$$\Rightarrow \frac{\partial^2 F}{\partial X_0^2}(p)\, X_0'(0)^2 + \frac{\partial^2 F}{\partial X_1^2}(p)\, X_1(0)^2 + \frac{\partial^2 F}{\partial X_2^2}(p)\, X_2(0)^2 + 2\frac{\partial^2 F}{\partial X_0 \partial X_1}(p)\, X_0(0) X_1(0)$$

$$+ 2\frac{\partial^2 F}{\partial X_1 \partial X_2}(p)\, X_1(0) X_2(0) + 2\frac{\partial^2 F}{\partial X_0 \partial X_2}(p)\, X_0(0) X_2(0) = 0 \quad \cdots \quad ③$$

⇒ By expanding ⊛, we have ⊛ = ① + ② + ③ = 0.

⇒ We get some idea why the tangent cone may be realized as the union of all tangent lines passing the given point.

Let's go back to the birational map $\psi : \mathbb{P}^2 \longrightarrow \mathbb{P}^2$ given by

$$x \longmapsto [\,G_2 G_3 l_1,\ G_1 G_3 l_2,\ G_1 G_2 l_3\,].$$

It remains to show that $\deg l_i = 1$ (See note P799).

First of all,

$$\begin{tikzcd} & \widetilde{\mathbb{P}}^2_{a_1,\dots,a_6} \arrow[dl] \arrow[dr, "\pi"] & \\ \mathbb{P}^2 \arrow[rr, "\psi"] & & \mathbb{P}^2 \end{tikzcd}$$

$\pi(E_i) \neq \{[0,*,*]\}$ since $E_i \cap \widetilde{G}_j \neq \phi$ if $i \neq j$, and $\pi(E_i)$ contains five of $\{\pi(\widetilde{G}_j)\}$ but no three of $\{\pi(\widetilde{G}_j)\}$ are collinear.

Second, $\psi : \mathbb{P}^2 - \bigcup G_j \longrightarrow \mathbb{P}^2 - \bigcup \pi(E_i)$ is biholomorphic.

⇒ Clearly $\{[0,*,*]\}$ contains $\underset{=\pi(\{G_2=0\})}{\pi(G_2)}$ and $\underset{\pi(\{G_3=0\})}{\overset{\|}{\pi(G_3)}}$.

$\Rightarrow \psi^{-1} : \mathbb{P}^1 - g_2 - g_3 \longrightarrow \{l_1 = 0\} - a_1 - a_2 - \cdots - a_6$