$$\begin{pmatrix} 1, & 0, & \cdots & 0, & \alpha_{11}-\alpha_{10}, & 0 & \cdots & 0 \\ 0, & 1, & \cdots & 0 & 0, & \alpha_{12}-\alpha_{10}, & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \\ 0, & 0, & \cdots & 1, & 0, & 0 & \cdots & \alpha_{1n}-\alpha_{10} \\ 1, & 0, & \cdots & 0, & \overset{\alpha_{21}}{-\alpha_{20}}, & 0 & \cdots & 0 \\ 0, & 1, & \cdots & 0, & 0, & \alpha_{22}-\alpha_{20}, & \cdots & 0 \\ \vdots & & & \vdots & \vdots & \vdots & & \\ 0 & 0, & \cdots & 1, & 0 & 0 & \cdots & \alpha_{2n}-\alpha_{20} \end{pmatrix}$$

is nonzero. Clearly $\operatorname{im} v_{1*} + \operatorname{im} v_{2*} \subset T_p E' + \operatorname{im} v_{q+1*}$ and from the above, we have $T_p'E' + \operatorname{im} v_{q+1*} = T_p \mathbb{C}^{2n}$ since $\operatorname{im} v_{1*} + \operatorname{im} v_{2*} = \mathbb{C}^{2n}$. ⌟

First, we see that v_1 vanishes at $X \in \mathbb{P}^n$ exactly when

$$[\alpha_{1,0} X_0, \cdots \alpha_{1n} X_n] = [X_0, \cdots X_n],$$

and since by assumption $\alpha_{1i} \neq 0$ for all i and $\alpha_{1i} \neq \alpha_{1j}$ for all $i \neq j$, this is the case only for $X = p_i$, where

$$p_i = [0, \cdots 0, 1, 0, \cdots 0], \quad i = 0, \cdots n.$$

Thus $C_n(\mathbb{P}^n) = n+1$.

⌜ $v_1 = \pi_*\left(\alpha_{1j} X_j \frac{\partial}{\partial X_j}\right) = 0$

$\Rightarrow \alpha_{1j} X_j \frac{\partial}{\partial X_j} = X_j \frac{\partial}{\partial X_j} \Rightarrow [\alpha_{10} X_0, \cdots \alpha_{1n} X_n] = [X_0, \cdots, X_n]$

$\Rightarrow$ Since $\alpha_{1i} \neq \alpha_{1j}$ for $i \neq j$, all but one of the homogeneous coordinates of X are zero. $\Rightarrow v_1 = 0$ at $p_i = [0, \cdots 0, 1, 0 \cdots 0]$ $i = 0, 1, \cdots n$.

$\Rightarrow$ By Gauss-Bonnet Formula II, since $C_n(\mathbb{P}^n)$ is Poincare dual to the degeneracy class $D_{n-n+1} = D_1 = \{p_i\}_{i=0}^n$. $\Rightarrow C_n(\mathbb{P}^n) = n+1$. ⌟

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