

These methods may be generalized to prove the Cayley - Bacharach theorem from Section 4.

4. Global Duality.

Global Ext.

Let M be a compact, complex manifold and \mathcal{L} the invertible sheaf associated to a positive line bundle $L \rightarrow M$.

Given coherent sheaves \mathcal{H}, \mathcal{G} on M , and using Theorem A discussed in the previous section, we may find a global syzygy

$$0 \rightarrow \mathcal{E}_n \rightarrow \mathcal{E}_{n-1} \rightarrow \dots \rightarrow \mathcal{E}_1 \rightarrow \mathcal{E}_0 \rightarrow \mathcal{H} \rightarrow 0$$

for \mathcal{H} . This gives rise to a complex of sheaves $\text{Hom}_{\mathcal{O}_M}(\mathcal{E}_i(\mathcal{H}), \mathcal{G})$, whose associated hypercohomology we take as the definition of global Ext, written

$$\text{Ext}(M; \mathcal{H}, \mathcal{G}) = H^*(M, \text{Hom}_{\mathcal{O}_M}(\mathcal{E}_i(\mathcal{H}), \mathcal{G})).$$

“ \mathcal{O}_I finite dimensional complex vector space with $\dim_{\mathbb{C}}(\mathcal{O}_I) = d = \text{degree of } f$.”

To show this, we need the following theorems and corollary, from Introduction to holomorphic functions of several variables.

5. Theorem. (P32). If $\pi: V \rightarrow W$ is a finite branched holo