the set of all effective divisors containing Λ,

$$\dim \pi_2^{-1}(\Lambda) = \dim \{F_2\} - 1 = \frac{(n+2)(n+3)}{2} - \frac{(k+1)(k+2)}{2} - 1 .$$

Thus $\dim I = \dim \pi_2^{-1}(\Lambda) + \dim G(k+1, n+2)$.

$$= \frac{(n+2)(n+3)}{2} - \frac{(k+1)(k+2)}{2} - 1 + (k+1)(n-k+1) .$$

$$\dim I = \dim \pi_1^{-1}(F) + \dim |F|$$
$$= \dim \pi_1^{-1}(F) + \frac{(n+2)(n+3)}{2} - 1$$

$$\Rightarrow \dim \pi_1^{-1}(F) = (k+1)(n-k+1) - \frac{(k+1)(k+2)}{2} .$$

$\Rightarrow$ Codimension in $G(k+1, n+2)$ is $\frac{(k+1)(k+2)}{2}$.

Let us now determine the class on $G(k+1, n+2)$ of the cycle $\Sigma_{k,n}$ of k-planes on a smooth quadric $F \subset \mathbb{P}^{n+1}$. Recall from Section 6 of Chapter 1 that for any flag $V_0 \subset V_1 \subset \cdots \subset V_{n+1} \subset \mathbb{C}^{n+2}$ the cohomology group $H^{(k+1)(k+2)}(G(k+1, n+2))$ is generated by the classes of the Schubert cycles

$$\sigma_{a_1, \dots a_{k+1}} = \{ \Lambda_{k+1} : \dim (\Lambda \cap V_{n-k+1+i-a_i}) \geq i \}$$

for all sequences

$$n-k+1 \geq a_1 \geq a_2 \geq \cdots \geq a_{k+1} \geq 0$$

with $\sum a_i = (k+1)(k+2)/2$.