$$= \quad t^n + \underbrace{(\textstyle\sum \lambda_i)}_{P^1(A)} t^{n-1} + \left(\sum_{i \neq j} \lambda_i \lambda_j\right) t^{n-2} + \cdots + \underbrace{\lambda_1 \lambda_2 \cdots \lambda_n}_{P^n(A)}.$$

In particular, $P^n(A) = \det(A)$ and $P^1(A) = \operatorname{trace}(A)$; in general, if for any multiindexes $I, J \subset \{1, 2, \ldots n\}$ we let $A_{I,J}$ denote the (I, J) th minor $(A_{ij})_{i \in I, j \in J}$ of A, we can write

$$p^k(A) = \sum_{\# I = k} \det(A_{I,I})$$

$$= \operatorname{trace}(\Lambda^k A).$$

Γ

$$\det(A + t \cdot I) = \det \begin{pmatrix} a_{11} + t, & a_{12}, & a_{13} \\ a_{21}, & a_{22} + t, & a_{23} \\ a_{31}, & a_{32} & a_{33} + t \end{pmatrix}$$

$$= (a_{11} + t) \begin{vmatrix} a_{22} + t, & a_{23} \\ a_{32}, & a_{33} + t \end{vmatrix} - a_{12} \begin{vmatrix} a_{21}, & a_{23} \\ a_{31}, & a_{33} + t \end{vmatrix}$$

$$+ a_{13} \begin{vmatrix} a_{21}, & a_{22} + t \\ a_{31}, & a_{32} \end{vmatrix}$$

For example, $n = 5$.

$$\det(A + t \cdot I) = \sum \epsilon(\sigma)\, a'_{1\sigma(1)} a'_{2,\sigma(2)} \cdot a'_{3,\sigma(3)} a'_{4\sigma(4)} a'_{5,\sigma(5)}.$$

where $a'_{ij} = (A + tI)_{ij}$.

$p^3(A)$ is the coefficient of t^2

Consider the coefficient part from $a'_{11} a'_{22} = (a_{11} + t)(a_{22} + t)$.

$\Rightarrow$ It is $\sum \epsilon(\sigma) \underset{a'_{3\sigma(3)}}{\underset{\parallel}{a_{3\sigma(3)}}} \underset{a'_{4\sigma(4)}}{\underset{\parallel}{a_{4\sigma(4)}}} \underset{a'_{5\sigma(5)}}{\underset{\parallel}{a_{5\sigma(5)}}} = \det A_{\{3,4,5\}\{3,4,5\}}$