$$\Rightarrow \quad \#(\sigma_{a^*-a_1^*} \cdot \sigma_{b^*-b^*_{n+k}} \cdot \sigma_{c^*-c^*_{n+k}}) = \#(\sigma_{a^*-a_1^*} \cdot \sigma_{b^*} \cdot \sigma_{c^*})$$

What is $(a^*-a_1^*)^*$? $\quad a_1-1,\ a_2-1,\ \cdots\ a_k-1.$

$$\Rightarrow \quad \#(\sigma_{a^*-a_1^*} : \sigma_{b^*} \cdot \sigma_{c^*})_{G(n-k-1,\, n-1)} = \#(\sigma_{a_1-1, \cdots a_k-1} \cdot \sigma_b \cdot \sigma_c)_{G(k-1,n)}$$

It remains to show $(a^*-a_1^*)^* = a_1-1, \cdots a_k-1.$

Again, $\cdots\ a_{m_1} > \cdots\ a_{m_2} > \cdots\ a_{m_3} > \cdots > \cdots > a_k \neq 0.$

$$\begin{array}{lccccccc}
\Rightarrow & a^*_{a_{m_1}} = \cdots < & a^*_{a_{m_2}} \cdots < & a^*_{a_{m_3}} \cdots < \cdots & & < & a^*_{a_k} = \cdots = a^*_1 \\
 & \| & \| & & & & \| \\
 & m_1 & m_2 & & & & k \\
 & \| & \| & & & & \| \\
\text{Let} & a'_{a_{m_1}-1} & a'_{a_{m_2}-1} & & & & a'_{a_k-1} = \cdots a'_{2-1}
\end{array}$$

$$\begin{array}{ccccccc}
a^*_{a'_{a_{m_1}-1}} > & \cdots & a^*_{a'_{a_{m_2}-1}} > & \cdots & a^*_{a'_{a_{m_3}-1}} > \cdots > & & a^*_{a'_{a_k-1}} \\
\| & & \| & & \| & & \| \\
a_{m_1}-1 > & \cdots & a_{m_2} > & \cdots & a_{m_3}-1 > & \cdots & a_k-1
\end{array}$$

Since $a'_{a_k-1} = \cdots = a'_1 = a^*_{a_k} = \cdots = a^*_1$ and

$$a^*_{a'_{a_k-1}} \geq a_k-1 \ \cdots \ a^*_{a'_1} \geq 1,$$

$$a^*_{a'_{a_k-1}} = a^*_{a'_1} = a_k-1.$$

Thus. $r=\beta=0$ & $a_k \neq 0$ case is valid in Reduction Formula II.