$\Rightarrow$ By the Jacobi relation

$$\sum_\nu \frac{g(P_\nu)}{J_f(P_\nu)} = 0, \quad \text{where } (g=0) = E, \qquad m+n-3 \le \Sigma d_i - (2+1) = m+n-3$$

$\Rightarrow$ $g(P_\nu)=0$, So if E passes all P_ν's except P_1

$$\sum_\nu \frac{g(P_\nu)}{J_f(P_\nu)} = \frac{g(P_1)}{J_f(P_1)} + \frac{g(P_2)}{J_f(P_2)} + \cdots + \frac{g(P_{mn})}{J_f(P_{mn})} = \frac{g(P_1)}{J_f(P_1)} = 0 \Rightarrow$$

$g(P_1) \neq 0$ gives a contradiction. $\lrcorner$

It is clear that the stronger relation (*) gives a more general statement than the Cayley-Bacharach theorem when C and D may not have transverse intersections. Rather than attempt to formalize this, we shall usually go ahead and use the Cayley-Bacharach theorem in degenerate cases where the proof will be an immediate consequence of (*).
To illustrate, we give an example of a degenerate case:

Suppose that the curves C and D above have intersection

$$C \cdot D = \sum_\nu m_\nu P_\nu,$$

where all the points P_ν are smooth points of C. If E is a curve of degree $m+n-3$ such that for some ν_0,

$$(C\cdot E)_{P_\nu} \ge m_\nu, \qquad \nu \neq \nu_0,$$
$$(C\cdot E)_{P_{\nu_0}} \ge m_{\nu_0} - 1,$$

then

$$(C\cdot E)_{P_{\nu_0}} \ge m_{\nu_0} .$$