Proof. By hypothesis, we may choose local holomorphic coordinates (z, w) around p_{ν_0} and defining functions $f(z,w)$, $g(z,w)$ for C, D, respectively, such that

$$\begin{cases} f(z,w) = z, \\ g(0,w) = w^{m_{\nu_0}} + \cdots \end{cases}.$$

Pf p_{ν_0} is a smooth point of C. $\Rightarrow$ We may choose local coordinates (z, w) around p_{ν_0} and defining functions $f(z,w)$ $g(z,w)$ for C, D respectively, s.t $(z=0) = C$.

$$f(z,w) = z. \qquad g(z,w)$$

Since $(z=0) \cap (g=0)$ is a set of isolated points, $(g=0)$ does not contain $\{z=0\}$, i.e w-axis. $\Rightarrow g(z,w) = (w^{m_{\nu_0}} + \cdots) h(z,w)$, where $h(z,w) \neq 0$ at $(z,w) = (0,0)$, by WPT.
$\Rightarrow g/h$ is a local defining function for D. m_{ν_0} is the multiplicity at p_{ν_0}. ⌟

The defining function $h(z,w)$ for E will then satisfy

$$h(0, w) = \alpha w^{m_{\nu_0}-1} + \cdots$$

and

$$(E \cdot C)_{p_{\nu_0}} \geq m_{\nu_0} \Leftrightarrow \alpha = 0.$$

Pf $(h=0) = E$ $\{h=0\}$ does not contain w-axis. $\Rightarrow$ By WPT, $h(z,w) = w^{(E\cdot C)_{p_{\nu_0}}} + a_1(z)\, w^{(E\cdot C)_{p_{\nu_0}}-1} + \cdots + \alpha w^{m_{\nu_0}-1} + \cdots$
Since if $z=0$, $h(0,w) = \beta w^{(E\cdot C)_{p_{\nu_0}}}$, it does not give anything. Consider $h(0,w)$. $\Rightarrow h(0,w) = \alpha w^{m_{\nu_0}-1} + \square\, w^{m_{\nu_0}} + \cdots$, since $(E\cdot C)_{p_{\nu_0}} \geq m_{\nu_0}-1$.

$$(E\cdot C)_{p_{\nu_0}} \geq m_{\nu_0} - 1 \Leftrightarrow \alpha = 0.$$