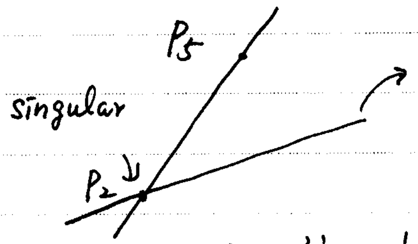
(i) $P_1 \in L_{24} \Rightarrow L_{24} = L_{12} = L \Rightarrow L$ contains P_3 & $P_4 \Rightarrow$
This contradicts ~~to~~ the assumption that only P_3 lies on L.
(ii) $P_1 \in L_{36} \Rightarrow P_2 \in L_{36} \Rightarrow L_{36} = L \Rightarrow P_6 \in L \Rightarrow *$ again.
(iii) $P_1 \in L^5{}_7 \Rightarrow P_2 \in L^5{}_7 \Rightarrow L^5{}_7 = L \Rightarrow P_5, P_7 \in L \Rightarrow *$
$L^5{}_7$ need not be L, since if the cubic cubic curve is singular at P_2, then everything is fine.
③ $P_1 \in L_{25} + L_{36} + L_{47}$.
(i) $P_1 \in L_{25} \Rightarrow$ By the argumet above, $\overline{P_2 P_5} = L_{25} = L$
$\Rightarrow P_5 \in L \Rightarrow$ Contradiction.
(ii) $P_1 \in L_{36} \Rightarrow$ Again $L_{36} = L$ $*$
(iii) $P_1 \in L_{47} \Rightarrow L_{47} = L$ $*$
Thus the only way-out is the singularity at P_2.



The line must be L_{36} or L_{47}.
In case L_{36}. $\Rightarrow L_{23} = L_{12} = L$ $*$
since $P_6 \in L$.
$\Rightarrow$ We have L_{47}. $\Rightarrow L_{47} \ni P_2$.
In this way, in ①, we have $P_2 \in L_{67}$ and in ② $P_2 \in L^5{}_7$. $\Rightarrow L_{27}$ contains P_6, P_4, P_5.

If, finally, none of the points $P_3, \cdots P_7$ lies on L, then since the cubic
$$L_{27} + L_{34} + L_{56}$$
contains P_1, either L_{34} or L_{56} – say L_{34} – must contain P_2.