

point $[0, 0, 0, 1]$. Thus, since any two distinct lines must intersect each other in case Q is $(X_0^2 + X_1^2 + X_2^2 = 0)$ or $(X_0^2 = 0)$, Q must be $(X_0^2 + X_1^2 = 0)$.

In case $Q = \text{Union of two hyperplanes}$, $E_L \subset Q$.
 \Rightarrow Since $E_L \cong B_L$, E_L must be in one of two planes, which is impossible since $g(B_L) = 2 \neq \frac{(5-1)(5-2)}{2} = 6$.
 \Rightarrow Thus Q must be smooth. \square

Two of the lines in the second ruling are also visible: at each point $p \in L$, one normal vector to $L \subset X$ will lie in the 3-plane $\bigcap_{x \in L} T_x(G)$, and the image of the points of $F \subset X_L$ corresponding to these normal vectors will be the line $(\bigcap_{x \in L} T_x(G)) \cap V_3$ in Q ; similarly the intersection of V_3 with $\bigcap_{x \in L} T_x(F)$ will be a line of the second ruling in Q .

Γ Consider $T_p(X) \cap (\bigcap_{x \in L} T_x(G)) = T_p(X) \cap (\bigcap_{x \neq p} T_x(G))$ by p222

$= T_p(X) \cap T_x(G) \Rightarrow \dim(T_p(X) \cap T_x(G)) \geq 2$
 \Rightarrow Since $T_p(X) \cap T_x(G) - L \neq \emptyset$, one normal vector to L at p lies in $\bigcap_{x \in L} T_x(G) = T_{x_1}(G) \cap T_{x_2}(G)$ which is a 3-plane. More precisely, choose $q \in T_p(X) \cap T_x(G) - L$.
 $\Rightarrow \overline{p, q} \subset T_p(X)$, and $\overline{p, q}$ is a normal vector to L