This correspondence reflects a basic aspect of the local analytic character of blow-ups: the infinitesimal behavior of functions, maps, or differential forms at the point x of M is transformed into global phenomena on $\tilde{M}$. Indeed, in classical terminology, a point in the exceptional divisor of the blow-up of M at x was called an "infinitely near point" of x; the exceptional divisor itself was called an "infinitesimal neihborhood" of x.

The next thing to do is to compute the curvature of the line bundles $[E]$ and $[-E]$ on $\tilde{M}$. We construct a metric on $[E]$ as follows: let h_1 be the metric on $[E]|_{\tilde{u}}$ given, in terms of the representation $(*)$ of E, by $|(l_1, \cdots l_n)|^2 = \|l\|^2$.

Let $\sigma \in H^0(\tilde{M}, \mathcal{O}([E]))$ be the above global section of $[E]$ on $\tilde{M}$ with $(\sigma) = E$, so that σ is nonzero on $\tilde{M} - E$; let h_2 be the metric on $[E]|_{\tilde{M}-E}$ given by $|\sigma(z)| \equiv 1$.

$\ulcorner$ $h_2(\sigma(z)) = 1$. $z \in \tilde{M} - E$, since $\sigma(z) \neq 0$.

$\tilde{M} = \tilde{M}_x$

Note that $[E]|_{\tilde{u}}$ can be extended to $\tilde{M}_x$, as follows;

$$\frac{\coprod_i \tilde{U}_i \times \mathbb{C} \amalg V \times \mathbb{C}}{((z,l)^{\in \tilde{U}_i}, \alpha) \sim ((z,l)^{\in \tilde{U}_j}, \frac{z_j}{z_i}\alpha) \quad ((z,l)^{\in \tilde{U}_i}, \alpha) \sim ((z,l)^{\in V}, z_i^{-1}\alpha)},$$ where $V \cup \tilde{u} - E = \tilde{M}_x - E$, V open in M.

$$V \xrightarrow{\sigma_0} \mathbb{C} \quad (z,l) \longmapsto 1 \qquad \sigma_i = z_i = g_{i0} 1 = g_{i0}\sigma_0$$

$$\tilde{U}_i \xrightarrow{\sigma_i} \mathbb{C} \quad (z,l) \longmapsto z_i \qquad g_{i0} = z_i \quad g_{j0} = z_j$$