An obvious but fundamental observation is that if V^* has dimension k, $A^{p,q}(V^*)=0$ for p or $q>k$; consequently for any form φ,

$$\int_V \varphi = \int_V \varphi^{(k,k)}.$$

⌈ Since V^* has dimension k, locally, any form φ is expressed by dz_i, $i=1,2,\cdots k$. ⌋

We can now prove

Stokes' Theorem for Analytic Varieties. For M a complex manifold, $V\subset M$ an analytic subvariety of dim. k, and φ a differential form of degree $2k-1$ with compact support in M,

$$\int_V d\varphi = 0.$$

Proof. The question is local, i.e., it will be sufficient to show that for every $p\in V$, there exists a nbd U of p such that for any $\varphi\in A_c^{2k-1}(U)$

$$\int_V d\varphi = 0.$$

⌈ Once we prove the claim above, given a differential form φ of degree $2k-1$ with compact support K in M, for every $p\in V\cap K$, $\exists$ U_p.
$\Rightarrow$ Since $V\cap K$ is compact, $\exists$ a finite # of U_p's,