$\Rightarrow$ We may see that $\dim(\Lambda_B \cap \Lambda_{B'}) - n \equiv (-1)^m \pmod 2$,
$I = \{i', \cdots i_m\} \Rightarrow$ Done.

Let

$$\Lambda_B = \begin{pmatrix} 1 & 0 & 0 & 0 & a_1 & a_2 \\ 0 & 1 & 0 & -a_1 & 0 & a_3 \\ 0 & 0 & 1 & -a_2 & -a_3 & 0 \end{pmatrix}. \qquad \Lambda_{B'} = \begin{pmatrix} 0 & 0 & 0 & 1 & a_1 & a_2 \\ -a_1 & 1 & 0 & 0 & 0 & a_3 \\ -a_2 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Note: ① Consider $(x_1, \cdots x_i \cdots x_j \cdots x_n)$ & $(x_1 \cdots \underset{i}{x_j} \cdots \underset{j}{x_i} \cdots x_n)$
$\Rightarrow (x_1 \cdots x_i \cdots x_j \cdots x_n) - (x_1 \cdots x_j \cdots x_i \cdots x_n) = (0, 0, \cdots 0\ \underset{i}{x_i - x_j} \cdots \underset{j}{x_j - x_i}, 0 \cdots 0)$
$= (x_i - x_j)(\overline{0, 0, \cdots \underset{i}{1}}\ \ \overline{0 \cdots 0, \underset{j}{-1}}, 0 \cdots 0)$

② Let $v_1 = (1, 0 \cdots 0\ a_1\ a_2)$ $v_2 = (0, 1, 0, -a_1, 0, a_3)$
$v_3 = (0, 0, 1, -a_2, -a_3, 0)$ $w_1 = (0, 0, 0, 1, a_1, a_2)$
$w_2 = (-a_1, 1, 0, 0, 0, a_3)$ $w_3 = (-a_2, 0, 1, 0, 0, 0)$, $v = (1\,0\,0, -1, 0\,0)$
$\Rightarrow \{w_1, w_2, w_3, v\}$ is linearly independent since w_1 & w_2 can take care of itself's respectively.

③ $\Lambda_B \cap \Lambda_{B'} \ni x_1 v_1 + x_2 v_2 + x_3 v_3 = y_1 w_1 + y_2 w_2 + y_3 w_3$
$\Rightarrow x_1(v_1 - w_1) + x_2(v_2 - w_2) + x_3(v_3 - w_3) = (-x_1 + y_1) w_1 + (-x_2 + y_2) w_2$
$+ (y_3 - x_3) w_3 = (x_1 + a_1 x_2 + a_2 x_3) v = (y_1 - x_1) w_1 + (y_2 - x_2) w_2$
$+ (y_3 - x_3) w_3$
$\Rightarrow y_1 = x_1$ $y_2 = x_2$, $y_3 = x_3$ and $x_1 + a_1 x_2 + a_2 x_3 = 0$
$\Rightarrow \dim(\Lambda_B \cap \Lambda_{B'}) = \dim\{(x_1, x_2, x_3) \mid x_1 + a_1 x_2 + a_2 x_3 = 0\}$
$= 1 = 2 - 1 = \dim\{(y_1, y_2, y_3) \mid y_1 + a_1 y_2 + a_2 y_3 = 0\}$ $\lrcorner$

We consider now the family of k-planes on a smooth quadric F in $\mathbb{P}^{n+1}$. The dimension of this family