and the inequality

$$\left\langle c_1(\Theta)^2 - 2c_2(\Theta),\ \left(\frac{2}{\sqrt{-1}}\right)^2 \tau_1\wedge\bar\tau_1\wedge\tau_2\wedge\bar\tau_2 \right\rangle \geq 0$$

for (1.0) vectors τ_1, τ_2 follows from the usual Cauchy-Schwartz inequality.

Γ $2c_2(\Theta) - 2c_2(\Theta)$

$$= \left(\frac{\sqrt{-1}}{2\pi}\right)^2 \left\{ 2A^1_\mu \bar A^1_\mu \cancel{A^2_\lambda \bar A^2_\lambda} - 2A^1_\mu \cancel{\bar A^2_\mu A^2_\lambda} \bar A^1_\lambda \right\} - \left(\frac{\sqrt{-1}}{2\pi}\right)^2 \left\{ 2\cancel{A^1_\mu \bar A^1_\mu} A^2_\lambda \cancel{\bar A^2_\lambda} - 2A^1_\mu \cancel{\bar A^2_\mu} A^2_\lambda A^1_\lambda \right\} = \left(\frac{\sqrt{-1}}{2\pi}\right)^2 \{ \cdots \quad = 0.$$

$$c_1(\Theta) - 2c_2(\Theta) = \left(\frac{\sqrt{-1}}{2\pi}\right)^2 \left\{ -(A^1_\mu A^1_\lambda \bar A^1_\mu \bar A^1_\lambda + A^2_\mu A^2_\lambda \bar A^2_\mu \bar A^2_\lambda) + 2A^1_\mu \bar A^1_\mu A^2_\lambda \cancel{\bar A^2_\lambda} - 2A^1_\mu \cancel{\bar A^1_\mu A^2_\lambda} \bar A^2_\lambda + 2A^1_\mu \bar A^2_\mu A^2_\lambda A^1_\lambda \right\}$$

$$= (2A^1_\mu \bar A^2_\mu A^2_\lambda \bar A^1_\lambda - A^1_\mu A^1_\lambda \bar A^1_\mu \bar A^1_\lambda - A^2_\mu A^2_\lambda \bar A^2_\mu \bar A^2_\lambda)\left(\frac{\sqrt{-1}}{2\pi}\right)^2.$$

$$\left\langle c_1(\Theta)^2 - 2c_2(\Theta),\ \left(\frac{2}{\sqrt{-1}}\right)^2 \tau_1\wedge\bar\tau_1\wedge\tau_2\wedge\bar\tau_2 \right\rangle$$

$$= \left(\frac{\sqrt{-1}}{2\pi}\right)^2 \left(\frac{2}{\sqrt{-1}}\right)^2 \left\{ 2\langle A^1_\mu \bar A^2_\mu A^2_\lambda \bar A^1_\lambda,\ \tau_1\wedge\bar\tau_1\wedge\tau_2\wedge\bar\tau_2\rangle - \langle A^1_\mu A^1_\lambda \bar A^1_\mu \bar A^1_\lambda,\ \tau_1\wedge\bar\tau_1\wedge\tau_2\wedge\bar\tau_2\rangle - \langle A^2_\mu A^2_\lambda \bar A^2_\mu \bar A^2_\lambda,\ \tau_1\wedge\bar\tau_1\wedge\tau_2\wedge\bar\tau_2\rangle \right\}$$

$$\Rightarrow\ 2\langle A^1_\mu \bar A^2_\mu A^2_\lambda \bar A^1_\lambda,\ \tau_1\wedge\bar\tau_1\wedge\tau_2\wedge\bar\tau_2\rangle$$

$$= 2\begin{vmatrix} A^1_\mu(\tau_1), & 0, & A^1_\mu(\tau_2), & 0 \\ 0, & \overline{A^2_\mu(\tau_1)}, & 0, & \overline{A^2_\mu(\tau_2)} \\ A^2_\lambda(\tau_1), & 0. & A^2_\lambda(\tau_2), & 0 \\ 0, & \overline{A^1_\lambda(\tau_1)}, & 0, & \overline{A^1_\lambda(\tau_2)} \end{vmatrix} = 2A^1_\mu(\tau_1)A^2_\lambda(\tau_2)\left(\overline{A^2_\mu(\tau_1)}\,\overline{A^1_\lambda(\tau_2)} - \overline{A^1_\lambda(\tau_1)}\,\overline{A^2_\mu(\tau_2)}\right)$$

$$- 2A^1_\mu(\tau_2)A^2_\lambda(\tau_1)\left(\overline{A^2_\mu(\tau_1)}\,\overline{A^1_\lambda(\tau_2)} - \overline{A^2_\mu(\tau_2)}\,\overline{A^1_\lambda(\tau_1)}\right)$$

$$= 2\det\begin{pmatrix} A^1_\mu(\tau_1), & A^1_\mu(\tau_2) \\ A^2_\lambda(\tau_1), & A^2_\lambda(\tau_2) \end{pmatrix} \det\begin{pmatrix} A^2_\mu(\tau_1), & A^2_\mu(\tau_2) \\ A^1_\lambda(\tau_1), & A^1_\lambda(\tau_2) \end{pmatrix}$$