This is not quite correct, since account may be taken of how the fundamental group $\pi_1(B, x_0)$ acts on the cohomology $H^q(F_{x_0}, Q)$ ($F_x = \pi^{-1}(x)$). More precisely, displacement of homology cycles in the fibers over a path γ from x_0 to x induces an isomorphism

$$H^q(F_x, Q) \cong H^q(F_{x_0}, Q)$$

that depends only on the homotopy class of γ. This is reasonably intuitive and is proven in standard books on topology. The upshot is that first there is a representation

$$\rho : \pi_1(B, x_0) \longrightarrow \mathrm{Aut}(H^q(F_{x_0}, Q))$$

that describes how cycles change when they are displaced around closed paths. Second, any representation of the fundamental group

$$\rho ; \pi_1(B, x_0) \longrightarrow \mathrm{Aut}(V)$$

gives locally constant sheaf $\mathcal{V}_\rho$ on B. To construct $\mathcal{V}_\rho$, we take the vector bundle

$$V_\rho = \tilde{B} \times_{\pi_1} V$$

associated to the universal covering $\tilde{B} \to B$, and then the sections of $\mathcal{V}_\rho$ over an open set $U \subset B$ are just those which lift to constant sections of $\tilde{B} \times V$.

Γ

$$\begin{tikzcd} \tilde{B} \times V \arrow[r] \arrow[d, "/\pi_1"] & \tilde{B} \arrow[l, dashed, bend left, "\tilde{\sigma}"] \arrow[d, "/\pi_1"] \\ \tilde{B} \times_{\pi_1} V \arrow[r] & B \arrow[l, dashed, bend left, "\sigma"] \end{tikzcd}$$

$$\tilde{\sigma} : \tilde{B} \longrightarrow \tilde{B} \times V$$
$$x \longmapsto (x, v)$$
for some $v \in V$.

$$\Rightarrow \sigma(\overset{*}{x}) = [(x, v)]$$