Now, since $d = \bar\partial$ on forms of type (n, q), there is a natural mapping

$$H_{\bar\partial}^{n,n-1}(U^*) \to H_{DR}^{2n-1}(U^*).$$

Γ

$$[\sigma] \longmapsto [[\sigma]]$$

$$\bar\partial\sigma = 0 \Leftrightarrow d\sigma = 0 \qquad \underset{[\sigma]}{[\sigma + \bar\partial\eta]} \longmapsto \underset{[[\sigma + d\eta]] = [[\sigma]]}{[[\sigma + \bar\partial\eta]]}$$

The punctured ball U^* is homotopically just the 2n-1 sphere, and so the right side is $\mathbb{C}$ with the isomorphism given by

$$\eta \longmapsto \int_{S^{2n-1}} \eta,$$

the orientation on the sphere being induced from that in $\mathbb{C}^n$. We shall prove that

Lemma. $\mathrm{Res}_{\{0\}}\,\omega = \eta_\omega$, or equivalently

$$\left(\frac{1}{2\pi\sqrt{-1}}\right)^n \int_P \omega = \int_{S^{2n-1}} \eta_\omega.$$

Γ $H_{DR}^{2n-1}(U^*) = H_{DR}^{2n-1}(U - \{0\}) \doteq H_{DR}^{2n-1}(S^{2n-1}) \cong \mathbb{C}$.

$$\begin{array}{ccc} H_{DR}^{2n-1}(U^*) & \xrightarrow{\cong} & \mathbb{C} \\ \Psi & & \Psi \\ [\eta] & \longmapsto & \int_{S^{2n-1}} \eta \end{array}$$

$$\begin{array}{ccccc} H^{n-1}(U^*, \Omega^n) \cong H_{\bar\partial}^{n,n-1}(U^*) & \longrightarrow & H_{DR}^{2n-1}(U^*) \cong \mathbb{C} \\ \omega \longmapsto \eta_\omega & \longmapsto & \int_{S^{2n-1}} \eta_\omega \end{array}$$