

Let  $W = \{ \bar{i}(\bar{Z}) \Lambda : \bar{Z} \in \Lambda^{k-1} V^* \} \Rightarrow W \stackrel{?}{=} \text{Ann}(\Lambda^\perp)$

$$\bar{i}(\bar{Z}) \Lambda \stackrel{?}{\in} \text{Ann}(\Lambda^\perp).$$

$$\langle \bar{i}(\bar{Z}) \Lambda, l \rangle \stackrel{?}{=} 0 \text{ for all } l \in \Lambda^\perp.$$

We may assume that  $\bar{Z}$  is decomposable i.e.  $\bar{Z} = v_1^* \wedge \dots \wedge v_{k-1}^*$ .

$$\langle \bar{i}(\bar{Z}) \Lambda, l \rangle = \langle \Lambda, \bar{Z} \wedge l \rangle = \pm \langle \Lambda, l \wedge \bar{Z} \rangle$$

$$= \pm \langle \bar{i}(l) \Lambda, \bar{Z} \rangle = 0 \text{ since } \bar{i}(l) \Lambda = 0 \text{ by the definition of } \Lambda^\perp \text{ and } l \in \Lambda^\perp.$$

$$\Rightarrow \bar{i}(\bar{Z}) \Lambda \in \text{Ann}(\Lambda^\perp) \Rightarrow W \subset \text{Ann}(\Lambda^\perp).$$

Let  $\tilde{W} = \text{Ann}(\Lambda^\perp)$ . Suppose  $\tilde{W} \neq W$ .

$$\Rightarrow \tilde{W}^\perp \subset W^\perp, \text{ where } W^\perp = \{ l \in V^* : l = 0 \text{ on } W \}$$

$$\Lambda^\perp = \tilde{W}^\perp = \{ l \in V^* : l = 0 \text{ on } \tilde{W} \}$$

$$\Rightarrow \exists l \in W^\perp - \tilde{W}^\perp.$$

$$\Rightarrow 0 = \langle \bar{i}(\bar{Z}) \Lambda, l \rangle = \langle \Lambda, \bar{Z} \wedge l \rangle = \pm \langle \Lambda, l \wedge \bar{Z} \rangle$$

$$= \pm \langle \bar{i}(l) \Lambda, \bar{Z} \rangle \text{ for all decomposable } \bar{Z} \in \Lambda^{k-1} V^*.$$

$$\Rightarrow \bar{i}(l) \Lambda = 0 \Rightarrow l \in \Lambda^\perp = \tilde{W}^\perp \Rightarrow \text{Contradiction}$$

$$\Rightarrow \tilde{W} = W.$$

To hold  $W = W'$ , for all  $w \in W$ ,  $\Lambda \wedge w = 0$ .

$\Rightarrow \Lambda \wedge (\bar{i}(\bar{Z}) \Lambda) = 0$  for all  $\bar{Z} \in \Lambda^{k-1} V^*$ , since every  $w$  can be expressed as  $\bar{i}(\bar{Z}) \Lambda$  for some  $\bar{Z} \in \Lambda^{k-1} V^*$ .  $\Rightarrow$

For example, suppose that

$$\Lambda = \frac{1}{2} \sum_{i,j} \lambda_{ij} e_i \wedge e_j, \quad \lambda_{ij} + \lambda_{ji} = 0,$$

is a bivector. Since for  $v^* \in V^*$

$$(\bar{i}(v^*) \Lambda) \wedge \Lambda = \frac{1}{2} \bar{i}(v^*) (\Lambda \wedge \Lambda),$$