$$\mathbb{Z}_{\delta_1} \oplus \mathbb{Z}_{\delta_1} \oplus \cdots \oplus \mathbb{Z}_{\delta_n} \oplus \mathbb{Z}_{\delta_n} \cong \mathbb{Z}_{\delta_1'} \oplus \mathbb{Z}_{\delta_1'} \oplus \cdots \oplus \mathbb{Z}_{\delta_n'} \oplus \mathbb{Z}_{\delta_n'}.$$

$\Rightarrow$ $\delta_1 = \delta_1'$ $\cdots$ $\delta_n = \delta_n'$.

$\Rightarrow$ δ_i's are invariants of Q.

$\lrcorner$

We see from the lemma that if ω is any integral invariant 2-form on $M = V/\Lambda$, we can find a basis $\lambda_1, \cdots \lambda_{2n}$ for Λ s.t. in terms of the dual coordinates $x_1, \cdots x_{2n}$ on V,

$$\omega = \sum_{i=1}^{n} \delta_i \, dx_i \wedge dx_{n+i}, \qquad \delta_i \in \mathbb{Z}.$$

$\ulcorner$ Since ω is an integral invariant 2-form on M, for some basis $\tau_1, \cdots \tau_{2n}$ for Λ and its dual basis $y_1, \cdots y_{2n}$,

$$\omega = \frac{1}{2} \sum p_{ij} \, dy_i \wedge dy_j.$$ Refer to P303.

$\Rightarrow$ By the lemma above, $\exists$ a basis $\lambda_1, \lambda_2, \cdots \lambda_{2n}$ for Λ s.t.

$$q_{ij} = \omega(\lambda_i, \lambda_j) = \omega(a_{il}\tau_l, a_{jk}\tau_k) = a_{il}\, \omega(\tau_l, \tau_k)\, a_{jk}$$
$$= a_{il} \, p_{lk} \, a_{jk} \Rightarrow {}^tAPA = Q.$$

$$Q = \left(\begin{array}{cc|cc} 0 & 0 & \delta_1 & 0 \\ \vdots & 0 & 0 & \delta_n \\ \hline -\delta_1 & 0 & & \\ 0 & -\delta_n & & 0 \end{array}\right) = (q_{ij})$$

Integration of dy_i over the i-th unit interval

$$= \int_{\tau_i} dy_i = dy_i\left(\frac{\partial}{\partial y_i}\right)$$

Here, $$\omega(\tau_l, \tau_k) = \frac{1}{2} \int_{\tau_l}\int_{\tau_k} \sum p_{ij} \, dy_i \wedge dy_j = \int_{\tau_k \times \tau_l} dy_k \, dy_l$$
$$\equiv \omega\left(\frac{\partial}{\partial y_l}, \frac{\partial}{\partial y_k}\right)$$