gives $E_1 \otimes_{\mathcal{O}} N \longrightarrow E_0 \otimes_{\mathcal{O}} N \longrightarrow M \otimes_{\mathcal{O}} N \longrightarrow 0$.

Next, we note that short exact sequences

$$\begin{cases} 0 \longrightarrow M' \longrightarrow M \longrightarrow M'' \longrightarrow 0 \\ 0 \longrightarrow N' \longrightarrow N \longrightarrow N'' \longrightarrow 0 \end{cases}$$

give rise to long exact sequences of Tor's

$$\begin{cases} \cdots \longrightarrow Tor_k^{\mathcal{O}}(M,N) \longrightarrow Tor_k^{\mathcal{O}}(M'',N) \longrightarrow Tor_{k-1}^{\mathcal{O}}(M',N) \longrightarrow \cdots \\ \cdots \longrightarrow Tor_k^{\mathcal{O}}(M,N) \longrightarrow Tor_k^{\mathcal{O}}(M,N'') \longrightarrow Tor_{k-1}^{\mathcal{O}}(M,N') \longrightarrow \cdots \end{cases}$$

for the same reason as for Ext.

$$\begin{tikzcd}
 & M'\otimes N \arrow[r] & M\otimes N \arrow[r] & M''\otimes N \arrow[r] & 0 \\
0 \arrow[r] & E_0'\otimes N \arrow[r] \arrow[u] & E_0\otimes N \arrow[r] \arrow[u] & E_0''\otimes N \arrow[r] \arrow[u] & 0 \\
0 \arrow[r] & E_1'\otimes N \arrow[r] \arrow[u] & E_1\otimes N \arrow[r] \arrow[u] & E_1''\otimes N \arrow[r] \arrow[u] & 0 \\
 & \vdots & & &
\end{tikzcd}$$

$\Rightarrow$

$Tor_1^{\mathcal{O}}(M'\otimes N) \longrightarrow Tor_1^{\mathcal{O}}(M\otimes N) \longrightarrow Tor_1^{\mathcal{O}}(M''\otimes N) \longrightarrow Tor_0^{\mathcal{O}}(M'\otimes N) \longrightarrow \cdots$

Now to prove the lemma. Observe that

$$M \otimes_{\mathcal{O}} \mathbb{C} \cong M/mM = M_0$$

is the fiber of M. Choose a free $\mathcal{O}$-module E such that $E_0 \cong M_0$.

$$\begin{tikzcd}
M \times \mathbb{C} \arrow[r] \arrow[dr, "\psi'"'] & M\otimes_{\mathcal{O}} \mathbb{C} \arrow[d, "\psi"] \ni x\otimes\alpha \\
 & M/mM
\end{tikzcd}$$

$(x, \alpha) \longmapsto \alpha x + mM$, $\quad x\otimes\alpha \longmapsto \alpha x + mM$