Next, consider the Gauss map $G \xrightarrow{\varphi} \mathbb{P}^{5*}$ given by $x \longmapsto T_x(G)$. $\Rightarrow$ By P735, φ is isomorphic, and $\varphi(G)$ is a smooth quadric of dim 4 in $\mathbb{P}^{5*}$. $\Rightarrow$ By P735, Proposition, $\varphi(G)$ can not contain a 3-plane, and especially $\varphi(G) \not\supset K$. $\Rightarrow$ This implies that, for generic hyperplane $H \supset L$, $H \cap G$ is smooth on L since $H + T_x(G) = \mathbb{P}^5$ for all $x \in L$, and $H \cap G$ is smooth. Similarly for F. $\Rightarrow$ We can conclude that, for generic $H \supset L$, $H \cap X$ is smooth, since H is spanned by L and $H \cap V_3$ ($\because$ $H \supset V_3 \Rightarrow H = \langle L, V_3 \rangle = \mathbb{P}^5$ $*$). $\Rightarrow$ For generic $V_2 \subset V_3$, $\overline{L, V_2} \cap X$ is the smooth intersection of two quadrics in 4-space $\overline{L, V_2}$. $\Rightarrow$ By P551. ⊛, $\overline{L, V_2} \cap V = (\overline{L, V_2} \cap F) \cap (\overline{L, V_2} \cap G)$ = intersection of two smooth quadrics in $\overline{L, V_2} = \mathbb{P}^4 \supset L$. $\Rightarrow$ L meets five lines on $\overline{L, V_2} \cap X$, which corresponds to the points of $V_2 \cap E_L$. $\Rightarrow$ $\deg E_L = 5$.

$\lrcorner$

Thus E_L is a quintic space curve.

F E_L is a quintic curve in $V_3 = \mathbb{P}^3$.

$*$ E_L is the cloure of $f_L(\bigcup_{L' \in B_L - L} L')$. This is well-defined, for consider $V = \bigcap_{x \in L} T_x(X)$ which is a 2-plane in case L is special. (In case L nonspecial, we have no problem).