$\Rightarrow \qquad \tilde{C} = \pi^* C - 2E_1 - \cdots \mp \pi^* C - E_1 - E_2$.

From the example above, we may assume that a generic curve in $|\mathcal{J}_{\Gamma_0}(n)|$ passes each point of Γ_0 once.

The canonical bundle of $\tilde{S}$ is

$$K_{\tilde{S}} = \pi^*(K_{\mathbb{P}^2}) + E$$
$$= \pi^* H^{-3} + E,$$

and the numerical characters for $L \to \tilde{S}$ in the Riemann - Roch formula are

$$h^2(L) = h^0(K_{\tilde{S}} - L) = 0, \qquad p_g = q = 0,$$
$$C \cdot C = C_0 \cdot C_0 - d = n^2 - d, \quad \text{where } d = \deg \Gamma_0,$$
$$C \cdot K_{\tilde{S}} = C \cdot (\pi^* H^{-3} + E) = -3n + d.$$

By the lemma on P187.

$$K_{\tilde{S}} = \pi^* K_{\mathbb{P}^2} + E, \quad \text{since } n = 1.$$

Here we may use the lemma d times or by following the argument in the proof we may prove it directly.

$\Rightarrow$ By P146. $K_{\mathbb{P}^2} = -3H$.

$h^2(L) = h^0(K_{\tilde{S}} - L)$ by Kodaira - Serre duality.

$K_{\tilde{S}} - L = -3\pi^* H + E - n\pi^* H + E = -(n+3)\pi^* H + 2E$

$\Rightarrow \deg(K_{\tilde{S}} - L) = -(n+3) + 2 = -n-1 < 0 \Rightarrow \nexists$ no holomorphic section on $\underset{K_{\tilde{S}} - L}{\downarrow} \to \tilde{S} \Rightarrow h^0(K_{\tilde{S}} - L) = 0$.

By the def on P494, $\quad P_g(M) = h^{n,0}(M)$

$\Rightarrow \quad P_g = h^{2,0}(\tilde{S}) = 0 \qquad$ by Theorem on P520 & remark on P536. Furthermore $q = h^{1,0}(\tilde{S}) = 0$, by the same

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