

$$\begin{array}{ccccccc}
 & 0 & & 0 & & 0 & \\
 & \downarrow & & \downarrow & & \downarrow & \\
 0 & \rightarrow & \text{Hom}(f(L), \Omega^2) & \rightarrow & \text{Hom}(\mathcal{O} \oplus \mathcal{O}, \Omega^2) & \rightarrow & \text{Hom}(\mathcal{O}(L^*), \Omega^2) \rightarrow 0 \\
 & \downarrow & & \downarrow & & \downarrow & \\
 \text{0-stage: } 0 & \rightarrow & \text{Hom}(\mathcal{O} \oplus \mathcal{O}, \Omega^2) & \rightarrow & \text{Hom}(\mathcal{O}(L^*) \oplus \mathcal{O} \oplus \mathcal{O}, \Omega^2) & \rightarrow & \text{Hom}(\mathcal{O}(L^*), \Omega^2) \xrightarrow{\text{Ext}^0(\mathcal{O}(L^*), \Omega^2)} 0 \\
 & \downarrow & & \downarrow & & \downarrow & \uparrow \text{"ker } \delta \\
 & & & & \downarrow \psi & \xrightarrow{\psi} & \downarrow \psi \\
 & & & & \downarrow \psi & \xrightarrow{\psi} & \downarrow \psi \\
 \text{1-stage: } 0 & \rightarrow & \text{Hom}(\mathcal{O}(L^*), \Omega^2) & \rightarrow & \text{Hom}(\mathcal{O}(L^*), \Omega^2) & \rightarrow & 0 \rightarrow 0 \\
 & & \downarrow -\psi & \xrightarrow{-\psi} & \downarrow -\psi & & \downarrow \\
 \text{2-stage: } 0 & \rightarrow & 0 & \rightarrow & 0 & \rightarrow & 0 \rightarrow 0
 \end{array}$$

$$\begin{array}{ccc}
 \Rightarrow & \text{Ext}^0(S; \mathcal{O}(L^*), \Omega^2) & \rightarrow \text{Ext}^1(S; f(L), \Omega^2) \\
 & \parallel & \downarrow \psi \\
 & H^0(\mathcal{O}(k+L)) & \xrightarrow{-\psi} \dots (*) \\
 & \downarrow \psi & \\
 & \psi & \xrightarrow{\quad} -\psi
 \end{array}$$

Second. (Roughly !!)

$$\begin{array}{ccccccc}
 & 0 & & 0 & & 0 & \\
 & \uparrow & & \uparrow & & \uparrow & \\
 0 & \rightarrow & f(L) & \rightarrow & f_2(L) & \rightarrow & \mathcal{O}(L) \rightarrow 0 \\
 & \uparrow & & \uparrow & & \uparrow & \\
 \text{0-stage: } 0 & \rightarrow & \mathcal{O} \oplus \mathcal{O} & \rightarrow & \mathcal{O} \oplus \mathcal{O} \oplus \mathcal{O}(L) & \rightarrow & \mathcal{O}(L) \rightarrow 0 \\
 & \uparrow & & \uparrow & & \uparrow & \\
 \text{1-stage: } 0 & \rightarrow & \mathcal{O}(L^*) & \rightarrow & \mathcal{O}(L^*) \oplus \mathcal{O} \oplus \mathcal{O} & \rightarrow & \mathcal{O} \oplus \mathcal{O} \rightarrow 0 \\
 & \uparrow & & \uparrow & & \uparrow & \\
 \text{2-stage: } 0 & \rightarrow & 0 & \rightarrow & \mathcal{O}(L^*) & \rightarrow & \mathcal{O}(L^*) \rightarrow 0 \\
 & \uparrow & & \uparrow & & \uparrow & \\
 & 0 & & 0 & & 0 & 
 \end{array}$$

$\psi = g_1 s + g_2 s' + \sigma$   
 $(\tau \otimes s', -\tau \otimes s, 0)$   
 $(-h_1, -h_2, h_1 s + h_2 s')$   
 $(\sigma, 0)$   
 $(\sigma, (f_1, f_2))$   
 $(-\eta, -\eta \otimes s', \eta \otimes s)$   
 $(f_1, f_2)$   
 $(-\eta \otimes s', +\eta \otimes s)$   
 $\text{id}$   
 $\eta$