

$\Rightarrow \pi^*(\frac{F}{G})$ is rational on $\mathbb{P}^n \Rightarrow$ Its restriction to V is rational. \square)

For the first part, choose a generic $(n-k-1)$ -plane \mathbb{P}^{n-k-1} in \mathbb{P}^n ; at this stage we require only that \mathbb{P}^{n-k-1} be disjoint from V . Let \mathbb{P}^k be a complementary k -plane, and $\pi: V \rightarrow \mathbb{P}^k$ the projection from \mathbb{P}^{n-k-1} . For each point p of \mathbb{P}^k , the inverse image $\pi^{-1}(p)$ is just the intersection of V with the $(n-k)$ -plane $\overline{\mathbb{P}^{n-k-1}, p}$; since $\overline{\mathbb{P}^{n-k-1}, p}$ will generically intersect V in $d = \deg(V)$ points, π expresses V as a d -sheeted branched cover of \mathbb{P}^k almost everywhere.

\square Choose a coordinates X_0, X_1, \dots, X_n on \mathbb{P}^n so that

$$\mathbb{P}^k = (X_{k+1} = X_{k+2} = \dots = X_n = 0)$$

$$\mathbb{P}^{n-k-1} = (X_0 = X_1 = \dots = X_k = 0)$$

$$\text{and } \pi([X_0, \dots, X_n]) = [X_0, X_1, \dots, X_k]$$

$$\pi: \mathbb{P}^n \longrightarrow \mathbb{P}^k$$

For each point $p \in \mathbb{P}^k$, $\pi^{-1}(p) \ni x$.

$$\Rightarrow x = [x_0, x_1, \dots, x_n] \Rightarrow \pi(x) = [x_0, x_1, \dots, x_k]$$

$$= p = [p_0, p_1, \dots, p_k]$$

$$\Rightarrow x = [p_0, p_1, \dots, p_k, x_{k+1}, \dots, x_n] = [p_0, p_1, \dots, p_k, 0, \dots, 0]$$

$$+ [0, \dots, 0, x_{k+1}, \dots, x_n]$$

$$\text{Since } [0, \dots, 0, x_{k+1}, \dots, x_n] \in \mathbb{P}^{n-k-1}$$

$$x \in \overline{\mathbb{P}^{n-k-1}, p} \Rightarrow \text{If we restrict } \pi \text{ to}$$