$$\Rightarrow \begin{pmatrix} 1, & -\frac{z_1}{z_3} \\ 0, & \frac{1}{z_3} \end{pmatrix} \begin{pmatrix} 1, & 0, & z_1, & z_2 \\ 0, & 1, & z_3 & z_4 \end{pmatrix}$$

$$= \begin{pmatrix} 1, & -\frac{z_1}{z_3}, & z_1 - z_1, & z_2 - \frac{z_1 z_4}{z_3} \\ 0, & \frac{1}{z_3}, & 1, & \frac{z_4}{z_3} \end{pmatrix}$$

$$\Rightarrow \quad \omega_1 = -\frac{z_1}{z_3}, \quad \omega_2 = z_2 - \frac{z_1 z_4}{z_3}$$

$$\omega_3 = \frac{1}{z_3}, \quad \omega_4 = \frac{z_4}{z_3}$$

$$\begin{array}{ccc} U_{12} \times \mathbb{C}^4 & \longrightarrow & U_{12} \times \mathbb{C}^4 \\ \cup & & \\ S|_{U_{12}} & \longrightarrow & U_{12} \times \mathbb{C}^2 \\ \Downarrow & & \\ (\Lambda, v) & \longmapsto & (\Lambda, {}^t(x_1, x_2)) \end{array}$$

In general,

$$\begin{array}{ccccc} U_{I'} \times \mathbb{C}^k & \longleftarrow & S|_{U_I \cap U_{I'}} & \longrightarrow & U_I \times \mathbb{C}^k \\ & & \Downarrow & & \\ (\Lambda, {}^t(y_1, \cdots y_k)) & \longleftarrow & (\Lambda, v) & \longmapsto & (\Lambda, {}^t(x_1, x_2, \cdots x_k)) \end{array}$$

Let $\Lambda^I = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ on $U_I$

$\Lambda^{I'} = \begin{pmatrix} v_1' \\ \vdots \\ v_n' \end{pmatrix}$ on $U_{I'}$.