We have to check the sequence is exact. To show the exactness, consider it at stalk level.

$$0 \longrightarrow \mathcal{O}_x(rH) \longrightarrow \mathcal{O}_x(kH) \oplus \mathcal{O}_x(lH) \longrightarrow I(F, G) \otimes \mathcal{O}_x(dH) \to 0$$

$$\tilde{\eta} \longmapsto (\tilde{\eta} G, -\eta F)$$

$$(\tilde{\xi}, \tilde{\psi}) \longmapsto F\tilde{\xi} + G\tilde{\psi}$$

which was proved exact on P698.

Thus the Koszul complex gives the global syzygy

$$0 \to \mathcal{O}(r) \longrightarrow \mathcal{O}(k) \oplus \mathcal{O}(l) \longrightarrow I(d) \longrightarrow 0.$$

By the way we may assume $r \geq 0$. 」

Next we recall that $H^1(\mathbb{P}^2, \mathcal{O}(r)) = 0$ for all r, since first $H^1(\mathbb{P}^2, \mathcal{O}(r)) = 0$ for $r < 0$ by the Kodaira vanishing theorem, and second

$$H^1(\mathbb{P}^2, \mathcal{O}(r)) \cong H^1(\mathbb{P}^2, \mathcal{O}(-r-3)) = 0$$

for $r \geq 0$ by Kodaira–Serre duality.

「 $1+0<2$. see p155. & p153 for Kodaira–Serre duality 」

The exact cohomology sequence then gives

$$H^0(\mathbb{P}^2, \mathcal{O}(k)) \oplus H^0(\mathbb{P}^2, \mathcal{O}(l)) \to H^0(\mathbb{P}^2, I(d)) \longrightarrow 0.$$

Our local assumptions (**) exactly mean that

$$H \in H^0(\mathbb{P}^2, I(d)) \subset H^0(\mathbb{P}^2, \mathcal{O}(d)),$$