Write $\psi = \psi(w)\, dw' \wedge d\bar{w}' + \varphi$, where $w' = (w_2, \cdots w_n)$ and all terms of φ contain either dw_1 or $d\bar{w}_1$; then in any polydisc Δ around z_0,

$$\lim_{\epsilon \to 0} \int_{\partial D(\epsilon) \cap \Delta} \partial \log f_\alpha \wedge \psi = \lim_{\epsilon \to 0} \int_{|w_1| = \epsilon} \frac{dw_1}{w_1} \cdot \psi(w) \cdot dw' \wedge d\bar{w}'$$

$$= 2\pi i \int_{w'} \psi(0, w')\, dw' \wedge d\bar{w}' = 2\pi i \int_{V \cap \Delta} \psi$$

and so $$\int_M \Theta \wedge \psi = -i\, Im\left(2\pi i \int_V \psi\right) = \frac{2\pi}{i} \int_V \psi.$$

Q. E. D.

① We can write $\psi = \psi(w)\, dw' \wedge d\bar{w}' + \varphi$, where $w' = (w_2 \cdots w_n)$ and all terms of φ contain either dw_1 or $d\bar{w}_1$.

Let Δ (in M) be a nbd of $z_0 \in V \cap U_\alpha$. z_0 is a smooth point.

② The poly-disc Δ is in the nbd of z_0.

$$f_\alpha = w_1 \Rightarrow \partial \log f_\alpha = \partial \log w_1 = \frac{dw_1}{w_1}$$

and $dw_1 \wedge \varphi = 0$.

③ $Im\left((2\pi i) \int_V \psi\right) = 2\pi \int_V \psi$ since ψ is real.

④ $$\lim_{\epsilon \to 0} \int_{\partial D(\epsilon) \cap \Delta} \partial \log f_\alpha \wedge \psi = 2\pi i \int_{V \cap \Delta} \psi$$

$$\Rightarrow \lim_{\epsilon \to 0} \int_{(\partial D(\epsilon) - V) \cap \Delta} \partial \log f_\alpha \wedge \psi - 2\pi i \psi = 0$$

$\Rightarrow$ Non sense.

$$\Rightarrow \lim_{\epsilon \to 0} \int_{\partial D(\epsilon) - V}$$