

For any two points $x \neq x' \in L$, let $p = l_x \cap l_{x'}$ be the point of intersection of the corresponding lines and $h = \overline{l_x, l_{x'}}$ the plane they span; the line $\sigma_{2,1}(p, h)$ in G then contains x and x' , and so equals L .

$\Gamma \quad \overline{x, x'} = L \subset G \Rightarrow$ By the result above,

$$l_x \cap l_{x'} \neq \emptyset \Rightarrow p = l_x \cap l_{x'}$$

Since $\sigma_{2,1}(p, h) = \{ \text{all lines passing } p \text{ and contained in } h = \overline{l_x, l_{x'}} \}$, $\sigma_{2,1}(p, h)$ is a line, and $\sigma_{2,1}(p, h) \ni x, x'$, $\sigma_{2,1}(p, h) = \overline{x, x'} = L$.

\sqcup

Finally, to see that

Every 2-plane $V_2 \subset \mathbb{P}^5$ contained in G is a Schubert cycle $\sigma_2(p)$ or $\sigma_{0,1}(h)$.

Observe that for any point $x \in V_2$ the tangent plane section $T_x(G) \cap G$ contains V_2 ; thus for x_1, x_2, x_3 any three noncollinear points of V_2 ,

$$V_2 \subset G \cap T_{x_1}(G) \cap T_{x_2}(G) \cap T_{x_3}(G) = \{ x \in G : l_x \cap l_{x'} \neq \emptyset, i = 1, 2, 3 \}.$$

Γ By the result above, $T_x(G) \cap G$ is the