$$
\begin{aligned}
T(\varphi) &= \lim_{\epsilon\to 0} T_\epsilon(\varphi) \\
&= \lim_{\epsilon\to 0} (T_\epsilon)_\delta(\varphi) \\
&= \lim_{\epsilon\to 0} T_\epsilon(\varphi_\delta) \\
&= T(\varphi_\delta) \qquad (\text{by property } 2) \\
&= T_\delta(\varphi)\ ;
\end{aligned}
$$

i.e., $T = T_\delta$

is a smooth distribution as desired. Q.E.D.

Γ $(T_\epsilon)_\delta = (T_{\psi_\epsilon})_\delta = T_{(\psi_\epsilon)_\delta}$ (by property 1)

$= T_{\psi_\epsilon}$ (since any harmonic function φ sati-

$= T_\epsilon$ sfies $\varphi_\delta = \varphi$ for $\delta > 0$)

and for $\varphi \in C_c^\infty(\mathbb{R}^n)$,

$$
\begin{aligned}
T(\varphi) &= \lim_{\epsilon\to 0} T_\epsilon(\varphi) && (\text{by } P\,375\ ) \\
&= \lim_{\epsilon\to 0} (T_\epsilon)_\delta(\varphi) && (\text{by the conclusion above}) \\
&= \lim_{\epsilon\to 0} T_\epsilon(\varphi_\delta) && (\text{by property } 2) \\
&= T(\varphi_\delta) && (\text{by } P\,375 \text{ again}) \\
&= T_\delta(\varphi) && (\text{by property } 2)
\end{aligned}
$$

$\Rightarrow$ $T = T_\delta$ which is the distribution on $\mathbb{R}^n$ defined by $T_\delta(x)$, see P 374. ⅃

We now extend regularity to the inhomogeneous equation.