Thus $G_i \subset S \subset \mathbb{P}^3$ is again a line, and

$$G_i \cdot G_i = (\pi^* C_i - \sum_{i \neq j} E_j)(\pi^* C_i - \sum_{i \neq j} E_j)$$

$$= C_i \cdot C_i - 5 = 4 - 5 = -1.$$

so $G_i$ is exceptional of the first kind.

Now if $L$ is any line in $S$, we consider the locus $\pi(L) \subset \mathbb{P}^2$. Assuming $L$ is not one of the exceptional divisors $E_i$, $L$ can meet each line $E_i$ at most once, and that transversely. Thus $\pi(L)$ will be a smooth rational curve in $\mathbb{P}^2$; hence by the genus formula either a line or a conic.

$$\begin{array}{cccc} \pi : & S & \longrightarrow & \mathbb{P}^2 \\ & \cup & & \cup \\ & L & \longrightarrow & \pi(L). \end{array} \quad \pi|_L \text{ is one to one}$$

$\Rightarrow$ Topologically, $\pi(L)$ is equal to $L$.

$\Rightarrow g(L) = \text{genus of } \mathbb{P}^1 = 0 = \text{genus of } \pi(L)$

$= \dfrac{(d-1)(d-2)}{2} \Rightarrow d = 1$ or $d = 2$ see p 221.

$\Rightarrow \pi(L)$ is a line or a conic.

Def : Rational curve means a curve biholomorphic to $\mathbb{P}^1$.

Since $L$ meets $E_i$ transversely, if $L \cap E_i \ni p_i$, then the tangent vector at $p_i$ is mapped into $T_{\pi(p_i)}\mathbb{P}^2$ or nonzero vector. This implies that the nonzero vector is the tangent vector of $\pi(L)$ at $\pi(p_i)$.

$\Rightarrow$ The gradient of $\pi(L)$ at $\pi(p_i)$ is nonzero. $\Rightarrow$ $\pi(p_i)$ is