$\alpha\,\delta\mu = \delta\alpha\mu = \delta\rho\delta\tau = \rho\delta^2\tau = 0$. and since α is injective, this implies $\delta\mu = 0$. Thus $\mu \in Z^{p+1}(\underline{\mathcal{U}}'', \mathcal{E})$ and we take $\delta^*\sigma = \mu \in H^{p+1}(M.\,\mathcal{E})$.

Basic Fact.

The sequence

$$\begin{aligned}
0 \to H^0(M, \mathcal{E}) &\longrightarrow H^0(M.\,\mathcal{F}) \longrightarrow H^0(M.\,\mathcal{G}) \\
\to H^1(M.\,\mathcal{E}) &\longrightarrow H^1(M.\,\mathcal{F}) \longrightarrow H^1(M.\,\mathcal{G}) \to \cdots \\
&\cdots \\
\to H^p(M.\,\mathcal{E}) &\longrightarrow H^p(M.\,\mathcal{F}) \longrightarrow H^p(M.\,\mathcal{G}) \to \cdots
\end{aligned}$$

is exact.

proof of sketch). For most exact sequence $0 \to \mathcal{E} \to \mathcal{F} \to \mathcal{G} \to 0$ that actually arise naturally, — and certainly for all sheaves with which we shall deal in this book. — it is the case that there exist arbitrarily fine coverings $\underline{\mathcal{U}}$ s.t for every open set $U = U_{i_0} \cap \cdots \cap U_{i_p}$ the sequence

$$0 \to \mathcal{E}(U) \xrightarrow{\alpha} \mathcal{F}(U) \xrightarrow{\beta} \mathcal{G}(U) \to 0 \quad \text{is exact.}$$

I think this is not correct. For simplicity, we can assume this.

We can construct an open covering as follows:

$\underline{\mathcal{U}} = \{U_\alpha\}$ open covering of M.

For each point $x \in M$, choose a nbd V_x of x. s.t.

① V_x lies in the intersection of the U_α's containg x.

② If $U_{\alpha_0} \cap \cdots \cap U_{\alpha_p} \neq \phi$ and $x \in U_{\alpha_0} \cap \cdots \cap U_{\alpha_p}$, then $V_x \subset U_{\alpha_0} \cap \cdots \cap U_{\alpha_p}$.

$r_{U_{\alpha_0} \cap \cdots \cap U_{\alpha_p},\, V_x}\, \sigma$ is in the image of β, for $\sigma \in \mathcal{F}(U_{\alpha_0} \cap \cdots \cap U_{\alpha_p})$