

By 1747,  $V_0(L) \sim 2\sigma_2 + 6\sigma_{1,1} \Rightarrow$  Since  $V_0(L) + V_1(W) \sim 9\sigma_{1,1} + 9\sigma_2$ ,  $V_1(W) = 3\sigma_{1,1} + 9\sigma_2$

□

The reader may find it an interesting exercise to prove that, for a generic web  $W$ , the surface  $V_1(W)$  is an Enriques surface.

□ Some time later.

□

### The Problem of Five Conics

To conclude this section, we will use the computation of Section 6, Chapter 4, for the cohomology ring of a blow-up to solve a classical problem in enumerative geometry:

Given  $C_1, \dots, C_5 \subset \mathbb{P}^2$  five smooth conic curves chosen generically, how many smooth conic curves in  $\mathbb{P}^2$  are tangent to all five?

To answer this question, consider first the linear system

$$W = |2H| \cong \mathbb{P}^5$$

of all conic curves in  $\mathbb{P}^2$ . For any smooth conic