⌈ Since $[\varphi] = [\varphi - d\eta]$, we may assume that $\varphi \equiv 0 \ (du)$. $\Rightarrow$ $\varphi = \sum \varphi_I \, du_I$.

$$d\varphi = \sum_{I,i} \frac{\partial \varphi_I}{\partial u_i} du_i \wedge du_I + \sum_{I,j} \frac{\partial \varphi_I}{\partial v_j} dv_j \wedge du_I = 0$$

$\Rightarrow$ For each j & I, $\frac{\partial \varphi_I}{\partial v_j} = 0$ since $dv_j \wedge du_I$'s are linearly independent. $\Rightarrow$ $\frac{\partial \varphi}{\partial v_j} = \sum \frac{\partial \varphi_I}{\partial v_j} du_I = 0$ ⌋

Inductively, we assume the theorem for $u' = (u_1, \cdots u_{k-1})$ and write

$$\varphi = \psi' + \psi'' \wedge du_k ,$$

where $\psi', \psi'' \equiv 0 \ (du')$. Consider the Laurent series

$$\psi'' = \sum_{\nu = -N}^{\infty} \psi''(u')_\nu \, u_k^\nu .$$

⌈ Since φ is holomorphic on $P^*(k,n)$, by Th 9.4 p242 Silverman ⌋

Then, by formally integrating the series insofar as possible, we may write

$$\psi'' - \frac{\psi''(u')_{-1}}{u_k} = \frac{\partial \eta}{\partial u_k} ,$$

where η has the same order pole in u' and one less order pole in u_k.

⌈ $$\psi'' = \sum_{\nu=-N}^{-2} \psi''(u')_\nu \, u_k^\nu + \frac{\psi''(u')_{-1}}{u_k} + \sum_{\nu=0}^{\infty} \psi''(u')_\nu \, u_k^\nu$$