$$= -\left(\frac{\sqrt{-1}}{2\pi}\right)^n \sum_i \int_{\partial B_\epsilon(P_\nu)} \Lambda,$$

and since our construction of Λ is essentially a local process, we will be able to evaluate the last integrals in terms of the local behavior of v at P_ν."

\\ Outline of Proof //

Since $\Theta \equiv 0$ in a ball $B_\epsilon(P_\nu)$,

$$\int_M P\left(\frac{\sqrt{-1}}{2\pi}\Theta\right) = \int_{M-\cup B_\epsilon(P_\nu)} P\left(\frac{\sqrt{-1}}{2\pi}\Theta\right).$$

$$\int_{M-\cup B_\epsilon(P_\nu)} P\left(\frac{\sqrt{-1}}{2\pi}\Theta\right) = \int_{M-\cup B_\epsilon(P_\nu)} \left(\frac{\sqrt{-1}}{2\pi}\right)^n P(\Theta) = \int_{M-\cup B_\epsilon(P_\nu)} \left(\frac{\sqrt{-1}}{2\pi}\right)^n d\Lambda$$

Stoke's theorem

$$= -\sum_\nu \int_{\partial B_\epsilon(P_\nu)} \left(\frac{\sqrt{-1}}{2\pi}\right)^n \Lambda.$$

So: let $\{P_\nu\}$ denote the zeros of v, and $z_1, \cdots z_n$ local holomorphic cordinates in $B_{2\epsilon}(P_\nu)$, and let h_ν be the Euclidean metric in $B_{2\epsilon}(P_\nu)$ given by

$$\left(\frac{\partial}{\partial z_j}, \frac{\partial}{\partial z_k}\right) = \delta_{jk}.$$

Let h_0 be any metric on $M^* = M - \{P_\nu\}$, and $\{\rho_0, \rho_\nu\}$ a partion of unity for the covering of M by $U_0 = M - \cup B_\epsilon(P_\nu)$ and $U_\nu = B_{2\epsilon}(P_\nu)$; we take as our metric on M

$$h = \rho_0 \cdot h_0 + \sum \rho_\nu h_\nu.$$

Let Θ hereafter be the curvature matrix of the associated metric connection D; clearly $\Theta \equiv 0$ in $B_\epsilon(P_\nu)$ for each ν.