$$(v_1, \cdots v_n, 0 \cdots 0)\left(\begin{array}{c|c} \frac{\partial^2 f}{\partial x_i \partial x_j} & * \\ \hline * & \frac{\partial^2 f}{\partial y_i \partial y_j} \end{array}\right)\begin{pmatrix} v_1 \\ \vdots \\ v_n \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$= \left(\left(v_i \frac{\partial^2 f}{\partial x_i \partial x_j} \right) \quad \cancel{*} \right)\begin{pmatrix} v_1 \\ \vdots \\ v_n \\ 0 \\ \vdots \\ 0 \end{pmatrix} = {}^t v \left(\frac{\partial^2 f}{\partial x_i \partial x_j} \right) v \geq 0$$

$$(0, \cdots 0, w_1 \cdots w_n)\left(\begin{array}{c|c} \frac{\partial^2 f}{\partial x_i \partial x_j} & * \\ \hline * & \frac{\partial^2 f}{\partial y_i \partial y_j} \end{array}\right)\begin{pmatrix} 0 \\ \vdots \\ 0 \\ w_1 \\ \vdots \\ w_n \end{pmatrix} = {}^t w \left(\frac{\partial^2 f}{\partial y_i \partial y_j} \right) w \geq 0$$

(iii). A, B positive semidefinite
$\Rightarrow$ $A+B$ is positive semidefinite

Thus by (i), (ii) & (iii), $\left(\frac{\partial^2}{\partial x_i \partial x_j}\right) \log \frac{1}{|s|^2}$ & $\left(\frac{\partial^2}{\partial y_i \partial y_j}\right) \log \frac{1}{|s|^2}$ are positive semidefinite. but $\left(\frac{\partial^2}{\partial x_i \partial x_j} + \frac{\partial^2}{\partial y_i \partial y_j}\right) \log \frac{1}{|s|^2}$ is negative defite $\Rightarrow$ Contradiction. //

In case M is a Riemann surface, the special case (*) is the general case, since $p+q<1 \Rightarrow p=q=0$. The theorem then is even more elementary: if Θ is a curvature form for L with $\frac{i}{2\pi}\Theta$ negative, we have

$$\langle c_1(L), [M] \rangle = \int_M \frac{i}{2\pi} \Theta < 0.$$