we see that the Gauss map on F is just the restriction to F of the rational map $\mathbb{P}^n \to \mathbb{P}^{n*}$ given by the matrix Q.

By ② on P.175,

$$0 = \sum_{i=0}^{n} \frac{\partial F}{\partial X_i}(p) X_i = \sum_{i=0}^{n} q_{ij} a_j X_i$$

Thus it remains to show that

$$\mathbb{P}^n \xrightarrow{\varphi} \mathbb{P}^{n*}$$
$$[b_0, \cdots b_n] \longmapsto \{ \Sigma q_{ij} b_j X_i = 0 \}$$

is a rational map.

φ is not defined on $\{ Qb = 0 \}$.

Since $\operatorname{rank} Q \neq 0$, and $\operatorname{rank} Q \geq 1$,

$\dim \{ \overset{=0}{Qb} \} \leq n. \Rightarrow \dim \{ [b] \mid Qb = 0 \} \leq n-1.$

Thus $\operatorname{rank} Q$ should be greater than 1.

If F is smooth, $\mathcal{G}$ is an isomorphism, and the dual variety $F^* = \mathcal{G}(F) \subset \mathbb{P}^{n*}$ is again a smooth quadric.

If F is smooth, $\operatorname{rank} Q$ is $n+1$.

$\Rightarrow$ If $Qa = kQb$, $k \in \mathbb{C}^*$, then $\{ \Sigma q_{ij} a_j X_i = 0 \} = \{ \Sigma q_{ij} b_j X_i = 0 \}$. Conversely, true.