

from the space of symmetric d -linear forms on \mathbb{C}^{n+1} — that is, homogeneous polynomials $F(X_0, \dots, X_n)$ of degree d in X_0, \dots, X_n — to the space of global sections of H^d .

$$\begin{array}{ccc} \boxed{F} & F \text{ is a } d\text{-linear form on } \mathbb{C}^{n+1} & \\ \Rightarrow & F: \mathbb{C}^{n+1} \times \dots \times \mathbb{C}^{n+1} \longrightarrow \mathbb{C} & \\ & \downarrow & \nearrow \\ \text{1-1} & & \\ \text{on } \mathbb{C} & & \\ \text{correspondence} & \bar{F}: \mathbb{C}^{n+1} \otimes \dots \otimes \mathbb{C}^{n+1} \longrightarrow \mathbb{C} & \end{array}$$

$$\begin{aligned} F(v, w) &= F(\lambda_1 X, \lambda_2 X) = \lambda_1 \lambda_2 F(X, X) \\ &= F(\lambda_2 X, \lambda_1 X) = F(w, v), \text{ since } v, w \in \{\lambda X\}. \end{aligned}$$

That F is alternating in any factors means $F(v, \dots, v, \dots) = 0 \Rightarrow F \equiv 0$ on $\{\lambda X\}$.

$$\begin{aligned} \text{Sym}^d(\mathbb{C}^{n+1*}) &= \text{Space of } d\text{-linear symmetric forms on } \mathbb{C}^{n+1} \\ &= \text{Space generated by } \{ \langle e_{i_1}^* \otimes e_{i_2}^* \otimes \dots \otimes e_{i_d}^* \rangle \} \end{aligned}$$

where $e_i^*(e_j) = \delta_{ij}$. $\{e_j\}$ standard base for \mathbb{C}^{n+1} .

$$\langle e_{i_1}^* \otimes \dots \otimes e_{i_d}^* \rangle = \frac{1}{d!} \sum_{\sigma \in S_d} e_{i_{\sigma(1)}}^* \otimes e_{i_{\sigma(2)}}^* \otimes \dots \otimes e_{i_{\sigma(d)}}^*$$

$S_d =$ symmetric group of degree d .