tal theorem of algebra the map

$$\sum q_i \longmapsto (\sigma_1\{z_i(q_i)\}, \dots \sigma_d\{z_i(q_i)\})$$

gives a coordinate chart on $\pi(U_1\times\cdots\times U_d) \subset S^{(d)}$.

┌ We are going to explain that for $d=2$ case.

We have to check that on $\pi(U_1\times U_2)\cap\pi(U_1'\times U_2')$
$\varphi\circ\psi^{-1}$ & $\psi\circ\varphi^{-1}$ are holomorphic, where

$$\varphi : \pi(U_1\times U_2) \longrightarrow \mathbb{C}\times\mathbb{C}$$
$$q_1+q_2 \longmapsto (\underbrace{z_1(q_1)+z_2(q_2)}_{=\sigma_1(z_1,z_2)},\ \underbrace{z_1(q_1)\,z_2(q_2)}_{=\sigma_2(z_1,z_2)})$$

$$\psi : \pi(U_1'\times U_2') \longrightarrow \mathbb{C}\times\mathbb{C}$$
$$q_1'+q_2' \longmapsto (\underbrace{z_1'(q_1')+z_2'(q_2')}_{=\sigma_1(z_1',z_2')},\ \underbrace{z_1'(q_1')\,z_2'(q_2')}_{=\sigma_2(z_1',z_2')}).$$

(i) $U_1\cap U_2=\phi$ & $U_1'=U_2'$

$\Rightarrow$ $z_1'=z_2'$. $\Rightarrow$ $q_1+q_2 \in \pi(U_1\times U_2)\cap\pi(U_1'\times U_2')$

$\Rightarrow$ $\exists$ open sets $V_1\ni q_1$, $V_2\ni q_2$ s.t. $V_1, V_2 \subset U_1'=U_2'$
$V_1\subset U_1$, $V_2\subset U_2$. $\pi(V_1\times V_2)\subset \pi(U_1\times U_2)$
$\cap\ \pi(U_1'\times U_2')$, and $V_1\cap V_2=\phi$.

$$\begin{array}{ccc} \mathbb{C} & & \mathbb{C} \\ {\scriptstyle z_1}\uparrow & & \uparrow{\scriptstyle z_2} \\ V_1 & \times & V_2 \\ {\scriptstyle z_1'}\downarrow & & \downarrow{\scriptstyle z_2'} \\ \mathbb{C} & , & \mathbb{C} \end{array}$$

$\Rightarrow$ z_1' is a function of z_1
z_2' " of z_2

But since $z_1'=z_2'$ as a variable, we may write

$z_1'(z_1)=f(z_1)$ ⇐ Nonsense
$z_2'(z_1)=f(z_2)$ . . .