But $\partial \pi^{-1}(T_\epsilon)$ is a finite cover of ∂T_ϵ ; so we need prove only that $\mathrm{vol}(\partial T_\epsilon) \to 0$ as $\epsilon \to 0$. To see this, let D_1 be the singular locus of D, D_2 the singular locus of D_1, and so on: let T_ϵ^i be the ϵ-nbd of $D_i^* = D_i - D_{i+1}$ in $\Delta' - D_{i+1}$. Then D_i^* is a submanifold of real dimension $\leq 2k-2$ having finite volume in $\Delta' - D_{i+1}$, and so the volume of ∂T_ϵ^i goes to 0 as $\epsilon \to 0$.

「 D_i^* is a complex submanifold of $\Delta' - D_{i+1}$ by the Remark between two proposition on P21.

$\Rightarrow$ D_i^* is a submanifold of real dimension $\leq 2k-2$ since $\dim(\Delta' - D_{i+1}) = 2k$.

$\Rightarrow$ By the proposition on P32, D_i^* has a finite volume in the bounded set $\Delta' - D_{i+1}$.

To show that the volume of ∂T_ϵ^i goes to 0 as $\epsilon \to 0$, since D_i^* is a submanifold in a compact set, we have only to consider the following case by dividing D_i^* into a finite # of pieces, which is homeomorphic to an open set of the Euclidean space.

For example, $\dim D_i^* = 2$ $\dim \Delta' = 4$.

$\Rightarrow$ We may assume $D_i^* = \Delta$ by the above.

$\Rightarrow$ $T_\epsilon^i = \Delta \times (-\epsilon, \epsilon)^2$.

$\Rightarrow$ $\partial T_\epsilon^i = \partial\Delta \times (-\epsilon,\epsilon)^2 \cup \Delta \times \partial(-\epsilon,\epsilon) \times (-\epsilon,\epsilon) \cup \Delta \times (-\epsilon,\epsilon) \times \partial(-\epsilon,\epsilon)$

$\Rightarrow$ Since $\partial(-\epsilon,\epsilon) = \{-\epsilon, \epsilon\}$ and $\lim_{\epsilon\to 0} \mathrm{vol}(-\epsilon,\epsilon) = 0$, $\lim_{\epsilon \to 0} \mathrm{vol}\, \partial T_\epsilon^i = 0$.

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