is spanned by the vectors

$$\left\{ \pi_* \frac{\partial}{\partial X_i} \right\}_{i=0,\cdots,n}$$

with the single relation

$$\sum X_i \frac{\partial}{\partial X_i} = 0.$$

$\ulcorner$ Since $\pi_* \frac{\partial}{\partial X_i} = \frac{1}{X_0} \frac{\partial}{\partial x_i}$, $i=1,\cdots n$ &

$$\pi_* \frac{\partial}{\partial X_0} = -\sum \frac{X_i}{X_0^2} \frac{\partial}{\partial x_i},$$

$\{ \pi_* \frac{\partial}{\partial X_0}, \cdots \pi_* \frac{\partial}{\partial X_i} \}$ spans the tangent space $T_x'(\mathbb{P}^n)$ and $\{ \pi_* \frac{\partial}{\partial X_1}, \cdots, \pi_* \frac{\partial}{\partial X_n} \}$ " too.

For example, consider $\{ \pi_* \frac{\partial}{\partial X_0}, \pi_* \frac{\partial}{\partial X_1}, \pi_* \frac{\partial}{\partial X_2} \}$.

$$\pi_* \frac{\partial}{\partial X_0} = -\frac{X_1}{X_0^2} \frac{\partial}{\partial x_1} - \frac{X_2}{X_0^2} \frac{\partial}{\partial x_2}$$

$$\pi_* \frac{\partial}{\partial X_1} = \frac{1}{X_0} \frac{\partial}{\partial x_1}$$

$$\pi_* \frac{\partial}{\partial X_2} = \frac{1}{X_0} \frac{\partial}{\partial x_2}$$

$$\Rightarrow \pi_* \frac{\partial}{\partial X_0} + \frac{X_1}{X_0} \pi_* \frac{\partial}{\partial X_1} + \frac{X_2}{X_0} \pi_* \frac{\partial}{\partial X_2} = \pi_* \frac{\partial}{\partial X_0} + x_1 \pi_* \frac{\partial}{\partial X_1} + x_2 \pi_* \frac{\partial}{\partial X_2} = 0$$

$\lrcorner$

Now, recalling that the fiber of the hyperplane line bundle $H \longrightarrow \mathbb{P}^n$ over a point $x = \pi(X) \in \mathbb{P}^n$ corresponds to linear functionals on the line $\mathbb{C}\{X\} \subset \mathbb{C}^{n+1}$, we can define a bundle map

$$H^{\oplus(n+1)} = \overbrace{H \oplus \cdots \oplus H}^{n+1} \xrightarrow{\mathcal{E}} T'(\mathbb{P}^n)$$