whose dimension is n.

$\Rightarrow \begin{pmatrix} A & D \\ C & B \end{pmatrix}$ has n negative eigenvalues on the vector space. V. $\Rightarrow$ It has at least n negative eigenvalues.

Clearly, this will also be true for functions ψ sufficiently close to φ in the C^2-topology.

Suppose ψ is a Morse function sufficiently close to φ in the C^2-topology.

$\Rightarrow \det(H(\psi) - tI) = f(t)$ $\quad g(t) = \det(H(\varphi) - tI)$ are very close together and f, g have all roots in R. $\Rightarrow$ # of negative roots of $f(t)=0$ = # of negative roots of $g(t)=0$.

Thus, by Morse theory, as far as homotopy type is concerned M is obtained from V by attaching cells of dimension at least n, and this gives the Lefschetz theorem on the homotopy level and for homology with $\mathbb{Z}$-coefficients. Q.E.D.

$$M \simeq V \cup e_{n_1} \cup e_{n_2} \cup \cdots \cup e_{n_\ell}$$

where e_{n_i} is n_i-cell, $n_i \geq n$.

$$\Rightarrow \quad H_q(M, \mathbb{Z}) = H_q(V \cup e_{n_1} \cup \cdots \cup e_{n_\ell}, \mathbb{Z})$$

$$= H_q(V) = H_q(V, \mathbb{Z}) \quad \text{for} \quad q \leq n-2.$$

since $\quad H_q(M^{n-1}) \cong H_q(V) \cong H_q(M, \mathbb{Z})$ for $q < n-1$, or