P648. P653

NO.
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m_w is an ideal. $\Rightarrow$ $a_i +$ $f^* m_w = f^* m_w = 0$ $\lrcorner$

F
$\uparrow$ Claim

$:(f_1, \cdots f_n) = f : \underset{\mathbb{C}^n}{\underset{\cap}{U}} \longrightarrow \underset{\mathbb{C}^n}{\underset{\cap}{W}}$ s.t $f^{-1}(0) = \{0\}$. See P648 & P653

$\Rightarrow \{ z \in U ; \ |f_i| = \epsilon \} \cong S^{2n-1}$. for sufficiently small $\epsilon > 0$.

Proof.

For example, $n = 3$. By Taylor expansion,

$f_1 = z^\alpha g_1$, $f_2 = z^\beta g_2$, $f_3 = z^\gamma g_3$, where $z^\alpha = z_1^{\alpha_1} z_2^{\alpha_2} z_3^{\alpha_3}$

and $g_1(0) \neq 0$ $g_2(0) \neq 0$, $g_3(0) \neq 0$.

Then

$$\{ z \in U ; \ |f_1| = |f_2| = |f_3| = \epsilon \}$$
$$\downarrow \phi$$
$$\{ z \in U ; \ |z^\alpha| = |z^\beta| = |z^\gamma| = \epsilon' \}$$

For $\underset{\alpha = 3}{n = 1}$, $z \overset{\phi}{\longmapsto} z|g|^{\frac{1}{3}}$, where $\epsilon' = \epsilon^{\frac{1}{3}}$

$\Rightarrow |J_0(\phi)| \neq 0 \Rightarrow \phi$ is diffeomorphism.

Define $\phi : \mathbb{C}^3 \longrightarrow \mathbb{C}^3$ by

$$(z_1, z_2, z_3) \longmapsto (z_1 |g_1|^{c_{11}} |g_2|^{c_{12}} |g_3|^{c_{13}}, \ z_2 |g_1|^{c_{21}} |g_2|^{c_{22}} |g_3|^{c_{23}}, \ z_3 |g_1|^{c_{31}} |g_2|^{c_{32}} |g_3|^{c_{33}})$$

where
$$\begin{pmatrix} \alpha_1, & \alpha_2, & \alpha_3 \\ \beta_1, & \beta_2, & \beta_3 \\ \gamma_1, & \gamma_2, & \gamma_3 \end{pmatrix} \begin{pmatrix} c_{11}, & c_{12}, & c_{13} \\ c_{21}, & c_{22}, & c_{23} \\ c_{31}, & c_{32}, & c_{33} \end{pmatrix} = I.$$