Note that $g \le \frac{m(m-1)}{2}(n-1) + m\epsilon$ holds for all d.

Note that if C achieves this bound, then equality must hold in the basic inequality (*) above, and it follows that the complete linear system $|kD|$ on C is cut out by hypersurfaces of degree k; or, in other words, the map

$$H^0(\mathbb{P}^n, \mathcal{O}(kH)) = Sym^k H^0(\mathbb{P}^n, \mathcal{O}(H)) \longrightarrow H^0(C, \mathcal{O}(kH))$$

must be surjective.

「 "Comment"

$$\begin{array}{ccc} H^0(\mathbb{P}^n, \mathcal{O}(kH)) & \xrightarrow{\phi} & H^0(C, \mathcal{O}(kH)) \\ \Psi & & \\ \sigma & \longmapsto & \sigma|_{(s=c)} \end{array}$$

$\Rightarrow$ ϕ may not be injective for if $\sigma| = 0$ on C, it is possible.

We want to examine for the case $n = 2$, $k = 2$.

$$\begin{array}{ccc} H^0(\mathbb{P}^2, \mathcal{O}(2H)) & \xrightarrow{\phi} & H^0(C, \mathcal{O}(2H)) \\ \Psi & & \\ \sigma & \longmapsto & \sigma|_C \end{array}$$

$\sigma : \mathbb{P}^2 \longrightarrow [H] \otimes [H]$.

$\Rightarrow \sigma|_U \longrightarrow [H] \otimes [H]|_U \cong [H]|_U \otimes [H]|_U \cong (U \times \mathbb{C}) \otimes (U \times \mathbb{C})$

$\qquad \qquad \qquad \qquad \qquad \cong U \times (\mathbb{C} \otimes \mathbb{C})$ $\qquad \cong U \times (\mathbb{C} \otimes \mathbb{C})$.

$\Rightarrow \sigma|_U = \sum \tau_i \otimes \tau_j \cdot a_{ij}$, where τ_i's are frame elements of $[H]$ on U locally.

Another point: Given $\sigma_\alpha = g_{\alpha\beta} h_{\alpha\beta} \sigma_\beta$, we can not conculde that $\exists$ $s_\alpha = g_{\alpha\beta} s_\beta$ for L, where $g_{\alpha\beta}$'s & $h_{\alpha\beta}$'s are transition functions for L & L', respectively.

Thus σ may not be expressed as $s_1 \otimes \cdots \otimes s_k$ where s_i's are sections of $[H]$, so that we can not