Let $v_1, v_2 \cdots v_{k-1}$ then be vector fields to σ_α^k along γ and $v_{k+1}, \cdots v_n$ vector fields to Δ_j^{n-k+1} along γ s.t

$v_1(0) \cdots v_{k-1}(0), \gamma'(0)$ is an oriented basis for $T_{\gamma(0)}(\sigma_\alpha^k)$ &
$v_{k+1}(0) \cdots v_n(0)$ is an " for $T_{\gamma(0)}(\Delta_\alpha^{n-k})$ &
$v_1(0) \cdots v_{k-1}(0) \in T_{\gamma(1)}(\sigma_j^{k-1})$
$v_{k+1}(0) \cdots v_n(1) \in T_{\gamma(1)}(\Delta_j^{n-k+1})$.

By hypothesis.

$$\iota_{\gamma(0)}(\sigma_\alpha^k \cdot \Delta_\alpha^{n-k}) = +1$$

the basis $v_1(0), \cdots v_{k-1}(0), \gamma'(0)\ v_{k+1}(0) \cdots v_n(0)$ is positive for the given orientation on M. Moreover, since $\gamma'(0)$ is inward normal to Δ_j^{n-k+1} at $\gamma(0)$. and since $v_{k+1}(0), \cdots v_n(0)$ is positively oriented for $T_{\gamma(0)}(\Delta_\alpha^{n-k})$,

the basis $\gamma'(0) \cdot v_{k+1}(0) \cdots v_n(0)$ will have sign $(-1)^{n-k}$ w.r.t the orientation on Δ_j^{n-k+1}
By continuity, these last two assersions will hold as well as $\gamma(1)$.
There, since $\gamma'(1)$ is outward normal to Δ_j^{n-k+1} at $\gamma(1)$, and since $v_1(1), \cdots \cancel{v_{k+1}}\ v_{k-1}(1), \gamma'(1)$ is positively oriented for $T_{\gamma(1)}(\sigma_\alpha^k)$, the basis $v_1(1), \cdots v_{k-1}(1)$ will be negatively oriented for σ_j^{k-1}.

Thus $\quad \bar{\iota}_{\gamma(1)}(\sigma_j^{k-1} \cdot \Delta_j^{n-k+1}) = (-1)^{n-k+1}$ as desired.

We see from this that the map

$\sigma_\alpha^k \longmapsto \tilde{\Delta}_\alpha^{n-k}$ induces an isomorphism between the complex (C_*, ∂) of chains in the original