┌ Take the following projective resolution.

$$\to 0 \to 0 \to \cdots \to 0 \to \underset{\overset{\|}{E_0}}{M} \to M \to 0$$

$\Rightarrow \ \mathrm{Ext}^q_{\mathcal{O}}(M, N) = H^q(\mathrm{Hom}(E_\cdot(M), N)) = \dfrac{\ker\delta}{\mathrm{Im}\,\delta}$ where

$$\mathrm{Hom}(E_{q-1}, N) \xrightarrow{\delta} \mathrm{Hom}(E_q, N) \xrightarrow{\delta} \mathrm{Hom}(E_{q+1}, N)$$

Since $q > 0$, $E_q = 0$. $\Rightarrow$ $\mathrm{Ext}^q_{\mathcal{O}}(M, N) = 0$. ┘

Conversely, suppose that $\mathrm{Ext}^1_{\mathcal{O}}(M, N) = 0$ for all N, and consider a diagram

$$\begin{tikzcd}
 & & M \arrow[d, dashed, "\beta"'] \arrow[dr, "\alpha"] & & \\
0 \arrow[r] & N \arrow[r] & P \arrow[r] & Q \arrow[r] & 0
\end{tikzcd}$$

in which the solid arrows are given. Applying (**) above and $\mathrm{Ext}^1_{\mathcal{O}}(M, N)$ gives

$$\mathrm{Hom}_{\mathcal{O}}(M, P) \longrightarrow \mathrm{Hom}_{\mathcal{O}}(M, Q) \longrightarrow 0,$$

so that the dotted arrow β can be filled in. Consequently M is projective. Q.E.D.

┌ $0 \to \mathrm{Hom}(M, N) \to \mathrm{Hom}(M, P) \to \mathrm{Hom}(M, Q) \to \underset{\overset{\|}{0}}{\mathrm{Ext}^1_{\mathcal{O}}(M, N)}$.

$\Rightarrow$ Given $\alpha \in \mathrm{Hom}(M, Q)$, $\exists$ $\beta \in \mathrm{Hom}(M, P)$ s.t
$f \circ \beta = \alpha$, where $f: P \to Q$. ┘

Finally we shall refine this to