a complete set of eigenforms for Δ_N, then the forms

$$\psi_i \otimes \eta_\alpha \quad \text{are eigenforms for } \Delta_{M\times N}.$$

By the lemma, they form a complete set.

⌈ Given any form $A^{p,q}(M\times N)$, it can be approximated by a decomposable form, and the decomposable form can be expressed as sum of $\psi_i \otimes \eta_\alpha$'s.
$\Rightarrow \{\psi_i \otimes \eta_\alpha\}$ is a dense set in $A^{p,q}(M\times N)$
$\Rightarrow$ By Th 4.18 (P90), Rudin. RCA, $\{\psi_i \otimes \eta_\alpha\}$ is a complete set of $A^{p,q}(M\times N)$, actually, the completion of $A^{p,q}(M\times N)$ with L^2-norm. ⌋

If $\Delta_M \psi_i = \lambda_i \psi_i, \quad \lambda_i \geq 0.$
$\Delta_N \eta_\alpha = \mu_\alpha \eta_\alpha, \quad \mu_\alpha \geq 0,$

then $\Delta_{M\times N}(\psi_i \otimes \eta_\alpha) = (\lambda_i + \mu_\alpha)(\psi_i \otimes \eta_\alpha).$

Since $\lambda_i + \mu_\alpha = 0 \Leftrightarrow \lambda_i = \mu_\alpha = 0$, the assertion (**) about the Harmonic forms follows.

Q.E.D for Künneth.

⌈ (**) $\mathcal{H}^{u,v}(M\times N) \cong \bigoplus_{\substack{p+r=u\\ q+s=v}} \left(\mathcal{H}^{p,q}(M) \otimes \mathcal{H}^{r,s}(N)\right)$

$$\bigoplus_{\substack{p+r=u\\ q+s=v}} \mathcal{H}^{p,q}(M) \otimes \mathcal{H}^{r,s}(N) \xrightarrow{\ \phi\ } \mathcal{H}^{u,v}(M\times N)$$
$$\psi_i \otimes \eta_\alpha \longmapsto \psi_i \otimes \eta_\alpha$$

ϕ is one to one onto on a dense set.

$\Rightarrow \phi$ can be extend to an isomorphism. ⌋