By ① & ②, $\pi_p(V) = \mathbb{P}^{n-1} \cap \overline{p.V}$.

For $\deg(\overline{p.V}) = \deg(H \cap \overline{p.V})$, since $H$ is a generic hyperplane, not containing $p$. $H \cap \overline{p.V}$ is a $k$-dimensional variety. By the definition of degree, $\deg(\overline{p.V}) = \#(\overline{p.V} \cap H_1 \cap H_2 \cap \cdots \cap H_{k+1}) = \#(\overline{p.V} \cap \mathbb{P}^{n-k-1}) = \#((\overline{p.V} \cap H_1) \cap H_2 \cap \cdots \cap H_{k+1})$
$= \#((\overline{p.V} \cap H_1) \cap \mathbb{P}^{n-k}) = \deg(\overline{p.V} \cap H_1) = \deg(\overline{p.V} \cap H)$.

92. 12. 31.

Another variety we may associate with a variety $V \subset \mathbb{P}^n$ is its chordal variety $C(V)$, defined to be the union of all lines meeting $V$ twice or, in the limiting case, tangent to $V$.

⌈ Comments on $\deg V = \deg(\pi_p V)$ which is wrong.

Proposition (5.5). Let $X^r \subset \mathbb{P}^n - \{x\}$ and let $p_x : \mathbb{P}^n - \{x\} \longrightarrow \mathbb{P}^{n-1}$ be the projection. Let $Y = p_x(X)$, let $\pi \doteq \text{res}\, p_x : X \longrightarrow Y$. Then $\deg X = \deg \pi \cdot \deg Y$
( D. Mumford, Algebraic Geometry I, Complex Projective Varieties ) 」

- $C(V)$ is the image under projection on the third factor of the closure of the incidence correspondence $I \subset \mathbb{P}^n \times \mathbb{P}^n \times \mathbb{P}^n$ defined by

$$I = \{(p, q, r) : p \neq q \in V, \quad p \wedge q \wedge r = 0\}.$$

⌈ Since $p \wedge q \wedge r = 0$, $r$ is a linear combination of $p$ and $q$. $\Rightarrow$ All linear combinations of $p$ and $q$ form the line through $p$ and $q$. As limiting case,