that generate each $\mathcal{O}_z$ - module $\mathcal{F}_z$ $(z \in U)$, and if moreover the relations among these generators are finitely generated over U.

⊓

$$\mathcal{O}_{z_0}^{(p)} \xrightarrow{\beta_{z_0}} \mathcal{O}_{z_0}^{(q)} \xrightarrow{\alpha_{z_0}} \mathcal{F}_{z_0} \longrightarrow 0 \quad \text{exact.}$$

Given any $[(\sigma, U')] \in \mathcal{F}_{z_0}$. $\exists$ $[(\tau, V')]$ s.t

$\alpha_{z_0}[(\tau, V')] = [(\sigma, U')]$.

$\tau \in \mathcal{O}^{(q)}(V') = \underbrace{\mathcal{O}(V') \oplus \cdots \oplus \mathcal{O}(V')}_{q}$

$(0 \cdots, 1 \cdots 0) = e_i$. $\quad \alpha(e_i) = f_i$

$\Rightarrow$ Clearly, $\{f_i\}_{i=1}^{q}$ is a set of generators for $\mathcal{F}_{z_0}$.

Actually, $\{f_i\}_{i=1}^{q}$ generate for any $\mathcal{F}_z$.

The same is true of the relations, i.e.,

$\beta(e_i') \in \mathcal{O}^{(q)}$ forms a set of generators for the relations.

Conversely, since M is compact, without loss of generality, $M = U_1 \cup U_2$, U_i satisfying the conditions above. $\mathcal{F}|U_1 = \{f_1, \cdots f_{\ell_1}\}$ $\mathcal{F}|U_2 = \{g_1, \cdots g_{\ell_2}\}$.

$\Rightarrow$ Define. $\alpha : \mathcal{O}^{(\ell_1)} \longrightarrow \mathcal{F}$ by

$$\alpha|_U : \mathcal{O}^{(\ell_1)}(U) \longrightarrow \mathcal{F}(U)$$

$$e_i \longmapsto f_i|_{U_1}$$

Misunderstanding !