$\Leftrightarrow \quad \Delta_d a = 0 \quad \Leftrightarrow \quad da = d^* a = 0 \quad \Rightarrow \quad [da] = 0$ in $H_{\bar\partial}^{p+1,q}(M)$.
See P44. $\Rightarrow$

$$H^q(M, \Omega^{p-1}) \xrightarrow{d} H^q(M, \Omega^p) \xrightarrow{d} H^q(M, \Omega^{p+1})$$

d zero maps $\Rightarrow$ $''E_2^{p,q} = H^q(M, \Omega^p) \cong H_{\bar\partial}^{p,q}(M)$

$\Rightarrow \quad ''E_3^{p,q} = H(''E_2) = H^q(M, \Omega^p) = \cdots = ''E_\infty^{p,q}$

$$\begin{aligned}\Rightarrow \quad \mathbb{H}^n(M, \Omega^*) &= ''E_2^{0,n} \oplus ''E_2^{1,n-1} \oplus \cdots \oplus ''E_2^{n,0} \\ &= H^n(M, \Omega^0) \oplus H^{n-1}(M, \Omega^1) \oplus \cdots \oplus H^0(M, \Omega^n) \\ &= \bigoplus_{p+q=n} H^q(M, \Omega^p)\end{aligned}$$

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In the stein case, $H^q(M, \Omega^*) = 0$ for $q > 0$ and $(*)$ reduces to the previously noted isomorphism

$$H^*(M, \mathbb{C}) \cong H^*_{DR}(M, hol).$$

┌ By the definition on P44 $H_{\bar\partial}^{p,q}(M) = H^q(M, \Omega^p) = 0, q > 0$

$$H^q(M, \Omega^{p-1}) \xrightarrow{d} H^q(M, \Omega^p) \xrightarrow{d} H^q(M, \Omega^{p+1})$$

$q > 0 \quad \Rightarrow \quad H^q(M, \Omega^p) = 0 \quad \Rightarrow \quad ''E_2^{p,q} = 0$

$q = 0 \quad \Rightarrow$

$$\Omega^{p-1}(M) \xrightarrow{d} \Omega^p(M) \xrightarrow{d} \Omega^{p+1}(M)$$

$\Rightarrow \quad ''E_2^{p,q} = H^p_{DR}(M, hol)$

$\Rightarrow \quad \mathbb{H}^n(M, \Omega^*) = H^n_{DR}(M, hol) = H^n(M, \mathbb{C})$

$\Rightarrow \quad \mathbb{H}^*(M, \Omega^*) = H^*(M, \mathbb{C})$.

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