Formally — i.e., ignoring the singularity — $\psi'(x) = 0$. However, the distributional derivative is given by

$$(DT_\psi)(\varphi) = -\int_{-\infty}^{+\infty} \varphi'(x)\,\psi(x)\,dx = -\int_0^{\infty} \varphi'(x)\,dx$$

$$= \varphi(0),$$

i.e., $\quad DT_\psi = \delta.$

The general principle will be

$$DT_\psi - T_{D\psi} = \text{"residue,"}$$

where $D\psi$ is the derivative of ψ computed formally. We will expound this in greater detail in a little while.

Another picture of distributions is obtained by looking at the torus $T = \mathbb{R}^n/(2\pi\mathbb{Z})^n$. In Section 6 of Chapter 0 on the proof of the Hodge theorem we defined the space $\mathcal{D}(T)$ of distributions on the torus and showed that

$$\mathcal{D}(T) = \bigcup_s H_s,$$

where H_s is the Sobolev space of formal Fourier series $T = \sum u_\xi e^{i(\xi,x)}$ satisfying $\sum (1+\|\xi\|^2)^s |u_\xi|^2 < \infty$.

☞ On P85, $\bigcup_s H_s = H_{-\infty}$ and on P91, we showed that $\mathcal{D}(T) = H_{-\infty}$. ⌟

By the Sobolev lemma proved there

$$C^\infty(T) = \bigcap_s H_s,$$

and for $\varphi = \sum \varphi_\xi e^{i(\xi,x)} \in C^\infty(T)$

$$T(\varphi) = \sum_\xi u_\xi \varphi_\xi.$$

☞ See P86 ~ P88