NO.

DATE . . ( )

$\Rightarrow \quad \sigma_1(V_3) = \{ \omega_1 \wedge \omega_2 \in \Lambda^2 \mathbb{C}^6 \mid \omega_1 \wedge \omega_2 \wedge v_1 \wedge v_2 \wedge v_3 \wedge v_4 = 0 \}$

where $V_3 = \langle v_1, v_2, v_3, v_4 \rangle$.

$\Rightarrow \quad V_3 \longmapsto \sigma_1(V_3)$ is the Plücker embedding

Thus $\{D_V\}$ is not a linear system, for $\sigma(V_3) + \sigma(V_3') = \sigma(V_3'')$ should be valid, but, as we saw, since, in $|\sigma_1| = \mathbb{P}(\Lambda^2\mathbb{C}^6)$, $G(4,6)$ is not linear subspace. $\Rightarrow$ $\{\sigma_1(V_3)\}$ is not a linear system. More precisely,

$\{\sigma_1(V_3)\} = G(4,6) \not\cong \mathbb{P}^k$.

$\lrcorner$

However, since the Schubert cycle

$$\sigma_{2,2,2,1}(V_2, V_4) = \{ V_3 \subset \mathbb{P}^5 : V_2 \subset V_3 \subset V_4 \}$$

in $G(4,6)$ has degree

$$\#(\sigma_{2,2,2,1} \cdot \sigma_1)_{G(4,6)} = 1$$

under the dual Plücker embedding, the family

$$\{D_V\}_{V \in \sigma_{2,2,2,1}(V_2, V_4)}$$

of divisors on A is in fact a pencil.

$\ulcorner$ $\sigma_{2,2,2,1} = \{ \Lambda \subset \mathbb{C}^6 \mid \dim(\Lambda \cap V'_{2+i-a_i}) \geq i \}$

$= \{ V_3 \subset \mathbb{P}^5 \mid V_2 \subset V_3 \subset V_4 \}$.

$\#(\sigma_{2,2,2,1} \cdot \sigma_1)_{G(4,6)} = 1$, since $b_{k-i+1} = n-k-a_i$.

i.e $b_{5-i} = 2 - a_i$.

Since $\tilde{\Phi}(\sigma_1(V_3))$ is a hyperplane section in $\mathbb{P}^2(\overset{4}{\Lambda}\mathbb{C}^6)$ by P209,