

etermined and must satisfy the following: For example,

$$\det \begin{pmatrix} d_f(z) g(A_1(z)), f(A_1(z))^{u-2}, \dots, 1 \\ \vdots \\ d_f(z) g(A_u(z)), f(A_u(z))^{u-2}, \dots, 1 \end{pmatrix} = \det \begin{pmatrix} a_1(z) f(A_1(z))^{u-1} + \dots + a_{u-1}(z), f(A_1(z))^{u-2}, \dots, 1 \\ \vdots \\ a_1(z) f(A_u(z))^{u-1} + \dots + a_{u-1}(z), f(A_u(z))^{u-2}, \dots, 1 \end{pmatrix}$$

$$= a_1(z) \det \begin{pmatrix} f(A_1(z))^{u-1}, f(A_1(z))^{u-2}, \dots, 1 \\ \vdots \\ f(A_u(z))^{u-1}, f(A_u(z))^{u-2}, \dots, 1 \end{pmatrix}$$

$$a_k(z) \det \{ f(A_j(z))^{u-1}, \dots, f(A_j(z)), 1 \}$$

$$= \det \{ f(A_j(z))^{u-1}, \dots, f(A_j(z))^{u-k+1}, d_f(z) g(A_j(z)), f(A_j(z))^{u-k}, \dots, 1 \}$$

where in each determinant only the  $j$ th row is written out. The determinant on the left-hand side of this equation is the van der Monde determinant  $\Delta(z)$ , and it is a familiar result from elementary algebra that  $\Delta(z)^2 = d_f(z)$ . Hence, factoring  $d_f(z)$  out of the determinant on the right-hand side and dividing by  $\Delta(z)$  lead to

$$a_k(z) = \Delta(z) \cdot \det \{ f(A_j(z))^{u-1}, \dots, g(A_j(z)), f(A_j(z))^{u-k-1}, \dots, 1 \}.$$

Both  $\Delta(z)$  and  $\det \{ \dots \}$  change signs upon interchanging