$\Rightarrow$ $\Theta_L = -\partial\bar\partial \log \|s\|^2$, $\quad \|\ \| = h_L(\ )$. $\quad s: M \longrightarrow L$.

$\Theta_{\pi^*L} = -\partial\bar\partial \log \|\pi^* s\|_0^2$, $\quad \|\ \|_0 = h_{\pi^*L}(\ )$. $\quad \pi^* s: \tilde M \longrightarrow \pi^* L$

$\pi^* h_L(v, w) = h_L(\pi_* v, \pi_* w)$

$\|\pi^* s\|_0^2 = \pi^* h_L(\pi^* s, \pi^* s) = h_L(\pi_* \pi^* s, \pi_* \pi^* s) = h_L(s, s)$

$= \|s\|^2$.

$\Rightarrow$ $\Omega_{\pi^*L} = -\partial\bar\partial \log \|s\|^2 \cdot \frac{i}{2\pi} = \frac{i}{2\pi} \Theta_{L} = \pi^* \Theta_L \cdot \frac{i}{2\pi}$

$= \frac{i}{2\pi} \pi^* \Theta_L = \pi^* \Omega_L$, $\Rightarrow$ Since $\pi: \tilde M - E \longrightarrow M - \{x\}$ is one to one, $\pi^* \Omega_L$ is positive defnite.

$\Rightarrow$ $\Omega_{\pi^*L} = \pi^* \Omega_L > 0$ on $\tilde M - E$.

"Comment: Here we have $\tilde M = \tilde{\mathcal U}_\epsilon$. $M = \mathbb P^{n-1}$.

$$\begin{tikzcd} {[-E]|_{\tilde{\mathcal U}_\epsilon} = \pi^*(H)} \arrow[r] \arrow[d] & {[H]} \arrow[d] \\ \tilde{\mathcal U}_\epsilon \arrow[r, "\pi"] & \mathbb P^{n-1} \end{tikzcd}$$

By P150, the hyperplane bundle [H] is a positive line bundle."

Moreover, for any $x \in E$ and tangent vector $v \in T'_x(\tilde M)$,

$$\langle \Omega_{\pi^*L} ; v, \bar v \rangle = \langle \Omega_L ; \pi_* v, \overline{\pi_* v} \rangle \geq 0$$

with equality holding $\Leftrightarrow$ $\pi_* v = 0$, i.e. $\Leftrightarrow$ v is tangent to E.

$\ulcorner$ $v \in T'_x(\tilde M)$ $\quad \pi_* \bar v = \overline{\pi_* v}$?

We want to prove ?

$f: M \xrightarrow{\text{holomorphic}} N$. $\quad f_*: T'_z M \longrightarrow T'_{f(z)} N$

$v \in T'_z M \Rightarrow v = \sum a_i \frac{\partial}{\partial z_i}$