Date . .

$$\Rightarrow \begin{pmatrix} \frac{\partial w_1}{\partial x_1}, & \frac{\partial w_1}{\partial x_2}, & \cdots & \frac{\partial w_1}{\partial x_n} \\ \frac{\partial w_2}{\partial x_1}, & \frac{\partial w_2}{\partial x_2} & \cdots & \frac{\partial w_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial w_N}{\partial x_1}, & \frac{\partial w_N}{\partial x_2}, & \cdots & \frac{\partial w_N}{\partial x_n} \end{pmatrix}$$

$$\frac{\partial w_i}{\partial x_j} = \frac{\partial\left(\frac{S_i}{S_0}\right)}{\partial x_j} = \frac{1}{S_0^2}\left(\frac{\partial S_i}{\partial x_j} S_0 - \frac{\partial S_0}{\partial x_j} S_i\right)$$

$$= \frac{1}{S_0^2}\begin{pmatrix} S_0\frac{\partial S_1}{\partial x_1} - S_1\frac{\partial S_0}{\partial x_1}, & S_0\frac{\partial S_1}{\partial x_2} - S_1\frac{\partial S_0}{\partial x_2}, & \cdots & S_0\frac{\partial S_1}{\partial x_n} - S_1\frac{\partial S_0}{\partial x_n} \\ S_0\frac{\partial S_2}{\partial x_1} - S_2\frac{\partial S_0}{\partial x_1}, & S_0\frac{\partial S_2}{\partial x_2} - S_2\frac{\partial S_0}{\partial x_2}, & \cdots & S_0\frac{\partial S_2}{\partial x_n} - S_2\frac{\partial S_0}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ S_0\frac{\partial S_N}{\partial x_1} - S_N\frac{\partial S_0}{\partial x_1}, & S_0\frac{\partial S_N}{\partial x_2} - S_N\frac{\partial S_0}{\partial x_2}, & \cdots & S_0\frac{\partial S_N}{\partial x_n} - S_N\frac{\partial S_0}{\partial x_n} \end{pmatrix}$$

For simplicity, let $n=2$. $N=3$.

Correction: $i_L$ has injective differential everywhere

$$U \xrightarrow{i_L} \mathbb{P}^3 . \qquad w_1 = \frac{Z_1}{Z_0}, \quad w_2 = \frac{Z_2}{Z_0} \cdots$$

$$T'U \xrightarrow{i_{L*}} T'\mathbb{P}^3 \text{ injective} \qquad w_3 = \frac{Z_3}{Z_0}$$

$$\Leftrightarrow \quad T^{*\prime}\mathbb{P}^3 \xrightarrow{i_L^*} T^{*\prime}U \text{ is surjective}$$

Given $v^* \in T^{*\prime}_{x_0}U$, $\quad v^* = a_1\, dx_1 + a_2\, dx_2$

What is $i_L^*(dw_1)$ ?

$$i_L^*(dw_1)\left(\frac{\partial}{\partial x_1}\right) = dw_1\left(i_{L*}\frac{\partial}{\partial x_1}\right) = dw_1\left(\frac{1}{S_0^2}\left(S_0\frac{\partial S_1}{\partial x_1} - S_1\frac{\partial S_0}{\partial x_1}\right)\frac{\partial}{\partial w_1}\right.$$

$$\left. + \quad \right) = \frac{1}{S_0^2}\left(S_0\frac{\partial S_1}{\partial x_1} - S_1\frac{\partial S_0}{\partial x_1}\right) = \frac{\partial\left(\frac{S_1}{S_0}\right)}{\partial x_1}$$

$$\Rightarrow \quad i_L^*(dw_1)\left(\frac{\partial}{\partial x_2}\right) = \frac{\partial\left(\frac{S_1}{S_0}\right)}{\partial x_2}$$