$\Rightarrow \quad \Delta \delta_{ij} = \sum_k a_{kj} A_{ik}$.

Then $\Delta f_i = \sum_j \Delta \delta_{ij} f_j = \sum_{j,k} a_{kj} f_j A_{ik} = \sum_k A_{ik} f_k'$,

so that $\Delta I \subset I'$ as desired.

F $\quad \Delta f_i = \sum_j \underset{\Uparrow}{\Delta} \delta_{ij} f_j = \sum_{j,k} a_{kj} A_{ik} f_j = \sum_{j,k} A_{ik} a_{kj} f_j$

since $\delta_{ij} = \begin{cases} 1, & i=j \\ 0 & i \neq j \end{cases}$

$= \sum_k A_{ik} f_k' \in (f_1', \cdots f_r') = I'$

" Since I' is regular, $\Delta \neq 0$. Otherwise, I' is not regular.

$\Delta \delta_{ij} = \sum_k a_{kj} A_{ik}$ holds even if $\Delta = 0$.

$$0 = \left| A^1, A^2, \cdots A^{j-1} \begin{pmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ a_{ii} \\ \vdots \\ a_{ni} \end{pmatrix} A^{j+1}, \cdots \right|$$

$= a_{1i} A_{1j} + a_{2i} A_{2j} \qquad = \Delta = 0$

$= a_{1i} A_{1j} + a_{2i} A_{2j} + \cdot \qquad = \sum a_{ki} A_{kj}$ "

Consider now the exact sequence

$$0 \longrightarrow I/I' \longrightarrow \mathcal{O}/I' \longrightarrow \mathcal{O}/I \longrightarrow 0.$$