Date

Pick. $X_0 X_3^{d-1}, \; X_1 X_3^{d-1}, \; X_2 X_3^{d-1}, \; X_3^d, \cdots X_n^d. \Rightarrow$

$$\begin{pmatrix} X_3^{d-1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ X_3^{d-1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ X_3^{d-1} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ dX_3^{d-1} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ dX_n^{d-1} \end{pmatrix}$$

Keep going $\Rightarrow$ We always get a linearly independent set of $(n+1)$- vectors, except $X_0 = X_1 = \cdots = X_n = 0$.

$\Rightarrow$ Thus the Veronese map is a smooth embedding.

Given a hypersurface V of degree d in $\mathbb{P}^n$,
$\Rightarrow \exists$ a homogeneous polynomial F s.t $(F=0) = V$
see p 172.
$\Rightarrow$ F may be expressed as a linear combination of $\{Z^\alpha\}$, set of monomials, i.e. $\sum a_\alpha Z^\alpha = F$.

Let H be the hyperplane in $\mathbb{P}^{N = {}_{d+n}C_n - 1}$ defined by $\sum a_\alpha Z_\alpha = 0$.

$\Rightarrow V = \iota_{dH}(\mathbb{P}^n) \cap H = (\sum a_\alpha Z^\alpha = F = 0)$.

Here are a few cases:

1. The Veronese map

$$\iota_{mH} : \mathbb{P}^1 \longrightarrow \mathbb{P}^{(n) = {}_{1+n}C_1 - 1}$$

is given, in terms of the Euclidean coordinate $t = Z_1/Z_0$ on $\mathbb{P}^1$, by

$$t \longmapsto [1, t, t^2, \cdots : t^n].$$

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