We now turn to quadrics in $\mathbb{P}^3$. Let $W \cong \mathbb{P}^9$ be the complete linear system of all quadrics, $W_1 \subset W$ the locus of quadrics of rank three or less, $W_2 \subset W_1 \subset W$ the locus of rank two or less and W_3 the set of rank-one quadrics. By our first of our previous arguments, W_1 is a hypersurface of degree 4 in W.

⌈ See P741, 1 $\quad \dim H^0(\mathbb{P}^3, \mathcal{O}(2H)) = {}_5C_2 = 10$ ⌋

Again, W_1 is smooth away from W_2: if F is any quadric of rank three — which we may take to be given by the matrix

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and L is the pencil generated by F and a generic quadric G, given by the matrix Q', then the polynomial

$$|\lambda Q + Q'|$$

has degree 3 in λ — i.e., L will contain three singular quadrics other than F, so $m_F(L \cdot W_1) = 1$ and F is a smooth point of W_1.