We may find a U' s.t φ is holomorphic in U' and is the image of a class $\Phi \in H^p_{DR}(M)$.

Since φ is a differential of the second kind, $\exists$ a divisor D_0 s.t if $D' \supset D_0$, then φ has no residues in $M-D'$. Consider $D'' = D + D_0$, where D is a divisor s.t $U = M - D$ is affine. $\Rightarrow$ By the argument above, $M-D''$ is also affine. Since φ is holomorph in $M-D_0$ and φ is the image of $H^p_{DR}(M-D_0) \leftarrow H^p_{DR}(M)$,

$$\begin{tikzcd} H^p_{DR}(M-D'') & H^p_{DR}(M-D_0) \arrow[l] \\ & H^p_{DR}(M) \arrow[u] \arrow[ul] \end{tikzcd}$$

$\varphi \in H^p_{DR}(M-D'')$, $\quad \Phi \in H^p_{DR}(M)$

Let $U' = M - D''$.

In the diagram

$$\begin{tikzcd} H^p_{DR}(M) \arrow[r] \arrow[dr] & H^p_{DR}(U', alg) \\ & H^p_{DR}(U, alg) \arrow[u] \end{tikzcd}$$

the restriction of Φ to U will be represented by a closed p-form η that is meromorphic on M and holomorphic in U.

$\Phi|U \in H^p_{DR}(U) \cong H^p_{DR}(U, alg)$
$\Rightarrow \Phi|U - \eta = d\alpha$, η closed meromorphic p-form on M and holomorphic in U. (α meromorphic $(p-1)$-form on M and holomorphic in U).