The cohomology of the complex is
$$H^*(K^*) = \bigoplus_{p\geq 0} H^p(K^*),$$
where
$$H^p(K^*) = \frac{Z^p}{dK^{p-1}}$$
with $Z^p = \ker\{d : K^p \longrightarrow K^{p+1}\}$ the group of cycles and $dK^{p-1} = B^p \subset Z^p$ the subgroup of boundaries. A subcomplex (J^*, d) is given by subgroups $J^p \subset K^p$ with $dJ^* \subset J^*$. The quotient complex (L^*, d) is defined by $L^* = K^*/J^*$ with the obvious differential. We then have an exact sequence of complexes
$$0 \longrightarrow J^* \longrightarrow K^* \longrightarrow L^* \longrightarrow 0;$$
by an easy and well-known argument, this gives rise to a long exact cohomology sequence
$$\cdots \longrightarrow H^p(J^*) \longrightarrow H^p(K^*) \longrightarrow H^p(L^*) \longrightarrow H^{p+1}(J^*) \rightarrow \cdots.$$
Generalizing the notion of a subcomplex is that of a filtered complex (F^pK^*, d), defined as a decreasing sequence of subcomplexes
$$K^* = F^0K^* \supset F^1K^* \supset \cdots \supset F^nK^* \supset F^{n+1}K^* = \{0\}.$$
The single subcomplex mentioned above corresponds to the filtration $K^* \supset J^* \supset \{0\}$,
and the spectral sequence of a filtered complex will generalize the long exact cohomology sequence. Before coming to this, we need a few more definitions.

The associated graded complex to a filtered complex (F^pK^*, d) is the complex
$$\mathrm{Gr}\, K^* = \bigoplus_{p\geq 0} \mathrm{Gr}^p K^*$$