$\Rightarrow [\lambda_1 \alpha_{10} X_0 + \lambda_2 \alpha_{20} X_0 + \lambda_3 \alpha_{30} X_0, \ \lambda_1 \alpha_{11} X_1 + \lambda_2 \alpha_{21} X_1 + \lambda_3 \alpha_{31} X_1,$
$\cdots, \lambda_1 \alpha_{1n} X_n + \lambda_2 \alpha_{2n} X_n + \lambda_3 \alpha_{3n} X_n] = [X_0, X_1, \cdots X_n].$

As we suggested, if

$$A = \begin{pmatrix} 1, & 1, & \cdots & 1 \\ \alpha_{10}, & \alpha_{11}, & \cdots & \alpha_{1n} \\ \vdots & \vdots & & \vdots \\ \alpha_{n0}, & \alpha_{n1}, & \cdots & \alpha_{nn} \end{pmatrix}$$ is an $(n+1) \times (n+1)$

matrix all of whose $l \times l$ $(l \geq 2)$ minor determinants are nonzero, we can conclude that v_1, v_2, v_3 are linearly dependent at X exactly when all but 3 of the homogeneous coordinates of X vanish as follows:

Suppose X_0, X_1, X_2, X_3 are not zero, but v_1 v_2, v_3 are linearly depent. $\Rightarrow$ Consider the 4×4 minor

$$\begin{pmatrix} 1, & 1, & 1, & 1 \\ \alpha_{10}, & \alpha_{11}, & \alpha_{12}, & \alpha_{13} \\ \alpha_{20}, & \alpha_{21}, & \alpha_{22}, & \alpha_{23} \\ \alpha_{30}, & \alpha_{31}, & \alpha_{32}, & \alpha_{33} \end{pmatrix} \Rightarrow \begin{vmatrix} 1, 1, 1, 1 \\ \alpha_{ij} \end{vmatrix} = \begin{vmatrix} 1, & 0, & 0, & 0 \\ \alpha_{10}, & \alpha_{11}-\alpha_{10}, & \alpha_{12}-\alpha_{10}, & \alpha_{13}-\alpha_{10} \\ \alpha_{20}, & \alpha_{21}-\alpha_{20}, & \alpha_{22}-\alpha_{21}, & \alpha_{23}-\alpha_{20} \\ \alpha_{30}, & \alpha_{31}-\alpha_{30}, & \alpha_{32}-\alpha_{30}, & \alpha_{33}-\alpha_{30} \end{vmatrix}$$

$\neq 0$ --- (**)

$$v_1 = (\alpha_{11} - \alpha_{10}) x_1 \frac{\partial}{\partial x_1} + (\alpha_{12} - \alpha_{10}) x_2 \frac{\partial}{\partial x_2} + (\alpha_{13} - \alpha_{10}) x_3 \frac{\partial}{\partial x_3} + \cdots$$
$$v_2 = (\alpha_{21} - \alpha_{20}) x_1 \frac{\partial}{\partial x_1} + (\alpha_{22} - \alpha_{20}) x_2 \frac{\partial}{\partial x_2} + (\alpha_{23} - \alpha_{20}) x_3 \frac{\partial}{\partial x_3} + \cdots$$
$$v_3 = (\alpha_{31} - \alpha_{30}) x_1 \frac{\partial}{\partial x_1} + (\alpha_{32} - \alpha_{30}) x_2 \frac{\partial}{\partial x_2} + (\alpha_{33} - \alpha_{30}) x_3 \frac{\partial}{\partial x_3} + \cdots$$

$\Rightarrow$ Since $v_1, v_2, \& v_3$ are linearly dependent at $X = [X_0, X_1, \cdots]$ where $X_0 \neq 0$ $X_1 \neq 0$, $X_2 \neq 0$, $X_3 \neq 0$. $\Rightarrow$ $x_1 \neq 0$