$U = M - D$ for sufficiently large divisors D.

「 ($\Rightarrow$) According to the definition of a differential of the second kind above, $\varphi = \psi|_U - d\eta$. $\psi \in \Omega(M)$
$\Rightarrow \psi|_U = \varphi + d\eta$. $\eta \in \Omega(U)$

$$\Rightarrow \int_\gamma \varphi = \int_\gamma \psi|_U - d\eta = \int_\gamma \psi|_U - \cancel{\int_\gamma d\eta}^{=0}$$

$$= \int_\gamma \psi|_U = \int_\gamma \psi = \int_{\partial\tau} \psi = 0$$ since γ is homologous to zero in M.

($\Leftarrow$) According to P79. Proposition 6.49 (Bott. Differential Forms in Algebraic Topology), there is an exact sequence

$$H^P_{DR}(M) \longrightarrow H^P_{DR}(U) \longrightarrow H^{P+1}_{DR}(M, U)$$

$\underset{[\varphi]=0}{\downarrow}$ since $H^P_{DR}(U) = \mathrm{Hom}(H_p(U), \mathbb{C})$ and $\int_\gamma \varphi = 0$ for all $\gamma \in H_p(U)$.

$\Rightarrow \exists\ [\psi] \in H^P_{DR}(M)$ s.t. $[\psi|_U] = [\varphi]$
$\Rightarrow \psi|_U = \varphi + d\eta$. 」

" Example, $M = \mathbb{C}$, $D = \{0\}$. $\varphi = \frac{dz}{z}$

$$\int_\gamma \frac{dz}{z}, \quad \gamma \in H_1(\mathbb{C} - \{0\}, \mathbb{Z}).$$
"