Here $0 = [(1,0,0)]$, $\infty = [(0,1,0)]$, $r = [(1,r,0)]$
$1 = [(1,1,0)]$.

By using translation and projective isomorphism, we have two polynomials $f(x,y)$, $g(x,y)$ of deg 4 representing C and C' respectively, s.t

$f(x,y)$ has slopes 0 and ∞ at $(0,0)$
$g(x,y)$ " 1 and $1/r$ at " .

$$\Rightarrow f(x,y) = a_{00} + a_{10}x + a_{01}y + a_{11}xy + a_{12}xy^2 + a_{21}x^2y + a_{22}x^2y^2 + a_{31}x^3y + a_{13}xy^3 + a_{40}x^4 + a_{04}y^4$$

$$\Rightarrow f(0,0) = a_{00} = 0.$$

By the slope conditions, $\frac{\partial f}{\partial x} = a_{10} = 0$ $\frac{\partial f}{\partial y} = a_{01} = 0$ at $(0,0)$. $\Rightarrow f(x,y) = xy(a_{11} + a_{12}y + \cdots)$, $a_{11} \neq 0$ since f has only two roots at $(0,0)$.

Similarly,

$$g(x,y) = b_{00} + b_{10}z + b_{01}w + b_{11}zw + \cdots$$
$$= G(z,w), \text{ where } z = x-y, \quad w = x-ry$$

$r \neq 1$. $\Rightarrow$ $b_{00} = 0$ since $g(0,0) = 0$

$$\Rightarrow G(x-y, x-ry) = g(x,y)$$

$$\Rightarrow \left(\frac{\partial G}{\partial z}, \frac{\partial G}{\partial w}\right) \cdot \left(1 - \frac{dy}{dx}, 1 - r\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{\partial G}{\partial z}(1-y') + \frac{\partial G}{\partial w}(1-ry') = 0.$$

At $(0,0)$, (i) $y' = 1$ (ii) $y' = 1/r$

$\Rightarrow \frac{\partial G}{\partial w} = 0 = b_{01}$ $\Rightarrow \frac{\partial G}{\partial z} = b_{10} = 0$