

Since $\sum C_i = a + \sum b_i$, it follows that $a \geq C_i$; and so there are three cases:

1. If $C_i > b_{i-1}$ for some i , then $a > C_i$ and so $\delta(a, b; c) = 0$.

$$\begin{aligned} \sqcap \quad \dim \sigma_a &= k(n-k) - a, \quad \dim \sigma_b = k(n-k) - \sum b_i \\ \dim \sigma_c &= k(n-k) - \sum C_i \end{aligned}$$

$$\Rightarrow \dim(\sigma_a \cdot \sigma_b) = k(n-k) - \sum b_i + k(n-k) - a - k(n-k) = k(n-k) - a - \sum b_i = k(n-k) - \sum C_i$$

$$\Rightarrow \sum C_i = a + \sum b_i$$

$$\Rightarrow C_1 + C_2 + \dots + C_k = a + b_1 + \dots + b_k$$

But since $C_2 \geq b_1, C_3 \geq b_2, \dots, C_k \geq b_{k-1}, \dots, b_k = 0$.

$C_1 \leq a$ the equality to be valid.

Note ^{that} $b_k = 0$ for some k .

If $C_i > b_{i-1}$ for some i , a can not be equal to C_1 , since $C_1 + C_2 + \dots + C_k = a + b_1 + \dots + b_k$.

\Rightarrow By Prop. (*) 1, since $a = C_1$, $\delta(a, b; c) = 0$

\sqcup

2. If $C_i < b_i$ for any i , then $C_i \geq b_{i-1}$ implies that $b_i > b_{i-1}$, i.e., the sequence b is not nonincreasing and σ_b is taken to be null; so $\delta(a, b; c) = 0$.

$$\begin{aligned} \sqcap \quad C_i < b_i &\Rightarrow \text{Since } C_i \geq b_{i-1}, \quad b_{i-1} \leq C_i < b_i \\ &\Rightarrow b \text{ is not nonincreasing.} \end{aligned}$$

\sqcup

3. If $b_i \leq C_i \leq b_{i-1}$ for all i , it follows that $C_i = b_{i-1}$ for all i , hence $a = C_1, b_k = 0$, and applying