

$$\begin{aligned} \mathbb{F} \quad \text{Hom}(E_p(L), M) \otimes \text{Hom}(E_q(M), N) &\longrightarrow \text{Hom}(E_{p+q}(L), N) \\ (\sigma_p, \tau_q) &\longmapsto \sigma_p \cup \tau_q \\ \Rightarrow \sigma \cup \tau(x) &\text{ is defined as follows: } x \in E_{p+q}(L) \end{aligned}$$

$$\begin{array}{ccccc} E_{p+q}(L) & \xrightarrow{\partial} & E_{p+q}(L) & \xrightarrow{\partial} & E_p(L) & \xrightarrow{\partial} & E_{p-1}(L) \\ \downarrow \sigma_{p+q} & & \downarrow \sigma_{p+q} & & \downarrow \sigma_p & & \\ E_q(M) & \xrightarrow{\partial} & E_{q-1}(M) & \xrightarrow{\partial} & E_0(M) & \rightarrow & M \rightarrow 0 \\ \tau \downarrow & & \downarrow \sigma_{p+q}(x) & & & & \\ N & & & & & & \end{array}$$

$$\Rightarrow \exists y \in E_q(M) \text{ s.t. } \partial(y) = \sigma_{p+q}(x) \Rightarrow \sigma \cup \tau(x) = \tau(y).$$

To check well-definedness, first. if another $y' \in E_q(M)$ s.t. $\partial(y') = \sigma_{p+q}(x)$. \Rightarrow By exactness, $\exists \delta$ s.t. $y - y' = \partial\delta$.

$$\begin{aligned} \Rightarrow \tau(y) &= \tau(y' + \partial\delta) = \tau(y') + \tau(\partial\delta) = (\partial\tau)(\delta) + \tau(y') \\ &= \tau(y'). \text{ since } \tau \in \text{Ext}^q(M, N), \text{ and so } \partial\tau = 0. \end{aligned}$$

Second, we need to show the independence of choice of σ_{p+q} . Problem! Since $\sigma_p \circ \partial = \partial^* \sigma_p = 0$, we may take $\sigma_{p+1} = 0$, which is meaningless.

There is another interpretation on $\text{Ext}_\Lambda^n(M, M')$.

See P35 ~ P37. Representations and cohomology by D. Benson.

Def: If M and N' are left Λ -modules, an n -fold extension of M by M' is an exact sequence

$$0 \rightarrow M' \rightarrow M_{n-1} \rightarrow M_{n-2} \rightarrow \dots \rightarrow M_0 \rightarrow M \rightarrow 0$$

beginning with M' and ending with M , and with n intermediate terms.