

and $F(N)$ for M and N , and consider the double complex

$$(E_*(M) \otimes F_*(N), \partial_M \otimes 1 \pm 1 \otimes \partial_N).$$

Recall from Chapter 3 that there are two spectral sequences with

$${}^{\prime}E_1 = H_*(E_*(M) \otimes_{\mathcal{O}_S} F_*(N), 1 \otimes \partial_N),$$

$${}^{\prime}E_i = H_*(E_i(M) \otimes_{\mathcal{O}} F_i(N), \partial_M \otimes 1),$$

both of which about to the hypercohomology

$$H_*(E.(M) \otimes_F F.(N), \partial_M \otimes 1 \pm 1 \otimes \partial_N).$$

Let $K_{p,q} = E_p \otimes F_q$, $K_n = \bigoplus_{p+q=n} E_p \otimes F_q$.

$$E.(M): \quad E_m \rightarrow E_{m-1} \rightarrow \dots \rightarrow E_0 \rightarrow M \rightarrow 0$$

$\begin{array}{ccccccc} & & & & & \parallel & \parallel \\ & & & & & E_{-1} & E_{-2} \end{array}$

$$F_*(N) : F_{2\ell} \rightarrow F_{2\ell-1} \rightarrow \dots \rightarrow F_0 \rightarrow N \rightarrow 0$$

$\begin{array}{ccccccc} & & & & & \parallel & \parallel \\ & & & & & F_{-1} & F_{-2} \end{array}$

Filtrations

$${}'_F P_{p+q} = \sum_{\substack{p'+q=p+q \\ p' \leq p}} K_{p',q} = K_{p,q} \oplus K_{p-1,q+1} \oplus \dots \oplus K_{0,p+q}$$

$$\bigoplus_{p+q} K_{-1, p+q+1} \oplus K_{-2, p+q+2} = E_p \otimes F_q \oplus E_{p-1} \otimes F_{q+1} \oplus \dots \oplus E_0 \otimes F_{p+q} \oplus E_{-1} \otimes F_{p+q+1} \oplus E_{-2} \otimes F_{p+q+2}$$

$${}''\overline{H}_q K_{p+q} = \sum_{\substack{p+q'=p+q \\ q' \leq q}} K_{p,q'} = K_{p,q} \oplus K_{p+1,q-1} \oplus \dots \oplus K_{p+q,0} \oplus K_{p+q+1,-1}$$

$$= E_p \otimes F_q \oplus E_{p+1} \otimes F_{q-1} \oplus \dots \oplus E_{p+q} \otimes F_0 \oplus E_{p+q+1} \otimes F_{-1}$$

See P 324 Rotman An Introduction To Homological Algebra