need to change the order of integration at all.

Proof of 2. Since T is linear,

$$
\begin{aligned}
T(\psi_\epsilon) &= T_y\left(\int \psi(x)\, \chi_\epsilon(x-y)\, dx\right)\\
&= \int \psi(x)\, T_y(\chi_\epsilon(x-y))\, dx\\
&= \int \psi(x)\, T_\epsilon(x)\, dx = T_\epsilon(\psi).
\end{aligned}
$$

$$T(\psi_\epsilon) = T_y(\psi_\epsilon(y)) = T_y\left(\int \psi(x)\, \chi_\epsilon(y-x)\, dx\right)$$

$= T_y\left(\int \psi(x)\, \chi_\epsilon(x-y)\, dx\right)$ since $\chi(x) = \chi(\|x\|)$.

$= \int \psi(x)\, T_y(\chi_\epsilon(x-y))\, dx$, by the linearity of T.

$$= \int \psi(x)\, T_\epsilon(x)\, dx = T_\epsilon(\psi)$$

Proof of 3. We may suppose that $D = \partial/\partial x_i$. If $T = T_\psi$ for $\psi \in C^\infty(\mathbb{R}^n)$ and $\varphi \in C_c^\infty(\mathbb{R}^n)$,

$$
\begin{aligned}
DT_\epsilon(\varphi) &= T_\epsilon(-D\varphi)\\
&= \iint -\frac{\partial \varphi}{\partial x_i}(x)\, \chi_\epsilon(x-y)\, \psi(y)\, dx\, dy\\
&= \int \chi_\epsilon(u)\left(\int -\frac{\partial \varphi}{\partial x_i}(x)\, \psi(x-u)\, dx\right) du\\
&= \int \chi_\epsilon(u)\left(\int \varphi(x)\, \frac{\partial \psi}{\partial x_i}(x-u)\, dx\right) du\\
&= \iint \frac{\partial \psi}{\partial y_i}(y)\, \varphi(x)\, \chi_\epsilon(x-y)\, dx\, dy\\
&= (DT)_\epsilon(\varphi).
\end{aligned}
$$