$\Rightarrow$ We have a sheaf $\mathcal{H}^q$ whose stalk is

$$\mathcal{H}_x^q = \lim_{U \ni x} \frac{\ker\{d : \mathcal{K}^q(U) \to \mathcal{K}^{q+1}(U)\}}{d\,\mathcal{K}^{q-1}(U)}$$

A section σ of $\mathcal{H}^q$ over an open set $U \subset X$ is given by a covering $\{U_\alpha\}$ of U and $\sigma_\alpha \in \mathcal{K}^q(U_\alpha)$ s.t

$$d\sigma_\alpha = 0,$$

$$\sigma_\alpha - \sigma_\beta = d\eta_{\alpha\beta}, \qquad \eta_{\alpha\beta} \in \mathcal{K}^{q-1}(U_\alpha \cap U_\beta);$$

the section is zero in case

$$\sigma_\alpha = d\eta_\alpha, \quad \eta_\alpha \in \mathcal{K}^{q-1}(U_\alpha),$$

after perhaps refining the given covering.

Γ $$\mathcal{H}^q = \bigcup_{x \in X} \mathcal{H}_x^q = \bigcup_{x \in X} \lim_{U \ni x} \frac{\ker\{d ; \mathcal{K}^q(U) \to \mathcal{K}^{q+1}(U)\}}{d\,\mathcal{K}^{q-1}(U)}$$

$\mathcal{H}^q(U) \ni \sigma \Rightarrow \sigma(x) \in \mathcal{H}_x^q$ satisfying the following:
For each $p \in U$, $\exists$ an open set $V_p \subset U$ s.t $p \in V_p$. & $\exists$ $s_p \in \dfrac{\ker\{d : \mathcal{K}^q(V_p) \to \mathcal{K}^{q+1}(V_p)\}}{d\,\mathcal{K}^{q-1}(V_p)}$ s.t (*) the germ of $s_p \overset{\text{at } y}{=} \sigma(y)$.

$\Rightarrow$ For each point $p \in U$, $\exists$ V_p and $s_p = \sigma_p + d\,\mathcal{K}^{q-1}(V_p)$ s.t $d\sigma_p = 0$

On $V_p \cap V_{p'}$, $s_p|_{V_p \cap V_{p'}} = s_{p'}|_{V_p \cap V_{p'}}$ by $\Rightarrow$

$$\sigma_p|_{V_p \cap V_{p'}} + d\,\mathcal{K}^{q-1}(V_p \cap V_{p'}) = \sigma_{p'} + d\,\mathcal{K}^{q-1}(V_p \cap V_{p'})$$

$$\Rightarrow \quad \sigma_p - \sigma_{p'} = d\eta_{pp'}, \quad \eta_{pp'} \in \mathcal{K}^{q-1}(V_p \cap V_{p'}).$$

In case, $\sigma(x) = 0 \Rightarrow \exists$ open set $U \ni x$ s.t $\sigma_U = 0$ on U