

due of P_0 with respect to curves of degree n .

Of course, P_0 is a set of distinct points in \mathbb{P}^2 .

As we saw on P713, $P_g = 0 = g$ $r = \dim |L| = \frac{n(n+3)}{2} - d + \omega$
 > 0 , since $d = \deg P_0 < \frac{n(n+3)}{2}$. ① $|L|$ has no base curve

Now we can apply this case to our previous assertion. $\omega = \dim |f_{P_0}(K_{\mathbb{P}^2} + L)| + 1$
 Since $P_g = 0$, and $h^0(\mathcal{O}(K_S - L)) = 0$, which implies $\dim |K_S - L| = -1$.

$$\Rightarrow \dim |f_{P_0}(n)| = \frac{n(n+3)}{2} - d + \omega \geq 2.$$

\Rightarrow The linear system $|f_{P_0}(n)|$ contains at least a pencil, actually a net.

$$P' = C \cdot C' = P_0 + P = n^2$$

Not quite right !!!

"We did not prove that if $P_g = 0$, then $\dim |K_S - L| = -1$, see P 673 note."

Suppose that P_0 is a set of distinct points in \mathbb{P}^2 and $|C_0| = |f_{P_0}(n)|$ is the linear system of curves of degree n passing through P_0 . This may be interpreted as follows: \exists a curve C_0 of degree n passing P_0 exactly once at each point in P_0 , and consider all curves of degree n passing through P_0 . Then everything fits with each other. In other words, $|C| = |L| = |\pi^*H - E|$.

Accept the version above. Then we can prove the application