To show that the left-hand side depends only on the image of $\bar{Z}$ under the natural projection

$$\bigoplus_{m=0}^{k+1}\left(\Lambda^{k+1-m}\Lambda^{\perp}\otimes \Lambda^{m}W^{*}\right)=\Lambda^{k+1}V^{*} \longrightarrow \Lambda^{k+1}W^{*},$$

we have only to prove that $\langle i(\bar{Z})\Lambda, w\rangle = 0$ for $\bar{Z}\in \Lambda^{k}W^{*}\otimes\Lambda^{\perp}$, $w\in W$, since we can see easily $\langle i(\bar{Z})\Lambda, w\rangle = 0$ for $\bar{Z}\in \Lambda^{m}W^{*}\otimes\Lambda^{n}\Lambda^{\perp}$ ($m+n=k+1$), $w\in W$.

By linearity, we may assume that $\bar{Z} = l\otimes w_1^{*}\wedge\cdots\wedge w_k^{*} = l\wedge w_1^{*}\wedge\cdots\wedge w_k^{*}$, (in $\Lambda^{k+1}V^{*}$).

$\Rightarrow \quad \langle i(l\wedge w^{*})\Lambda, w\rangle = \langle l\wedge w^{*}, \Lambda\wedge w\rangle$

For simplicity, $k=2$, and let

$$\Lambda = v_1\wedge v_2.$$

$\Rightarrow \langle l\wedge w^{*}, \Lambda\wedge w\rangle = \langle l\wedge w_1^{*}\wedge w_2^{*}, v_1\wedge v_2\wedge w\rangle$

$$= \det\begin{pmatrix} l(v_1), & l(v_2), & \overset{=0}{l(w)} \\ w_1^{*}(v_1), & w_1^{*}(v_2), & w_1^{*}(w) \\ w_2^{*}(v_1), & w_2^{*}(v_2), & w_2^{*}(w)\end{pmatrix} \qquad \text{since } l=0 \text{ on } W \ (\because\ l\in\Lambda^{\perp})$$

$$= -w_1^{*}(w)\det\begin{pmatrix} l(v_1), & l(v_2) \\ w_2^{*}(v_1), & w_2^{*}(v_2)\end{pmatrix} + w_2^{*}(w)\det\begin{pmatrix} l(v_1), & l(v_2) \\ w_1^{*}(v_1), & w_1^{*}(v_2)\end{pmatrix}$$

$$= -w_1^{*}(w)\langle l\wedge w_2^{*}, v_1\wedge v_2\rangle + w_2^{*}(w)\langle l\wedge w_1^{*}, v_1\wedge v_2\rangle$$

$$= -w_1^{*}(w)\langle l\wedge w_2^{*}, \Lambda\rangle + w_2^{*}(w)\langle l\wedge w_1^{*}, \Lambda\rangle$$

$$= -w_1^{*}(w)\langle \Lambda, l\wedge w_2^{*}\rangle + w_2^{*}(w)\langle \Lambda, l\wedge w_1^{*}\rangle$$

$$= -w_1^{*}(w)\langle i(l)\Lambda, w_2^{*}\rangle + w_2^{*}(w)\langle i(l)\Lambda, w_1^{*}\rangle \cdots \circledast$$

But by the definition of $\Lambda^{\perp} = \{v^{*}\in V^{*} \mid i(v^{*})\Lambda = 0\}$ and the fact that $l\in\Lambda^{\perp}$, $\circledast$ becomes zero,