$(A_1^2(\tau_1), A_1^2(\tau_2)) = (0, 2)$ & $(A_2^2(\tau_1), A_2^2(\tau_2)) = (0, 2)$, then

$$* = 0 + 0 - 2 \begin{vmatrix} 1, & 1 \\ 0, & 2 \end{vmatrix}^2 - 2 \begin{vmatrix} 0, & |t| \\ 1, & 1 \end{vmatrix}^2$$

$$+ 2 \det \begin{pmatrix} 1, & 1 \\ 0, & 2 \end{pmatrix} \det \begin{pmatrix} 0, & 2 \\ 1, & 1 \end{pmatrix} + 2 \det \begin{pmatrix} 1, & 1 \\ 0, & 2 \end{pmatrix} \det \begin{pmatrix} 0, & 2 \\ 1, & 1 \end{pmatrix}$$

$$= 0 + 0 - 2 \cdot 2^2 - 2 \cdot 2^2 + 2 \cdot 2 (-2) + 2 (2)(-2)$$
$$= -8 - 8 - 8 - 8 = -32$$

$\Rightarrow$ $c_1(\Theta)^2 - 2c_2(\Theta) \geq 0$ does not hold. I don't know whether my computation is correct or not. 끝

The inequalities

$$\begin{cases} c_q(E) \geq 0, \\ c_1^2(E) \geq 2c_2(E), \text{ etc.}, \end{cases}$$

are valid for any holomorphic vector bundle that is positive in a suitable sense. We shall not give the proof here, but the reader may consult S. Bloch and D. Gieseker, The positivity of the Chern classes of an ample vector bundle, Invent. Math., Vol. 12 (1971), 112-117.

"Comment.

$c_1^2 - 2c_2 = \left(\frac{\sqrt{-1}}{2\pi}\right)^2 \operatorname{trace}(\Theta^2)$, since

$\left(\frac{i}{2\pi} \operatorname{tr}(\Theta)\right)^2 = c_1^2$. and

$$\det\left(I + t\Theta \cdot \frac{\sqrt{-1}}{2\pi}\right) = 1 + c_1 t + c_2 t^2.$$

$c_2 = \sum x_i x_j \Rightarrow 2c_2 = \sum 2x_i x_j$. $(x_1 + \cdots + x_n)^2 =$