$f : U \longrightarrow R^n$. and f has a compact support in U.

$$\|f(x)\|^2 = \sum |f_\alpha(x)|^2$$

$$\nabla f(x) = (\nabla f_\alpha) = (df_\alpha + \sum_\beta \theta_{\beta\alpha} f_\beta)$$

Again, for simplicity, $n = 2$

$$\nabla f(x) = (df_1 + \sum \theta_{\beta 1} f_\beta, \quad df_2 + \sum \theta_{\beta 2} f_\beta)$$

In $E|_U$,
$$\nabla f = \nabla(f_1 e_1 + f_2 e_2) = df_1 \otimes e_1 + f_\beta \theta_{\beta 1} \otimes e_1 + df_2 \otimes e_2 + f_\beta \theta_{\beta 2} \otimes e_2$$
$$= df_1 \otimes e_1 + f_\beta \theta_{\beta 1} \otimes e_1 + df_2 \otimes e_2 + f_\beta \theta_{\beta 2} \otimes e_2$$
$$= (v_1(f_1)\varphi_1 + v_2(f_1)\varphi_2) \otimes e_1 + (f_\beta \theta_{\beta 1}(v_1)\varphi_1 + f_\beta \theta_{\beta 1}(v_2)\varphi_2) \otimes e_1$$
$$+ (v_1(f_2)\varphi_1 + v_2(f_2)\varphi_2) \otimes e_2 + (f_\beta \theta_{\beta 2}(v_1)\varphi_1 + f_\beta \theta_{\beta 2}(v_2)\varphi_2) \otimes e_2$$

$\Rightarrow \|\nabla f\|^2 =$
$$= |v_1(f_1) + f_\beta \theta_{\beta 1}(v_1)|^2 + |v_2(f_1) + f_\beta \theta_{\beta 1}(v_2)|^2$$
$$+ |v_1(f_2) + f_\beta \theta_{\beta 2}(v_1)|^2 + |v_2(f_2) + f_\beta \theta_{\beta 2}(v_2)|^2$$
$$= |(v_1(f_1) + f_\beta \theta_{\beta 1}(v_1), \quad v_1(f_2) + f_\beta \theta_{\beta 2}(v_1))|^2$$
$$+ |(v_2(f_1) + f_\beta \theta_{\beta 1}(v_2), \quad v_2(f_2) + f_\beta \theta_{\beta 2}(v_2))|^2$$

$$\leq K(|v_1(f_1)|^2 + |v_1(f_2)|^2 + |v_2(f_1)|^2 + |v_2(f_2)|^2 + \|f\|^2)$$
$$= K(\|v_1(f_1, f_2)\|^2 + \|v_2(f_1, f_2)\|^2 + \|f\|^2)$$
$$\leq K'(\|D_1(f_1, f_2)\|^2 + \|D_2(f_1, f_2)\|^2 + \|f\|^2)$$

$$\Rightarrow \int_U \|\nabla f\|_0^2 \, dx \leq K' \sum_{|\alpha| \leq 1} \int_U \|D_\alpha(f_1, f_2)\|^2 dx .$$

$\quad \| \quad (f_1, f_2)$.