⸢ $v_0, v_1 \in V. \Rightarrow \exists\ \Lambda(v_0)\ \Lambda(v_1) \in U \Rightarrow t\Lambda(v_0)+(1-t)\Lambda(v_1)$
$\in U \Rightarrow \Lambda(tv_0+(1-t)v_1) \in U \Rightarrow tv_0+(1-t)v_1 \in \Lambda^{-1}(U).$
$\alpha v_0 \Rightarrow \alpha\Lambda(v_0) \in U \Rightarrow \alpha v_0 \in \Lambda^{-1}(U) = V$ if $|\alpha| \leq 1$. ⸥

By (a) of Theorem 6.5, $V$ is open in $\mathcal{D}(\Omega) \Leftrightarrow \mathcal{D}_K \cap V$ is open in $\mathcal{D}_K$, for every $\mathcal{D}_K \subset \mathcal{D}(\Omega)$. This proves the equivalence of (a) & (b). ///

Corollary. Every differential operator $D^\alpha$ is a continuous mapping of $\mathcal{D}(\Omega)$ into $\mathcal{D}(\Omega)$.

proof. Since $\|D^\alpha\phi\|_N \leq \|\phi\|_{N+|\alpha|}$ for $N = 0.1.2.\cdots$, $D^\alpha$ is continuous on each $\mathcal{D}_K$. ///

⸢ Since $D^\alpha$ is continuous on each $\mathcal{D}_K$, by (d) $\Leftrightarrow$ (a), $D^\alpha$ is continuous on $\mathcal{D}(\Omega)$.

$$\begin{array}{cccc} D^\alpha: & \mathcal{D}_K & \longrightarrow & \mathcal{D}(\Omega). \\ & \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ & \phi & \longmapsto & D^\alpha\phi \end{array}$$

$V_N \subset \mathcal{D}(\Omega)$, $(D^\alpha)^{-1}(V_N)$ is open ?
$\Rightarrow \phi \in (D^\alpha)^{-1}(V_N) \Rightarrow D^\alpha\phi \in V_N \Rightarrow \|D^\alpha\phi\|_N = \max\{$
$|D^\beta D^\alpha\phi(x)|;\ x \in \Omega,\ |\beta| \leq N\} \leq \|\phi\|_{N+|\alpha|}.$
$\Rightarrow V_{N+|\alpha|} \subset (D^\alpha)^{-1}(V_N). \Rightarrow D^\alpha$ is continuous at 0.
$\Rightarrow D^\alpha$ is continuous at every point, by P14. Th 1.17. ⸥

6.7 Definition A linear functional on $\mathcal{D}(\Omega)$ which is continuous (with respect to the topology $\tau$ described in Def. 6.3) is called a distribution in $\Omega$.

The space of all distributions in $\Omega$ is denoted by $\mathcal{D}'(\Omega)$.

Note that Theorem 6.6 applies to linear functionals on $\mathcal{D}(\Omega)$.