

$$= \frac{(-1)^{n-1}}{\langle v, v \rangle^n} (-1)^{\frac{(n-1)n}{2}} \sum_{i=0}^n \frac{(-1)^{i-1} \bar{v}^i d\bar{v}^1 \wedge \dots \wedge d\hat{\bar{v}}^i \wedge \dots \wedge d\bar{v}^n}{\det\left(\frac{\partial v^i}{\partial z_j}\right)} dz_1 \wedge \dots \wedge dz_n \cdot (n-1)!$$

$\Rightarrow$

$$\frac{\beta(v, \bar{v})}{\det A_p} = - \left( \frac{1}{\det A_p} \right)^{-1} C_n \sum_{i=0}^n (-1)^{i-1} \bar{v}^i d\bar{v}^1 \wedge \dots \wedge d\bar{v}^n dz_1 \wedge \dots \wedge dz_n \quad (*)$$

This means that

$$\beta(v, \bar{v}) = \left( \frac{\sqrt{-1}}{2\pi} \right)^{+n} \partial \log \|v\|^2 \wedge (\partial \bar{\partial} \log \|v\|^2)^{n-1}$$

$$\text{for } \left( \frac{\sqrt{-1}}{2\pi} \right)^n \partial \log \|v\|^2 \wedge (\partial \bar{\partial} \log \|v\|^2)^{n-1}$$

$$= \left( \frac{\sqrt{-1}}{2\pi} \right)^n (-1) (-1)^{\frac{n(n+1)}{2}} (n-1)! \det\left(\frac{\partial v^i}{\partial z_j}\right) \sum_{i=0}^n \frac{(-1)^{i-1} \bar{v}^i d\bar{v}^1 \wedge \dots \wedge d\hat{\bar{v}}^i \wedge \dots \wedge d\bar{v}^n}{\langle v, v \rangle^n} dz_1 \wedge \dots \wedge dz_n$$

$$= (-1) C_n \det A_p \sum_{i=0}^n \frac{(-1)^{i-1} \bar{v}^i d\bar{v}^1 \wedge \dots \wedge d\bar{v}^n}{\langle v, v \rangle^n} dz_1 \wedge \dots \wedge dz_n$$

$$= \beta(v, \bar{v}). \quad \text{by } (*).$$

$\square$

Putting everything together,

$$\int_M P\left(\frac{\sqrt{-1}}{2\pi} \Theta\right) = \int_{M \cup B_c(p_0)} P\left(\frac{\sqrt{-1}}{2\pi} \Theta\right)$$

$$= - \sum_v \int_{\partial B_c(p_0)} \Phi$$