Let $\mathbb{P}^2 \subset \mathbb{P}^3$ be a plane not containing p and $I \subset \mathbb{P}^2 \times N_p$ the incidence correspondence defined by

$$I = \{ (q, F) : \overline{pq} \subset F \}.$$

The projection of I on the second factor expresses I as a double cover of $N_p \cong \mathbb{P}^2$, branched over the locus of singular quadrics in N_p.

「 p is a generic point $\Rightarrow$ N_p = the set of quadrics in $W \cong \mathbb{P}^2$, and, since $\dim W_2 = 6$, in $W \cong \mathbb{P}^9$ $N_p \cap W_2 = \phi$ ($\because$ N_p is a generic net).

F is singular $\Rightarrow$ rank $F = 3$, i.e., $F \in W_1$.

(Refer to p744 & p745 for W_1 & W_2.)

We may assume $F = X_0^2 + X_1^2 + X_2^2 = 0$ and $p = [1, a, b, c] \in F$ $\Rightarrow$ $1 + a^2 + b^2 = 0$

$\Rightarrow$ $T_p(F) = X_0 + aX_1 + bX_2 = 0$.

$F \cap T_p(F) = \{ X_0 + aX_1 + bX_2 = 0 \} \cap \{ X_0^2 + X_1^2 + X_2^2 = 0 \}$

$$\begin{aligned}
\Rightarrow \quad & (aX_1 + bX_2)^2 + X_1^2 + X_2^2 = 0 \\
= \quad & (a^2+1) X_1^2 + (b^2+1) X_2^2 + 2ab X_1 X_2 \\
= \quad & -b^2 X_1^2 - a^2 X_2^2 + 2ab X_1 X_2 = -(bX_1 - aX_2)^2 \\
\Rightarrow \quad & bX_1 = aX_2 \\
\Rightarrow \quad & F \cap T_p(F) = (X_0 + aX_1 + bX_2 = 0) \cap (bX_1 - aX_2 = 0)
\end{aligned}$$

which is a line passing through p.

In case $p = [0, 0, 0, 1]$,

Tangent cone $T_p(F) = 0$.

Thus we have only to show that p can not