image of

$$H^p_{DR}(M) \longrightarrow H^p_{DR}(\mathcal{U}).$$

Equivalently, a differential of the second kind is a closed meromorphic p-form on M, holomorphic on $M-D$, which can be extended, up to an exact form on $M-D$, to a C^∞ closed form on M. We let P_p be the dimension of the space

$$\frac{(p\text{-forms of the second kind})}{d(\text{meromorphic } (p-1)\text{-forms})}.$$

⌈ ($\Downarrow$) ($\Rightarrow$) φ closed meromorphic p-form on M and holomorphic in $\mathcal{U} = M-D$.

$$\begin{array}{ccc} H^p_{DR}(M) & \longrightarrow & H^p_{DR}(\mathcal{U}) \\ \Psi & & \Psi \\ [\psi] & \longmapsto & [\varphi] \\ & & \| \\ & & [\psi|_{\mathcal{U}}] \end{array}$$

$\Rightarrow$ $\psi|_{\mathcal{U}} - \varphi = d\eta$, η C^∞ form on $\mathcal{U} = M-D$

$\Rightarrow$ $\psi|_{\mathcal{U}} = \varphi + d\eta$

$\Rightarrow$ φ can be extended to a C^∞ closed form on M, up to an exact form $\underset{d\eta}{\wedge}$ on $\mathcal{U} = M-D$.

($\Uparrow$) ($\Leftarrow$). Suppose φ closed meromorphic p-form on M and holomorphic on $M-D$.

$\psi|_{\mathcal{U}} = \varphi + d\eta$ $\qquad$ ψ C^∞ closed form on M.

$\Rightarrow$ $\qquad$ $d\eta$ exact C^∞ form on $\mathcal{U}$.

$$\begin{array}{ccc} H^p_{DR}(M) & \longrightarrow & H^p_{DR}(\mathcal{U}) \\ \Psi & & \Psi \\ [\psi] & \longmapsto & [\psi|_{\mathcal{U}}] = [\varphi + d\eta] = [\varphi] \end{array}$$