Pf

$$ds^2 = \sum_{i,j} (\delta_{i\bar{j}} + [1]) \, dz_i \otimes d\bar{z}_j$$
$$= \sum \varphi_i \otimes \overline{\varphi_i}.$$

$$h\left(\frac{\partial}{\partial z_i}, \frac{\partial}{\partial z_i}\right) = \sum \varphi_j\left(\frac{\partial}{\partial z_i}\right) \overline{\varphi_j\left(\frac{\partial}{\partial z_i}\right)} = \sum_j \left|\varphi_j\left(\frac{\partial}{\partial z_i}\right)\right|^2 = h_{i\bar{i}}$$

$$\varphi_i = \sum a_{ji} \frac{\partial}{\partial z_j}$$

$$\varphi_i\left(\frac{\partial}{\partial z_j}\right) = a_{ji} \qquad a_{ij} = \varphi_j\left(\frac{\partial}{\partial z_i}\right)$$

$$h\left(\frac{\partial}{\partial z_i}, \frac{\partial}{\partial z_j}\right) = \sum_k \varphi_k\left(\frac{\partial}{\partial z_i}\right) \overline{\varphi_k\left(\frac{\partial}{\partial z_j}\right)} \qquad (a_{ij}) = A$$

$$= \sum a_{ik} \overline{a_{jk}} = (A\,{}^t\bar{A})_{i\bar{j}} = h_{i\bar{j}}$$

$$\Rightarrow \quad h = A\,{}^t\bar{A}$$
$$dh = dA\,{}^t\bar{A} + A\,d\,{}^t\bar{A}$$
$$\Rightarrow \text{At } z_0, \quad dh(z_0) = 0 = dA(z_0)\,\underbrace{{}^t\bar{A}(z_0)}_{=I} + A(z_0)\, d\,{}^t\bar{A}(z_0)$$
$$= dA(z_0) + d\,{}^t\bar{A}(z_0) = 0 \quad \text{때!}$$

Construct. them as follows;

$$\varphi_1 = \left(\frac{1}{h_{11}}\right)^{\frac{1}{2}} \frac{\partial}{\partial z_1}$$

$$\varphi_2' = a \frac{\partial}{\partial z_2} - \varphi_1$$

$$\langle \varphi_2', \varphi_1 \rangle = \langle a \frac{\partial}{\partial z_2}, \varphi_1 \rangle - 1 = 0$$

$$\Rightarrow a\, h_{11}^{-\frac{1}{2}} h_{21} = 1 \qquad a = h_{11}^{\frac{1}{2}} h_{21}^{-1}$$

$$\Rightarrow \quad \varphi_2' = h_{11}^{\frac{1}{2}} h_{21}^{-1} \frac{\partial}{\partial z_2} - \varphi_1$$

$\Rightarrow$ Let $\varphi_2 = \dfrac{\varphi_2'}{\|\varphi_2'\|}$. .. Continue this Gram-Schmidt process. $\Rightarrow$ As we can see $d\, h_{11}^{-\frac{1}{2}} = h_{11}^{-1} d h_{11} = 0$ at z_0 ($\Rightarrow d\varphi_1(z_0) = 0$)

since $dh_{i\bar{j}}(z_0) = 0$. for all i, j. $\Rightarrow d\varphi_i = 0$ at z_0.