$\Rightarrow \quad (-1)^{k-1}\, \sigma_{a_{k-1}+d-(k-1)} \cdot \sigma_{a_1,\cdots a_{k-2},\, a_k-1,\cdots a_d-1}$ — ①

$+ (-1)^{k}\, \sigma_{a_k+d-k} \cdot \sigma_{a_1,\cdots a_{k-1},\, a_{k+1}-1,\cdots a_d-1}$ — ②

$\Rightarrow$ ① $= (-1)^{k-1} \sum \sigma_c$.

$\sum c_i = a_1+\cdots+a_{k-1}+d-(k-1)+a_k-1+\cdots+a_d-1 = \sum a_i$.

$a_i \le c_i \le a_{i-1}$ if $i \le k-2$.

$a_{i+1}-1 \le c_i \le a_i-1$ if $i > k-1 \Rightarrow i \ge k$

$a_k-1 \le c_{k-1} \le a_{k-2}$

By ⊛⊛, $a_i \le c_i \le a_{i-1}$ $\quad i \le k-1$

$a_{i+1}-1 \le c_i \le a_i-1$ $\quad i \ge k$.

For $i \le k-2$, O.K.

$i = k-1$. $a_{k-1} \le c_{k-1} \le a_{k-2}$. $\Rightarrow a_k-1 \le c_{k-1} \le a_{k-2}$

since $a_k-1 \le a_{k-1}$.

For $i \ge k$, $a_{i+1}-1 \le c_i \le a_i-1 \Rightarrow$ O.K.

② $= (-1)^k \sum \sigma_c$

$\sum c_i = a_1+\cdots+a_{k-1}+a_k+d-k+a_{k+1}-1+\cdots+a_d-1$

$= \sum a_i$

$a_i \le c_i \le a_{i-1}$ if $i \le k-1$

$a_{i+1}-1 \le c_i \le a_i-1$ if $i > k$

$a_{k+1}-1 \le c_k \le a_{k-1}$.

By ⊛⊛, $a_i \le c_i \le a_{i-1}$ $\quad i \le k-1$

$a_{i+1}-1 \le c_i \le a_i-1$ $\quad i \ge k$

For $i \le k-1$ al $i \ge k$, O.K.

If $i = k$, $a_{k+1}-1 \le c_k \le a_k-1 \le a_k \le a_{k-1}$

$\Rightarrow$ c_k lies between $a_{k+1}-1$ and a_{k-1}.