Consider the sequence

$$0 \longrightarrow \mathcal{O}_M(E\otimes L^{m-1}) \longrightarrow \mathcal{O}_M(E\otimes L^{m}) \longrightarrow \mathcal{O}_V(E\otimes L^{m}) \to 0.$$

「 Since V is smooth, as P174, V is connected.
See P139, for the sequence. 」

By Theorem B, there exists m_1 such that for $m > m_1$, $H^1(M, \mathcal{O}(E\otimes L^{m-1})) = 0$, so that the restriction map

$$H^0(M, \mathcal{O}(E\otimes L^m)) \longrightarrow H^0(V, \mathcal{O}(E\otimes L^m))$$ will be surjective.

「 $H^0(M, \mathcal{O}_M(E\otimes L^{m-1})) \longrightarrow H^0(M, \mathcal{O}_M(E\otimes L^m)) \longrightarrow H^0(M, \mathcal{O}_V(E\otimes L^m)) \longrightarrow H^1(M, \mathcal{O}_M(E\otimes L^{m-1})) \longrightarrow \cdots$

See P159, Theorem B. $\Rightarrow \exists\ m_1$ s.t if $m > m_1$, then $H^1(M, \mathcal{O}_M(L^m\otimes E)) = 0$. 」

On the other hand, by induction there exists m_2 such that for $m > m_2$,

$$H^0(V, \mathcal{O}_V(E\otimes L^m)) \to (E\otimes L^m)_x \oplus (E\otimes L^m)_y$$

is surjective.

「 We are using induction on $\dim M$. Since $\dim V < \dim M$, $\exists\ m_2$ s.t if $m > m_2$,

$$H^0(V, \mathcal{O}_V(E\otimes L^m)) \longrightarrow (E\otimes L^m)_x \oplus (E\otimes L^m)_y$$

is surjective 」

For $m > m_0 = \max(m_1, m_2)$, then the map $(*)$ will be surjective.

Similarly, for each of a generating set of cot-