$$f(z) = u(z)\, w(z)$$

with $u \in \mathcal{O}^*$. In additon to the Weierstrass preparation theorem, we also proved the division theorem: For $g \in \mathcal{O}_n$

$$g = hf + r,$$

where $r \in \mathcal{O}_{n-1}[z_n]$ has degree less than that of w. These two results provide the basic tools for studying the local ring $\mathcal{O}$ — especially the ideals in $\mathcal{O}$.

The method is frequently by induction on n. For example, the inductive hypothesis and Gauss lemma imply that $\mathcal{O}_{n-1}[z_n]$ is a unique factorization domain, and using the preparation theorem we deduced that

$\mathcal{O}_n$ is a unique factorization domain.

「 See P10 Proposition. $\Rightarrow$ $\mathcal{O}_{n-1}[z_n]$ is UFD by Gauss lemma 」

Similarly, we shall prove

$\mathcal{O}_n$ is a Noetherian ring.

Proof. We must show that any ideal $I \subset \mathcal{O}$ has a finite number of generators.

「 By Proposition 4.1 on P375. Th. 1.9 P388. Algebra by Hungerford, A commutative ring R with identity is Noetherian $\Leftrightarrow$ every prime ideal of R is finitely generated. So if we prove every ideal $I \subset \mathcal{O}$ has a finite number of generators, then it is done. 」