

characteristic of the trivial line bundle, i.e

$$\chi(\mathcal{O}_M) = \sum (-1)^p h^p(M, \mathcal{O}(M \times \mathbb{C})).$$

$$\begin{aligned} \mathbb{F} \quad \dim H^p(M, \mathcal{O}(M \times \mathbb{C})) &= \dim H^p(M, \mathcal{O}) = h^p(M, \mathcal{O}) = \\ \dim H^{0,p}(M) &= h^{0,p}(M) \quad \text{see P151.} \end{aligned}$$

Now for a line bundle L on a Riemann surface S , by Kodaira - Serre duality we have

$$\begin{aligned} \chi(L) &= h^0(S, \mathcal{O}(L)) - h^1(S, \mathcal{O}(L)) \\ &= h^0(L) - h^0(K-L) \\ \chi(\mathcal{O}_S) &= h^{0,0}(S) - h^{0,1}(S) \\ &= 1 - g, \end{aligned}$$

and so the Riemann - Roch formula reads simply

$$\chi(L) = \chi(\mathcal{O}_S) + c_1(L).$$

$$\begin{aligned} \mathbb{F} \quad \chi(L) &= h^0(S, \mathcal{O}(L)) - h^1(S, \mathcal{O}(L)) \\ &= h^0(L) - \dim H^1(S, \mathcal{O}(L)) = h^0(L) - \dim H^{0,1}(S, L) \\ &= h^0(L) - \dim H^{0,1}(L) = h^0(L) - h^{0,1}(L) \end{aligned}$$

$$\begin{aligned} \text{Kodaira - Serre} &= h^0(L) - \dim H^1(S, \Omega^1(K^* \otimes L)) = h^0(L) - \dim H^0(S, \\ \text{duality} \Rightarrow \Omega^1(L^*)) &= h^0(L) - \dim H^0(S, \mathcal{O}(K \otimes L^*)) \\ &= h^0(L) - h^0(K-L) \end{aligned}$$

$$\begin{aligned} \chi(\mathcal{O}_S) &= h^{0,0}(S) - h^{0,1}(S) = \dim H^{0,0}(S) - \dim H^{0,1}(S) \\ &= \dim H^1(S) - \dim H^0(S, \Omega^1) = 1 - g \end{aligned}$$

$$\begin{aligned} \Rightarrow \chi(L) &= h^0(L) - h^0(K-L) = d - g + 1 = d + \chi(\mathcal{O}_S) \\ &= \chi(\mathcal{O}_S) + c_1(L). \end{aligned}$$