「 $\eta_1 - \eta_2 = \omega$ holomorphic since ω has no pole at all.

$$\Rightarrow \int_{\delta_i} \eta_1 - \eta_2 = \int_{\delta_i} \omega \quad \Rightarrow \quad \int_{\delta_i} \eta = a_i$$

$$\Rightarrow \quad \eta - \sum a_i \omega_i \quad \Rightarrow \quad \int_{\delta_i} \eta - \sum a_i \omega_i = 0$$

Suppose $\exists$ two such meromorphic one forms η_1, η_2.

$\Rightarrow \int_{\delta_i} \eta_1 - \eta_2 = \int_{\delta_i} \omega = 0 \quad \Rightarrow \quad \omega = 0$ since $\{\delta_i\}$ generates $H_1(S, \mathbb{Z})$. ⌟

Let $W \cong \mathbb{C}^d$ denote the vector space of such forms, and consider the linear map

$$\psi : W \longrightarrow \mathbb{C}^g$$

obtained by integration over the B-cycles of S:

$$\psi : \varphi_a \longmapsto \left(\int_{\delta_{g+1}} \varphi_a, \cdots \int_{\delta_{2g}} \varphi_a \right).$$

Clearly, the vector space V above is just the kernel of the map ψ.

「 $\psi(\varphi_a) = 0$ φ_a has a double pole at each p_λ and holomorphic on $S - \{p_\lambda\}$, and has no residues.
$\Rightarrow$ φ_a has no periods $\Rightarrow$ $\varphi_a \in V$. $\psi = 0$ on V. ⌟

To describe ψ explicitly, let $\omega_1, \omega_2, \cdots \omega_g$ be a normalized basis for $H^0(S, \Omega^1)$. By the reciprocity law for differentials of the first and second kinds, . . .