Given any point $x \in M$, $\exists$ an open set $x \in U_x \subset M$ s.t

$$f|_{U_x} \in m^*(U_x) \longrightarrow j \circ f|_{U_x} \in m^*/\mathcal{O}^*(U_x) \quad \text{s.t}$$

$$f|_{U_x}\,\mathcal{O}^*(U_x) = \overline{f|_{U_x}} \quad \text{and} \quad \varinjlim_{U_p} \overline{f|_{U_x}} = (j \circ f|_{U_x})(p)$$

where $U_p \ni p$. al $\forall p \in U_x$.

But since $\dfrac{f|_{U_x}}{f|_{U_y}} = 1$ if $U_x \cap U_y \neq \phi$,

$\{ f|_{U_x}\,\mathcal{O}^*(U_x) \}$ gives an global section of the quotient sheaf $m^*/\mathcal{O}^*$.

$\{ f|_{U_x}\,\mathcal{O}^*(U_x) \} \equiv \{ (f|_{U_x}, U_x) \}$ where

$$\frac{f|_{U_x}}{f|_{U_y}} = 1 \in \mathcal{O}^*(U_x \cap U_y)$$

(2) $\delta D = [D]$,

$$\begin{array}{ccc} H^0(M, m^*/\mathcal{O}^*) & \xrightarrow{\delta} & H^1(M, \mathcal{O}^*). \\ \parallel & & \\ H^0(\underline{U}, m^*/\mathcal{O}^*) & & \end{array}$$

$D = (f)$, $f$ is a global section of $m^*/\mathcal{O}^*$. $\Rightarrow$

$f$ is represented by $\{ (f_\alpha, U_\alpha) \}$ $f_\alpha \in m^*(U_\alpha)$. $\frac{f_\alpha}{f_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta)$