Take an element $\{(f_\alpha, U_\alpha)\}$ in $C^0(\underline{U}, M^*)$.
Consider $\delta\{(f_\alpha, U_\alpha)\}$

$\Rightarrow$ $\dfrac{f_\alpha}{f_\beta} = g_{\alpha\beta} \in C^1(\underline{U}, M^*)$ and $g_{\alpha\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta)$ by the property of $\{(f_\alpha, U_\alpha)\}$.

$\Rightarrow$ $\{g_{\alpha\beta}\} \in C^1(\underline{U}, M^*)$ is clearly 1-cocycle.

$\Rightarrow$ $\{g_{\alpha\beta}\}$ defines a line bundle, which is exactly same as $[D]$.

We now wish to discuss holomorphic and meromorphic sections of line bundles. Let $L \longrightarrow M$ be a holomorphic line bundle, with trivializations $\varphi_\alpha : L_{U_\alpha} \longrightarrow U_\alpha \times \mathbb{C}$ over an open cover $\{U_\alpha\}$ of M and transition functions $\{g_{\alpha\beta}\}$ relative to $\{\varphi_\alpha\}$. As we have seen, the trivialization φ_α induce isomorphisms

$$\underbrace{\varphi_\alpha^* : \mathcal{O}(L)(U_\alpha)}_{L\text{-valued holomorphic sections of } L \text{ over } U_\alpha} \longrightarrow \mathcal{O}(U_\alpha)$$

we see via the correspondence

$$s \in \mathcal{O}(L)(U) \longmapsto \{s_\alpha = \varphi_\alpha^*(s) \in \mathcal{O}(U \cap U_\alpha)\}$$

that a section of L over $U \subset M$ is given exactly by a collection of functions $s_\alpha \in \mathcal{O}(U \cap U_\alpha)$ satisfying

$$s_\alpha = g_{\alpha\beta} \cdot s_\beta \quad \text{in} \quad U \cap U_\alpha \cap U_\beta .$$

$$\varphi_\alpha : L_{U_\alpha} \longrightarrow U_\alpha \times \mathbb{C}$$

$$\mathcal{O}(L)(U_\alpha) \ni s \longmapsto \varphi_\alpha^*(s) \in \mathcal{O}^*(U_\alpha)$$

$$\begin{tikzcd} L_{U_\alpha} \arrow[r, "\varphi_\alpha"] & U_\alpha \times \mathbb{C} \ni (x, \varphi_\alpha^*(s)) \\ U_\alpha \ni x \arrow[u, "s"] \arrow[ur] & \end{tikzcd}$$

$$s_\alpha = \varphi_\alpha^*(s), \quad (x, \varphi_\alpha^*(s)) \longleftrightarrow (x, \varphi_\beta^*(s)) \in U_\beta \times \mathbb{C}, \quad \varphi_\beta^*(s) = s_\beta$$

$$\varphi_\alpha^* \varphi_\beta^{*-1}(s_\beta) = \underline{\underline{g_{\alpha\beta}(x) s_\beta}} = s_\alpha$$

$$\varphi_\beta^*(s) \longleftarrow \varphi_\alpha^*(s) = s_\alpha$$

$$\text{"} \varphi_\beta \varphi_\alpha^{*}(s_\alpha) = g_{\beta\alpha}(s_\alpha) = s_\beta .$$