Definition. A distribution on $\mathbb{R}^n$ is a linear map $T: C_c^\infty(\mathbb{R}^n) \to \mathbb{C}$ that is continuous in the C^∞ topology. The vector space of distributions on $\mathbb{R}^n$ is denoted $\mathcal{D}(\mathbb{R}^n)$.

We say that a distribution is of order p if it is continuous in the C^p-topology. Now any linear map from a topological vector space V to $\mathbb{C}$ is continuous $\Leftrightarrow$ the inverse of the unit ball in $\mathbb{C}$ is open in the topology of V.

$\ulcorner$ $T^{-1}(U) \ni v_0 \Rightarrow T(v_0) \in U. \Rightarrow T(v_0) + rB(0,1) \subset U.$ for r sufficiently small. $\Rightarrow v_0 + rT^{-1}(B(0,1)) \subset U$.

Since for the space $C_c^\infty(K)$ of functions supported in a compact set K the C^∞ topology is the union of the C^p topologies, we see that any distribution is locally of finite order.

" I think the explanation in this book is not good. So I go to Rudin's Functional Analysis, (P136 ~ P141).

Test Function Spaces

6.2. The space $\mathcal{D}(\Omega)$. Consider a nonempty open set $\Omega \subset \mathbb{R}^n$. For each compact $K \subset \Omega$, the Fréchet space $\mathcal{D}_K$ was described in Section 1.46.

$\ulcorner$ $\mathcal{D}_K$ denotes the space of all $f \in C^\infty(\mathbb{R}^n)$ whose support lies in K.

We now define a topology on $C^\infty(\Omega)$ which makes $C^\infty(\Omega)$ into a Fréchet space with the Heine-Borel property, such that $\mathcal{D}_K$ is a closed subspace of $C^\infty(\Omega)$ whenever $K \subset \Omega$.