" I think: the compatibility of $\equiv$ with the group operation of $Div(M)$ means as follows:

$$D_1 \equiv D_2 \ \& \ D_1' \equiv D_2' \Rightarrow D_1 + D_1' \equiv D_2 + D_2' \quad "$$

" Comment on the finiteness of $NS(M)$"

If $\eta_D \in H^2(M, \mathbb{Z}) \subset H^2(M, \mathbb{C})$ is zero in $H^2(M, \mathbb{C})$, see P61, then η_D is a torsion element. $\Rightarrow$ By Severi and Picard, λD is algebraically equivalent to zero for some $\lambda \in \mathbb{Z}$. $\Rightarrow$ $\bar{D} \in NS(M) = Div(M)/\equiv$, $\bar{D} = D + \{\}$.

$\Rightarrow$ $\bar{D}$ is a torsion element of $NS(M)$. $\Rightarrow$ $\dim_{\mathbb{C}} NS(M) < \infty$, since $\dim H^2(M, \mathbb{C}) < \infty$. ⌟

The basic result we need is the

Lemma. Two divisors D_1, D_2 are algebraically equivalent if and only if they are homologous.

Proof. It is clear that $D_1 \equiv D_2 \Rightarrow \eta_{D_1} = \eta_{D_2}$ in $H^2(M, \mathbb{Z})$.

⌜ $D_1 \equiv D_2 \Rightarrow \exists\ D$ s.t $D_1 + D \equiv D_2 + D$. $\Rightarrow \exists\ T$ and L s.t $L \cdot M \times \{t_i\} = D_i + D$.

$\Rightarrow \eta_{D_1+D} = \eta_{D_2+D} \Rightarrow \eta_{D_1} + \eta_D = \eta_{D_2} + \eta_D \Rightarrow \eta_{D_1} = \eta_{D_2}$ ⌟

For the converse we assume that $\eta_{D_1} = \eta_{D_2}$, which is