Date

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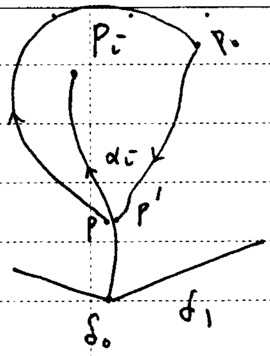

$$\int_p^{p_0} d\log f + \int_{p_0}^{p'} d\log f = -\int_{B_\epsilon(p_i)} d\log f = -\mathrm{Res}_{p_i}(d\log f) \times 2\pi\sqrt{-1}$$

$$= -\mathrm{ord}_{p_i}(f)\cdot 2\pi\sqrt{-1}$$

$$= \log f(p_0) - \log f(p) + \log f(p') - \log f(p_0)$$

$$= \log f(p') - \log f(p)$$

$$\Rightarrow \quad \log f(p') - \log f(p) = -2\pi\sqrt{-1}\ \mathrm{ord}_{p_i}(f)$$

$$\int_{\alpha_i+\alpha_i^{-1}} \varphi = \int_{\alpha_i+\alpha_i^{-1}} \log f\cdot\frac{dg}{g} = \int_{\alpha_i} (\log f(p) - \log f(p'))\,\frac{dg}{g}$$

$$= 2\pi\sqrt{-1}\ \mathrm{ord}_{p_i}(f) \int_{\alpha_i} d\log g$$

$$= 2\pi\sqrt{-1}\ \mathrm{ord}_{p_i}(f) \int_{s_0}^{p_i} d\log g .$$

Thus

$$\sum_i \int_{\alpha_i+\alpha_i^{-1}} \varphi = 2\pi\sqrt{-1}\ \Big(\sum \mathrm{ord}_{p_i}(f)\cdot(\log g(p_i) - \log g(s_0))\Big) = 2\pi\sqrt{-1}\ \sum \mathrm{ord}_{p_i}(f)\cdot\log g(p_i),$$

since $\sum_{p_i} \mathrm{ord}_{p_i}(f) = 0$.

Γ $$\sum_i \int_{\alpha_i+\alpha_i^{-1}} \varphi = 2\pi\sqrt{-1}\ \sum \mathrm{ord}_{p_i}(f)\ (\log g(p_i) - \log g(s_0))$$

$$= 2\pi\sqrt{-1}\ \sum \mathrm{ord}_{p_i}(f) \log g(p_i) - \log g(s_0)\cdot 2\pi\sqrt{-1}\cdot \sum \mathrm{ord}_p(f)$$