

prove assertion 2, leaving 3 and 4 for the reader to carry out or look up in the references.

Given the solid arrow in the diagram

$$\begin{array}{ccccc} E_0 & \longrightarrow & M & \longrightarrow & 0 \\ \downarrow \mathbb{I}_0 & & \downarrow \varphi & & \\ F_0 & \longrightarrow & N & \longrightarrow & 0 \end{array}$$

the dotted arrow \mathbb{I}_0 exists by the definition of projective. Proceeding to the next step, if R_0 and S_0 are defined by

$$\begin{array}{ccccccc} 0 & \longrightarrow & R_0 & \longrightarrow & E_0 & \longrightarrow & M \longrightarrow 0 \\ & & \downarrow \mathbb{I}_0 & & \downarrow \mathbb{I}_0 & & \downarrow \varphi \\ 0 & \longrightarrow & S_0 & \longrightarrow & F_0 & \longrightarrow & N \longrightarrow 0 \end{array}$$

then what we have is the solid arrows in the diagram

$$\begin{array}{ccccc} E_1 & \longrightarrow & R_0 & \longrightarrow & 0 \\ \downarrow \mathbb{I}_1 & & \downarrow \mathbb{I}_0 & & \\ F_1 & \longrightarrow & S_0 & \longrightarrow & 0 \end{array}$$

and the dotted arrow fills in by projectivity. Continuing in this manner gives assertion 2. Q.E.D.

$$\begin{array}{ccccccc} \Gamma & E_2 & \longrightarrow & E_1 & \longrightarrow & E_0 & \longrightarrow M \longrightarrow 0 \\ & & & & & \downarrow \mathbb{I}_0 & \downarrow \varphi \\ & F_2 & \longrightarrow & F_1 & \longrightarrow & F_0 & \longrightarrow N \longrightarrow 0 \end{array}$$