for some holomorphic matrix $A(z) = (a_{ij}(z))$. Equivalently, the ideals should satisfy

$$\{g_1 \cdots g_n\} \subset \{f_1 \cdots f_n\}.$$

Then, for $h(z) \in \mathcal{O}(\overline{U})$

$$\mathrm{Res}_{\{0\}}\left(\frac{h\, dz_1 \wedge \cdots \wedge dz_n}{f_1 \cdots f_n}\right) = \mathrm{Res}_{\{0\}}\left(\frac{h \det A\, dz_1 \wedge \cdots \wedge dz_n}{g_1 \cdots g_n}\right).$$

$\ulcorner$ If $g_i(z) = \sum a_{ij} f_j$, $\Rightarrow \{g_1 \cdots g_n\} \subset \{f_1 \cdots f_n\}$.
If $\{g_1 \cdots g_n\} \subset \{f_1 \cdots f_n\}$. then $\{f_1 \cdots f_n\} \ni g_1$. $\Rightarrow$
$g_1 = \sum f_{i_1} \cdots f_{i_\ell} \cdot g'_{i_1 \cdots i_\ell}$. $\Rightarrow g_1 = \sum a_{ij} f_j$. $\lrcorner$

Proof. We prove this in cases of increasing difficulty.
Case 1: $J_f(0) \neq 0$ and $\det A(0) \neq 0$.
Then $J_g(0) = J_f(0) \det A(0)$, and by the evaluation of the residue integral in the nondegenerate case

$$\begin{aligned}
\mathrm{Res}_{\{0\}}\left(\frac{h\, dz_1 \wedge \cdots \wedge dz_n}{f_1 \cdots f_n}\right) &= (2\pi\sqrt{-1})^n \frac{h(0)}{J_f(0)} \\
&= (2\pi\sqrt{-1})^n \frac{h(0) \det A(0)}{J_g(0)} \\
&= \mathrm{Res}_{\{0\}}\left(\frac{h \det A\, dz_1 \wedge \cdots \wedge dz_n}{g_1 \cdots g_n}\right)
\end{aligned}$$

$\ulcorner$ $\dfrac{\partial g_i}{\partial z_k} = \dfrac{\partial a_{ij}}{\partial z_k} f_j + a_{ij} \dfrac{\partial f_j}{\partial z_k} \Rightarrow \dfrac{\partial g_i}{\partial z_k}(0) = a_{ij}(0) \dfrac{\partial f_j}{\partial z_k}(0)$
since $f_j(0) = 0$.
$\Rightarrow J_g(0) = J_f(0) \det A(0)$. Easy! $\lrcorner$