

$$\Lambda = \{ m_1 \pi_1 + \dots + m_{2g} \pi_{2g}, m_i \in \mathbb{Z} \}$$

in  $\mathbb{C}^g$ ; we define the Jacobian variety  $J(S)$  of  $S$  to be the complex torus  $\mathbb{C}^g / \Lambda$ . The Jacobian is a natural range for Abelian integrals: whereas the integral  $\int_P^q \omega$  of a single holomorphic differential  $\omega$

is defined only modulo the  $2g$  periods of  $\omega$ , which are usually dense in  $\mathbb{C}$ , the vector

$$\left( \int_P^q \omega_1, \dots, \int_P^q \omega_g \right),$$

is well-defined as a vector in  $\mathbb{C}^g$  modulo the discrete lattice  $\Lambda \subset \mathbb{C}^g$ .

“which are usually dense in  $\mathbb{C}$ ”

$\{ \int_{\delta_1} \omega, \int_{\delta_2} \omega, \dots, \int_{\delta_{2g}} \omega \} \subset \mathbb{C}$  should be discrete.

Picking a base point  $p_0 \in S$ , accordingly, we have a natural map

$$\mu: S \longrightarrow J(S)$$

given by

$$\mu(p) = \left( \int_{p_0}^p \omega_1, \dots, \int_{p_0}^p \omega_g \right) \in J(S).$$

More generally, if  $\text{Div}^0(S)$  denotes the group of divisors of degree 0 on  $S$ , we define  $\mu: \text{Div}^0(S) \rightarrow J(S)$  by

$$\mu \left( \sum p_\lambda - \sum q_\lambda \right) = \left( \sum \int_{q_\lambda}^{p_\lambda} \omega_1, \dots, \sum \int_{q_\lambda}^{p_\lambda} \omega_g \right).$$

