_{ij}(x)\text{'s are } C^\infty \text{ on } U$$
$$v_2(x) = a_{12}(x) D_1 + a_{22}(x) D_2 \qquad \Rightarrow |a_{ij}(x)| < k \text{ for } {}^\forall x \in V \subset \overline{V}$$

$$v_1(x)(\varphi(x)) = a_{11}(x)\, D_1\varphi + a_{21}(x)\, D_2\varphi$$
$$v_2(x)\varphi = a_{12}(x)\, D_1\varphi + a_{22}(x)\, D_2\varphi$$

$$\Rightarrow |v_1(x)\varphi| = |a_{11} D_1\varphi + a_{21} D_2\varphi| \leq |a_{11}|\,|D_1\varphi| + |a_{21}|\,|D_2\varphi|$$
$$\leq k\,|D_1\varphi| + k\,|D_2\varphi| = k\,(|D_1\varphi| + |D_2(\varphi)|)$$

In the same way, we have $|v_2(x)\varphi| \leq k\,(|D_1\varphi| + |D_2\varphi|)$

$$\Rightarrow \sum_i |v_i(x)\varphi|^2 \leq (2k)^2 (|D_1\varphi| + |D_2\varphi|)^2 \leq (2k)^2\, 2\,(|D_1\varphi|^2 + |D_2\varphi|^2)$$

$$\Rightarrow |\varphi|^2 + \sum_i |v_i(x)\varphi|^2 \leq |\varphi|^2 + 8k^2(|D_1\varphi|^2 + |D_2\varphi|^2)$$
$$\leq (8k^2+1)\left(|\varphi|^2 + \sum_{[\alpha]=1} |D^\alpha\varphi|^2\right)$$

$$\Rightarrow \int_V \rho(x) \left\{ |\varphi(x)|^2 + \sum_i |v_i(x)\cdot\varphi(x)|^2 \right\} dx$$
$$\leq K \int_V \rho(x) \left\{ \sum_{[\alpha]\leq 1} |D^\alpha\varphi(x)|^2 \right\} dx$$
$$\leq KM \int_V \sum_{[\alpha]\leq 1} |D^\alpha\varphi(x)|^2\, dx = KM \sum_{[\alpha]\leq 1} \int_V |D^\alpha\varphi(x)|^2\, dx$$