Thus,

Any smooth algebraic variety of dimension k may be embedded in $\mathbb{P}^{2k+1}$.

Γ $V^k \subset \mathbb{P}^n \Rightarrow$ By the above argument, V may be embedded into a hyperplane of $\mathbb{P}^n$ if $n > 2k+1$.
Continue this process, $V^k \subset \mathbb{P}^{2k+2} \Rightarrow$ Since $2k+2 > 2k+1$, V^k may be embedded into $\mathbb{P}^{2k+1}$ which is a hyperplane of $\mathbb{P}^{2k+2}$. 」

As we shall see, the degree of the chordal variety $C(V)$ of a variety does not depend on the degree of V alone.

A variety $V \subset \mathbb{P}^n$ is called nondegenerate if it does not lie in a hyperplane. We have the following condition on the degree of a nondegenerate variety:

If $V \subset \mathbb{P}^n$ is an irreducible, nondegenerate, k-dimensional variety, then $\deg(V) \geq n-k+1$.

We prove this first for V a curve in $\mathbb{P}^n$. Any n points of V lie in a hyperplane H, and if the degree of V were less than n, then H, having n points in common with V, would have a curve in common with V; being irreducible, V would then lie in H.

Γ V a curve in $\mathbb{P}^n \Rightarrow V$ 1-dimensional variety
$\Rightarrow \deg(V) \geq n-1+1 = n$. Suppose $\deg V \leq n-1$.
$\Rightarrow$ Any n points of V lie in a hyperplane H, since we can find a $(a_0, \cdots a_n)$ which is perpendicular to