

Setting $D_i' = (f_i' - \omega_i')$, the local intersection number $(D_2' \cdots D_n')_{z_0} > 0$ by induction hypothesis. This is in contradiction to $df_2' \wedge \cdots \wedge df_n' \equiv 0$.

By induction hypothesis, we assumed that $\partial(f_1, \dots, f_{n-1}) / \partial(z_1, \dots, z_n) \neq 0$ and $(D_1, \dots, D_{n-1})_{z_0} > 0$. But if we set $D_i' = (f_i' - \omega_i')$, then $\bigcap_{i=1}^{n-1} D_i' = \{z_0\} = \{z_0'\}$, where $z_0 = (0, z_0')$ and $(D_1, D_2, \dots, D_n)_{z_0} = (D_2', \dots, D_n')_{z_0'}$ by (d), since z_0 is a smooth point of $D_1 \cap (D_2', \dots, D_n')_{z_0'}$ by the induction hypothesis.

Furthermore

$$\Rightarrow (D_2', \dots, D_n')_{z_0'} = \left(\frac{1}{2\pi\sqrt{-1}} \right)^{n-1} \int_{|f_i' - \omega_i'| = \varepsilon} \frac{df_2'}{f_2'} \wedge \cdots \wedge \frac{df_n'}{f_n'} = 0 \quad *$$

since $df_2' \wedge \cdots \wedge df_n' \equiv 0$ by assuming $df_1 \wedge \cdots \wedge df_n \equiv 0$.

Now, assuming that $df_1 \wedge \cdots \wedge df_n \neq 0$, we shall prove that $(D_1, \dots, D_n)_{z_0} > 0$. The Dolbeault representative is

$$\begin{aligned} \gamma(f_1, \dots, f_n) &= C_n \frac{\sum (-1)^{i-1} \bar{f}_i df_1 \wedge \cdots \wedge \widehat{df_i} \wedge \cdots \wedge d\bar{f}_n \wedge df_1 \wedge \cdots \wedge df_n}{\|f\|^{2n}} \\ &= f^*(\beta), \end{aligned}$$

where, according to the Bochner - Marti nelli formula in Section 1 of Chapter 3,

$$\beta = C_n \frac{\sum (-1)^{i-1} \bar{w}_i d\bar{w}_1 \wedge \cdots \wedge \widehat{d\bar{w}_i} \wedge \cdots \wedge d\bar{w}_n \wedge dw_1 \wedge \cdots \wedge dw_n}{\|W\|^{2n}}$$

is a closed $(n, n-1)$ -form in $\mathbb{C}^n - \{0\}$ whose restriction to every sphere $\|W\| = \varepsilon$ is a $(2n-1)$ -form with total integral