We set
$$\chi_\epsilon(x) = \frac{1}{\epsilon^n}\,\chi\!\left(\frac{x}{\epsilon}\right).$$

If $\operatorname{supp}\chi = K$, then $\operatorname{supp}\chi_\epsilon = \epsilon K$ and
$$\int_{\mathbb{R}^n} \chi_\epsilon(x)\,dx = 1.$$

Γ $x \in \overline{K}^c \Rightarrow \chi(x) = 0 \quad \chi_\epsilon(\epsilon x) = \frac{1}{\epsilon^n}\chi\!\left(\frac{x\epsilon}{\epsilon}\right) = \frac{1}{\epsilon^n}\chi(x)$
$= 0 \Rightarrow \epsilon x \in \overline{(\epsilon K)}^c$

Suppose $\epsilon x \notin \operatorname{supp}\chi_\epsilon$ when $x \in \operatorname{supp}\chi$.
$\Rightarrow \chi_\epsilon(\epsilon x) = 0$ and $\chi(x) = 0$ or not 0.
If $\chi(x) \neq 0$, since $\chi_\epsilon(\epsilon x) = \frac{1}{\epsilon^n}\chi(x) = 0$, contradiction.
$\Rightarrow \chi(x) = 0 \Rightarrow$

Let $L_1 = \{x \in \mathbb{R}^n \mid \chi(x) \neq 0\}$ $\quad L_2 = \{x \in \mathbb{R}^n \mid \chi_\epsilon(x) \neq 0\}$.
$\Rightarrow \forall x \in L_1 \Rightarrow \chi(x) \neq 0 \Rightarrow \chi_\epsilon(\epsilon x) \neq 0 \Rightarrow \epsilon x \in L_2$
$\Rightarrow \epsilon L_1 \subset L_2$ $\quad$ Similarly, $\epsilon L_1 \supset L_2 \Rightarrow \epsilon L_1 = L_2$
$\Rightarrow$ Since $\operatorname{supp}\chi = \overline{L_1}$ & $\operatorname{supp}\chi_\epsilon = \epsilon\overline{L_2}$, $\Rightarrow \overline{L_1} = \epsilon\overline{L_2}$.

$$\int_{\mathbb{R}^n} \chi_\epsilon(x)\,dx = \int_{\mathbb{R}^n} \frac{1}{\epsilon^n}\chi\!\left(\frac{x}{\epsilon}\right)dx = \int_{\mathbb{R}^n} \chi(x')\,dx' = 1$$
⅃

We remark that
$$T_{\chi_\epsilon} \longrightarrow \delta \quad \text{as } \epsilon \to 0$$

in the sense that for any test function $\varphi \in C_c^\infty(\mathbb{R}^n)$
$$\lim_{\epsilon \to 0} \int_{\mathbb{R}^n} \chi_\epsilon(x)\,\varphi(x)\,dx = \varphi(0).$$

To see this, simply note that
$$\min_{x \in \epsilon K} \varphi(x) \leq \int_{\mathbb{R}^n} \chi_\epsilon(x)\,\varphi(x)\,dx \leq \max_{x \in \epsilon K} \varphi(x),$$