$$\sum_i (-1)^{d_1+\cdots+d_{i-1}} \tilde{P}(A_1, \cdots, DA_i, \cdots A_q)$$

$$= \sum_i (-1)^{d_1+\cdots+d_{i-1}} \tilde{P}((a^1_{\alpha\beta}), \cdots\cdots, (da^i_{\alpha\beta} + (-1)^{d_i+1} \bar{\theta}_{\alpha k} \wedge a^i_{\alpha\beta} + (-1)^{d_i} a^i_{\alpha\ell} \wedge \theta_{\ell\beta}) \cdots\cdots, (a^q_{\alpha\beta}))$$

Let $A^1, \cdots A^q$ be matrices $(a^1_{\alpha\beta}), \cdots (a^q_{\alpha\beta})$, by abuse of notation. And let $\theta = (\theta_{\alpha\beta})$.

$$= \sum_i (-1)^{d_1+\cdots+d_{i-1}} \tilde{P}(A^1, \cdots, dA^i + (-1)^{d_i+1} \bar{\theta} \wedge A^i + (-1)^{d_i} A^i \wedge \theta, \cdots A^q)$$

$$= \sum_i (-1)^{d_1+\cdots+d_{i-1}} \tilde{P}(A^1, \cdots, dA^i, \cdots A^q)$$

$$+ \sum_i (-1)(-1)^{d_1+\cdots+d_{i-1}+d_i} \tilde{P}(A^1, \cdots, \bar{\theta} \wedge A^i, \cdots A^q)$$

$$+ \sum_i (-1)^{d_1+\cdots+d_{i-1}+d_i} \tilde{P}(A^1, \cdots, A^i \wedge \theta, \cdots A^q)$$

Again since we can choose $\theta(x_0) = 0$ at any point x_0,

$$\tilde{P}(A^1, \cdots \bar{\theta} \wedge A^i, \cdots A^q) = \tilde{P}(A^1, \cdots, A^i \wedge \theta, \cdots A^q) = 0 \text{ at } x_0.$$

$$\Rightarrow \sum_i (-1)^{d_1+\cdots+d_{i-1}} \tilde{P}(A_1, \cdots DA_i, \cdots A_q)$$

$$= \sum_i (-1)^{d_1+\cdots+d_{i-1}} \tilde{P}(A^1, \cdots, dA^i, \cdots A^q)$$

$$= d\tilde{P}(A^1, \cdots A^q) \quad \cdots \text{①}$$

$$\Rightarrow d\tilde{P}(A_1, \cdots, A_q) = d\tilde{P}(A^1, \cdots A^q) \cdots \text{②}$$

By ① & ②, $d\tilde{P}(A_1, \cdots A_q) = \sum (-1)^{d_1+\cdots+d_{i-1}} \tilde{P}(A_1, \cdots, DA_i, \cdots A_q)$