2. Applications of Residues.

Intersection Numbers

Recall that our discussion of the local structure of those analytic varieties defined by a single function - i.e., analytic hypersurfaces - was based on the one-variable Cauchy formula and subsequent residue theorem. It is similarly possible to use the n-variable residue theorem to derive the local properties of analytic varieties of codimension n defined by n holomorphic functions in $U \subset \mathbb{C}^N$. We shall now carry this out in case the variety is zero-dimensional - i.e., $N = n$. By allowing dependence on parameters, it is possible to adapt the method to the more general situation just mentioned.

We begin by discussing intersection numbers, thereby complementing our previous definitions, which were either topological or used the theory of currents - cf. Section 4 of Chapter 0 and section 2 of Chapter 3.

Consider an ideal $I(f) = \{f_1, \cdots f_n\}$ of holomorphic functions $f_i \in \mathcal{O}(\overline{U})$ whose divisors D_i have the origin as set-theoretic intersection - i.e., $f^{-1}(0) = \{0\}$, where $f = \{f_1, \cdots f_n\}$. As usual, we allow ourselves to shrink U when necessary. Doing this, we may assume that $f^{-1}(\omega)$ is a discrete set of points in U for $\|\omega\| < \varepsilon$, since we will have $\|f(z)\| \geq C > 0$ for $z \in \partial U$.

☞ See P.657 & note P534