

Now we will describe the geometry of \tilde{M}_x near E in more detail. First, we give local coordinates near E on \tilde{M}_x : let $z = (z_1, z_2, \dots, z_n)$ be local coordinates on $U \ni x$ with center x . Then

$$\tilde{U} = \pi^{-1}(U) = \{ (z, l) \in U \times \mathbb{P}^{n-1} : z_i l_j = z_j l_i \};$$

and we set

$$\tilde{U}_i = (l_i \neq 0) \subset \tilde{U}.$$

In this way we obtain an open cover of the mbd \tilde{U} of E , and in each open set \tilde{U}_i we have local coordinates $z(i)_j$:

$$z(i)_j = \frac{l_j}{l_i} = \frac{z_j}{z_i}, \quad j \neq i,$$

$$\text{and } z(i)_i = z_i.$$

The map $\pi: \tilde{M}_x \rightarrow M$ is given in \tilde{U}_i by

$$\begin{aligned} (z(i)_1, \dots, z(i)_i, \dots, z(i)_n) &\longmapsto (z_1, z_2, \dots, z_n) \\ \left(\frac{z_1}{z_i}, \frac{z_2}{z_i}, \dots, z_i, \dots, \frac{z_n}{z_i} \right) &= \left(\frac{z(i)_1}{z(i)_i} \cdot z(i)_i, \frac{z(i)_2}{z(i)_i} \cdot z(i)_i, \dots, z(i)_i, \dots, \frac{z(i)_n}{z(i)_i} \cdot z(i)_i \right) \end{aligned}$$

and the divisor E is given by

$$E = (z(i)_i = 0) \quad \text{in } \tilde{U}_i.$$

⌈ If $z(i)_i = z_i = 0$, since $z \in l \Rightarrow$ all z_j 's must be zero. $\Rightarrow E = (z(i)_i = 0) \text{ in } \tilde{U}_i. \quad \rfloor$

$$\begin{aligned} \text{In } \tilde{U}_i \cap \tilde{U}_j, \quad z(i)_k &= z(j)_i^{-1} \cdot z(j)_k, \\ z(i)_j &= z(j)_i^{-1}, \\ z_i &= z(j)_i \cdot z_j. \end{aligned}$$