$\ulcorner$ $i\langle[\Lambda,\Theta]\eta,\eta\rangle = i\langle\Lambda\Theta\eta,\eta\rangle - i\langle\Theta\Lambda\eta,\eta\rangle$
$= \langle D'\eta, D'\eta\rangle + \langle D''^*\eta, D''^*\eta\rangle \geq 0.$ $\lrcorner$

But $\Theta = (2\pi/i)\,L$, and so

$$2i\langle[\Lambda,\Theta]\eta,\eta\rangle = 4\pi\langle[\Lambda,L]\eta,\eta\rangle = 4\pi(n-p-q)\|\eta\|^2 \geq 0.$$

Thus, $p+q>n \Rightarrow \eta = 0$. Q.E.D.

$\ulcorner$ $i\langle[\Lambda,\Theta]\eta,\eta\rangle = 4\pi\langle[\Lambda,L]\eta,\eta\rangle$
$= \langle 4\pi(n-p-q)\eta,\eta\rangle = 4\pi(n-p-q)\|\eta\|^2$
since $[\Lambda,L] = (n-p-q)$ by p121.

Λ, L are algebraic operators, and we have only to show that $[\Lambda,L] = n-p-q$ at some arbitrary z_0.

Don't forget that Λ, L act only on forms, not on sections of L. $\lrcorner$

As was suggested when we first introduced cohomology, the groups $H^q(M, \Omega^p(E))$ $(q\geq 1)$ most frequently arise as obstructions to globally solving analytic problems – this is specially true for $q=1$ as in the Mittag-Leffler problem, but once one admits H^1's, then all the rest become involved. The Kodaira vanishing theorem – together with its variants to be discussed later – is the best general method for eliminating cohomology.

Dualizing the Kodaira vanishing theorem, we obtain:

$H^q(M, \Omega^p(L)) = 0$ for $p+q<n$ in case $L\to M$ is a negative line bundle.