$$-\overline{\partial}_M \, *_{M\times N} \, \overline{\partial}_N \, *_{M\times N} + (-1) \, *_{M\times N} \, \overline{\partial}_N \, *_{M\times N} \, \overline{\partial}_M = 0$$

$$\Rightarrow -\overline{\partial}_M \, *_{M\times N} \, \overline{\partial}_N \, *^2_{M\times N} + (-1) \, *_{M\times N} \, \overline{\partial}_N \, *_{M\times N} \, \overline{\partial}_M \, *_{M\times N} = 0$$

$$= -\overline{\partial}_M \, *_{M\times N} \, \overline{\partial}_N \, (-1)^* + (-1)^2 \, *_{M\times N} \, \overline{\partial}_N \, \overline{\partial}_M^*$$

$$\Rightarrow -*_{M\times N} \, \overline{\partial}_M \, *_{M\times N} \overline{\partial}_N \epsilon \pm (*_{M\times N})^2 \, \overline{\partial}_N \, \overline{\partial}_M^* = 0$$

$$\Rightarrow \overline{\partial}_M^* \, \overline{\partial}_N + \overline{\partial}_N \, \overline{\partial}_M^* = 0.$$

In case, $*_{M\times N} \, \overline{\partial}_N \, *_{M\times N} \, \overline{\partial}_M : \{(p,q),(p',q')\} \longrightarrow (p, q+1)(p',q')$

$\longrightarrow (m-p, m-q-1), (n-p', n-q') \longmapsto (m-p, m-q-1)(n-p', n-q'+1)$

$$\Rightarrow 2m-p-q-\cancel{1}+2n-p'-q'+\cancel{1} = 2(m+n)-(p+q)-(p'+q')$$

$$\Rightarrow *^2_{M\times N} = (-1)^{2(m+n)-(p+q)-(p'+q')}$$

$(m-p)$ $(m-q)$ $(n-p')$ $(n-q')$

In case. $-\overline{\partial}_M \, *_{M\times N} \, \overline{\partial}_N \, *_{M\times N}$: $(-1)^{p+q+p'+q'}$

$\Rightarrow \epsilon$ has the same sign as $(*_{M\times N})^2$.

To show $\overline{\partial}_M \, *_{M\times N} \, \overline{\partial}_N \, *_{M\times N} + *_{M\times N} \, \overline{\partial}_N \, *_{M\times N} \, \overline{\partial}_M = 0$,
we have only to check it for $\varphi \wedge \psi$, where
$\varphi \in A^{p,q}(M)$ $\psi \in A^{p',q'}(N)$.

$$*_{M\times N}(\varphi\wedge\psi) = \epsilon \; *_M\varphi \wedge *_N\psi.$$

$$\Rightarrow \varphi\wedge\psi \wedge *_{M\times N}(\varphi\wedge\psi) = \epsilon\, \varphi\wedge\psi\wedge *_M\varphi \wedge *_N\psi$$

$$= \epsilon\,(-1)^{(p'+q')(m-p+m-q)} \varphi\wedge *_M\varphi \wedge \psi\wedge *_N\psi = \epsilon\,(-1)^{(p+q)(p'+q')} \langle\varphi,\varphi\rangle \Phi_M \wedge$$

$$\langle\psi,\psi\rangle \Phi_N = \epsilon\,(-1)^{(p+q)(p'+q')} \langle\varphi,\varphi\rangle\langle\psi,\psi\rangle \Phi_M\wedge\Phi_N$$

$$\Rightarrow \epsilon = (-1)^{(p+q)(p'+q')}$$

Thus $*_{M\times N}(\varphi\wedge\psi) = (-1)^{(p+q)(p'+q')} *_M\varphi \wedge *_N\psi$.

where $p+q$ is the degree of φ
$p'+q'$ is the degree of ψ,