

generate line  $L'' \neq L$ ,  $F_{\lambda(L'')} \supset \Lambda$ .

□

$F_{\lambda(L)}$  is clearly unique: if  $\Lambda$  lay on two quadrics of the pencil  $F_\lambda$ , it would be contained in  $X$ ; but  $X$ , as we saw, contains no  $\alpha$ -planes.

□  $\Lambda \subset \lambda_1 F + G$  and  $\Lambda \subset \lambda_2 F + G \Rightarrow \Lambda \subset (\lambda_1 - \lambda_2) F = 0$   
 $\Rightarrow \Lambda \subset F \Rightarrow \Lambda \subset G \Rightarrow \Lambda \subset F \cap G = X$   
 $\Rightarrow$  Since  $\Lambda = \sigma(\mathbb{C}P^1)$  or  $\sigma(\mathbb{C}H^1)$ , and by 762,  $X$  can not contain  $\Lambda$ ,  $F_{\lambda(L)}$  is unique.

□

We may thus define the map  $\pi$  by sending any line  $L' \in B_L$  to  $\lambda(L')$ .

$$\begin{array}{ccccc} \square & B_L & \longrightarrow & \{F_\lambda\} & \longrightarrow & \mathbb{P}^1 \\ & \downarrow \psi & & \downarrow \psi & & \downarrow \psi \\ & L' & \longmapsto & F_{\lambda(L')} & \longmapsto & \lambda \end{array}$$

□

Now let  $F_\lambda$  be any quadric in our pencil, and consider the inverse image  $\pi^{-1}(\lambda)$ . If  $\Lambda$  is any  $\alpha$ -plane in  $F_\lambda$  containing  $L$ , then the intersection of  $\Lambda$  with  $X$  — that is, the intersection of  $\Lambda$  with any second element  $F_\mu$  of the pencil — will consist of  $L$  plus a second line  $L'$ ; the inverse image  $\pi^{-1}(\lambda)$