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$$\Rightarrow \quad \iota(v)\,\big(\omega \wedge (\bar\partial \omega)^{n-r-1} \wedge \iota(v)\, P_{r+1}(E, \Theta)\big)$$
$$= \iota(v)\,(\omega \wedge (\bar\partial \omega)^{n-r-1}) \wedge \iota(v)\, P_{r+1}(E, \Theta) \quad (\text{since } \iota(v)\,\iota(v) = 0.)$$
$$= (\bar\partial \omega)^{n-r-1} \wedge \iota(v)\, P_{r+1}(E, \Theta) \quad (\text{since } \iota(v)\,\bar\partial\omega = 0 \;\&\; \iota(v)\,\omega = 1)$$

$\iota(v)\, P_0(E, \Theta)$ is trivially zero, and so if we set

$$\Phi = \sum_{i=0}^{n-1} \Phi_i,$$

we see that

$$\iota(v)\,\bar\partial \Phi = \sum_{i=1}^{n-1} (\bar\partial\omega)^{n-r} \wedge \iota(v)\, P_i(E, \Theta) - \sum_{i=1}^{n} (\bar\partial\omega)^{n-i} \wedge \iota(v) \cdot P_i(E, \Theta) = -\iota(v)\, P_n(\Theta) = -\iota(v)\, P(\Theta).$$

$$P_0(E, \Theta) = \tilde P(E, \cdots E) \in A^{0,0}(M)$$
$$\iota(v)\, P_0(E, \Theta) = 0$$ by definition (We had better think this way, no! see below*)

$$\iota(v)\,\bar\partial\Phi = \sum_{i=0}^{n-1} \iota(v)\,\bar\partial\Phi_i = \sum_{i=0}^{n-1} (\bar\partial\omega)^{n-i} \wedge \iota(v)\, P_i(E,\Theta) - (\bar\partial\omega)^{n-i-1} \wedge \iota(v)\, P_{i+1}(E,\Theta)$$

$$\begin{aligned}
= \; & (\bar\partial\omega)^{n} \wedge \iota(v)\, P_0(E, \Theta) - \cancel{(\bar\partial\omega)^{n-1} \wedge \iota(v)\, P_1(E, \Theta)} \\
& + \cancel{(\bar\partial\omega)^{n-1} \wedge \iota(v)\, P_1(E, \Theta)} - \cancel{(\bar\partial\omega)^{n-2} \wedge \iota(v)\, P_2(E, \Theta)} \\
& + \cancel{(\bar\partial\omega)^{n-2} \wedge \iota(v)\, P_2(E, \Theta)} - (\bar\partial\omega)^{n-3} \wedge \iota(v)\, P_3(E, \Theta) \\
& \quad \vdots \\
& + (\bar\partial\omega) \wedge \iota(v)\, P_{n-1}(E, \Theta) - (\bar\partial\omega)^{0} \wedge \iota(v)\, P_n(E, \Theta) \\
= \; & -\iota(v)\, P_n(E, \Theta) = -\iota(v)\, P(\Theta).
\end{aligned}$$

* $$\Phi_0 = \omega \wedge (\bar\partial\omega)^{n-1}\, \tilde P(E, \cdots E) \Rightarrow \bar\partial \Phi_0 = (\bar\partial\omega)^{n}\, \tilde P(E, \cdots E) \quad = 0$$
$$- \omega \wedge (\bar\partial\omega)^{n-1} \wedge \bar\partial \tilde P(E, \cdots E) \Rightarrow \iota(v)\,\bar\partial\Phi_0 = \cancel{\iota(v)\,(\bar\partial\omega)^{n}}\, \tilde P(E \cdots E)$$
$$- (\bar\partial\omega)^{n-1} \wedge \iota(v)\, P_1(E, \Theta) = -(\bar\partial\omega)^{n-1} \wedge \iota(v)\, P_1(E, \Theta).$$