space of definition $Z = X \times Y$ is the product of two varieties, the changes of local coordinates being then of the type

$$x'_k = \psi_k(x_1^*, \cdots x_p^*) \,;\; y'_j = \phi_j(y_1^*, \cdots y_q^*). \qquad (3)$$

Then

$$d_x d_y V = \sum \frac{\partial^2 V}{\partial x_p \partial y_q} dx_p\, dy_q$$

is invariant, as a bilinear form, under the transformation (3).

In the case of a complex analytic variety W^n, we have on each coordinate patch coordinates $z_k, \bar{z}_k$ and the change of coordinates

$$z'_k = \psi_k(z_1, \cdots z_n) \qquad \bar{z}'_k = \bar{\psi}_k(\bar{z}_1, \cdots \bar{z}_n) \qquad (4)$$

are of type (3), the function ψ_k being holomorphic functions. The bilinear form

$$L(V) = d_z d_{\bar{z}} V$$

is invariant under (4) and the condition $L(V) \geq 0$ is independent of the choice of the local coordinates on W^n.

Γ $\quad L(V) = \partial \bar{\partial} V$

$$\begin{tikzcd} & W \arrow[r, "V"] \arrow[dl, "\psi"'] \arrow[d, "\phi"] & \mathbb{C} \\ \mathbb{C}^n & \mathbb{C}^n & \end{tikzcd}$$

$(z_1, \cdots z_n) \qquad (z'_1, z'_2, \cdots z'_n).$

$$\phi_k \circ \psi^{-1}(z_1, z_2, \cdots z_n) = z'_k$$

$$\bar{\partial} V = \frac{\partial V}{\partial \bar{z}'_j} d\bar{z}'_j = \frac{\partial V}{\partial \bar{z}_l} \frac{\partial \bar{z}_l}{\partial \bar{z}'_j} \frac{\partial \bar{z}'_j}{\partial \bar{z}_k} d\bar{z}_k \quad \text{since } d\bar{z}'_j = \frac{\partial \bar{z}'_j}{\partial \bar{z}_k} d\bar{z}_k$$

$$\left(\because \frac{\partial z_p}{\partial \bar{z}_k} = 0\right).$$