$G + E_m = G' + E_m \Rightarrow G = G'$ since a divisor is a formal sum of hypersurfaces. $\Rightarrow G \longmapsto G + E_m$ is injective for all $m \geq 0$. Since $\dim |2K|$ is finite, so is $\dim |K - mD|$, see P137 $|2K| \cong \mathbb{P}(H^0(M, \mathcal{O}(2K)))$ and $\dim H^0(M, \mathcal{O}(2K)) = \dim \mathcal{H}^{0,0}([2K]) < \infty$ see P152.
But as we saw above, $\dim H^0(K - mD) \to \infty$ as $m \to \infty$
$\Rightarrow$ Contradiction.

Blowing Up and Down

We recall some definitions from Chapter 1. Section 4:
let M be a complex manifold of dimension n, $z = (z_1, z_2, \cdots z_n)$ holomorphic coordinates in an open set $U \subset M$ centered around the point $p \in M$. See P182.
The blow-up $\tilde{M}$ of M at p is then taken to be the complex manifold obtained by adjoining to $M - \{p\}$ the manifold

$$\tilde{U} = \{(z, l) : z \in l\} \subset U \times \mathbb{P}^{n-1}$$

via the isomorphism

$$\tilde{U} - (z = 0) \cong U - \{p\}$$

given by $(z, l) \longmapsto z$. There is a natural projection map $\pi : \tilde{M} \longrightarrow M$ extending the identity on $M - \{p\}$.

$$\begin{array}{ccccc} \tilde{M} = & M - \{p\} \underset{\pi}{\cup} \tilde{U} & [(z,l)] = [z], & z \neq 0 \\ & \pi \downarrow & \downarrow & \\ & M & z. & \end{array}$$