$$(*) \qquad \varphi = \sum_i \lambda_i \frac{df_i}{f_i} + dg.$$

⌈ When we choose a polycylindrical nbd W of $x_0 \in M$, we may assume that, for example, $n=2$

$$\varphi = \frac{h_1(z)}{g_1(z)} dz_1 + \frac{h_2(z)}{g_2(z)} dz_2$$

h_1 & g_1 are relatively prime in $\mathcal{O}_z$, for all $z \in W$
h_2 & g_2 " " "
by the proposition on P10.

$\Rightarrow \quad g_1 = f_{11} \cdots f_{1k} \cdot g_1'$, $\quad g_1'(x_0) \neq 0 \quad f_1$: irreducible
$\quad\;\; g_2 = f_{21} \cdots f_{2\ell} \cdot g_2'$, $\quad g_2'(x_0) \neq 0. \quad f_{2,j}$ "

since $\mathcal{O}_{2,x_0}$ is a UFD by P10.
If necessary, choose a polycylindrical nbd W' smaller than W, we can assume that $D_1 = (f_{11}=0)$
$D_2 = (f_{12}=0)$ .... $D_k = (f_{1k}=0)$ .. $D_{k+1} = (f_{21}=0)$ ...
$D_{k+\ell} = (f_{2\ell}=0)$ are irreducible in W', by P12.

Since h_1 & g_1 are relatively prime, if $f_{1i}(x_0)=0$ then $h_1(x_0) \neq 0$.

Assume that D_i's are distinct, $D_i = (f_i = 0)$

$$\Rightarrow \frac{1}{2\pi\sqrt{-1}} \int_{\gamma_i} \varphi = \frac{1}{2\pi\sqrt{-1}} \int_{\gamma_i} \frac{h'}{f_i} dz_j,$$

$$= \begin{cases} h'(z_0) \Big/ \frac{\partial f_i}{\partial z_i}(z_0) = \lim_{z \to z_0} \frac{f_i(z_1,z_2)}{z_i} & \text{where } z_0 \text{ is a smooth pt of } D_i, \text{ if } i=j \\ 0 & \text{otherwise.} \end{cases}$$

And $$\frac{1}{2\pi\sqrt{-1}} \int_{\gamma_i} \frac{df_i}{f_i} = 1.$$

For example, $f_i(z_1, z_2) = z_1\, k(z_1, z_2)$. $\qquad x_0 = (0,0)$,