

Note that,
$$v(z) = \sum_{\substack{j=1 \\ k=1}}^n (a_{jk}^v + \sqrt{-1} b_{jk}^v) z_j \frac{\partial}{\partial z_k}$$

$$= \sum (a_{jk}^v + \sqrt{-1} b_{jk}^v) (x_{2j-1} + \sqrt{-1} x_{2j}) \frac{\partial}{\partial z_k}$$

$$= \sum (a_{jk}^v x_{2j-1} - b_{jk}^v x_{2j} + \sqrt{-1} b_{jk}^v x_{2j-1} + \sqrt{-1} a_{jk}^v x_{2j}) \frac{\partial}{\partial z_k}$$

$$= \sum (a_{jk}^v + \sqrt{-1} b_{jk}^v) x_{2j-1} \frac{\partial}{\partial z_k} + \sum (-b_{jk}^v + \sqrt{-1} a_{jk}^v) x_{2j} \frac{\partial}{\partial z_k}$$

$$\Rightarrow A^v = \begin{pmatrix} a_{11}^v, a_{12}^v, \dots, a_{1n}^v \\ -b_{11}^v, -b_{12}^v, \dots, -b_{1n}^v \\ a_{21}^v, a_{22}^v, \dots, a_{2n}^v \\ \vdots \\ a_{n1}^v, a_{n2}^v, \dots, a_{nn}^v \\ -b_{n1}^v, -b_{n2}^v, \dots, -b_{nn}^v \end{pmatrix}$$

$$B^v = \begin{pmatrix} b_{11}^v, b_{12}^v, \dots, b_{1n}^v \\ a_{11}^v, a_{12}^v, \dots, a_{1n}^v \\ \vdots \\ a_{n1}^v, a_{n2}^v, \dots, a_{nn}^v \\ b_{n1}^v, b_{n2}^v, \dots, b_{nn}^v \end{pmatrix}$$

According to the definition of the index on $P_{4,2,2}$, \tilde{sgn} the index of v' at P_v is the determinant of the following matrix.

$$C = \begin{pmatrix} a_{11}^v, b_{11}^v, a_{12}^v, b_{12}^v, \dots, a_{1n}^v, b_{1n}^v \\ -b_{11}^v, a_{11}^v, -b_{12}^v, a_{12}^v, \dots, -b_{1n}^v, a_{1n}^v \\ \vdots \\ a_{n1}^v, b_{n1}^v, a_{n2}^v, b_{n2}^v, \dots, a_{nn}^v, b_{nn}^v \\ -b_{n1}^v, a_{n1}^v, -b_{n2}^v, a_{n2}^v, \dots, -b_{nn}^v, a_{nn}^v \end{pmatrix}$$