$$\begin{array}{ccc} (p, \Lambda_n) & \in I \subset F \times G(n+1, 2n+3) \\ \downarrow & \downarrow \pi_2 \\ \Lambda_n & \in \Sigma_n' \subset G(n+1, 2n+3) \end{array}$$ and $\Lambda_n \cong \mathbb{P}^n$

On the other hand, consider the fibers of the projection $\pi_1 : I \longrightarrow F$, that is, the n-planes on F passing through a point p.

$$\begin{array}{c} I \subset F \times G(n+1, 2n+3) \\ \downarrow \pi_1 \\ p \in F \end{array} \Rightarrow \pi_1^{-1}(p) = \{ (p, \Lambda) \mid p \in \Lambda, \ \Lambda \text{ n-plane on } F \}$$

Clearly, any such n-plane Λ lies in the tangent plane to F at p, and hence in the intersection $F \cap T_p(F)$.

Quite clear Since $\Lambda \subset F$, $T_p(F) \supset \Lambda$, and $\Lambda \subset F \cap T_p(F)$.

But we have seen that $F \cap T_p(F)$ is just the cone through p over a smooth quadric $\tilde{F}_{2n-1} \subset \mathbb{P}^{2n}$, and so the n-planes in F through p are exactly the n-planes spanned by p together with $(n-1)$-planes in $\tilde{F}$.