the intersection index $i_p(A \cdot B)$ of A and B. at p to be $+1$ if the tangent vectors to A & B in tern form an oriented basis for $T_p(M)$, -1 if not.

Define the intersection number $\#(A \cdot B)$ of cycles A and B meeting transversely in smooth points to be the sum

$$\#(A \cdot B) = \sum_{p \in A \cap B} i_p(A \cdot B)$$

$\#(A \cdot B)$ depends only on the homology classes of A and B.

reverse orientation of A'

If A is homologous to A', $A \cup (-A')$ is the boundary of regions $C_i \subset T$ with the tangent vector to $A \cup (-A')$ and the inward normal vector to ∂C_i. always forming an oriented basis for $T(M)$. $M =$ Torus.

$\Rightarrow$ The path B will intersect $A \cup (-A')$ positively every time it enters a region C_i and negatively everytime it leaves;

thus. $\#\{(A \cup (-A')) \cdot B\} = 0 = \#(A \cdot B) - (A' \cdot B)$

$$= \sum_{p \in A \cup (-A') \cap B} i_p(A \cup (-A') \cdot B) = \sum_{p \in A \cap B} i_p(A \cdot B) + \sum_{p \in -A' \cap B} i_p((-A') \cdot B)$$

We can deform so that $(A \cup (-A')) \cap B = (A \cap B)$ or $(-A') \cap B$. i.e $A \cap (-A') \cap B = \phi$.

$$\Rightarrow \quad \#(A \cdot B) = \#(A' \cdot B) \quad //$$

Finally, since, for any two homology classes $\alpha, \beta \in H_1(T, \mathbb{Z})$, we can find cycles A and B on T representing α and β and intersecting transversely, we have defined a bilinear pairing

$$H_1(T, \mathbb{Z}) \times H_1(T, \mathbb{Z}) \longrightarrow \mathbb{Z}.$$