Date

$\Rightarrow$ By the argument above $\exists$ $\overset{\text{constant}}{\alpha}$ s.t $\alpha(h_{11}, h_{12}) = (h_{21}(x), h_{22}(y))$.

$$\begin{pmatrix} h_{11}(x), & h_{12}(y), & h_{13}(z) \\ h_{21}(x), & h_{22}(y), & h_{23}(z) \\ h_{31}(x), & h_{32}(y), & h_{33}(z) \end{pmatrix} = \begin{pmatrix} h_{11}(x), & h_{12}(y), & h_{13}(z) \\ \alpha h_{11}(x), & \alpha h_{12}(y), & h_{23}(z) \\ h_{31}(x), & h_{32}(y), & h_{33}(z) \end{pmatrix}$$

$$p(x,y,z)\, h_{11} + \alpha h_{11}\, q(x,y,z) = r(x,y,z)\, h_{31}$$

$\Rightarrow$ $\dfrac{h_{11}}{h_{31}} = \dfrac{(p+\alpha q)^{-1}}{r^{-1}} = \dfrac{r}{p+\alpha q}$ is independent of z & y

$\Rightarrow$ Similarly, $\dfrac{h_{12}}{h_{32}} = \dfrac{r}{p+\alpha q}$ is " of z, x

$\Rightarrow$ $\begin{pmatrix} h_{11}, & h_{12} \\ \alpha h_{11} & \alpha h_{12} \\ h_{31}, & h_{32} \end{pmatrix}$ linear indepent.

(ii) No minor 2×2 matrix has determinat zero.

$\det\begin{pmatrix} h_{11}, & h_{12} \\ h_{21}, & h_{22} \end{pmatrix} \neq 0$. on some open set

$\Rightarrow$ $\begin{aligned} h_{31}(x) &= a h_{11} + b h_{21} \\ h_{32}(y) &= a h_{12} + b h_{22} \end{aligned}$ $\Rightarrow$ $\begin{pmatrix} h_{11}, & h_{21} \\ h_{12} & h_{22} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} h_{31} \\ h_{32} \end{pmatrix}$

$\Rightarrow$ $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} h_{11}, & h_{21} \\ h_{12}, & h_{22} \end{pmatrix}^{-1}\begin{pmatrix} h_{31} \\ h_{32} \end{pmatrix}$

$\Rightarrow$ $a(x,y,z)$ is a function of x, y

$b(x,y,z)$ " of x, y

$\Rightarrow$ a & b must be constant if we do this for $\begin{pmatrix} h_{12}, & h_{13} \\ h_{22}, & h_{23} \end{pmatrix}$ and so on.