dependent of u_k. otherwise $d\xi'$ contains $f_J\, du_J$ term.
where $J \ni k$, $\Rightarrow$ but f_J can not have a simple pole in u_k and so $f_J\, du_J$ can not be cancelled.
$\Rightarrow$ ξ' is a form in only $u_1, u_2, \cdots u_{k-1}$. ⌟

Now $\tilde{\varphi} = 0$ in $H^q_{DR}((\Delta^*)^k)$, and thus the restriction of $\tilde{\varphi}$ to $H^q_{DR}((\Delta^*)^{k-1})$ is zero, where $(\Delta^*)^{k-1} \subset (\Delta^*)^k$ is given by $u_k =$ constant.

⌜

$$(\Delta^*)^{k-1} \xhookrightarrow{\iota} (\Delta^*)^k$$

$$\begin{array}{ccc} H^q_{DR}((\Delta^*)^k) & \xrightarrow{\iota^*} & H^q_{DR}((\Delta^*)^{k-1}) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ \tilde{\varphi} & \longmapsto & \iota^*(\tilde{\varphi}) \\ \parallel & & \parallel \\ 0 & & 0 . \end{array}$$

⌟

This restriction is just ξ', and by induction $\xi' = d\gamma'$, where γ' has at most a pole in u'.

⌜ $\iota^* \tilde{\varphi} = \iota^*\left(\xi' + \xi'' \wedge \dfrac{du_k}{u_k}\right)$

$$= \iota^*(\xi') + \iota^*\xi'' \wedge \iota^*\left(\frac{du_k}{u_k}\right)$$

$\Rightarrow$ Since $\iota^*(du_k) = d\,\iota^*(u_k) = d(\text{constant}) = 0$,
and $\iota^*\xi' = \xi'$ ($\because$ ξ' is a form in only $u_1, \cdots u_{k-1}$),

$$\iota^*\tilde{\varphi} = \xi' .$$

$\Rightarrow$ By induction, the theorem $(*)$ holds for $u' = (u_1, \cdots u_{k-1})$. ξ' is a closed form in u'.
$\Rightarrow$ $\xi' = d\gamma'$, where γ' has no essential singu-