

pf). For example, assume V has dimension ≥ 2 in \mathbb{P}^n . \Rightarrow As in the proof of Stokes' Theorem for Analytic Varieties, P33, let V_1 be the singular locus of V , V_2 the singular locus of V_1 , ... and so on. $\Rightarrow V^* = V - V_1$, $V_1^* = V_1 - V_2$, ..., $V_2^* = V_2 - V_3$

Assume $\dim V_1^* = 1$, $\dim V_2^* = 0$

$\Rightarrow V_2^*$ is a set of discrete points.

$$V_3^* = \emptyset$$

$\Rightarrow \{H \subset \mathbb{P}^n \mid H \text{ hyperplane, } H \cap V_2^* \neq \emptyset\} = A_2$ is open and dense in $G(n-1, n)$.

$\Rightarrow \{H \subset \mathbb{P}^n \mid H \text{ hyperplane, } H \cap V_1^* = \text{a set of discrete points, and } H \text{ meets } V_1^* \text{ transversely}\} = A_1$

is open and dense in $G(n-1, n)$ by Bertini Theorem

$\Rightarrow \{H \subset \mathbb{P}^n \mid H \text{ hyperplane, } H \text{ meets } V^* \text{ transversely}\} = A_0$ is open and dense in $G(n-1, n)$, by Bertini.

$\Rightarrow A_0 \cap A_1 \cap A_2$ is open and dense in $G(n-1, n)$.

\Rightarrow For a generic hyperplane H , $H \cap V^* = (H \cap V)^*$ (mod 0-dimensional variety). More precisely,

$H \cap V^* = \text{smooth one-dimensional manifold}$
and $(H \cap V)^* - (H \cap V^*) = \text{set of discrete points.}$

\Rightarrow By using the argument above, for a generic hyperplane H , $H \cap V^* \cap H$ is a smooth 0-dimensional manifold & $H \cap H$ meets V^* transversely.

Note that $H \cap H$ does not intersect V_1 , i.e., a generic \mathbb{P}^2 intersect V^* transversely without meeting V_1 .