line, is tangent to S if and only if it lies on a pencil of lines of X all tangent to S.

F Suppose $l_x \cap S = \{p_1, p_2, p_3, p_4\}$ p_i's distinct.
$\Rightarrow$ For each i, $\sigma(p_i) \cap X$ is singular. Let $\sigma(p_i, h_i)$ be one of two lines of $\sigma(p_i) \cap X$, which contains x.
$\Rightarrow$ $x \in \sigma(p_i, h_i) \subset T_x(X) \cap X$.
$\Rightarrow$ $T_x(X) \cap X = \bigcup_{i=1}^{4} \sigma(p_i, h_i)$, for otherwise

$\sigma(p_1, h_1) = \sigma(p_2, h_2)$. $\Rightarrow$ $p_1 = p_2$. $\Rightarrow$ Contradiction.
$\Rightarrow$ l_x is tangent to S. $\Rightarrow$ By the result above, all the lines $\{l_x\}_{x \in L}$ are tangent to S.
Suppose l_x is tangent to S. If l_x is not singular, then by the result on p.764, l_x meets S in four distinct points, unless $T_x(X) \cap X$ contains a multiple component. $\Rightarrow$ Say L is the multiple component. $\Rightarrow$ l_x lies on L.

⅃

Note that if $L \subset X$ is any pencil of lines, all tangent to S, then by Bertini they must all be tangent at the focus p_L of the pencil, i.e., the plane h_L of the pencil must be the tangent plane to S at p_L.