$$L^k : H^{n-k}(M) \xrightarrow{\sim} H^{n+k}(M)$$

and primitive decomposition

$$\left\{ \begin{aligned} H^{\ell}(M) &= \bigoplus_{j \le [\ell/2]} L^j P^{\ell-2j}(M), \text{ where} \\ P^{n-k}(M) &= \ker \{ L^{k+1} : H^{n-k}(M) \to H^{n+k+2}(M) \} \end{aligned} \right\},$$

both of which were proved in Section 6 of Chapter 0. We recall also that the primitive decomposition is compatible with the Hodge (p, q) decomposition.

Note, as noted above, the diagram

$$\begin{array}{ccc} {}'E_1^{p,q} & \xrightarrow{{}'d_1} & {}'E_1^{p+1,q} \\ \wr\| & & \wr\| \\ H^{n-p,q}(M) & \xrightarrow{\iota(v)} & H^{n-p-1,q}(M) \end{array}$$

is commutative, at least up to ± 1.

Γ By the diagram on p 709

$$\begin{array}{ccc} \Omega^{n-p} & \xrightarrow{\iota(v)} & \Omega^{n-p-1} \\ \uparrow & \circlearrowleft & \uparrow \text{ up to } (-1)^p . \\ \underline{\mathrm{Hom}}_{\mathcal{O}}(\Omega^p, \Omega^n) & \xrightarrow{\iota(v)^*} & \underline{\mathrm{Hom}}_{\mathcal{O}}(\Omega^{p+1}, \Omega^n) \end{array}$$

$\Rightarrow$

By the result on p 443, $d_1' = (-1)^p \iota(v)^*$

since $D = \delta + (-1)^p \iota(v)^*$, where p is the degree of Ω^p. ($\because$ $D^2 = 0$, $\delta \iota(v)^* = \iota(v)^* \delta$ see p446)