

$$(b) \quad H_1 \neq H_2$$

We may assume  $H_1 = (X_0 = 0)$  &  $H_2 = (X_1 = 0)$

$$\Rightarrow F \cap H_1 = G \cap H_1 \Rightarrow F(0, X_1, X_2, X_3) = G(0, X_1, X_2, X_3)$$

$$F \cap H_2 = G \cap H_2 \Rightarrow F(X_0, 0, X_2, X_3) = G(X_0, 0, X_2, X_3)$$

Express  $F$  &  $G$  as follows:

$$F = X_0 (a_0 X_0 + a_2 X_2 + a_3 X_3) + X_1 (b_1 X_1 + b_2 X_2 + b_3 X_3) + c X_0 X_1 + F'(X_2, X_3)$$

$$G = X_0 (a'_0 X_0 + a'_2 X_2 + a'_3 X_3) + X_1 (b'_1 X_1 + b'_2 X_2 + b'_3 X_3) + c' X_0 X_1 + G'(X_2, X_3)$$

$$X_0 = 0$$

$$\Rightarrow F(0, X_1, X_2, X_3) = k G(0, X_1, X_2, X_3)$$

$$\Rightarrow \text{Assume } k = 1$$

$$\Rightarrow (b_1, b_2, b_3, c) = (b'_1, b'_2, b'_3, c') \text{ and } F' = G'$$

$$\Rightarrow F - G = X_0 (a''_0 X_0 + a''_2 X_2 + a''_3 X_3)$$

$$X_1 = 0 \Rightarrow 0 = X_0 (a''_0 X_0 + a''_2 X_2 + a''_3 X_3)$$

$$\Rightarrow a''_0 = a''_2 = a''_3 = 0$$

$\Rightarrow F = G \Rightarrow$  Contradiction to the fact that  $F$  &  $G$  are generic elements.

Thus the only case <sup>(iv)</sup>  $4=4$  remains.

$\Rightarrow F \cap G$  is irreducible and is of degree 4. //

$\Rightarrow$  Combining this with (2), we have  $F \cap G$  which is of degree 4 and is of genus 1, i.e.,  $F \cap G$  is an elliptic curve of degree 4. //

Let  $F_\lambda = F + \lambda G$ . If  $l \subset F_\lambda$  for some  $\lambda$ ,