

ity of  $\Delta'$ , the varieties  $V$  and  $W + \epsilon$  will meet transversely in  $\mu$  points in  $\Delta'$ . The number  $\mu$  is called the intersection multiplicity of  $V$  and  $W$  at 0 and is written

$$\mu = m_0(V \cdot W).$$

$$\Gamma \quad \tilde{V} \cap \tilde{W}$$

$$= V \times \Delta' \cap \{(z, w) : z - w \in W\} \ni (x, y)$$

$$\Rightarrow x \in V, \quad x - y \in W.$$

$\Rightarrow x$  varies over  $V$  of dim  $k$ . and for each  $x \in V$ ,  $\{y \in x - W\}$  has dimension  $n - k$

$$\Rightarrow \tilde{V} \cap \tilde{W} \text{ has dimension } n = k + n - k.$$

$$\textcircled{1} \pi_2: \tilde{V} \cap \tilde{W} \rightarrow \Delta' \text{ is proper}$$

$$\text{For, } \pi_2^{-1}(0) \cap \tilde{V} \cap \tilde{W} = V \times \{0\} \cap W \times \{0\} = V \cap W \times \{0\}$$

$$= \{0, 0\}. \Rightarrow \text{A point set.}$$

$\Rightarrow$  For a small nbd  $U$  of  $(0, 0)$ ,  $\pi_2^{-1}(\epsilon) \cap \tilde{V} \cap \tilde{W}$  is a set of discrete points,  $\epsilon \in U$ . For such nbd see P68. The Transversality Theorem (Differential Topology by V. Guillemin).

$\textcircled{2} \pi_2: \tilde{V} \cap \tilde{W} \rightarrow \Delta'$  has <sup>the Jacobian of</sup> rank  $n$  for generic points, since any  $n \times n$  minor matrix of

$$\begin{pmatrix} \frac{\partial(\pi_2)_1}{\partial z} & \dots & \dots \\ \vdots & & \end{pmatrix}_{2n}$$

$\Rightarrow$  For generic points, by the inverse function theorem,  $\pi_2$  is a finite-sheeted covering map.