case 1

$$\mathrm{Res}_{\{0\}}\left(\frac{h\,dz_1\wedge\cdots\wedge dz_n}{f_1\cdots f_n}\right)=\lim_{t\to 0}\sum_{P_t\in f_t^{-1}(0)\cap U}\mathrm{Res}_{P_t}\left(\frac{h\,dz_1\wedge\cdots\wedge dz_n}{f_{t,1}\cdots f_{t,n}}\right)$$

$$=\lim_{t\to 0}\sum_{P_t\in g_t^{-1}(0)\cap U}\mathrm{Res}_{P_t}\left(\frac{h\det A\,dz_1\wedge\cdots\wedge dz_n}{g_{t,1}\cdots g_{t,n}}\right)$$

$$=\mathrm{Res}_{\{0\}}\left(\frac{h\det A\,dz_1\wedge\cdots\wedge dz_n}{g_1\cdots g_n}\right).$$

「 Since $\det A(z)\neq 0$ and $J_g(z)=\det A(z)\,J_f(z)$, $g_t=A\cdot f_t$ is a good perturbation of $g=g_0$.

Once we accept the principle of continuity, then.

$$\mathrm{Res}_{\{0\}}\left(\frac{h\,dz_1\wedge\cdots\wedge dz_n}{f_1\cdots f_n}\right)\overset{\text{by continuity}}{=}\lim_{t\to 0}\sum_{P_t\in f_t^{-1}(0)\cap U}\mathrm{Res}_{P_t}\left(\frac{h\,dz_1\wedge\cdots\wedge dz_n}{f_{t,1}\cdots f_{t,n}}\right)$$

$$\overset{\text{by case}}{=}\lim_{t\to 0}\sum_{P_t\in g_t^{-1}(0)\cap U}\mathrm{Res}_{P_t}\left(\frac{h\det A\,dz_1\wedge\cdots\wedge dz_n}{g_{t,1}\cdots g_{t,n}}\right)$$

$$=\mathrm{Res}_{\{0\}}\left(\frac{h\det A\,dz_1\wedge\cdots\wedge dz_n}{g_1\cdots g_n}\right).$$

$$\sum_{P_t\in f_t^{-1}(0)\cap U}\mathrm{Res}_{P_t}\left(\frac{h\,dz_1\wedge\cdots\wedge dz_n}{f_{t,1}\cdots f_{t,n}}\right)=\sum_{P_t\in g_t^{-1}(0)\cap U}\mathrm{Res}_{P_t}\left(\frac{h\det A\,dz_1\wedge\cdots\wedge dz_n}{g_{t,1}\cdots g_{t,n}}\right)$$

since $g_t=A\cdot f_t$ and g_t & f_t are nondegenerate 」

「 When we prove the residue theorem, we used the lemma on P651. To prove the lemma, we did not need the nondegeneracy of f, but we want $\Gamma_I=\{z \mid |f_i(z)|=\epsilon$ for $i\in I$, $|f_j(z)|\le\epsilon$ for $j\notin I\}$ a chain with orientation $d\arg f_{i_1}\wedge\cdots\wedge d\arg f_{i_q}\wedge\left(\bigwedge_{j\notin I}\frac{\sqrt{-1}}{2}df_j\wedge d\bar f_j\right)\ge 0$