Putting this all together, what we might call the distinguished Dolbeault representative of $(1/2\pi\sqrt{-1})^n \omega$ is

$$\eta_\omega = g(z)\left[\frac{C_n \sum (-1)^{i-1} \bar{f}_i\, d\bar{f}_1 \wedge \cdots \wedge \widehat{d\bar{f}_i} \wedge \cdots \wedge d\bar{f}_n \wedge dz_1 \wedge \cdots \wedge dz_n}{\|f\|^{2n}}\right],$$

where C_n is a constant depending only on n.

Pf $\quad \eta_\omega = n!\,(-1)^{\frac{(n+1)(n+2)}{2}} \qquad (-1)^i \dfrac{\bar\partial \rho_1 \cdots \wedge \widehat{\bar\partial \rho_i} \wedge \cdot \bar\partial \rho_n \wedge dz_1 \wedge \wedge dz_n}{f_1 \cdots f_n}\, g(z)$

$$= n!\,(-1)^{\frac{(n+1)(n+2)}{2}} \frac{(-1)^i}{f_1 \cdots f_n} (-1)^i g(z) \frac{f_1 \cdots f_n \sum_k (-1)^k \bar{f}_k \bigwedge_{j \neq k} d\bar{f}_j}{\|f\|^{2n}} \wedge dz_1 \wedge \cdots \wedge dz_n$$

$$= n!\,(-1)^{\frac{(n+1)(n+2)}{2}} \left[g(z) \frac{\sum_k (-1)^k \bar{f}_k\, d\bar{f}_1 \wedge \cdots \wedge \widehat{d\bar{f}_k} \wedge \cdots d\bar{f}_n \wedge dz_1 \wedge \cdots \wedge dz_n}{\|f\|^{2n}}\right]$$

$$= n!\,(-1)^{\frac{(n+1)(n+2)}{2}} (-1) \left[g(z) \frac{\sum_k (-1)^{k-1} \bar{f}_k\, d\bar{f}_1 \wedge \cdots \wedge \widehat{d\bar{f}_k} \wedge \cdot\, d\bar{f}_n \wedge dz_1 \wedge \cdots \wedge dz_n}{\|f\|^{2n}}\right]$$

$$C_n = n!\,(-1)^{\frac{n(n-1)}{2}}$$

At this juncture, recall from Section 1 of Chapter 3 the Bochner-Martinelli kernel

$$K(z,\zeta) = C_n \frac{\sum (-1)^{i-1} \bigwedge_{j \neq i} (d\bar{z}_j - d\bar{\zeta}_j) \wedge d\zeta_1 \wedge \cdots \wedge d\zeta_n}{\|z-\zeta\|^{2n}}$$

on $\mathbb{C}^n \times \mathbb{C}^n$. If $F: U \to \mathbb{C}^n \times \mathbb{C}^n$ is defined by

$F(z) = (z + f(z), z)$

then $\eta_\omega = g\, F^* K$. Pf $\quad g(z)\, F^* K(z) = g(z) = K(z+f, z) = \eta_\omega$

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