since $*$ is an algebraic operator, not differential operator.

$\Rightarrow \quad \partial^*(f\,\varphi_I \wedge \overline{\varphi_J}) = -\sum \bar{\partial}_k\, i_k(f\,\varphi_I \wedge \overline{\varphi_J})$ at z_0.

$\qquad = -\sum \bar{\partial}_k f\, i_k(\varphi_I \wedge \overline{\varphi_J})$ at z_0.

Thus $\quad [\Lambda, \bar{\partial}] \overset{?}{=} -i\,\partial^*$.

On $\mathbb{C}^n$, $\quad [\Lambda_E, \overline{\partial_E}] = -i\,\partial_E^*$,

Question is $\quad [\Lambda, \bar{\partial}] = -i\,\partial^*$ on M ?

Choose $\{\varphi_1 \cdots \varphi_n\}$ s.t at z_0, $d\varphi_i$'s $= 0$.

$\Rightarrow \quad \Lambda\partial = \Lambda_E \partial_E$ at z_0.

$\qquad \partial\Lambda = \partial_E \Lambda_E$ at z_0

and. $\quad -i\,\partial_E^* = -i\,\partial$ at z_0.

$\ulcorner$ Let's make more clear. on next pages. $\lrcorner$