By Bertini's, away from L_a, generic element $H \cap X$ is smooth and transverse intersection.

Assume $L_a = \{[*, *, 0, 0, 0, 0]\}$

$$\begin{array}{ccc} L_a & \longrightarrow & G(4,6) \\ \Psi & & \Psi \\ x & \longmapsto & T_x X \end{array}$$

$\Rightarrow$ If $H \supset L_a$, then $H = (a_2 X_2 + \cdots + a_5 X_5 = 0)$.

Suppose $H \supset T_x X$ ($\Rightarrow H \cap X$ is singular at x).

$\Rightarrow$ Since $T_x X = T_x F \cap T_x G$, $\nabla F(x) = (\frac{\partial F}{\partial X_0}(x), \cdots \frac{\partial F}{\partial X_5}(x))$, $\nabla G(x) = (\frac{\partial G}{\partial X_0}(x), \cdots \frac{\partial G}{\partial X_5}(x))$ and $(0, 0, a_2, a_3, \cdots a_5)$ are linearly independent. $\Rightarrow [(0, 0, a_2, a_3, a_4, a_5)]$ $\in \langle \nabla F(x), \nabla G(x) \rangle = l_x$, which is a line.

$\Rightarrow \{l_x\}_{x \in L_a}$ is a variety of dim 2 at most.

But $\{[0, 0, a_2, a_3, \cdots a_5]\}$ is of dim 3.

$\Rightarrow$ For generic H, $H \cap X$ is smooth on L_a.

$\lrcorner$

But we have seen in Section 4 of Chapter 4 that any smooth intersection of two quadrics in $\mathbb{P}^4$ contains exactly 16 lines, so that the Schubert cycle $\sigma_{1,1}(V_4) \subset G(2,6)$ will meet A in 16 distinct points, including a.

$\ulcorner$ See P550-P551. $H \cap X$ is smooth $\Rightarrow$ $H \cap G$ & $H \cap F$ are smooth quadrics in $H = \mathbb{P}^4$.

$\sigma_{1,1}(V_4) = \{\Lambda \mid \dim(\Lambda \cap V_{4+i-a_i}) \geq i\} = \{L \mid L \text{ line } L \subset V_4\}$ $\Rightarrow$ $\sigma_{1,1}(V_4) \cap A =$ Set of lines in $F \cap G$,