$$\begin{tikzcd} i_E : U \arrow[r] \arrow[dr] & G(k, V^*) \\ & U_I \arrow[u, phantom, "\cup"] \arrow[d] \\ & \mathbb{C}^{k(n-k)} \end{tikzcd}$$

$$x' \longmapsto A^{-1}(x')\, B(x'), \quad \text{where } A = \begin{pmatrix} a_{11}(x'), \cdots a_{1k}(x') \\ \vdots \\ a_{k1}(x'), \cdots a_{kk}(x') \end{pmatrix}$$

$$\text{and } B(x') = \begin{pmatrix} a_{11}(x'), \cdots & a_{1n}(x') \\ \vdots & \\ a_{k1}(x'), \cdots & a_{kn}(x') \end{pmatrix}$$

$$\Rightarrow \quad d\iota_E = d(A^{-1}(x')\, B(x'))$$

$$= (dA^{-1}(x))\; B(x') + A^{-1}(x')\; dB(x')$$

$$d_x \iota_E(v) = dA^{-1}(x)(v)\; B(x) + A^{-1}(x)\; dB(x)(v) = 0$$

Since $\sigma(x) = 0 = c_i \sigma_i = c_i\, a_{\alpha i}\, e_\alpha$,

$$B(x)\, C = 0.$$

$$\Rightarrow \quad dA^{-1}(x)(v)\; B(x)\; C + A^{-1}(x)\; dB(x)(v)\; C = 0$$

$$\Rightarrow \quad dB(x)(v)\; C = 0 \quad \cdots \quad ①$$

$$d\sigma(x)(v) = d(c_i\, a_{\alpha i}\, e_\alpha) = c_i\, (d a_{\alpha i})\, e_\alpha + \underbrace{(c_i\, a_{\alpha i})}_{=0}\, d e_\alpha \neq 0 \quad \Rightarrow \quad dB(x)(v)\, C \neq 0 \quad \cdots ②$$

① contradicts to ②. $\Rightarrow$ $v = 0$ must hold.

The compactness argument used in the proof of the Kodaira embedding theorem again assures us that to prove the result, it is sufficient to show that