s.t for any closed $(n-k)$ - form ψ,

$$\int_M \varphi \wedge \psi = \int_A \psi .$$

A note: The ordinary cup product $\alpha \cup \beta$ of two cohomology classes $\alpha \in H^k(M.Q)$ and $\beta \in H^{k'}(M.Q)$ may be defined as the pull back

$$\alpha \cup \beta = \Delta^*(\alpha \otimes \beta)$$ via the diagonal map $\Delta : M \longrightarrow M \times M$. of the the class $\alpha \otimes \beta$ on $M \times M$ defined by $\alpha \otimes \beta\,(\sigma \times \tau) = \alpha(\sigma) \cdot \beta(\tau)$ for all cycles $\sigma . \tau$ on M.

With this definition, it is clear that if φ and ψ are closed forms on M representing α and β, the form $\pi_1^*\varphi \wedge \pi_2^*\psi$ on $M \times M$ represents $\alpha \otimes \beta$;

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$$\begin{tikzcd} H_{DR}^{k+k'}(M \times M, Q) \arrow[r] & H^{k+k'}(M \times M . Q) \ni \alpha \otimes \beta \\ \pi_1^*\varphi \wedge \pi_2^*\psi \arrow[u, phantom, "\in" rotate=90] & \end{tikzcd}$$

$$\int_{\mu \times \nu} \pi_1^*\varphi \wedge \pi_2^*\psi = \int_\mu \varphi \cdot \int_\nu \psi = \alpha(\mu) \cdot \beta(\nu)$$

$\int_\mu \varphi = \alpha(\mu)$ $\quad \int_\nu \psi = \beta(\nu)$ by definition of ~~re~~ de Rham isomorphism. ⌟)

Hence $\varphi \wedge \psi$ represents the class $\alpha \cup \beta$,

$$\begin{tikzcd} \pi_1^*\varphi \wedge \pi_2^*\psi \in H_{DR}^{k+k'}(M \times M, Q) \arrow[r, "\Delta^*"] \arrow[d, leftrightarrow] & H_{DR}^{k+k'}(M . Q) \ni \varphi \wedge \psi \arrow[d, leftrightarrow] \\ \cancel{\varphi \wedge \psi} \in H^{k+k'}(M \times M, Q) \arrow[r, "\Delta^*"] & H^{k+k'}(M, Q) \\ \alpha \otimes \beta \arrow[r, mapsto] \arrow[u, phantom, "\in" rotate=90] & \alpha \cup \beta \end{tikzcd}$$

by functoriality of the de Rham isomorphism. P45

⇒ P67