$$\Rightarrow \quad \iota(v_{i_\alpha})(\omega_1)\wedge\omega_2 + (-1)^p\,\omega_1\wedge\iota(v_{i_\beta})(\omega_2)$$

$$= (-1)^{\alpha-1}\,\varphi_{i_1}\wedge\cdots\wedge\hat{\varphi}_{i_\alpha}\wedge\cdots\,\varphi_{i_p}\wedge\varphi_{j_1}\wedge\cdots\wedge\varphi_{j_q}$$
$$+ (-1)^p\,\varphi_{i_1}\wedge\cdots\,\varphi_{i_\alpha}\wedge\varphi_{i_p}\wedge\varphi_{j_1}\wedge\cdots\wedge\hat{\varphi}_{j_\beta}\wedge\cdots\varphi_{j_q}\,(-1)^{\beta-1}$$

$$= (-1)^{\alpha-1}\,\varphi_{i_1}\wedge\cdots\wedge\varphi_{j_\beta}\wedge\cdots\wedge\varphi_{i_p}\wedge\varphi_{j_1}\wedge\hat{\varphi}_{j_\beta}\wedge\varphi_{j_q}\quad(-1)^{\beta-1}\cdot(-1)^{p-\alpha}$$

$$+ (-1)^p\,\varphi_{i_1}\wedge\cdots\wedge\varphi_{i_\alpha}\wedge\cdots\wedge\varphi_{i_p}\wedge\varphi_{j_1}\wedge\cdots\wedge\hat{\varphi}_{j_\beta}\cdots\wedge\varphi_{j_q}\quad(-1)^{\beta-1}$$

$$= (-1)^p(-1)^\beta\,\varphi_{i_1}\wedge\cdots\wedge\varphi_{j_\beta}\wedge\cdots\wedge\varphi_{i_p}\wedge\varphi_{j_1}\wedge\cdots\wedge\hat{\varphi}_{j_\beta}\wedge\cdots\wedge\varphi_{j_q}$$
$$+ (-1)^p(-1)^{\beta-1}\,\varphi_{i_1}\wedge\cdots\wedge\varphi_{i_\alpha}\wedge\cdots\wedge\varphi_{i_p}\wedge\varphi_{j_1}\wedge\cdots\wedge\hat{\varphi}_{j_\beta}\wedge\cdots\wedge\varphi_{j_q} = 0$$

② $I\cap J=\phi$

$$\iota(v_j)(\omega_1\wedge\omega_2) = \begin{cases} 0, & j = \text{any of } i_\alpha \text{ or } j_\beta\text{'s} \\ (-1)^{\alpha-1}\,\varphi_{i_1}\wedge\cdots\wedge\hat{\varphi}_{i_\alpha}\wedge\cdots\varphi_{i_p}\wedge\varphi_{j_1}\wedge\cdots\wedge\varphi_{j_q} & \text{if } j=i_\alpha. \\ (-1)^p(-1)^{\beta-1}\,\varphi_{i_1}\wedge\cdots\wedge\varphi_{i_p}\wedge\varphi_{j_1}\wedge\cdots\wedge\hat{\varphi}_{j_\beta}\wedge\cdots\wedge\varphi_{j_q} & \text{if } j=j_\beta. \end{cases}$$

$$\iota(v_j)\,\omega_1\wedge\omega_2 = \begin{cases} 0. & j = \text{any of } i_\alpha\text{'s} \\ (-1)^{\alpha-1}\,\varphi_{i_1}\wedge\cdots\wedge\hat{\varphi}_{i_\alpha}\wedge\cdots\varphi_{i_p}\wedge\varphi_{j_1}\wedge\cdots\wedge\varphi_{j_q} & \text{if } j=i_\alpha. \end{cases}$$

$$(-1)^p\,\omega_1\wedge\iota(v_j)\,\omega_2 = \begin{cases} 0. & j = \text{any of } j_\beta\text{'s} \\ (-1)^p(-1)^{\beta-1}\,\varphi_{i_1}\wedge\cdots\wedge\varphi_{i_p}\wedge\varphi_{j_1}\wedge\cdots\hat{\varphi}_{j_\beta}\cdots\wedge\varphi_{j_q} & \text{if } j=j_\beta. \end{cases}$$

$$\Rightarrow \quad \iota(v_j)(\omega_1\wedge\omega_2) = \iota(v_j)(\omega_1)\wedge\omega_2 + (-1)^{\deg\omega_1}\,\omega_1\wedge\iota(v_j)(\omega_2)$$