

p2010 \simeq p2015

$$= \{ p \in M-B \mid \lambda_2 s_1(p) - \lambda_1 s_2(p) = 0 \}.$$

Since $f(V)$ has measure zero, for a generic^{regular} element $\lambda \in \mathbb{P}^1$, $f^{-1}(\lambda)$ is submanifold of $M-B$.

In general, we can easily prove the above.

For example,

$$\begin{array}{ccc} M-B & \xrightarrow{f} & \mathbb{P}^2 \\ \downarrow \text{p} & \longrightarrow & [(s_{\alpha 1}(p), s_{\alpha 2}(p), s_{\alpha 3}(p))] \end{array}$$

$$B = \{ s_1 = s_2 = s_3 = 0 \}.$$

\Rightarrow By the same argument, we can show that the set of regular values forms an open dense subset in \mathbb{P}^2 .

Let $\lambda \in \mathbb{P}^2$ be a regular value.

$\Rightarrow \exists \mathbb{P}^1$ s.t. $\mathbb{P}^1 \subset \mathbb{P}^2$, and $\lambda \in \mathbb{P}^1$.

$\Rightarrow \exists$ open set $U \ni \lambda$ s.t. $U \subset \mathbb{P}^2$, all points in U are regular values.

$\Rightarrow f^{-1}(U)$ is submanifold in $M-B$.

$\Rightarrow f^{-1}(U) \xrightarrow{g \circ f|} \mathbb{P}^1 \cap U$ is regular, since

\Rightarrow $f|$ $U \xrightarrow{g} \mathbb{C} \xrightarrow{\cong} \mathbb{C}$ g is regular.

$$\begin{array}{ccc} & U & \xrightarrow{g} \mathbb{C} \\ f| \swarrow & \nearrow & \downarrow \cong \\ & \mathbb{C} & \nearrow \\ & (w_1, w_2) & \nearrow (aw_1 + bw_2) + c \end{array}$$

$\Rightarrow (g \circ f|)^{-1}(g(u))$ is smooth divisor away from B .

* $\mathbb{C}^n \xrightarrow{\phi} \mathbb{C} \quad (w_1 \dots w_n) \mapsto a_1 w_1 + \dots + a_n w_n + a_0$ regular, for $(a_1 \dots a_n) \neq 0$.

This represents a hyperplane in \mathbb{P}^2 or \mathbb{P}^n (considering $a_0 s_0 + \dots + a_n s_n = 0$)