Now $\omega = \frac{i}{2} \sum \varphi_i \wedge \overline{\varphi_i}$,

so $\omega^{n-q} = C_q (n-q)! \sum_{\# K = n-q} \varphi_K \wedge \overline{\varphi_K}$;

thus, for suitable $C_q' \neq 0$,

$$\eta \wedge \overline{\eta} \wedge \omega^{n-q} = C_q' \sum_I |\eta_I|^2 \Phi .$$

where Φ is the volume form.

$\ulcorner$ $\eta \wedge \overline{\eta} \wedge \omega^{n-q} = \sum_{I,J} \eta_I \overline{\eta_J} \, \varphi_I \wedge \overline{\varphi_J} \wedge C_q (n-q)! \sum_{\# K=n-q} \varphi_K \wedge \overline{\varphi_K}$

$= \sum_{\substack{I = K^\circ \\ \# I = p}} \eta_I \overline{\eta_I} \, \varphi_I \wedge \overline{\varphi_I} \wedge C_q (n-q)! \sum_{\# K = n-q} \varphi_K \wedge \overline{\varphi_K}$.

$= C_q (n-q)! \sum_I |\eta_I|^2 \varphi_I \wedge \overline{\varphi_I} \wedge \varphi_K \wedge \overline{\varphi_K}$

$= C_q (n-q)! \sum_I |\eta_I|^2 \varphi_I \wedge \varphi_K \wedge \overline{\varphi_I} \wedge \overline{\varphi_K}$.

$\Rightarrow$ $\varphi_I \wedge \varphi_K = \epsilon \, \Phi' = \varphi_1 \wedge \cdots \wedge \varphi_n$
$\overline{\varphi_I} \wedge \overline{\varphi_K} = \epsilon \, \overline{\Phi'} = \overline{\varphi_1} \wedge \cdots \wedge \overline{\varphi_n}$

$\Rightarrow$ $C_q' \sum_I |\eta_I|^2 \Phi$ where $\Phi = \Phi' \wedge \overline{\Phi'}$. $\lrcorner$

Consequently,

$$\int_M \eta \wedge \overline{\eta} \wedge \omega^{n-q} \neq 0 \quad \text{if } \eta \neq 0$$

Now suppose $\eta = d\psi$. Then $d\eta = d\overline{\eta} = 0$, and since $d\omega = 0$, we have

$$\int_M \eta \wedge \overline{\eta} \wedge \omega^{n-q} = \int_M d(\psi \wedge \overline{\eta} \wedge \omega^{n-q}) = 0 .$$

Thus $\eta = d\psi$ implies that $\eta \equiv 0$. Finally, since $d\eta = \partial\eta$ is a holomorphic $(q+1)$-form and is exact, it follows that $d\eta = 0$.