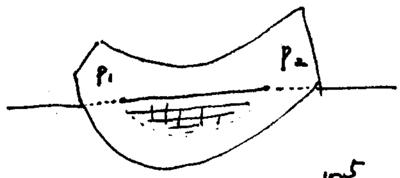
for any open set U

More precisely, $\forall$ $\exists$ $p_1, p_2 \in W_2$ s.t $\overline{p_1 p_2} \not\subset W_2$, $p_1, p_2 \in U$.



$\Rightarrow \dim C(W_2) \geq 3$.

$\Rightarrow$ $C(W_2) \subset W_1 \subset \mathbb{P}^5$ $\Rightarrow$ $C(W_2)$ is a cubic hypersurface in $\mathbb{P}^5$.

$\lrcorner$

Finally, it is interesting to observe that there are two distinct four-dimensional families of lines on the variety W_1 – that is, two kinds of pencils of singular conics. First, there are the pencils formed by a fixed line l_0 plus a pencil l_λ of lines; for example

$$L = \{ (\lambda X_0 X_1 + X_0 X_2) \}_\lambda \quad (l_0 = (X_0 = 0)).$$

Pf

for exaple $\{\lambda X_0^2 + X_1 X_2\}$ is not singular pencil since $\mathrm{rk}\begin{pmatrix}\lambda & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} = 3$

$$\lambda X_0 X_1 + X_0 X_2 = X_0 (\lambda X_1 + X_2)$$

$l_\lambda = \lambda X_1 + X_2$ // Note: Divide the cases into two, i.e $\lrcorner$ the pencil contains one or two double lines.

(See Figure 3.) Such a pencil will either miss W_2 altogether or meet it in a single point, depending on whether the base point of the pencil l_λ lies on l_0.

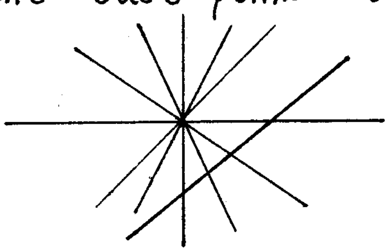


Figure 3