Consider the following

$$\mathbb{P}^2 \xleftarrow{\varphi} \mathbb{P}^2 \xrightarrow{\tilde{\psi}} \mathbb{P}^2$$

$$[X_0X_1, X_0X_2, X_1X_2] \longleftarrow [X_0, X_1, X_2] \longmapsto [(b_0X_2^2 + b_1X_0X_1 + b_2X_0X_2 + b_3X_1X_2)X_0X_1,$$
$$(c_0X_1^2 + c_1X_0X_1 + c_2X_0X_2 + c_3X_1X_2)X_0X_2,$$
$$(d_0X_0^2 + d_1X_0X_1 + d_2X_0X_2 + d_3X_1X_2)X_1X_2]$$

Let $X_0X_1 = A$, $X_0X_2 = B$, $X_1X_2 = C$.

$\Rightarrow$

$$\tilde{G}_3 X_0X_1 = (b_0X_2^2 + b_1X_0X_1 + b_2X_0X_2 + b_3X_1X_2)X_0X_1$$
$$= b_0(X_0X_2)(X_1X_2) + b_1A^2 + b_2AB + b_3AC$$
$$= b_0BC + b_1A^2 + b_2AB + b_3AC.$$
$$\tilde{G}_2 X_0X_2 = (c_0X_1^2 + c_1X_0X_1 + c_2X_0X_2 + c_3X_1X_2)X_0X_2$$
$$= c_0AC + c_1AB + c_2B^2 + c_3BC$$
$$\tilde{G}_1 X_1X_2 = (d_0X_0^2 + d_1X_0X_1 + d_2X_0X_2 + d_3X_1X_2)X_1X_2$$
$$= d_0AB + d_1AC + d_2BC + d_3C^2$$

$\Rightarrow$ $\tilde{\psi} \circ \varphi^{-1}$ is a birational quadratic transformation.
$\Rightarrow$ $\tilde{\psi}$ is a composition of two quadratic transformations, since $\tilde{\psi} \circ \varphi^{-1}$ is a quadratic transformation. ㅛ

We will return later in this section to prove a structure theorem for birational maps on surfaces: before we do that, however, we need to know some more about curves on surfaces.