Remark : ①

$$\begin{tikzcd} {[-V]|_{U\cap V}} \arrow[r] \arrow[d] & U\cap V \times \mathbb{C} \arrow[dl] \\ U\cap V \arrow[u, bend left, "df"] & \end{tikzcd}$$

$\Rightarrow \exists$ a section σ s.t

$$\begin{tikzcd} {[V]|_{U\cap V}} \arrow[r, "\varphi|"] \arrow[d] & U\cap V \times \mathbb{C} \arrow[dl] \\ U\cap V \arrow[u, bend left, "\sigma"] & \end{tikzcd}$$

s.t $\sigma \cdot df = 1$. formally

$\sigma \otimes df = 1$.

where

$$\begin{tikzcd} {[V]|_U} \arrow[r, "\varphi"] \arrow[d] & U \times \mathbb{C} \arrow[dl] \\ U & \end{tikzcd}$$

is a trivialization over U.

Then $\exists$ an extension to U i.e $\exists$ $\tau : U \longrightarrow [V]|_U$
s.t $\tau|_{V\cap U} = \sigma$

for σ can be considered a holomorphic function from $U\cap V$ to $\mathbb{C}$. $\Rightarrow$ $\sigma : \mathbb{C}^{n-1} \longrightarrow \mathbb{C}$ holomorphic.

$$\begin{tikzcd} U \arrow[d] & \supset & U\cap V \arrow[d] & \\ \mathbb{C}^n \arrow[rrr, dashed, bend right, "?"'] & \supset & \mathbb{C}^{n-1} \ni \{(z_1, \cdots z_{n-1}, 0)\} \arrow[r, "\sigma"] & \mathbb{C} \end{tikzcd}$$

$? = \tau$ may be defined by

$\tau(z_1, z_2, \cdots z_n)$

$= \sigma(z_1, z_2, \cdots z_{n-1})$