$\mathcal{L}|S^*$. $\Rightarrow \left\{ \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & id \end{pmatrix} \right\}$ are transition functions for $\mathcal{L}|S^* \oplus \mathcal{O}_{S^*} = \mathcal{E}^*|S^*$

$\Rightarrow$ The transition functions for $\Lambda^2\mathcal{E}^*|S^*$ are, by P67,

$$\left\{ \det \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & id \end{pmatrix} \right\} = \{ g_{\alpha\beta} \} \Rightarrow \Lambda^2\mathcal{E}^*|S^* = \mathcal{L}|S^*.$$

$\Rightarrow$
$$\begin{tikzcd} \Lambda^2 E^* \arrow[d, shift left] & L \arrow[d, shift left] \\ S^* \arrow[u, shift left, "\sigma_1 \to \text{meromorphic section}"] & S^* \arrow[u, shift left, "\sigma_2"'] \end{tikzcd} \qquad \mathcal{O}(L) = \mathcal{L}$$

$\Rightarrow$ To show that $\Lambda^2 E^* = L$, we have only to prove that σ_1/σ_2 is meromorphic function on S by the results on P136.

Since $\Lambda^2 E^*|S^* = L|S^*$, $\sigma_1/\sigma_2|S^*$ is meromorphic function on S^*. $\Rightarrow$ By Levi extension theorem (I) on P396, $\sigma_1/\sigma_2|S^*$ extends to σ_1/σ_2, which is meromorphic on S. (Refer to P414.)

Referring to (*), we may rephrase the problem as follows: Given $(Z, \mathcal{O}_Z)$ and $\mathcal{L} \in \mathrm{Pic}(S)$, we seek

(**) $\left\{ \begin{array}{l} e \in \mathrm{Ext}^1(S; I, \mathcal{L}), \text{ such that } e_p \text{ is a unit} \\ \text{in } \underline{\mathrm{Ext}}^1(I, \mathcal{L})_p \cong \mathcal{O}_{Z,p} \text{ for each point } p \in Z. \end{array} \right.$