The Riemann – Roch formula gives us immediately a picture of the behavior of generic linear systems: for generic effective divisors $D = \sum_{\lambda=1}^{d} p_\lambda$ of degree d the matrix $((\omega_i/dz_\lambda)(p_\lambda))$ has maximal rank, and so

$$h^0(D) = \begin{cases} 1, & d \leq g, \\ d-g+1, & d > g, \end{cases}$$

for D outside an analytic subvariety in $S^{(d)}$.

Pf $\quad$ rank $\varphi = d \Rightarrow h^0(D) = 1$, if $d \leq g$ since $\max \leq d$.
rank $\varphi = g \Rightarrow h^0(D) = d-g+1$, if $g < d$.

$$\det\begin{pmatrix} h_{11}(x), & h_{12}(y) \\ h_{21}(x), & h_{22}(y) \end{pmatrix} = 0$$ for all x, y, where each h_{ij} C^∞.

$$\Rightarrow h_{11}(x)h_{22}(y) = h_{21}(x)h_{12}(y) \Rightarrow \frac{h_{11}(x)}{h_{21}(x)} = \frac{h_{12}(y)}{h_{22}(y)} = a(x,y)$$

$$\Rightarrow \frac{h_{11}(x)}{h_{21}(x)} = a \text{ must be constant}$$

$$\Rightarrow (h_{11}(x), h_{12}(y)) = a(h_{21}(x), h_{22}(y))$$

$$\det\begin{pmatrix} h_{11}(x) & h_{12}(y), & h_{13}(z) \\ h_{21}(x), & h_{22}(y), & h_{23}(z) \\ h_{31}(x), & h_{32}(y), & h_{33}(z) \end{pmatrix} = 0 \quad \text{for all } x, y, z.$$

(i) a minor 2×2 matrix has determinant 0.
$\Rightarrow$ For example, $\det\begin{pmatrix} h_{11}(x), & h_{12}(y) \\ h_{21}(x), & h_{22}(y) \end{pmatrix} = 0$.