

If  $f$  &  $f'$  satisfy the three equations, then  $f+f'$  satisfies. Clearly  $\alpha f$  satisfies for  $\alpha \in \mathbb{C}$ . Here we have a parametric representation of  $C$  near each point  $P_U$ , since  $\frac{\partial f}{\partial y} \neq 0$ , so  $\mathbb{C}^2 \xrightarrow{F} \mathbb{C}^2$   $f(F) \neq 0$  at  $P_U$ .  
 $(x, y) \mapsto (x, f)$

$$\Rightarrow G(x, f) = (x, y) \Rightarrow G(x, 0) = (x, y) = (x, g(x)).$$

Since finding a curve  $C$  of degree  $n$  and with prescribed second-order behavior at points  $P_U$  on the line  $\{x=0\}$  is equivalent to finding a point satisfying ① ② ③ at each point  $P_U$ , and each ①, ②, ③ at each  $P_U$  represents <sup>three</sup> linear relations on  $\mathbb{P}^n(V) = \mathbb{P}^{3n-1}$ , we have to find a point in  $\mathbb{P}^{3n-1}$  satisfying the  $3n$  linear conditions. So we have  $3n$  linear conditions on  $\mathbb{P}^{3n-1}$ . In general,  $3n$  linear conditions determine a point on  $\mathbb{P}^{3n-1}$ , or  $\exists$  no point. I don't understand well enough on the sufficiency of Reiss relation. Is that the meaning that  $3n$  linear conditions plus Reiss relation are enough for determining a point in  $\mathbb{C}^{3n} \rightarrow \mathbb{P}^{3n-1}$ . Let's see what comes next.  $\Downarrow$

Finally, we wish to point out that the residue theorem from Section 1 applies to configurations of points on general algebraic surfaces, not just  $\mathbb{P}^2$ . More precisely, suppose that  $L, L'$  are holomorphic line bundles over a surface  $S$  and  $C \in |L|, C' \in |L'|$  are curves meeting transversely at  $d = L \cdot L'$  points. Then we have the  
 $\mathbb{P}^1$   $d = L \cdot L' = \text{intersection number} \Rightarrow$