$\Rightarrow V$ lies in a $\mathbb{P}^{d+k-1}$.

(ii) V degenerate $\Rightarrow V \subset \mathbb{P}^{n-1} \Rightarrow V \subset \mathbb{P}^{\ell}$
Suppose $V \subset \mathbb{P}^{\ell}$ nondegenerate
$\Rightarrow d \geq \ell - k + 1 \Rightarrow d + k - 1 \geq \ell \Rightarrow V$ lies in a $(d+k-1)$ - plane $\mathbb{P}^{d+k-1}$.

($\Leftarrow$). Given $V \subset \mathbb{P}^n$ irreducible, nondegenerate, then can we conclude that $d \geq n-k+1$?
Suppose not. $\Rightarrow d < n-k+1 \Rightarrow$ By the assumption, since $V \subset \mathbb{P}^{d+k-1} \subsetneq \mathbb{P}^n$, V is degenerate $\Rightarrow$ Contradiction. //

Suppose V has degree 1 in $\mathbb{P}^n$. Assume that V is irreducible and $\dim V = k$. $\Rightarrow V \subset \mathbb{P}^{1+k-1} = \mathbb{P}^k \Rightarrow$ Since $\mathbb{P}^k$ is irreducible and $\dim(\mathbb{P}^k \cap V) = \dim \mathbb{P}^k$, $\mathbb{P}^k \subset V$ locally, $\mathbb{P}^k \subset V \Rightarrow V = \mathbb{P}^k$. by p 77 Th 1.K. H. Whitney. If V is not irreducible, $V = \mathbb{P}^k \cup \cdots \cup \mathbb{P}^k$ can not be of deg 1.))

We shall see that varieties that realize this lower bound on the degree — e.g., curves of degree n in $\mathbb{P}^n$, surfaces of degree $n-1$ in $\mathbb{P}^n$, etc. — are of a special character.

Tangent Spaces to Algebraic Varieties

To a variety $V \subset \mathbb{P}^n$ and a smooth point $p \in V$ is associated a linear subspace of $\mathbb{P}^n$, the tangent cone to V at p. This may be defined in several