and this proves the theorem. Q.E.D.

F Already H is a global section on $\mathbb{P}^2$, i.e. $H \in H^0(\mathbb{P}^2, \mathcal{O}(d))$. $H_p \in I_p$ means that $H \in H^0(\mathbb{P}^2, I(d))$. But by the argument above, we have an exact sequence

$$H^0(\mathbb{P}^2, \mathcal{O}(k)) \oplus H^0(\mathbb{P}^2, \mathcal{O}(\ell)) \longrightarrow \underset{\underset{H\ (=\sigma_H)}{\psi}}{H^0(\mathbb{P}^2, I(d))} \longrightarrow 0$$

$\Rightarrow \exists\ \xi \in H^0(\mathbb{P}^2, \mathcal{O}(k))$, $\psi \in H^0(\mathbb{P}^2, \mathcal{O}(\ell))$ s.t
$(\sigma_H \Rightarrow) H = \xi \otimes \sigma_F + \psi \otimes \sigma_G$.

$\Rightarrow H = AF + BG$ globally.

" F', G' polynomials. $\deg F' = \deg G'$
$(F'=0) = (G'=0) \Rightarrow F'/G'$ holomorphic on $\mathbb{P}^2 \Rightarrow F'/G' =$ constant "

In order to apply Noether's theorem, it is useful to have numerical criteria for when the local conditions (**) are satisfied. It is pretty clear that the local duality theorem is relevant to this question, and we shall pursue this lead in one rather simple case here.

Suppose that $f(z,w)$ is holomorphic in a nbd of the origin and has divisor a nonsingular curve C passing through the origin. If $g(z,w)$ is holomorphic near the origin, then we define $\mathrm{Ord}_C(g)$ to be the order of vanishing of $g|_C$ at the origin. Suppose now that $g(z,w)$ has divisor D and that the set-theoretic intersection $C \cap D = \{0\}$. Denote by $I \subset \mathcal{O}$ the ideal $\{f, g\}$ in the local ring at the origin.

F Compare $\mathrm{Ord}_C(g)$ with $\mathrm{ord}_C(g)$ on P130.