



At (x_0, y_0) , assume $f(x, k(x)) = 0$ locally.

$$\Rightarrow \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) \cdot (1, k'(x)) = 0$$

Similarly, assume $g(x, k(x)) = 0$ locally

$$\Rightarrow \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \right) \cdot (1, k'(x)) = 0.$$

$$\Rightarrow \text{Since } k_1'(x_0) \neq k_2'(x_0), \quad \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = A = (a_1, a_2)$$

& $\left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \right) = B = (b_1, b_2)$ are linearly independent.

Consider $f(x, k(x)) + \gamma g(x, k(x)) = 0$.

$$\Rightarrow \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) (1, k'(x)) + \gamma \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \right) (1, k'(x)) = 0$$

\Rightarrow

$$(a_1, a_2) \cdot (1, k'(x)) + \gamma (b_1, b_2) \cdot (1, k'(x)) = 0$$

$$\Rightarrow a_1 + a_2 k'(x_0) + \gamma (b_1 + b_2 k'(x_0)) = 0$$

$$\Rightarrow \gamma = - \frac{a_1 + a_2 k'(x_0)}{b_1 + b_2 k'(x_0)}$$

If $b_1 + b_2 k'(x_0) = 0$, consider $\gamma f(x, k(x)) + g(x, k(x)) = 0$.