$$Z(2)_1 = Z(1)_2^{-1}$$

$$Z_2 = Z(1)_2 \cdot Z_1, \qquad Z(2)_i = Z(1)_i \cdot Z(1)_2^{-1}, \quad i \neq 1, 2.$$

and so the Jacobian matrix for the change of coordinates is

$$J_{12} = \begin{bmatrix} 0 & -Z(1)_2^{-2}, & 0 & \cdots & & 0 & & \\ Z(1)_2, & Z_1, & 0 & \cdots & & & & \\ 0 & & & & & & & \\ \vdots & & & & & & & \\ 0, & -Z(1)_j \cdot Z(1)_2^{-2}, & 0 & \cdots & 0 & Z(1)_2^{-1} & 0 \cdots & 0 \end{bmatrix};$$

in general $\quad g_{ij} = \det J_{ij} = Z(1)_j^{-n+1}$.

$$J_{12} = \begin{bmatrix} \frac{\partial Z(2)_1}{\partial Z_1}, & \frac{\partial Z(2)_1}{\partial Z(1)_2}, & \frac{\partial Z(2)_1}{\partial Z(1)_3}, & \cdots 0 \\ \frac{\partial Z_2}{\partial Z_1}, & \frac{\partial Z_2}{\partial Z(1)_2} & \frac{\partial Z_2}{\partial Z(1)_3} & 0 \\ & \frac{\partial Z(2)_3}{\partial Z(1)_2} & \frac{\partial Z(2)_3}{\partial Z(1)_3} & 0 \end{bmatrix}$$

$$Z_2 = Z(1)_2 \cdot Z_1 \Rightarrow \frac{\partial Z_2}{\partial Z_1} = Z(1)_2. \qquad Z(2)_3 = Z(1)_3 \cdot Z(1)_2^{-1} \Rightarrow \frac{\partial Z(2)_3}{\partial Z(1)_3} = Z(1)_2^{-1}.$$

let $Z(1)_2 = x$

$$J_{12} = \begin{bmatrix} 0 & -x^{-2} & 0 & & \cdots 0 & 0 \\ x & Z_2 & 0 & & 0 & 0 \\ 0 & -x^{2} Z(1)_3, & x^{-1} & 0 & \cdots 0 & 0 \\ \vdots & -x^{2} Z(1)_4 & 0 & x^{-1} & 0 & 0 \\ \vdots & \vdots & & & & \vdots \\ 0 & -x^{2} Z(1)_n, & 0 & \cdots 0 & 0 & x^{-1} \end{bmatrix}$$

$\det J_{12} =$

$$= (x^{-1})^{n-2} \cdot \det\begin{pmatrix} 0, & -x^{-2} \\ x & Z_2 \end{pmatrix} = (x^{-1})^{n-2} x^{-1} = x^{-n+1}$$

$= Z(1)_2^{-n+1} \Rightarrow$ We can easily guess that $g_{ij} = \det J_{ij}$

$= Z(1)_j^{-n+1}$.