

and so we obtain  $'E_1^{p,q} \cong 'E_2^{p,q} \cong \dots \cong 'E_\infty^{p,q}$ .

By the result in the proof of Proposition on P442, for  $r$  sufficiently large

$$E_r^{p,q} \cong \frac{F^p H_{DR}^{p+q}(M)}{F^{p+1} H_{DR}^{p+q}(M)} \dots \quad (*)$$

Note that since  $'F^p A^n(M)$  means " $n$ -forms having at least  $p$ -dz's", if  $p \geq n+1$ , then  $'F^p A^n(M) = 0$  which implies that  $'F^p H_{DR}^{p+q}(M) = 0$  if  $p \geq n+1$ .

By  $(*)$ , since  $'F^p H_{DR}^{p+q}(M)$  is  $\mathbb{C}$ -free abelian group, and  $'E_1 \cong \dots \cong 'E_\infty$ ,

$$0 \rightarrow F^{p+1} H_{DR}^{p+q}(M) \rightarrow F^p H_{DR}^{p+q}(M) \rightarrow E_r^{p,q} \rightarrow 0$$

$$\text{and } F^p H_{DR}^{p+q}(M) \cong E_r^{p,q} \oplus F^{p+1} H_{DR}^{p+q}(M).$$

$$\begin{aligned} \Rightarrow F^p H_{DR}^n(M) &\cong E_r^{p,n-p} \oplus F^{p+1} H_{DR}^n(M) \cong E_r^{p,n-p} \oplus E_r^{p+1,n-p-1} \oplus F^{p+2} H_{DR}^n(M) \\ &\cong \dots \cong E_r^{p,n-p} \oplus E_r^{p+1,n-p-1} \oplus \dots \oplus E_r^{n,0} \oplus F^{n+1} H_{DR}^n(M) \\ &\cong E_r^{p,n-p} \oplus \dots \oplus E_r^{n,0} \oplus 0 \cong H_2^{p,n-p}(M) \oplus \dots \oplus H_2^{n,0}(M) \\ &\cong H^{p,n-p}(M) \oplus \dots \oplus H^{n,0}(M). \end{aligned} \quad \square$$

If  $M$  is compact but not Kähler, it may happen that  $'E_1 \neq 'E_2$ , but no example seems to be known where  $'E_2 \neq 'E_\infty$ . An example of  $'E_1 \neq 'E_2$  is provided by the Iwasawa manifold

$$M = \frac{G}{\Gamma},$$

where  $G$  is the Lie group of all complex matrices

$$g = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$