$\Rightarrow \tau_1 = \tau_3$ by the second observation $\Rightarrow$ Contradiction

$\Rightarrow \eta_{12} \neq \eta_{24}$. $\Rightarrow$ ϕ is one to one on $\{F_{ij}\}$.

And as we saw above, $\eta_{ij} \neq \tau_i, l_j$.

$\Rightarrow$ ϕ is one to one, and ϕ is an automorphism, uniquely determined by the assignment $E_i \longrightarrow \tau_i$.

Thus there are $72 \cdot \#$ of permutations of τ_i's $= 72 \cdot 6! = 51840$ symmetries of the configuration of lines on S. ◢

2. If two of the lines L, L' on intersect, then there is a unique other line on S that intersects them both: the hyperplane in $\mathbb{P}^3$ containing L and L' must intersect S in a cubic curve including L and L', hence in a third line. In fact, the planes in $\mathbb{P}^3$ that meet S in a union of three lines are

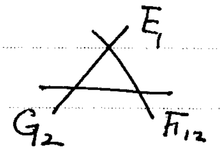
$$H_{ij} = \overline{E_i G_j F_{ij}} \quad \text{and} \quad H_{ijklmn} = \overline{F_{ij} F_{kl} F_{mn}}.$$

◸ Let H be the hyperplane containing L and L'.

$\Rightarrow$ $\iota_{\tilde{C}}^{-1}(H \cap S)$ is a curve which is equivalent to $\pi^* C - E_1 - \cdots - E_6$. $\Rightarrow$ $\iota_{\tilde{C}}(\pi^* C - E_1 - E_2 - \cdots - E_6)$ is a cubic curve since S is a cubic surface, and, for any line l in H, $\#(l \cap S) = 3 = \#(H \cap S \cap l) = \#((H \cap S) \cap l)$.

$\Rightarrow$ $H \cap S = L + L' + L''$, L'' a third line

E_1, G_2, F_{12} span a hyperplane.



$\Rightarrow$ Let $H_{12} = \overline{E_1 G_2 F_{12}}$.