$\Rightarrow$ Since $\partial\Lambda = \sum_k e_k \partial_k \left(-\frac{i}{2}\sum \bar{\iota}_\ell \, i_\ell\right)$

$= -\frac{i}{2}\sum_{k,\ell} \partial_k e_k \bar{\iota}_\ell \, i_\ell = -\frac{i}{2}\sum_{k,\ell} \partial_\ell e_\ell \bar{\iota}_k \, i_k,$

$\Lambda\partial = \partial\Lambda - i\sum \partial_k \bar{\iota}_k$.

$\cdot\ \bar{\partial}^* = -\sum \partial_k \bar{\iota}_k \quad \Rightarrow \quad \Lambda\partial = \partial\Lambda + i\,\bar{\partial}^*$

$\Rightarrow\ [\Lambda . \partial] = i\,\bar{\partial}^*$ so the identity is proved on $\mathbb{C}^n$.

To prove the result on a Kähler manifold M, we us the condition of osculation to show that the identity holds at any point: for $z_0 \in M$, we can choose a coframe $\varphi_1, \varphi_2 \cdots \varphi_n$ for the metric such that $d\varphi_i(z_0) = 0$.

The expression for Λ holds with dz_I replaced by φ_I: we can make essentially the same computation for $[\Lambda, \bar{\partial}]\eta$ as on $\mathbb{C}^n$ except that we will get terms involving $\bar{\partial}\varphi_i$. Since $[\Lambda . \bar{\partial}]$ involves only first derivatives, however, all the additional terms will have a factor $\bar{\partial}\varphi_i$ and hence will vanish at z_0.

Likewise, we have computed $\partial^*\eta = C_n * \partial * \eta$ on $\mathbb{C}^n$, where it agrees with $i\,[\Lambda, \bar{\partial}]\eta$: the computation on M in terms of the φ_i will again be the same except for additional terms involving $\bar{\partial}\varphi_i$, which vanish at z_0. Thus we see that the identity holds at z_0, hence everywhere.

This argument is just one instance of a general principle: any intrinsically defined identity that involves the metric together with its first derivatives and which is valid on $\mathbb{C}^n$ with the Euclidean metric, is also valid on a Kähler manifold.