2. By $\bar\partial$-Poincare lemma,

$$\underset{\overset{\|}{\Omega^0}}{H^q(\mathbb{C}^n, \mathcal{O})} = H^{0,q}_{\bar\partial}(\mathbb{C}) \doteq 0 \quad \text{for } q>0.$$

and more generally, $H^q((\mathbb{C})^k \times (\mathbb{C}^*)^\ell, \mathcal{O}) = 0$ for $q>0$.

Since $\mathbb{C}^n$ is contractible, $\check{H}^q(\mathbb{C}^n, \mathbb{Z}) = 0$. for $q>0$.

$$\check{H}^q(\mathbb{C}^n, \mathbb{Z}) = H^q_{\text{simp}}(\mathbb{C}^n, \mathbb{Z})$$

Now, from the long exact sequence associated to the exponential sheaf sequence on $\mathbb{C}^n$, $(0 \to \mathbb{Z} \to \mathcal{O} \to \mathcal{O}^* \to 0)$

$$\underset{0}{H^q(\mathbb{C}^n, \mathcal{O})} \longrightarrow H^q(\mathbb{C}^n, \mathcal{O}^*) \longrightarrow \underset{0}{H^{q+1}(\mathbb{C}^n, \mathbb{Z})} \longrightarrow \quad \text{is exact,}$$

$\Rightarrow H^q(\mathbb{C}^n, \mathcal{O}^*) = 0$ for $q>0$.

As an immediate consequence, we have the answer to the following problem:

Cousin problem: Any analytic hypersurface in $\mathbb{C}^n$ is the zero locus of an entire function.

pf) From P12, we know that, in a nbd of any point $p \in \mathbb{C}^n$, an analytic hypersurface $V \subset \mathbb{C}^n$ may be given as the zero locus of a holomorphic function $f \in \mathcal{O}_p$.

Choose f not divisible by the square of any nonunit in $\mathcal{O}_p$, which implies, f is unique <u>up to multiplication by a unit.</u>
($\mathcal{O}_p$ is UFD)

$f = f_1^2 f_2 \qquad g = f_1 f_2 \quad \Rightarrow \quad \{f=0\} = \{g=0\}$
where f_1 is not a unit in $\mathcal{O}_p$.

Thus we can find a cover $\underline{\mathcal{U}} = \{U_\alpha\}$ of $\mathbb{C}^n$ and