$\Rightarrow$ By Oka's lemma, $\ker F|_U$ is locally finitely generated as a sheaf of $\mathcal{O}$-modules. $\Rightarrow \exists\ \sigma_1, \sigma_2 \cdots \sigma_m \in \ker F|_{U'}$, $U' \ni z_0$, $U' \subset U$. s.t $\sigma_1 . \sigma_2 \cdots \sigma_m$ generate the $\mathcal{O}_z$-module $\ker F_z$ for $z \in U'$. Consider $\mathcal{O}^{(m)}(U')$.
$\Rightarrow \quad \mathcal{O}^{(m)}(U') \longrightarrow \ker F|_{U'} \longrightarrow \mathcal{O}^{(p)}(U') \longrightarrow \mathcal{O}^{(q)}(U') \rightarrow \mathcal{F}(U') \rightarrow 0$ is exact. Let $r = m$.

Applying it again, we obtain

$$\mathcal{O}^{(s)} \longrightarrow \mathcal{O}^{(r)} \longrightarrow \mathcal{O}^{(p)} \longrightarrow \mathcal{O}^{(q)} \longrightarrow \mathcal{F} \longrightarrow 0$$

in $U'' \subset U'$. After at most n steps, the syzygy theorem assures us that the kernel on the left will have as stalk at z_0 a free $\mathcal{O}_{z_0}$-module, and this then gives our local syzygy. Q.E.D.

Continue to apply Oka's lemma, then we get

$$0 \longrightarrow \underset{\ker\alpha}{\overset{\shortparallel}{F}} \longrightarrow \mathcal{O}^{(k_n)} \xrightarrow{\alpha} \mathcal{O}^{(k_{n-1})} \longrightarrow \cdots \longrightarrow \mathcal{O}^{(k_0)} \longrightarrow \mathcal{F} \longrightarrow 0 \quad \text{in } U_0 \subset U.$$

$\Rightarrow$ At $z \in U_0$, $0 \longrightarrow F_z \longrightarrow \mathcal{O}_z^{(k_n)} \longrightarrow \cdots \rightarrow \mathcal{O}_z^{(k_0)} \longrightarrow \mathcal{F}_z \rightarrow 0$ and $\mathcal{O}_z^{(k_n)}, \cdots \mathcal{O}_z^{(k_0)}$ are free $\mathcal{O}$-modules $\Rightarrow$ By Syzygy theorem on P694, $F_z = 0$. $\Rightarrow F = 0$.
$\Rightarrow \quad 0 \rightarrow \mathcal{O}^{(k_n)} \longrightarrow \cdots \rightarrow \mathcal{O}^{(k_0)} \longrightarrow \mathcal{F} \longrightarrow 0$ in $U_0 \subset U$.

As an application we have:

For a coherent sheaf $\mathcal{F}$, the cohomology sheaves

$$\mathcal{H}^q(\mathcal{F}) = 0 \quad \text{for } q > 0.$$