for $\phi \in \mathcal{D}(\Omega)$ and $N = 0, 1, 2, \cdots$; see Section 1.46 for the otations D^α and $|\alpha|$.

The restrictions of these norms to any fixed $\mathcal{D}_K \subset \mathcal{D}(\Omega)$ induce the same topology on $\mathcal{D}_K$ as do the seminorms p_N of Section 1.46. To see this, note that to each K corresponds an integer N_0 s.t $K \subset K_{N_0}$ for all $N \geq N_0$. For these N, $\|\phi_N\| = p_N(\phi)$ if $\phi \in \mathcal{D}_K$. Since

$$(2) \qquad \|\phi\|_N \leq \|\phi\|_{N+1} \quad \text{and} \quad p_N(\phi) \leq p_{N+1}(\phi),$$

the topologies induced by either sequence of seminorms are unchanged if we let N start at N_0 rather than at 1.

Γ $V_N' = \{ \phi \in \mathcal{D}_K ; \|\phi\|_N < \frac{1}{N} \}$ $\qquad V_N = \{ \phi \in \mathcal{D}_K ; p_N(\phi) < \frac{1}{N} \}$

$\Rightarrow$ $V_N' \supset V_{N+1}'$. $\qquad V_{N+1} \supset V_N$.

$\Rightarrow$ $\{V_N'\}_{N=N_0}^{\infty}$ forms a local base and $\{V_N'\}_{N=1}^{\infty}$ forms a local base too for the same topology.

$\Rightarrow$ The same is true for V_N, and since $\{V_N'\}_{N=N_0}^{\infty} = \{V_N\}_{N=N_0}^{\infty}$, the topologies induced by either sequence of seminorms are unchanged. $\lrcorner$

These two topologies of $\mathcal{D}_K$ coincide therefore: a local base is formed by the sets

$$(3) \qquad V_N = \{ \phi \in \mathcal{D}_K ; \|\phi\|_N < \frac{1}{N} \} \qquad (N = 1, 2, 3, \cdots).$$

The same norms (1) can be used to define a locally convex metrizable topology on $\mathcal{D}(\Omega)$; see Theorem 1.37 and (b) of Section 1.38. However, this topology has the disadvantage of not being complete. For example, take $n=1$, $\Omega = R$, pick $\phi \in \mathcal{D}(R)$ with support in $[0,1]$,