⌈ By Bertini's theorem, $\exists$ a hyperplane H meeting $\mathcal{S}$ and $\mathcal{S}_0$ transversely.

Consider the set $\mathcal{S} = \{ (p, [(a_1, \cdots a_{20})]) \in \mathbb{P}^3 \times \mathbb{P}^{19} \mid a_1 \sigma_1 + \cdots + a_{20}\sigma_{20} = 0$ at p, $\nabla\sigma(p) \perp H \}$, $\sigma = a_i \sigma_i$ and $W = |\langle \sigma_1, \cdots \sigma_{20} \rangle|$. $\Rightarrow$ $\mathcal{S}$ is an analytic subvariety of $\mathbb{P}^3 \times \mathbb{P}^{19}$. $\Rightarrow$ By the proper mapping theorem, $\pi_2(\mathcal{S})$ is an analytic subvariety of $\mathbb{P}^{19} = W$. But since $H \notin \pi_2(\mathcal{S})$, $V' = \pi_2(\mathcal{S})$ is a proper subvariety of W. ⌋

We may therefore choose our path γ to lie in $W - V - V'$, so that $Y' = X' \cap (W \times H)$ will be a submanifold of X' mapping smoothly onto I.

⌈ $X' \cap W \times H \xrightarrow{\gamma^{-1} \circ \pi} I$

For any $t \in I$, $\pi^{-1}(\gamma(t)) = \{ (\gamma(t), p) \mid p \in \gamma(t) \}$

$\gamma(t)$ is a nonsingular cubic meeting H transversely.

$\Rightarrow$ Choose $p_0 \in \gamma(t) \cap H$. $\Rightarrow$ $\gamma^{-1} \circ \pi(p_0) = t$

$\Rightarrow$ $\gamma^{-1} \circ \pi$ is onto.

$X' = \{ a_1(t)\sigma_1 + \cdots + a_{20}(t)\sigma_{20} = 0 \}_{t \in I}$.

$\Rightarrow$ $X' \cap (W \times H) = \{ a_1(t)\sigma_1 + \cdots + a_{20}(t)\sigma_{20} = 0 \} \cap H$

Let $H = (b_0 X_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 = 0)$, and $x = X_1/X_0$, $y = X_2/X_0$, $z = X_3/X_0$. Since H intersects with $\{ a_1(t)\sigma_1 + \cdots a_{20}\sigma_{20} = 0 \}$ transversely for each t,

$$\begin{pmatrix} \frac{\partial\sigma}{\partial t}, & \frac{\partial\sigma}{\partial x}, & \frac{\partial\sigma}{\partial y}, & \frac{\partial\sigma}{\partial z} \\ 0, & \frac{\partial(b_0 + b_1 x + \cdot)}{\partial x}, & b_2, & b_3 \end{pmatrix} \text{ has rank } 2,$$