To be linear, g_0, g_1 & g_2 are constants. For simplicity, $g_0 = g_1 = g_2 = 1$ $\Rightarrow$ We have the map above.
For example,

$$[X_0, X_1, X_2] \xrightarrow{g} [X_1^2 X_2,\ X_0 X_2^2,\ X_0^2 X_1]$$

$\Rightarrow$ $\{X_1^2 X_2 = X_0 X_2^2 = X_0^2 X_1 = 0\} = \{a, b, c\}$ is of codimension 2.

$$\Rightarrow \text{Let } \frac{X_1}{X_0} = x,\ \frac{X_2}{X_0} = y. \Rightarrow \frac{X_0 X_2^2}{X_1^2 X_2} = \frac{X_0 X_2}{X_1^2} = \frac{X_0}{X_1}.$$

$$\frac{X_0}{X_1} \frac{X_2}{X_0} = \frac{1}{x^2} y \quad \& \quad \frac{X_0^2 X_1}{X_1^2 X_2} = \frac{X_0\ X_0}{X_1\ X_2} = \frac{1}{xy}.$$

$$\Rightarrow \frac{y}{x^2} = \frac{b}{a^2},\ \frac{1}{xy} = \frac{1}{ab} \Rightarrow \frac{xy}{x^2 \cdot x} = \frac{b}{a^2} = \frac{ab}{x^3}$$

$$\Rightarrow \frac{1}{a^2} = \frac{a}{x^3} \Rightarrow x^3 = a^3 \Rightarrow x = a,\ x = e^{i\alpha} a$$

$$\Rightarrow a e^{i\alpha} y = ab \Rightarrow b = y e^{i\alpha} \quad y = b e^{-i\alpha}.$$

In this way, we can show that g_0, g_1 & g_2 must be constants to get a birational map. ⌟

Defined as it is by a linear system of conics, $\varphi_{a,b,c}$ is called a quadratic transformation of the plane. Note that a general line L through the point a is carried over into a line through the image point of L_{bc}, while a line L not containing any of the points a, b, c is carried over into a conic passing through all three image points $d = \varphi(L_{ab})$, $e = \varphi(L_{bc})$ and $f = \varphi(L_{ac})$. This reflects the fact that $\varphi_{edf} \circ \varphi_{abc}$ is holomorphic.