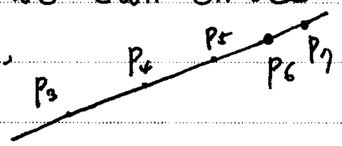
We have 5 equations and 6 unknown a_i's. $\Rightarrow \exists$ a nontrivial solution, i.e., not all a_i's zero. $\Rightarrow$ This implies that such C exists. As in the original case, the cubics

$$C + L_{34}, \quad C + L_{35}, \quad C + L_{45}$$

each contain all eight points; since each of the points P_3, P_4 and P_5 lies outside one of the lines L^{34}, L_{35} and L_{45}, it follows that C contains all eight.

Here we can choose three noncollinear points $\neq p_1, p_2$.

If not, 

$\Rightarrow$ some five are collinear.

We leave to the reader the proof of the lemma in three additional cases: when p_1 and p_2 are infinitely near p_3, when p_1 is infinitely near p_2 and p_3 is infinitely near p_4, and when p_1 is infinitely near p_2, which is itself infinitely near p_3.

① p_1 & p_2 are infinitely near p_3.

First, we want to show that

(i) Any seven points p_j $p_3 . p_4 \cdots p_8$ with $p_j (= p_1 \text{ or } p_2)$ infinitely near p_3, impose independent conditions on cubics unless five are collinear.

(ii) Any seven points $p_1, p_2, p_3, \cdots p_7$ with p_1 and p_2 infinitely near p_3, impose independent conditions on cubics