$$\mathrm{res}_f(g, h) = \mathrm{Res}_{\{0\}}\left(\frac{g\cdot h\, dz_1\wedge dz_2}{f_1 f_2}\right) \quad \text{for } n=2$$

$$= \mathrm{res}((g+I)\otimes \varphi), \qquad \varphi \in \mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I, \Omega^n). \qquad g+I \in \mathcal{O}/I.$$

$$\mathrm{res}_{f'}(g, h') = \mathrm{Res}_{\{0\}}\left(\frac{g\, h'\, dz_1\wedge dz_2}{f_1' f_2'}\right)$$

where $h' = \Delta h$, $\Delta = \det(a_{ij})$.
$f_i' = \sum a_{ij} f_j$.

According to Proposition on p680 ~ p681,

$$\begin{tikzcd}
\varphi \in \mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) \arrow[r, "\sim"] \arrow[d] & \mathcal{O}/I \ni h+I \arrow[d, "\Delta"] \\
\varphi \in \mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I', \mathcal{O}) \arrow[r] & \mathcal{O}/I' \ni \Delta h + I' = h'+I
\end{tikzcd}$$

$$\Rightarrow \quad \mathrm{res}_{f'}(g, h') = \mathrm{Res}_{\{0\}}\left(\frac{g\, h'\, dz_1\wedge dz_2}{f_1' f_2'}\right)$$

$$= \mathrm{Res}_{\{0\}}\left(\frac{g\, h\, \Delta\, dz_1\wedge dz_2}{f_1' f_2'}\right)$$

$$= \mathrm{Res}_{\{0\}}\left(\frac{g\, h\, dz_1\wedge dz_2}{f_1 f_2}\right)$$ by transformation law on p657 ~ p658.

$$= \mathrm{res}_f(g, h).$$

$(z_1, z_2) \longmapsto (w_1, w_2)$. $\quad w_1(z)$. $w_2(z)$

$$\mathrm{res}_f(g, h)_z = \mathrm{Res}_{\{0\}}\left(\frac{g\, h\, dz_1\wedge dz_2}{f_1 f_2}\right)$$

$$\mathrm{res}_f(g, h')_w = \mathrm{Res}_{\{0\}}\left(\frac{g\, h'\, dw_1\wedge dw_2}{f_1 f_2}\right)$$