

type  $(p, p)$  are the continuous linear functionals on the compactly supported forms  $A_c^{n-p, n-p}(M)$ . A  $(p, p)$  current  $T$  is real in case  $T = \bar{T}$  in the sense that  $\overline{T(\varphi)} = T(\bar{\varphi})$  for all  $\varphi \in A_c^{n-p, n-p}(M)$ , and a real current is positive in case

$$(\sqrt{-1})^{p(p-1)/2} T(\eta \wedge \bar{\eta}) \geq 0, \quad \eta \in A_c^{n-p, 0}(M).$$

Comment on  $A_c^{p, q}(U) \xrightarrow{T} \mathbb{C}$ , from P126 note

$$\begin{array}{ccc} & \downarrow \psi^{-1*} & \\ & A_c^{p, q}(V) \xrightarrow{T'} \mathbb{C} & \end{array} \quad U \xrightarrow{\psi} V \subset \mathbb{C}^n$$

$$\Rightarrow T'(\varphi) = T(\psi^* \varphi).$$

Suppose  $\bar{\partial} \eta' = T' \Rightarrow \exists \eta$  on  $A_c^{p, q-1}(U)$

Question:  $\bar{\partial} \eta = T$ ? Yes.

$$\begin{aligned} (\bar{\partial} \eta)(\varphi) &= (-1)^q \eta(\bar{\partial} \varphi) = (-1)^q \eta'(\psi^{-1*}(\bar{\partial} \varphi)) \\ &= (-1)^q \eta'(\bar{\partial}(\psi^* \varphi)) = \bar{\partial} \eta'(\psi^* \varphi) = T'(\psi^* \varphi) \\ &= T(\varphi) \quad \Rightarrow \quad \bar{\partial} \eta = T. \quad \square \end{aligned}$$

Especially, noteworthy are the closed, positive currents. Note that for real  $T \in \mathcal{D}^{p, p}(M)$ ,

$$dT = 0 \Leftrightarrow \partial T = \bar{\partial} T = 0.$$

I think the note should be changed into the following statement, for real  $T \in \mathcal{D}^{p, p}(M)$ ,  $\partial T = 0 \Leftrightarrow \bar{\partial} T = 0$ .