Since D^* is compatible with the complex structure, $D^{*''} = \bar\partial$.
$\Rightarrow$ The two operators agree on the factor $A^{1,0} \otimes A^{0,1}$.

As will now be seen, this gives us an effective means of computing the connection matrix of D.

Let $ds^2 = \sum h_{i\bar j}\, dz_i \otimes d\bar z_j = \sum \varphi_i \otimes \bar\varphi_i$ be the hermitian metric on M.

lemma: $\exists$ a unique matrix ψ_{ij} of 1-forms s.t. $\psi + {}^t\bar\psi = 0$

and $d\varphi_i = \sum_j \psi_{ij} \overset{(\wedge)}{\otimes} \varphi_j + \tau_i$. where τ_i is of type $(2,0)$

$\otimes \longleftrightarrow \wedge$ identification

pf) Write $\psi = \psi' + \psi''$ for the type decomposition of ψ.

$\Rightarrow \bar\partial \varphi_i = \sum \psi_{ij}'' \otimes \varphi_j$ determines ψ'', and

$\psi + {}^t\bar\psi = 0 \Rightarrow \psi' = -{}^t\bar\psi''$.

Explicitly, $\varphi_i = \sum a_{ij}\, dz_j$. where $a\,{}^t\bar a = h$, (φ_i orthonormal)

$\Rightarrow \bar\partial\varphi_i = \sum_k \bar\partial a_{ik} \otimes dz_k \overset{\text{identification}}{=} \sum \bar\partial a_{ik} \wedge dz_k$

$= \sum_{j,k} \bar\partial a_{ik} \otimes a_{kj}^{-1} \varphi_j$. $\Rightarrow$ $\psi'' = \bar\partial a\, a^{-1}$))

Note: $\wedge T'^*M \otimes \wedge T''^*M = \wedge T'^*M \wedge \wedge T''^*M$

$\varphi_i = \sum a_{ij}\, dz_j$. $\quad \sum h_{i\bar j}\, dz_i \otimes d\bar z_j = \sum \varphi_i \otimes \bar\varphi_i$

$= \sum_i \left(\sum_j a_{ij}\, dz_j \otimes \sum_j \bar a_{ij}\, d\bar z_j \right) = \sum_{i,k,l} a_{ik}\, dz_k \otimes \bar a_{il}\, d\bar z_l$

$= \sum_i \sum_{k,l} a_{ik} \bar a_{il}\, dz_k \otimes d\bar z_l = \sum_{k,l} \sum_i {}^t a_{ki} \bar a_{il}\, dz_k \otimes d\bar z_l$

$= \sum_{k,l} ({}^t a \bar a)_{kl}\, dz_k \otimes d\bar z_l \Rightarrow {}^t a \bar a = h$))