$\ulcorner$ Let $\langle \eta \rangle = H^0(M, \mathcal{O}(D))$. Suppose $\varphi/\eta^2 = c\, d(\frac{1}{\eta})$, where $\varphi \in H^0(M, \Omega^1(2D))$.
$\Rightarrow$ $\varphi = c\eta^2 d(\frac{1}{\eta})$. $\Rightarrow$ φ is non-vanishing 1-form over M with values in $[2D]$. $\Rightarrow$ $[K+2D]$ is trivial, $\Rightarrow \deg(K+2D) = 2g-2+2g = 4g-2 = 0$
$\Rightarrow$ $g = \frac{1}{2}$. $\Rightarrow$ contradiction. Thus

$$\left\{\begin{array}{l}\text{1-forms } \varphi \text{ having} \\ \text{no residues and} \\ \text{polar divisor } 2D\end{array}\right\} \cong \frac{\{\text{1-forms of the second kind}\}}{\{\text{exact forms}\}}$$

$\lrcorner$

We turn now to the case $p=2$. To first explain how the relation

$$\rho_2 = b_2 - \rho$$

was used classically, we refer to the exact sequence

$$\mathbb{H}^1(\Omega(*)) \xrightarrow{R} H^0(\bigoplus_{D\,\mathrm{irr}} \mathbb{C}_D) \xrightarrow{i} H^2(M, \mathbb{C}),$$

which appeared in the proof above. We may interpret it in the following manner:

If D is a divisor on M with fundamental class $\eta_D \in H^2(M, \mathbb{Z})$, then η_D is a torsion element if and only if there exists a closed, meromorphic 1-form φ whose residue $R(\varphi) = D$.
$\ulcorner$ Clear from the exact sequence $\lrcorner$