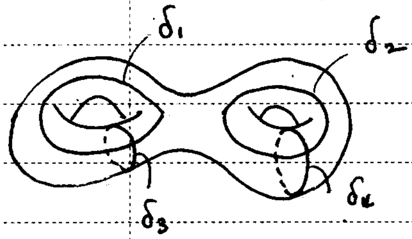

Now let $\omega_1, \cdots \omega_g \in H^0(S, \Omega^1)$ be a basis for the space of holomorphic 1-forms on S. The period matrix of S is the $g \times 2g$ matrix

$$\Omega = \begin{bmatrix} \int_{\delta_1} \omega_1, & \cdots & , \int_{\delta_{2g}} \omega_1 \\ \vdots & & \vdots \\ \int_{\delta_1} \omega_g, & \cdots & \int_{\delta_{2g}} \omega_g \end{bmatrix}.$$

The (transposed) column vectors $\Pi_i = (\int_{\delta_i} \omega_1 \cdots \int_{\delta_i} \omega_g) \in \mathbb{C}^g$ of the period matrix are called the periods: we first check that they are linearly independent over R: If we have $\sum k_i \Pi_i = 0$, $k_i \in \mathbb{R}$, then

$$\sum k_i \int_{\delta_i} \omega_j = 0 \text{ for all } j \Rightarrow \sum k_i \int_{\delta_i} \overline{\omega}_j = 0 \text{ for all } j,$$

$$\Rightarrow \sum k_i [\delta_i] = 0 \in H_1(S, \mathbb{R}),$$

since $\{\omega_j, \overline{\omega}_j\}$ span $H^1_{DR}(S)$; this is impossible, since $\{\delta_i\}$ is a basis for $H_1(S, \mathbb{Z})$.

$H^0(S, \Omega^1) = H^{1,0}(S) \Rightarrow H^1(S) = H^{1,0}(S) \oplus H^{0,1}(S)$ by Hodge decomposition theorem P116. See P226

The $2g$ periods $\Pi_i \in \mathbb{C}^g$ thus generate a lattice.