

$$\begin{aligned}\Rightarrow \log h(z + \lambda_{n+r}) &= \log h(z) - 4\pi \operatorname{Im}(z_r) \\ \Rightarrow h(z + \lambda_{n+r}) &= e^{-4\pi \operatorname{Im}(z_r)} h(z) = |e^{2\pi i z_r}| h(z).\end{aligned}$$

□

Now we can compute the curvature form Θ_L associated to the metric in L given by h :

$$\begin{aligned}\Theta_L &= \partial \bar{\partial} \log \frac{1}{h} \\ &= -\frac{\pi}{2} \partial \bar{\partial} \left(\sum_{\alpha, \beta} W_{\alpha\beta} (z_\alpha - \bar{z}_\alpha)(z_\beta - \bar{z}_\beta - 2i Y_{\beta\beta}) \right) \\ &= \frac{\pi}{2} \partial \left(\sum_{\alpha, \beta} W_{\alpha\beta} ((z_\alpha - \bar{z}_\alpha) d\bar{z}_\beta + (z_\beta - \bar{z}_\beta - 2i Y_{\beta\beta}) d\bar{z}_\alpha) \right) \\ &= \pi \sum_{\alpha, \beta} W_{\alpha\beta} dz_\alpha \wedge d\bar{z}_\beta.\end{aligned}$$

□ See P13_Λ & P14 & P15.

By the facts $\bar{\partial} z_\alpha = 0$, $\bar{\partial} \bar{z}_\alpha = d\bar{z}_\alpha$, $\partial z_\alpha = dz_\alpha$ and $\partial \bar{z}_\alpha = d\bar{z}_\alpha$, we get the above. $dz = \partial + \bar{\partial}$. □

We want to express this in terms of the basis $\{dx_\alpha, dx_{n+\alpha}\}$; we have

$$dz_\alpha = \delta_\alpha dx_\alpha + \sum_\beta z_{\alpha\beta} dx_{n+\beta},$$

$$d\bar{z}_\alpha = \delta_\alpha d\bar{x}_\alpha + \sum_\beta \bar{z}_{\alpha\beta} d\bar{x}_{n+\beta},$$

so

$$\Theta_L = \pi \sum W_{\alpha\beta} dz_\alpha \wedge d\bar{z}_\beta$$