subspace of $\mathbb{P}(H^0(\mathbb{P}^n, \mathcal{O}(H)))$. $\Rightarrow$ By Bertini theorem to V^*, generic elements $\{H \cap V\}$ are smooth and are of dim $(k+n-1)-n = k-1$ where $\dim V^* = k$. Continue this process, we get a generic $\mathbb{P}^{n-k}$ through p meeting V transversely. (Refer to P 431 Note)

Coning is an operation we have not previously encountered. If $V \subset \mathbb{P}^n$ is any variety, $p \in \mathbb{P}^n$ at any point lying off V, we take the cone $\overline{p,V}$ through p over V to be the union of the lines through p meeting V. That $\overline{p,V}$ is a variety is easy to see: it is the image under projection on the first factor of the incidence correspondence $I \subset \mathbb{P}^n \times \mathbb{P}^n$ defined by

$$I = \{(q, r) : r \in V,\ p \wedge q \wedge r = 0\},$$ itself an analytic subvariety of $\mathbb{P}^n \times \mathbb{P}^n$.

Since $p \wedge q \wedge r = 0$, q is a linear combination of p & r. $\Rightarrow$ $q \in \overline{p,V}$. $\Rightarrow$ $\overline{p,V} = \pi(I)$ where $I \xrightarrow{\pi_1} \mathbb{P}^n$.

Since $f(q,r) = p \wedge q \wedge r$ is holomorphic and r satisfies some polynomials,

$$\{f=0\} \cap \mathbb{P}^n \times V = I$$ is an analytic subvariety of $\mathbb{P}^n \times \mathbb{P}^n$.

(Alternatively, if, in homogeneous coordinate, $p = [0, 0, \cdots 1]$, let $\mathbb{P}^{n-1}$ be the hyperplane $X_n = 0$; if the image $\pi_p(V) \subset \mathbb{P}^{n-1}$ of V under projection from p is cut out in $\mathbb{P}^{n-1}$ by polynomials $\{F_\alpha(X_0, X_1, \cdots X_{n-1})\}$, then the cone $\overline{p,V}$ is cut out by the polynomials $\{\tilde{F}_\alpha(X_0, \cdots X_n) = F_\alpha(X_0, \cdots X_{n-1})\}$).