$$\begin{array}{ccc} {}'E_1^{p,q} & \xrightarrow{d_1' = (-1)^p \iota(v)^*} & {}'E_1^{p+1,q} \\ \| & & \| \\ \dfrac{\operatorname{Ker}\delta \subset C^q(\underline{U}, \operatorname{Hom}_{\mathcal{O}}(\Omega^p, \Omega^n))}{\operatorname{Im}\delta} & & \dfrac{\operatorname{Ker}\delta}{\operatorname{Im}\delta} \end{array}$$

$$\begin{tikzcd}
C^q(\underline{U}, \operatorname{Hom}(\Omega^p, \Omega^n)) \arrow[r, "(-1)^p \iota(v)^*"] \arrow[d, "\delta"'] \arrow[dd] & C^q(\underline{U}, \operatorname{Hom}(\Omega^{p+1}, \Omega^n)) \arrow[d] \arrow[dd] \\
C^{q+1}(\underline{U}, \operatorname{Hom}(\Omega^p, \Omega^n)) \arrow[r, "(-1)^{p+1}\iota(v)^*"] \arrow[dd] & C^{q+1}(\underline{U}, \operatorname{Hom}(\Omega^{p+1}, \Omega^n)) \arrow[dd] \\
C^q(\underline{U}, \Omega^{n-p}) \arrow[r, "\iota(v)"] \arrow[d] & C^q(\underline{U}, \Omega^{n-p-1}) \arrow[d] \\
C^{q+1}(\underline{U}, \Omega^{n-p}) \arrow[r] & C^{q+1}(\underline{U}, \Omega^{n-p-1})
\end{tikzcd}$$

commute

$\Rightarrow$

$$\begin{tikzcd}
{}'E_1^{p,q} \arrow[r, "d_1'"] \arrow[d] & {}'E_1^{p+1,q} \arrow[d] \\
H^q(M, \Omega^{n-p}) \arrow[r, "\iota(v)"] & H^{n-p-1}(M, \Omega^q)
\end{tikzcd}$$

commute

Thus, we must prove that $\iota(v)$ induces zero as a map on cohomology. For a holomorphic 1-form $\varphi \in H^{1,0}(M)$, $\iota(v)\varphi$ is a holomorphic function on M that vanishes on $Z_{\neq\phi}$.

Γ $(\iota(v)\varphi)(p) = (\varphi_i v_i)(p)$ locally $\Rightarrow$ $v_i(p) = 0$ $\Rightarrow$ $(\iota(v)\varphi)(p) = 0$.
where $\varphi = \varphi_i dz_i$ $\quad v = v_j \frac{\partial}{\partial z_j}$

Since constants are only holomorphic functions on M, $\iota(v)\varphi = 0$, since $\iota(v)\varphi$ vanishes on $Z \neq \phi$.

Thus $\iota(v)\varphi = 0$. Using this we shall prove

Lichnerowicz' Lemma. $\iota(v)\omega = 0$ in $H^{0,1}(M)$.