(since $\omega_{u_1 \wedge \cdots u_\ell \wedge \cdots \wedge u_\ell \wedge \cdots u_n} = 0$)

$= \rho_i (-1)^{i-1} \omega_{u_1 \wedge \cdots \wedge u_i \wedge \cdots \wedge \hat{u}_j \wedge \cdots u_n} + \rho_j (-1)^{j-2} \omega_{u_1 \wedge \cdots \wedge \hat{u}_i \wedge \cdots \wedge u_n}$

$= (-1)^{i-1} \rho_i \, \omega_{\{j\}^c} + (-1)^{j-2} \rho_j \, \omega_{\{i\}^c}$

$= (-1)^{i-1} (-1)^{j-1} \rho_i \bar{\partial} \rho_j \omega + (-1)^{j-2} \rho_j (-1)^{i-1} \bar{\partial} \rho_i \omega$

$= (-1)^{i} (-1)^{j} \rho_i \bar{\partial} \rho_j \omega - (-1)^{i+j} \rho_j \bar{\partial} \rho_i \omega$

$= (-1)^{i+j} (\rho_i \bar{\partial} \rho_j \omega - \rho_j \bar{\partial} \rho_i \omega)$

$\Rightarrow \bar{\partial} \xi_{\{i,j\}^c} = \omega_{\{i,j\}^c} = (-1)^{i+j} \, 2 \, \bar{\partial} \rho_i \wedge \bar{\partial} \rho_j \wedge \omega$. //

$\xi_{\{i,j,k\}^c} = \xi_{u_1 \wedge \cdots \wedge \hat{u}_i \wedge \cdots \wedge \hat{u}_j \wedge \cdots \wedge \hat{u}_k \wedge \cdots \wedge u_n}$

$= \sum \rho_\ell \, \omega_{u_\ell \wedge u_1 \wedge \cdots \wedge \hat{u}_i \wedge \cdots \wedge \hat{u}_j \wedge \cdots \wedge \hat{u}_k \wedge \cdots \wedge u_n}$

$= \rho_i \, \omega_{u_i \wedge u_1 \wedge \cdots \hat{u}_i \wedge \cdots \hat{u}_j \cdots \wedge \hat{u}_k \wedge u_n} + \rho_j \, \omega_{u_j \wedge \cdots \hat{u}_i \, \hat{u}_j \, \hat{u}_k \wedge \cdots u_n}$

$+ \rho_k \, \omega_{u_k \wedge u_1 \wedge \cdots \hat{u}_i \wedge \cdots \hat{u}_j \wedge \cdots \hat{u}_k \wedge u_n}$

$= (-1)^{i-1} \rho_i \, \omega_{u_1 \wedge \cdots \hat{u}_j \cdots \hat{u}_k \wedge \cdots u_n} + (-1)^{j-2} \rho_j \, \omega_{u_1 \wedge \cdots \hat{u}_i \cdots \hat{u}_k \cdots u_n}$

$+ (-1)^{k-3} \rho_k \, \omega_{u_1 \wedge \cdots \hat{u}_i \cdots \hat{u}_j \wedge \wedge u_n}$

$= (-1)^{i-1} \rho_i \, \omega_{\{j,k\}^c} + (-1)^{j-2} \rho_j \, \omega_{\{i,k\}^c} + (-1)^{k-3} \rho_k \, \omega_{\{i,j\}^c}$

$= (-1)^{i-1} \rho_i \, (-1)^{j+k} \, 2 \, \bar{\partial} \rho_j \wedge \bar{\partial} \rho_k \wedge \omega$

$+ (-1)^{j-2} \rho_j \, (-1)^{i+k} \, 2 \, \bar{\partial} \rho_i \wedge \bar{\partial} \rho_k \wedge \omega$

$+ (-1)^{k-3} \rho_k \, (-1)^{i+j} \, 2 \, \bar{\partial} \rho_i \wedge \bar{\partial} \rho_j \wedge \omega$

$= (-1)^{i+j+k-1} \, 2 \, (\rho_i \bar{\partial} \rho_j \wedge \bar{\partial} \rho_k \wedge \omega - \rho_j \bar{\partial} \rho_i \wedge \bar{\partial} \rho_k \wedge \omega$

$+ \rho_k \bar{\partial} \rho_i \wedge \bar{\partial} \rho_j \wedge \omega)$

$\Rightarrow \bar{\partial} \xi_{\{i,j,k\}^c} = \omega_{\{i,j,k\}^c} = (-1)^{i+j+k+1} \, 2 \, (\bar{\partial} \rho_i \wedge \bar{\partial} \rho_j \wedge \bar{\partial} \rho_k \wedge \omega$

$- \bar{\partial} \rho_j \wedge \bar{\partial} \rho_i \wedge \bar{\partial} \rho_k \wedge \omega + \bar{\partial} \rho_k \wedge \bar{\partial} \rho_i \wedge \bar{\partial} \rho_j \wedge \omega)$

$= (-1)^{i+j+k-1} \, 3! \, \bar{\partial} \rho_i \wedge \bar{\partial} \rho_j \wedge \bar{\partial} \rho_k \wedge \omega$

Continue this process, then we obtain

$\bar{\partial} \xi_{\{i_1, i_2, \cdots i_{n-1}\}^c} = \bar{\partial} \xi_{\{i\}} = (-1)^{i_1 + \cdots + i_{n-1} + n-1} \, n! \, \bar{\partial} \rho_{i_1} \wedge \bar{\partial} \rho_{i_2} \wedge \cdots \wedge \bar{\partial} \rho_{i_{n-1}} \wedge \omega$

$= (-1)^{1+2+\cdots+n-i+n-1} \, n! \, \bar{\partial} \rho_1 \wedge \cdots \wedge \widehat{\bar{\partial} \rho_i} \wedge \cdots \wedge \bar{\partial} \rho_n \wedge \omega$

$= (-1)^{i} (-1)^{\frac{(n+1)(n+2)}{2}} \, n! \, \bar{\partial} \rho_1 \wedge \cdots \wedge \widehat{\bar{\partial} \rho_i} \wedge \cdots \wedge \bar{\partial} \rho_n \wedge \omega = \omega_{\{i\}} \in H^{n,n-1}($