

$\Rightarrow g^{-1}(U)$ is an open subset of $N-W$, where W is an analytic subvariety of N , s.t. $\text{cod } W \geq 2$

$g: N-W \rightarrow M$ is holomorphic.

\Rightarrow Clearly $g^{-1}(D)$ is analytic in $g^{-1}(U)$.

$\Rightarrow g^{-1}(D)$ is analytic in N , since analyticity is a local problem, and whenever we have a point $q \in \overline{g^{-1}(D)} \cap W$, consider an open set $O_q \ni q$ s.t. $O_q \cong \Delta$ O_q is small, and then, by using the Levi extension theorem II, $O_q \cap \overline{g^{-1}(D)}$ is analytic in O_q . "

Examples of Rational and Birational Maps

1. Any holomorphic map $M \rightarrow \mathbb{P}^n$ is trivially rational.

2. If $\tilde{M} \xrightarrow{\pi} M$ is the blow-up of an algebraic variety M at a collection of points $\{p_i\}$, then the inverse map

$$M - \{p_i\} \xrightarrow{\pi^{-1}} \tilde{M}$$

is clearly rational, so π is a rational isomorphism.

By Prop 4, $\pi: \tilde{U}_i \rightarrow U$

$$(z(i), \dots, z(i)_n) \mapsto (z(i)_i \cdot z(i), \dots, z(i)_i \cdot z(i)_i \cdot z(i)_n).$$

$$\Rightarrow U \rightarrow \tilde{U}_i$$

$$(z_1, \dots, z_n) \mapsto \left(\frac{z_1}{z_i}, \frac{z_2}{z_i}, \dots, z_i, \dots, \frac{z_n}{z_i} \right)$$

$\Rightarrow \frac{z_1}{z_i}, \dots, \frac{z_n}{z_i}$ are meromorphic functions. \square

A holomorphic map $f: \tilde{M} \rightarrow \mathbb{P}^n$ thus gives a rational map