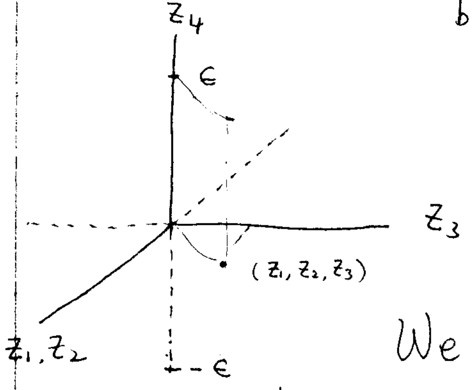
Let $\Delta' = \{ |z_i| < 1, \quad z_3 \neq 0 \} \cong \Delta^2 \times \Delta^* \times \Delta$.
Let $V = \Delta^2 \times \{0\} \times \{0\}$.

$\Rightarrow \quad H^1(\Delta', \mathcal{O}) \underset{\uparrow}{=} H^2(\Delta', \mathbb{Z}) = 0 \quad \Rightarrow H^1(\Delta', \mathcal{O}^*) = 0.$
by P27.



If $D^* = D \cap \Delta'$, then the line bundle $[D^*] \longrightarrow \Delta'$ is trivial and we have $h \in \mathcal{O}(\Delta')$ s.t $(h = 0) = D^*$.

We may assume that D doesn't contain the z_4-axis, and therefore $D \cap \underset{\{z_1 = z_2 = z_3 = 0\}}{\underbrace{\{z_1 = z_2 = z_3 = 0\}}}$ consists of a finite number of points in the punctured disc $0 < |z_4| < 1$. We may find a circle $|z_4| = \epsilon$ that does not meet these points. It follows by continuity (tube lemma) that, for δ sufficiently small, the locus

$\{ |z_4| = \epsilon, \quad |(z_1, z_2, z_3)| \leq \delta \}$ is compact since $\{(0, 0, 0\ z_4) : |z_4| = \epsilon\}$!

For (z_1, z_2, z_3) with $0 < |(z_1, z_2, z_3)| \leq \delta$ and $z_3 \neq 0$, the integral

$$\frac{1}{2\pi\sqrt{-1}} \int_{|z_2| = \epsilon} \frac{dh(z_1, z_2, z_3\ z_4)}{h(z_1, z_2, z_3\ z_4)}$$

is well-defined, continuous, and integer-valued. Compare with P8. Thus, projecting $\overline{D}$ on the $(z_1, z_2\ z_3)$ plane gives a proper mapping $\pi : \overline{D} \longrightarrow \{ \quad |(z_1, z_2, z_3)| \leq \delta \}$