not lie in $T_{\iota(x)} \underbrace{\sigma_{1,1,\cdots,1}}_{r}(V)$.

$\lrcorner\!\lrcorner$

If α is any cycle of real dimension $2r$ on M meeting D_{k-r+1} transversely at points p_i, then $\iota_*\alpha$ will meet $\sigma_{1,1,\cdots,1}(V)$ transversely at the points $\iota(p_i)$, and by our choice of orientation for D_{k-r+1} the intersection number of $\iota_*\alpha$ with $\sigma_{1,\cdots,1}(V)$ at $\iota(p_i)$ will be that of α with D_{k-r+1} at p_i. Thus

$$\#(\iota_*\alpha \cdot \sigma_{1,\cdots,1}) = \#(\alpha \cdot D_{k-r+1}),$$

and we see that

$$\begin{aligned} c_r(E)(\alpha) &= \iota^*(c_r(S^*))(\alpha) \quad (\text{since } \iota^*(S^*)=E) \\ &= c_r(S^*)(\iota_*\alpha) \\ &= \#(\iota_*\alpha \cdot \sigma_{1,1,\cdots,1}) \quad (\text{by } c_r(S^*) = (-1)^r c_r(S) \text{ and G-B.Th I}) \\ &= \#(\alpha \cdot D_{k-r+1}). \end{aligned}$$

We thus have the

Gauss-Bonnet Formula II. The r-th Chern class $c_r(E)$ is Poincaré dual to the degeneracy cycle D_{k-r+1}.

$\ulcorner$ Since $\iota_*(T_x\alpha + T_x D_{k-r+1}) = \iota_*(T_x M) = \iota_*(T_x\alpha)$ ($\because$ $\iota_*(T_x D_{k-r+1}) = 0$) and $\iota_*(T_x M) + T_{\iota(x)}\sigma_{1,1,\cdots,1}(V) = T_{\iota(x)} G(k,n)$,

$$\iota_*(T_x\alpha) + T_{\iota(x)}\sigma_{1,\cdots,1}(V) = T_{\iota(x)} G(k,n).$$

Let $\dim M = n$, $p \in D_{k-r+1} - D_{k-r}$ and $(x_1, \cdots x_{n-2r}, \cdots g_{k-r+1}, h_{k-r+1}, \cdots g_k, h_k)$ where $(g_{k-r+1} + i h_{k-r+1} = g_{k-r} + i h_{k-r} = \cdots = g_k + i h_k = 0) = D_{k-r+1} - D_{k-r}$ near p.