A subspace of V fixed under $\rho(sl_2)$ is called a submodule; V (or ρ) is called irreducible if V has no non-trivial submodules. By a fundamental result, which we won't prove here, every submodule W of an sl_2-module V has a complementary submodule $W^\perp$; thus every sl_2-module is the direct sum of irreducible sl_2-modules, and to study representations of sl_2 we need only look at irreducible ones.

Suppose then that V is an irreducible sl_2-module. The key to analyzing the structure of V is to look at the eigenspaces for $\rho(H)$ (from now on, we will omit the ρ's). These are called weighted spaces. First of all, note that if $v \in V$ is an eigenvector of H with eigenvalue λ, then Xv and Yv are also eigenvectors of H, with eigenvalues $\lambda+2$ and $\lambda-2$, respectively: this follows from

$$H(Xv) = XHv + [H.X]v = XHv + 2Xv$$
$$= X(\lambda v) + 2Xv = (\lambda+2)Xv$$

and.

$$H(Yv) = YHv - 2Yv = Y\lambda v - 2Yv$$
$$= (\lambda-2)Yv.$$

Since H can have only a finite number of eigenv , we see from this that X and Y are nilpotent. We say that $v \in V$ is primitive if v is an eigenvector for H and $Xv = 0$; clearly primitive elements exist.

「 X, nilpotent means $\exists$ $k > 0$ s.t $X^k = 0$. 」

Proposition. If $v \in V$ is primitive, then V is generated as a vector space by $v, Yv, Y^2v, \cdots$.