dependent $\Rightarrow$ $v_1 \wedge v_2 \wedge e_0 \wedge e_1 \wedge e_2 \wedge e_3 = 0$ --- ⊛
$\tilde{\phi}(L) = [\, h_{ij} \,]$ $\Rightarrow$ From ⊛, we have a linear equation $\Rightarrow$ $\tilde{\phi}(\sigma_1(V_3)) \subset H$, where H is the hyperplane corresponding to the linear equation obtained above.
$\Rightarrow$ Since H is irreducible, $\tilde{\phi}(\sigma_1(V_3)) = H$. (Recall that $\sigma_1(V_3)$ is irreducible, see P196.)

Given $H \in \mathbb{P}(\Lambda^2\mathbb{C}^6)^*$, then H may be considered as a linear functional on $\Lambda^2\mathbb{C}^6$. $\Rightarrow$ Since $\wedge : \Lambda^2\mathbb{C}^6 \times \Lambda^4\mathbb{C}^6 \longrightarrow \mathbb{C} \cong \Lambda^6\mathbb{C}^6$ is nondegenerate, see P156, $\exists$ a unique $\omega \in \Lambda^4\mathbb{C}^6$ s.t $H(\_) = (\_) \wedge \omega$.
$\Rightarrow$ $\mathbb{P}(\Lambda^2\mathbb{C}^6)^* = \mathbb{P}(\Lambda^4\mathbb{C}^6)$.

Given a hyperplane $H \in \mathbb{P}(\Lambda^2\mathbb{C}^6)^*$, then $\exists$ $\omega \in \Lambda^4\mathbb{C}^6$ s.t $H(\_) = (\_) \wedge \omega$, by the argument above.

Suppose $G(2,6) \cap H_1 = G(2,6) \cap H_2$. Let ω_1, ω_2 be the elements in $\Lambda^4\mathbb{C}^6$ corresponding to H_1 & H_2 respectively.
$\Rightarrow$ For all $v_1 \wedge v_2 \in \Lambda^2\mathbb{C}^6$, $v_1 \wedge v_2 \wedge \omega_1 = v_1 \wedge v_2 \wedge \omega_2$.
$\Rightarrow$ $v_1 \wedge v_2 \wedge (\omega_1 - \omega_2) = 0$ for all $v_1 \wedge v_2$
$\Rightarrow$ Let $\omega = \omega_1 - \omega_2 =$ $a_{ijkl}\, e_i \wedge e_j \wedge e_k \wedge e_l$.
$\Rightarrow$ For example, $a_{0123} = 0$, for, choose $v_1 = e_4$, $e_5 = v_2$
$\Rightarrow$ $v_1 \wedge v_2 \wedge \omega = a_{0123} = 0$. In the same way, we get $a_{ijkl} = 0$. $\Rightarrow$ $\omega = 0$ $\Rightarrow$ $\omega_1 = \omega_2$
$\Rightarrow$ $\exists$ one to one correspondence between $\mathbb{P}(\Lambda^2\mathbb{C}^6)^*$ and $|\sigma_1|$, i.e. $H \longleftrightarrow H \cap G(2,6)$.

$$\begin{array}{ccccc} G(4,6) & \longrightarrow & |\sigma_1| & \longleftrightarrow & \mathbb{P}(\Lambda^4\mathbb{C}^6) \\ \Psi & & \Psi & & \Psi \\ V_3 & \longmapsto & \sigma_1(V_3) & \longleftrightarrow & v_1 \wedge v_2 \wedge v_3 \wedge v_4 \end{array}$$