$$\begin{tikzcd} \mathbb{R}^m \supset A \ni x_0 \arrow[r, bend left, "F"] & B \ni y_0, \ B \subset \mathbb{R}^m \arrow[l, bend left, "G"] \end{tikzcd}$$

$$F(x_1, x_2 \cdots x_m) = (\psi\circ f\circ\varphi^{-1}(x_1 \cdots x_m),\ x_{n+1}, \cdots x_m)$$

$\Rightarrow$ Restriction to $(\psi\circ f\circ\varphi^{-1})^{-1}(y_0)$, F is again diffeomorphic.

$\Rightarrow$ $F| : (\psi\circ f\circ\varphi^{-1})^{-1}(y_0) \longrightarrow (y_0, x_{n+1}, \cdots x_m)$

$\Rightarrow$ $A \cap (\psi\circ f\circ\varphi^{-1})^{-1}(y_0) \cong \{y_0\} \times \mathbb{R}^{m-n}$.

$$\begin{tikzcd} M \supset U'' \arrow[r, "f"] \arrow[d, "\varphi"'] & V'' \subset N \arrow[d] \\ \mathbb{R}^m \supset A = U''' \arrow[r, "\psi\circ f\circ\varphi^{-1}"] \arrow[dr, "F"] & V''' \subset \mathbb{R}^n \\ & B \subset \mathbb{R}^m \end{tikzcd}$$

Thus $(F\circ\varphi, U'')$ is an element of charts of M s.t.

$$U'' \cap f^{-1}(y) \xrightarrow{F\circ\varphi} \{y_0\} \times \mathbb{R}^{n-m}.$$

③ Theorem. If the smooth (C^∞) map $f: M \longrightarrow N$ is transversal to a submanifold $Z \subset N$, then the preimage $f^{-1}(Z)$ is a submanifold of M. Moreover, the codimension of $f^{-1}(Z)$ in M equals the codimension of Z in N.

pf)

$$\begin{tikzcd} U' \ni x_0' \arrow[r, "f"] \arrow[d, "\varphi"'] \arrow[d, bend left] & V' \ni y_0' \arrow[d, "\psi"] \arrow[d, bend left] \\ \mathbb{R}^m \supset U \ni x_0 \arrow[r, "\psi\circ f\circ\varphi^{-1}"] & V \ni y_0, \ V \subset \mathbb{R}^n \end{tikzcd}$$

$$\psi(V' \cap Z) \subset V$$
$$\parallel$$
$$V \cap \{0\} \times \mathbb{R}^{n-k}.$$