

$$\Rightarrow 'E_3^{0,1} = \ker d_2 / \text{im} d_2 = \ker d_2 \subset 'E_2^{0,1}$$

$$'E_{3,0}^{-3,3} \longrightarrow 'E_3^{0,1} \xrightarrow{d_3} 'E_3^{3,1-3+1} = 0 \Rightarrow 'E_k^{0,1} = 'E_3^{0,1}$$

$$\Rightarrow 'E_\infty^{0,1} = 'E_3^{0,1} \subset 'E_2^{0,1} \quad \text{inclusion.}$$

$$\text{"ker } d_2$$

③ Note, from $'F^1 H^1 / 'F^2 H^1 \cong 'E_\infty^{1,0}$

$$0 \rightarrow 'F^2 H^1 \rightarrow 'F^1 H^1 \rightarrow 'E_\infty^{1,0} \rightarrow 0.$$

④ Note, let $K^{p,q} = C^p(\underline{U}, \Omega^q(*))$ $K^n = \bigoplus_{p+q=n} K^{p,q}$

$$'F^p K^{p+q} = K^{p,q} \oplus K^{p+1,q-1} \oplus \dots \oplus K^{p+q,0}$$

$$\Rightarrow \text{If } n > p, \text{ then } 'F^n K^p = K^{n,p-n} \oplus K^{n+1,p-n-1} \oplus \dots$$

$$K^{n,p-n} = C^n(\underline{U}, \Omega^{p-n}(*)) = K^{n+1,p-n-1} = \dots = 0$$

$$\Rightarrow 'F^n K^p = 0$$

Similarly, we get $'F^p K^{p+q} = K^{q,p} \oplus \dots \oplus K^{0,p+q}$

and $'F^n K^p = C^{p-n}(\underline{U}, \Omega^n(*)) \oplus C^{p-n-1}(\underline{U}, \Omega^{n+1}(*)) \oplus \dots$

$$= 0$$

Thus $'F^1 H^1 \cong 'E_\infty^{1,0}$

⑤ Note, $'E_{2,0}^{1,1} \rightarrow 'E_2^{1,0} \xrightarrow{d_2} 'E_2^{3,0-2+1} = 0$

$$\Rightarrow 'E_3^{1,0} = \ker d_2 = 'E_2^{1,0} \Rightarrow 'E_2^{1,0} = 'E_\infty^{1,0}$$

\Rightarrow By notes ① ~ ⑤, we have an exact sequence

$$0 \rightarrow 'E_2^{1,0} \rightarrow H^1 \rightarrow 'E_2^{0,1} \xrightarrow{d_2} 'E_2^{2,0} \rightarrow \text{coker } d_2 \rightarrow 0$$

$$\begin{array}{c} \parallel \\ 'E_\infty^{1,0} = 'F^1 H^1 \end{array} \nearrow \quad \searrow \quad \begin{array}{c} \parallel \\ 'E_\infty^{0,1} = \ker d_2 \end{array} \nearrow$$

⑥