Date . .

To show $f^* H_{\mathbb{P}^5} = 2 H_{\mathbb{P}^2}$, we have to show that f has multiplicity 2.

$$f : \mathbb{P}^2 \longrightarrow \mathbb{P}^5$$
$$[Z_0, Z_1, Z_2] \longmapsto [Z_0^2, Z_0Z_1, Z_0Z_2, Z_1^2, Z_1Z_2, Z_2^2]$$

$$f^{-1}(\mathbb{P}^4) = f^{-1}([0, * * \cdots]) = 2\,\mathbb{P}^1.$$

See also P177. $f^*(H_{\mathbb{P}^5}) = 2 H_{\mathbb{P}^2}$. $(L = \iota_E^*(H))$

If $V \subset \mathbb{P}^5$ is an irreducible, nondegenerate, 2-dimensional variety, then

$$\deg V \geq 5 - 2 + 1 = 4 \qquad \text{See } P173$$

We digress for a moment to discuss a curious feature of the Veronese surface $S \subset \mathbb{P}^5$: it is the unique nondegenerate surface in $\mathbb{P}^5$ whose variety of chords

$$C(S) = \bigcup_{p,q \in S} \overline{pq} \quad \text{is a proper subvariety of } \mathbb{P}^5.$$

To see this, note that the line $L = \overline{uu'} \subset \mathbb{P}^2$ is mapped into a curve of degree

$$\#(H_{\mathbb{P}^5} \cdot f(L)) = \#(2H_{\mathbb{P}^2} \cdot L) = 2$$

in $\mathbb{P}^5$, hence by the result of P173, is a conic lying in a 2-plane $V_2 \subset \mathbb{P}^5$.

See P173 & P174 (restatement of the result of P173).
Any irreducible k-dimensional variety $V \subset \mathbb{P}^5$ of degree d

PAGE