it follows that $r$ vanishes at all the zeros of $g$ in $\pi^{-1}(z)$; since $\deg(r) < \deg(g)$, this implies that $r$, and hence $\beta + hf$, vanish identically on $\pi^{-1}(z)$.

F We claimed that the image of $W$ under the projection map $\pi : \mathbb{C}^n \longrightarrow \mathbb{C}^{n-1}$ is the locus of $\gamma$.

Suppose it is not true. $\Rightarrow \exists\ z \in \mathbb{C}^{n-1}$ s.t $\gamma(z) = 0$, but $\pi^{-1}(z) \cap W = \phi$.

$$\begin{array}{ccc} \mathbb{C}^n & \supset & W \\ \pi \downarrow & & \downarrow \\ \mathbb{C}^{n-1} & \supset & \pi(W) \overset{?}{=} (\gamma = 0) \end{array}$$

Note that $\pi(W) \subset (\gamma = 0)$.

If $a \in \pi^{-1}(z)$ satisfies $g = 0$, since $a$ can not satisfy $f = 0$, but $a$ must satisfy $rf + (\beta + hf)g = 0$ $r(a) = 0$. $\Rightarrow$ This implies that $r$ vanishes at all the zero's of $g$ in $\pi^{-1}(z)$.

$r, g \in \mathcal{O}_{n-1}[z_n]$.

$r(z_n) = b_0(z) z_n^{\ell} + b_1(z) z_n^{\ell-1} + \cdots + b_\ell(z)$

$g(z_n) = c_0(z) z_n^{k} + c_1(z) z_n^{k-1} + \cdots + c_\ell(z)$. $\qquad k > \ell$

$\Rightarrow$ For fixed $z \in \mathbb{C}^{n-1}$, $r(z_n)$ & $g(z_n)$ are simply polynomials of $z_n$. Since $\deg r < \deg g$, and $r$ vanish at all the zeros of $g$ in $\pi^{-1}(z)$, $r \equiv 0$ along the $\pi^{-1}(z)$. Since $g$ is Weierstrass polynomial in $z_n$, $g$ can not be identically zero on $\pi^{-1}(z)$. $\Rightarrow$ $\beta + hf$ must be identically zero on $\pi^{-1}(z)$.

$\lrcorner$