On $W' \cap U'$,

$g\Gamma \in \phi^{-1}(W' \cap U')$. have two representative, i.e.

if $g = \begin{pmatrix} 1, & a, & b \\ 0, & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$ s.t $0 < \mathrm{Re}\, a < 1$ $\quad 0 < \mathrm{Im}\, a < 1$

$\frac{1}{2} < \mathrm{Re}\, c < \frac{3}{2}$, $\quad 0 < \mathrm{Im}\, c < 1$,

then $\begin{pmatrix} 1, & a, & b \\ 0, & 1, & c \\ 0, & 0, & 1 \end{pmatrix} \begin{pmatrix} 1, & m, & n \\ 0 & 1 & -l \\ 0 & 0, & 1 \end{pmatrix} = \begin{pmatrix} 1, & a+m, & n+b-al \\ 0, & 1, & c-l \\ 0, & 0, & 1 \end{pmatrix}$

where $\mathrm{Re}\, l = 1$, i.e $l = 1$.

$$\Rightarrow \quad \varphi_{U'}(g\Gamma) = ([(a+m, c-l)], [n+b-al]) = ([(a, c)], [b-al])$$

$$\Rightarrow \quad \varphi_{U'} \circ \varphi_{W'}([(a,c)], [b]) = ([(a,c)], [b-al]) = ([(a,c)], [b-a])$$

$\Rightarrow$ Clearly the transition function is holomorphic.
As we can see. $n+b-al$, the trivialization depends on l, not m, n. $M \to \mathbb{C}/\mathbb{Z}^4$ is not trivial. □

Since $\dim E_r \geq \dim E_{r+1}$ and the Euler characteristic is invariant under taking cohomology, we have the Fröhlicher relations

$$\sum_{p+q=r} h^{p,q} \geq b_r,$$

$$\sum_{p,q} (-1)^{p+q} h^{p,q} = \sum_r (-1)^r b_r = \chi(M),$$

where $h^{p,q} = \dim H^{p,q}_{\bar\partial}(M)$ and b_r is the rth Betti number.