as follows: $\sigma \longrightarrow \sigma/s_0$, where $(s_0=0) = D$, see p138 ~p139.

$\Rightarrow$ $h^\circ(KD) - \dim V \geq K(n-1)+1 \iff h^\circ(kD) - h^\circ((k-1)D) \geq k(n-1)+1$. ∎

The same argument likewise shows that for $k > m$ we can find k hyperplanes in $\mathbb{P}^n$ containing all but any one of the points of D, so that

$$h^\circ(kD) - h^\circ((k-1)D) = d \quad \text{for } k > m.$$

Pf. $p_0 \in D \Rightarrow \#(D - \{p_0\}) = d-1. \Rightarrow$ Partition $D - \{p_0\}$ into k sets $A_1, A_2, \cdots A_k$ so that A_i's are nonempty, $\bigcup A_i = D - \{p_0\}$, $\# A_i = p-1$, all elements of A_i are distinct. $\Rightarrow$ Since the points of D are in general position each A_i is linearly independent and its linear span will not contain p_0 $\Rightarrow$ We can find hyperplane $H_1, \cdots H_k$ in $\mathbb{P}^n$ containing the set A_i but p_0. The sum $H_1 + \cdots + H_k$ is the hypersurface of deg k not containing p_0

$\Rightarrow$ $h^\circ(kD) - h^\circ((k-1)D) \geq d$, for $k > m$. --- ①

Let $D = \{p_1, p_2 \cdots p_d\}$ and $\{\tau_1, \cdots \tau_d\}$ be the sections of those hypersurfaces not containing p_i's respectively.

$\Rightarrow$ Given $\sigma \in H^\circ(S, \mathcal{O}(kD)) - V$, consider

$$\tau = \sum \frac{\sigma(p_i)}{\tau_i(p_i)} \tau_i \quad \Rightarrow \quad \tau(p_i) = \sigma(p_i)$$

$\Rightarrow$ $\sigma - \tau = 0$ on D $\Rightarrow$ $\sigma \in V + \langle \tau_1 \cdots \tau_d \rangle$

$\Rightarrow$ $\dim H^\circ(S, \mathcal{O}(kD)) \leq \dim V + d = h^\circ(S, \mathcal{O}((k-1)D)) + d$

$\leq h^\circ((k-1)D) + d$ -- ② $\Rightarrow$ By ① & ②, $h^\circ(kD) - h^\circ((k-1)D) = d$. ∎

Thus we have
$$h^\circ(D) \geq n+1,$$
$$h^\circ(2D) \geq n+1 + 2(n-1) + 1 = 3(n-1)+3,$$
$$h^\circ(3D) \geq 6(n-1) + k,$$