

$$Td_1(p^1) = \frac{p^1}{2} \quad Td_2(p^1, p^2) = \frac{p^1{}^2 + p^2}{12}$$

$$Td_3(p^1, p^2, p^3) = \frac{p^1 p^2}{24} \quad \text{by the following computation}$$

$$\prod_{i=1}^n \frac{x_i}{1 - e^{-x_i}}$$

$$f(x) = \frac{x}{1 - e^{-x}} = 1 - \frac{1}{2}(-x) + \frac{1}{6} \cdot \frac{1}{2!} \cdot (-x)^2 + \dots$$

$$= 1 + \frac{x}{2} + \frac{x^2}{12} + \dots$$

$$\Rightarrow \prod_{i=1}^n \frac{x_i}{1 - e^{-x_i}} = \prod_{i=1}^n \left(1 + \frac{x_i}{2} + \frac{x_i^2}{12} + \dots \right)$$

$$= 1 + \frac{\sum x_i}{2} + \frac{(\sum x_i)^2 + \sum x_i x_j}{12} + \frac{(\sum x_i)(\sum x_i x_j)}{24}$$

According to Griffiths & Harris, the Todd polynomials are different a little bit.

$$g(t) = \frac{-t\lambda}{1 - e^{-t\lambda}} = -1 - \frac{1}{2}(t\lambda) - \frac{t^2\lambda^2}{12} + \dots$$

$$\Rightarrow \prod_{i=1}^n \frac{-t\lambda_i}{1 - e^{-t\lambda_i}} = \prod_{i=1}^n \left(-1 - \frac{1}{2}(t\lambda_i) - \frac{t^2\lambda_i^2}{12} + \dots \right)$$

$$= (-1)^n \left(1 + \frac{\sum \lambda_i}{2} t + \frac{(\sum \lambda_i)^2 + \sum \lambda_i \lambda_j}{12} t^2 + \frac{(\sum \lambda_i)(\sum \lambda_i \lambda_j)}{24} t^3 + \dots \right)$$

$$\Rightarrow Td_1(p^1) = (-1)^n \frac{p^1}{2} \quad Td_2(p^1, p^2) = \frac{p^1{}^2 + p^2}{12} (-1)^n$$

$$Td_3(p^1, p^2, p^3) = \frac{p^1 p^2}{24} (-1)^n$$

$$\left(\begin{aligned} \prod_{i=1}^n \frac{-t\lambda_i}{1 - e^{-t\lambda_i}} &= (-t)^n \prod_{i=1}^n \frac{\lambda_i}{1 - e^{-t\lambda_i}} = (-1)^n \left(1 + \frac{p^1}{2} t + \dots \right) \\ \Rightarrow \prod_{i=1}^n \frac{\lambda_i}{1 - e^{-t\lambda_i}} &= (-1)^n (-1)^n t^{-n} \left(1 + \frac{p^1}{2} t + \dots \right) \end{aligned} \right)$$