then clearly $d_f(B) \neq 0$. To complete the proof, it is only necessary to find for any function $g \in {}_\nu\mathcal{O}_V$, a polynomial $Q_{f,g}(X) = a_1 X^{\nu-1} + \cdots + a_{\nu-2} X + a_{\nu-1} \in {}_n\mathcal{O}_W[X]$, ${}_n\mathcal{O}_W = \pi^*({}_n\mathcal{O}_W)$, s.t

$$d_f(z)\, g(A_j(z)) = a_1(z)\, f(A_j(z))^{\nu-1} + \cdots + a_{\nu-2}(z)\, f(A_j(z)) + a_{\nu-1}(z)$$

$$\Leftrightarrow \quad (\pi^* d_f)\cdot g = (\pi^* a_1)\, f^{\nu-1} + \cdots + (\pi^* a_{\nu-2})\, f + \pi^* a_{\nu-1}.$$

for all $z \in W$ and all points $A_j(z) \in \pi^{-1}(z)$, and to see that it is uniquely determined. For any fixed point $z \in W$, these conditions can be viewed as a set of $\nu$ linear equations in the $\nu$ unknown values of $a_k(z)$, i.e

$$\begin{aligned}
d_f(z)\, g(A_1(z)) &= a_1(z)\, f(A_1(z))^{\nu-1} + \cdots + a_{\nu-2}(z)\, f(A_1(z)) + a_{\nu-1}(z)\\
d_f(z)\, g(A_2(z)) &= a_1(z)\, f(A_2(z))^{\nu-1} + \cdots + a_{\nu-2}(z)\, f(A_2(z)) + a_{\nu-1}(z)\\
&\vdots\\
d_f(z)\, g(A_\nu(z)) &= a_1(z)\, f(A_\nu(z))^{\nu-1} + \cdots + a_{\nu-2}(z)\, f(A_\nu(z)) + a_{\nu-1}(z)
\end{aligned}$$

$$\Leftrightarrow \begin{pmatrix} f(A_1(z))^{\nu-1}, & f(A_1(z))^{\nu-2}, & \cdots & f(A_1(z)), & 1 \\ f(A_2(z))^{\nu-1}, & f(A_2(z))^{\nu-2}, & \cdots & f(A_2(z)), & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ f(A_\nu(z))^{\nu-1}, & f(A_\nu(z))^{\nu-2}, & \cdots & f(A_\nu(z)), & 1 \end{pmatrix} \begin{pmatrix} a_1(z) \\ a_2(z) \\ \vdots \\ a_{\nu-1}(z) \end{pmatrix} = \begin{pmatrix} d_f(z)\, g(A_1(z)) \\ d_f(z)\, g(A_2(z)) \\ \vdots \\ d_f(z)\, g(A_\nu(z)) \end{pmatrix}.$$

Then by Cramer's rule the solutions $a_k(z)$ are uniquely det