= the # of lines tangent to C_1, C_2.

$\lrcorner$

Since the tangent lines to a conic in $\mathbb{P}^2$ form a conic curve in $\mathbb{P}^{2*}$, this number is

$$\tilde{I}_p^3 \cdot \tilde{I}_\ell^2 = 4.$$

$\ulcorner$ By the classical Plücker formulas on P280

$d^* = d(d-1) - 2\delta - 3\kappa$,

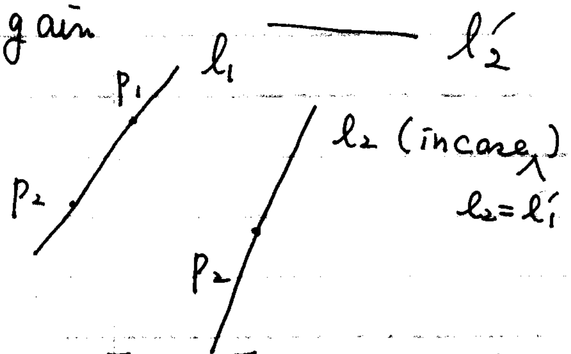
since we are dealing with a smooth curve C (generic), $\delta = \kappa = 0$. $\Rightarrow$ $d = 2$ $\Rightarrow$ $d^* = 2$ $\Rightarrow$ C^* is a conic.

$\ell \subset \mathbb{P}^2$ is tangent to C_1 and C_2 $\Leftrightarrow$ ℓ as a point $\in C_1^* \cap C_2^* \subset \mathbb{P}^{2*}$

$\Rightarrow$ For generic C_1 & C_2, $\#(C_1^* \cap C_2^*) = 4$.

$\Rightarrow$ $I_p^3 \cdot I_\ell^2 = 4$.

Again



But ℓ_2' is not tangent to $\ell_1 + \ell_2$.

$\Rightarrow$ $I_{p_1} \cap I_{p_2} \cap I_{p_3} \cap I_{\ell_1'} \cap I_{\ell_2'} \cap W_1 = \phi$.

$\Rightarrow$ $I_p^3 \cdot I_\ell^2 = \tilde{I}_p^3 \cdot \tilde{I}_\ell^2 = 4$

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