$\Rightarrow$ Here since we mean the space is a projective space, $d-1-\ell$ is the dimension of the space.
Let ℓ be the number of independent relations on $\{P_i\}$, as above.

$$h^0(D) = r+1 = d - \dim(\overline{D}) = d - (d-1-\ell)$$

$$\Rightarrow \quad r+1 = 1+\ell \Rightarrow r = \ell.$$

$\Rightarrow$ $\dim(\overline{D}) = d-r-1 = d-\ell-1$. $\Rightarrow$ The points of D span exactly a $(d-r-1)$-plane. ⅃

Indeed, the Riemann-Roch formula may be quite easily proved in this geometric form. To start, we prove the inequality

$$(*) \qquad \dim \overline{D} \leq (d-1) - \dim |D|$$

Proof. Suppose that $D = \sum P_i$ moves in an r-dimensional linear system: that is, there exist $r+1$ independent meromorphic functions $f_0, f_1, \cdots f_r$ on S with

$$(f_\nu) + D \geq 0.$$

⌈

$$H^0(S, \mathcal{O}(D)) \longleftrightarrow |D|$$

$$\cup \qquad\qquad\qquad r\text{-dim. l.s.} \cong \mathbb{P}^r.$$

$$V = \langle \sigma_0, \sigma_1, \cdots \sigma_r \rangle$$

$$\sigma_i = (f_i) + D. \qquad f_i\text{'s meromorphic functions on } S. ⅃$$

We may take $f_0 = 1$; then no nontrivial linear combination of the functions $f_1, f_2, \cdots f_r$ will be holomorphic.
⌈ $V = \langle \sigma_0, \sigma_1, \cdots \sigma_r \rangle$, $\Rightarrow$ $(\sigma_0 = 0) = D$. $(\sigma_i = 0) = D + (f_i)$. $f_0 = 1$.

~~$\frac{f_i}{f_0} = g_i \Rightarrow g_i$ is a meromorphic function again. $\Rightarrow$ Let g_i.~~
~~Take $D' = D + (f_0) \Rightarrow D' + (f_1/f_0) \geq 0, \cdots D' + (f_r/f_0) \geq 0.$~~