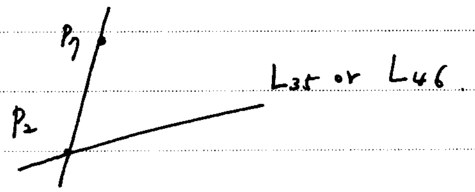
⌈ $L_{27} + L_{35} + L_{46}$ is singular at P_2.



(i) $P_2 \in L_{35} \Rightarrow P_2 \in L_{56} \Rightarrow P_2, P_3, P_5, P_6, P_4$ are collinear since $P_2 \in P_{34}$.

(ii) $P_2 \in L_{46}. \Rightarrow P_2 \in L_{56} \Rightarrow P_2, P_4, P_3, P_5, P_6$ are collinear. ⌋

The lemma now follows just as in the original case: given eight points $P_1, \cdots P_8$, with P_1 infinitely near P_2 and no five collinear, by the first step any conic containing all but any three noncollinear points P_i contains all eight. Q.E.D.

⌈ Choose three noncollinear points, call them P_3, P_4, P_5. Let C be a conic containing the remaining points P_1, P_2, P_6, P_7, P_8. Such C exists. For,
$H^0(\mathbb{P}^2, \mathcal{O}(2H)) = \langle \tau_1, \tau_2, \cdots \tau_6 \rangle$.

$$\left.\begin{array}{l} a_1 \tau_1(P_2) + \cdots + a_6\tau_6(P_2) = 0 \\ a_1 \tau_1(P_6) + \cdots + a_6\tau_6(P_6) = 0 \\ a_1 \tau_1(P_7) + \cdots + a_6\tau_6(P_7) = 0 \\ a_1 \tau_1(P_8) + \cdots + a_6\tau_6(P_8) = 0 \end{array}\right\} (*)$$

Let $P_1 = [(1, P_{11}, P_{12})] \Rightarrow \left(\frac{\partial\tau}{\partial x}, \frac{\partial\tau}{\partial y}\right)\cdot(P_{11}, P_{12}) = 0 \cdots (**)$
where $\tau = a_1\tau_1 + \cdots + a_6\tau_6$.