Since Λ, d and d^c are real operators, either of these implies the other; we will prove $[\Lambda, \partial] = i\bar{\partial}^*$.

$\ulcorner$ $\overline{[\Lambda, \partial]} = -i\partial^* = [\Lambda, \bar{\partial}]$

$\bar{\Lambda}(\varphi) = \overline{\Lambda(\bar{\varphi})} \overset{?}{=} \Lambda(\varphi)$

$L(\eta) = \eta \wedge \omega \qquad \bar{L}(\eta) = \overline{L(\bar{\eta})} = \overline{L(\bar{\eta})}$

$= \overline{\bar{\eta} \wedge \omega} = \eta \wedge \bar{\omega} = L(\eta) = \eta \wedge \omega$

Since $\omega = \frac{i}{2} \Sigma \varphi_i \wedge \bar{\varphi}_i$ and, $\bar{\omega} = -\frac{i}{2} \Sigma \bar{\varphi}_i \wedge \varphi_i$
$= \frac{i}{2} \Sigma \varphi_i \wedge \bar{\varphi}_i = \omega$, $\quad L$ is real operator. i.e
$\bar{L}(\eta) = L(\eta)$.

$\langle \eta, \bar{\Lambda}(\zeta) \rangle = \langle \eta, \overline{\Lambda(\bar{\zeta})} \rangle = \overline{\langle \bar{\eta}, \Lambda(\bar{\zeta}) \rangle}$
$= \overline{\langle L(\bar{\eta}), \bar{\zeta} \rangle} = \langle \overline{L(\bar{\eta})}, \bar{\bar{\zeta}} \rangle = \langle \bar{L}(\eta), \zeta \rangle$
$= \langle L(\eta), \zeta \rangle = \langle \eta, \Lambda(\zeta) \rangle \Rightarrow \bar{\Lambda} = \Lambda$.

We use the following simple fact:

$\langle v, \omega \rangle = \langle \bar{v}, \bar{\omega} \rangle$

$v = a\, e_I \quad \omega = b\, e_J \Rightarrow \langle v, \omega \rangle$
$= a\bar{b} \langle e_I, e_J \rangle$

$\Rightarrow \langle e_I, e_J \rangle = 0$. or $1 = \langle \bar{e}_I, \bar{e}_J \rangle \Rightarrow$ We are done. $\lrcorner$

We make the computations first on $\mathbb{C}^n$ with the Euclidean metric. Here it is messy but straightforward and will be facilitated by our breaking it up into component steps. To do this, we introduce some new operators on forms in $\mathbb{C}^n$: for each $k = 1, 2, \cdots n$, let
$e_k : A_c^{p,q}(\mathbb{C}^n) \longrightarrow A_c^{p+1,q}(\mathbb{C}^n)$ be the operator on compactly supported forms defined by

$$e_k(\varphi) = dz_k \wedge \varphi;$$