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$$H^0(M, \mathcal{O}_M(L + (\mu-1)H)) \rightarrow H^0(M, \mathcal{O}_M(L + \mu H)) \rightarrow H^0(M, \mathcal{O}_V(L + \mu H)) \rightarrow H^1(M, \mathcal{O}_M(L + (\mu-1)H)) \rightarrow H^1(M, \mathcal{O}_M(L + \mu H)) \rightarrow H^1(M, \mathcal{O}_V(L + \mu H)) \rightarrow \dots$$

$\Rightarrow$  By induction assumption,

$$H^0(M, \mathcal{O}_V(L + \mu H)) = H^0(V, \mathcal{O}(L + \mu H)) \neq 0 \quad (\text{see p157 \& compare with it})$$

$$\Rightarrow \text{By Theorem B, } H^1(M, \mathcal{O}(L + (\mu-1)H)) = 0.$$

$$\Rightarrow H^0(M, \mathcal{O}_M(L + \mu H)) \rightarrow H^0(V, \mathcal{O}(L + \mu H)) \rightarrow 0$$

$$\Rightarrow H^0(M, \mathcal{O}_M(L + \mu H)) \neq 0.$$

Thus  $H^0(M, \mathcal{O}_M(L + \mu H)) \neq 0$ , and the result is proved. Q. E. D.

We now consider for a moment the general problem of analytic cycles. On a compact Kähler manifold  $M$ , the Hodge decomposition

$$H^n(M, \mathbb{C}) = \bigoplus_{p+q=n} H^{p,q}(M)$$

on complex cohomology gives a slightly coarser decomposition of real cohomology

$$H^n(M, \mathbb{R}) = \bigoplus_{\substack{p+q=n \\ p \leq q}} (H^{p,q}(M) \oplus H^{q,p}(M)) \cap H^n(M, \mathbb{R}).$$