Since $\psi(x_0) \neq 0$, without loss of generality, assume $a_{11}(x_0) \neq 0$. and $a_{11}(x) > 0$. for all $x \in \overline{V} \subset U$ and $x_0 \in V$.

Let $\rho(x) = 1$ on $\overline{V}$, and $\rho = 0$ on U^c.

Consider $s = a_{11}(x) \rho e_1$, $\sigma = \rho \sigma_1$ which are defined globally.

$$\Rightarrow \int_M \langle \psi, a_{11} \rho e_1 \otimes \rho \sigma_1 \rangle dx$$

$$= \int_M |a_{11}|^2 \rho^2 \, dx > 0 \quad \Rightarrow \quad \text{Contradiction}$$

Thus we can conclude that $\Gamma(E) \otimes \Gamma(F)$ is dense in $\Gamma(E \otimes F)$ w.r.t L^2-norm.

③ $\quad \eta = \eta_\alpha \otimes e_\alpha \in A^{p+1, q} \quad \tau = \tau_\alpha \otimes e_\alpha . \in A^{p,q}$

$$D'\eta = \partial \eta_\alpha \otimes e_\alpha + \Sigma(\eta_\alpha \wedge \theta'_{\alpha\beta}) \otimes e_\beta .$$

For convenience, we write $\theta_{\alpha\beta}$ instead of $\theta'_{\alpha\beta}$

"Note : If $\eta = \eta'_\alpha \otimes e'_\alpha$, $\tau = \tau'_\alpha \otimes e'_\alpha$, where $\eta_\alpha = \eta'_\beta g_{\beta\alpha}$, $e'_\alpha = g_{\alpha\beta} e_\beta$, ${}^t\overline{g} = g^{-1}$, $\theta' = (\partial g) g^{-1} + g \theta g^{-1}$.

then $\eta_\alpha \wedge * \tau_\alpha = \eta'_\beta g_{\beta\alpha} \wedge * \tau'_\gamma g_{\gamma\alpha}$

$= \eta'_\beta g_{\beta\alpha} \wedge \overline{g}_{\gamma\alpha} * \tau'_\gamma = \eta'_\beta g_{\beta\alpha} \overline{g}_{\gamma\alpha} \wedge * \tau'_\gamma$

$= \eta'_\beta (g \, {}^t\overline{g})_{\beta\gamma} \wedge * \tau'_\gamma = \eta'_\beta \delta_{\beta\gamma} \wedge * \tau'_\gamma$

$= \eta'_\beta \wedge * \tau'_\beta \quad \Rightarrow \sum_\alpha \eta_\alpha \wedge * \tau_\alpha$ is well-defined global differential form"