

$P_1, P_2, P_3, P_4, P_5, \dots, P_8$  fail to impose independent conditions on cubics. Assume no five are collinear.

①  $P_5, P_6, P_7, P_8$  non collinear, say  $P_6, P_7, P_8$ .

Choose a conic  $\tau$  containing  $P_1, P_2, P_3, P_4, P_5$ . by (i)

Consider  $\tau + L_{68}, \tau + L_{78}, \tau + L_{67}$ , and  $\tau + L_{68} \Rightarrow P_7 \dots$  ✓

$\Rightarrow$  By the collinearity,  $P_7, P_8, P_6 \in \tau$ .

Here we can always choose such  $\tau$  since

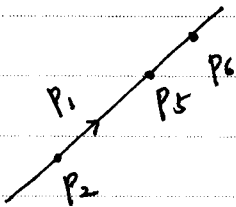
$$\left( \frac{\partial \tau}{\partial x}, \frac{\partial \tau}{\partial y} \right) \cdot P_1 = 0 \quad \left( \frac{\partial \tau}{\partial x}, \frac{\partial \tau}{\partial y} \right) \cdot P_3 = 0$$

$\tau(P_2) = \tau(P_4) = \tau(P_5) = 0$  are 5-equations.

Thus it remains to prove (i).

Consider  $L = L_{12}$ .

①  $L$  contains two points, say  $P_5, P_6$ .



$L + L_{67} + L_4$  where  $L_4$  is a line passing  $P_4$  missing all the other points.

$\Rightarrow L + L_{67} + L_4 \ni P_1, P_2, P_4, P_5, P_6, P_7$ .

$\Rightarrow$  As we observed on P747 note,  $P_1, P_2, P_4, P_5, P_6, P_7$  impose independent conditions unless no five are collinear.

Implicitly, we assumed that  $P_1, P_2, P_3, P_4, P_5, P_6, P_7$  fail to impose independent conditions. And we are going to show that  $\exists$  five collinear points.