rals of this type was made by Abel in 1826. Abel noted that, while the single integral above is a highly intractable function of the point $p=(x,y)$ on C, the qualitative behavior of the more general Abelian sums

$$\sum \int_{p_0}^{p_i} \omega \quad)$$

was in fact subject to easily expressed relations.

A special case of what Abel proved is the following: for C and ω as above, and for any line $L \subset \mathbb{P}^2$, let $p_1(L)$; $p_2(L)$, and $p_3(L)$ denote the three points of intersection of L with C (the ordering of these points, of course, is not well-defined). Let $\psi(L)$ denote the Abelian sum

$$\psi(L) = \sum_{i=1}^{3} \int_{p_0}^{p_i} \omega ;$$

as before, $\psi(L)$ is well-defined modulo the periods Λ of ω. Then we have

Abel's Theorem (First Version)

$$\psi(L) = \text{constant} \pmod{\Lambda}$$

Proof. A modern version of the proof is deceptively easy. We consider ψ as a map

$$\psi : \mathbb{P}^{2*} \longrightarrow \mathbb{C}/\Lambda$$

from the space $\mathbb{P}^{2*}$ of lines in $\mathbb{P}^2$ to the complex torus $\mathbb{C}/\Lambda$; clearly it is holomorphic.

$$[(a_0, a_1, a_2)] \longleftrightarrow \mathbb{P}^2 \ni \ell = (a_0 z_0 + a_1 z_1 + a_2 z_2 = 0)$$

$$\Gamma \quad \mathbb{P}^{2*} = G(1,3) \longrightarrow \mathbb{C}/\Lambda$$

$$L \longmapsto \int_{p_0}^{p_1} \omega + \int_{p_0}^{p_2} \omega + \int_{p_0}^{p_3} \omega$$

parametrization of the lines in $\mathbb{P}^2$.