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$(\bar{\partial} T)(\bar{\varphi})$

$\overline{\partial T(\varphi)} = \overline{T(\partial \varphi)} = T(\bar{\partial} \bar{\varphi}) \Rightarrow$ Since $\bar{A} \xrightarrow[\text{on } T_0]{p+n+1, n-p} A \xrightarrow[\text{on } T_0]{1-1} A^{n-p, n-p+1}$

under the conjugation, $\partial T = 0 \Leftrightarrow \bar{\partial} T = 0.$ \Rightarrow

The positivity of a current implies that it is order zero in the sense of distributions. For example, a current $T \in \mathcal{D}'(M)$ is locally written as

$$T = \frac{\sqrt{-1}}{2} \sum_{i,j} t_{ij} dz_i \wedge d\bar{z}_j,$$

a differential form with distribution coefficients defined by

$$t_{ij}(\alpha) = (-1)^{n+\bar{i}+j} (\alpha dz_1 \wedge \dots \wedge \hat{dz}_i \wedge \dots \wedge dz_n \wedge d\bar{z}_1 \wedge \dots \wedge \hat{d\bar{z}}_j \wedge \dots \wedge d\bar{z}_n).$$

$$\begin{aligned} \int T(\alpha dz_1 \wedge \dots \wedge \hat{dz}_i \wedge \dots \wedge dz_n \wedge d\bar{z}_1 \wedge \dots \wedge \hat{d\bar{z}}_j \wedge \dots \wedge d\bar{z}_n) \Phi(z) \wedge \bar{\Phi} \\ = \frac{\sqrt{-1}}{2} \sum_{i,j} t_{ij}(\alpha) dz_i \wedge d\bar{z}_j \wedge dz_1 \wedge \dots \wedge \hat{dz}_i \wedge \dots \wedge dz_n \wedge d\bar{z}_1 \wedge \dots \wedge \hat{d\bar{z}}_j \wedge \dots \wedge d\bar{z}_n \\ = \frac{\sqrt{-1}}{2} t_{ij}(\alpha) dz_i \wedge d\bar{z}_j \wedge dz_1 \wedge \dots \wedge \hat{dz}_i \wedge \dots \wedge dz_n \wedge d\bar{z}_1 \wedge \dots \wedge \hat{d\bar{z}}_j \wedge \dots \wedge d\bar{z}_n \\ = \frac{\sqrt{-1}}{2} t_{ij}(\alpha) (-1)^{\bar{i}} d\bar{z}_j \wedge \Phi(z) \wedge d\bar{z}_1 \wedge \dots \wedge \hat{d\bar{z}}_j \wedge \dots \wedge d\bar{z}_n \\ = \frac{\sqrt{-1}}{2} t_{ij}(\alpha) (-1)^{\bar{i}} (-1)^{n+j-1} \Phi(z) \wedge \bar{\Phi}(z) \\ = \frac{\sqrt{-1}}{2} t_{ij}(\alpha) (-1)^{n+\bar{i}+j-1} \Phi(z) \wedge \bar{\Phi}(z) \end{aligned}$$

$$\Rightarrow t_{ij}(\alpha) = (-1)^{n+\bar{i}+j-1} \frac{2}{\sqrt{-1}} T(\alpha dz_1 \wedge \dots \wedge \hat{dz}_i \wedge \dots \wedge dz_n \wedge d\bar{z}_1 \wedge \dots \wedge \hat{d\bar{z}}_j \wedge \dots \wedge d\bar{z}_n).$$

\Rightarrow

The current is real if $\bar{t}_{ij} = t_{ji}$ and positive if for