As before, choose $\Lambda_0 \subset \Gamma_0'$ with $\#\Lambda_0 = 3$.

(i) Case 2.

$\dim |\mathcal{J}_{\Lambda_0}(2)| = \frac{2(2+3)}{2} - 3 + 0 = 2$ Contradiction

(ii) Case 1

$\Rightarrow \exists$ a fixed line $\Rightarrow \exists$ linearly independent lines $l_1, l_2, l_3, l_4$, which contradicts to the fact $\dim H^0(\mathbb{P}^2, \mathcal{O}(H)) = 3$. ⌟

If, finally, $C_0$ is a line containing $\leq 5$ points from $\Gamma_0$, then $|C - C_0|$ will be a linear system of $\infty^{3+\rho}$ $(\rho \geq 0)$ cubics passing through $\geq 7$ points.

⌈ Let $l_0$ be a line representing $C_0$.

$\Gamma_0 - l_0 \cap \Gamma_0 = \Gamma_0'$ $\Rightarrow \exists$ a correspondence between $H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0}(4))$ and $H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0'}(3))$. Guess! $\infty^{3+\rho}$ means dimension $3+\rho$ of $|\ |$. ⌟

$\Downarrow$ $h^0(\mathcal{J}_{\Gamma_0}(4)) = \dim H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0'}(3))$

By our previous result, either five will be on a line — in which case 10 points from $\Gamma_0$ are on a (degenerate) conic — or eight will be on a conic.

⌈ ① $\#(C_0 \cap \Gamma_0) = 5$

$\Rightarrow \#\Gamma_0' = 7$

$\Rightarrow$ By the result on p714, since $\Gamma_0'$ does not impose independent conditions on $|\mathcal{O}_{\mathbb{P}^2}(3)|$, five of $\Gamma_0'$ lie on a fixed line $l_0'$. $\Rightarrow$ $l_0 + l_0'$ is a