Obiviously f takes every value except a_0.

We don't need the argumet above.

By P217, since $\deg f^*(p') = n = \sum_{q \in f^{-1}(p')} v(q)$, for any $p' \in \mathbb{P}^1$,

if we choose $p' = \infty = [(1, 0)] \in \mathbb{P}^1$,

$\deg f^*(p') = 1$. $\Rightarrow$ This implies that $\exists$ no branch point in S. $\Rightarrow$ f is a covering map from S to $\mathbb{P}^1$. $\Rightarrow$ Since f is a map of degree 1, f is one to one, and f is isomorphism. ⌟

Next, we consider curves of genus 1. The full story on these curves will not be available to us until the next section; for the time being we will start by proving that any compact Riemann surface S of genus 1 can be realized as a nonsingular cubic curve in $\mathbb{P}^2$.

The proposition is easy to prove: we know that $\deg K_S = 0$, and so, by the embedding theorem, for any $p \in S$ the complete linear system of the line bundle $L = [3p]$ gives an embedding of S as a cubic curve in $\mathbb{P}^N$ where $N = h^0(S, \mathcal{O}(L)) - 1 \geq 2$.

Γ $\quad \iota_L : S \longrightarrow \mathbb{P}^N$

$\qquad q \longmapsto [(\sigma_1(q), \cdots \sigma_\ell(q))]$, $\quad \langle \sigma_1, \cdots \sigma_\ell \rangle = H^0(S, \mathcal{O}(L))$

$\Rightarrow \ell = h^0(S, \mathcal{O}(L))$ $\Rightarrow$ $\ell - 1 = N$. $\Rightarrow$ $N = h^0(S, \mathcal{O}(L)) - 1$

If $N = 1$, $S \subset \mathbb{P}^1$. $\Rightarrow$ $\mathbb{P}^1 = S$ $\Rightarrow$ $g(S) = 0$ $\times$.

According to P177, ⊛, the degree of $\iota_L(S)$ is given by $(c_1(L))^n$, where $n = 1$. in our case, since $\dim S = 1$.