let $x \in G$ be any point. Since $G \subset \mathbb{P}^5$ is a quadric, by what we have seen, the intersection $T_x(G) \cap G$ is just the locus of lines in G through x.

⌈ G is smooth in $\mathbb{P}^5$ $\Rightarrow$ By P735, $T_x(G) \cap G$ is the cone through x over a smooth quadric in $\mathbb{P}^3$. (The smooth quadric has the dimension 2.) $\Rightarrow$ $T_x(G) \cap G$ is the locus of lines in G through x. ⌋

But if $x' \in G$ is any point whose corresponding line (in $\mathbb{P}^3$), $l_{x'}$ meets l_x, then x and x' both lie on the Schubert cycle $\sigma_{2,1}(p, h)$ of lines in $\mathbb{P}^3$ through the point $p = l_x \cap l_{x'}$ and contained in the hyperplane $h = \overline{l_x, l_{x'}}$.

⌈

$$
\begin{array}{ccc}
G(2,4) & \xrightarrow{\phi} & \mathbb{P}^5 \\
\Psi & & \Psi \\
l_x & \longmapsto & x \\
l_{x'} & \longmapsto & x'
\end{array}
$$

$\Rightarrow$ $\sigma_{2,1}(p, h) = \{ l \in G(2,4) \mid p \in l \subset h \}$

$\Rightarrow$ Clearly, $l_x \ni p = l_x \cap l_{x'}$ and $l_x, l_{x'} \subset h = \overline{l_x, l_{x'}}$. ⌋

Since $\sigma_{2,1}(p, h)$ is a line, it follows that $\sigma_{2,1}(p, h)$