$\Vdash \quad 0 = \sum a_{\nu i} \left(\frac{\omega(p)}{dz_i} \right) = a_{\nu 1} \frac{\omega}{dz_1}(p_1) + \cdots + a_{\nu d} \frac{\omega}{dz_d}(p_d).$

Plug ω_i in ω.

$\Rightarrow \quad a_{\nu 1} \frac{\omega_i}{dz_1}(p_1) + \cdots + a_{\nu d} \frac{\omega_i}{dz_d}(p_d), \quad i = 1, 2, \cdots g.$

$$\Rightarrow \quad a_{\nu 1} \begin{pmatrix} \frac{\omega_1}{dz_1}(p_1) \\ \vdots \\ \frac{\omega_g}{dz_1}(p_1) \end{pmatrix} + a_{\nu 2} \begin{pmatrix} \frac{\omega_1}{dz_2}(p_2) \\ \vdots \\ \frac{\omega_g}{dz_2}(p_2) \end{pmatrix} + \cdots + a_{\nu d} \begin{pmatrix} \frac{\omega_1}{dz_d}(p_d) \\ \vdots \\ \frac{\omega_g}{dz_d}(p_d) \end{pmatrix} = 0$$

for $\nu = 1, 2, \cdots r$.

$\Rightarrow$ This gives r independent relations on the points p_i on the canonical curve since $\{(a_{11}, a_{12}, \cdots a_{1d}), \cdots (a_{r1}, a_{r2}, \cdots a_{rd})\}$ is a linearly independent set. Thus since $\{i_K(p_i)'s\}$ has at least r independent relations, the dimension of the space $|D|$ spanned by $\{i_K(p_i)'s,\ i = 1, 2, \cdots d\}$ is $\leq d - 1 - r$.

$\Rightarrow \quad \dim|D| \leq d - 1 - r \leq d - 1 - \dim|D|$, since $r \leq \dim|D|$ = dimension of the complete linear system. $\lrcorner\!\lrcorner$

We may now prove the opposite inequality by applying $(*)$ to the residual series $K - D$ of D. Suppose that on the canonical curve

$$\dim \overline{D} = d - s - 1.$$

The hyperplanes in $\mathbb{P}^{g-1}$ containing D then cut out a linear subseries of $|K - D|$ of dimension $(g-1) - (d-s-1) - 1 = g - d + s - 1$.