Now we compute

$$\bar\partial\psi = \bar\partial f\wedge\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q + A^\circ(f)\,\bar\varphi_*$$

$$= \sum_k \bar v_k(f)\,\bar\varphi_k\wedge\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q + A^\circ(f)\,\bar\varphi_*$$

$$\equiv \sum_k f_{\bar k}\,(-1)^q\,\bar\varphi_1\wedge\cdots\bar\varphi_q\wedge\bar\varphi_k = (-1)^q\sum_{k>q} f_{\bar k}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q\wedge\bar\varphi_k$$

Notation: $f_{\bar k} = \bar v_k(f) = \bar\partial f(\bar v_k)$

$$*\bar\partial\psi \equiv (-1)^q\sum_{k>q} *\left(f_{\bar k}\,\bar\varphi_1\wedge\cdots\wedge\bar\varphi_q\wedge\bar\varphi_k\right) = (-1)^q\sum_{k>q}\bar f_{\bar k}\,\bar\varphi_{q+1}\wedge\bar\varphi_{q+2}\wedge\cdots$$

$$\cdots\hat{\bar\varphi}_k\cdots\bar\varphi_n\wedge\Phi'\,(-1)^{k-1-q}\,(-1)^{\frac{n(n-1)}{2}}\,i^{-n}$$

$$\bar f_{\bar k} = \overline{\bar v_k(f)} = v_k(\bar f) = f_k.$$ omit

$$= (-1)^q\sum_{k>q}(-1)^{k-1-q}(-1)^{\frac{n(n-1)}{2}}\,i^{-n}\,\bar f_k\,\bar\varphi_{q+1}\wedge\bar\varphi_{q+2}\wedge\cdots\hat{\bar\varphi}_k\cdots\bar\varphi_n\wedge\Phi'$$

$$\bar\partial*\bar\partial\psi \equiv (-1)^q\sum_{k>q}(-1)^{k-1-q}\,\bar\partial\bar f_k\wedge\bar\varphi_{q+1}\wedge\bar\varphi_{q+2}\wedge\cdots\hat{\bar\varphi}_k\cdots\bar\varphi_n\wedge\Phi'$$

$$= (-1)^q\sum_{k>q,\,l}(-1)^{k-1-q}\,(\bar f_k)_{\bar l}\,(=\bar f_{k,\bar l})\;\bar\varphi_l\wedge\bar\varphi_{q+1}\wedge\bar\varphi_{q+2}\wedge\cdots\hat{\bar\varphi}_k\cdots\bar\varphi_n\wedge\Phi'$$

$$= (-1)^q\sum_{\substack{k>q\\ l\le q}}(-1)^{k-1-q}\,\bar f_{k,\bar l}\;\bar\varphi_l\wedge\bar\varphi_{q+1}\wedge\bar\varphi_{q+2}\wedge\cdots\hat{\bar\varphi}_k\cdots\bar\varphi_n\wedge\Phi'$$

$$+ (-1)^q\sum_{k>q}(-1)^{2(k-1-q)}\,\bar f_{k,\bar k}\;\bar\varphi_{q+1}\wedge\cdots\bar\varphi_n\wedge\Phi'$$

$$= (-1)^q\sum_{k>q}\bar f_{k,\bar k}\,\bar\varphi_{q+1}\wedge\cdots\wedge\bar\varphi_n\wedge\Phi'$$

$$+ \sum_{\substack{k>q\\ l\le q}}(-1)^{k-1}\,\bar f_{k,\bar l}\;\bar\varphi_l\wedge\bar\varphi_{q+1}\wedge\cdots\hat{\bar\varphi}_k\cdots\bar\varphi_n\wedge\Phi'.$$