「 All right. I understand. 」

2. Letting L be, as above, a generic pencil of conics, we have by the formula of p. 509

$$\chi(\mathbb{P}^2) = 2\chi(F_\lambda) + \mu - n,$$

where F_λ is a generic element of L, $n = F_\lambda \cdot F_\lambda$ the number of base points of L, and μ the number of singular conics in L.

「 As above, $L = \{ Q^\lambda(X) = 0 \mid Q^\lambda = Q^0 + \lambda Q^\infty \}$

$$Q^\lambda = Q^0 + \lambda Q^\infty$$
$$= Q^0 (I + \lambda Q^{0-1} Q^\infty).$$

$\Rightarrow$ $Q^{0-1} Q^\infty$ is nonsingular

$\Rightarrow$ $Q^{0-1} Q^\infty$ is similar to a Jordan form.

$$\begin{pmatrix} a & * & 0 \\ 0 & b & * \\ 0 & 0 & c \end{pmatrix}$$

$\Rightarrow$ To know the rank of Q^λ, we have only to consider the rank of $I + \lambda Q^{0-1} Q^\infty$.

$\Rightarrow$ $C (I + \lambda Q^{0-1} Q^\infty) C^{-1}$, for a proper nonsingular C

$$= I + \lambda \begin{pmatrix} a & * & 0 \\ 0 & b & * \\ 0 & 0 & c \end{pmatrix}$$

Since L is generic, a, b, c are distinct (and are nonzero, since $Q^{0-1} Q^\infty$ is nonsingular.).

$\Rightarrow$ Any curve in L has rank, at least, 2.