Now we notice something unexpected: the intersection of $T_x(X)$ with X will fail to be transverse — that is, fail to consist of four distinct lines — everywhere along a line $L \subset X$ if the intersection

$$T_x(X) \cap T_y(X) = T_x(F) \cap T_y(F) \cap T_x(G) \cap T_y(G)$$

is two-dimensional for all $y \in L$.

⌜ If $T_x(X) \cap T_y(X)$ is 2-dim, then, at y, $T_x(X)$ intersects X nontransversely, since $T_x(X) + T_y(X) \neq \mathbb{C}^5$. ⌟

But the family of hyperplanes

$$\{T_x(F)\}_{x \in L}$$

forms a pencil, as does the family $\{T_x(G)\}_{x\in L}$; thus for any $x \neq x' \in L$,

$$T_x(G) \cap T_{x'}(G) = \bigcap_{y \in L} T_y(G)$$

and

$$T_x(F) \cap T_{x'}(F) = \bigcap_{y \in L} T_y(F).$$

⌜ See P766. for a proof of pencils.

Let $\langle \sigma_1, \sigma_2 \rangle = \{T_x(F)\}_{x \in L}$.

$\Rightarrow (a_1\sigma_1 + b_1\sigma_2 = 0) \cap (a_1'\sigma_1 + b_1'\sigma_2 = 0) = (\sigma_1 = 0) \cap (\sigma_2 = 0)$ is the base locus of the pencil. $\Rightarrow T_x(F) \cap T_{x'}(F) = \bigcap_{y \in L} T_y(F)$. ⌟