By the factorization of π into π' and projection from $\mathbb{P}^{k+1}$ to $\mathbb{P}^{k}$, $\pi^{-1}(p)$ consists of d' points. But $\pi^{-1}(p)$ must be a d number of points generically. 」

Note, as a consequence, that the field of rational functions on an algebraic variety V is independent of the embedding.

「 I guess, the field of rational functions on an algebraic variety V is obtained by pulling back the restriction of $K(\mathbb{P}^n)$ to $i(V)$, where $i : V \longrightarrow \mathbb{P}^n$ is an embedding. 」

「 Comment on $\pi' : V \longrightarrow \mathbb{P}^{k+1}$.

$$\begin{tikzcd} V \arrow[r, "\pi'"] \arrow[dr, "\pi"'] & \mathbb{P}^{k+1} \arrow[d] \\ & \mathbb{P}^{k} \end{tikzcd}$$

For a generic point $p = [\alpha_0, \cdots \alpha_k]$ in $\mathbb{P}^k$
$\exists$ d points $\{[\alpha_0, \alpha_1, \cdots \alpha_k, X_{k+1,i}, X_{k+2,i} \cdots X_{n,i}]\}_{i=1}^{d}$
$= \pi^{-1}(p)$.

$\Rightarrow$ For each point $\overbrace{[\alpha_0, \alpha_1, \cdots \alpha_k, X_{k+1,i}, \cdots X_{n,i}]}^{= q_i}$ in V,
$\exists$ a generic $(n-k-2)$ plane in $\mathbb{P}^{n-k-1}$
s.t.
$$\overline{\mathbb{P}^{n-k-2}, q_i} \cap V = \{q_i\}.$$
$\Rightarrow$ Since $\{q_i\}_{i=1}^{d}$ is finite, $\exists$ a generic