$\Rightarrow \{f_1 = 0\} \cap \{g_1 = g_2\} \supset \Gamma_0$. and $\#(\{f_1=0\} \cap \{g_1 = g_2\}) \leq 12$, since $\deg(g_1 - g_2) = 3$ $\Rightarrow \{f_1 = 0\} \cap \{g_1 = g_2\} = \Gamma_0$

$\Rightarrow$ Let $C_4 = \{f_1 = 0\}$ $\quad C_3 = \{g_1 = g_2\}$.

$\Rightarrow \Gamma_0 = C_4 \cdot C_3$

Now assume that any two curves from $|\mathcal{J}_{\Gamma_0}(4)| = |C|$ have a common component C_0. It is not possible that C_0 is a cubic, since otherwise the residual system $|C - C_0|$ would consist of lines and have dimension ≥ 3.

$\dim |\mathcal{J}_{\Gamma_0}(4)| \geq 3 \Rightarrow \sigma_1 = \tau_0 l_1$, $\sigma_2 = \tau_0 l_2$, $\sigma_3 = \tau_0 l_3$, $\sigma_4 = \tau_0 l_4$ where $(\tau_0) = C_0$, $\{l_1, l_2, l_3, l_4\}$ is linearly independent. Contradiction to $\dim H^0(\mathbb{P}^2, \mathcal{O}(H)) = \binom{2+1}{1} = 3$

Suppose next that C_0 is a conic containing ≤ 9 points from Γ_0. Then the linear system of conics $|C - C_0|$ will pass through ≥ 3 points and have dimension at least 3, which is a contradiction. So 10 or more points from Γ_0 lie on a conic.

Let $\Gamma_0' = \Gamma_0 - C_0 \cap \Gamma_0$. $\Rightarrow \# \Gamma_0' \geq 3$.

$\Rightarrow \dim |\mathcal{J}_{\Gamma_0'}(2)| = \dim |\mathcal{J}_{\Gamma_0}(4)| \geq 3$