

$$\text{Codim } 2. \quad a_1=1, a_2=2 \Rightarrow b_1=2, b_2=2+2-1=3.$$

$$\dim(\Lambda \cap V_2) \geq 1 \quad \dim(\Lambda \cap V_3) \geq 2. \Rightarrow \Lambda \subset V_3, \text{ since } \Lambda \cap V_2 \neq (0).$$

$$a_1=2, a_2=0. \Rightarrow b_1=1, b_2=4 \Rightarrow \dim(\Lambda \cap V_1) \geq 1 \Rightarrow \Lambda \supset V_1$$

$$\text{Codim } 3. \quad a_1=2, a_2=1 \Rightarrow b_1=1, b_2=3$$

$$\Rightarrow \dim(\Lambda \cap V_1) \geq 1 \quad \dim(\Lambda \cap V_3) \geq 2. \quad V_1 \subset \Lambda \subset V_3. \quad \rfloor$$

Alternatively, if we think of $G(2,4)$ as the set of lines ℓ in \mathbb{P}^3 and fix the projective flag $p \in \ell_0 \subset h$ consisting of a point, line and hyperplanes in \mathbb{P}^3 , then

$$\sigma_{1,0}(\ell_0) = \{\ell : \ell \cap \ell_0 \neq \emptyset\},$$

$$\sigma_{2,0}(p) = \{\ell : p \in \ell\}$$

$$\sigma_{1,1}(h) = \{\ell : \ell \in h\}, \quad \sigma_{2,1}(p,h) = \{\ell : p \in \ell \subset h\}.$$

$$\Gamma \quad V_1 \leftrightarrow p \quad V_2 \leftrightarrow \ell_0 \quad V_3 \leftrightarrow h.$$

$$\Lambda \text{ is a line } \ell \text{ in } \mathbb{P}^3. \Rightarrow \dim(\Lambda \cap V_2) \geq 1$$

$$\Leftrightarrow \ell \cap \ell_0 \neq \emptyset \quad \dots \quad \rfloor$$

The Schubert Calculus.

Now that we have determined the additive cohomology of $G(k,n)$, we would like to describe its multiplicative structure — that is, to express the intersection of general Schubert cycles σ_a, σ_b as a linear combination of other Schubert cycles in homology.

The first task is to write down the intersection pairing in complementary dimensions. To do this, let