⌈ By the proposition on P690~P691,

$$\begin{tikzcd} \mathrm{Ext}^2_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) \arrow[r] \arrow[d] & \mathcal{O}/I \arrow[d, "\Delta"] \\ \mathrm{Ext}^2_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) \arrow[r] & \mathcal{O}/I \end{tikzcd}$$

$$f_i' = a_{ij} f_j \quad \Rightarrow \quad f_k = b_{ki} f_i'$$

$\Rightarrow \quad f_k = b_{ki} a_{ij} f_j = (BA)_{kj} f_j$

$\Rightarrow$ Since $I = \{f_1, f_2\}$ is regular, $BA = I$.

$\Rightarrow \quad \Delta = \det(a_{ij})$ is a unit. $\Rightarrow \quad \Delta e' = e$ is a unit. ⌋

For our construction of rank-two vector bundles on a surface, we will use the

Lemma. Suppose that $e \in \mathrm{Ext}^1_{\mathcal{O}}(I, \mathcal{O})$ gives an extension

$$0 \longrightarrow \mathcal{O} \longrightarrow E \longrightarrow I \longrightarrow 0.$$

Then E is projective $\Leftrightarrow$ e is a unit.

Proof. The exact sequence of Ext gives

$$\mathrm{Hom}_{\mathcal{O}}(\mathcal{O}, \mathcal{O}) \xrightarrow{\partial} \mathrm{Ext}^1_{\mathcal{O}}(I, \mathcal{O}) \longrightarrow \mathrm{Ext}^1_{\mathcal{O}}(E, \mathcal{O}) \longrightarrow 0.$$

⌈ See the first exact sequence on P686. ⌋

MOOKEUK CO