$\Rightarrow \quad (f^*g)(0) = g(f(0)) = g(0) = 0. \;\Rightarrow\; g(w)=0$ at $w=0$.
$\Rightarrow \quad g \in M_w$.

By the result above, $\{z_1^d, z_2^d, \cdots z_n^d\} \subset \{f_1, \cdots f_n\}$.
Consider $V = \{z_1+I, \cdots z_1^{d-1}+I, z_2+I, \cdots z_2^{d-1}+I, \cdots\cdots, z_n^{d-1}+I\}$ in $\mathcal{O}/I$. We claim V span $\mathcal{O}/I$. Given $h \in \mathcal{O}(U)$, s.t $h(0)=0$, without loss of generality, we may assume $h=0$ does not contain z_n-axis, since, if not, $h=0$ does not contain $a_1z_1+\cdots+a_nz_n$ for some a_i's. $\Rightarrow$ By the Weierstrass preperation theorem, $h = z_n^{\ell} + a_1(z_1\cdots z_{n-1})z_n^{\ell-1} + \cdots + a_\ell(z_1,\cdots z_{n-1})$
$\Rightarrow h+I \equiv a_\ell(z_1,\cdots z_{n-1}) \pmod V$. Continue this, until we get $h+I \equiv \text{constant}+I \pmod V$ finally. $\Rightarrow \mathcal{O}/I$ is finite dimensional complex vector space, since

$$f^*\left(\frac{\mathcal{O}_w}{M_w}\right) \times \mathcal{O}_z/I \longrightarrow \mathcal{O}_z/I \quad \text{is a module}$$

over $f^*\left(\mathcal{O}_w/m_w\right) \cong \mathbb{C}$.

In case one of D_i's is nonsingular at the origin, by using induction on n, we can prove $\dim_{\mathbb{C}} \mathcal{O}/I = n$ as follows:

Assume that D_1 is nonsingular at the origin
$\Rightarrow$ As in the proof of (d) on P665, let $D_i' = D_1 \cap D_i$ for $i \geq 2$. $\Rightarrow \quad (D_1, \cdots D_n)_{\{0\}} = (D_2', \cdots D_n')_{\{0\}} = \dim_{\mathbb{C}}(\mathcal{O}_{z'}/I')$
where $I' = \{f_2', \cdots f_n'\}$. $f_i'(z') = f_i(0,z') = f_i|_{D_1}$.