we see that formally $\bar\partial k(z,w)=0$.

$$\bar\partial\left(\frac{\sum \overline{\Phi_i(\zeta)}}{\|\zeta\|^{2n}}\right)=\frac{n\,\overline{\Phi(\zeta)}}{\|\zeta\|^{2n}}+\bar\partial\left(\frac{1}{\|\zeta\|^{2n}}\right)\wedge\sum\overline{\Phi_i(\zeta)}$$

$$=\frac{n\,\overline{\Phi(\zeta)}}{\|\zeta\|^{2n}}+(-n)\frac{\bar\partial\|\zeta\|^2\wedge\overline{\sum\Phi_i(\zeta)}}{\|\zeta\|^{2n+2}}$$

$$=\frac{n}{\|\zeta\|^{2n}}\left(\overline{\Phi(\zeta)}-\frac{\left(\sum\zeta_i\,d\bar\zeta_i\right)\wedge\overline{\sum\Phi_i(\zeta)}}{\|\zeta\|^2}\right)$$

$$=\frac{n}{\|\zeta\|^{2n}}\left(\overline{\Phi(\zeta)}-\frac{1}{\|\zeta\|^2}\left(\sum|\zeta_i|^2\,\overline{\Phi(\zeta)}\right)\right)=0.$$

$$\bar\partial k(z,w)=\bar\partial\left(\frac{\sum\overline{\Phi_i(z-w)}\wedge\Phi(w)}{\|z-w\|^{2n}}\right)=\bar\partial_z\left(\frac{\sum\overline{\Phi_i(z-w)}\wedge\Phi(w)}{\|z-w\|^{2n}}\right)$$

$$+\bar\partial_w\left(\frac{\sum\overline{\Phi_i(z-w)}\wedge\Phi(w)}{\|z-w\|^{2n}}\right)=0\wedge\Phi(w)+(\quad)\wedge\overbrace{\bar\partial_z\Phi(w)}^{=0}$$

$$+\;0\wedge\Phi(w)+(\quad)\wedge\bar\partial_w\overbrace{\Phi(w)}^{=0}=0,\ \text{since}$$

$$\bar\partial_z\left(\frac{\sum\overline{\Phi_i(z-w)}}{\|z-w\|^{2n}}\right)=\bar\partial_{z-w}\left(\frac{\sum\overline{\Phi_i(z-w)}}{\|z-w\|^{2n}}\right)\ \text{and}\ \bar\partial_w\left(\frac{\sum\overline{\Phi_i(z-w)}}{\|z-w\|^{2n}}\right)$$

$$=\pm\,\bar\partial_{z-w}\left(\frac{\sum\overline{\Phi_i(z-w)}}{\|z-w\|^{2n}}\right).\quad\Rightarrow\quad\bar\partial k(z,w)=0 \text{ on } \mathbb{C}^n\times\mathbb{C}^n-\Delta$$

where $\Delta=\{(z,w)\in\mathbb{C}^n\times\mathbb{C}^n;\ z=w\}$.

Ignoring for a moment the singularities, for a test form $\psi\in A_c^{n,n-q+1}(\mathbb{C}^n)$ Stokes' theorem gives