Theorem B. Our second vanishing theorem for the cohomology of holomorphic vector bundles is less precise but broader in scope than the Kodaira Vanishing Theorem:

Theorem B. Let M be a compact, complex manifold and $L \to M$ a positive line bundle. Then for any holomorphic vector bundle E, there exists μ_0 s.t

$$H^q(M, \mathcal{O}(L^\mu \otimes E)) = 0 \quad \text{for } q>0, \ \mu \geq \mu_0.$$

pf). Before we prove this, note that in case E is a line bundle the result is already implied by the Kodaira theorem: just take μ_0 s.t $L^\mu \otimes E \otimes K_M^*$ is positive for $\mu \geq \mu_0$: then since $c_1(L^\mu \otimes E) = \mu c_1(L) + c_1(E)$

$$H^q(M, \mathcal{O}(L^\mu \otimes E)) = H^q(M, \Omega^n(L^\mu \otimes E \otimes K_M^*)) = 0$$

for $q>0$, $\mu \geq \mu_0$.

$\ulcorner$ Since $c_1(L^\mu \otimes E) = \mu c_1(L) + c_1(E)$, we have only to show that $\exists\ \mu \geq \mu_0$ s.t

If A positive definite, $\mu A + B$ is positive definite.

$v \in \mathbb{C}^n$

$$\langle (\mu A + B) v, v \rangle = \langle \mu A v, v \rangle + \langle B v, v \rangle$$
$$= \mu \langle A v, v \rangle + \langle B v, v \rangle$$
$$= \langle A v, v \rangle \left(\mu + \frac{\langle B v, v \rangle}{\langle A v, v \rangle} \right)$$