$$\Delta\psi = *\bar\partial *\bar\partial\psi + \bar\partial *\bar\partial *\psi$$

$$\equiv (-1)^{q+q^2}\sum_{K>q} f_{\bar K K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q + \sum_{\substack{K>q\\ l\le q}}(-1)^{q+l+1} f_{\bar K, l}\,\bar\varphi_1\wedge\cdots\wedge\hat{\bar\varphi_l}\cdots\bar\varphi_q\wedge\bar\varphi_K$$

$$+\sum_{l\le q} f_{l\bar l}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q + \sum_{\substack{l\le q\\ K>q}}(-1)^{q+l} f_{l,\bar K}\,\bar\varphi_1\wedge\cdots\wedge\hat{\bar\varphi_l}\cdots\wedge\bar\varphi_q\wedge\bar\varphi_K$$

$$= \sum_{K>q} f_{\bar K,K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q + \sum_{K\le q} f_{K\bar K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q$$

$$\left(f_{\bar K K} = v_K(\bar v_K f) = \bar v_K(v_K f) + A'(f) = f_{K\bar K} + A'(f)\right)$$

$$= \sum_{K>q} f_{\bar K,K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q + \sum_{K\le q} f_{\bar K,K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q + A'(f)$$

$$= \sum_{K} f_{\bar K,K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q$$

$$\Rightarrow\quad \Delta\psi \equiv \sum_K f_{\bar K K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q$$

This proves the Weitzenböck formular.

Γ $\quad\psi = f\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q$.

$$\nabla\psi = df\otimes\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q + A^\circ(f)\ \square$$

$$\equiv \bar\partial f\otimes\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q + \partial f\wedge\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q$$

$$\Rightarrow\quad \nabla_{\bar K}\psi \equiv \bar v_K(f)\quad \bar\varphi_1\wedge\cdots\wedge\bar\varphi_q = f_{\bar K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q$$

$$\nabla_K(\nabla_{\bar K}\psi) \equiv v_K(f_{\bar K})\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q = f_{\bar K,K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q$$

$$\Rightarrow$$

$$\sum_{K=1}^{n}\nabla_K\nabla_{\bar K}\psi \equiv \sum_K f_{\bar K K}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q$$

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