NO.
DATE / /

Let $H^0(\mathcal{F}) = \{\tau_1, \tau_2\}$. For a fixed point $x \in M$, given $\sigma_x \in \mathcal{F}_x$. We have to show that σ_x may be expressed as a linear combination of $(\tau_1)_x$ & $(\tau_2)_x$ with coefficients in $\mathcal{O}_x$.

By the sequence $\mathcal{O} \xrightarrow{\phi} \mathcal{F} \longrightarrow 0$ locally, on a nbd U of x, $\exists\ f \in \mathcal{O}$ s.t $\phi(f) = \sigma$.

$\Rightarrow$ Since $\phi(1)$ is a generator for $\mathcal{F}|U$, $\exists\ g_1, g_2 \in \mathcal{O}(U)$ s.t

$$\tau_1|_U = g_1\,\phi(1) \quad \text{and} \quad \tau_2|_U = g_2\,\phi(1).$$

Since we can choose the tangent plane to H at x arbitrarily, by choosing a proper coordinate system on U, we may assume that $g_1 = \varphi_1(z', z_n)\, h_1(z', z_n)$, $g_2 = \varphi_2(z', z_n)\, h_2(z', z_n)$. $\varphi_1(0) \neq 0, \varphi_2(0) \neq 0$

h_1, h_2 are Weierstrass polynomials in z_n.

and $\{z_n = 0\} = H$, $\phi(1)|_H \neq 0$ in $\mathcal{O}_{H,x}$.

Let $\phi(1) = \eta$.

$$\Rightarrow \left.\begin{aligned} \tau_1 &= g_1\,\eta = \varphi_1 \cdot (z_n^{l_1} + a_1(z')\, z_n^{l_1 - 1} + \cdots + a_{l_1}(z'))\,\eta \\ \tau_2 &= g_2\,\eta = \varphi_2 \cdot (z_n^{l_2} + a_1'(z')\, z_n^{l_2 - 1} + \cdots + a_{l_2}(z'))\,\eta. \end{aligned}\right\} \circledast$$

$\Rightarrow$ Since $\tau_{1,H}$ & $\tau_{2,H}$ generate $(\mathcal{F}_H)_x$, and $\eta_H = \eta_{|\{z_n=0\}} \in (\mathcal{F}_H)_x$

$$\eta_H = p(z')\,\tau_{1,H} + q(z')\,\tau_{2,H}.$$

$\Rightarrow$ From $\circledast$,

$$\tau_{1,H} = \varphi_1(z',0)\ a_{l_1}(z')\ \eta_H$$

$$\tau_{2,H} = \varphi_2(z',0)\ a_{l_2}(z')\ \eta_H$$

and $\quad p(z')\,\tau_{1,H} + q(z')\,\tau_{2,H} = p(z')\,\varphi_1(z',0)\, a_{l_1}(z')\,\eta_H + q(z')\,\varphi_2(z',0)\, a_{l_2}(z')\,\eta_H = \eta_H$

$$\Rightarrow \quad p(z')\,\varphi_1(z',0)\, a_{l_1}(z') + q(z')\,\varphi_2(z',0)\, a_{l_2}(z') = 1. \text{ unless}$$