1982. 10. 29. Date

Everything is same as before.

Note on multilinear form

lemma : $n!\, x_1 x_2 \cdots x_n$

$$= (x_1+x_2+\cdots+x_n)^n - \sum_{i_1<\cdots<i_{n-1}} (x_{i_1}+\cdots+x_{i_{n-1}})^n + \sum_{i_1<\cdots<i_{n-2}} (x_{i_1}+\cdots+x_{i_{n-2}})^n$$

$$- \sum_{i_1<\cdots<i_{n-3}} (x_{i_1}+\cdots+x_{i_{n-3}})^n + \cdots \qquad + (-1)^{n+1} \sum_{i=1}^{n} x_i^n$$

pf). Note that the right-hand side is symmetric and homogeneous of degree n.
This implies that if we expand the right-hand side and have a term containg x_i^d with $d \geq 2$, we have to have at least one term with no x_n.

Thus if we show that the right-hand side is zero if $x_n = 0$, this proves that $\exists$ no term of type

$x_1^{d_1} \cdots x_i^{d_i} \cdots x_n^{d_n}$ where at least one of d_i's is greater than 1.

So we have only one term $x_1 x_2 \cdots x_n$ coming out of $(x_1+\cdots+x_n)^n$, which has exactly $n!$