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Def: A directed set J is a set with a relation $<$ such that

(1) $\alpha < \alpha$ for all $\alpha \in J$

(2) $\alpha < \beta$ and $\beta < \gamma$ implies $\alpha < \gamma$

(3) Given α and β, $\exists$ δ such that $\alpha < \delta$ and $\beta < \delta$.

Def: If J is a directed set, a subset J_0 of J is said to be cofinal in J if for each $\alpha \in J$, there exists $\delta \in J_0$ with $\alpha < \delta$. see P432 & P435 Munkres.

Given a divisor D on M, and let D_0 be an ample divisor on M, for sufficiently large k,

$kD_0 + D$ is an ample divisor. i.e., $[kD_0 + D] \to M$ is positive line bundle. $\Rightarrow$ $kD_0 + D \supset D$

$$H^q(M, \Omega(*)) = 0 \quad \text{for } q > 0.$$

pf). ~~$C^q(M, \Omega^p(*)) \ni \varphi$~~

~~$\Rightarrow$ φ has poles on $D_1, D_2, \cdots D_k$, where D_i's are effective, $\Rightarrow$ Since M is compact, the # of poles of φ is finite. $\Rightarrow$ We can find an ample divisor D s.t. φ has poles on D.~~

~~and $\underline{\mathcal{U}}$ is locally~~ ~~finite open covers~~

$$C^q(\underline{\mathcal{U}}, \Omega^p(*)) \ni \varphi$$

$$\Rightarrow \quad \varphi_{\alpha_0, \cdots \alpha_q} \in \Omega^p(*)(U_{\alpha_0} \cap \cdots \cap U_{\alpha_q}) = \bigcup_{D \in \mathrm{Div} M} \bigcup_{k \geq 0} \Omega^p(kD)(U_{\alpha_0} \cap \cdots \cap U_{\alpha_q})$$

$\Rightarrow$ Since M is compact & $\underline{\mathcal{U}}$ is locally finite open cover, since $\# \underline{\mathcal{U}} < \infty$, $\exists$ finite # of irreducible effective divisors $D_1, D_2 \cdots D_k$,