is finite.

A curve forms a cycle in $\mathbb{P}^n$, which represents an element of $H_2(\mathbb{P}^n)$. The element is non-zero, since any curve is an analytic subvariety of $\mathbb{P}^n$. $\Rightarrow$ By P64, the fundamental class of the curve is nonzero in the homology of $\mathbb{P}^n$. $\Rightarrow$ The curve is a multiple of a line $\mathbb{P}^1 \subset \mathbb{P}^n$. $\Rightarrow$ The curve must intersect with $\mathbb{P}^{n-k-1}$.))

Note that if we choose homogeneous coordinates $X = [X_0, X_1, \cdots X_n]$ on $\mathbb{P}^n$ such that $\mathbb{P}^{n-k-1}$ is given as $(X_0 = X_1 = \cdots = X_k = 0)$ and $\mathbb{P}^k$ as $(X_{k+1} = \cdots = X_n = 0)$, the map π is given by

$$\pi([X_0, \cdots X_n]) = [X_0, X_1, \cdots X_k].$$

In particular, the pullback $\pi^* f$ to V of any rational function f on $\mathbb{P}^k$ is clearly rational, so that on V we have inclusions

$$\pi^* K(\mathbb{P}^k) \subset K(V) \subset \mathcal{M}(V).$$

Now, to see that $\pi^* K(\mathbb{P}^k)$ has index at most d in $\mathcal{M}(V)$, let φ be any meromorphic function on V and let $D = (\varphi)_\infty$. Let $B \subset \mathbb{P}^k$ be the branch locus of π; on $\mathbb{P}^k - B$, we can define functions ψ_i by

$$\psi_1(p) = \sum_{q \in \pi^{-1}(p)} \varphi(q),$$