



⌈ Since L is a generic pencil, if $L = \{a s_1 + b s_2\}$, $(s_1=0) \cap (s_2=0)$ is a set of four distinct points. \Rightarrow By the Reciprocity Formula I on P 714,

$$\dim |f_{P_0}(2)| = \left\{ \frac{2(2+3)}{2} - 4 \right\} + h^0(f_P(2-3))$$

$$= 1 + 0 = 1$$

where $P_0 = \{p_1, p_2, p_3, p_4\}$, $n=2$, $d = \deg P_0 = 4$

$$\Rightarrow L = |f_{P_0}(2)|$$

\Rightarrow This implies that any conic passing P_0 is a linear combination of s_1 and s_2 . \square

But since no three of the points p_i are collinear, if F is a conic consisting of two lines l, l' and containing $\{p_i\}$, then l and l' must each contain two of the points $\{p_i\}$.

⌈ Suppose three of p_i 's are collinear.

\Rightarrow Say $\overline{p_1 p_2 p_3} = l$, $\Rightarrow \#(l \cap S_1) = 3 \Rightarrow$ Since S_1 is a quadric, $S_1 \supset l$. Similarly, $S_2 \supset l \Rightarrow$
 $S_1 = l \cup \sigma_1$, $S_2 = l \cup \sigma_2$, σ_1, σ_2 lines.