P1. If T is of finite order p, T can be extended to the space $\mathcal{D}^p(G)$ of functions of class (C^p) with compact support in G; for $\mathcal{D}^{(\infty)}(G)$ is dense in $\mathcal{D}^p(G)$ when $\mathcal{D}^p(G)$ is given the topology of uniform convergence on each compact set of all the derivatives of order less than or equal to p.

In particular, we can identify a distribution of order zero with a Radon measure.

Γ First of all, we need to review the def. of order of a distribution T. According to P141, Rudin's F.A., for every compact $K \subset G$, $\exists$ a smallest nonnegative integer N and a constant $C_K < \infty$ s.t the inequality

$$|\Lambda \phi| \leq C_K \|\phi\|_N$$

holds for every $\phi \in \mathcal{D}_K$.

$\Rightarrow$ If T is of order 0, for $\phi \in \mathcal{D}_K$, $\exists$ universal $\overset{\text{smallest}}{N}$ and a constant C_K s.t

$$|\Lambda \phi\| \leq C_K \|\phi\|_0.$$

Γ I want to practice by proving the following.

(a) $|\Lambda \phi| \leq C \|\phi\|_N$ for $\forall \phi \in \mathcal{D}_K$

$\Leftrightarrow$ (b) Λ is continuous on $\mathcal{D}_K$.

Pf) ($\Rightarrow$ (a)) $\Lambda^{-1}(B(\alpha,\epsilon)) \ni \phi_0 \Rightarrow \Lambda(\phi_0) \in B(\alpha,\epsilon)$.

Consider $\phi_0 + N(0,\delta)$, where $N(0,\delta) = \{\phi \in \mathcal{D}_K : \|\phi\|_N < \delta\}$