Taking $f_i(z) = z_i$ and applying our lemma, we obtain another proof of the Bochner-Martinelli formula.

$$\int_{\|z\|=\epsilon} g(z)\, \beta(z,\bar{z}) = g(0).$$

⌈ If we let $f_i(z) = z_i$, then

$$\eta_\omega = g(z) \frac{C_n \sum (-1)^{i-1} \bar{z}_i \, d\bar{z}_1 \wedge \cdots \widehat{d\bar{z}_i} \wedge \cdots d\bar{z}_n \wedge dz_1 \wedge \cdots \wedge dz_n}{\|z\|^{2n}}$$

$$= g(z)\, \beta(z,\bar{z})$$

$$\int_{\|z\|=\epsilon} \eta_\omega = \int_{\|z\|=\epsilon} g(z)\, \beta(z,\bar{z}) = \mathrm{Res}_{\{0\}}\, \omega \quad \text{by lemma on p651.}$$

$$\int_{S^{2n-1}} \eta_\omega \qquad = \frac{g(0)}{J_f(0)}.$$

But since $f_i(z) = z_i$, $J_f(0) = \begin{vmatrix} 1 & 0 & & 0 \\ 0 & 1 & & \\ & & \ddots & \\ & 0 & & 1 \end{vmatrix} = 1$.

$$\Rightarrow \int_{\|z\|=\epsilon} \eta_\omega = \int_{\|z\|=\epsilon} g(z)\, \beta(z,\bar{z}) = g(0). \qquad \lrcorner$$

• Recall also from Section 2 of Chapter 3 on the holomorphic Lefschetz fixed-point formula that we proved that if the origin is an isolated nondegenerate fixed point of a map $f : U \to \mathbb{C}^n$, then

$$\int_{\|z\|=\epsilon} F^* K = J_f(0)^{-1}.$$