Again, the projection $\pi: \tilde{M}_x - \{\pi^{-1}(x)\} \longrightarrow M - \{x\}$ is an isomorphism; the inverse image $\pi^{-1}(x)$ in $\tilde{M}_x$ is called the exceptional divisor of the blow-up, and is usually denoted E or E_x.

Note that the blow-up $\tilde{M} \to M$ is independent of the coordinates used in the disc Δ: if $\{z_i' = f_i(z)\}$ are other coordinates in Δ with $f_i(0) = 0$, $\tilde{\Delta}'$ the blow-up of Δ in terms of these coordinates, then the isomorphism

$$f: \tilde{\Delta} - E \longrightarrow \tilde{\Delta}' - E'$$

may be extended over E by setting $f(0, l) = (0, l')$, where

$$l_j' = \sum \frac{\partial f_j}{\partial z_i}(0) \cdot l_i .$$

$\ulcorner$ $(z_1 \cdots z_n) \longmapsto (f_1(z), f_2(z), \cdots f_n(z))$

$$\begin{pmatrix} \frac{\partial f_1}{\partial z_1}(0) & \cdots & \frac{\partial f_1}{\partial z_n}(0) \\ \vdots & & \\ \frac{\partial f_n}{\partial z_1}(0), & \cdots & \frac{\partial f_n}{\partial z_n}(0) \end{pmatrix} \text{ is the map as } (z_1 \cdots z_n) \to 0.$$

$\Rightarrow$ $z_i = t\, l_i$. $t \to 0$

$$(f_1(tl), \cdots f_n(tl)) \longrightarrow \begin{pmatrix} \frac{\partial f_1}{\partial z_1}(0) & \cdots & \frac{\partial f_1}{\partial z_n}(0) \\ & & \\ \frac{\partial f_n}{\partial z_1}(0), & \cdots & \frac{\partial f_n}{\partial z_n}(0) \end{pmatrix} \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} = \begin{pmatrix} l_1' \\ \vdots \\ l_n' \end{pmatrix}$$

$\Rightarrow$ $l_j' = \sum \frac{\partial f_j}{\partial z_i}(0) \cdot l_i$. And since f is biholomorphic,

$\exists$ an inverse of $\left(\frac{\partial f_j}{\partial z_i}(0)\right)$. $\Rightarrow$ $\exists$ an inverse $f^{-1}: \tilde{\Delta}' - E' \longrightarrow \tilde{\Delta} - E$ which may be extended over E'.