$L \cap L' = \sigma(p_L, h_L) \cap \sigma(p_{L'}, h_{L'}) \ni \ell \Rightarrow \ell \ni p_{L'}$ and $\ell \subset h_L$.

The map

$$j : A \longrightarrow S$$

sending each line $L \subset X$ to its focus p_L thus maps the curve B_L

$$j : B_L \longrightarrow h_L \cap S$$

onto the hyperplane section $h_L \cap S$ of S; $j|_{B_L}$ is clearly generically one to one.

⌈ $K = \{ L' \subset A \mid L' \cap L \neq \phi \} \ni L'$. $\Rightarrow L' = \sigma(p', h')$
$\Rightarrow$ Since $\sigma(p', h') \cap \sigma(p_L, h_L) \neq \phi$, $\exists \ \ell_x \ni p', p_L$.
$\Rightarrow \ell_x = \overline{p' p_L}$ and $\ell_x \subset h_L \Rightarrow p' \in h_L$
$\Rightarrow \quad j(L') = p' \in h_L \cap S$.

Given a point $p \in h_L \cap S$, consider $\sigma(p)$ and $\sigma(p_L, h_L)$. $\Rightarrow \sigma(p) \cap \sigma(p_L, h_L) \ni \overline{p\, p_L}$
$\Rightarrow$ Since $\sigma(p_L, h_L) \subset X$, $\overline{p\, p_L} \in X$. $\Rightarrow$
$\sigma(p) \cap \sigma(p_L, h_L) \cap X \ni \overline{p\, p_L} \Rightarrow \overline{p\, p_L} \in \sigma(p, h_1)$ or $\sigma(p, h_2)$, where $\sigma(p) \cap X = \sigma(p, h_1) \cup \sigma(p, h_2)$.
$\Rightarrow j$ is onto $h_L \cap S$. Since $B_L = \overline{K}$, $j(B_L) = h_L \cap S$.

Suppose that $j|_{B_L}$ is not generically one to one.
$\Rightarrow \exists$ dense set $U \subset h_L \cap S - R$, s.t for each $p \in U$.
$\sigma(p) \cap X = \sigma(p, h_1) \cup \sigma(p, h_2)$, $\sigma(p, h_i) \cap \sigma(p_L, h_L) \neq \phi$.