

$$'E_{\infty}^{p,q} \Rightarrow \text{Ext}^{p+q}(M; \mathcal{F}, \mathcal{G}).$$

$$\sqcap \quad 'E_2^{p,q} = H^p(M, \mathcal{H}^q(\text{Hom}_{\mathcal{O}}(E(\mathcal{F}), \mathcal{G}))) \quad \text{by P446.}$$

$$\Rightarrow \text{Since } \mathcal{H}^q(\text{Hom}_{\mathcal{O}}(E(\mathcal{F}), \mathcal{G})) = \underline{\text{Ext}}_{\mathcal{O}}^q(\mathcal{F}, \mathcal{G}),$$

$$'E_2^{p,q} = H^p(M, \underline{\text{Ext}}_{\mathcal{O}}^q(\mathcal{F}, \mathcal{G})).$$

By the property of spectral sequence $'E_{\infty}^{p,q} \Rightarrow H^{p+q}(M, \text{Hom}_{\mathcal{O}}(E(\mathcal{F}), \mathcal{G})) = \text{Ext}_{\mathcal{O}}^{p+q}(M; \mathcal{F}, \mathcal{G}).$

More clearly, locally,

$$0 \rightarrow \mathcal{E}_n \rightarrow \mathcal{E}_{n-1} \rightarrow \dots \rightarrow \mathcal{E}_0 \rightarrow \mathcal{F} \rightarrow 0$$

$$\Rightarrow 0 \rightarrow \text{Hom}(\mathcal{F}, \mathcal{G}) \xrightarrow{\partial^*} \text{Hom}(\mathcal{E}_0, \mathcal{G}) \xrightarrow{\partial^*} \dots \rightarrow \text{Hom}(\mathcal{E}_n, \mathcal{G}) \rightarrow 0$$

$$\Rightarrow \text{We have } \underline{\text{Ext}}_{\mathcal{O}}^p(\mathcal{F}, \mathcal{G})_x = \mathcal{H}_x^p = \lim_{u \ni x} \frac{\ker \partial^*}{\text{Im } \partial^*} = \text{Ext}_{\mathcal{O}}^p(\mathcal{F}_x, \mathcal{G}_x) \quad (u) \quad (u)$$

$$\begin{aligned} \text{Actually, } \mathcal{H}_x^p &= \lim_{u \ni x} \mathcal{H}^p(u) = \lim_{u \ni x} \frac{\ker \{ \partial^*: \text{Hom}(\mathcal{E}_p, \mathcal{G})^u \rightarrow \text{Hom}(\mathcal{E}_{p+1}, \mathcal{G})^u \}}{\partial^* \text{Hom}(\mathcal{E}_{p-1}, \mathcal{G})(u)} \\ &= \frac{\ker \{ \partial^*: \varprojlim_{u \ni x} \text{Hom}(\mathcal{E}_p, \mathcal{G})(u) - \varprojlim_{u \ni x} \text{Hom}(\mathcal{E}_{p+1}, \mathcal{G})(u) \}}{\partial^* \varprojlim_{u \ni x} \text{Hom}(\mathcal{E}_{p-1}, \mathcal{G})(u)} = \text{Ext}_{\mathcal{O}}^p(\mathcal{F}_x, \mathcal{G}_x) \end{aligned}$$

Two applications of this spectral sequence will be useful. The first one is:

For \mathcal{E} a locally free sheaf on M ,

$$\text{Ext}^i(M; \mathcal{E}, \mathcal{G}) \cong H^i(M, \mathcal{E}^* \otimes_{\mathcal{O}} \mathcal{G}).$$

In particular, for any coherent sheaf \mathcal{F} ,

$$\text{Ext}^i(M; \mathcal{O}, \mathcal{F}) \cong H^i(M, \mathcal{F}).$$

$$\sqcap \quad \text{Ext}^i(M; \mathcal{O}, \mathcal{F}) \cong H^i(M, \mathcal{O}^* \otimes_{\mathcal{O}} \mathcal{F}) = H^i(M, \mathcal{F}), \text{ since}$$