By (b), $\{\phi_i\}$ is then also a Cauchy sequence relative to τ_K. This proves (e).

$\ulcorner$ V is a local base elmt for τ_K. $\Rightarrow$ $V = \mathcal{D}_K \cap E$, $E \in \mathcal{D}(\Omega)$.
$\Rightarrow \exists\ W \subset E$, $W \ni 0$. $\Rightarrow$ $\exists\ N$ s.t if $i, j > N$, $\phi_i - \phi_j \in W$
(W: local base elmt)
$\Rightarrow$ $\phi_i - \phi_j \in W \cap \mathcal{D}_K$ since $\{\phi_i\} \subset \mathcal{D}_K$. $\lrcorner$

Statement (f) is just a restatement of (e).
Finally, (g) follows from (b), (e), and the completenes of $\mathcal{D}_K$. (Recall that $\mathcal{D}_K$ is a Fréchet space.) ///

$\ulcorner$ $\{\phi_i\}$ Cauchy sequence in $\mathcal{D}(\Omega)$. $\Rightarrow$ By (e), $\exists\ \mathcal{D}_K$ s.t $\{\phi_i\} \subset \mathcal{D}_K$ and Cauchy sequence in $\mathcal{D}_K$. $\mathcal{D}_K$ is complete. See P 33 bottom line. $\lrcorner$

6.6 Theorem. Suppose Λ is a linear mapping of $\mathcal{D}(\Omega)$ into a locally convex space Y. Then each of the following four properties implies the others:

(a) Λ is continuous.
(b) Λ is bounded.
(c) If $\phi_i \to 0$ in $\mathcal{D}(\Omega)$, then $\Lambda\phi_i \to 0$ in Y
(d) The restrictions of Λ to every $\mathcal{D}_K \subset \mathcal{D}(\Omega)$ are continuous.

proof. The implication (a) $\longrightarrow$ (b) is contained in Th. 1.32.
E bounded in X $\Rightarrow$ Since Λ is continuous, for a nbd W of 0, $\exists$ a nbd V of 0 s.t $\Lambda(V) \subset W$. Since E is bounded, $E \subset tV$ for some t. $\Lambda(E) \subset \Lambda(tV) = t\Lambda(V) \subset tW$ $\Rightarrow$ $\Lambda(E)$ is bounded in Y.
Assume Λ is bounded and $\phi_i \to 0$ in $\mathcal{D}(\Omega)$. By Th. 6.5,