is, the integrals of dx/y over closed loops $\gamma \in H_1(C,\mathbb{Z})$. More precisely, note that from the preceding section the form $\omega = dx/y$ is everywhere holomorphic on C and so is a generator of $H^0(C,\Omega^1)$.

Pf By p 221, the Poincare residue map: $H^0(\mathbb{P}^2, \Omega^2(S)) \longrightarrow H^0(S,\Omega^1)$ is isomorphic.

If we let $C = S$, $(S = C)$,

$$H^0(\mathbb{P}^2, \Omega^2(\underset{C}{\overset{\shortparallel}{S}})) \longrightarrow H^0(C,\Omega^1)$$

$$\frac{g(x,y)}{f(x,y)}\, dx\wedge dy \longmapsto -\frac{g(x,y)}{\frac{\partial f}{\partial y}(x,y)}\, dx$$

where, $f(x,y) = y^2 - x^3 - ax^2 - bx - c$.

Since $g(C)=1$, $\dim H^0(C,\Omega^1) = 1$.

g is a polynomial of degree $d^{=3}-3\,(=0) \Rightarrow g$ is a constant $\Rightarrow g = 1 \times 2 \times (-1) = -2$.

$$\frac{\partial f}{\partial y} = 2y$$

$$\Rightarrow \text{We have } \omega = \frac{dx}{y} \in H^0(C,\Omega^1).$$

$$f(x,y) = y^2 - x^3 - ax^2 - bx - c = 0$$

On $C = (f(x,y)=0)$, $f(x(t),y(t)) = 0 \Rightarrow \frac{d}{dt} f(x(t),y(t)) = 0 \Rightarrow \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = 0$

$$\Rightarrow \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = 0 \text{ on } C$$

$$\Rightarrow \text{On } C,\ \frac{dx}{\frac{\partial f}{\partial y}} = -\frac{dy}{\frac{\partial f}{\partial x}}$$