By P30, $\omega = \frac{\sqrt{-1}}{2\pi}\left[\frac{\sum dw_i \wedge d\bar{w}_i}{1+|w|^2} - \frac{(\sum \bar{w}_i dw_i)\wedge(\sum w_i d\bar{w}_i)}{(1+|w|^2)^2}\right]$

we will do for $n=3$.

$$\Rightarrow \omega = \frac{\sqrt{-1}}{2\pi}\left[\frac{dw_1\wedge d\bar{w}_1 + dw_2\wedge d\bar{w}_2}{1+|w_1|^2+|w_2|^2} - \frac{(\bar{w}_1 dw_1 + \bar{w}_2 dw_2)\wedge(w_1 d\bar{w}_1 + w_2 d\bar{w}_2)}{(1+|w|^2)^2}\right]$$

$$= \frac{\sqrt{-1}}{2\pi}\left[\frac{dw_1\wedge d\bar{w}_1 + dw_2\wedge d\bar{w}_2}{1+|w|^2} - \frac{|w_1|^2 dw_1\wedge d\bar{w}_1 + \bar{w}_2 w_1 dw_2 \wedge d\bar{w}_1 \cdots\cdots}{(1+|w|^2)^2}\right]$$

$$= \omega = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial \log\|f(w_1,w_2)\|^2.$$

$$\begin{tikzcd} \mathbb{C}^3-\{0\} \arrow[r, shift left, "p"] \arrow[rd, shift left, "p"] & \mathbb{P}^2 \arrow[l, shift left, "f"] \arrow[d] \\ & \mathbb{C}^2 \arrow[lu, shift left, "f"] \end{tikzcd}$$

$$(z_1,z_2,z_3) \mapsto \left(\frac{z_2}{z_1}, \frac{z_3}{z_1}\right)$$
$$(1, w_1, w_2) \leftarrow (w_1, w_2)$$

$\Rightarrow p^*\omega = \frac{\sqrt{-1}}{2\pi} p^*\partial\bar\partial \log\|f(w_1,w_2)\|^2$. $\Rightarrow$ Since p is holomorphic, $p^*\circ\bar\partial = \bar\partial\circ p^*$ and $p^*\circ\partial = \partial\circ p^*$ by P24.

$\Rightarrow p^*\omega = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\, p^*\log\|f(w_1,w_2)\|^2 = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log\|f\circ p^*(w_1,w_2)\|^2 = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log\left\|f\left(\frac{z_2}{z_1},\frac{z_3}{z_1}\right)\right\|^2$

$$= \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log\left(1+\left|\frac{z_2}{z_1}\right|^2+\left|\frac{z_3}{z_1}\right|^2\right)$$

$$= \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log(|z_1|^2+|z_2|^2+|z_3|^2) - \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log|z_1|^2$$

$$// \qquad \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log z_1 - \frac{\sqrt{-1}}{2\pi}\bar\partial\partial\log\bar{z}_1 = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial(\log z_1 + \log\bar{z}_1) = 0$$

$$\underset{0}{\parallel} \qquad \underset{0}{\parallel}$$

$$= \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log(|z_1|^2+|z_2|^2+|z_3|^2) = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log\|z\|^2 = \Omega \times 2.$$

$\Rightarrow$ In general, we get $2\Omega = p^*\omega$.