$\ulcorner$ $(\bar\partial(KT))(\psi) + (-1)(K(\bar\partial T))(\psi) = (-1)^q (KT)(\bar\partial\psi) + (\bar\partial T)(K\psi)$ (since $\bar\partial T$ is compactly supported)

$= (-1)^q(KT)(\bar\partial\psi) + (-1)^q(T)(\bar\partial(K\psi))$

$= (-1)^q\ (T)(K(\bar\partial\psi)) + (-1)^q(T)(\bar\partial(K\psi))$

$= (-1)^q\ T(\ K(\bar\partial\psi) + (-1)\bar\partial(K\psi)) = (-1)^q T\circ(K\circ\bar\partial + \bar\partial\circ K)(\psi)$

$= (-1)^q T(\psi)$ by P122 note. $\lrcorner$

In particular, the proof of the $\bar\partial$-Poincaré lemma for smooth forms may be extended verbatim to prove the result for currents.

$\ulcorner$ Actually, all the arguments above are for proving

$$0 \to \Omega^0 \longrightarrow \mathcal{D}^{0,0} \longrightarrow \mathcal{D}^{0,1} \longrightarrow \mathcal{D}^{0,1} \longrightarrow \cdots \mathcal{D}^{0,2} \to \cdots \to \mathcal{D}^{0,n} \to 0$$

is exact, and the arguments can be extended to prove the general complex of sheaves

$$0 \to \Omega^p \longrightarrow \mathcal{D}^{p,0} \xrightarrow{\bar\partial} \mathcal{D}^{p,1} \xrightarrow{\bar\partial} \mathcal{D}^{p,2} \to \cdots \to \mathcal{D}^{p,n} \to 0$$

is exact.

Thus

$$\mathcal{D}^{0,q-1} \xrightarrow{\bar\partial} \mathcal{D}^{0,q} \xrightarrow{\bar\partial} \mathcal{D}^{0,q+1}$$

given $T \in \mathcal{D}^{0,q}(U)$ s.t $\bar\partial T = 0$, for any pt $x \in U$ we have only to find an open subset $V \subset U$ s.t $x \in V \subset \bar V \subset U$, $\bar V$ compact, $\exists\ T' \in \mathcal{D}^{0,q-1}(V)$ s.t $\bar\partial T' = T$ on V.

Consider the restriction T to V, and extend T to