$$\Rightarrow \sum_\alpha \int_{U_\alpha} \phi_\alpha \psi \wedge * \Delta \eta = \sum_\alpha \int_{U_\alpha} \phi_\alpha \psi \wedge * \sum_I (C \Delta_d \eta_I + \text{lower ord.}) \, dx_I$$

$$= \sum_\alpha \int_{U_\alpha} \phi_\alpha \varphi \wedge * \eta = \sum_\alpha \int_{U_\alpha} \phi_\alpha \varphi \wedge * \sum_I \eta_I \, dx_I$$

$$\Rightarrow \sum_\alpha \int_{\mathbb{R}^n} \phi_\alpha \Sigma \psi_J dx_J \wedge \sum_I (C \Delta_d \eta_I + \text{lower order}) \, dx_I \circ f_I$$

$$= \sum_\alpha \int_{\mathbb{R}^n} \phi_\alpha \Sigma \varphi_J dx_J \wedge \sum_I \eta_I \, dx_I \circ f_I$$

where f_I comes from $|\langle dx_i, dx_j \rangle|_{i,j \in I}$ ← determinat.

$\Rightarrow$ Lets simplify this whole thing.

$$\Rightarrow \int_{\mathbb{R}^n} \phi \Sigma \psi dx_J \wedge C (\Delta_d \eta + \text{lower order derivatre}) \, dx_I \circ f.$$

$$= \int_{\mathbb{R}^n} \phi \Sigma \varphi_J dx_J \wedge \eta \, dx_I \circ f.$$

$$\Rightarrow \int_{\mathbb{R}^n} \psi \wedge \phi (C \Delta_d + \text{lower order derivatives}) \eta \, dx_I$$

$$= \int_{\mathbb{R}^n} \phi \varphi \wedge \eta \, dx_I = \int_{\mathbb{R}^n} \varphi \wedge \phi \eta \, dx_I$$

Let $P = C \Delta_d + \text{lower order derivatives}$

$$\Rightarrow \int_{\mathbb{R}^n} \phi \psi \wedge (P \eta) \, dx_I = \int_{\mathbb{R}^n} \phi \varphi \wedge \eta \, dx_I.$$