

From the above argument,

in $V \cap W \longrightarrow V \longrightarrow \mathbb{P}^n$, V is positive divisor of \mathbb{P}^n . and $V \cap W$ is a positive divisor of V .

\Rightarrow Consecutively, we can use the Lefschetz Hyperplane Theorem. to this case. \searrow

A final remark on the Lefschetz theorem: Lefschetz's method was insofar as possible to study the topology of an algebraic variety M inductively, reducing questions about the homology of M to questions about the homology of a smaller-dimensional variety. His original proof of the last theorem asserted that for a hyperplane section V of M , the map $H_q(V, \mathbb{Z}) \rightarrow H_q(M, \mathbb{Z})$ is an isomorphism for $q < n-1$, and surjective in dimension $n-1$. By the hard Lefschetz theorem, the homology of M in dimension above n is mirrored in dimensions less than n , and by the Lefschetz decomposition, any nonprimitive cycle in dimension n can be obtained by intersecting a cycle in dimension greater than n with hyperplanes.

\square By p. 100. $L^k: H^{n-k}(M) \longrightarrow H^{n+k}(M)$ is isomorphic.

\Rightarrow By Poincare duality,

$$\begin{array}{ccc} H^{n-k}(M) & \xrightarrow{L^k \cong} & H^{n+k}(M) \\ \text{P.D.} \downarrow & & \downarrow \text{P.D.} \\ H_{n+k}(M) & \xrightarrow[\cong]{\cap \mathbb{P}^{n-k}} & H_{n-k}(M) \end{array}$$

\nwarrow

The homology of M in dimension above n is mirrored in dimensions