is in fact a homomorphism.

Γ

$$\begin{array}{ccc} H^0(M, \mathcal{M}^*/\mathcal{O}^*) & \xrightarrow{\phi} & \mathrm{Div}(M) \\ \Psi & & \\ f & \longmapsto & D = \sum \mathrm{ord}_{V_i}(f_\alpha) V_i \end{array}$$

where $V_i \cap U_\alpha \neq \phi$. $\quad \dfrac{f_\alpha}{f_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta)$

" Given any holomorphic function g, $g \equiv 0$ or locally, $g = f \cdot h$ where f is a Weierstrass polynomial and. $h(p) = 0$.

$\Rightarrow$ The zero set of g projects locally onto some hyperplane as a finite sheet cover branched over the zero set of some analytic function. (See P 8 ~ P 9)

$\Rightarrow$ Locally, near p, $\exists$ only finite number of hypersurfaces V_i's s.t $g = f_i^{a_i} h_i$. $a_i \neq 0$.
where f_i is a local defining function of V_i
Since a meromorphic function f is locally a quotient of two holomorphic functions, $\exists$ only finite # of hypersurfaces V_i's s.t $\mathrm{ord}_{V_i}(f) \neq 0$. "

$$\begin{array}{ccc} \mathrm{Div}(M) & \xrightarrow{\psi} & H^0(M, \mathcal{M}^*/\mathcal{O}^*) \\ D = \sum a_i V_i & \longmapsto & (f_\alpha), \quad f_\alpha = \prod_i g_{i\alpha}^{a_i} \end{array}$$

$$\psi \circ \phi(f) = \psi\left(\sum \mathrm{ord}_V(f_\alpha) \cdot V\right)$$

(f_α, U_α). $\quad V \cap U_\alpha \neq \phi$

We can find an open cover $\{U_{\alpha,\beta}\}$ s.t $\bigcup_\beta U_{\alpha,\beta} = U_\alpha$
$U_{\alpha,\beta} \cap V_i \neq \phi$, $\Rightarrow \exists$ a local defining function $g_{\alpha\beta i} \in \mathcal{O}(U_{\alpha\beta})$