$$S \longrightarrow \mathbb{P}^r.$$

We denote by $\bar{Z}$ the linear span in $\mathbb{P}^r$ of a set Z of d distinct points on S. For generic Z, $\dim \bar{Z} = d-1$; the Cayley – Bacharach property implies that

$$\dim \bar{Z} = d-2-\rho \quad (\rho \geq 0),$$

so that configurations Z satisfying that property may be roughly thought of as "multisecant planes" such as trichords, etc.

☞ By P174, $i(S) \cap H$ is nondegenerate curve in $\mathbb{P}^n$ for generic hyperplane H. $\Rightarrow$ For generic set Z of d points, by P249 lemma, $\dim \bar{Z} = d-1$.

$$\begin{tikzcd} K \otimes L \arrow[r] \arrow[d] & {[H]} \arrow[d] \\ S \arrow[r, hook, "i"] & \mathbb{P}^r \end{tikzcd}$$

$$i = i_{H^0(S, \Omega^2(L))} = i_{H^0(S, \mathcal{O}(K \otimes L))}$$

Here clearly $1+r \geq d$. Consider the following equations

$$\left.\begin{array}{c} a_1 \psi_1(p_1) + a_2 \psi_2(p_1) + \cdots + a_{r+1} \psi_{r+1}(p_1) = 0 \\ \vdots \\ a_1 \psi_1(p_{d+1}) + a_2 \psi_2(p_{d+1}) + \cdots + a_{r+1} \psi_{r+1}(p_{d+1}) = 0 \end{array}\right\} \circledast, \quad H^0(S, \mathcal{O}(K \otimes L)) = \langle \psi_1, \psi_2, \cdots \psi_{r+1} \rangle$$

$$Z = \{p_1, \cdots p_d\}$$

$\Rightarrow \exists$ not all zero a_i's satisfying $\circledast$