$(\{\pi^{-1}(U_\beta), \{\pi^* f_\beta\})$, since $(\{\pi^{-1}(V_\beta \cap U_\alpha)\}, \{\pi^* f_\beta\}) =$ $(\{\pi^{-1}(U_\alpha)\}, \{\pi^* f_\alpha\})$ and $(\{\pi^{-1}(V_\beta \cap U_\alpha)\}, \{\pi^* g_\beta\})$. ⌟

Note that for a divisor on N given by an analytic hypersurface $V \subset M$, the pull back divisor $\pi^* V$ on M lies over V but need not coincide with the analytic hypersurface $\pi^{-1}(V) \subset M$ — multiplicities may occur.

⌜ $\{\pi^{-1}(U_\alpha)\}$ should be an open cover of M, and $\pi^{-1}(V_i)$ should be an analytic subvariety in M of codimension 1.

To get codimension 1, I think. $\pi(M)$ should be transverse to D.

We need to review the inverse function theorem and implicit function theorem.

① $M \xrightarrow{f} R$. C^∞-diff.

If $a \in R$ is a regular value of f, $f^{-1}(a)$ is a sub-manifold of M with $\dim f^{-1}(a) = \dim M - 1$.

pf)

$$\begin{tikzcd} U \arrow[r, "f"] \arrow[d, "\varphi"'] & R \\ \hat{R}^n \arrow[ur, dashed, "f\circ\varphi^{-1}"] & \end{tikzcd}$$

$f \circ \varphi^{-1} : R^n \longrightarrow R$

$x_0' \in f^{-1}(a)$. $\Rightarrow$ $\varphi(x') = x_0$

$\Rightarrow$ Since a is a regular value, $D(f\circ\varphi^{-1})$ has rank one at x. Suppose $D_1(f\circ\varphi^{-1}) \neq 0$ at x.

Define a function. $F : R^n \longrightarrow R \times R^{n-1}$ by

$(x_1, x_2, \cdots x_n) \longmapsto (f\circ\varphi^{-1}(x_1 \cdots x_n), x_2, x_3 \cdots x_n)$

$\Rightarrow$ DF is non-singular at x_0

$\Rightarrow$ By the inverse function theorem, $\exists$ G open sets W, V in R^n s.t $F\circ G = id$. $G \circ F = id$.