

Thus all lines in Q pass through $[0,0,0,1]$.
 \Rightarrow But $L_1 \cap L'_1 = \emptyset \Rightarrow$ Contradiction to the fact that $L_1 \cap L'_1 \ni [0,0,0,1]$.

(iii) Q rank 2.

$$\Rightarrow Q = X_0^2 + X_1^2 \Rightarrow X_0 + iX_1 = 0 \cup X_0 - iX_1 = 0$$

$\Rightarrow Q$ is the union of two planes in $V_3 = \mathbb{P}^3$.

$\Rightarrow Q = V_2 \cup V'_2$, V_2, V'_2 are $\sigma_2(p)$ or $\sigma_{ii}(h)$ by the result on P757 \Rightarrow By the results on P759, any hyperplane containing $\sigma(p)$ or $\sigma(h)$ is tangent to G
 \Rightarrow Since $H \cap G = T_x(G) \cap G$, for some $x \in G$, is singular, this is impossible ($\because H \cap G$ is nonsingular).

(iv) Q rank 1 $\Rightarrow Q = X_0^2$, which is impossible since $Q = G \cap V_3 = (G \cap H) \cap H = (X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2) \cap H$.

$$\Rightarrow (X_0^2 + \dots + X_4^2 = 0) \cap (a_0 X_0 + \dots + a_4 X_4 = 0)$$

$$\Rightarrow \text{Assume } a_4 \neq 0. \Rightarrow a_0 X_0 + \dots + a_3 X_3 = X_4$$

$$\Rightarrow X_0^2 + \dots + (a_0 X_0 + \dots + a_3 X_3)^2 = 0 \Leftrightarrow (b_0 X_0 + \dots + b_3 X_3)^2 = 0$$

$$\Rightarrow 1 + a_i^2 = b_i^2 \text{ \& \> } a_i a_j = b_i b_j \Rightarrow a_i^2 = -\frac{1}{2}, b_i^2 = \frac{1}{2}$$

$$\Rightarrow (X_0 \pm X_1 \pm X_2 \pm X_3)^2 + (X_0 \pm X_1 \pm X_2 \pm X_3)^2 = 2X_0^2 + 2X_1^2 + 2X_2^2 + 2X_3^2 \text{ which is impossible.}$$

Thus $\{L_i\}$ & $\{L'_i\}$ span a 4-plane.

Second, if $\{L_i\}$ & $\{L'_i\}$ span another 4-plane H' , then $\{L_i\}$ & $\{L'_i\}$ lie in $H \cap H' = \mathbb{P}^3 \Rightarrow$ It is impossible by the arguments above.