

$$\begin{array}{ccc} \mathbb{P}^3 & \xrightarrow{\psi} & G(3,6) \text{ is holomorphic} \\ \downarrow & & \downarrow \\ \mathbb{P} & \xrightarrow{\quad} & \sigma(\mathbb{P}) \end{array}$$

and ψ is one to one $\Rightarrow \psi(\mathbb{P}^3) = \tau$ is a 3-dimensional cycle in $G(3,6)$.

W_F is an analytic subvariety of $G(3,6)$, for given any Λ_2 , by changing coordinates, we may assume $\Lambda_2 = \mathbb{P}^2 \subset \mathbb{P}^5 \Rightarrow$ If we let $F = (F=0)$, where F is a quadratic in \mathbb{P}^5 , then, by the coordinate change, $F = \sum_{0 \leq i, j \leq 2} q_{ij} x_i x_j$, where each

q_{ij} is a function (polynomial) in some variables z_j 's representing Λ_2 . \Rightarrow For F to be a double line, $\text{rank } F = 1 \Rightarrow \exists$ analytic equations in z_j 's. $\Rightarrow W_F$ is analytic in $G(3,6)$.

* Consider the following map ϕ defined by

$$\begin{array}{ccc} \phi : G(3,6) & \longrightarrow & W = \mathbb{P}^5 \\ \downarrow & & \downarrow \\ \text{rank 1 conics} & \Lambda_2 \longmapsto & \Lambda_2 \cap F \text{ (in fact } \Lambda_2 \cdot F) \end{array}$$

\nearrow all conics in a plane

$\Rightarrow \phi$ is holomorphic, and ϕ is transverse to W_2 since $d\phi(G(3,6)) = 5$.

$\Rightarrow \phi^{-1}(W_2)$ has codimension 3 in $G(3,6)$

$\Rightarrow \dim \phi^{-1}(W_2) = 6 \Rightarrow \phi^{-1}(W_2) = W_F$ is of dimension 6.* Note: the 'argument' is not rigorous!!