

$\Rightarrow [\omega] \in H^2(M, \mathbb{Z})$ . In other words,  $\int_{\sigma} \omega = \#(V \cdot \sigma)$  is an integer, where  $\sigma$  is a 2-cycle in  $M$ .  $\square$

A metric whose  $(1,1)$ -form is rational is called a Hodge metric.

Corollary. If  $M, M'$  are algebraic varieties, then  $M \times M'$  is.

pf) If  $\omega, \omega'$  are closed, integral, positive  $(1,1)$ -form on  $M, M'$ , respectively, and  $\pi: M \times M' \rightarrow M, \pi': M \times M' \rightarrow M'$  are the projection maps, then  $\pi^*\omega + \pi'^*\omega'$  is again closed, integral, and positive of type  $(1,1)$ . Q.E.D.

$\square$   $d(\pi^*\omega + \pi'^*\omega') = \pi^*d\omega + \pi'^*d\omega' = 0 \Rightarrow \pi^*\omega + \pi'^*\omega'$  is closed.  $(\pi^*\omega + \pi'^*\omega')(v+v', \bar{v}+\bar{v}') = \pi^*\omega(v+v', \bar{v}+\bar{v}') + \pi'^*\omega'(v+v', \bar{v}+\bar{v}') = \omega(v, \bar{v}) + \omega'(v', \bar{v}') \geq 0$   
The equality holds <sup>only</sup> if  $v = v+v' = 0 \Leftrightarrow v=0=v'$ .

Given  $f: M \rightarrow N$  and  $\omega \in H^2(N, \mathbb{Z})$ , then  $f^*\omega$  is in  $H^2(M, \mathbb{Z})$  since  $f^*\omega(\sigma) = \omega(f_*\sigma)$  is an integer ( $\because \omega$  is integral &  $f_*\sigma$  is a cycle)  
 $\Rightarrow$  By the previous theorem,  $M \times M'$  is algebraic.  $\square$

A classical example of this is the Segre map  $\mathbb{P}^n \times \mathbb{P}^m \rightarrow \mathbb{P}^{n+m}$  given by the complete linear system of the line bundle  $\pi_1^*H \otimes \pi_2^*H$  on  $\mathbb{P}^n \times \mathbb{P}^m$ . For example, the Segre map  $\mathbb{P}^1 \times \mathbb{P}^1 \rightarrow \mathbb{P}^3$  is given, in terms of homogeneous coordinates  $[z_0, z_1]$  and  $[w_0, w_1]$  on  $\mathbb{P}^1$ , by  
 $([z_0, z_1], [w_0, w_1]) \rightarrow [z_0w_0, z_0w_1, z_1w_0, z_1w_1]$ .