$$= -\frac{2-(\mu+\lambda)y'}{2y'\mu\lambda-(\mu+\lambda)} \quad \text{by L'Hospital's rule}$$

$\Rightarrow$
$$-2+(\mu+\lambda)y' = 2y'^2\mu\lambda-(\mu+\lambda)y'$$

$\Rightarrow \quad 2y'^2\mu\lambda - 2(\mu+\lambda)y' + 2 = 0$

$\Rightarrow \quad (y'\mu-1)(y'\lambda-1)=0 \qquad y' = \frac{1}{\mu}$ or $\frac{1}{\lambda}$.

$\Rightarrow$ The tangent directions to the curve C_f defined by f will be $\frac{1}{\mu}$ or $\frac{1}{\lambda}$. $\quad \mu\lambda = r$. r fixed. ⌟

Geometrically, the curve C_f must have an ordinary double point at p_i and the cross-ratio of its tangents together with those of C is prescribed.

⌜ I think, the general C_f has an ordinary double point at p_i and the cross-ratio of its tangents together with those of C is $r = \mu\lambda$. ⌟

We now write
$$C \cdot C' = \Gamma_0 + \Gamma,$$
where Γ_0 and Γ are zero-dimensional schemes with $\mathcal{J}_{\Gamma_0} = \mathcal{J}_{p_1} \cap \mathcal{J}_{p_2} \cap \mathcal{J}_{p_3}$ and Γ is the residue of Γ_0 relative to the pencil $|C+\lambda C'|$. We note that $\deg \Gamma_0 = 12$ and consequently $\deg \Gamma = 4$.

⌜ multiplicity at $p_i = 4$. $\Rightarrow 3 \times 4 = \deg \Gamma_0$
$\Rightarrow \deg C \cdot C' = \deg \Gamma_0 + \deg \Gamma = 16$ ⌟