

$$H_{\bar{\partial}}^{p,q}(M) \xrightarrow{\cong} H^{p,*}(\mathcal{O}^{p,*}(M), \bar{\partial})$$

$$H_{\text{PR}}^p(M) \longrightarrow H^*(\mathcal{O}^*(M), d), \quad \text{p 385.}$$

I think the authors were confused by their notations. If T_ϵ is the smoothing of T , as they claimed in the proof of Lemma. P390, T_ϵ should be positive and $\lim_{\epsilon \rightarrow 0} T_\epsilon = T$ by P375. But this T_ϵ is obtained through the isomorphisms above. $\lim_{\epsilon \rightarrow 0} T_\epsilon$ does not make sense at all. They chose bad notations so they made a mistake. \Rightarrow

However, we can not say that the T_ϵ are positive forms. For example, suppose that $M = \tilde{N}_p$ is the blow-up of a two-dimensional complex manifold N at a point p . The fiber over p in $M \rightarrow N$ is a curve $E \cong \mathbb{P}^1$ with normal bundle H^* , where $H \rightarrow \mathbb{P}^1$ has Chern class $+1$.

$$\Gamma \quad M = \tilde{N}_p = N - p \cup_{\pi} \tilde{\Delta}, \quad \tilde{\Delta} \subset \Delta \times \mathbb{P}^{n-1} \\ \{(z, l) : z \in \Delta\}$$

$\tilde{\Delta} \rightarrow \mathbb{P}^{n-1}$ is the universal bundle $J = H^*$, see P142 and P145. \Rightarrow

Thus the self-intersection number $E \cdot E = -1$.