m of its image $\pi(C)$ in M; thus

(*) $\quad Div(\tilde{M}) = \pi^* Div(M) \oplus \mathbb{Z}\{E\}.$

「 $C \cap E$ = Set of points since $C \not\supset E$, refer to P64

$\pi: \tilde{M} - E \longrightarrow M - \{p\}$ is isomorphic

$\quad \cup \qquad\qquad \cup$

$\quad C - E \longrightarrow \pi(C) - \{p\}$

$\Rightarrow \pi(C)$ is an algebraic subvariety of M by the proper mapping theorem and clearly $C = \overline{\pi^{-1}(\pi(C) - \{p\})}$, which implies that C is the proper transform of $\pi(C)$.

Given any divisor C on $\tilde{M}$, then (i) C contains not E (ii) $C \supset E$.

(i) $C = \pi^*(\pi(C)) + mE$ by $\widetilde{\pi(C)} = \pi^*(\pi(C)) - mult_p(\pi(C)) \cdot E$ for some $m \in \mathbb{Z}$.

(ii) $C \supset E$. & C irreducible. $\Rightarrow$ $C = E$.

Thus from (i) & (ii), $Div(\tilde{M}) = \pi^* Div(M) \oplus \mathbb{Z}\{E\}$. 」

We can now compute intersection numbers readily. We have seen in Chapter 1 that the normal bundle to E in $\tilde{M}$ is just the dual of the hyperplane bundle on $E \cong \mathbb{P}^1$; thus

$$(E \cdot E) = deg([E]|_E) = deg(N_E) = -1.$$

「 By the Adjuction Formula I on P146, $N_E^* = [-E]|_E$, where $E \subset \tilde{M}$, and N_E is the normal bundle to E.

$\Rightarrow$ By the note on P185, $[-E]|_E$ is the hyperplane bundle on E. $\Rightarrow$ N_E is the dual bundle of the hyperplane bundle on E.