point $p \in S$, the projection map $\pi_p : S - \{p\} \longrightarrow \mathbb{P}^{N-1} \subset \mathbb{P}^N$ to a hyperplane can be extended continuously, hence holomorphically, over all of S by sending p to the point of intersection of its tangent line with $\mathbb{P}^{N-1}$.

$\ulcorner$ Let $p = [0, 0, \cdots 0, 1] \in \mathbb{P}^N$. Given $q \in S - \{p\}$,
$q = [q_0, \cdots q_N] \in \mathbb{P}^N$. $\Rightarrow$ $\pi_p(q) = [q_0, q_1, \cdots q_{N-1}, 0] \in \mathbb{P}^{N-1}$.
$\Rightarrow$ $\pi_p(q) = \overline{p.q} \cap \mathbb{P}^{N-1}$, i.e. $\overline{p.q} = [(q_0, \cdots q_N) + t\,(0, 0 \cdots 0\, 1)]$
$\cap\ \mathbb{P}^{N-1} = \{[*, *, \cdots, *, 0]\} = [(q_0, \cdots q_{N-1}, 0)]$ when $-q_N = t \in \mathbb{C} \cup \{\infty\}$.

Since the tangent line is the limit of $\overline{pq}$ as $q \to p$, $\pi_p(p) = \lim_{q \to p} \overline{pq} \cap \mathbb{P}^{N-1}$ is naturally defined uniquely. $\Rightarrow$ π_p is continuous at p, because $\pi_p(q) \to \pi_p(p)$ as $p \to q$. $\Rightarrow$ By Riemann's removable singularity theorem and $\dim S = 1$, π_p is holomorphic over all of S. $\lrcorner$

The intersection of a general hyperplane $H \subset \mathbb{P}^{N-1}$ with the image $\pi_p(S)$ will be just the intersection of the hyperplane $\overline{H.p} \subset \mathbb{P}^N$ with $S - \{p\}$, so that

$$\deg \pi_p(S) = \deg S - 1 \quad \text{for } p \in S.$$

The simplest case here is the stereographic projection of a plane conic C from a point of C onto a line.

C