where $\quad c_{p,\mu,\nu} = \int_\Delta \varphi^*_{\mu,\nu,n-p,p} = (-1)^{n-p}\,\delta_{\mu,\nu}$,

i.e., η_Δ is represented by

$$\varphi_\Delta = \sum_{p,\mu} (-1)^{n-p}\, \varphi_{\mu,\mu,p,n-p}.$$

$\ulcorner$

$$\varphi_\Delta \in H^n_{DR}(M\times M).$$

$\Rightarrow$ Put $\varphi_\Delta = \sum_{p,\mu,\nu} c_{p,\mu,\nu}\, \varphi_{\mu,\nu,p,n-p}$.

To obtain $c_{p,\mu,\nu}$, wedge product $\varphi^*_{\mu,\nu,p,n-p}$ both sides.

$\Rightarrow$

$$\varphi_\Delta \wedge \varphi^*_{\mu,\nu,p,n-p} = \sum_{p',\mu',\nu'} c_{p',\mu',\nu'}\, \varphi_{\mu',\nu',p',n-p'} \wedge \varphi^*_{\mu,\nu,p,n-p}$$

Integrate over $M\times M$, then we get

$$\int_{M\times M} \varphi_\Delta \wedge \varphi^*_{\mu,\nu,p,n-p} = c_{p,\mu,\nu} = \int_\Delta \varphi^*_{\mu,\nu,p,n-p} \quad \text{by P59}$$

$$= \int_\Delta (-1)^{(n-p)(p+n-p)}\, \pi_1^*\psi^*_{\mu,n-p} \wedge \pi_2^*\psi_{\nu,p}$$

$$= (-1)^{n(n-p)} \int \psi^*_{\mu,n-p} \wedge \psi_{\nu,p} \quad \text{by P59}$$

$$= (-1)^{n(n-p)} \int \psi_{\nu,p} \wedge \psi^*_{\mu,n-p}\ (-1)^{p(n-p)}$$

$$= (-1)^{(n+p)(n-p)} \int \psi_{\nu,p} \wedge \psi^*_{\mu,n-p} = (-1)^{(n+p)(n-p)}\,\delta_{\nu\mu} = (-1)^{n-p}\,\delta_{\mu\nu}$$

since $(n+p)(n-p) - (n-p) = n^2 - p^2 - n + p = n(n-1) - p(p-1)$ is even.