$\Rightarrow \quad \|\phi_0 + \phi - \phi_0\| \leq \delta. \qquad |\Lambda(\phi_0+\phi)| \leq |\Lambda\phi_0| + |\Lambda\phi|$

$\leq \qquad C\|\phi\|_N + |\Lambda\phi_0| < C\delta + |\Lambda\phi_0|$.

$\Rightarrow$ If we choose δ small enough, then we can make $|\Lambda(\phi_0+\phi) - \alpha| < \epsilon. \Rightarrow \Lambda(\phi_0+\phi) \in B(\alpha, \epsilon)$.

$\Rightarrow \quad \Lambda(\phi_0 + N(0,\delta)) \subset B(\alpha, \epsilon). \Rightarrow \Lambda$ is continuous.

$(\Leftarrow)$ $\Lambda^{-1}(-\epsilon, \epsilon)$ is open in $\mathcal{D}_K$. $\Rightarrow \exists \; V_N = \{\phi \in \mathcal{D}_K ; \|\phi\|_N < \frac{1}{N}\} \subset \Lambda^{-1}(B(0,\epsilon))$

$\Rightarrow$ Let $\psi \in \mathcal{D}_K$. $\Rightarrow \quad \dfrac{\psi}{\|\psi\|_N} \times \dfrac{1}{N+1} \in V_N$.

$$\Rightarrow \quad \Lambda\left(\frac{\psi}{\|\psi\|_N} \frac{1}{N+1}\right) = \frac{1}{(N+1)\|\psi\|_N} \Lambda(\psi) \in B(0,\epsilon)$$

$\Rightarrow \quad |\Lambda(\psi)| < (N+1)\|\psi\|_N \epsilon = C\|\psi\|_N$ where $C = \epsilon(N+1)$.

Proposition proof.

Indeed, suppose $\phi \in L$; its support being compact, there exists $\alpha(x) \in L$ which is majorant for the constant 1 on $\operatorname{supp}\phi$, and

$$-\|\phi\|\,\alpha(x) \leq \phi(x) \leq \|\phi\|\,\alpha(x)$$

where $\|\phi\| = \sup|\phi(x)|$.

We deduce, since $\ell(\phi)$ is positive, that

$$-\|\phi\|\,\ell(\alpha) \leq \ell(\phi) \leq \|\phi\|\,\ell(\alpha).$$

By the assumption, if χ is the characteristic