we must show that $\mathbb{H}^p(M, \mathcal{H}om_{\mathcal{O}}(\mathcal{E}(\mathcal{F}), \mathcal{G}))$
$= \mathbb{H}^p(M, \mathcal{H}om_{\mathcal{O}}(\mathcal{E}'(\mathcal{F}), \mathcal{G}))$.

$$\mathbb{H}^p(M, \mathcal{H}om_{\mathcal{O}}(\mathcal{E}(\mathcal{F}), \mathcal{G})) = \varinjlim_{\underline{\mathcal{U}}} H^p(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}(\mathcal{F}), \mathcal{G}))$$

$\mathcal{H}om(\mathcal{E}(\mathcal{F}), \mathcal{G}))$: $0 \to \mathcal{H}om(\mathcal{F}, \mathcal{G}) \to \mathcal{H}om(\mathcal{E}_0, \mathcal{G}) \to \mathcal{H}om(\mathcal{E}_1, \mathcal{G}) \to \cdots \to 0$

Here

$$\delta_p : C^p(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}_q, \mathcal{G})) \to C^{p+1}(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}_p, \mathcal{G}))$$
$$\partial_q^* : C^p(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}_q, \mathcal{G})) \to C^p(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}_{q+1}, \mathcal{G}))$$
$$D = \delta + \partial$$

$$\Rightarrow \quad C^n(\underline{\mathcal{U}}, D) = \bigoplus_{p+q=n} C^p(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}_q, \mathcal{G}))$$

$$\begin{tikzcd}
\sigma \in C^p(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}_q, \mathcal{G})) \arrow[rr, "\delta_p"] \arrow[dr, "\partial_q^*"] \arrow[dd, "\phi_q^*"'] & & C^{p+1}(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}_q, \mathcal{G})) \ni \delta_p\sigma \arrow[dr, "\partial_q^*"] \arrow[dd, "\phi_q^*"'] & \\
& C^p(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}_{q+1}, \mathcal{G})) \ni \partial_q^*\sigma \arrow[rr, "\delta_p"] \arrow[dd, "\phi_{q+1}^*"'] & & C^{p+1}(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}_{q+1}, \mathcal{G})) \ni \partial_q^*(\delta_p\sigma) \arrow[dd, "\phi_{q+1}^*"] \\
C^p(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}'_q, \mathcal{G})) \ni \phi_q^*\sigma \arrow[rr, "\delta'_p"] \arrow[dr, "\partial'^*_q"] & & C^{p+1}(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}'_q, \mathcal{G})) \ni \phi_q^*(\delta_p\sigma) \arrow[dr] & \\
& C^p(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}'_{q+1}, \mathcal{G})) \ni \phi_{q+1}^*(\partial_q^*\sigma) \arrow[rr, "\delta'_p"] & & C^{p+1}(\underline{\mathcal{U}}, \mathcal{H}om(\mathcal{E}'_{q+1}, \mathcal{G}))
\end{tikzcd}$$

$$\begin{tikzcd}
0 \arrow[r] & \mathcal{E}'_n \arrow[r] \arrow[d, "\phi_n"] & \mathcal{E}'_{n-1} \arrow[r] \arrow[d, "\phi_{n-1}"] & \cdots \xrightarrow{\partial'_q} \mathcal{E}'_q \xrightarrow{\partial'_{q-1}} \cdots \arrow[r] \arrow[d, "\phi_q"] & \mathcal{E}'_0 \arrow[r] \arrow[d, "\phi_0"] & \mathcal{F} \arrow[r] \arrow[d, equal] & 0 \quad \text{locally.} \\
0 \arrow[r] & \mathcal{E}_n \arrow[r] & \mathcal{E}_{n-1} \arrow[r] & \cdots \xrightarrow{\partial_q} \mathcal{E}_q \xrightarrow{\partial_{q-1}} \cdots \arrow[r] & \mathcal{E}_0 \arrow[r] & \mathcal{F} \arrow[r] & 0
\end{tikzcd}$$