

is holomorphic since $f d\varphi$ must be holomorphic and $j \in L$. (If not, f does not cancel $z_L \cdot z_j$ and $f d\varphi$ is not holomorphic.) \Rightarrow

It follows that φ_{jk}/z_j is holomorphic, as was to be proved. Q.E.D.

I think we don't need $\sum_{L,k} \varphi_{Lk} \frac{dz_L}{z_L} \wedge dz_k$,

$$\text{from } d\varphi \equiv - \sum_{j,k} \sum_{j \in J} \frac{\varphi_{jk}}{z_j} \frac{dz_j}{z_j} \wedge \frac{dz_{i-j}}{z_{i-j}} \wedge dz_k.$$

Since $z_i d\varphi$ is holomorphic, $\frac{\varphi_{jk}}{z_j}$ must be holomorphic for otherwise.

since $\{ dz_j \wedge dz_{i-j} \wedge dz_k \mid j \in J \}$ is a set of linearly independent, and $j \in J$ $\frac{\varphi_{jk}}{z_j} \neq 0$

From the conclusion $z_{i-L} \varphi_{Lk} = \pm \sum_{j \in L} \frac{\varphi_{(i-L) \cup \{j\}, k}}{z_j}$ is

holomorphic, we see that

$\frac{\varphi_{(i-L) \cup \{j\}, k}}{z_j}$ is holomorphic for each $j \in L$,

for suppose $\frac{\varphi_{(i-L) \cup \{j\}, k}}{z_j}$ is not holomorphic \Rightarrow

it has singularities along $z_j=0$, \Rightarrow these singularities can not be cancelled by other $\frac{\varphi_{(i-L) \cup \{i\}, k}}{z_i}$ $i \neq j$ except itself, i.e. $\frac{\varphi_{(i-L) \cup \{j\}, k}}{z_j}$ must be holomorphic.

Put $L_j = (I - J) \cup \{j\} \Rightarrow J = (I - L_j) \cup \{j\}$, where $j \in L$ and $j \in J$.