$$H^r(M.\mathbb{C}) = \bigoplus_{p+q=r} H^{p.q}(M), \qquad \text{and}$$

by Dolbeault

$$H^{p.q}(M) = H^{p.q}_{\bar\partial}(M) \cong H^q(M.\Omega^p_M).$$

⌈ Universal Coefficient Theorem. $\Rightarrow$ $H^q(M.\mathbb{Q})\otimes\mathbb{C} \cong H^q(M.\mathbb{C})$.
or by elementary way. $\mathbb{C}\otimes_{\mathbb{Q}}\mathbb{Q} = \mathbb{C}$. ⌋

The same holding for V, it is sufficient to prove that the map

$$H^p(M.\Omega^q_M) \longrightarrow H^p(V.\Omega^q_V)$$

is an isomorphism for $p+q \leqslant n-2$, and injective for $p+q = n-1$.

⌈

$$H^r(M.\mathbb{C}) = \bigoplus H^{p.q}(M) = \bigoplus H^q(M.\Omega^p_M)$$
$$H^r(V.\mathbb{C}) = \bigoplus H^{p.q}(V) = \bigoplus H^q(V.\Omega^p_V)$$

$\Rightarrow$ If $H^q(M.\Omega^p_M) \longrightarrow H^q(V.\Omega^p_V)$ satisfies the condition, $H^r(M.\mathbb{C}) \longrightarrow H^r(V.\mathbb{C})$ satisfies the same condition. ⌋

To see this, we factor the restriction $\Omega^p_M \longrightarrow \Omega^p_V$ by

$$\Omega^p_M \xrightarrow{r} \Omega^p_M|_V \xrightarrow{i} \Omega^p_V,$$

where $\Omega^p_M|_V$ is the sheaf of sections of $(\Lambda^p T^*M)|_V$ — considered either as a sheaf on V or, by extension, as a sheaf on M — r is the restriction map, and i is the pullback map induced by the natural projection $(\Lambda^p T^*M)|_V \longrightarrow \Lambda^p T^*V'$.

⌈ i is the pullback map induced by the natural inclusion. ⌋