I think, the conditions of 'inscribe' and 'circumscribe' are not enough. So the conditions '$L_i = \sigma(p_i, h_i)$ $i=1..4$ $L'_i = \sigma(p'_i, h_i)$, $i=1,..4$ lie in a hyperplane H' must be included.

The Quadric Line Complex and Associated Kummer Surface I

We come now to the main object of study in this chapter: the quadric line complex, defined to be the family of lines in $\mathbb{P}^3$ corresponding to the smooth intersection $X = G \cap F$ of the Grassmannian $G \subset \mathbb{P}^5$ with a quadric hypersurface F. As in the case of the linear complex, our initial problem in regard to the quadric line complex is to identify the pencils of lines in X and to determine, for any point p and any hyperplane h in $\mathbb{P}^3$, the locus of lines in our complex passing through p or contained in h. We first check that

Lemma. No 2-plane $\sigma(p)$ or $\sigma(h)$ lies in the quadric line complex $X = F \cap G$.

Proof. We will give two proofs of this fact. First, in an elementary but rather special vein, we can