「 The fact that $H^0(M, \mathcal{F}(k))$ generate the $\mathcal{O}_x$-modules $\mathcal{F}(k)_x$ for x near to x_0 does not follow from Oka's lemma.

Let $g_1, \cdots g_r$ generate $\mathcal{F}(k)$ in a nbd of z_0. (This is plausible since $\mathcal{F}(k)$ is coherent.) Let $\{f_1, \cdots f_\ell\} = H^0(M, \mathcal{F}(k))$. ⇒ Since $\{(f_1)_{z_0}, \cdots (f_\ell)_{z_0}\}$ generates $\mathcal{F}(k)_{z_0}$, ∃ holomorphic functions a_{ij}, actually, $(a_{ij})_{z_0} \in \mathcal{O}_{z_0}$ s.t

$$(g_i)_{z_0} = \sum (a_{ij})_{z_0} (f_j)_{z_0}.$$

⇒ We have

$$g_i = \sum a_{ij} f_j \quad \text{in a nbd of } z_0.$$

This proves the fact. See P162. Lemma. 7.1.3. Introduction to complex analysis in several variables by. L. Hörmander.

⇒ By the compactness of M, ∃ large k_0 satisfying Th. A.」

Proof of 3. The proof is by induction on $n = \dim_{\mathbb{C}} M$. Given a point $x \in M$ and hyperplane ξ in the tangent space $T'_x(M)$, we may find a non singular hypersurface passing through x and with tangent plane ξ.

「 For example. $M^3 \subset P^4$.

$x^0 = (x_1^0, x_2^0, x_3^0, x_4^0) \in M^3$. $U_0 = (z_0 \neq 0) \ni x^0$

$\xi \subset T'_{x^0} M$ is spanned by two vectors $(\xi_1^1, \xi_2^1, \xi_3^1, \xi_4^1)$ & $(\xi_1^2, \xi_2^2, \xi_3^2, \xi_4^2) \in T'_{x^0} U_0 \cong \mathbb{C}^4$.

Since we have to find a hyperplane, we have only to find a_0, a_1, a_2, a_3, a_4 satisfying the following: