

In fact, we have already encountered this surface in Section 6 of Chapter 4; we saw there that S is the image under projection to \mathbb{P}^3 of the Veronese surface in \mathbb{P}^5 .

⌈ See P629 ~ P632. Section 6 of Chapter 4 ⌋

Indeed, we can see this directly in the present context: Since the transformation φ of \mathbb{P}^2 which blows up P_1, P_2, P_3 and blows down the lines \tilde{L}_{ij} is given by the linear system $|f_{P_1} \otimes f_{P_2} \otimes f_{P_3}(2)|$ of conics through the points P_i , the composition $i_{10(2)} \circ \varphi : \mathbb{P}^2 \rightarrow \mathbb{P}^5$ of φ with the map is given by the linear system $|f_{P_1}^2 \otimes f_{P_2}^2 \otimes f_{P_3}^2(4)|$ of quartics with double points at the points P_i ; the map f , given by the sublinear system $|f_{P_3}(4)| \subset |f_{P_1}^2 \otimes f_{P_2}^2 \otimes f_{P_3}^2(4)|$, is just the composition of $i_{10(2)} \circ \varphi$ with a projection to \mathbb{P}^3 .

⌈ According to P496 ~ P497, if $P_1 = [(1, 0, 0)]$, $P_2 = [(0, 1, 0)]$, $P_3 = [(0, 0, 1)]$,

$\varphi : \mathbb{P}^2 \rightarrow \mathbb{P}^2$ is given by

$$[(X_0, X_1, X_2)] \mapsto [(X_1 X_2, X_0 X_2, X_0 X_1)]$$

$$i_{10(2)} : \mathbb{P}^2 \rightarrow \mathbb{P}^5$$

$$[(X_0, X_1, X_2)] \mapsto [(X_0^2, X_1^2, X_2^2, X_0 X_1, X_0 X_2, X_1 X_2)].$$