$G(k,n)$ as a linear combination of Schubert cycles, by computing intersections, i.e.,

$$\gamma = \sum \#(\gamma\cdot\sigma_{n-k-a_k,\cdots,n-k-a_1})\cdot\sigma_a,$$

and in particular reduces the problem of computing the intersection of pairs of Schubert cycles in arbitrary dimension to the problem of computing triple intersections in complementary dimension:

$$(\sigma_a\cdot\sigma_b) = \sum \#(\sigma_a\cdot\sigma_b\cdot\sigma_{n-k-c_k,\cdots,n-k-c_1})\cdot\sigma_c .$$

⌜ $\gamma = \sum x_a\,\sigma_a.$

$\Rightarrow \#(\gamma\cdot\sigma_{n-k-a_k\cdots n-k-a_1}) = x_a$

$\Rightarrow \gamma = \sum \#(\gamma\cdot\sigma_{n-k-a_k\cdots n-k-a_1})\cdot\sigma_a .$

Suppose $\sigma_a\cdot\sigma_b = \sum x_c\,\sigma_c$

$\Rightarrow x_c = \#(\sigma_a\cdot\sigma_b\cdot\sigma_{n-k-c_k,\,n-k-c_{k+1},\cdots n-k-c_1})$ ⌟

As an example, for any hypersurface $W \subset \mathbb{P}^n$ of degree 2, let $\tau(W) \subset G(2,n+1)$ denote the set of lines in $\mathbb{P}^n$ lying on W. $\tau(W)$ is clearly an analytic cycle in $G(2,n+1)$, and since a line $l \subset \mathbb{P}^n$ lies on $W \Leftrightarrow$ three points of l lie on W, $\tau(W)$ has complex codimension 3.

⌜ $I = \{1\le i_1, i_2 \le n+1\}$, $\quad U_I \cap \tau(W) \xrightarrow{\varphi} \mathbb{C}^{2(n-1)}$

$\qquad\qquad\qquad\qquad\qquad\qquad \overset{\cup}{\Lambda}$

$\Lambda \in U_I \Rightarrow \Lambda \in G(2,n+1)$ and Λ is expressed by

$$\begin{array}{c} \begin{array}{ccccccccc} 1 & & & i_1 & & i_2 & & & n+1 \end{array} \\ \begin{pmatrix} * & *\cdots* & 1 & *\cdots* & 0 & * & \cdots & * \\ * & *\cdots* & 0 & *\cdots* & 1 & * & \cdots & * \end{pmatrix} \end{array} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \quad v_1, v_2 \in \mathbb{C}^{n+1}$$