$\eta_H \neq 0$. (We can avoid this situation since we can choose H arbitrarily) }.

Consider $p(z')\tau_1 + q(z')\tau_2$.

$\Rightarrow$ $p(z')\tau_1 + q(z')\tau_2 = \eta\,(z_n\, h(z', z^n) + \varphi_1(z', z_n)\, a_{e_1}(z')\, p(z') + \varphi_2(z', z_n)\, a_{e_2}(z')\, q(z'))$. $\Rightarrow$ Observe that

$z_n h(z', z^n) + \varphi_1(z', z_n)\, a_{e_1}(z')\, p(z') + \varphi_2(z', z_n)\, a_{e_2}(z')\, q(z')|_{(0,0)} \neq 0$

which implies the holomorphic function is a unit in $\mathcal{O}_{M,x}$. $\Rightarrow$ $\eta = \dfrac{p(z')\tau_1 + q(z')\tau_2}{(\quad)}$ $\Rightarrow$ η may be expressed as a linear combination of τ_1 & τ_2 with coefficients in $\mathcal{O}_{M,x}$. In the general case, we can easily apply the argument above. ⌟

Noether's "AF + BG" Theorem. As an illustration of a particular global syzygy and application of the local residue theorem, we shall discuss a classical result of Max Noether, which is traditionally used as a cornerstone in the algebraic treatment of plane curves.

In $\mathbb{P}^2$ with homogeneous coordinates $X = [X_0, X_1, X_2]$ let $F(X)$ and $G(X)$ be homogeneous polynomials of respective degrees m and n whose divisors are plane curves C and D, which we assume to have no common component. Given a homogeneous polynomial $H(X)$ of degree $d = m + k = n + l$ with $k, l \geq 0$, we ask when there is a relation

$$(*) \qquad H = AF + BG.$$

⌜ A & B are homogeneous polynomials of respective degrees k, l ⌟