equivalent to $c_1([D_1]) = c_1([D_2])$ by the proposition in Section 1 of Chapter 1, and shall show that $D_1 \equiv D_2$. Let $D_i^-$ be the part of $D_i$ appearing with negative coefficients and add $E = D_1^- + D_2^-$ of each of $D_1, D_2$ to obtain effective divisors, thereby reducing us to proving that $D_1 \equiv D_2$ for effective divisors in the same homology class.

We have only to prove that if $D_1 + E \equiv D_2 + E$, then $D_1 \equiv D_2$. Since $D_1 + E \equiv D_2 + E$, $\exists$ a divisor $D$ s.t $D_i + E + D$ effective and $D_1 + E + D \equiv D_2 + E + D$. $\Rightarrow$ Let $D' = E + D$. $\Rightarrow$ $D_i + D'$ effective and $D_1 + D' \equiv D_2 + D'$.

Now we come to the point. Recall that the Picard variety $Pic^\circ(M) = H^1(M, \mathcal{O})/H^1(M, \mathbb{Z})$ parametrizes line bundles with first Chern classe zero; we denote by $\{P_\xi \to M\}$ $(\xi \in H^1(M, \mathcal{O})/H^1(M, \mathbb{Z}))$ this family.

$$0 \longrightarrow \mathbb{Z} \longrightarrow \mathcal{O} \longrightarrow \mathcal{O}^* \longrightarrow 0$$

$$0 \to H^1(M, \mathbb{Z}) \longrightarrow H^1(M, \mathcal{O}) \longrightarrow H^1(M, \mathcal{O}^*) \xrightarrow{c_1} H^2(M, \mathbb{Z})$$

$$\Rightarrow 0 \to H^1(M, \mathbb{Z}) \longrightarrow H^1(M, \mathcal{O}) \longrightarrow \underset{\substack{\| \\ Pic^\circ(M)}}{\operatorname{Ker} c_1} \longrightarrow 0$$

see P313

$$\Rightarrow Pic^\circ(M) = H^1(M, \mathcal{O})/H^1(M, \mathbb{Z}).$$

$\Rightarrow$ $\xi \in H^1(M, \mathcal{O})/H^1(M, \mathbb{Z})$, and let $\{P_\xi \to M\}$ the family of line bundles with $c_1 = 0$.