top and bottom by powers of X_0, we may write the function φ as

$$\varphi(x_1, x_2, \cdots x_n) = \frac{f(x_1, x_2, \cdots x_n)}{g(x_1, x_2, \cdots x_n)},$$

where f and g are polynomials (not necessarily both of degree d) in the Euclidean coordinates x_i.

$$X_i / X_0 = x_i$$

Example. $F(X_0, X_1) = X_0^{\,2} X_1^{\,2}$, $G(X_0, X_1) = X_0^{\,4} + X_1^{\,4}$

$$\Rightarrow f(x_1) = \left(\frac{X_1}{X_0}\right)^2 = x_1^{\,2} \qquad g(x_1) = 1 + \left(\frac{X_1}{X_0}\right)^4 = 1 + x_1^{\,4}$$

Thus the field $K(\mathbb{P}^n)$ of rational functions on $\mathbb{P}^n$ is isomorphic to $\mathbb{C}(x_1, x_2, \cdots x_n)$.

For the def. of $\mathbb{C}(x_1, x_2, \cdots x_n)$, see P231 ~ P232 Hungerford.

$$\begin{array}{ccc} \varphi(X) & \longmapsto & \varphi(x_1, x_2 \cdots x_n) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ K(\mathbb{P}^n) & \longrightarrow & \mathbb{C}(x_1, x_2 \cdots x_n) \end{array}$$

See P166.

It is not hard to see that any meromorphic function on $\mathbb{P}^n$ is rational. By Chow's theorem, both the zero-divisor $(\varphi)_0$ and polar divisor