

$$a_5 X_5 = 0 \Rightarrow \dim(\text{Such hyperplanes}) = 1.$$

By the argument above, <sup>the</sup> dimension of <sup>the set of</sup> generic hyperplanes with possible singular point  $r$  is 2.  $\Rightarrow$

$\{ \text{generic hyperplanes with possible singular point } r \}$   
 $\cap \{ \text{hyperplanes containing } \text{Tr}(X) \}$  has dimension 2.

$\Rightarrow$  For generic  $x \in \sigma(q)$ ,  $U_x$  is smooth, since  $T_x G + T_x X = \mathbb{P}^5$  at the only possible singular point  $r$ .  $\square$

We have thus shown that for  $l_x$  a generic line in  $\mathbb{P}^3$  through  $q \in S - R$ , the surface  $U_x$  is smooth.

The argument above then shows that  $l_x$  meets  $S$  in four distinct points, and hence meets  $S$  transversely; a fortiori, it shows that  $q$  is a smooth point of  $S$ .

$\square$  By P765,  $\#(l_x \cap S) = \deg S = \mu$ ,  $\mu$  the number of singular curves in the pencil  $\{X_p\}_{p \in l_x}$ .

$\Rightarrow l_x \cap S = \text{Set of distinct points.}$

$\Rightarrow l_x$  meets  $S$  transversely.

$\Rightarrow l_x$  meets  $S$  at  $q$  once.  $\Rightarrow q$  is not singular since otherwise any line meets  $S$  at  $q$  <sup>more than</sup> twice.

$\Rightarrow q$  is a smooth point of  $S$ .

Note: If  $l_x$  meets  $S$  at  $q$  more than once, then  $F$  is tangent to  $\sigma(q)$  at points more than one (maybe at one point with multiplicity  $\geq 2$ )

$\Rightarrow F$  is tangent to  $\sigma(q)$  along a line for

this is impossible since  $\deg F = 2$ .