embedding,
$$\begin{array}{ccc} [\tilde{C}] & \longrightarrow & [H] \\ \downarrow & & \downarrow \\ \tilde{\mathbb{P}}^2 & \xrightarrow{\iota_{\tilde{C}}} & \mathbb{P}^3 \end{array}$$

by P177. $\deg \iota_{\tilde{C}}(\tilde{\mathbb{P}}^2) = \tilde{C}\cdot\tilde{C} = 3$. $\tilde{C} \in |\tilde{C}|$. $\Rightarrow \iota_{\tilde{C}}(\tilde{\mathbb{P}}^2) = S$ is a cubic.

3. $p \neq q \in \tilde{\mathbb{P}}^2$ Since p, q impose linearly independent conditions on cubics, $[(\tilde{\sigma}_1(p), \tilde{\sigma}_2(p), \tilde{\sigma}_3(p), \tilde{\sigma}_4(p))] \neq [(\tilde{\sigma}_1(q), \tilde{\sigma}_2(q), \tilde{\sigma}_3(q), \tilde{\sigma}_4(q))]$

where $H^0(\mathbb{P}^2, \mathcal{J}_P(3)) = \langle \sigma_1, \sigma_2, \sigma_3, \sigma_4 \rangle$ $P = \{P_1, P_2, \cdots P_6\}$

$H^0(\tilde{\mathbb{P}}^2, \mathcal{O}([\tilde{C}])) = \langle \tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\sigma}_3, \tilde{\sigma}_4 \rangle$, $\tilde{\sigma}_i$ proper transform of σ_i. Note that since $P_1, \cdots P_6$ impose 6 linearly independent conditions on cubics, we have $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ passing $P_1 \cdots P_6$

$\dim H^0(\mathbb{P}^2, \mathcal{O}(3)) = 10 - 6 = 4$. More precisely, let $H^0(\mathbb{P}^2, \mathcal{O}(3)) = \langle \sigma_1', \cdots \sigma_{10}' \rangle$

$$a_1\sigma_1' + \cdots + a_{10}\sigma_{10}' = 0 \quad \text{at } P_1$$
$$\vdots$$
$$a_1\sigma_1' + \cdots + a_{10}\sigma_{10}' = 0 \quad \text{at } P_6.$$

$\Rightarrow \{(a_1 \cdots a_{10})\}$ has dimension 4.

$\Rightarrow \langle (a_{11}, \cdots a_{1,10}), \cdots (a_{41} \cdots a_{4,10}) \rangle = \{(a_1, \cdots a_{10})\}$.

$\Rightarrow$ Let $\sigma_1 = a_{11}\sigma_1' + \cdots + a_{1.10}\sigma_{10}'$
$$\vdots$$
$$\sigma_4 = a_{41}\sigma_1' + \cdots + a_{4.10}\sigma_{10}'.$$

Now p, q give more restrictions on $\langle \sigma_1 \cdots \sigma_4 \rangle$.
Let U be a small neighborhood of p, so that $\tilde{\sigma}_1, \cdots \tilde{\sigma}_4$ are expressed as holomorphic functions $f_1 \cdots f_4$