$$\{\ ,\ \} : A^{p,r}(M) \otimes A^{q,s}(M) \longrightarrow A^{p+q,\ r+s}(M)$$

given by
$$\{\psi.\ \eta\} = \psi \wedge \eta.$$
satisfies
$$\bar\partial\{\psi.\ \eta\} = \{\bar\partial\psi.\ \eta\} + (-1)^{\deg\psi}\{\psi.\ \bar\partial\eta\}$$
and so induces

$$(**).\qquad H^{p.*}_{\bar\partial}(M) \otimes H^{q.*}_{\bar\partial}(M) \longrightarrow H^{p+q.*}_{\bar\partial}(M).$$

The pairings $(*)$ and correspond under the Dolbeault isomorphism, at least modulo signs, for the same reason as in the discussion at the end of de Rham's theorem.

$\ulcorner$

$$\begin{array}{ccc} H^{p.q}_{\bar\partial}(M) & \xrightarrow[\cong]{\text{Dolbeault Isomorphism}} & H^q(M.\ \Omega^p) \\ \psi & \longmapsto & \Psi \end{array}$$

$\Rightarrow$ For this, see Bott & Tu Differential Forms in Algebraic Topology.

s.t $\Psi_{i_0, i_1 \cdots i_q} \in \Omega^p(U_{i_0} \cap \cdots \cap U_{i_q})$

where $\Psi_{i_0 \cdots i_q} = \psi|U_{i_0} \cap \cdots \cap U_q$.

$$(\Psi \wedge \Phi)_{i_0 \cdots i_{p+q}} = \Psi_{i_0 \cdots i_p} \wedge \Phi_{i_p.\, i_{p+1} \cdots i_{p+q}}$$

$$\|$$

$$(\overline{\psi \wedge \varphi})_{i_0 \cdots i_{p+q}} = \psi \wedge \varphi | U_{i_0} \cap \cdots \cap U_{i_{q+p}} = \psi|U_{i_0} \cdots U_{i_{p+q}} \wedge \varphi|U_{i_0} \cdots U_{i_{p+q}}$$

Thus the two pairings correspond each other under the Dolbeault isomorphism. $\lrcorner$

With this understood, we have the