"Def: X locally convex if $\exists$ a local base $\mathcal{B}$ whose members are convex

" F-space if its topology τ is induced by a complete invariant metric d.

" Fréchet space if X is locally convex F-space

X has Heine-Borel property if every closed and bounded subset of X is compact.

Here, local base means a local base at 0.

A metric d on X is called invariant if $d(x+z, y+z) = d(x, y)$. "

See P8.

To prove the statement above, choose compact sets K_i ($i=1,2,\cdots$) such that K_i lies in the interior of K_{i+1} and $\Omega = \bigcup K_i$. Define seminorms P_N on $C^\infty(\Omega)$, $N = 1, 2, \cdots$ by setting

(4). $$P_N(f) = \max\{\, |D^\alpha f(x)| : x \in K_N,\ |\alpha| = [\alpha] \leq N \,\}.$$

They define a metrizable locally convex topology on $C^\infty(\Omega)$ by Th 1.37 and remark (c) of 1.38, See P26 ~ P27.

「 The metric d is given by

$$d(f, g) = \sum_{N=1}^{\infty} \frac{2^{-N} P_N(f-g)}{1 + P_N(f-g)}$$ see P27 ~ P28 」

For each $x \in \Omega$, the functional $f \longmapsto f(x)$ is continuous in this topology.

「 Let N_0 be the number s.t $x \in K_{N_0}$.

To show the functional $f \longmapsto f(x)$ is continuous,