

(Note that L' positive implies that L is positive).

$$\int \eta_{L'} = n \int \eta_L \quad \text{by P139.}$$

$$c_1(L') = n c_1(L).$$

Since $c_1(L')$ is represented by a positive form $n \cdot \frac{\sqrt{-1}}{2\pi} \Theta$, by the proposition on P148, L' is positive. \square

By hypothesis, $H^2(M, \mathbb{C}) = \mathbb{C}$ and since $H^{1,1}(M, \mathbb{C}) = \mathbb{C}$, we have $h^{2,0}(M) = 0$; $b^1(M) = 0$ implies $h^{1,0}(M) = 0$ and hence $\chi(\mathcal{O}_M) = 1$.

$$\int H^2(M, \mathbb{C}) = H^{2,0}(M) \oplus H^{0,2}(M) \oplus H^{1,1}(M, \mathbb{C}) = \mathbb{C}$$

$\Rightarrow H^{2,0}(M) = H^{0,2}(M) = 0$ by the previous argument, and $H^{1,1}(M, \mathbb{C}) = \mathbb{C}$.

$$\chi(\mathcal{O}_M) = h^{0,0}(M) - h^{0,1}(M) + h^{0,2}(M) = h^{0,0}(M) = 1 \quad \square$$

The topological Euler characteristic $\chi(M) = 3$, and so by Noether's formula

$$1 = \chi(\mathcal{O}_M) = \frac{K_M \cdot K_M + \chi(M)}{12} \Rightarrow K_M \cdot K_M = 9.$$

$$\int 12 = K_M \cdot K_M + 3 \Rightarrow K_M \cdot K_M = 9 \quad \square$$

Since $c_1(L)$ generates $H^2(M, \mathbb{Z})$, by Poincaré duality

$$L \cdot L = (c_1(L) \cup c_1(L)) [M] = \pm 1,$$

and since L^k is effective for $k \gg 0$ and L is positive,

$$L \cdot L^k = k(L \cdot L) > 0 \Rightarrow L \cdot L = 1.$$