?

we have only to prove that, given a sequence $\{f_n\}$ s.t $f_n \longrightarrow f$, $f_n(x) \longrightarrow f(x)$.

Given $\epsilon > 0$, $d(f_n, f) = \sum_{i=1}^{\infty} \frac{2^{-i} P_i(f_n - f)}{1 + P_i(f_n - f)}$

$$= \frac{2^{-1} P_1(f_n - f)}{1 + P_1(f_n - f)} + \frac{2^{-2} P_2(f_n - f)}{1 + P_2(f_n - f)} + \cdots\cdots + \cdots\cdots < \delta .$$

$$\Rightarrow \frac{2^{-N_0} P_{N_0}(f_n - f)}{1 + P_{N_0}(f_n - f)} < \delta . \qquad \text{Assume } 2^{-N_0} > \delta$$

$$\Rightarrow 2^{-N_0} P_{N_0}(f_n - f) < \delta + \delta P_{N_0}(f_n - f)$$

$$\Rightarrow (2^{-N_0} - \delta) P_{N_0}(f_n - f) < \delta$$

$$\Rightarrow P_{N_0}(f_n - f) < \frac{\delta}{2^{-N_0} - \delta} .$$

$$\Rightarrow |(f_n - f)(x)| \leq \frac{\delta}{2^{-N_0} - \delta} .$$

$\Rightarrow$ Choose δ so that $\frac{\delta}{2^{-N_0} - \delta} < \epsilon$. $\Rightarrow \exists N_\delta$ s.t $d(f_n, f) < \delta$ if $n > N_\delta$. $\Rightarrow$ If $n > N_\delta$, $d(f_n, f) < \delta$, $\Rightarrow |f_n(x) - f(x)| < \epsilon$. $\Rightarrow f_n(x) \longrightarrow f(x)$. //

Since $\mathcal{D}_K$ is the intersection of the null spaces of these functionals, as x ranges over the complement of K, it follows that $\mathcal{D}_K$ is closed in $C^\infty(\Omega)$.

Γ Denote ϕ_x by the functional above for each $x \in \Omega$.

$$\phi_x : \quad \begin{matrix} C^\infty(\Omega) & \longrightarrow & \mathbb{C} \\ \Psi & & \Psi \\ f & \longmapsto & f(x) \end{matrix} \qquad \Rightarrow \mathcal{D}_K = \bigcap_{x \in \Omega - K} \phi_x^{-1}(0) .$$

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