divisor D, since on U_α. $f=0 \Rightarrow f_\alpha = 0$. (since $h_\alpha \neq 0$) ⟫

Thus the line bundle $[D]$ associated to a divisor D on M is trivial $\Leftrightarrow$ D is the divisor of a meromorphic function. We say that two divisors D, D' on M are linearly equivalent and write $D \sim D'$ if $D = D' + (f)$. for some $f \in \mathcal{M}^*(M)$, or equivalently if $[D] = [D']$.

Also, note that $[\ ]$ is functorial; that is, if $\pi : M \to N$ is a holomorphic map of complex manifolds, it is easy to check that for any $D \in Div(N)$,

$$\pi^*([D]) = [\pi^* D].$$

⟦

$$\pi : M \longrightarrow N \Rightarrow \begin{array}{ccc} Div(N) & \xrightarrow{\pi^*} & Div(M) \\ \Psi & & \Psi \\ D & \longmapsto & \pi^* D \end{array}.$$

$$Div(N) = H^0(M, \mathcal{M}^*/\mathcal{O}^*)$$

For D, $\exists$ $f \in H^0(M, \mathcal{M}^*/\mathcal{O}^*)$. $\Rightarrow \exists \{(f_\alpha, U_\alpha)\}$ s.t

$$\frac{f_\alpha}{f_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta).$$

$\Rightarrow$ $\pi^* D$ is represented by $\{(\pi^{-1}U_\alpha, \pi^* f_\alpha)\}$.

$\Rightarrow$ What are the transition functions of the bundle defined by $\pi^* D$?

On $\pi^{-1}(U_\alpha) \cap \pi^{-1}(U_\beta)$, $\dfrac{\pi^* f_\alpha}{\pi^* f_\beta} = \dfrac{f_\alpha \circ \pi}{f_\beta \circ \pi} = g_{\alpha\beta} \circ \pi = \pi^* g_{\alpha\beta}$.

$[D]$ represents the line bundle of which transition functions are given by $g_{\alpha\beta} = \frac{f_\alpha}{f_\beta}$ on $U_\alpha \cap U_\beta$.

See P6f.

$\Rightarrow$ $\pi^*[D]$ has the transition functions $\pi^* g_{\alpha\beta}$ on $\pi^{-1}(U_\alpha \cap U_\beta) = \pi^{-1}(U_\alpha) \cap \pi^{-1}(U_\beta)$.

$\Rightarrow$ $\pi^*[D] = [\pi^* D]$.

⟫