

$$\tilde{f}: \mathbb{R}^M \longrightarrow P_f \subset M \times M.$$

$[\tilde{f}_*(v_i)]_{i=1}^n$ is an orientation of P_f when $[v_i]$ is an orientation of M . \square

Let p be a fixed point of f , x_1, \dots, x_n an oriented coordinate system for M centered around p ; take as coordinates around $(p, p) \in M \times M$ the functions

$$y_i = \pi_1^* x_i \text{ and } z_i = \pi_2^* x_i.$$

An oriented basis for $T_{(p,p)}(\Delta) \subset T_{(p,p)}(M \times M)$ is then given by

$$\Delta_* \left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n} \right) = \left(\frac{\partial}{\partial y_1} + \frac{\partial}{\partial z_1}, \dots, \frac{\partial}{\partial y_n} + \frac{\partial}{\partial z_n} \right),$$

where Δ is the diagonal map $x \mapsto (x, x)$, and an oriented basis for $T_{(p,p)}(P_f) \subset T_{(p,p)}(M \times M)$ is given by

$$\tilde{f}_* \left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n} \right) = \left(\frac{\partial}{\partial y_1} + \sum \frac{\partial f_i}{\partial x_1} \cdot \frac{\partial}{\partial z_i}, \dots, \frac{\partial}{\partial y_n} + \sum \frac{\partial f_i}{\partial x_n} \cdot \frac{\partial}{\partial z_i} \right).$$

$$\Gamma \quad \Delta: \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \times \mathbb{R}^2$$

$$(x_1, x_2) \longmapsto (x_1, x_2, x_1, x_2)$$

$$\Delta_* \left(\frac{\partial}{\partial x_1} \right) = \frac{\partial}{\partial y_1} + \frac{\partial}{\partial z_1} \quad \dots$$

$$\tilde{f}: \mathbb{R}^2 \longrightarrow \mathbb{R}^2 \times \mathbb{R}^2$$

$$(x_1, x_2) \longmapsto (x_1, x_2, f_1(x_1, x_2), f_2(x_1, x_2))$$

$$\tilde{f}_* \left(\frac{\partial}{\partial x_1} \right) = \frac{\partial}{\partial y_1} + \frac{\partial f_1}{\partial x_1} \frac{\partial}{\partial z_1} + \frac{\partial f_2}{\partial x_1} \frac{\partial}{\partial z_2}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \frac{\partial f_1}{\partial x_1} \\ \frac{\partial f_2}{\partial x_1} \end{pmatrix}$$