

described as follows. Let $p_0 \in R$ be any of the double points of S , let \tilde{S} be the blow-up of S at p_0 , and consider the map

$$r: \tilde{S} \longrightarrow \mathbb{P}^2$$

obtained by projection from p_0 onto a hyperplane.

⌈ See P475.

For the projection map, see P479. Refer to P176. \sqcup

The generic hyperplane section C_h of S through p_0 is a plane quartic curve with one double point at p_0 , its proper transform $\tilde{C}_h \subset \tilde{S}$ its desingularization.

⌈ By Bertini theorem, generic hyperplane section C_h of S through p_0 is smooth away p_0 . Furthermore, generic C_h has a double point at p_0 . Since $\deg S = 4$, C_h is quartic. \Rightarrow Since C_h has an ordinary double point, \tilde{C}_h is smooth, and is the desingularization.

\sqcup

\tilde{C}_h thus has genus 2, and since r expresses \tilde{C}_h as a 2-sheeted cover of its image $L = h \cap \mathbb{P}^2 = \mathbb{P}^1$, r must be branched at exactly six points over