Explicitly, $df = \sum \frac{\partial f}{\partial z_i} dz_i$

$\Rightarrow$ If we take $\omega' = (-1)^{i-1} \dfrac{g(z)\, dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n}{\frac{\partial f}{\partial z_i}}$

for any i such that $\frac{\partial f}{\partial z_i} \neq 0$. The map

$$\frac{g(z)\, dz_1 \wedge \cdots \wedge dz_n}{f(z)} \longmapsto (-1)^{i-1} \frac{g(z)\, dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n}{\frac{\partial f}{\partial z_i}} \Bigg|_{f=0}$$

is called the Poincaré residue map, denoted P.R.

Pf $\omega' \wedge \dfrac{df}{f} = (-1)^{i-1} \dfrac{g(z)\, dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n}{\frac{\partial f}{\partial z_i}} \wedge \dfrac{df}{f}$

$$= \frac{g(z)\, dz_1 \wedge \cdots \wedge dz_n}{f}$$

Note that $df = 0$ on $V = \{f=0\}$, since
if $\alpha(t)$ curve on V, $df(\alpha'(t)) = \alpha'(t)(f) = \dfrac{d f(\alpha(t))}{dt} = 0$

Thus $df \wedge dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge \widehat{dz_j} \wedge \cdots \wedge dz_n = 0$

$= \left(\sum \frac{\partial f}{\partial z_k} dz_k\right) \wedge dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge \widehat{dz_j} \wedge \cdots \wedge dz_n$

$= \frac{\partial f}{\partial z_i} dz_i \wedge dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge \widehat{dz_j} \wedge \cdots \wedge dz_n$

$\quad + \frac{\partial f}{\partial z_j} dz_j \wedge dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge \widehat{dz_j} \wedge \cdots \wedge dz_n$

$= (-1)^{i-1} \frac{\partial f}{\partial z_i} dz_1 \wedge \cdots \wedge \widehat{dz_j} \wedge \cdots \wedge dz_n + (-1)^{j-2} \frac{\partial f}{\partial z_j} dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n.$