$L'(m) = L_1 \circ \overset{\psi}{(m')} = L_2(m') \Rightarrow \psi$ is well-defined, and ψ is onto. $L_2(m) = \qquad L_1 \circ \psi(m) \Rightarrow \psi$ is one to one $\Rightarrow$ Thus ψ is isomorphic.

Now consider the map $Q : \mathbb{Z}^{2n} \longrightarrow \mathbb{Z}^{2n}$ defined by

$$Q(x) = \left(\begin{array}{c|c} 0 & \begin{matrix} \delta_1 & & 0 \\ & \ddots & \\ 0 & & \delta_n \end{matrix} \\ \hline \begin{matrix} -\delta_1 & & 0 \\ & \ddots & \\ 0 & & -\delta_n \end{matrix} & 0 \end{array}\right) \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n} \end{pmatrix}.$$

$$\Rightarrow \quad \mathbb{Z}^{2n}/\operatorname{im} Q \cong \mathbb{Z}_{\delta_1} \oplus \mathbb{Z}_{\delta_1} \oplus \mathbb{Z}_{\delta_2} \oplus \mathbb{Z}_{\delta_2} \oplus \cdots \oplus \mathbb{Z}_{\delta_n} \oplus \mathbb{Z}_{\delta_n}$$

$\delta_1 | \delta_2, \ \cdots \ \delta_{n-1} | \delta_n.$

Consider the map ${}^tAQA = Q' : \mathbb{Z}^{2n} \longrightarrow \mathbb{Z}^{2n}$ defined by

$$Q'(x) = \left(\begin{array}{c|c} 0 & \begin{matrix} \delta_1' & & 0 \\ & \ddots & \\ 0 & & \delta_n' \end{matrix} \\ \hline \begin{matrix} -\delta_1' & & 0 \\ & \ddots & \\ 0 & & -\delta_n' \end{matrix} & 0 \end{array}\right) \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix},$$

where A is an isomorphism of $\mathbb{Z}^{2n}$.

$$\Rightarrow \quad \mathbb{Z}^{2n}/\operatorname{im} Q' \cong \mathbb{Z}_{\delta_1'} \oplus \mathbb{Z}_{\delta_1'} \oplus \cdots \oplus \mathbb{Z}_{\delta_n'} \oplus \mathbb{Z}_{\delta_n'}$$

Here, by assuming that Q & Q' are given by different bases for $\mathbb{Z}^{2n}$, we have ${}^tAQA = Q'$.

$\Rightarrow$ By the lemma above, $\mathbb{Z}^{2n}/\operatorname{im} Q' \cong \mathbb{Z}^{2n}/\operatorname{im} {}^tAQA \cong \mathbb{Z}^{2n}/\operatorname{im} Q \quad \Longrightarrow$