Proof. Since $\mathcal{E}'(-k)$ is locally free, $\underline{\mathrm{Ext}}^1_{\mathcal{O}}(\mathcal{E}'(-k), \cdot) = 0$, and so the exact sequence of $\underline{\mathrm{Ext}}$'s gives

$$0 \to \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{R}) \longrightarrow \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{E}) \longrightarrow \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{F}) \to 0.$$

⊓ By P686, $\mathrm{Ext}^q_{\mathcal{O}}(M, N) = 0$ for a free M-module. and from the exact sequence

$$\underset{\substack{\| \\ 0}}{\mathrm{Ext}^1_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{F})} \to \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{R}) \to \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{E}) \to \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{F}) \to 0$$

we get the sequence above. ⊔

By Theorem B,

$$H^1(M, \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{R})) = H^1(M, \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}', \mathcal{R})(k)) = 0$$

for $k \geq 0$.

⊓ $\mathrm{Hom}_R(M_1 \otimes M_2, N) \cong M_1^* \otimes M_2^* \otimes N = M_1^* \otimes N \otimes M_2^*$
$= \mathrm{Hom}_R(M_1, N) \otimes M_2^*$
$\Rightarrow \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k)_x, \mathcal{R}_x) = \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'_x \otimes \mathcal{L}_x^{-k}, \mathcal{R}_x) \cong \mathcal{E}'^*_x \otimes \mathcal{L}_x^{-k*} \otimes \mathcal{R}_x$
$= \mathcal{E}'^*_x \otimes \mathcal{R}_x \otimes \mathcal{L}_x^{k} = \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'_x, \mathcal{R}_x) \otimes \mathcal{L}_x^{k} = (\mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'_x, \mathcal{R}) \otimes \mathcal{L}^k)_x$
$= \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}', \mathcal{R})(k)_x$
$\Rightarrow$ By Theorem B, on P 700, $H^1(M, \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{R})) =$
$H^1(M, \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}', \mathcal{R})(k)) = 0$. ⊔

Consequently, we obtain a surjection

$$H^0(M, \mathrm{Hom}_{\mathcal{O}}(\mathcal{E}'(-k), \mathcal{E})) \longrightarrow H^0(M, \mathrm{Hom}(\mathcal{E}'(-k), \mathcal{F})) \longrightarrow 0.$$

Q.E.D.