$$\varphi_u \varphi_v^{-1} : T_z\mathbb{C}^n \longrightarrow T_z\mathbb{C}^n$$

$$(\varphi_u \varphi_v^{-1})_* \left(\frac{\partial}{\partial z_i}\right) = \sum \circ \frac{\partial}{\partial w_i}$$ see p18.

and so we see that $T'(M)$ has naturally the structure of a holomorphic vector bundle. ($\because$ $J_{\mathbb{C}}(\varphi_u \varphi_v^{-1}) = \left(\frac{\partial w_i}{\partial z_j}\right)$ is holomorphic)

$$\mathbb{C}^2 \longrightarrow \mathbb{C}^2$$
$$(1, -i) \longmapsto (1, 0)$$
$$(1, i) \longmapsto (0, 1).$$

$\Rightarrow$ ~~$\varphi_u : U$~~

$$\varphi_u : \pi^{-1}(U) \longrightarrow U \times \mathbb{C}^{2n} \qquad (1, 0 \cdots i \cdots 0) \text{ (i at position } n+1)$$
$$\downarrow \cong \qquad \qquad \downarrow$$
$$U \times \mathbb{C}^{2n} \qquad (1, 0 \cdots 0)$$

let φ_u just φ_u.

$$(1, 0 \cdots 0, i\, 0 \cdots 0) \longmapsto (0 \cdots 0 \overset{n+1}{1} 0 \cdots 0)$$
$$(0\,1\,0 \cdots 0\,0\,i\,0 \cdots 0) \longmapsto (0\,1\,0 \cdots \cdots 0) \qquad (0, 1, 0 \cdots 0\,0\,i\,0 \cdots 0) \mapsto (0\,0 \cdots 1\,0)$$

$$\Rightarrow \quad \varphi_u\big|_{T_x'(M)} : \pi^{-1}(U)\big|_{T_x'(M)} \longrightarrow U \times \mathbb{C}^n \subset \mathbb{C}^{2n}.$$

so that $T'(M)$ is a subbundle of $T(M)$.

Similarly, we define

$T^*(M) = T(M)^*$: complex cotangent bundle.
$T^{*\prime}(M) =$ holomorphic cotangent, $\quad T^{*\prime\prime}(M) =$ antiholomorphic cotangent bundle.

$$T^{*(p,q)}(M) = \Lambda^p T^{*\prime}(M) \otimes \Lambda^q T^{*\prime\prime}(M)$$

The tensor, symmetric and exterior product of the holomorphic and complexified tangent and cotangent bundles are called tensor bundles.