Conversely, any such function h defines a metric on L.

「 Define a metric on $\pi^* L$, and $\exists_1$ the induced metric on L. 」

Now write $Z = X + \sqrt{-1}\, Y$ as before; since $Y > 0$, we can set $W = (W_{\alpha\beta}) = Y^{-1}$.

「 $Y > 0 \Rightarrow D(Y) \neq 0$ since all eigenvalues are not zero. $\Rightarrow W = (W_{\alpha\beta}) = Y^{-1}$. 」

Then we claim that the function

$$h(z) = e^{(\pi/2) \sum W_{\alpha\beta}(z_\alpha - \bar{z}_\alpha)(z_\beta - \bar{z}_\beta - 2i Y_{\beta\beta})}$$

satisfies the functional equations above. Clearly $h(z + \lambda_\alpha) = h(z)$; for the others, write

$$\log h(z + \lambda_{n+r}) = \frac{\pi}{2} \sum_{\alpha,\beta} W_{\alpha\beta}(z_\alpha - \bar{z}_\alpha + 2i Y_{\alpha r})(z_\beta - \bar{z}_\beta + 2i(Y_{\beta r} - Y_{\beta\beta})) = \frac{\pi}{2} \sum_{\alpha,\beta} W_{\alpha\beta}(z_\alpha - \bar{z}_\alpha)(z_\beta - \bar{z}_\beta - 2i Y_{\beta\beta}) + \frac{\pi}{2} \sum_{\alpha,\beta} W_{\alpha\beta}(z_\alpha - \bar{z}_\alpha) \cdot 2i Y_{\beta r} + \frac{\pi}{2} \sum_{\alpha,\beta} W_{\alpha\beta} \cdot 2i Y_{\alpha r}(z_\beta - \bar{z}_\beta + 2i(Y_{\beta r} - Y_{\beta\beta}))$$

$$= \log h(z) + \frac{\pi}{2} \sum_{\alpha} \delta_{\alpha r} \cdot 2i(z_\alpha - \bar{z}_\alpha) + \frac{\pi}{2} \sum_{\beta} \delta_{\beta r} \cdot 2i(z_\beta - \bar{z}_\beta + 2i(Y_{\beta r} - Y_{\beta\beta}))$$