

$$\Rightarrow 'E_1^{p,q} \cong H_\delta^q(K^{p,*}).$$

□

The differential d_1 is computed from $D = d + \delta$ on $'E_1$. Since $\delta = 0$ on $'E_1$, we see that $d_1 = d$ and

$$'E_2^{p,q} = H^*('E_1^{p,q}, d_1) \cong H_d^p(H_\delta^q(K^{*,*})).$$

The last expression denotes the cohomology of

$$\cdots \rightarrow H_\delta^q(K^{p+1,*}) \xrightarrow{d} H_\delta^q(K^{p,*}) \xrightarrow{d} H_\delta^q(K^{p-1,*}) \rightarrow \cdots,$$

which has meaning, since $d\delta + \delta d = 0$. Summarizing:

Associated to a bigraded complex $(K^{*,*}; d, \delta)$ are two spectral sequences both abutting to the cohomology of the total complex and where

$$\begin{cases} 'E_2^{p,q} \cong H_d^p(H_\delta^q(K^{*,*})) \\ ''E_2^{p,q} \cong H_\delta^q(H_d^p(K^{*,*})) \end{cases}$$

□

$$\begin{array}{ccc} a + D \in 'E_0^{p,q} & \xrightarrow{D} & 'E_0^{p,q+1} \\ \uparrow & \uparrow \cong & \uparrow \cong \\ a \in K^{p,q} & \xrightarrow{\delta} & K^{p,q+1} \end{array}$$

$$\Rightarrow \delta a = 0.$$

$$\begin{array}{ccc} a + \text{im } D \in \frac{\text{ker } D}{\text{im } D} = 'E_1^{p,q} & \xrightarrow{D} & 'E_1^{p+1,q} \\ \uparrow & \uparrow \cong & \uparrow \cong \\ a + \text{im } \delta \in \frac{\text{ker } \delta}{\text{im } \delta} & \xrightarrow{D = d + \delta} & \frac{\text{ker } \delta}{\text{im } \delta} \end{array}$$

$$\Rightarrow \text{We only need } d. \Rightarrow d_1 = d : H_\delta^q(K^{p,*}) \rightarrow H_\delta^q(K^{p+1,*})$$