Proof. By the $\bar{\partial}$-Poincare lemma, $\mathcal{H}^q(\mathcal{O}) = 0$ for $q > 0$. Arguing by induction on the length of a local syzygy, we may assume that we have

$$0 \to \mathcal{R} \to \mathcal{O}^{(k)} \longrightarrow \mathcal{F} \longrightarrow 0,$$

where $\mathcal{H}^q(\mathcal{R}) = 0$ for $q > 0$. Our result then follows by the exact sequence of cohomology. Q.E.D.

「 I think: Definition of $\mathcal{H}^q(\mathcal{F})$.

$$\mathcal{H}^q_z(\mathcal{F}) = \lim_{z \in U} H^q(U, \mathcal{F}) \quad \text{stalk of } \mathcal{H}^q(\mathcal{F}) \text{ at } z.$$

$$\Rightarrow \quad \mathcal{H}^q_z(\mathcal{O}) = \lim_{z \in U} H^q(U, \mathcal{O}) = \lim_{z \in U} H^{0,q}_{\bar{\partial}}(U) = 0 \quad \text{for } q \geq 1$$

by the $\bar{\partial}$-Poincare lemma on P25 ($U \cong \Delta$.).
We are going to use the induction on the length n of a local syzygy. $n = 1$. Fix $z_0 \in M$,

$$\Rightarrow \quad 0 \to \mathcal{O}^{(k_1)} \longrightarrow \mathcal{O}^{(k_0)} \longrightarrow \mathcal{F} \longrightarrow 0, \; z_0 \in U \subset M.$$

$\Rightarrow$ By the exact sequence of cohomology.

$$H^q(U, \mathcal{O}^{(k_1)}) \longrightarrow H^q(U, \mathcal{O}^{(k_0)}) \longrightarrow H^q(U, \mathcal{F}) \to H^{q+1}(U$$
$$H^q(U, \mathcal{O}^{(k_1)}) = \oplus H^q(U, \mathcal{O}) = 0, \qquad H^q(U, \mathcal{O}^{(k_0)}) = \oplus H^q(U, \mathcal{O})$$

$$\mathcal{O}^{(k_1)}) = 0 \quad \Rightarrow \quad H^q(U, \mathcal{F}) = 0 \quad q \geq 1.$$
$$\Rightarrow \quad \mathcal{H}^q_z(\mathcal{F}) = \lim_{z \in U} H^q(U, \mathcal{F}) = 0.$$

Suppose it is true for $n-1$. We have a local syzygy of length n for $\mathcal{F}$.