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a secant line through p in h, it would follow that X contained the pencil $\sigma(p,h)$, hence all of $\sigma(h)$. Thus, X can contain a priori only one pencil in $\sigma(h)$, and so $h \in R^*$.

II By the result on P765 ⊛ (see P905 note),
since $T_x(X) \cap X = L_1 \cup L_2 \cup L_3 \cup L_4$, where
$L_1 \cup L_2 = \sigma(p) \cap F$, $L_3 = \sigma(p'', h'')$ $L_4 = \sigma(p''', h''')$
$\Rightarrow$ $l_x \cap S = \{p, p''\}$ or $\{p, p'', p'''\}$ or $\{p, p'''\}$ or $\{p\}$. and $p'' = p'''$ means that $p'' \in l_x \cap S$,
and, so at p'' or p, l_x is tangent to S.
(In case p, p'', p''' are distinct.
$\sigma(p'') \cap F$ need not be contained in $T_x(X) \cap X$, but
$\sigma(p'') \cap F$ is a singular curve. $\Rightarrow$ $p'' \in S$.)
$\Rightarrow$ In any case, the multiplicity of p is ≥ 2.
$\Rightarrow$ l_x is tangent to S at p. (Remember p is generic $\Rightarrow$ $h \cap S$ is smooth at generic p)
$\Rightarrow$ Since h is tangent to S at p, and
the common line is tangent to S at p, the line lies in h.
'$\sigma(p) \cap X = \sigma(p) \cap F = L_1 \cup L_2 = \sigma(p, h_1) \cup \sigma(p, h_2)$
$\Rightarrow$ The common line is $h_1 \cap h_2$'
Now consider $\overline{pp''}$, $\overline{pp'''}$, and $h_1 \cap h_2$.
$\Rightarrow$ They all line in $\sigma(p, h)$, and since for generic $p \in C_h$, they are <u>distinct</u>, and they all lie in F, $\#(F \cdot \sigma(p, h)) = 3$.

since $\deg C_h = \deg(h \cap S) = 2$, and $h_1 \cap h_2$ can not be equal to $\overline{pp''}$ or $\overline{pp'''}$ ($\because \overline{pp^{\circ}} \cap C_h \ni p^{\circ}$.)