tion will be a reference to "the generic k-plane in $\mathbb{P}^n$": until the section on Grassmannians, we have – at least officially – no way of parametrizing linear subspaces of projective space. The fastidious reader may substitute "the linear span of the generic $(k+1)$-tuple of points in $\mathbb{P}^n$."

A basic fact about analytic subvarieties is the

Proposition. V_s is contained in an analytic subvariety of M not equal to V.

Proof. For $p \in V$ let k be the largest integer such that there exist k functions $f_1 \cdots f_k$ in a nbd U of p vanishing on V and such that $\mathcal{J}(f)$ has a $k \times k$ minor not everywhere singular on V; we may assume that

$$|(\partial f_i/\partial z_j)_{1\le i,j\le k}| \not\equiv 0 \text{ on } V.$$

$\ulcorner$ $\exists$ open set $O \ni p$ s.t $O \cap V = \{f_1 = \cdots = f_\ell = 0\}$, $f_1, \cdots f_\ell$ holomorphic on O. Say

$$\left(\frac{\partial f_i}{\partial z_j}\right)_{1\le i,j\le k} \text{ is not everywhere singular on } V.$$

But $\det\left(\frac{\partial f_i}{\partial z_j}\right) \equiv 0$ on V if $\# i\text{'s} = \# j\text{'s} > k$. $\lrcorner$