$$\Rightarrow \quad v \wedge \left(\frac{\partial}{\partial z}\right)_J = v^{i_\alpha} (-1)^{\alpha-1} \left(\frac{\partial}{\partial z}\right)_I.$$

$$\Rightarrow \quad \langle f(z)\, dz_I,\ v \wedge \left(\tfrac{\partial}{\partial z}\right)_J \rangle = v^{i_\alpha} (-1)^{\alpha-1}.$$

② $\#(J \cap I) \leq p-2$. $\Rightarrow$ $\langle f(z)\, dz_I,\ v \wedge (\frac{\partial}{\partial z})_J \rangle = 0$
since no term of $v \wedge (\frac{\partial}{\partial z})_J$ is of type $dz_I$. $\Rightarrow b_J = 0$

Thus $\iota(v)(f(z)\, dz_I) = \sum_\alpha (-1)^{\alpha-1} v^{i_\alpha}(z) f(z)\, dz_{I-\{i_\alpha\}}$. ⌟

In particular, since the coefficient functions $v^i$ are holomorphic it follows from sign considerations that

$$\bar\partial \cdot \iota(v) + \iota(v) \cdot \bar\partial = 0.$$

⌜ $$\bar\partial\, \iota(v)(f(z)\, dz) = \bar\partial \sum_\alpha (-1)^{\alpha-1} v^{i_\alpha}(z) f(z)\, dz_{I-\{i_\alpha\}}.$$

$$= \sum_\alpha (-1)^{\alpha-1} v^{i_\alpha}(z)\, \bar\partial f(z) \wedge dz_{I-\{i_\alpha\}} \quad \text{--- ①}$$

Put $\iota(v)\left(\frac{\partial f}{\partial \bar z_\ell} d\bar z_\ell \wedge dz_I\right) = \iota(v)\left(\bar\partial f \wedge dz_I\right) =$

$$= \sum_{K,J} b_{KJ}\, d\bar z_K \wedge dz_J$$

$$\Rightarrow \langle \sum b_{KJ}\, d\bar z_K \wedge dz_J,\ \eta \rangle = \langle \sum_\ell \frac{\partial f}{\partial \bar z_\ell} d\bar z_\ell \wedge dz_I,\ v \wedge \eta \rangle$$

Plug in $\eta = \frac{\partial}{\partial \bar z_K} \wedge \frac{\partial}{\partial z_J}$.

$LHS = b_{KJ}$ $\qquad RHS = \langle \sum_\ell \frac{\partial f}{\partial \bar z_\ell} d\bar z_\ell \wedge dz_I,\ v \wedge \frac{\partial}{\partial \bar z_K} \wedge \frac{\partial}{\partial z_J} \rangle$

$$= \begin{cases} \dfrac{\partial f}{\partial \bar z_K} (-1)^{\alpha-1} \underset{\uparrow \text{ because of } \frac{\partial}{\partial \bar z_K}.}{(-1)} v^{i_\alpha}(z) & \text{if } J = I - \{i_\alpha\} \\ 0 & \text{otherwise.} \end{cases}$$

$$\Rightarrow \quad b_{KJ} = \begin{cases} (-1)^\alpha \dfrac{\partial f}{\partial \bar z_K} v^{i_\alpha}(z) & \text{if } J = I - \{i_\alpha\} \\ 0 & \text{otherwise.} \end{cases}$$

$$\Rightarrow \quad \iota(v)(\bar\partial f \wedge dz_I) = \sum_\alpha (-1)^\alpha\, \bar\partial f \wedge v^{i_\alpha}(z)\, dz_{I-\{i_\alpha\}}. \quad \text{--- ②}$$

$\Rightarrow$ ① + ② $= 0$ ⌟