Note in particular that
There are no nonzero global holomorphic forms on $\mathbb{P}^n$.

⌈ $H^0(M, \Omega^p) = H^{p,0}_{\bar\partial}(M) = 0$ if $M = \mathbb{P}^n$ $p>0$. ⌋

The Lefschetz Decomposition

Another important application of the Hodge identities is the Lefschetz decomposition of the cohomology of a compact Kähler manifold. To put this in proper perspective, we must first digress for a moment and discussion representations of sl_2.

Representations of sl_2. sl_2 is the Lie Algebra of the group SL_2; it is realized as the vector space of 2x2 complex matrices with trace 0, and with the bracket

$$[A, B] = AB - BA.$$

⌈ Def. of SL_2 (special linear group)

$$\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1 \right\}$$

$a(0) = d(0) = 1$

$b(0) = c(0) = 0$

$$a(t)\,d(t) - b(t)\,c(t) = 1$$

$$a'(t)\,d(t) + a(t)\,d'(t) - b'(t)\,c(t) - b(t)\,c'(t) = 0$$

$$a'(0) + d'(0) = 0$$

$$sl_2 = \left\{ \begin{pmatrix} a'(0), & b'(0) \\ c'(0) & d'(0) \end{pmatrix} \right\}$$

$\Rightarrow$ Every element of sl_2 has trace 0.

From the fact that $\det e^A = e^{\operatorname{trace} A}$, $\det(\exp sl_2) = 1$.

$\Rightarrow sl_2 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ s.t } a+d=0 \right\}$ since $\exp t \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2$ and