by $\mathcal{O}$-linearity and the usual boundary formula

$$\partial(e_J) = \sum_{\nu=1}^{k} (-1)^{\nu-1} f_{j_\nu} e_{j_1} \wedge \cdots \wedge \hat{e}_{j_\nu} \wedge \cdots \wedge e_{j_k}.$$

For $k=1$ we set $E_0 = \mathcal{O}$ and $\partial(e_i) = f_i$.

$$E_0 = \mathcal{O} \otimes_{\mathbb{C}} \Lambda^0 \mathbb{C}^r = \mathcal{O} \otimes_{\mathbb{C}} \mathbb{C} \cong \mathcal{O}.$$

This defines the Koszul complex $E.(f)$ for any set of functions $f = (f_1, \cdots f_r)$, and we have the

Lemma. In case $(f_1, \cdots f_r)$ is a regular sequence,

$$H_q(E.(f)) = 0 \quad \text{for } q > 0$$

and

$$H_0(E.(f)) \cong \mathcal{O}/I.$$

Consequently, $E.(f)$ gives a projective resolution of $\mathcal{O}/I$.

$$E_k \longrightarrow E_{k-1} \longrightarrow \cdots \longrightarrow E_0 = \mathcal{O}$$

$\Rightarrow$

$$H_0(E.(f)) = \mathcal{O}/\partial E_1 \cong \mathcal{O}/I$$

$$E_2 \xrightarrow{\partial} E_1 \xrightarrow{\partial} \underset{\mathcal{O}}{\overset{\|}{E_0}} \xrightarrow{\pi} \mathcal{O}/I \longrightarrow 0 \quad \cdots (*)$$

$\ker \pi = I \subset E_0$. $\partial E_1 = I \Rightarrow (*)$ is a projective resolution for $\mathcal{O}/I$. We should prove $\partial E_1 \subset I$