Date

Then since we choose a set Γ from D, and no n points of D are linearly independent, for each i, $\{p_1^i, p_2^i, \cdots p_{n-1}^i, q\}$ is a set of linearly independent.
$\Rightarrow \{p_1^i, p_2^i, \cdots p_{n-1}^i\}$ is linearly independent and its linear span does not contain q. $\lrcorner\!\lrcorner$

We can thus find hyperplanes $H_1, H_2, \cdots H_k$ in $\mathbb{P}^n$ containing the points $\{p_\alpha^i\}_\alpha$ but not q: the sum $H_1 + \cdots + H_k$ is the desired hypersurface of degree k.

Γ $H_i = (a_{i0} Z_0 + \cdots + a_{in} Z_n = 0)$.
$\prod_{i=1}^{k} a_{i0} Z_0 + \cdots + a_{in} Z_n = F(Z_0, \cdots Z_n) \Rightarrow (F=0) = H_1 + \cdots + H_k$ $\lrcorner\!\lrcorner$

We see from this that the vector space of sections of $[kD]$ vanishing on all the points of D has codimension at least $k(n-1)+1$ in $H^0(C, \mathcal{O}(kD))$, i.e.

$$h^0(kD) - h^0((k-1)D) \geq k(n-1)+1, \quad \text{for } k \leq m.$$

Γ $\Gamma = \{q_1, q_2, \cdots q_{k(n-1)+1} \in D\}$.
Let F_i be a hypersurface (i.e. homogeneous polynomial on $\mathbb{P}^n$) of deg k, which does not contain q_i.
$\Rightarrow x_1 F_1 + \cdots + x_{k(n-1)+1} F_{k(n-1)+1} = 0$
$\Rightarrow$ Plug in q_i. $\Rightarrow x_i F_i(q_i) = 0 \Rightarrow x_i = 0$ since $F_i(q_i) \neq 0$, and other F_j's contain q_i.
$\Rightarrow \{F_i\}$ is linearly independent and they are not in the vector space $\Rightarrow$ codimension of the v.s $\geq k(n-1)+1$.
Let $V = \{\sigma \in H^0(S, \mathcal{O}(kD)) ; \sigma = 0 \text{ on } D\}$. $\Rightarrow$ We have a correspondence between V and $H^0(S, \mathcal{O}((k-1)D))$, given by