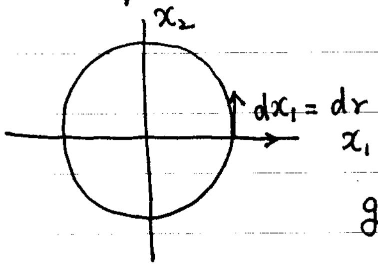
Point : $\sigma = c\, dx_2 \wedge \cdots \wedge dx_n$ at $(1, 0 \cdots 0)$ for some constant c. Since σ is invariant under $SO(n)$, $\sigma = R_g^*(c\, dx_2 \wedge \cdots \wedge dx_n)$ for all $g \in SO(n)$.

For example $n=2$.



At $(1,0)$, $dr = dx_1$

$\sigma = c\, dx_2$.

$$g = \begin{pmatrix} g_{11}, & g_{12} \\ g_{21}, & g_{22} \end{pmatrix} \in SO(2)$$

$$R_g^* dx_2 = a\, dx_1 + b\, dx_2$$

$$\Rightarrow a = R_g^* dx_2\left(\frac{\partial}{\partial x_1}\right) = dx_2\left(\begin{pmatrix} g_{11}, & g_{12} \\ g_{21}, & g_{22} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right)$$

$$= dx_2\left(g_{11}\frac{\partial}{\partial x_1} + g_{21}\frac{\partial}{\partial x_2}\right) = g_{21}$$

$$b = R_g^* dx_2\left(\frac{\partial}{\partial x_2}\right) = dx_2\left(\begin{pmatrix} g_{11}, & g_{12} \\ g_{21}, & g_{22} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = dx_2\left(g_{12}\frac{\partial}{\partial x_1} + g_{22}\frac{\partial}{\partial x_2}\right)$$

$$= g_{22} \qquad \Rightarrow \qquad R_g^* dx_2 = g_{21}\, dx_1 + g_{22}\, dx_2$$

Since R_g^* is the pull-back, at $g^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = {}^t g \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} g_{11} \\ g_{12} \end{pmatrix}$, $R_g^* dx_2$ should be considered. $\Rightarrow$ $x_1 = g_{11}$, $x_2 = g_{12}$.

Since $x_1^2 + x_2^2 = 1$, $\begin{vmatrix} x_1 & x_2 \\ g_{21}, & g_{22} \end{vmatrix} = 1$, & $g_{21}^2 + x_1^2 = 1$,

$$g_{21} = \mp x_2 \quad \& \quad g_{22} = \pm x_1.$$

$$\Rightarrow \quad R_g^* dx_2 = c\,(x_2\, dx_1 - x_1\, dx_2)$$

(e) In fact it proves more. If we consider $f = (f_1, \cdots f_n)$ as a mapping

$$f : U^* \longrightarrow \mathbb{C}^n - \{0\},$$

then we have essentially shown that