

thus corresponds to the  $\mathbb{A}^2$ -planes in  $F_\lambda$  containing  $L$ .

$\square$  Note that, if  $\lambda \neq \mu$ ,  $F_\mu \cap F_\lambda = X$  by the argument above.  $\Rightarrow \Lambda \cap X = \Lambda \cap (F_\lambda \cap F_\mu) = \Lambda \cap F_\mu$  and since  $\Lambda \not\subset F_\mu$ ,  $\Lambda \cap F_\mu$  is a conic curve  $\simeq \Lambda$ .  $\Rightarrow \Lambda \cap F_\mu \supset L \Rightarrow \Lambda \cap F_\mu = L \cup L'$ .

$$\pi^{-1}(\lambda) \xrightarrow{\phi} \{ \Lambda \mid \Lambda \subset F_\lambda \}$$

$$\downarrow \quad \quad \quad \downarrow$$

$$L' \longmapsto L', L$$

by the previous argument.

Given  $\Lambda \supset L$ , then  $\Lambda \cap F_\mu = L \cup L' \Rightarrow \Lambda \cap X = L \cup L' \Rightarrow \Lambda = \overline{L, L'} \Rightarrow \exists$  inverse of  $\phi$ , s.t.

$\phi(\Lambda) = L' \Rightarrow \phi$  is one to one, onto.

$\square$

There are two possibilities: first, if  $F_\lambda$  is smooth, then, as we have seen, the  $\mathbb{A}^2$ -planes on  $F_\lambda$  fall into two connected three-dimensional components.

$\square$  See P135, Proposition.  $\Rightarrow F_\lambda$  contains two irreducible 3-dimensional families of  $\mathbb{A}^2$ -planes.  $\square$

Now if  $p \in L \subset F_\lambda$  is any point of  $L$ , the intersection  $T_p(F_\lambda) \cap F_\lambda$  will be a cone over the smooth quadric surface  $\tilde{F}_\lambda$  cut out on  $F_\lambda$  by any 3-plane in  $T_p(F_\lambda)$  missing  $p$ , and the  $\mathbb{A}^2$ -planes on  $F_\lambda$  through the point  $p$  will be spanned by the lines on  $\tilde{F}_\lambda$  together with  $p$ .