by

$$(K\varphi)(z) = \int_{w \in \mathbb{C}^n} K(z,w) \wedge \varphi(w) .$$

This integral makes sense, since k is locally integrable and φ has compact support. With the usual change of variables $u = z - w$,

$$(K\varphi)(z) = \int_{u \in \mathbb{C}^n} k(z, z-u) \wedge \varphi(z-u)$$

is C^∞ in z since only $\|u\|^{2n}$ will appear in the denominator of the integrand.

If $\varphi = \varphi \, d\bar{w}_1 \wedge \cdots \wedge d\bar{w}_q$

$$\Rightarrow \quad K(z,w) \wedge \varphi(w) = \frac{\sum \overline{\Phi_i(z-w) \wedge \Phi(w)}}{\|z-w\|^{2n}} \wedge \varphi \, d\bar{w}_1 \wedge \cdots \wedge d\bar{w}_q$$

Let $n = 2$. $q = 1$.

$$\overline{\Phi_1(z-w)} \wedge dw_1 \wedge dw_2 \wedge d\bar{w}_1 + \overline{\Phi_2(z-w)} \wedge dw_1 \wedge dw_2 \wedge d\bar{w}_1$$

$$= (z_1 - w_1)(d\bar{z}_2 - d\bar{w}_2) \wedge dw_1 \wedge dw_2 \wedge d\bar{w}_1 + (-1)(z_2 - w_2)(d\bar{z}_1 - d\bar{w}_1)$$

$$\wedge \, dw_1 \wedge dw_2 \wedge d\bar{w}_1 = -(z_1 - w_1)\, d\bar{z}_2 \wedge dw_1 \wedge d\bar{w}_1 \wedge dw_2$$

$$+ (z_1 - w_1)\, dw_1 \wedge dw_2 \wedge d\bar{w}_1 \wedge d\bar{w}_2 - (z_2 - w_2)\, d\bar{z}_1 \wedge dw_1 \wedge dw_2 \wedge d\bar{w}_1$$

$$+ (z_2 - w_2)\, d\bar{w}_2 \wedge dw_1 \wedge dw_2 \wedge d\bar{w}_1$$

What is the definition of $\displaystyle\int_{w \in \mathbb{C}^2} d\bar{z}_2 \wedge dw_1 \wedge d\bar{w}_1 \wedge dw_2$?

I think it is zero. There is no other way.

$\Rightarrow \quad K : A_c^{0,1}(\mathbb{C}^2) \longrightarrow A^{0,0}(\mathbb{C}^2)$ makes sense.