$$k\,\tilde{P}\left(\frac{\partial}{\partial t}\Theta_t, \Theta_t, \cdots \Theta_t\right) = \frac{\partial}{\partial t} P(\Theta_t)$$

since $\frac{\partial}{\partial t} P(\Theta_t) = \frac{\partial}{\partial t}\tilde{P}(\Theta_t, \cdots \Theta_t) = \sum \tilde{P}(\Theta_t, \cdots \frac{\partial}{\partial t}\Theta_t, \cdots \Theta_t)$ and Θ_t & $\frac{\partial}{\partial t}\Theta_t$ are 2-forms. ⌟

To restate the lemma: If we let Φ denote the graded algebra of invariant polynomials, then for any vector bundle $E \to M$, we obtain a well-defined homomorphism of algebras

$$\Phi \xrightarrow{W} H^{2*}_{DR}(M)$$

given by

$$P \overset{W}{\longmapsto} [P(\Theta)],$$

where Θ is the curvature matrix of any connection in E; W is called the Weil homomorphism.

In particular, let P^i denote again the elementary invariant polynomials. We define the Chern forms $c_i(\Theta)$ of the curvature Θ in E by

$$c_i(\Theta) = P^i\left(\frac{\sqrt{-1}}{2\pi}\Theta\right),$$

and we define the Chern class $c_i(E)$ by

$$c_i(E) = \left[P^i\left(\frac{\sqrt{-1}}{2\pi}\Theta\right)\right] \in H^{2i}_{DR}(M).$$

The total Chern class $c(E)$ is the sum of the Chern classes:

$$c(E) = \sum_{i \geq 0} c_i(E) \in H^{2*}_{DR}(M),$$

where we set $c_0(E) = 1 \in H^0_{DR}(M)$. Also, for M a complex manifold, we take the Chern classes $c_i(M)$