5. The final possibility is $L_1 = L_2 = L$. This can occur only when L is a special line and

$$V_2(r) = \bigcap_{x \in L} T_x(X).$$

☞ $L_1 = L_2 = L \Rightarrow V_2(r) \cap X = 2L \Rightarrow V_2(r)$ is tangent to X at all $x \in L$. $\Rightarrow V_2(r) \subset \bigcap_{x \in L} T_x(X)$
$\Rightarrow$ By P979 & P981 note, $\bigcap_{x \in L} T_x(X)$ is of dim 2.

$\Rightarrow V_2(r) = \bigcap_{x \in L} T_x(X)$.

In this case, $V_2(r)$ contains a normal vector to $L \subset X$ at each point of L, and the map $\tilde{f}_L$ sends the curve consisting of these normal vectors to the point r.

☞ $V_2(r)/L$ represents a vector at each point $x \in L$.
$\Rightarrow$ If we let $V_2(r)/L$ (at x) $= (x, \eta_x)$, then $\overline{L, \eta_x} = V_2(r)$.
$\Rightarrow (\overline{L, \eta_x} = V_2(r)) \cap V_3 = \tilde{f}_L((x, \eta_x)) = \tilde{f}_L(\eta_x) = r$.
$\{(x, \eta_x)\}$ is a curve in $\pi^{-1}(L)$.

Suppose now that the line L of projection is not a special line. Then by the above, the map $\tilde{f}_L$ is one-to-one away from the proper transforms in $\tilde{X}_L$ of the lines on X meeting L, and maps each of