s.t $\eta \in \mathcal{D}'^{1,0}$. $\bar\partial \eta = T$. $\Rightarrow$ Take $\frac{1}{\sqrt{-1}}\eta$. ⌟

The current $\partial\eta$ is of type $(2,0)$, and $\bar\partial(\partial\eta) = -\partial\bar\partial\eta = (1/\sqrt{-1})\partial T = 0$. By the regularity theorem for the $\bar\partial$-operator, $\partial\eta$ is a closed holomorphic 2-form, and so by the d-Poincaré lemma for holomorphic forms $\partial\eta = d\xi$ for a holomorphic 1-form ξ.

⌜ η is a holomorphic one form by the argument above. $\Rightarrow \partial\eta$ is a holomorphic 2-form. $\Rightarrow$ Since $\bar\partial\partial\eta = \partial\partial\eta = 0$, $\partial\eta$ is closed. $\Rightarrow d\partial\eta = 0$.

$$0 \to \mathbb{C} \to C^\infty \xrightarrow{d} \mathcal{A}^1 \xrightarrow{d} \mathcal{A}^2 \longrightarrow$$

$\Rightarrow \exists\, \xi \in \mathcal{A}^1$ s.t $d\xi = \partial\eta$. $\Rightarrow \xi$ should be a holomorphic 1-form. ⌟

Then $T = -\sqrt{-1}\,\bar\partial\eta'$, where $\eta' = \eta - \xi$ satisfies $\partial\eta' = 0$.

⌜ $T = -\sqrt{-1}\,\bar\partial\eta = -\sqrt{-1}\,\bar\partial(\eta' + \xi) = -\sqrt{-1}(\bar\partial\eta' + \bar\partial\xi)$
$= -\sqrt{-1}\,\bar\partial\xi + (-\sqrt{-1})\,\bar\partial\eta' = -\sqrt{-1}\,\bar\partial\eta'$ since $\bar\partial\xi = 0$ ($\because\ \xi$ is holomorphic)

$\partial\eta' = \partial\eta - \partial\xi = \partial\eta - d\xi = 0$. ⌟

Now, by the ∂-Poincaré lemma, $\eta' = \partial\gamma$ for some distribution γ; $\varphi = \frac{1}{2}(\gamma + \bar\gamma)$ is then a real distribution satisfying $\sqrt{-1}\,\partial\bar\partial\varphi = T$.