$$H^q(M, \Omega^p_M|_V) \overset{?}{=} H^q(V, \Omega^p_M|_V)$$

Let $\underline{U} = \{U_\alpha\}$ open covering of M. $\Rightarrow$ $\underline{U} \cap V = \{U_\alpha \cap V\}$ open covering of V.

$$C^q(\underline{U}, \Omega^p_M|_V) = \prod_{\alpha_0 \cdots \alpha_p} \Omega^p_M|_V (U_{\alpha_0} \cap \cdots \cap U_{\alpha_p})$$

$$\Omega^p_M|_V (U_{\alpha_0} \cap \cdots \cap U_{\alpha_p}) = \Omega^p_M|_V ((U_{\alpha_0} \cap V) \cap \cdots \cap (U_{\alpha_p} \cap V))$$

$$\Rightarrow \quad C^p(\underline{U}, \Omega^p_M|_V) = C^p(\underline{U} \cap V, \Omega^p_M|_V)$$

$$\Rightarrow \quad H^q(M, \Omega^p_M|_V) = H^q(V, \Omega^p_M|_V).$$ by Greenberg P211 (25.11)

Given an open covering $\{O_\beta\}$ of V, $\{O'_\beta\} \cup \{M-V\}$ is an open covering of M, where $O'_\beta \cap V = O_\beta$, O'_β open in M.

$\Rightarrow$ We can give a map from $C^p(\underline{O}, \mathcal{F})$ to $C^p(\underline{O}', \mathcal{F})$, more precisely, $C^p(\underline{O}, \mathcal{F}) = C^p(\underline{O}', \mathcal{F})$.

But if V is not closed, $\exists$ a lot of choices of $\{O'\}$ which do not make $C^p(\underline{O}', \mathcal{F}) = C^p(\underline{O}, \mathcal{F})$.

E $M = \mathbb{C}$, $V = \mathbb{C} - \{0\}$. $\mathcal{F}$ is constat sheaf Z.

$\Rightarrow$ $H^1(M, Z) = 0 \neq H^1(\mathbb{C}-\{0\}, Z) = Z$.

this is not an extension of the constat sheaf Z.
$\mathcal{F}(U) = Z$ for any open set. $\Rightarrow$ This does not make sense.

This can not be an counterexample.
In case $M = \mathbb{C}$. $V = \mathbb{C} - \mathring{D}^2$.
$\mathcal{F}$: constat sheaf Z. $\Rightarrow$ The extended sheaf