$$\frac{\partial \phi}{\partial z_1} = (0,\ z_0,\ 2z_1).$$

$$\begin{pmatrix} 2z_0, & z_1, & 0 \\ 0, & z_0, & 2z_1 \end{pmatrix} \text{ has rank } 2.$$

For the general case, $\exists$ no zero component.

$\Rightarrow$ Since $[X_0, \cdots X_n] \xrightarrow{\phi} [X_0^d, \cdots X_1^d, \cdots X_n^d]$, $(d > 1)$

$$\left(\frac{\partial \phi}{\partial X_i}\right) = \begin{array}{c} \\ \begin{pmatrix} dX_0^{d-1} & 0 & & 0 \\ 0 & dX_1^{d-1} & & \vdots \\ \vdots & 0 & & 0 \\ 0 & \vdots\ 0 & & dX_n^{d-1} \end{pmatrix} \end{array} \begin{array}{l} \to \frac{\partial}{\partial X_0}, \\ \frac{\partial}{\partial X_1} \\ \vdots \\ \frac{\partial}{\partial X_n} \end{array} \text{ has rank } (n+1).$$

$$dX_0^{d-1} = \frac{\partial X_0^d}{\partial X_0}, \quad 0 = \frac{\quad}{\partial X_0}$$

Pick $X_1^d, \cdots X_n^d, \ X_0 X_1^{d-1}$. in case $X_0 = 0$, $X_i \neq 0$ $i = 1, \cdots n$.

$$\begin{pmatrix} \vdots & X_1^{d-1} \neq 0 & 0 & 0 \\ & & dX_1^{d-1} & \vdots \\ & & 0 & 0 \\ & & \vdots & \\ & & 0 & dX_n^{d-1} \end{pmatrix} \text{ has rank } (n+1).$$

In case $X_0 = X_1 = 0$. $X_i \neq 0$ $i = 2, \cdots n$.

Pick $X_0 X_2^{d-1}, \ X_1 X_2^{d-1}, \ X_2^d, \cdots X_n^d \Rightarrow$

$$\begin{pmatrix} X_2^{d-1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ X_2^{d-1} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ dX^{d-1} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ dX_n^{d-1} \end{pmatrix} \text{ at } X_0 = X_1 = 0.$$

In case $X_0 = X_1 = X_2 = 0$. $X_i \neq 0$, $i = 3, \cdots n$.