

where $\langle x, y \rangle$ is the line containing x & y .

$$\Rightarrow \dim(\langle x, y \rangle \cap M) = 1 \text{ or } 0.$$

$$(i) \quad \dim(H \cap M)_s = 0.$$

$\Rightarrow (H \cap M)_s$ is a set of finite points. i.e

$$\Rightarrow (H \cap M)_s = \{a_1, a_2, \dots, a_k\}.$$

For the generic hyperplane $H_i \ni x, y$,

$$T_{a_i} M + T_{a_i} H_i = T_{a_i} \mathbb{P}^N.$$

$\Rightarrow \{H_1\} \cap \{H_2\} \cap \dots \cap \{H_n\}$ are the set of generic hyperplanes s.t the intersection with M is smooth.

$$(ii) \quad \dim(H \cap M)_s = 1.$$

$$\Rightarrow \dim(\langle x, y \rangle \cap M) = 1 \Rightarrow \langle x, y \rangle \subset M \Rightarrow \langle x, y \rangle \cap M = \mathbb{P}^1 \Rightarrow (H \cap M)_s = \mathbb{P}^1 \text{ since } \mathbb{P}^1 \text{ is irreducible. (see p. 77 Th. 1.8 Whitney)}$$

\Rightarrow For each point $a \in \langle x, y \rangle$, \exists generic hyperplanes $\{H_a\}$ s.t $T_a H_a + T_a M = T_a \mathbb{P}^N$.

\Rightarrow For a fixed $a \in \langle x, y \rangle$, the generic hyperplanes $\{H_a\} \cap \{H\} = \{H'\}$ have $(H' \cap M)_s \neq \mathbb{P}^1$.

$\Rightarrow \dim(H' \cap M)_s = 0 \Rightarrow$ We can apply the arguments as (i) again. \leftarrow the facts that

Here we used the generic hyperplanes form an open dense subset of $G(N-1, N)$, and a finite intersection of open dense subsets is again open dense.