$$\begin{tikzcd} & U_\alpha \cap U_\beta \arrow[dl] \arrow[dr] & \\ \mathbb{C}^2 \ni (z_\alpha, w_\alpha) & & \mathbb{C}^2 \ni (z_\beta, w_\beta) \arrow[ll, "\phi"'] \end{tikzcd}$$

$\phi(z_\beta, w_\beta) = (z_\alpha, w_\alpha) \Rightarrow z_\alpha, w_\alpha$ are functions of z_β, w_β. $\Rightarrow$ We may write $z_\alpha = z_\alpha(z_\beta, w_\beta)$ $w_\alpha = w_\alpha(z_\beta, w_\beta)$. $\Rightarrow$ $w_\beta(z_\beta, 0) = 0$, obviously. 」

$w_\alpha = 0 \Leftrightarrow L \Leftrightarrow w_\beta = 0 \Rightarrow w_\alpha = 0 = w_\alpha(z_\beta, 0)$

In U_α, the sections of the sheaf $\mathcal{O}_{(\mu)}$ are just the holomorphic functions $f_\alpha(z_\alpha, w_\alpha) = f_0(z_\alpha) + f_1(z_\alpha) w_\alpha + \cdots + f_\mu(z_\alpha) w_\alpha^{\mu}$ taken modulo $w_\alpha^{\mu+1}$. The data in the Reiss relation are given by $f_\alpha \in \mathcal{O}_{(2)}(U_\alpha)$ with $f_\alpha / f_\beta = f_{\alpha\beta} \in \mathcal{O}^*_{(2)}(U_\alpha \cap U_\beta)$. Thus, giving the second-order elements of arc is the same as giving an invertible sheaf $\mathcal{L}_{(2)} \in H^1(L_{(2)}, \mathcal{O}^*_{(2)})$ and section $\sigma_{(2)} \in H^0(\mathcal{L}_{(2)})$.

「 Giving a prescribed second-order elemats of arc is choosing an element in $\mathcal{O}/I^3$. I hope this is correct understanding. $\Rightarrow$ In U_α, $f_\alpha + I$, $f_\alpha \in \mathcal{O}(U_\alpha)$.
In $U_\alpha \cap U_\beta$, $f_\alpha + I \equiv f_\beta + I \pmod{\mathcal{O}^*(U_\alpha \cap U_\beta)}$.

$\Rightarrow$ $g_{\alpha\beta} f_\beta + I = f_\alpha + I \Rightarrow g_{\alpha\beta} f_\beta - f_\alpha \in I$

$\Rightarrow$ $\left(g_{\alpha\beta} - \frac{f_\alpha}{f_\beta}\right) f_\beta \in I$. $\Rightarrow$ Since I is prime ideal and $f_\beta \notin I$, $g_{\alpha\beta} - \frac{f_\alpha}{f_\beta} \in I$. $\Rightarrow$ $g_{\alpha\beta} + I = \frac{f_\alpha}{f_\beta} + I$.

" Since $f_\alpha + D_\alpha$ & $f_\beta + D_\beta$ must represent the same curve, $\frac{f_\alpha + D_\alpha}{f_\beta + D_\beta} = g_{\alpha\beta}$. "