$$0 \to \mathcal{O}(K_S - L) \longrightarrow \mathcal{O}(K_S) \oplus \mathcal{O}(K_S) \longrightarrow \mathcal{J}_P(K_S + L) \to 0.$$

☞ I guess that "$P = C \cdot C'$ for general curves" means that C intersects C' transversely. Then why do they need that the complete linear system $|L|$ has no base curves? Strange!
No base curves condition guarantes the regularity.

$$\begin{array}{ccccccc} 0 \to & \mathcal{O}(K_S - L) & \longrightarrow & \mathcal{O}(K_S) \oplus \mathcal{O}(K_S) & \longrightarrow & \mathcal{J}_P(K_S + L) & \to 0 \\ & \sigma & \longmapsto & (\sigma s', -\sigma s) & & & \\ & & & (\tau_1, \tau_2) & \longmapsto & \tau_1 s + \tau_2 s' & \end{array}$$

To show the exactness at $\mathcal{J}_P(K_S+L)$, given $\delta \in \mathcal{J}_P(K_S + L)$ at a point $p \in P \Rightarrow \delta(p) = 0$. We have to show that $\exists\ \tau_1, \tau_2 \in \mathcal{O}_p(K_S)$ s.t $\tau_1 s + \tau_2 s' = \delta$.
Local nature make me assume $\delta \in \mathcal{O}$ s.t $\delta(0) = 0$.
$\{s, s'\} = I$. $(s, s') : \mathbb{C}^2 \to \mathbb{C}^2$ is degree 1, since $(s=0)$ meets $(s'=0)$ transversely. $\Rightarrow$ $\delta \in I$ by 66p. corollary. ⌟

By our assumption
$$h^1(\mathcal{O}(K_S)) = h^{2,1}(S) = h^{0,1}(S) = 0,$$
so we find
$$\begin{aligned} h^1(\mathcal{O}(L)) &= h^1(\mathcal{O}(K_S - L)) \qquad (\text{by duality}) \\ &= h^0(\mathcal{J}_P(K_S + L)) - 2h^0(\mathcal{O}(K_S)) + h^0(\mathcal{O}(K_S - L)), \end{aligned}$$
which gives our assertion. Q.E.D.

☞ $q = h^{1,0}(S) = h^{0,1}(S) = h^{2,1}(S)$ by duality
$h^1(\mathcal{O}(K_S)) = \dim H^1(S, \mathcal{O}(K_S)) = \dim H^1(S, \Omega^2) = h^{2,1}(S)$.