$$h^1(\mathcal{O}(L)) = h^1(\mathcal{O}(K_S-L))$$
$$= h^0(\mathcal{J}_P(K_S+L)) - 2h^0(\mathcal{O}(K_S)) + h^0(\mathcal{O}(K_S-L)), \quad \text{for}$$

$$0 \to H^0(S,\mathcal{O}(K_S-L)) \to H^0(S,\mathcal{O}(K_S)) \oplus H^0(S,\mathcal{O}(K_S)) \to H^0(S,\mathcal{J}_P(K_S+L))$$

$$\to H^1(S,\mathcal{O}(K_S-L)) \to \underbrace{H^1(S,\mathcal{O}(K_S))}_{=0} \oplus \underbrace{H^1(S,\mathcal{O}(K_S))}_{=0}$$

$$\Rightarrow \left\{ \begin{array}{l} h^0(\mathcal{O}(K_S-L)) - 2h^0(\mathcal{O}(K_S)) + h^0(\mathcal{J}_P(K_S+L)) - h^1(\mathcal{O}(K_S-L)) = 0. \\ h^0(\mathcal{J}_P(K_S+L)) = \dim|\mathcal{J}_P(K_S+L)| + 1 \\ h^0(\mathcal{O}(K_S-L)) = \dim|K_S-L| + 1 \\ h^0(\mathcal{O}(K_S)) = h^{2,0}(S) = p_g. \end{array} \right.$$

These give the assertion.

In case $p_g = 0$, $\dim|K_S - L| = -1$, and the formula simplifies to

$$\omega = \dim|\mathcal{J}_P(K_S+L)| + 1.$$

As an application, suppose that Γ_0 is a set of $d < n(n+3)/2$ points in $\mathbb{P}^2$. Then the linear system $|\mathcal{J}_{\Gamma_0}(n)|$ of curves of degree n passing through Γ_0 contains at least a pencil, and either

1. this linear system has a fixed curve of degree less than n; or
2. general curves $C, C' \in |\mathcal{J}_{\Gamma_0}(n)|$ will have intersection

$$C \cdot C' = \Gamma_0 + \Gamma,$$

where Γ is a set of $n^2 - d$ points that we call a resi