

$n$  self-intersection.  $\Rightarrow$  Since  $\chi(X_p) = 2$ ,  $\chi(U) = 4 + \mu - n$ . Note that  $\{X_p\}_{p \in \mathbb{A}^1}$  is a pencil in  $U$ .  $\Rightarrow$   $X_p$  changes continuously as  $p$  varies in  $\mathbb{A}^1$ .  $\Rightarrow$  But we know that  $\{X_p\}_{p \in \mathbb{A}^1}$  has no base point. In particular, by assuming  $p_0 \neq 0$ ,  $(X_1 + p_1 X_3 + X_2 + p_1 X_4 = 0) \cap (X_1 + p_1' X_3 + X_2 + p_1' X_4 = 0) \cap (X_1 + p_1' X_3 = 0) = \sigma(p)$ ,  $(X_1 + p_1' X_3 + X_2 + p_1' X_4 = 0) \cap (X_1 + p_1' X_3 = 0) = \sigma(p')$   $\Rightarrow \sigma(p) \cap \sigma(p') = \emptyset$ , for we get  $(p_1 - p_1')X_3 + (p_1 - p_1')X_4 = 0$ , since  $p_1 \neq p_1'$ ,  $X_3 + X_4 = 0 \Rightarrow X_1 = X_2 = 0$ .  $\Rightarrow$  The self-intersection of a generic  $X_p$  is  $*((\sigma(p) \cap F) \cap (\sigma(p') \cap F)) = 0$ ,  $p, p' \in \{[*], [0, 0]\}$  see note P906 for notations. Thus  $n = 0 \Rightarrow \chi(U) = 4 + \mu$ .

$\square$

But  $U$ , being the smooth intersection of two quadrics in  $\mathbb{P}^4$ , is biholomorphic to  $\mathbb{P}^2$  blown up five times (Section 4, Chapter 4) and so has Euler characteristic 8. Consequently  $\deg S = \mu = 4$ .

$\square$   $U = T_x(G) \cap X \subset T_x(G) = \mathbb{P}^4$  and  $T_x(G) \cap X = T_x(G) \cap F \cap G = (T_x(G) \cap G) \cap (T_x(G) \cap F) \Rightarrow$  Since  $T_x(G) \cap G$  &  $T_x(G) \cap F$  are quadrics in  $T_x(G) = \mathbb{P}^4$ ,  $U$  is the smooth intersection of two quadrics in  $\mathbb{P}^4$ .  $\Rightarrow U$  is  $\mathbb{P}^2$  blown up in five distinct points by the result on P550.  $\Rightarrow H_0(U) = H_4(U) = \mathbb{Z}$  and  $H_2(U) =$