$$H^q(C^*(\underline{U}), D) = {}''E_r^{0,q}(\underline{U}) \oplus {}''E_r^{1,q-1}(\underline{U}) \oplus \cdots \oplus {}''E_r^{q,0}(\underline{U}) \oplus F^{q+1} H^q(C^*(\underline{U}), D).$$

$$= {}''E_2^{0,q}(\underline{U}) \oplus {}''E_2^{1,q-1}(\underline{U}) \oplus \cdots \oplus {}''E_2^{q,0}(\underline{U}) \oplus F^{q+1} H^q(C^*(\underline{U}), D).$$

$$= {}''E_2^{q,0}(\underline{U}) \oplus F^{q+1} H^q(C^*(\underline{U}), D).$$

$\Rightarrow$ Since $F^{q+1} H^q(C^*(\underline{U}), D) = 0$, $H^q(C^*(\underline{U}), D) = {}''E_2^{q,0}(\underline{U})$.

$\Rightarrow$ Passing to the limit, then

$$\varinjlim_{\underline{U}} H^q(C^*(\underline{U}), D) = \mathbb{H}^q(M, \mathcal{A}^*(*D)) = {}''E_2^{q,0}$$

$$= H_d^q(H^0(M, \mathcal{A}^q(*D))) = H^q_{DR}(M-D) \quad \text{by } \circledast$$

$$= H^q(M-D, \mathbb{C}) \quad \text{by deRham isomorphism.} \quad ⌟$$

Using this together with the lemma on quasi-isomorphisms, we deduce the isomorphisms

$$\begin{array}{lccc} & \mathbb{H}^*(M, \Omega^*(\log D)) & \cong & H^*(M-D, \mathbb{C}) \\ (*) & \downarrow & & \| \\ & \mathbb{H}^*(M, \Omega^*(*D)) & \cong & H^*(M-D, \mathbb{C}). \end{array}$$

⌈ By the lemma (on P 456) & the lemma (on P 447),

$$\mathbb{H}^q(M, \Omega^*(\log D)) \cong \mathbb{H}^q(M, \mathcal{A}^*(*D)) \cong H^q(M-D, \mathbb{C}).$$
$$\mathbb{H}^q(M, \Omega^*(*D)) \cong \mathbb{H}^q(M, \mathcal{A}^*(*D)) \cong \quad ⌟$$

This gives a method for computing the cohomology of the complement of a divisor with normal crossing by using meromorphic forms that are holomorphic in $M-D$ and have poles along D.