Given A nxn matrix, A^{-1} varies holomorphically with A. $\Rightarrow$ g varies with the entries of $\varphi_{I'}(\Lambda)$.

Thus. $\varphi_I \circ \varphi_{I'}^{-1}(\varphi_{I'}(\Lambda)) = \varphi_I(\Lambda) = g^{-1}\varphi_{I'}(\Lambda)$. since $\varphi_{I'}(\Lambda) = g\,\varphi_I(\Lambda)$.

$\Rightarrow$ $\varphi_I \circ \varphi_{I'}^{-1}(\varphi_{I'}(\Lambda)) = g^{-1}\varphi_{I'}(\Lambda)$ is holomorphic. $\lrcorner$

With this topology $G(k,n)$ is compact and connected, since the unitary group U_n maps surjectively and continuously onto $G(k,n)$ by the map $g \longmapsto g(V_k)$, where $V_k = \{e_1, e_2, \cdots e_k\} \subset \mathbb{C}^n$. The full linear group GL_n likewise acts transitively on $G(k,n)$.

$\ulcorner$ $U_n \longrightarrow G(k,n)$

$g \longmapsto g(V_k)$. is onto since any k-plane in $\mathbb{C}^n$ is spanned by a set of orthonormal vectors. $\lrcorner$

Note in particular that $G(1,n)$ is biholomorphic to $\mathbb{P}^{n-1}$ as a complex manifold: the "matrix representative" $(v_1, \cdots v_n)$ for a line $\Lambda \in G(1,n)$ corresponds, via the natural set-theoretic identification of $G(1,n)$ with $\mathbb{P}^{n-1}$, to the homogeneous coordinates of $\Lambda \in \mathbb{P}^{n-1}$, and

$$\Lambda^{\{i\}} = \left(\frac{v_1}{v_i}, \cdots, 1, \cdots \frac{v_n}{v_i}\right),$$

so

$$\varphi_{\{i\}} = \Lambda \longmapsto \left(\frac{v_1}{v_i}, \cdots \frac{v_n}{v_i}\right),$$

i.e., the coordinates on $G(1,n)$ given by $\varphi_{\{i\}}$ are just the Euclidean coordinates on $\mathbb{P}^{n-1}$.

$\ulcorner$ $I = \{i\}$

$\varphi_{\{i\}} : U_{\{i\}} \longrightarrow \mathbb{C}^{1(n-1)} = \mathbb{C}^{n-1}$. $U_{\{i\}} = \{\Lambda \in G(1,n) :$