$$\Rightarrow \left(\sum \sigma_{2n-i,i}\right)^2 = n+1.$$

In a similar vein, the number of lines in $\mathbb{P}^4$ meeting six 2-planes in general position is given by σ_1^6 in $G(2,5)$: we have

so

$$\sigma_1^3 = \sigma_1 \cdot (\sigma_{1,1} + \sigma_2) = 2\sigma_{2,1} + \sigma_3,$$

$$\sigma_1^6 = (2\sigma_{2,1} + \sigma_3)^2 = 4 + 1 = 5.$$

∵

$$\sigma_1^2 = \sum_{\sum c_i = 2} \sigma_c = \sigma_{1,1} + \sigma_2.$$

$$\sigma_1 \cdot (\sigma_{1,1} + \sigma_2) = \sigma_1 \cdot \sigma_{1,1} + \sigma_1 \cdot \sigma_2.$$

$$\sigma_1 \cdot \sigma_{1,1} = \sum_{\substack{\sum c_i = 3 \\ 1 \le c_2 \le 1 \\ 1 \le c_1 \le 3}} \sigma_c = \sigma_{2,1}$$

$$\sigma_1 \cdot \sigma_2 = \sum_{\substack{\sum c_i = 3 \\ 0 \le c_2 \le 2 \\ 2 \le c_1 \le 2+1}} \sigma_c = \sigma_{2,1} + \sigma_3$$

$$\Rightarrow \quad \sigma_1^3 = 2\sigma_{2,1} + \sigma_3.$$

$$\sigma_1^6 = (\sigma_1^3)^2 = (2\sigma_{2,1} + \sigma_3)^2 = 4 \cdot \sigma_{2,1}^2 + \sigma_3^2 + 4\underbrace{\sigma_{2,1} \cdot \sigma_3}_{=0} = 4 \cdot \underbrace{\sigma_{2,1} \cdot \sigma_{3-1,3-2}}_{=4} + \underbrace{\sigma_3 \cdot \sigma_3}_{= \sigma_3 \cdot \sigma_{3-0} = 1}$$

$$\sigma_{2,1} \cdot \sigma_{2,1} = 1$$

$$\sigma_3 \cdot \sigma_3 = 1$$

$$= 4 + 1 = 5$$