Now consider the bundle map

$$\Lambda^{p-1}T' \otimes \Lambda^q T'' \xrightarrow{\wedge v} \Lambda^p T' \otimes \Lambda^q T''$$

given by wedge product with $v(z) \in T'_z$. We define the contraction operator

$$\iota(v) : A^{p,q}(M) \longrightarrow A^{p-1,q}(M)$$

to be the adjoint of $\wedge v$; such that for any $\varphi \in A^{p,q}(M)$ and $\eta \in C^\infty(\Lambda^{p-1}T' \otimes \Lambda^q T'')$,

$$\langle \iota(v)\varphi, \eta \rangle = \langle \varphi, v \wedge \eta \rangle.$$

Thus, if in terms of local coordinates z_i

$$v(z) = \sum_j v^j(z) \frac{\partial}{\partial z_j},$$

we have $\quad \iota(v)(dz_i) = v^i.$

and in general

$$\iota(v)(f(z)\cdot dz_I) = \sum_\alpha (-1)^{\alpha-1} v^{i_\alpha}(z) f(z)\, dz_{I-\{i_\alpha\}}.$$

$\ulcorner$ $\quad \langle \iota(v)(dz_i), \eta \rangle = \langle dz_i, v \wedge \eta \rangle$

$= \langle dz_i, v^j \frac{\partial}{\partial z_j} \wedge \eta \rangle = \langle dz_i, \eta v^j \frac{\partial}{\partial z_j} \rangle$

$= \eta v^j = \iota(v)(dz_i)\eta \Rightarrow \iota(v)(dz_i) = v^j.$

In general, $\quad \iota(v)(f(z)\,dz_I) = \sum b_J\, dz_J. \quad \#I = p. \quad \#J = p-1.$

$\Rightarrow \quad \langle \iota(v)(f\,dz_I), \eta \rangle = \langle f(z)\,dz_I, v \wedge \eta \rangle.$

$\quad = \langle \sum b_J\, dz_J, \eta \rangle.$

Choose $\quad \eta = (\frac{\partial}{\partial z})_J.$

$\Rightarrow \quad LHS = b_J \qquad RHS = \langle f(z)\,dz_I, v \wedge (\frac{\partial}{\partial z})_J \rangle$

① $J = I - \{i_\alpha\}$, where $i_\alpha \in I$.

ith number