

$$\begin{aligned}
\frac{\partial}{\partial t} P(\Theta_t) &= \frac{\partial}{\partial t} \tilde{P}(\Theta_t, \dots, \Theta_t) \\
&= k \cdot \tilde{P}\left(\frac{\partial}{\partial t} \Theta_t, \Theta_t, \dots, \Theta_t\right) \\
&= k \tilde{P}(d\eta, \Theta_t, \dots, \Theta_t) - k \tilde{P}(\tilde{\Theta} \wedge \eta + \eta \wedge \tilde{\Theta}, \Theta_t, \dots, \Theta_t) \\
&\quad - 2kt \tilde{P}(\eta \wedge \eta, \Theta_t, \dots, \Theta_t).
\end{aligned}$$

$$\begin{aligned}
\Gamma \quad \Theta_t &= d(\tilde{\Theta} + t\eta) - (\tilde{\Theta} + t\eta) \wedge (\tilde{\Theta} + t\eta) \\
&= d\tilde{\Theta} + t d\eta - t\eta \wedge \tilde{\Theta} - t\tilde{\Theta} \wedge \eta - \tilde{\Theta} \wedge \tilde{\Theta} - t^2 \eta \wedge \eta
\end{aligned}$$

$$\Rightarrow \frac{\partial}{\partial t} \Theta_t = d\eta - (\eta \wedge \tilde{\Theta} + \tilde{\Theta} \wedge \eta) - 2t\eta \wedge \eta.$$

$$\frac{\partial}{\partial t} P(\Theta_t) = \frac{\partial}{\partial t} \tilde{P}(\Theta_t, \dots, \Theta_t), \quad \tilde{P} \text{ is the polarization of } P.$$

$$= \sum_i \tilde{P}(\Theta_t, \dots, \frac{\partial}{\partial t} \Theta_t, \dots, \Theta_t) \quad \text{since } \tilde{P} \text{ is linear}$$

$$= \sum_i \tilde{P}\left(\frac{\partial}{\partial t} \Theta_t, \Theta_t, \dots, \Theta_t\right), \quad \text{since } \tilde{P} \text{ is symmetric and } \frac{\partial}{\partial t} \Theta_t, \Theta_t \text{'s are 2-forms.}$$

$$= k \tilde{P}\left(\frac{\partial}{\partial t} \Theta_t, \Theta_t, \dots, \Theta_t\right) = k \tilde{P}(d\eta - (\eta \wedge \tilde{\Theta} + \tilde{\Theta} \wedge \eta) - 2t\eta \wedge \eta, \dots, \Theta_t)$$

$$\begin{aligned}
&= k \tilde{P}(d\eta, \Theta_t, \dots, \Theta_t) - k \tilde{P}((\eta \wedge \tilde{\Theta} + \tilde{\Theta} \wedge \eta), \Theta_t, \dots, \Theta_t) \\
&\quad - 2kt \tilde{P}(\eta \wedge \eta, \Theta_t, \dots, \Theta_t). \quad \square
\end{aligned}$$

Applying the identity (\*) with  $\theta = \eta$ ,

$$\begin{aligned}
&\tilde{P}(\eta \wedge \eta, \Theta_t, \dots, \Theta_t) - (k-1) \tilde{P}(\eta, \eta \wedge \Theta_t, \Theta_t, \dots, \Theta_t) \\
&= -\tilde{P}(\eta \wedge \eta, \Theta_t, \dots, \Theta_t) - (k-1) \tilde{P}(\eta, \Theta_t \wedge \eta, \Theta_t, \dots, \Theta_t),
\end{aligned}$$

so that