a frame $\sigma = (\sigma_1, \sigma_2 \cdots \sigma_k)$ is called holomorphic if each σ_i is, and in terms of a holomorphic frame $\{\sigma_i\}$ any section

$$\sigma(x) = \sum f_i(x)\, \sigma_i(x)$$ is holomorphic $\Leftrightarrow$ the functions f_i are.

One important difference between C^∞ and holo. v.b is this: while there is no naturally defined exterior derivative d on the space of sections of a v.b, on a holo. v.b, E, the $\bar\partial$-operator

$$\bar\partial : A^{p,q}(E) \longrightarrow A^{p,q+1}(E)$$ from E-valued

(p,q)-forms to E-valued $(p,q+1)$-forms is well-defined: we take $\{e_1, e_2, \cdots e_k\}$ any local holo. frame for E over $\mathcal{U}$, write $\sigma \in A^{p,q}(E)$ as

$$\sigma = \sum \omega_i \otimes e_i, \qquad \omega_i \in A^{p,q}(E).$$

and set

$$\bar\partial\sigma = \sum \bar\partial\omega_i \otimes e_i.$$

If $\{e_1', \cdots e_k'\}$ is any other holomorphic frame for E over $\mathcal{U}$, with

$$e_i = \sum g_{ij}\, e_j', \quad \text{then} \quad \sigma = \sum g_{ij}\,\omega_i \otimes e_j'$$

and $$\bar\partial\sigma = \sum \bar\partial(g_{ij}\,\omega_i) \otimes e_j' = \sum g_{ij}\,\bar\partial\omega_i \otimes e_j' = \sum \bar\partial\omega_i \otimes e_i$$

so $\bar\partial\sigma$ does not depend on the frame.

Examples.

M complex manifold, let $T_x(M)$ be the complex tangent space to M at x. For $x \in \mathcal{U} \subset M$ and $\varphi_{\mathcal{U}} : \mathcal{U} \longrightarrow \mathbb{C}^n$