But $\sum W_{\alpha\beta}\delta_\beta Z_{\alpha r}\, dx_{n+r}\wedge dx_\beta = -\sum W_{\alpha\beta}\delta_\alpha Z_{\beta r}\, dx_\alpha \wedge dx_{n+r}$ since $W_{\alpha\beta}=W_{\beta\alpha}$, and $\sum_{\alpha,\beta,\varepsilon} W_{\alpha\beta}\delta_\alpha \bar{Z}_{\beta\varepsilon}\, dx_\alpha\wedge dx_{n+\varepsilon} = \sum_{\alpha,\beta,r} W_{\alpha\beta}\delta_\alpha \bar{Z}_{\beta r}\, dx_\alpha\wedge dx_{n+r}$. $\Rightarrow$ We have what we want.

Since $W={}^tW$ and $Z={}^tZ$, the first and last of these three terms are zero, and hence

$$\Theta = \pi \sum_{\alpha,\beta,r} \delta_\alpha W_{\alpha\beta}(\bar{Z}_{\beta r}-Z_{\beta r})\, dx_\alpha\wedge dx_{n+r}$$

$$= -2\pi\sqrt{-1}\sum_{\alpha,\beta,r}\delta_\alpha W_{\alpha\beta} Y_{\beta r}\, dx_\alpha \wedge dx_{n+r}$$

$$= -2\pi\sqrt{-1}\sum_\alpha \delta_\alpha\, dx_\alpha\wedge dx_{n+\alpha},$$

and so finally

$$c_1(L) = \left[\frac{\sqrt{-1}}{2\pi}\Theta\right] = [\omega].$$

Q.E.D.

Γ $\alpha\neq\beta$ $W_{\alpha\beta}=W_{\beta\alpha}$ $\delta_\alpha\delta_\beta=\delta_\beta\delta_\alpha$ and $dx_\alpha\wedge dx_\beta + dx_\beta\wedge dx_\alpha = 0$ $\Rightarrow$ the first term is 0. ($\alpha=\beta$, O.K, since $dx_\alpha\wedge dx_\alpha=0$). $Z_{\alpha r}\bar{Z}_{\beta\varepsilon}W_{\alpha\beta} = W_{\beta\alpha}Z_{\alpha r}\bar{Z}_{\beta\varepsilon} = (WZ)_{\beta r}\bar{Z}_{\beta\varepsilon} = \bar{Z}_{\varepsilon\beta}(WZ)_{\beta r} = (\bar{Z}WZ)_{\varepsilon r}$
$Z_{\alpha r}\bar{Z}_{\beta\varepsilon}W_{\alpha\beta} = Z_{\alpha r}\bar{Z}_{\varepsilon\beta}W_{\beta\alpha} = Z_{\alpha r}(\bar{Z}W)_{\varepsilon\alpha} = (\bar{Z}W)_{\varepsilon\alpha}Z_{\alpha r} = (\bar{Z}WZ)_{\varepsilon r} = Z_{\alpha r}W_{\alpha\beta}\bar{Z}_{\beta\varepsilon} = Z_{\alpha r}(W\bar{Z})_{\alpha\varepsilon} = (ZW\bar{Z})_{r\varepsilon} = ({}^t(ZW\bar{Z}))_{\varepsilon r} = (\bar{Z}WZ)_{\varepsilon r}$ But!!