Since $\deg P \leq \nu-1$ and P has distinct roots $f(A_i)$ $i=1 \cdots \nu$ on some nbd. $\Rightarrow \{1, f, \cdots, f^{\nu-1}\}$ is linearly independent set. "

The inclusion $d_f \cdot {}_\nu\mathcal{O} \subseteq {}_n\mathcal{O}[f]$ implies immediately that ${}_\nu\mathcal{O} \subseteq {}_nM[f]$; since f is algebraic over ${}_nM$, the extension ${}_nM[f]$ is also a field, and consequently ${}_\nu M \subseteq {}_nM[f]$.

" $d_f \cdot {}_\nu\mathcal{O} \subseteq {}_n\mathcal{O}[f] \subseteq {}_nM[f]$, since ${}_nM$ is a quotient field of ${}_n\mathcal{O}$, and ${}_n\mathcal{O} \subset {}_nM$. $d_f^{-1} \cdot d_f \cdot {}_\nu\mathcal{O} \subseteq d_f^{-1} \cdot {}_nM[f] = {}_nM[f] \Rightarrow {}_\nu\mathcal{O} \subseteq {}_nM[f]. \Rightarrow$ Since ${}_\nu M$ is the smallest field containing ${}_\nu\mathcal{O}$ and ${}_nM[f]$ is a field containing ${}_\nu\mathcal{O}$ ($\because$ f is algebraic over ${}_nM$), ${}_\nu M \subset {}_nM[f]$. "

On the other hand, since ${}_nM \subseteq {}_\nu M$ and $f \in {}_\nu\mathcal{O} \subseteq {}_\nu M$, then ${}_nM[f] \subseteq {}_\nu M$, so that actually ${}_\nu M = {}_nM[f]$.

" $\{1, f, \cdots f^{\nu-1}\}$ generates ${}_\nu M$ over ${}_nM$ $\Rightarrow [{}_\nu M, {}_nM] = [{}_nM[f], {}_nM] = \nu$. together with the linearly independence of $\{1, f, \cdots f^{\nu-1}\}$ "

That suffices to conclude the proof.

Now let's go back to the original question:

$$\dim_{\mathbb{C}}(\mathcal{O}_z/I) = d.$$

Let $\pi = (\underset{f}{\underbrace{f_1, \cdots f_n}}) : U^{\subset \mathbb{C}^n} \longrightarrow W^{\subset \mathbb{C}^n}$