⌈Γ $p = [0, t']$ ($\uparrow L_0$) $q = [s, t] \in \mathbb{P}^2$.

$$C(S) = \bigcup_{\substack{p \in L_0 \\ q \in \mathbb{P}^2}} \overline{f(p), f(q)} = \{[\alpha \cdot f(s,t) + (1-\alpha) f(0,t')]\}$$

$$= \{[(1, \alpha s, \alpha t, \alpha s^2, \alpha s t, \alpha t^2) + (1-\alpha)(1, 0, t', 0, 0, t'^2)]\} = \{[1, \alpha s, \alpha t + (1-\alpha)t', \alpha s^2, \alpha s t, \alpha t^2 + t'^2]\}$$ ⌋

Now we solve for $\alpha$, $s$, $t$ and $t'$: given $X = [X_0, X_1 \cdots X_5] \in C(S)$, $X$ must be the point $\alpha \cdot f(s,t) + (1-\alpha) f(0,t')$ for the values

$$s = \frac{X_3}{X_1}, \quad t = X_4/X_1, \quad \alpha = X_1^2/X_0 X_3,$$

$$t' = (X_2 X_3 - X_1 X_4)/(X_0 X_3 - X_1^2).$$

⌈Γ $\alpha s = X_1/X_0$, $\alpha t + (1-\alpha) t' = \dfrac{X_2}{X_0}$, $\alpha s^2 = X_3/X_0$

$\alpha s t = \dfrac{X_4}{X_0}$, $\alpha t^2 + t'^2 = \dfrac{X_5}{X_0}$ ... ⊛

$$t = X_4/X_0 \times \frac{X_0}{X_1} = X_4/X_1 \quad \text{from } \frac{\alpha s t}{\alpha s}.$$

$$s = X_3/X_0 \times \frac{X_0}{X_1} = X_3/X_1 \Rightarrow \alpha = X_1/X_0 \times \frac{X_1}{X_3} = \frac{X_1^2}{X_0 X_3}$$

... ⌋

Consequently the coordinates of $X \in C(S)$ must satisfy

$$X_5/X_0 = \alpha t^2 + (1-\alpha) t'^2 = X_4^2/X_0 X_3 + (X_2 X_3 - X_1 X_4)^2/(X_0 X_3 (X_0 X_3 - X_1^2)),$$

i.e., $(X_0 X_3 - X_1^2) X_5 = X_0 X_4^2 + X_2^2 X_3 - 2 X_1 X_2 X_4$, and we see that the variety of chords of the Veronese surface in $\mathbb{P}^5$ is a cubic hypersurface.