The map

$$\frac{g(z)\, dz_1 \wedge \cdots \wedge dz_n}{f(z)} \longmapsto (-1)^{i-1} \left. \frac{g(z)\, dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n}{\partial f / \partial z_i} \right|_{f=0}$$

$$\omega \longmapsto \omega'$$

is called the Poincaré residue map, denoted by P.R.

「 ω' is well-defined once ω is well-defined. since

$$\omega = \frac{df_\alpha}{f_\alpha} \wedge \omega' = \frac{g_{\alpha\beta}\, df_\beta}{g_{\alpha\beta} f_\beta} \wedge \omega' = \frac{df_\beta}{f_\beta} \wedge \omega'. \quad 」$$

「 Note that from the isomorphism

$$K_V = (K_M \otimes [V])|_V \iff K_M|_V = K_V \otimes N_V^* \atop \qquad\qquad\qquad\qquad\qquad\quad \| \atop \qquad\qquad\qquad\qquad\qquad K_V \otimes [-V]|_V$$

$\omega \in \Omega_M^n(V) \Rightarrow$

개소리

$\omega \in \Gamma(M, K_M)$. with a single pole along V.

On U_α.

$$\omega|_{U_\alpha} = \frac{df_\alpha}{f_\alpha} \wedge \omega'|_{U_\alpha} = \omega'|_{U_\alpha} \wedge \frac{df_\alpha}{f_\alpha}$$

$$\Rightarrow \left(\frac{df_\alpha}{f_\alpha}\right) \notin \Gamma(M, N_V^*) \qquad (\omega'|_{U_\alpha}) \in \Gamma(M, K_V)$$

↓ meromorphic section for $N_V^* \atop {\| \atop [-V]}$

since $\dfrac{df_\alpha}{f_\alpha} \in \mathrm{Hom}\left(\dfrac{TM|_V}{TV}, \mathbb{C}\right)$ and $\dfrac{df_\alpha}{f_\alpha} = \dfrac{g_{\alpha\beta}\, df_\beta}{g_{\alpha\beta} f_\beta} = \dfrac{df_\beta}{f_\beta}$.

Note that $\left(\dfrac{df_\alpha}{f_\alpha} \otimes \varphi_\alpha^{-1}(x, f_\alpha)\right) \in N_V^* \otimes [V]$ defines a non-vanishing section. $\Rightarrow$ $\omega \otimes \varphi_\alpha^{-1}(x, f_\alpha)|_V$ corresponds to an element in $K_M|_V$.

$\omega \otimes \varphi_\alpha^{-1}(x, f_\alpha) \in \mathcal{O}(K_M)$