

$$H^0(\mathbb{P}^2, \mathcal{O}(\mathfrak{z}H)) = \langle X_0^2, X_1^2, X_2^2, X_0X_1, X_0X_2, X_1X_2 \rangle.$$

$\Rightarrow (X_0=0) \times \mathbb{P}^2 + \mathbb{P}^2 \times (X_0=0) \stackrel{is}{\sim} \text{cut by } \Delta \text{ as } X_0^2 \text{ in } \mathbb{P}^2$
 $(\cong \Delta)$. More precisely,

$$\begin{array}{ccccc} [f^* \bar{i}^* D] & \longrightarrow & [\bar{i}^* D] & \longrightarrow & [(X_0=0) \times \mathbb{P}^2 + \mathbb{P}^2 \times (X_0=0)] = [D] \\ \downarrow & & \downarrow & & \downarrow \\ \mathbb{P}^2 & \xrightarrow{\bar{f}'} & \Delta & \xrightarrow{\bar{i}} & \mathbb{P}^2 \times \mathbb{P}^2 \end{array}$$

$$\Rightarrow f^* \bar{i}^* D = (X_0^2=0).$$

Similarly we get

$$f^* \bar{i}^* ((X_0=0) \times \mathbb{P}^2 + \mathbb{P}^2 \times (X_1=0)) = (X_0X_1=0),$$

$$\text{since } f^* \bar{i}^* ((X_0=0) \times \mathbb{P}^2) = (X_0=0).$$

Thus $|W_1 + W_2|$ cuts out the complete series $|\mathcal{O}_{\mathbb{P}^2}(\mathfrak{z}H)|$ on Δ .

$$\begin{array}{ccc} \Delta & \xrightarrow{\quad} & \mathbb{P}^{2*} \times \mathbb{P}^{2*} \xrightarrow{f} W_2 \subset W (\cong \mathbb{P}^5) \\ \cong \uparrow & \xrightarrow{\quad} & ((a_0X_0 + a_1X_1 + a_2X_2=0), (a_0X_0 + a_1X_1 + a_2X_2=0)) \mapsto ((a_0X_0 + a_1X_1 + a_2X_2)^2=0) \\ & \uparrow & (a_0^2X_0^2 + a_1^2X_1^2 + a_2^2X_2^2 + 2a_0a_1X_0X_1 + 2a_0a_2X_0X_2 \\ & & + 2a_1a_2X_1X_2=0) \uparrow \\ \mathbb{P}^2 \ni [a_0, a_1, a_2] & \xrightarrow{\varphi} & [a_0^2, a_1^2, a_2^2, 2a_0a_1, 2a_0a_2, 2a_1a_2] \in \mathbb{P}^5 \end{array}$$

$\psi \circ \varphi$ is the Veronese map, see P179, where
 $\psi: \mathbb{P}^5 \rightarrow \mathbb{P}^5$ $[X_0, X_1, X_2, X_3, X_4, X_5] \mapsto [X_0, X_1, X_2, \frac{1}{2}X_3, \frac{1}{2}X_4, \frac{1}{2}X_5]$
 is defined by