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(ii) If $\ell' \supset \Lambda_0$,

(a) $\Gamma_0'' - \Lambda_0 \neq \phi$

$\Rightarrow$ ℓ_1, ℓ_2, ℓ_3 pass some common point $p \in \Gamma_0'' - \Lambda_0$

$\Rightarrow$ This is impossible since $\dim H^0(\mathbb{P}^2, \mathcal{J}_p(H)) = 2$.

(b) $\Gamma_0'' = \Lambda_0$.

Consider $\ell' l_0$, where $\{\tau_1 l_0, \tau_2 l_0, \tau_3 l_0\}$ is a basis for $H^0(\mathbb{P}^2, \mathcal{J}_{\Gamma_0}(3))$.

$\Rightarrow$ Since l_0 passes all points $\Gamma_0 - \Gamma_0''$, $\ell' l_0$ passes all points of Γ_0. $\Rightarrow$ $\exists$ a conic $\ell' l_0$ passing Γ_0.

$\Rightarrow$ We proved that if Γ_0 does not impose independent conditions on $|\mathcal{O}_{\mathbb{P}^2}(3)|$, then Γ_0 lie on a conic or five of Γ_0 lie on a line.

The result we needed in the section on cubic surfaces now follows easily:

Let Δ be six points in $\mathbb{P}^2$, no three of which are on a line and which are not on a conic. Then $\Gamma_0 = \Delta + p + q$ imposes independent conditions on $|\mathcal{O}_{\mathbb{P}^2}(3)|$ for any $p, q \in \mathbb{P}^2$.

Proof. If not, then either five points from Γ_0 must be collinear or all eight must be on a conic – this contradicts the assumption on Δ. Q.E.D.

☞ See P481~P483.