

Since  $T_x(F)$  is nowhere tangent to  $G$ ,  $T_x(F) \cap G$  is smooth, and  $T_y(T_x(F) \cap G) = T_x(F) \cap T_y(G)$ . In particular, the tangent space of  $T_x(F) \cap G$  at  $x$  is  $T_x(F) \cap T_x(G)$ .  $\Rightarrow T_x(F) \cap G \cap T_x(F) \cap T_x(G) = T_x(F) \cap G \cap T_x(T_x(F) \cap G) = T_x(F) \cap T_x(G) \cap G$  is the cone through  $x$  over a smooth quadric  $T_x(F) \cap G \cap T_x(G) \cap H \cap T_x(G) = T_x(F) \cap G \cap T_x(G) \cap H$ ,  $H \ni x$ , since  $T_x(G) \cap H$  is a hyperplane not containing  $x$  in  $T_x(G)$ .

$\Rightarrow$  By the argument on P734,  $T_x(F) \cap G \cap T_x(G) \cap H = T_x(F) \cap (T_x(G) \cap G \cap H) = T_x(F) \cap Q$  is smooth conic.

Suppose two lines of  $T_x(X) \cap X$  lie on some  $\sigma(p)$ .  $\Rightarrow$  By the recall above,  $\sigma(p) = \overline{x, L}$ , where  $L \subset Q$ .

But  $L \cap T_x(F) \cap Q = \text{one point}$   $\Rightarrow$  Since any line of  $T_x(X) \cap X$  is of  $\overline{xy}$ ,  $y \in T_x(F) \cap Q$ ,  $\sigma(p)$  contains double lines.  $\Rightarrow$

$F$  must be tangent to  $T_x(F) \cap Q$ ,

$$\begin{array}{ccc} T_x(G) \cap T_x(F) \cap G & & T_x(G) \cap T_x(F) \cap G \cap F \\ \downarrow \text{cone} & \Rightarrow & \downarrow \text{cone} \end{array}$$

$T_x(F) \cap Q = \text{smooth conic} = (T_x(F) \cap Q) \cap F = \text{Set of points}$   
since any line of  $T_x(X) \cap X$  lies in  $F$ .

$\Rightarrow$

One corollary of our lemma implied by this picture is that the locus  $T_x(X) \cap X$  will contain two lines from the same  $\sigma(p) \Leftrightarrow$  it contains