

$$f: M \longrightarrow \Delta^{n+1}$$

that has maximal rank n at some point $p_0 \in M^*$. We let $W \subset M$ be the union of the singular set of M and subvariety where the Jacobian of f has rank $< n$.

$$\Gamma \quad W = M_s \cup K, \quad K = \{x \in M: J(f)(x) \text{ has rank } < n\}$$

By the induction assumption $f(W)$ is an analytic subvariety of codimension ≥ 2 in Δ^{n+1} .

Γ M_s is an analytic subvariety of M by P21.

K is " " M , too, since

for $\forall p \in M$, $\exists U \cong \mathbb{C}^n$ s.t.

$$U \cap K = \left(\det \left(\frac{\partial f_i}{\partial z_j} \right) = 0 \right), \text{ where } \left(\frac{\partial f_i}{\partial z_j} \right)$$

is a $\begin{pmatrix} n & n \\ k & k \end{pmatrix}$ -minor matrix of the $J(f)$ on U .

The image of a sufficiently small neighborhood of a point $p \in M - W$ is a piece of smooth analytic hypersurface in Δ^{n+1} , and the closure

$$\overline{f(M-W)} = f(M-W) \cup f(W).$$

Γ We have only to consider the following case

$$\mathbb{C}^2 \xrightarrow{f} \Delta^3 \text{ with } \left(\frac{\partial f_i}{\partial z_j} \right) \text{ has rank } \geq 2 \text{ al } \det \left(\frac{\partial f_i}{\partial z_j} \right) \neq 0, 1 \leq i, j \leq 2, \\ f(0) = 0 \quad \text{at the origin.}$$