

$$Gr H^*(K^*) = \bigoplus_{p,q} Gr^p H^q(K^*),$$

where $Gr^p H^q(K^*) = \frac{F^p H^q(K^*)}{F^{p+1} H^q(K^*)}.$

By the previous filtration,

$$\rightarrow F^2 H^q(K^*) \rightarrow F^1 H^q(K^*) \rightarrow F^0 H^q(K^*). \quad \hookrightarrow$$

// Comment on "Holomorphic Lefschetz fixed point formula".

If $L(f, O) = \sum_{f(p_i)=p_i} \frac{(-1)^n \det(B_{\alpha})}{\det(I - B_{\alpha})},$

then by p416, the Hirzebruch - Riemann - Roch formula derivation is correct. I think $(-1)^n$ comes from consideration of the orientation in the proof of the holomorphic Lefschetz fixed-point formula. //

Definition. A spectral sequence is a sequence $\{E_r, d_r\}$ ($r \geq 0$) of bigraded groups

$$E_r = \bigoplus_{p,q \geq 0} E_r^{p,q}$$

together with differentials

$$d_r : E_r^{p,q} \rightarrow E_r^{p+r, q-r+1}, \quad d_r^2 = 0,$$

such that

$$H^*(E_r) = E_{r+1}.$$

When working with spectral sequences it is useful — even essential — to draw the "picture" (Figure 3).

In practice we will always have $E_r = E_{r+1} = \dots$ for $r \geq r_0$;