We claim that

$$[\Lambda, \bar\partial] = [\Lambda, \bar\partial'] \quad \text{at } z_0$$
$$-i\,\partial^* = -i\,\partial'^* \quad \text{at } z_0.$$

$$[\Lambda, \bar\partial](f\,\varphi_I \wedge \varphi_J)$$
$$= \Lambda\,\bar\partial(f\,\varphi_I \wedge \varphi_J) - \bar\partial\,\Lambda(f\,\varphi_I \wedge \varphi_J)$$
$$= \Lambda(\bar v_K(f)\,\varphi_K \wedge \varphi_I \wedge \varphi_J) + \Lambda(\text{terms of } d\varphi_i\text{'s})$$
$$- \bar\partial\,\Lambda(f\,\varphi_I \wedge \bar\varphi_J)$$

$$= \bar v_K(f)\,\Lambda(\varphi_K \wedge \varphi_I \wedge \varphi_J) + \Lambda(\text{terms of } d\varphi_i\text{'s})$$
$$- \bar\partial f \wedge (\varphi_I \wedge \varphi_J)$$

$$\Rightarrow \sum_K \bar v_K(f)\,\Lambda(\varphi_K \wedge \varphi_I \wedge \varphi_J) = \Lambda\bar\partial'(f\,\varphi_I \wedge \varphi_J)$$
$$\parallel$$
$$\Lambda(\bar v_K(f)\,\varphi_K \wedge \varphi_I \wedge \varphi_J)$$

$$\bar\partial f \wedge (\varphi_I \wedge \varphi_J) = \bar v_K(f)\,\varphi_K \wedge \Lambda(\varphi_I \wedge \varphi_J)$$
$$+ f\,(\text{terms of } d\varphi_i\text{'s})$$
$$= \bar\partial'\Lambda(f\,\varphi_I \wedge \varphi_J)$$

Since $d\varphi_i$'s are zero at z_0, $[\Lambda, \bar\partial](f\,\varphi_I \wedge \varphi_J)$
$= [\Lambda, \bar\partial'](f\,\varphi_I \wedge \varphi_J)$ at z_0.

$$\partial^*(f\,\varphi_I \wedge \bar\varphi_J) = -*\partial*(f\,\varphi_I \wedge \bar\varphi_J)$$
$$= -*\partial(\epsilon\,\bar f\,\varphi_{I^\circ} \wedge \bar\varphi_{J^\circ})$$

$$= -*\{\epsilon\,v_K(\bar f)\,\varphi_K \wedge \varphi_{I^\circ} \wedge \bar\varphi_{J^\circ} + \epsilon\,\bar f\,(\text{terms of } d\varphi_i\text{'s})\}$$

At z_0, $\partial^*(f\,\varphi_I \wedge \bar\varphi_J) = -*\,\epsilon\,v_K(\bar f)\,\varphi_K \wedge \varphi_{I^\circ} \wedge \bar\varphi_{J^\circ}$
since $*$ is algebraic.
$\Rightarrow -*\,\epsilon\,v_K(\bar f)\,\varphi_K \wedge \varphi_{I^\circ} \wedge \bar\varphi_{J^\circ} = -\overline{v_K(\bar f)}\,*(\varphi_K \wedge \varphi_{I^\circ} \wedge \bar\varphi_{J^\circ})\,\epsilon$
where ϵ = sign of $\varphi_I \wedge \bar\varphi_J \wedge \varphi_{I^\circ} \wedge \bar\varphi_{J^\circ}$.