

From the exact sequence of sheaves above, we have the long exact sequence

$$H^0(M, \text{Hom}_\mathcal{O}(\mathcal{E}'(-k), \mathcal{R})) \rightarrow H^0(M, \text{Hom}_\mathcal{O}(\mathcal{E}'(-k), \mathcal{E})) \rightarrow H^0(M, \text{Hom}_\mathcal{O}(\mathcal{E}'(-k), \mathcal{F}))$$

$$\rightarrow H^1(M, \text{Hom}_\mathcal{O}(\mathcal{E}'(-k), \mathcal{R})) \rightarrow$$

$\overset{0}{\parallel}$  by the result above for  $k \geq k_0$ .

$$\Rightarrow \exists \text{ a surjection } H^0(M, \text{Hom}_\mathcal{O}(\mathcal{E}'(-k), \mathcal{E})) \rightarrow H^0(M, \text{Hom}_\mathcal{O}(\mathcal{E}'(-k), \mathcal{F}))$$

$\Rightarrow$  This implies that  $\forall$  the diagram the dotted arrow is filled.  $\Rightarrow$

From this we may draw the following conclusion: When working globally with coherent sheaves on  $M$ , the four properties of projective resolutions of  $\mathcal{O}$ -modules carry over to global syzygies, at least provided we allow ourselves to tensor with  $L^{-k}$ . As a consequence,  $\text{Ext}(M; \mathcal{F}, \mathcal{G})$  is well-defined and has functorial properties analogous to those of local  $\text{Ext}$ . The most important of these are the two long exact sequences.

Comment on  $\underline{\text{Ext}}_\mathcal{O}^p(\mathcal{F}, \mathcal{G})$

$$\text{Def: } \underline{\text{Ext}}_\mathcal{O}^p(\mathcal{F}, \mathcal{G})(U) \cong \bigcup_{x \in U} \text{Ext}_\mathcal{O}^p(\mathcal{F}_x, \mathcal{G}_x) = \bigcup_{x \in U} H^p(\text{Hom}(\mathcal{E}(\mathcal{F})_x, \mathcal{G}_x))$$

$\{ \sigma: U \rightarrow \bigcup_{x \in U} \text{Ext}_\mathcal{O}^p(\mathcal{F}_x, \mathcal{G}_x) \text{ satisfying some conditions see Hartshorne Algebraic Geometry p60~p62 } \}$   
See note P646.

Here we have to prove the following:

Given two global syzygies of  $\mathcal{F}$ , i.e.,

$$0 \rightarrow \mathcal{E}_n \rightarrow \mathcal{E}_{n-1} \rightarrow \dots \rightarrow \mathcal{E}_0 \rightarrow \mathcal{F} \rightarrow 0 \text{ \& } 0 \rightarrow \mathcal{E}'_n \rightarrow \mathcal{E}'_{n-1} \rightarrow \dots \rightarrow \mathcal{E}'_0 \rightarrow \mathcal{F} \rightarrow 0,$$