and $\langle \sigma_U, U \rangle$ = the germ σ_U at x = $\sigma(x)$. See Greenberg p211-p213

We note that essentially by definition:
The cohomology sheaves $\mathcal{H}^q = 0$ for $q>0$ $\Leftrightarrow$ the Poincare lemma holds for the complex of sheaves $(\mathcal{K}^*, d)$.

The Poincare lemma for $(\mathcal{K}^*, d)$ is the following:

$$\mathcal{K}^{q-1} \xrightarrow{d} \mathcal{K}^q \xrightarrow{d} \mathcal{K}^{q+1} \text{ is exact.}$$

$\mathcal{H}^q = 0 \Rightarrow \mathcal{H}^q(U) = 0$ for any open set $U \subset X$.
$\Rightarrow$ For any $\sigma \in \mathcal{H}^q(U)$ given by $\{(\sigma_\alpha, U_\alpha)\}$ satisfying the conditions above, $\exists\ \eta_\alpha \in \mathcal{K}^{q-1}(U_\alpha)$ s.t $d\eta_\alpha = \sigma_\alpha$.
This implies $\sigma = 0$. and more.
Suppose $\sigma \in \mathcal{K}^q(U)$ s.t $d\sigma = 0$ in $\mathcal{K}^{q+1}(U)$.
$\Rightarrow$ By the assumption above, for each point $x \in U$,
$\exists\ x \in V_x \subset U$ s.t, (since $\mathcal{H}^q(U) = 0$,
$\langle \sigma, U \rangle = 0$ at x)
$\sigma|V_x = d\eta_x$, $\eta_x \in \mathcal{K}^{q-1}(V_x)$.
$\Rightarrow$ $\mathcal{K}^{q-1} \xrightarrow{d} \mathcal{K}^q \xrightarrow{d} \mathcal{K}^{q+1}$ is exact, see P37 ⊛
Conversely, if $\mathcal{K}^{q-1} \xrightarrow{d} \mathcal{K}^q \xrightarrow{d} \mathcal{K}^{q+1}$ is exact, then the sheaf associated to the presheaf $U \mapsto \dfrac{\ker\{d : \mathcal{K}^q(U) \to \mathcal{K}^{q+1}(U)\}}{d\,\mathcal{K}^{q-1}(U)}$ is zero. See P382

Now let $\underline{U} = \{U_\alpha\}$ be a covering of X and $C^p(\underline{U}, \mathcal{K}^q)$ the Cech cochains of degree p with values in $\mathcal{K}^q$. The two operators

$$\delta : C^p(\underline{U}, \mathcal{K}^q) \longrightarrow C^{p+1}(\underline{U}, \mathcal{K}^q),$$