

$$\text{Res}_p \left( \frac{\eta \psi}{s \cdot s'} \right)$$

has intrinsic meaning since  $\eta \psi \in \mathcal{O}(K+2L)_p$  and  $s \cdot s' \in \mathcal{O}(2L)_p$ .

①  $p \notin P$

$$\mathcal{O}_{P(L)_p} = \frac{\mathcal{O}(L)_p}{\mathcal{I}_P(L)_p} = 0 \quad \text{i.e.} \quad \mathcal{I}_P(L)_p = \mathcal{O}(L)_p$$

$$\Rightarrow \eta \in \mathcal{I}_P(L)_p = \mathcal{O}(L)_p \quad \eta(p) \neq 0.$$

$$\Rightarrow \text{Res}_p \left( \frac{\eta \psi}{s \cdot s'} \right) = 0$$

②  $p \in P$

As P674 note, we may assume that

$$s(z_1, z_2) = z_1^2 \quad s'(z_1, z_2) = z_2^3 \quad \text{locally.}$$

$\Rightarrow \eta \in \mathcal{I}_P(L)_p$  may be written as  $f_1 s + f_2 s'$  where  $f_1, f_2$  holomorphic functions of  $z_1, z_2$ .

$$\begin{aligned} \Rightarrow \frac{\psi \eta}{s s'} &= \frac{\eta (f_1 s + f_2 s')}{s s'} dz_1 \wedge dz_2 \\ &= \frac{\eta f_1}{s'} dz_1 \wedge dz_2 + \frac{\eta f_2}{s} dz_1 \wedge dz_2 \\ &= \frac{\eta f_1}{z_2^3} dz_1 \wedge dz_2 + \frac{\eta f_2}{z_1^2} dz_1 \wedge dz_2 \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{Res}_0 \left( \frac{\psi \eta}{s s'} \right) &= \text{Res}_{(0,0)} \left( \frac{\psi \eta}{s s'} \right) = \int \int_{|z_1|=e_1, |z_2|=e_2} \frac{\eta f_2}{z_1^2} dz_1 \wedge dz_2 \\ &+ \int_{|z_1|=e_1} \int_{|z_2|=e_2} \frac{\eta f_1}{z_2^3} dz_1 \wedge dz_2 = 0, \quad \text{since} \quad \int_{|z_1|=e_1} \frac{\eta f_2}{z_1^2} dz_1 = 0 \end{aligned}$$