$H^{p,q}(M) \cong H^{q,p}(M)$." Refer to p117.

4. Of interest also are the dimensions

$$P_n(M) = h^0(M, \mathcal{O}(K_M^n)),$$

of the space of sections of the nth powers of the canonical bundle, called collectively the plurigenera of M.

⊩ n th tensor product of $K_M = K_M^n$.

5. The fundamental group $\pi_1(M)$ of an algebraic variety is also a birational invariant: suppose

$$f: M \longrightarrow N$$

is a birational map, defined away from the subvariety $U \subset M$ and one to one away from the subvariety $V \subset M$. If γ is any loop on M, then we can find a loop γ' in M homotopic to γ and disjoint from U; and the class of $f(\gamma')$ on N will be independent of the choice of γ': since U has real codimension at least 4, if $\gamma' - \gamma''$ is the boundary of a disk in M, then it is the boundary of a disk in $M - U$.

⊩ $\gamma' - \gamma'' = \partial D^2$, $D^2 \subset M$. We may choose γ' so that $\gamma' \cap U = \phi$, $\gamma' \cap V = \phi$.

$\Rightarrow$ By perturbing D^2 slightly, we make D^2 in general position with U. $\Rightarrow$ Since $\dim U + \dim D^2 < \dim M$, $U \cap D^2 = \phi$.

$\Rightarrow$ $f(\gamma') - f(\gamma'') = \partial f(D^2)$,

since $\mathrm{codim}_{\mathbb{R}} V \geq 2$.