it is clearly enough to show that

(1) $$\phi + W \subset V_1 \cap V_2 \qquad \text{for some } W \in \beta.$$

「 This claim implies that $\{\phi + W\}$ forms a base for τ.
$\Rightarrow$ τ is a topology, since every element can be expressed as a union of $(\phi + W)$'s. 」

The definition of τ shows that there exist $\phi_i \in \mathcal{D}(\Omega)$ and $W_i \in \beta$ such that

(2) $$\phi \in \phi_i + W_i \subset V_i \qquad (i = 1, 2).$$

Choose K so that $\mathcal{D}_K$ contains ϕ_1, ϕ_2 and ϕ.
「 This is possible since $\mathcal{D}_K \subset \mathcal{D}_{K'}$ if $K \subset K'$. 」

Since $\mathcal{D}_K \cap W_i$ is open in $\mathcal{D}_K$, we have

(3) $$\phi - \phi_i \in (1 - \delta_i) W_i$$

for some $\delta_i > 0$.

「 $W_i \in \beta \iff \mathcal{D}_K \cap W_i \in \tau_K$ for all K
$\iff \mathcal{D}_K \cap W_i$ is open in $\mathcal{D}_K$ for all K.

Consider $W_i \cap \{\alpha\psi ; \alpha \in R\}$. $\Rightarrow$ Since W_i is open, and $W_i \ni \psi$, $\exists\ \epsilon > 0$ s.t for $r \in (1-\epsilon, 1+\epsilon)$, $\{r\psi\} \subset W_i$.
$\Rightarrow$ $\exists\ \delta_i$ s.t $\phi - \phi_i \in (1 - \delta_i) W_i$. Here $\psi = \phi - \phi_i$. 」

The convexity of W_i implies therefore that

(4) $$\phi - \phi_i + \delta_i W_i \subset (1 - \delta_i) W_i + \delta_i W_i = W_i.$$