$|J(F)| \neq 0$ at the origin.
$$\parallel \quad \left|\frac{\partial f_i}{\partial z_j}\right| \quad 1 \le i, j \le 2$$

$\Rightarrow$ By the inverse function theorem, $\exists$ an inverse $G: \mathbb{C}^7 \longrightarrow \mathbb{C}^7$ around the origin.

$G(f_1', f_2', f_3'+\omega_1, f_4'+\omega_2, a_1, a_2, z_3) = (z_1, z_2, z_3, a_1, a_2, \omega_1, \omega_2)$

Let $\omega_1 = \omega_2 = 0$, and $f_1' = f_2' = f_3' = 0$.

$\Rightarrow G(0, 0, 0, 0, a_1, a_2, z_3) = (z_1, z_2, z_3, a_1, a_2, 0, 0)$.

$\Rightarrow$ $z_1 = g_1(a_1, a_2, z_3)$
$z_2 = g_2(a_1, a_2, z_3)$ on some open set $U \times V$
$U \ni (0,0)$. $V \ni 0$.

Clearly, $f_i'(z_1, z_2, z_3, a_1, a_2) = 0$ if $z_1 = g_1(a_1, a_2, z_3)$
$z_2 = g_2(a_1, a_2, z_3)$.

$\Rightarrow$ For each (a_1, a_2), $f_i(a_1, a_2, V) = f_i(a_1, a_2, 0)$

$\Rightarrow$ Let $W = U \times V$.

$\Rightarrow$ $f(W) = f(U) = f(W \cap \mathbb{C}^2)$.

In general, it follows from the argument by changing the numbers.

Now it remains to show that $f(M) = f(S)$.

By the induction, since $k < n$, $f(S)$ is an analytic subvariety of N. $\Rightarrow$ $f^{-1}(f(S))$ is analytic subvariety, for. let $x \in M$. $f(x) \in N$.

$\Rightarrow \exists$ open set $U \ni f(x)$ s.t $U \cap f(S) = \{g_1 = \cdots = g_e = 0\}$

g_i: holomorphic on U. $\Rightarrow$ $f^{-1}(U \cap f(S)) = f^{-1}(U) \cap f^{-1}(f(S))$