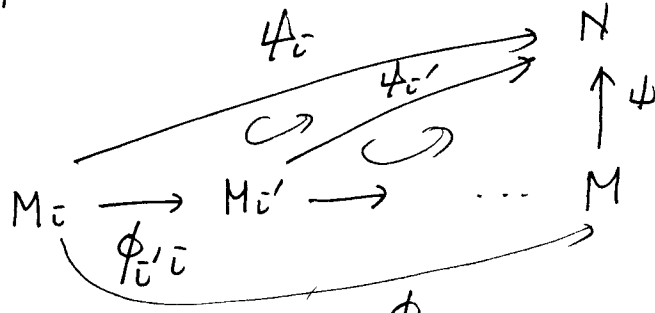


In part (25.12 sublemma is extremely important)



If $\phi_{\bar{c}}(x_{\bar{c}}) = 0$, $\exists \bar{c}'$ with $\bar{c} \leq \bar{c}'$ s.t. $\phi_{\bar{c}'\bar{c}} x_{\bar{c}} = 0$.

Def: \mathcal{F}, \mathcal{G} presheaves on X , a morphism $\varphi: \mathcal{F} \rightarrow \mathcal{G}$ consists of a morphism of abelian groups

$\varphi(U): \mathcal{F}(U) \rightarrow \mathcal{G}(U)$ for each open set U ,
s.t. whenever $V \subseteq U$, is an inclusion, the diagram

$$\begin{array}{ccc} \mathcal{F}(U) & \xrightarrow{\varphi(U)} & \mathcal{G}(U) \\ \rho_{UV} \downarrow & & \downarrow \rho'_{UV} \\ \mathcal{F}(V) & \xrightarrow{\varphi(V)} & \mathcal{G}(V) \end{array} \quad \text{is commutative.}$$

An isomorphism is a morphism which has a two-sided inverse.

The following proposition (which would be false for presheaves) illustrates the local nature of a sheaf.

Proposition: Let $\varphi: \mathcal{F} \rightarrow \mathcal{G}$ be a morphism of sheaves on top X .

Then φ is an isomorphism \iff the induced map on the stalk $\varphi_p: \mathcal{F}_p \rightarrow \mathcal{G}_p$ is an isomorphism for every $p \in X$.