Before we go further, we have to verify that

$$\det \begin{pmatrix} \alpha_1, & \alpha_2, & \alpha_3 \\ \beta_1, & \beta_2, & \beta_3 \\ \gamma_1, & \gamma_2, & \gamma_3 \end{pmatrix} \neq 0$$

to conclude the existence of (C_{ij}).

Assume that the matrix is of det 0.

$\Rightarrow \exists$ m, n, l s.t

$$(z_1^{\alpha_1} z_2^{\alpha_2} z_3^{\alpha_3})^m (z_1^{\beta_1} z_2^{\beta_2} z_3^{\beta_3})^n = (z_1^{\gamma_1} z_2^{\gamma_2} z_3^{\gamma_3})^l$$

m, n, l integers.

$\Rightarrow$ This implies that $\{z_1^{\alpha_1} z_2^{\alpha_2} z_3^{\alpha_3} = z_1^{\beta_1} z_2^{\beta_2} z_3^{\beta_3} = 0\} \supset \{z_1^{\gamma_1} z_2^{\gamma_2} z_3^{\gamma_3} = 0\}$. $\Rightarrow$ $\operatorname{codim}\{z_1^{\alpha_1} z_2^{\alpha_2} z_3^{\alpha_3} = z_1^{\beta_1} z_2^{\beta_2} z_3^{\beta_3} = 0\} = \operatorname{cod}\{f_1 = f_2 = 0\} \geq 1.$, which contradicts to the fact that $\operatorname{codim}\{f_1 = f_2 = f_3 = 0\} = \operatorname{codim}\{f_1 = f_2 = 0\} = 0$ since $f^{-1}(0) = \{0\}$. This can be applied to the general case n.

$$\begin{tikzcd} \phi : \{z : |z^{\alpha} g_1| = |z^{\beta} g_2| = |z^{\gamma} g_3| = \epsilon\} \arrow[d] & \ni z \arrow[d] \\ \{w : |w^{\alpha}| = |w^{\beta}| = |w^{\gamma}| = \epsilon\} & \ni (z_1 \cdots, \end{tikzcd}$$

is diffeomorphic since

$$|J_0(\phi)| = \begin{vmatrix} * & * & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{vmatrix} \neq 0$$