

x , $x' = Q^{-1}Q'x$, and $x'' = (Q^{-1}Q')^2 x$ would then likewise be collinear in \mathbb{P}^5 ; since all three lie on G , the line L they span would lie on G .

⌈ Since Qx , Qx' and Qx'' are collinear, x , x'' and x' are collinear.

① $x' = Q^{-1}Q'x \in G$.

For, $(Qx', x') = (Q Q^{-1}Q'x, Q^{-1}Q'x) = (Q'x, Q^{-1}Q'x)$
 $= {}^t(Q'x) Q^{-1}Q'x = {}^t x Q' Q^{-1}Q'x = (x, Q'Q^{-1}Q'x)$
 $= 0$ since $x \in H$.

② $x'' \in G$.

$L \cap G = \{x, x', x''\} \Rightarrow \#(L \cdot G) = 3 \Rightarrow$ Since $\deg G = 2$,
 $L \subset G$. \sqcup

But now the linear transformation

$M: x \mapsto Q^{-1}Q'x$

taking G into G takes x and x' (distinct, since by hypothesis $Qx \neq Q'x$ for any $x \in F \cap G$) into L , and so takes L into itself; thus $L \subset F \cap G$.

⌈ \Rightarrow ② $x'' \in G$.

$\Rightarrow x'' = ax + bx'$

$\Rightarrow (Qx'', x'') = 0$, for $(Qx'', x'') = (a^t x + b^t x') Q(ax + bx') = a^t x Qx + b^t x' Qx' + ba^t x Qx' + ab^t x' Qx$