5. M Riemann Surface. $\mathcal{PP}$ quotient sheaf of the sheaf $\mathcal{M}$ by the subsheaf $\mathcal{O} \xrightarrow{i} \mathcal{M}$.
then. for $U \subset M$ open.

$$\mathcal{PP}(U) = \{ (p_n, f_n) \} : \left| \begin{array}{l} \{p_n\} \subset U \text{ discrete} \\ f_n \in \mathcal{M}_{p_n} / \mathcal{O}_{p_n} \end{array} \right.$$

i.e. giving a section of $\mathcal{PP}$ over U is the same as specifying the data of a Mittag-Leffler problem for U.

$$\mathcal{PP}(U) = \{ s : U \longrightarrow \bigcup_{p \in U} \mathcal{M}_p / \mathcal{O}_p \}.$$

$$s(p) \in \mathcal{M}_p / \mathcal{O}_p . \quad \Rightarrow \exists \ (V, f) \text{ s.t } \quad f \in \mathcal{M}(V) / \mathcal{O}(V)$$

If f has no pole at p. $s(p) = 0$. since f is holomorphic around p.

If s is not identically zero, $\exists$ a discrete set $\{p_n\} \subset U$ s.t $s(p_n) \neq 0$, since s can not be non-vanishing.
(Non-vanishing means $\frac{*}{0}$ impossible.

Except $\{p_n\}$, $s = 0$. and, at each pt p_n, $\exists$ $s(p_n) = f_n \in \mathcal{M}_{p_n} / \mathcal{O}_{p_n}$.