

and hence $l \in V_1(W)$; or
 (2) $F_1 = F_2 \in L$, i.e., $l \in V_0(L)$.

Since, conversely, both $V_0(L)$ and $V_1(W)$ are contained in $V_0(N_1) \cap V_0(N_2)$,

$$V_0(N_1) \cap V_0(N_2) = V_1(W) \cup V_0(L).$$

Clearly, $V_0(L) \subset V_0(N_1) \cap V_0(N_2)$.

$l \in V_1(W) \Rightarrow l$ lies on a pencil of quadrics in W

\Rightarrow If we let K be the pencil, $K \cap N_1 \neq \emptyset$

$\Rightarrow l \in N_1$. Similarly $l \in N_2$

$\Rightarrow V_0(L) \cup V_1(W) \subset V_0(N_1) \cap V_0(N_2)$.

By the argument above, $V_0(N_1) \cap V_0(N_2) \subset V_1(W) \cup V_0(L)$.

$\Rightarrow V_0(N_1) \cap V_0(N_2) = V_0(L) \cup V_1(W)$.

□

Now $V_0(N_1) \sim V_0(N_2) \sim 3\sigma_1$, so the intersection $V_0(N_1) \cap V_0(N_2)$ has class

$$(3\sigma_1)^2 = 9\sigma_{1,1} + 9\sigma_2;$$

On the other hand, $V_0(L) \sim 2\sigma_2 + 6\sigma_{1,1}$, so we find again that

$$V_1(W) \sim 7\sigma_2 + 3\sigma_{1,1}.$$

By the result on p747, $V_0(N_1) \sim V_0(N_2) \sim 3\sigma_1$,

$$\Rightarrow (3\sigma_1)^2 = 9(\sigma_2 + \sigma_{1,1}) = 9\sigma_{1,1} + 9\sigma_2 \text{ by p746.}$$