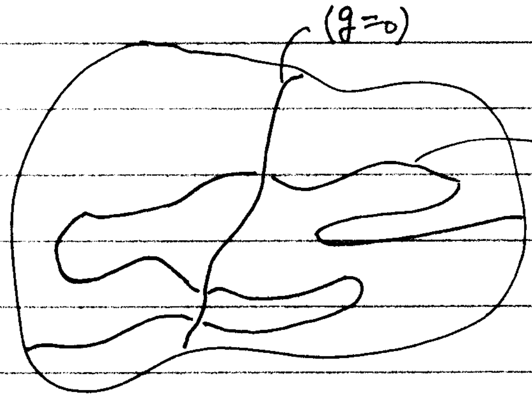
alytic hypersurface, and in terms of holomorphic coordinates $z_1, \cdots z_n$ on M, centered around p we write

$$f(z_1, z_2, \cdots z_n) = f_m(z_1, \cdots z_n) + f_{m+1}(z_1, \cdots z_n) + \cdots$$

with $f_k(z_1, \cdots z_n)$ a homogeneous polynomial of degree k in $z_1, z_2, \cdots z_n$, then the tangent cone to V at p is taken to be the subvariety of $T'_p(M) = \mathbb{C}\{\frac{\partial}{\partial z_i}\}$ defined by

$$\{\sum \alpha_i \frac{\partial}{\partial z_i} : f_m(\alpha_1, \cdots \alpha_n) = 0\}.$$

⌈ Comment on $\{f_i = 0\} \cap \Delta = \overline{V_i} \cap \Delta$.



⌟

⌈ If V is smooth, then since $df \neq 0$,

$m = 1$. $\Rightarrow$ $f_m(z_1, \cdots z_n) = c_1 z_1 + \cdots + c_n z_n$

$$\Rightarrow \left.\frac{\partial f}{\partial z_i}\right|_p = \left.\frac{\partial f_m}{\partial z_i}\right|_p = c_i.$$

$$\Rightarrow T_p V = \{\sum \alpha_i \frac{\partial}{\partial z_i} : c_i \alpha_i = 0 = f_m(\alpha_1, \cdots \alpha_n) = 0\}$$

$\Rightarrow$ The definition of the tangent cone is reasonable. ⌟