It is possible to prove these formulas by reducing to the one-variable case in a manner that sheds some additional light on the expression for β. First, we shall show that

$$\beta = C_n \, (\partial \log \|z\|^2) \wedge (\partial \bar\partial \log \|z\|^2)^{n-1}.$$

Proof. Denote by γ the form on the right-hand side of this equation.
Since

$$\partial \log \|z\|^2 = \frac{(dz, z)}{(z, z)},$$

$$\partial \bar\partial \log \|z\|^2 = \partial \left(\frac{(z, dz)}{(z, z)} \right) = \frac{(dz, dz)}{(z, z)} - \frac{(dz, z) \wedge (z, dz)}{(z, z)^2},$$

and since $(dz, z) \wedge (dz, z) = 0$,

$$\gamma = C_n' \, \frac{(dz, z) \wedge (dz, dz)^{n-1}}{\|z\|^{2n}}.$$

Definition of (α, β)

$(\alpha, \beta) = \sum \alpha_i \bar\beta_i$, where $\alpha = (\alpha_1, \cdots \alpha_n)$, $\beta = (\beta_1, \beta_2, \cdots \beta_n)$.

$$\partial \log \|z\|^2 = \frac{1}{\|z\|^2} \partial \|z\|^2 = \frac{1}{\|z\|^2} \sum \bar z_i \, dz_i = \frac{(dz, z)}{(z, z)}$$

where $dz = (dz_1, \cdots dz_n)$, $z = (z_1, z_2, \cdots z_n)$.

$$\partial \bar\partial \log \|z\|^2 = \partial \left(\frac{(z, dz)}{(z, z)} \right) = \partial \left(\frac{\sum z_i \, d\bar z_i}{\|z\|^2} \right) = \frac{(dz, d\bar z)}{\|z\|^2} + (+1)$$

$$\sum z_i \, d\bar z_i \wedge \sum \bar z_i \, dz_i \; \|z\|^{-4}$$

$$= \frac{\langle dz, d\bar z \rangle}{\langle z, z \rangle} + \frac{\langle z, dz \rangle \wedge \langle dz, z \rangle}{\langle z, z \rangle^2}, \quad \sum z_i \, d\bar z_i \wedge \sum z_i d$$

$$z_i = (z_1 \, d\bar z_1 + z_2 \, d\bar z_2) \wedge (z_1 \, d\bar z_1 + z_2 \, d\bar z_2) = 0 \quad \text{for } i = 1, 2.$$