Given $f: M \longrightarrow \mathbb{P}^N$, modulo projective transformations,

$f(p) = [s_0(p), \cdots s_N(p)]$, we get the pullback bundle $f^*(H)$,

$$\begin{tikzcd} f^*(H) \arrow[r] \arrow[d] & H \arrow[d] \\ M \arrow[r, "f"] & \mathbb{P}^N \end{tikzcd}.$$

Each s_i defines a line bundle. Since the transition functions are same, each s_i determines the same line bundle $f^*(H)$. $\Rightarrow$ $f^*(H)$ is a line bundle over M with $\langle s_0, \cdots s_N \rangle = E \subset H^0(M, \mathcal{O}(L))$. E has no base points.

Suppose $f, g : M \longrightarrow \mathbb{P}^N$ s.t $f \equiv g$ (modulo projective transformations). $\Rightarrow$ $f^*H = g^*H$, for, $\exists$ T s.t $T \circ f = g$

$$\begin{tikzcd} f^*H \arrow[r] \arrow[d] & \overset{\overset{T^*H}{\shortparallel}}{H} \arrow[r] \arrow[d] & H \arrow[d, dash] \\ M \arrow[r, "f"] & \mathbb{P}^N \arrow[r, "T"] & \mathbb{P}^N \end{tikzcd}$$

$\Rightarrow$ $T^*H = H$ and $g^* = f^* \circ T^*$ $\Rightarrow$ $g^*H = f^*H$.

$$\left\{ \begin{array}{c} L \\ \downarrow \\ M \end{array} \text{ with } \begin{array}{c} E \subset H^0(M, \mathcal{O}(L)) \\ \text{no base points} \end{array} \right\} \xrightarrow{\ \phi\ } \left\{ \begin{array}{c} \iota_E : M \longrightarrow \mathbb{P}^N \\ \text{modulo projective} \\ \text{transformations} \end{array} \right\} \xrightarrow{\ \psi\ } \begin{array}{c} L \\ \downarrow \\ M \end{array}$$