Date . .

$$= \int_V \psi^{n-p,\,n-p} = \int_M \psi^{n-p,\,n-p} \wedge \eta = \int_M \psi \wedge \eta^{p,p}$$

since $\psi^{p',q'} \wedge \eta^{p,p} = 0$ unless $p' = q' = n-p$.

$\Rightarrow$
$\langle \psi, \eta^{p,p} \rangle = \langle \psi, \eta \rangle$

$\Rightarrow \langle \psi, \eta - \eta^{p,p} \rangle = 0$ for all ψ

$\Rightarrow \quad \eta = \eta^{p,p}$

The famous Hodge Conjecture asserts that the converse is also true: On $M \subset \mathbb{P}^N$ a submanifold of projective space every rational cohomology class of type (p,p) is analytic. Whether the Hodge conjecture is true or false is at present unknown; it is a very beautiful and very difficult problem. The only case which has been proved in general is the case $p = 1$; this is the

Lefschetz Theorem on $(1,1)$ classes.

For $M \subset \mathbb{P}^N$ a submanifold, every cohomology class

$$\gamma \in H^{1,1}(M) \cap H^2(M, \mathbb{Z})$$

is analytic; in fact

$$\gamma = \eta_D$$

for some divisor D on M.