$$\begin{aligned}\mathrm{Ext}^n(M;\, \mathcal{O}_Z, \Omega^n) &= \mathbb{H}^n(M, \Omega^{n-}) \\ &= {}'E_\infty^{0,n} \oplus {}'E_\infty^{1,n-1} \oplus {}'E_\infty^{2,n-2} \oplus \cdots \oplus {}'E_\infty^{n,0} \\ &= {}'E_1^{0,n} \oplus {}'E_1^{1,n-1} \oplus {}'E_1^{2,n-2} \oplus \cdots \oplus {}'E_1^{n,0} \\ &= H^n(M, \Omega^n) \oplus H^{n-1}(M, \Omega^{n-1}) \oplus \cdots \oplus H^0(M, \Omega^0)\end{aligned}$$

$\Rightarrow \dim \mathrm{Ext}^n(M;\, \mathcal{O}_Z, \Omega^n) = \dim H^n(M, \Omega^n) + \dim H^{n-1}(M, \Omega^{n-1}) + \cdots + \dim H^0(M, \Omega^0) = h^{n,n} + h^{n-1,n-1} + \cdots + h^{0,0} = \sum h^{p,p}$

$\Rightarrow \deg(Z) = \sum_p h^{p,p}$ $\lrcorner$

Since $h^{p,q}(M) = 0$ for $p \neq q$ the left-hand side is just the topological Euler characteristic, what we have is just a special case of the Hopf index theorem.

$\ulcorner$ $\sum h^{p,q} = \sum \dim H^{p,q}(M)$

$\chi(M) = \sum_\ell (-1)^\ell \dim H^\ell(M) = \sum h^{p,p}(M) = \sum \dim H^{p,q}(M)$

since if ℓ odd, $\dim H^\ell(M) = 0$ ($\because \dim H^{\ell-x,x}(M) = 0$ since $\ell - x \neq x$.).

$\chi(M) = \deg Z \Rightarrow$ This tells more because it allows multiplicity, on the contrary the definitions on P127 & P122 don't. Topologically, it is a special case of the theorem. $\lrcorner$

More substantial applications, including a proof of Bott's residue formula, arise by keeping track of the filtrations induced by the spectral sequences. These are given in the paper of Carrell and Liebermann.