

Again by 139, $\Lambda \cap \bar{V}_6 \geq \mathbb{P}^3$, otherwise $\Lambda \cap \bar{V}_6 > \mathbb{P}^3$
 $3 > \frac{6-2}{2}$. \square

But \bar{V}_6 meets \mathbb{P}_{ij}^{n-3} in a line, and F_{ij} in a pair of points P_{ij1} and P_{ij2} ; and writing any k -plane $\Lambda \subset F$ through P_i and P_j as

$$\Lambda = \overline{P_i, P_j, \Lambda \cap \mathbb{P}_{ij}^{n-3}},$$

we see that Λ satisfies this condition if and only if it contains either P_{ij1} or P_{ij2} .

IF

Let $i=1, j=1$.

First:

By the choice of \bar{V}_4 , $F \cap \bar{V}_4$ is a smooth quadric in \bar{V}_4 . We want to know about $T_{p_i}(F) \cap \bar{V}_4 \cap F$.

$$T_{p_i}(F) \cap \bar{V}_4 \cap F = (T_{p_i}(F) \cap \bar{V}_4) \cap (\bar{V}_4 \cap F)$$

$$= T_{p_i}(F \cap \bar{V}_4) \cap (F \cap \bar{V}_4) \text{ since } p_i \in F \cap \bar{V}_4$$

and if M & N intersect transversely at p , then $T_p(M \cap N) = T_p(M) \cap T_p(N)$ by counting the dimensions. (\because Clearly $T_p(M \cap N) \subset T_p(M) \cap T_p(N)$. $\dim T_p(M \cap N) = \dim (T_p(M) \cap T_p(N))$.)

Thus since \mathbb{P}_i^{n-1} is an $(n-1)$ -plane not containing p_i , by 134 $(\mathbb{P}_i^{n-1} \cap \bar{V}_4) \cap (F \cap \bar{V}_4)$ is a smooth quadric of dimension 0. \Rightarrow Since $\mathbb{P}_i^{n-1} \cap \bar{V}_4$ is a line in $T_{p_i}(F \cap \bar{V}_4) = \mathbb{P}^2$, $\mathbb{P}_i^{n-1} \cap F \cap \bar{V}_4$ is a set of distinct