$$\Rightarrow \quad \eta(0) = C_n \int_{\mathbb{R}^n} \frac{\Delta \eta(x)}{\|x\|^{n-2}} dx = \varphi(u) = C_n \int_{\mathbb{R}^n} \frac{\Delta_x \varphi(x+u)}{\|x\|^{n-2}} dx$$

$$= C_n \int_{\mathbb{R}^n} \frac{\Delta_u \varphi(x+u)}{\|x\|^{n-2}} dx \overset{(-1)^n}{\uparrow}{=} \pm C_n \int_{\mathbb{R}^n} \frac{\Delta_u \varphi(u-x)}{\|x\|^{n-2}} dx$$

See & compare with P376. ⌟

In polar coordinates $x = r\omega$, where $r = \|x\|$ and $\omega \in S^{n-1}$,

$$\frac{\Delta \eta(x)}{\|x\|^{n-2}} dx = \frac{d * d\eta}{r^{n-2}}$$

$$= d\left(\frac{* d\eta}{r^{n-2}}\right) \pm \left(\frac{1}{n-2}\right) \frac{dr \wedge * d\eta}{r^{n-1}}$$

$$= d\left(\frac{* d\eta}{r^{n-2}}\right) \pm \left(\frac{1}{n-2}\right) \frac{d\eta \wedge *(r dr)}{r^{n}}$$

$$= d\left(\frac{* d\eta}{r^{n-2}}\right) \pm \left(\frac{1}{n-2}\right) d(\eta\sigma).$$

⌜ By the middle of P377, $\Delta\eta \, dx = \pm \, d * d\eta$

see P78 note. $\Rightarrow \quad \Delta \eta \, dx = - d * d\eta.$

$$\frac{d * d\eta}{r^{n-2}} = \frac{1}{r^{n-2}} d(* d\eta) = d\left(\frac{1}{r^{n-1-1}} * d\eta\right)$$

$$- d(r^{-(n-2)}) \wedge * d\eta = d\left(\frac{* d\eta}{r^{n-2}}\right) + (n-2) r^{-n+1} dr \wedge * d\eta$$

$$= d\left(\frac{* d\eta}{r^{n-2}}\right) + \frac{n-2}{r^{n-1}} dr \wedge * d\eta =$$