

$$\Rightarrow \#(V \cdot \mathbb{P}^{N-n}) = \int_M c_1(L)^n = \deg V \in \mathbb{Z}$$

$$\begin{aligned} \#(\bar{i}_E(M) \cdot \mathbb{P}^{N-n}) &= \#(M \cdot \bar{i}_E^{-1}(\mathbb{P}^{N-n})) = \#(\bar{i}_E^{-1}(\mathbb{P}^{N-n})) \\ &= \#(\bar{i}_E^{-1}(\mathbb{P}^{N-1}) \cap \dots \cap \bar{i}_E^{-1}(\mathbb{P}^{N-1})) = \#(\overline{D} \cap \dots \cap \overline{D}). \end{aligned}$$

(We should assume that \bar{i}_E is injective, i.e. embedding.) 1) PPS. 6.29. If we count the multiplicity then we need not assume i_E is injective.

A variety $V \subset \mathbb{P}^n$ is called normal if the linear system on V giving the embedding $i: V \rightarrow \mathbb{P}^n$ is complete, that is, if the restriction map

$$H^0(\mathbb{P}^n, \mathcal{O}(H)) \longrightarrow H^0(V, \mathcal{O}(H)) \text{ is surjective.}$$

Note that any smooth hypersurface $V \subset \mathbb{P}^n$ is normal: from the exact sheaf sequence

$$0 \rightarrow \mathcal{O}_{\mathbb{P}^n}(H-V) \rightarrow \mathcal{O}_{\mathbb{P}^n}(H) \xrightarrow{r} \mathcal{O}_V(H) \rightarrow 0$$

we have an exact sequence of cohomology groups

$$H^0(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(H)) \xrightarrow{r} H^0(V, \mathcal{O}_V(H)) \rightarrow H^1(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(H-V)),$$

$$\text{but } H^1(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(H-V)) = H^1(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}((1-d)H)) = 0$$

so r must be surjective.

IF If $\deg V = d$, then $V = dH$.

$$\Rightarrow H^1(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(H-V)) = H^1(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}((1-d)H))$$

(i) $d > 1$, $(1-d)H$ is negative line bundle.

Assume $n > 1$.