$P(\rho\theta) \neq P(\rho)\wedge\theta + \rho P(\theta)$, since $\bar{\partial}^*(\rho\theta) \neq \rho\bar{\partial}^*\theta$. ⌟

It will consequently suffice to prove the regularity assertion about weak solutions of the equation (*) for forms compactly supported in a fixed coordinate patch on M. This coordinate patch may be taken to be diffeomorphic to $\mathbb{R}^n$, so that what we must show is the following.

⌜ I think we change the whole thing into the original equation. $\Delta\psi = \varphi$.

Start:

$$\psi \in \mathcal{H}_0^{p,q}(M), \quad \eta \in A^{p,q}(M). \quad \varphi \in \mathcal{H}_s^{p,q}(M).$$

$$\langle \psi, \Delta\eta\rangle = \langle \varphi, \eta\rangle$$

For simplicity, we will assume that. $\psi \in \mathcal{H}_0^p$, $\eta \in A^p(M)$, $\varphi \in \mathcal{H}_s^p(M)$, M. real.

$$\Rightarrow \int_M \psi\wedge *\Delta\eta = \int_M \varphi\wedge *\eta.$$

Let $\{\phi_\alpha\}$ be a partition of unity, s.t $\operatorname{supp}\phi_\alpha \subset U_\alpha$ and $\sum\phi_\alpha = 1$, $U_\alpha \cong \mathbb{R}^n$

$$\int_M \psi\wedge *\Delta\eta = \sum_\alpha \int_M \phi_\alpha \psi\wedge *\Delta\eta = \sum_\alpha \int_{U_\alpha} \phi_\alpha \psi\wedge *\Delta\eta$$

$$= \int_M \varphi\wedge *\eta = \sum_\alpha \int_M \phi_\alpha \varphi\wedge *\eta = \sum_\alpha \int_{U_\alpha} \phi_\alpha \varphi\wedge *\eta$$

On U_α, locally, $\eta = \sum_{\#I = p} \eta_I \, dx_I$

$$\Rightarrow \Delta\eta = \{\underset{\text{constant}}{C}\,\underset{\text{standard Laplacian on } \mathbb{R}^n}{\Delta_d}\,\eta_I + \text{lower order terms of } \eta_I\}\, dx_I$$ by Weitzenbock formula P97