Fixing a base point $p_0 \in S$, there are inclusions

$$\iota : S^{(d)} \longrightarrow \mathrm{Div}^0(S)$$

given by

$$\sum p_\lambda \longmapsto \sum (p_\lambda - p_0).$$

and, correspondingly, holomorphic maps

$$\mu^{(d)} : S^{(d)} \longrightarrow J(S)$$

$$\sum p_\lambda \longmapsto \Big(\sum_\lambda \int_{p_0}^{p_\lambda} \omega_1 , \cdots \sum_\lambda \int_{p_0}^{p_\lambda} \omega_g \Big).$$

$\ulcorner$ $\iota : S^{(d)} \longrightarrow \mathrm{Div}^0(S)$

$$\sum p_\lambda \longrightarrow \sum p_\lambda - d p_0 = \sum (p_\lambda - p_0)$$

$$\downarrow$$

$$\Big(\sum_\lambda \int_{p_0}^{p_\lambda} \omega_1 , \cdots \sum_\lambda \int_{p_0}^{p_\lambda} \omega_g \Big) \in J(S)$$

$\lrcorner$

In this context, the Jacobi inversion theorem asserts that for S of genus g, the map $\mu^{(g)}$ is surjective and generically one to one. (See the statement of The Jacobi Inversion Theorem P 235).

Now let $D = \sum p_i$ be a point of $S^{(g)}$ with all p_i distinct, z_i a local coordinate on S centered at p_i, and $(z_1, \cdots z_g)$ corresponding coordinates on $S^{(g)}$ near D. For $D' = \sum z_i$ near D, by calculus

$$\frac{\partial}{\partial z_i} \big(\mu^{(g)}(D') \big) = \frac{\partial}{\partial z_i} \Big(\int_{p_0}^{z_i} \omega_j \Big)$$

$$= \omega_j / d z_i ,$$