

$$\text{From } H^{p,p+\bar{j}} = \bigoplus L^k P^{p-k, p-k+\bar{j}}$$

$$\begin{aligned} (-1)^p h^{p,p+\bar{j}} &= (-1)^p \dim P^{p,p+\bar{j}} + (-1)^{p-1} \dim P^{p-1,p-1+\bar{j}} + (-1)^{p-2} \dim P^{p-2,p-2+\bar{j}} + \dots + (-1)^0 \dim P^{0,0+\bar{j}} \\ 2 (-1)^{p-1} h^{p-1,p-1+\bar{j}} &= 2 (-1)^{p-1} \dim P^{p-1,p-1+\bar{j}} + 2 (-1)^{p-2} \dim P^{p-2,p-2+\bar{j}} + 2 (-1)^{p-3} \dim P^{p-3,p-3+\bar{j}} + \dots + 2 (-1)^0 \dim P^{0,0+\bar{j}} \\ 2 (-1)^{p-2} h^{p-2,p-2+\bar{j}} &= 2 (-1)^{p-2} \dim P^{p-2,p-2+\bar{j}} + 2 (-1)^{p-3} \dim P^{p-3,p-3+\bar{j}} + 2 (-1)^{p-4} \dim P^{p-4,p-4+\bar{j}} + \dots + 2 (-1)^0 \dim P^{0,0+\bar{j}} \\ &\vdots \\ + 2 (-1)^0 h^{0,0+\bar{j}} &= 2 (-1)^0 \dim P^{0,0+\bar{j}} \end{aligned}$$

$$\text{LHS} = (-1)^p h^{p,p+\bar{j}} + 2 \sum_{\bar{i}=0}^{p-1} (-1)^{\bar{i}} h^{\bar{i}, \bar{i}+\bar{j}}$$

$$\begin{aligned} \text{RHS} &= (-1)^p \dim P^{p,p+\bar{j}} + (-1)^{p-1} \dim P^{p-1,p-1+\bar{j}} + (-1)^{p-2} \dim P^{p-2,p-2+\bar{j}} + \dots + (-1)^0 \dim P^{0,0+\bar{j}} \\ &= \sum_{\bar{i}=0}^p (-1)^{\bar{i}} \dim P^{\bar{i}, \bar{i}+\bar{j}} \end{aligned}$$

$$\Rightarrow (-1)^p h^{p,p+\bar{j}} + 2 \sum_{\bar{i}=0}^{p-1} (-1)^{\bar{i}} h^{\bar{i}, \bar{i}+\bar{j}} = \sum_{\bar{i}=0}^p (-1)^{\bar{i}} \dim P^{\bar{i}, \bar{i}+\bar{j}} \quad \Rightarrow \text{LHS} = \text{RHS}$$

$$\textcircled{2} \quad I(M) = \sum_{\substack{p+q \equiv 0(2) \\ \leq 2n \\ p \leq q}} (-1)^p \dim P^{p,q}(M)$$

$$= \sum_{\substack{p+q=2n \\ p \leq q}} (-1)^p \dim P^{p,q}(M) + \sum_{\substack{p+q \equiv 0(2) \\ p+q < 2n}} (-1)^p \dim P^{p,q}(M)$$

$$\begin{aligned} \Rightarrow & \begin{array}{cccccc} p+q=0 & p+q=2 & p+q=4 & p+q=6 & p+q=8 & \dots & p+q=2n-2 & p+q=2n \\ (-1)^0 \dim P^{0,0}(M) & + (-1)^0 \dim P^{0,2}(M) & + (-1)^0 \dim P^{0,4} & + (-1)^0 \dim P^{0,6} & + (-1)^0 \dim P^{0,8} & \dots & + (-1)^0 \dim P^{0,2n-2} & + (-1)^0 \dim P^{0,2n} \\ & + (-1)^1 \dim P^{1,1} & + (-1)^1 \dim P^{1,3} & + (-1)^1 \dim P^{1,5} & + (-1)^1 \dim P^{1,7} & & + (-1)^1 \dim P^{1,2n-3} & + (-1)^1 \dim P^{1,2n-1} \\ & & + (-1)^2 \dim P^{2,2} & + (-1)^2 \dim P^{2,4} & + (-1)^2 \dim P^{2,6} & & + (-1)^2 \dim P^{2,2n-4} & + (-1)^2 \dim P^{2,2n-2} \\ & & & + (-1)^3 \dim P^{3,3} & + (-1)^3 \dim P^{3,5} & & + (-1)^3 \dim P^{3,2n-5} & + (-1)^3 \dim P^{3,2n-3} \\ & & & & + (-1)^4 \dim P^{4,4} & & + (-1)^4 \dim P^{4,2n-6} & + (-1)^4 \dim P^{4,2n-4} \\ & & & & & & \vdots & \vdots \\ & & & & & & + (-1)^n \dim P^{n,n} \end{array} \end{aligned}$$