⌈ $E_2^{0,1} = H^1_{DR}(S^1) \otimes H^0_{DR}(\mathbb{P}^n) = H^1_{DR}(S^1) \cong \mathbb{C}$.

If d_2 is a zero map, then $E_2^{0,1} = E_3^{0,1} = \cdots = E_\infty^{0,1}$.

$\Rightarrow \dfrac{H^q(S^{2n+1})}{F^1H^q(S^{2n+1})} = E_\infty^{0,q}$

$\Rightarrow$ For $q=1$, $H^1(S^{2n+1}) = E_\infty^{0,1} \oplus F^1H^1(S^{2n+1})$.

$\Rightarrow 0 = E_2^{0,1} \oplus F^1H^1(S^{2n+1}) \Rightarrow E_2^{0,1}$ must be zero

$\Rightarrow$ Contradiction $\Rightarrow d_2\eta \neq 0$. ⌋

Thus $d_2\eta = \omega$, where $\omega \in E_2^{2,0} \cong H^2(\mathbb{P}^n)$ is a generator.

⌈
$$\begin{array}{ccc} E_2^{0,1} & \xrightarrow{d_2} & E_2^{2,0} \\ \parallel & & \parallel \\ \mathbb{C} & & H^0(S^1)\otimes H^2(\mathbb{P}^n) = \mathbb{C} \\ \parallel & & \\ H^1(S^1) & & \end{array}$$
⌋

If we represent S^{2n+1} as the unit sphere $\{z : \|z\|=1\}$ in $\mathbb{C}^{n+1}$, then
$$\omega = dd^c \log \|z\|^2$$
is the standard Kähler form on $\mathbb{P}^n$.

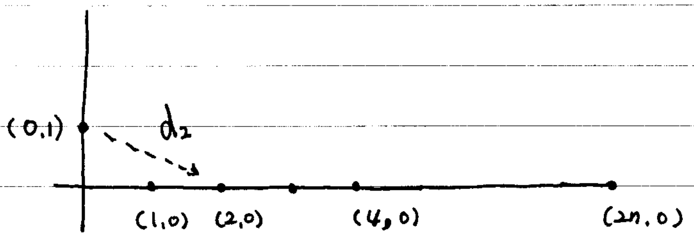

Figure 6.

Up on S^{2n+1}, $\omega = d\eta$, where it is straightforward to