$$\Rightarrow \iota(x) = \left\langle \begin{array}{c|l} & \qquad\qquad\qquad\qquad \ell \qquad\qquad\quad i \\ & (1, 0, \cdots\cdots 0 \ * \ \cdots \ * \qquad\qquad *) \\ & (0, 1 \cdots\cdots 0 \ * \ \cdots \ * \qquad\qquad *) \\ & \vdots \\ & (0, 0, \cdots\cdots 1, * \ \cdots \ * \qquad\qquad *) \\ \ell+1 & (0, 0, \cdots\cdots 0 \ 0 \quad 0 \ * \ * \ \cdots \ *) \\ & \vdots \\ n & (0, 0, \cdots\cdots 0. 0, \cdots 0. *, * \cdots\cdots *) \end{array} \right\rangle$$

In general,

$$\iota(x) = \left\langle \begin{array}{c|l} & \quad \textcircled{1} \quad \textcircled{2} \cdots \textcircled{\ell} \qquad i \\ & (* \cdots 1 \ * \cdots 0 \ * \cdots 0 \ * \quad * \ * \qquad\qquad *) \\ & (* \quad 0 \ * \ 1 \qquad 0 \qquad\qquad\qquad\qquad *) \\ & \vdots \\ & (* \quad 0 \ * \ 0 \qquad 1 \quad\ \ * \ * \qquad\qquad *) \\ \ell+1 & (0, 0, 0 \cdots\cdots 0 \ 0 \cdots 0, *, * \cdots\cdots *) \\ & \vdots \\ n & (0, 0, 0 \cdots\cdots 0. 0, \cdots 0, *, *, \cdots\cdots *) \end{array} \right\rangle \quad \textcircled{*}$$

$$\Lambda \cap V_{n-k+1-j} = \Lambda \cap \{ e_{k+j}, e_{k+j+1}, \cdots e_n \}$$

$$\dim \left\langle \begin{array}{c|l} \ell+1 & (0 \qquad\quad 0 * \cdots *) \\ & \vdots \\ n & (0 \ \cdots \quad 0 * \cdots *) \end{array} \right\rangle = k - \ell \cdots \textcircled{**}$$

By elementary row operations, we can make entries between $i+1$ and $k+j-1$ zero as many as possible.

$\Rightarrow$ The possible maximal # of nonzero entries is $k-i+j-1$.

$$\begin{array}{l} \qquad\qquad\qquad\qquad \sim k-i+j-1 \\ (0 \cdots\cdots 0, 1 . \ 0 \ 0 * * \quad *) \\ \vdots \qquad\qquad\qquad \quad * \\ (\qquad\qquad\qquad 0 \quad 1 \ * \cdots) \end{array}$$

$$\Rightarrow \dim(\Lambda \cap V_{n-k+1-j}) \geq k - \ell - (k-i+j-1) = -\ell + i - j + 1 \geq 1$$

In the same way, we get

$$\dim(\Lambda \cap V_{n-k+p-j}) \geq k - \ell - (k+j-\cancel{p}^{\,-i}\,p) = -\ell + i - j + p \geq p$$

for $p \leq k-i+1$.