

$$\begin{cases} \Omega^*(\log D)_p = \Omega^*(\ast D)_p = \Omega_p^* \\ \mathcal{A}^*(\ast D)_p = \mathcal{A}_p^* \end{cases}$$

and the result follows from the usual holomorphic and C^∞ Poincare lemmas, respectively.

Since D is closed in M , \exists a nbd U of p s.t. $U \cap D = \emptyset$. $\Rightarrow \Omega^*(\log D)(U)$ is the set of holomorphic forms on U .

$$\begin{aligned} \Rightarrow \Omega^*(\log D)_p &= \lim_{U' \ni p} \Omega^*(\log D)(U') \\ &= \lim_{U' \ni p} \Omega^*(\ast D)(U') = \lim_{U' \ni p} \Omega^*(U') = \Omega_p^* \\ &\quad \text{"} \\ &\quad \Omega^*(\ast D)_p. \end{aligned}$$

By the same reason, $\mathcal{A}^*(\ast D)_p = \lim_{U' \ni p} \mathcal{A}^*(U' - U' \cap D)$

$$= \lim_{U' \ni p, U' \cap D = \emptyset} \mathcal{A}^*(U') = \mathcal{A}_p^*.$$

$$0 \rightarrow \mathbb{C} \rightarrow \mathcal{A}^0 \xrightarrow{d} \mathcal{A}^1 \xrightarrow{d} \mathcal{A}^2 \xrightarrow{d} \mathcal{A}^3 \rightarrow \dots$$

\Rightarrow By the Poincare lemma (the ordinary Poincare lemma for the complex differential forms),

$$\mathcal{H}^q(\mathcal{A}^*)_p = 0 \quad q > 0, \quad \mathcal{H}^0(\mathcal{A}^*)_p = \mathbb{C}$$

$$0 \rightarrow \mathbb{C} \rightarrow \Omega^0 \xrightarrow{d} \Omega^1 \xrightarrow{d} \Omega^2 \xrightarrow{d} \Omega^3 \rightarrow \dots$$

\Rightarrow By the holomorphic Poincare lemma (P448. the example 3),

$$\mathcal{H}^q(\Omega^*)_p = 0 \quad q > 0, \quad \mathcal{H}^0(\Omega^*)_p = \mathbb{C}.$$