

$$\begin{aligned}
 * \bar{\partial} * \bar{\partial} \psi &\equiv (-1)^q \sum_{k>q} f_{\bar{k},k} \bar{\varphi}_1 \wedge \dots \wedge \bar{\varphi}_q (-1)^{q^2} \\
 &+ \sum_{\substack{k>q \\ l \leq q}} (-1)^{k-1} f_{\bar{k},l} \bar{\varphi}_1 \wedge \dots \wedge \hat{\bar{\varphi}}_l \wedge \dots \wedge \bar{\varphi}_q \wedge \bar{\varphi}_k (-1)^{q+l+k} \\
 &\quad \underbrace{\hspace{10em}}_{(-1)^{q+l-1}}
 \end{aligned}$$

This gives $* \bar{\partial} * \bar{\partial} \psi$, and the other term $\bar{\partial} * \bar{\partial} * \psi$ is similar but shorter:

$$* \psi = \bar{f} \bar{\varphi}_{q+1} \wedge \dots \wedge \bar{\varphi}_n \wedge \Xi'$$

$$\bar{\partial} * \psi = \bar{\partial} \bar{f} \wedge \bar{\varphi}_{q+1} \wedge \dots \wedge \bar{\varphi}_n \wedge \Xi' = \sum_{l \leq q} \bar{v}_l(\bar{f}) \bar{\varphi}_l \wedge \bar{\varphi}_{q+1} \wedge \dots \wedge \bar{\varphi}_n \wedge \Xi'$$

$$\equiv \sum_{l \leq q} \bar{f}_l \bar{\varphi}_l \wedge \bar{\varphi}_{q+1} \wedge \dots \wedge \bar{\varphi}_n \wedge \Xi'$$

$$\begin{aligned}
 * \bar{\partial} * \psi &\equiv \sum_{l \leq q} f_l \bar{\varphi}_1 \wedge \dots \wedge \hat{\bar{\varphi}}_l \wedge \dots \wedge \bar{\varphi}_q (-1)^{l-1} \\
 &= \sum_{l \leq q} (-1)^{l-1} f_l \bar{\varphi}_1 \wedge \dots \wedge \hat{\bar{\varphi}}_l \wedge \dots \wedge \bar{\varphi}_q
 \end{aligned}$$

$$\bar{\partial} * \bar{\partial} * \psi \equiv \sum_{\substack{l \leq q \\ k \leq q}} (-1)^{l-1} \bar{\partial} f_l(\bar{v}_k) \bar{\varphi}_k \wedge \bar{\varphi}_1 \wedge \dots \wedge \hat{\bar{\varphi}}_l \wedge \dots \wedge \bar{\varphi}_q$$

$$+ \sum_{\substack{l \leq q \\ k > q}} (-1)^{l-1} \bar{\partial} f_l(\bar{v}_k) \bar{\varphi}_1 \wedge \dots \wedge \hat{\bar{\varphi}}_l \wedge \dots \wedge \bar{\varphi}_q \wedge \bar{\varphi}_k (-1)^{q-1}$$

$$= \sum_{l \leq q} f_{l,\bar{q}} \bar{\varphi}_1 \wedge \dots \wedge \bar{\varphi}_q + \sum_{\substack{l \leq q \\ k > q}} (-1)^{q+l} f_{l,\bar{k}} \bar{\varphi}_1 \wedge \dots \wedge \hat{\bar{\varphi}}_l \wedge \dots \wedge \bar{\varphi}_q \wedge \bar{\varphi}_k$$

Now $\bar{v}_i(\bar{v}_j f) - \bar{v}_j(\bar{v}_i f) \equiv A'(f) \Rightarrow \frac{\partial^2}{\partial \bar{z}_i \partial \bar{z}_j} = \frac{\partial^2}{\partial \bar{z}_j \partial \bar{z}_i}$ so that modulo first-order terms.