Recall that on a general oriented compact real manifold X of dimension $2k$, we have a bilinear form on $H^k(X, \mathbb{R}) = H^k_{DR}(X)$ defined by

$$\underline{Q}(\eta, \zeta) = \int \eta \wedge \zeta;$$

by Poincare duality $\underline{Q}$ is nondegenerate. If k is even, $\underline{Q}$ is symmetric, and we can associate to X as a topological invariant the signature of Q, defined as

the number of positive eigenvalues minus the # of negative eigenvalues in a matrix representation $\underline{Q}$. The signature of $\underline{Q}$ is called the index $I(X)$ of the manifold X. Of course, if M is a compact Kähler manifold of dim $2n$ then $Q = \underline{Q}$ on $H^{2n}(M, \mathbb{R})$, and we may use the bilinear relations to compute the index of M:

Γ
$$Q : H^{2n}(M) \times H^{2n} \longrightarrow \mathbb{C}$$

$$\underline{Q}(\zeta, \eta) = \int \zeta \wedge \eta \wedge \omega \quad \Rightarrow \text{Obvious}$$ ⅃

$$H^{2n}(M) = \bigoplus L^k P^{2(n-k)}(M)$$
$$= \bigoplus_{\substack{p+q \equiv 0 (2) \\ \leq 2n}} L^{n-(p+q)/2} P^{p,q}(M).$$

Γ $p+q = 2(n-k) \Rightarrow k = n - \frac{p+q}{2}$.

$$\bigoplus_k L^k P^{2(n-k)}(M) = \bigoplus_k L^k\Big(\bigoplus_{p+q=2(n-k)} P^{p,q}(M)\Big) = \bigoplus_{\substack{p+q=2(n-k)\\ k=0\cdots n}} L^k P^{p,q}(M)$$
$$= \bigoplus_{\substack{p+q=2(n-k)\\ k=0\cdots n}} L^{n-\frac{p+q}{2}} P^{p,q}(M) = \bigoplus_{\substack{p+q\equiv 0(2)\\ \leq 2n}} L^{n-\frac{p+q}{2}} P^{p,q}(M).$$ ⅃