$z_1/z_2 = \frac{\xi_1}{\xi_2}$ is meromorphic function on M, especially, z_1/z_2 has a simple pole at p_2 and has a simple zero at p_1, on C. $\Rightarrow$ The restriction of z_1/z_2 to C gives an isomorphism from C to $\mathbb{P}^1$, as we saw on P 222.
$\Rightarrow$ $d(z_1/z_2) : TC \longrightarrow T\mathbb{P}^1$ is isomorphic. $\Rightarrow$ Clearly $d(z_1/z_2)(p) \neq 0$ on T_pC, $p \in C$.

$\xi_2 : M \longrightarrow L'$

Take a small enough open set $U \ni p$, as follows.

$$\begin{tikzcd} p \in U\cap C \subset U \arrow[d] \arrow[r, phantom] & U \arrow[d, "\cong"] \arrow[r] & L'|_U \cong U \times \mathbb{C} \\ 0 \in \mathbb{C} \arrow[r, hook] & \mathbb{C}^2 \arrow[ur] & \end{tikzcd}$$

$\omega_1 \longmapsto (\omega_1, 0)$

$\Rightarrow$ $\xi_2|_U$ can be expressed as a holomorphic function $f_2 : \mathbb{C}^2 \longrightarrow \mathbb{C}$.

$f_2(\omega_1, \omega_2) = \omega_2\, g_2(\omega_1, \omega_2)$ since ξ_2 vanishes on C identically. Here $g_2(0,0) \neq 0$, since $\xi_2 \otimes \tau^{-1}|_C$ has a simple pole at $p_2 \neq p \in U_1 = C - \{p_2\}$.

Note that $\xi_2 \otimes \tau^{-1}$ may be given by $g_2(\omega_1, \omega_2)$ locally near p.

$\Rightarrow$ $z_2 = \dfrac{\omega_2\, g_2(\omega_1,\omega_2)}{h(\omega_1,\omega_2)} = \dfrac{f_2}{h}$, where h is nonvanishing on U and h represents ξ_0.

$\Rightarrow$ $dz_2 = \dfrac{g_2}{h}\, d\omega_2 + \omega_2\, d\left(\dfrac{g_2}{h}\right)$