If $\operatorname{res}_f(g,h)=0$ for all h, then by the assumed nondegeneracy of $\operatorname{res}_{f'}$ it follows that $b_1 g \in I(f')$.

$$\operatorname{res}_f(g,h) = \operatorname{res}_{f'}(b_1 g, h) = 0 \text{ for all } h \Rightarrow b_1 g \in I(f')$$

Thus

$$b_1 g = c_1 f_1' + \sum_{i\geq 2} c_i f_i$$

$$= c_1 b_1 f_1 + \sum_{i \geq 2} (b_1 c_i - b_i c_1) f_i,$$

so that

$$b_1 g = c_1 b_1 f_1 \quad \text{in } \mathcal{O}/\{f_2, \cdots f_n\}.$$

This implies that either

$$g = c_1 f_1 \quad \text{in } \mathcal{O}/\{f_2, \cdots f_n\}$$

or

$$b_1 \text{ is a zero-divisor in } \mathcal{O}/\{f_2, \cdots f_n\}.$$

$$b_1 g = c_1 b_1 f_1 \Rightarrow b_1(g - c_1 f_1) = 0 \text{ in } \mathcal{O}/\{f_2 \cdots f_n\}.$$
$\Rightarrow$ $g = c_1 f_1$ or b_1 is a zero-divisor

In the first case, $g \in \{f_1, f_2, \cdots f_n\}$ as desired. In the second case, $b_1 f_1$ is a zero-divisor in $\mathcal{O}/\{f_2 \cdots f_n\}$, and hence so is $f_1' = b_1 f_1 + (b_2 f_2 + \cdots + b_n f_n)$.

$b_1(g - c_1 f_1) \in \{f_2, \cdots f_n\} \Rightarrow b_1 f_1 (g - c_1 f_1) \in \{f_2 \cdots f_n\}$ by the definition of an ideal. $f_1'(g - c_1 f_1) = b_1 f_1 (g - c_1 f_1) + (b_2 f_2 + \cdots + b_n f_n)(g - c_1 f_1) \in \{f_2, \cdots f_n\} \Rightarrow$ Since $g \neq c_1 f_1$, f_1' is a zero-divisor in $\mathcal{O}/\{f_2 \cdots f_n\}$.