cuts out on C a linear system of degree 4 without base points. Not clear!!!)

Let m be the # of tangent points of C' & C, counting multiplicity. $\Rightarrow$ m is the # of branch points of C'.

① $m = 1$

By Bertini's theorem, for generic line $l \ni C'$, $\text{mult}_{C'}(l, V_C) = 1$, and $\text{mult}_{C'}(V_C) = 1$ which implies that C' is a smooth point of V_C. See P750 for another proof.

② $m = 2$

Again, by Bertini's theorem, $\text{mult}_{C'}(V_C) = 2$.

Otherwise,

(i) $\text{mult}_{C'}(V_C) = 1$.

$\Rightarrow \exists\ l \subset W$ s.t $V_C \cap l$ is a set of 6 distinct points, which is a smooth manifold.

$\Rightarrow$ Counting the # of branch points of the covering map induced by l, we get $\# > 6$, since $m=2$ and $\exists$ five more branch points.

(ii) $\text{mult}_{C'}(V_C) = 3$.

Again, $\exists$ generic line $l \subset W$ s.t $V_C \cap l - C'$ is a set of 3 distinct points, which are smooth points of V_C.

By the argument ① ($m=1$) above, the # of