it follows that a fiber of $\pi_1 : I \longrightarrow |F|$ – the family of k-planes on a quadric F – has dimension

$$(k+1)(n-k+1) - \frac{(k+1)(k+2)}{2}$$

or, alternatively, codimension $(k+1)(k+2)/2$ in the Grassmannian $G(k+1, n+2)$.

Γ Given any quadric F' in Λ, we have F quadric in $\mathbb{P}^{n+1}$ s.t $F|\Lambda = F'$. We can see this easily as follows: By changing the coordinates, we may assume $\Lambda = \mathbb{P}^k$ i.e., $\Lambda = \{[\overbrace{*,\cdots *}^{k+1}, \overline{0\cdots 0}]\} \subset \mathbb{P}^{n+1}$. Given $F' = \sum_{i,j=0}^{k} q_{ij} X_i X_j$, let $F = \sum_{i,j=0}^{k} q_{ij} X_i X_j$ in $\mathbb{P}^{n+1}$. $\Rightarrow$ $F|\Lambda = F'$.

For series, see P249.

Given a quadric F in $\mathbb{P}^{n+1}$, F may be expressed as $F_1 + F_2$, where

$$F = \sum_{i,j=0}^{n} q_{ij} X_i X_j = \underbrace{\sum_{i,j=0}^{k} q_{ij} X_i X_j}_{F_1} + \underbrace{\sum_{\substack{i>k \\ 0\le j\le k}} q_{ij} X_i X_j + \sum_{\substack{j>k \\ 0\le i\le k}} q_{ij} X_i X_j + \sum_{\substack{i>k \\ j>k}} q_{ij} X_i X_j}_{F_2}$$

$\Rightarrow$ Clearly, since $\Lambda = \{[\overbrace{*\cdots *}^{k+1}\overline{0\cdots 0}]\}$, $\Lambda \subset F_2$.
As we see above, $\dim\{F_1\} = (k+1)(k+2)/2$, and $\dim\{F_2\} = \frac{(n+2)(n+3)}{2} - \frac{(k+1)(k+2)}{2}$. $\Rightarrow$ Since $\pi_2^{-1}(\Lambda)$ is