On the other hand, take the image of $i_* z$ under the Dolbeault isomorphism: we write

$$z_{\alpha\beta\gamma} = f_{\alpha\beta} + f_{\beta\gamma} - f_{\alpha\gamma},$$
$$\bar\partial f_{\alpha\beta} = \omega_\beta^{0,1} - \omega_\alpha^{0,1},$$

and we see that $\bar\partial \omega_\alpha^{0,1} = (d\omega_\alpha)^{0,2}$ represents z in $H_{\bar\partial}^{0,2}(M)$.

$\ulcorner$ $$df_{\alpha\beta} = \omega_\beta - \omega_\alpha = \partial f_{\alpha\beta} + \bar\partial f_{\alpha\beta} = \omega_\beta - \omega_\alpha$$
$$= \omega_\beta^{1,0} + \omega_\beta^{0,1} - \omega_\alpha^{1,0} - \omega_\alpha^{0,1} = \omega_\beta^{1,0} - \omega_\alpha^{1,0} + \omega_\beta^{0,1} - \omega_\alpha^{0,1}$$

$\Rightarrow$ $\bar\partial f_{\alpha\beta} = \omega_\beta^{0,1} - \omega_\alpha^{0,1}$ by comparing the types.

$\Rightarrow$ $\bar\partial \omega_\alpha^{0,1} = \bar\partial \omega_\beta^{0,1}$ represents the global $(0,2)$ type form and in $H_{\bar\partial}^{0,2}(M)$.

$$(d\omega_\alpha)^{0,2} = (\partial\omega_\alpha + \bar\partial\omega_\alpha)^{0,2} = (\partial\omega_\alpha)^{0,2} + (\bar\partial\omega_\alpha)^{0,2}$$
$$= (\partial(\omega_\alpha^{1,0} + \omega_\alpha^{0,1}))^{0,2} + (\bar\partial\omega_\alpha^{1,0} + \bar\partial\omega_\alpha^{0,1})^{0,2}$$
$$= (\bar\partial\omega_\alpha^{0,1})^{0,2} = \bar\partial\omega_\alpha^{0,1}.$$

$$H^2(M, \mathcal{O}) = H^1(M, Z_{\bar\partial}^{0,1}) = \frac{H^0(M, Z_{\bar\partial}^{0,2})}{\bar\partial H^0(M, \mathcal{A}^{0,1})}$$

$$= \frac{Z_{\bar\partial}^{0,2}(M)}{\bar\partial \mathcal{A}^{0,1}(M)} = H_{\bar\partial}^{0,2}(M)$$

$$0 \to \mathcal{O} \to \mathcal{A}^{0,0} \xrightarrow{\bar\partial} Z_{\bar\partial}^{0,1} \to 0$$

$$0 \to Z_{\bar\partial}^{0,1} \to \mathcal{A}^{0,1} \xrightarrow{\bar\partial} Z_{\bar\partial}^{0,2} \to 0$$