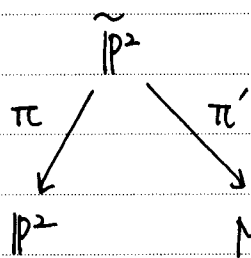


The resulting surface S , by the last lemma of the previous section, is isomorphic to \mathbb{P}^2 ; so we have given, up to an automorphism of \mathbb{P}^2 , a birational map φ_{abc} of \mathbb{P}^2 to itself.

IF



$$\begin{aligned} H_i(\tilde{\mathbb{P}}^2) &= H_i(M) \oplus H_i(\tilde{L}_{ab}) \oplus H_i(\tilde{L}_{bc}) \oplus H_i(\tilde{L}_{ac}) \\ &= H_i(M) \oplus H_i(\mathbb{P}^1) \oplus H_i(\mathbb{P}^1) \end{aligned}$$

$$\Rightarrow H_1(\tilde{\mathbb{P}}^2) = H_1(M) = 0$$

$$H_2(\tilde{\mathbb{P}}^2) = H_2(M) \oplus \mathbb{C}^3 = \mathbb{C}^4, \text{ since}$$

$$H_2(\tilde{\mathbb{P}}^2) = H_2(\mathbb{P}^2) \oplus \mathbb{C}^3. \Rightarrow b^1(M) = 1 = b^3(M)$$

$$b^0(M) = b^4(M) = 1 \quad b^1(M) = b^3(M) = 0.$$

$$K_{\tilde{\mathbb{P}}^2} = \pi'^* K_M + \tilde{L}_{ab} + \tilde{L}_{bc} + \tilde{L}_{ac}$$

$$\begin{aligned} \Rightarrow K_M \cdot D &= \pi'^* D \cdot \pi'^* K_M \\ &= \pi'^* D \cdot (K_{\tilde{\mathbb{P}}^2} - \tilde{L}_{ab} - \tilde{L}_{bc} - \tilde{L}_{ac}) \\ &= \pi'^* D \cdot K_{\tilde{\mathbb{P}}^2} \leq 0 (?) \text{ for any } D \subset M. \end{aligned}$$

To show this,

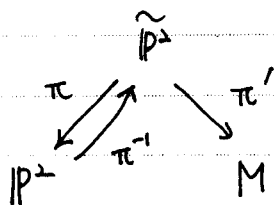
$$\text{since } K_{\tilde{\mathbb{P}}^2} = \pi^* K_{\mathbb{P}^2} + E_a + E_b + E_c,$$

$$(\pi'^* D' + (n_1 E_a + n_2 E_b + n_3 E_c)) \cdot K_{\tilde{\mathbb{P}}^2}$$

$$= D' \cdot K_{\mathbb{P}^2} - n_1 - n_2 - n_3 \leq 0 \text{ for } n_1, n_2, n_3 \geq 0 \text{ and } D' \subset \mathbb{P}^2 \text{ divisor.}$$

$$\Rightarrow K_{\tilde{\mathbb{P}}^2} \text{ is not positive} \Rightarrow \pi'^* D \cdot K_{\tilde{\mathbb{P}}^2} \leq 0 \Rightarrow K_M$$

$$\text{is not positive.} \Rightarrow \text{By the lemma on p487, } M \cong \mathbb{P}^2.$$



$$\Rightarrow \pi' \circ \pi^{-1}: \mathbb{P}^2 \rightarrow M \text{ is a birational map with an inverse}$$