$H^n(M, \Omega^n) \cong \mathbb{C}$ and $(*)$ becomes

$$H^p(M, \mathcal{F}) \otimes \mathrm{Ext}^{n-p}(M; \mathcal{F}, \mathcal{G}) \longrightarrow \mathbb{C}.$$

$\vdash$ $H^n(M, \Omega^n) = H^{n,n}_{\bar\partial}(M) = H^0(M, \mathcal{O}) \cong \mathbb{C}$.

Where are the proofs ? ╜

Global Duality Theorem II. The above pairing is nondegenerate and is functorial in the following sense: A sheaf mapping ρ: $\mathcal{F} \to \mathcal{G}$ induces $\rho_*: H^*(M, \mathcal{F}) \longrightarrow H^*(M, \mathcal{G})$ and ρ^*: $\mathrm{Ext}^*(M; \mathcal{G}, \Omega^n) \longrightarrow \mathrm{Ext}^*(M; \mathcal{F}, \Omega^n)$ s.t the diagram

$$\begin{array}{ccccc} H^p(M, \mathcal{F}) & \otimes & \mathrm{Ext}^{n-p}(M; \mathcal{F}, \Omega^n) & \longrightarrow & \mathbb{C} \\ \rho_* \downarrow & & \uparrow \rho^* & & \\ H^p(M, \mathcal{G}) & \otimes & \mathrm{Ext}^{n-p}(M; \mathcal{G}, \Omega^n) & \longrightarrow & \mathbb{C} \end{array}$$

is commutative.

As mentioned before, we have proved this in the two extreme cases $\mathcal{F} \cong \mathcal{O}(E)$ and $\mathcal{F} = \mathcal{O}_z$, which is all that we shall have geometric applications for. The general result can also be proved without ~~too~~ much additional effort – and most of this in the nature of formalism – from our local duality theorem.

$\vdash$

$$\begin{array}{ccccc} H^p(M, \mathcal{O}(E)) & \otimes & \mathrm{Ext}^{n-p}(M; \mathcal{O}(E), \Omega^n) & \longrightarrow & \mathbb{C} \\ \rho_* \downarrow & & \uparrow \rho^* & & \\ H^p(M, \mathcal{G}) & \otimes & \mathrm{Ext}^{n-p}(M; \mathcal{G}, \Omega^n) & \longrightarrow & \mathbb{C} \end{array}$$