

above line 11 on P208, \exists a smooth hyperplane section, since we can find a point $y \in M$ s.t. $\overline{x, y} \ni v$.

We will prove the surjectiveness by experimenting an example.

Let $\dim M = 2$. $\dim E_x = 2$.

$T_x^* M = \langle \omega_1, \omega_2 \rangle$. $E_x = \langle e_1, e_2 \rangle$

Since $\dim(T_x^* V \otimes E_x) = 2$, and the natural projection $T_x^* M \otimes E_x \rightarrow T_x^* V \otimes E_x$ is surjective, at least \dim of the image of d is 2.

Suppose $\dim \text{im } d \neq 4 \Rightarrow \exists$ a smooth hyperplane section V s.t. the natural projection $T_x^*(M) \otimes E_x \rightarrow T_x^*(V) \otimes E_x$ is not onto.

Suppose $\text{im } d = \langle \omega_1 \otimes e_1 + \omega_2 \otimes e_2, \omega_2 \otimes e_1 + \omega_1 \otimes e_2 \rangle$.
 $\Rightarrow \exists$ a hyperplane section V s.t. $T_x^* V = \langle u_1 + u_2 \rangle$
 where $\langle u_1 \rangle = \ker \omega_2$ $\ker \omega_1 = \langle u_2 \rangle$.

Let $\tau: T_x^* V \rightarrow E_x$ be a map defined by

$$\tau(u_1 + u_2) = 2e_1 + e_2.$$

\Rightarrow We have $\sigma \in H^0(M, f_x(E))$ s.t.

$$d\sigma = a\omega_1 \otimes e_1 + a\omega_2 \otimes e_2 + b\omega_2 \otimes e_1 + b\omega_1 \otimes e_2$$

(Without loss of generality, assume that $\omega_i(u_j) = \delta_{ij}$.)

$$\text{s.t. } d\sigma|_V = \tau.$$

$$d\sigma(u_1 + u_2) = ae_1 + ae_2 + be_1 + be_2$$

$$= (a+b)e_1 + (a+b)e_2 \neq \tau(u_1 + u_2) = 2e_1 + e_2$$