theorem to $\mathcal{U} = M - D$, see P 453. and refer to P96, (6.28) Theorem, Differential Geometry of Complex Vector Bundles by S. Kobayashi. ⌟

In fact, we may take $\mathcal{U}$ to be an affine nbd of x_0 as discussed at the end of the preceding section.

⌜ If we take $D = \mathbb{P}^{N-1} \cap M$, $M - D = M - (\mathbb{P}^{N-1} \cap M)$
$= M \cap (\mathbb{P}^{N-1} \cap M)^c = M \cap (\mathbb{P}^{N-1\,c} \cup M^c) = M \cap (\mathbb{P}^{N-1})^c$
$= M \cap \mathbb{C}^N$, since $\mathbb{P}^N = \mathbb{P}^{N-1} \cup \mathbb{C}^N$, see P15 ~ P16,
$\Rightarrow M - D = \mathcal{U} \subset \mathbb{C}^N \subset \mathbb{P}^N$. $\Rightarrow$ $\mathcal{U}$ is defined by a finite # of homogeneous polynomials. ⌟

Then for any divisor $D' \supset D$, $M - D' = \mathcal{U}' \subset \mathcal{U}$ will also be affine and consequently

$$H^*_{DR}(\mathcal{U}') \cong H^*_{DR}(\mathcal{U}', alg).$$

⌜ $M - D' = \mathcal{U}' \subset \mathcal{U} \subset \mathbb{C}^N$

Suppose $M = (f_1(z_0, \cdots z_N) = \cdots = f_K(z_0, \cdots z_N) = 0)$
and $D' = M \cap (z_0 = 0)$
$\Rightarrow \mathcal{U}' = M - D' = (f_1 = f_2 = \cdots = f_K = 0$ but $z_0 \neq 0)$.
$\Rightarrow \mathcal{U}' = (f_1(1, \frac{z_1}{z_0}, \cdots \frac{z_N}{z_0}) = \cdots = f_K(1, \frac{z_1}{z_0}, \cdots \frac{z_N}{z_0}) = 0)$
$\Rightarrow \mathcal{U}' = (\tilde{f}_1(\omega_1, \cdots \omega_N) = \cdots = \tilde{f}_K(\omega_1, \cdots \omega_N) = 0)$ in $\mathbb{C}^N$
where

$$\tilde{f}_i(\omega_1, \cdots \omega_N) = f_i(1, \frac{z_1}{z_0}, \cdots \frac{z_N}{z_0}), \quad \frac{z_\ell}{z_0} = \omega_\ell$$

$\Rightarrow \mathcal{U}'$ is defined by polynomial equations. ⌟

⌜ If $\mathcal{U}'' \subset \mathcal{U}'$, then since $M \cap D'' = (f_1 = \cdots = f_K = g_1 = \cdots = g_\ell = z_0 = 0)$ and $D'' \subset D'$, $z_0 \neq 0 \Rightarrow \mathcal{U}'' = M - D''$ is defined by polynomial equations ⌟