$$A \cong \frac{(H^{1,0}(A))^*}{H_1(A,\mathbb{Z})} \cong \frac{(H^{1,0}(B_L))^*}{H_1(B_L,\mathbb{Z})} = J(B_L)$$ see P327

$$(H^{1,0}(B_L))^* = (H^0(B_L,\Omega^1))^* = H^1(B_L,\mathcal{O}) = H^{0,1}(B_L).$$

Note that since the analytic representative $B_L$ of the cohomology class $[B_L] \in H^2(A,\mathbb{Z})$ is unique up to translations, all the curves $B_L \subset A$ are translates of one another. Hence all the curves $B_L$ are smooth, and $A = J(B_L)$ for any $L \in A$.

see P957 note.

To relate the various curve $B_L$ on $A$, let $L_0$ be one of the 16 lines in $j'^{-1}(R^*)$ and take $L_0$ to be the origin in $A$. Since clearly

$$\iota'(L) \in B_L \quad \text{for } L \neq \iota'(L),$$

by continuity,

$$L_0 \in B_{L_0};$$

so we can also take $L_0$ as base point on the curve $B_{L_0}$.

By the definition $\iota'(L) \cup L = \sigma(h) \cap X$ for some