We now express this condition by duality in two ways. For the first we use the operator

$$i(\overline{Z}) : \Lambda^k V \longrightarrow V^*$$

defined for $\overline{Z} \in \Lambda^{k+1} V^*$ by $\langle i(\overline{Z})\Lambda, v \rangle = \langle \overline{Z}, \Lambda \wedge v \rangle$ for all $v \in V$. We observe that, by the definition of $\Lambda^\perp$, for $v \in W$ the left-hand side depends only on the image of $\overline{Z}$ under the natural projection

$$\Lambda^{k+1} V^* \longrightarrow \Lambda^{k+1}\left(\frac{V^*}{\Lambda^\perp}\right) \cong \Lambda^{k+1} W^*.$$

「 First of all, we will show that $V^*/\Lambda^\perp \cong W^*$.

Define a map $\phi : V^* \longrightarrow W^*$ by

$$\phi(l) = l|_W.$$

Consider an inner product $\langle\, ,\, \rangle$ on V, which is positive definite. $\Rightarrow$ For each $l \in V^*$, $\exists$ a unique vector v_l s.t $\langle \_, v_l \rangle = l(\,)$.

$\Rightarrow$ Note that, given a subspace $K \subset V^*$, $\exists$ a unique subspace $L \subset V$ s.t $K = \{\langle\, , a\rangle \mid a \in L\}$.

$\Rightarrow$ $\ker\phi = \{ l \in V^* \mid l = 0$ on $W \}$

$\Rightarrow$ By the correspondence between V^* and V, $\ker\phi$ corresponds to the subspace of V perpendicular to W. $\Rightarrow$ Since $\Lambda^\perp = \{ v^* \in V^* \mid v^*(\omega) = 0$ for all $\omega \in W \}$ ($\because \mathrm{Ann}(\Lambda^\perp) = W$),

$\ker\phi = \Lambda^\perp$.

$\Rightarrow$ $V^*/\Lambda^\perp \cong W^*$ since ϕ is onto.