The data of the Mittag-Leffer problem consists of a global section $g \in \mathcal{PP}(M) = H^0(M, \mathcal{PP})$: the question is whether $g = \beta^* f$ for some global meromorphic function $f$.
If $\{f_U\}$ are the local solutions of the problem, we have seen that

$$(\delta^* g)_{U,V} = f_V - f_V \quad \text{and} \quad \text{that}$$

$$g = \beta^* f \iff \delta^* g = 0 \quad \text{in} \quad H^1(M, \mathcal{O}).$$

There are "roughly speaking" three kinds of sheaves.

1. Holomorphic sheaves — such as $\mathcal{O}$, $\mathcal{J}_V$, $\mathcal{O}(E)$ and $\Omega^p$. locally sections are by $n$-tuples of holomorphic functions.

These contain for us the most information and are the principal objects of interest.

2. $C^\infty$ sheaves, — $\mathcal{Q}^{p,q}$, locally $C^\infty$-functions (n-tuples)
Generally used in an auxillary manner.

3. Constant sheaves — $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{C}$.
These contain topological information about the underlying manifold.

Observations.

1. $H^p(M, \mathcal{Q}^{r,s}) = 0$ for $p > 0$.

pf) Given a locally finite cover $\underline{\mathcal{U}} = \{U_\alpha\}$ of $M$, we can find a partition of unity subordinate to $\underline{\mathcal{U}}$ i.e $C^\infty$-functions $\rho_\alpha$ on $M$ s.t $\Sigma \rho_\alpha = 1$ and $\operatorname{supp}(\rho_\alpha) \subset U_\alpha$.