On the product $M \times N$, with projection maps π_1, π_2, we have

$$\int_{\mu\times\nu} \pi_1^*\varphi \wedge \pi_2^*\psi = \int_\mu \varphi \cdot \int_\nu \psi = {}^\#(\sigma\cdot\mu)\cdot {}^\#(\tau\cdot\nu)$$

⌈ $\mu : \Delta^{n-k} \longrightarrow M$. $\mu = \sum a_i f_i$. $f_i : \Delta^{n-k} \longrightarrow M$.

$\nu = \sum b_j g_j$. $g_j : \Delta^k \longrightarrow N$.

$$\mu\times\nu = \sum a_i b_j \, f_i \times g_j .$$

$$\int_{\mu\times\nu} \pi_1^*\varphi \wedge \pi_2^*\psi = \int_{\sum a_i b_j f_i\times g_j} \pi_1^*\varphi \wedge \pi_2^*\psi = \sum a_i b_j \int_{\Delta^{n-k}\times\Delta^k} (f_i\times g_j)^*(\pi_1^*\varphi\wedge\pi_2^*\psi)$$

$$= \sum a_i b_j \int_{\Delta^{n-k}\times\Delta^k} (f_i\times g_j)^*\pi_1^*\varphi \wedge (f_i\times g_j)^*\pi_2^*\psi = \sum a_i b_j \int_{\Delta^{n-k}\times\Delta^k} (\pi_1\circ f_i\times g_j)^*\varphi \wedge (\pi_2\circ f_i\times g_j)^*\psi$$

$\Rightarrow$ Since $\pi_1 \circ f_i \times g_j = f_i \circ P_1$ and $\pi_2 \circ f_i\times g_j = g_j \circ P_2$,

$$\begin{tikzcd}
\Delta^{n-k}\times\Delta^k \arrow[r, "f_i\times g_j"] \arrow[d, "P_1"'] & M\times N \arrow[r, "\pi_1"] & M \\
\Delta^{n-k} \arrow[urr, "f_i"'] & &
\end{tikzcd}$$

$$\Rightarrow \int_{\Delta^{n-k}\times\Delta^k} (\pi_1\circ f_i\times g_j)^*\varphi \wedge (\pi_2\circ f_i\times g_j)^*\psi = \int_{\Delta^{n-k}\times\Delta^k} (f_i\circ P_1)^*\varphi \wedge (g_j\circ P_2)^*\psi$$

$$= \int_{\Delta^{n-k}\times\Delta^k} P_1^*(f_i^*\varphi)\wedge P_2^*(g_j^*\psi) = \int_{\Delta^{n-k}} f_i^*\varphi \cdot \int_{\Delta^k} g_j^*\psi .$$

for, $\int_{R\times R} h(x)dx \wedge k(x)dy = \int_R h\,dx \int_R k(x)dy$. and

$P_1^*(f_i^*\varphi)$ $(n-k)$ – form on Δ^{n-k}

$P_2^*(g_j^*\varphi)$ k – form on Δ^k.