

$$(*) \quad 0 \rightarrow \text{Ext}^1(\mathcal{S}; \mathcal{I}_P(L), \Omega^2) \rightarrow \text{Ext}^1(\mathcal{S}; \mathcal{I}(L), \Omega^2) \xrightarrow{P} \text{Ext}^1(\mathcal{S}; \mathcal{O}_P(L), \Omega^2).$$

By \otimes on p 706 & p 686, we have

$$\text{Ext}^1(\mathcal{S}; \mathcal{O}_P(L), \Omega^2) \rightarrow \text{Ext}^1(\mathcal{S}; \mathcal{I}_P(L), \Omega^2) \rightarrow \text{Ext}^1(\mathcal{S}; \mathcal{I}(L), \Omega^2) \xrightarrow{P} \text{Ext}^1(\mathcal{S}; \mathcal{O}_P(L), \Omega^2) \rightarrow \dots$$

\Downarrow \Downarrow

For the middle term we use the second exact sheaf sequence, noting that $\text{Ext}^1(\mathcal{S}; \mathcal{O} \oplus \mathcal{O}, \Omega^2) \cong H^1(\mathcal{S}; \Omega^2 \oplus \Omega^2) = 0$ since \mathcal{S} is regular, and also that

$$\left\{ \begin{array}{l} \text{Ext}^0(\mathcal{S}; \mathcal{O}(L^*), \Omega^2) \cong H^0(\mathcal{O}(k+L)), \\ \text{Ext}^0(\mathcal{S}; \mathcal{O} \oplus \mathcal{O}, \Omega^2) \cong H^0(\mathcal{O}(k)) \oplus H^0(\mathcal{O}(k)), \\ \text{Ext}^0(\mathcal{S}; \mathcal{I}(L), \Omega^2) \cong H^0(\underline{\text{Hom}}(\mathcal{I}(L), \Omega^2)) \cong H^0(\mathcal{O}(k-L)) \end{array} \right.$$

to obtain

$$\begin{aligned} 0 &\rightarrow H^0(\mathcal{O}(k-L)) \rightarrow H^0(\mathcal{O}(k)) \oplus H^0(\mathcal{O}(k)) \\ &\rightarrow H^0(\mathcal{O}(k+L)) \rightarrow \text{Ext}^1(\mathcal{S}; \mathcal{I}(L), \Omega^2) \rightarrow 0. \end{aligned}$$

By the note p 644 and the first property on p 706, since $\mathcal{O} \oplus \mathcal{O}$ is locally free,

$$\begin{aligned} \text{Ext}^1(\mathcal{S}; \mathcal{O} \oplus \mathcal{O}, \Omega^2) &= H^1(\mathcal{S}, (\mathcal{O}^* \oplus \mathcal{O}^*) \otimes_{\mathcal{O}} \Omega^2) \\ &= H^1(\mathcal{S}, (\mathcal{O}^* \otimes_{\mathcal{O}} \Omega^2) \oplus (\mathcal{O}^* \otimes_{\mathcal{O}} \Omega^2)) = H^1(\mathcal{S}, \Omega^2 \oplus \Omega^2) \\ &(\because \mathcal{O}^* \otimes_{\mathcal{O}} \mathcal{I} \cong \mathcal{I}) \\ &= H^1(\mathcal{S}, \Omega^2) \oplus H^1(\mathcal{S}, \Omega^2) \Rightarrow \text{By regularity of } \mathcal{S}, \\ 0 &= h^{2,1}(\mathcal{S}) = \dim H^1(\mathcal{S}, \Omega^2) \Rightarrow \text{Ext}^1(\mathcal{S}; \mathcal{O} \oplus \mathcal{O}, \Omega^2) = 0. \end{aligned}$$