$\Rightarrow$ Since $(g_F^{-1} \circ g_G)^* = (\mathrm{id})^*$, $L(g_F^{-1} \circ g_G) = \sum_p (-1)^p \mathrm{tr}$ id.

$$\mathrm{id} : H^p_{DR}(\mathbb{P}^2) \longrightarrow H^p_{DR}(\mathbb{P}^2)$$
$$\begin{array}{l} \parallel \\ \mathbb{R} \quad \text{if } p = 0, 2, 4 \\ 0 \quad \text{otherwise.} \end{array}$$

$\Rightarrow$ $L(g_F^{-1} \circ g_G) = 3 > 0$ $\Rightarrow$ By the note on p421, $g_F^{-1} \circ g_G$ has a fixed point x. $\Rightarrow$ $g_F^{-1} \circ g_G(x) = x$ $\Rightarrow$ $g_G(x) = g_F(x) = T_x(G) = T_x(F)$ $\Rightarrow$ $x \in F \cap G$, and $T_x(F) + T_x(G) = T_x\mathbb{P}^5$ by the transversality $\Rightarrow$ $\times$.

⌟

Alternatively, we see by the Lefschetz theorem on hyperplane sections that the generator of

$$H^2(X, \mathbb{Z}) \overset{i^*}{\cong} H^2(G, \mathbb{Z}) \overset{i^*}{\cong} H^2(\mathbb{P}^5, \mathbb{Z})$$

is the restriction to X of the hyperplane class ω in $\mathbb{P}^5$; in particular, that every surface on X has even degree.

⌜ By p156, Lefschetz theorem, since $G \subset \mathbb{P}^5$ is homologous to $2H$, and $[G \cap F] = L$, where

$$\begin{array}{ccccc} i^*[2H] & \longrightarrow & i^*[2H] & \longrightarrow & [2H] \\ \downarrow & & \downarrow & & \downarrow \\ G \cap F & \longrightarrow & G & \overset{i}{\hookrightarrow} & \mathbb{P}^5 \end{array} \qquad i^*[2H] = L,$$

$[G \cap F]$ & $[G]$ are positive, and $H^2(X, \mathbb{Z}) \cong H^2(G, \mathbb{Z}) \cong H^2(\mathbb{P}^5, \mathbb{Z})$ ($\because$ $2 \leq 4-2$ and $5-2$). Since the generator of $H^2(\mathbb{P}^5, \mathbb{Z})$ is