essentially transition functions. $\Rightarrow$ By P67 & P133, we get the above. Thus, once we proved the claim, done.

To this end we first note that the inclusion of exact sheaf sequences

$$\begin{tikzcd} 0 \arrow[r] & \mathbb{Z} \arrow[r] & \mathcal{O} \arrow[r, "\exp"] & \mathcal{O}^* \arrow[r] & 0 \\ 0 \arrow[r] & \mathbb{Z} \arrow[r] \arrow[u, equal] & \mathbb{C} \arrow[r] \arrow[u] & \mathbb{C}^* \arrow[r] \arrow[u] & 0 \end{tikzcd}$$

on any compact Kähler manifold X induces a commutative diagram

$$\begin{tikzcd} H^1(X, \mathcal{O}) \arrow[r] & H^1(X, \mathcal{O}^*) \arrow[r, "c_1"] & H^2(X, \mathbb{Z}) \\ H^1(X, \mathbb{C}) \arrow[r] \arrow[u, "\iota_1^*"'] & H^1(X, \mathbb{C}^*) \arrow[r] \arrow[u, "\iota_2^*"'] & H^2(X, \mathbb{Z}) \arrow[u] \end{tikzcd}.$$

The map ι_1^* represents projection of $H^1(X, \mathbb{C}) = H^{1,0}(X) \oplus H^{0,1}(X)$ on the second factor, and so is surjective.

$[z] \in H^1(X, \mathbb{C}) \Rightarrow z = (z_{\alpha\beta}) \in Z^1(\underline{U}, \mathbb{C})$

Consider the following sheaf exact sequence

$$0 \to \mathbb{C} \longrightarrow \mathcal{O} \longrightarrow \mathcal{O}^* \longrightarrow 0$$

$\Rightarrow$ We have a long exact sequence redundant

$$H^1(X, \mathbb{C}) \xrightarrow{\iota_1^*} H^1(X, \mathcal{O}) \longrightarrow$$

We have to find what ι_1^* is.

To know this, we have to close look at