Restricting to U', we find the desired presentation

$$\varphi - \eta = d\psi,$$

where ψ is a meromorphic $(p-1)$-form on M that is holomorphic in U'.

┌ Since $\Phi|U' - \varphi = d\gamma$, and $\Phi|U' - \eta|U' = d\alpha|U'$,

$-\varphi + \eta|U' = d\gamma - d\alpha|U'$, $\overset{\text{since } U' \text{ is open}}{\Rightarrow}$ $-\varphi + \eta = d\gamma - d\alpha$. by the property of a meromorphic form

$\Rightarrow \varphi - \eta = d(\gamma - \alpha)$, $\Rightarrow$ let $\gamma - \alpha = \psi$. $\Rightarrow \varphi - \eta = d\psi$.

$\Rightarrow$ Clearly since γ, α meromorphic $(p-1)$-form on M and holomorphic in U', ψ is meromorphic $(p-1)$-form on M and holomorphic in U'. ┘

When $p=1$, it is clear from our above description of residue cycles that a closed meromorphic 1-form that has local presentations

$$\varphi = d\psi + \eta$$

will have no residues, and consequently φ is of the second kind. Q.E.D.

┌ Suppose a closed meromorphic p-form φ has local representations $\varphi = d\psi + \eta$. $\Rightarrow \exists$ a divisor D s.t. φ is holo. in $M-D$. D " φ_∞ (see p131).

$$\int_\gamma \varphi = \int_{\sum m_i \gamma_{D_i}} \varphi = m_i \int_{\gamma_{D_i}} \varphi = m_i \int_{\gamma_{D_i}} (d\psi + \eta)$$

$$= m_i \underbrace{\int_{\gamma_{D_i}} d\psi}_{=0} + m_i \underbrace{\int_{\gamma_{D_i}} \eta}_{=0} = 0,$$

by Stoke's theorem. since η is holomorphic.

where $D = D_1 + \cdots + D_k$, each D_i irreducible.