"Comment on the definition of a birational map"

① $f: M \to N$ birational if $\exists$ an inverse rational map $g: N \to M$ s.t $f \circ g = id$ as a rational map.

(*) Consider $\mathbb{P}^2 \times \mathbb{P}^2 \xrightarrow{\pi} \mathbb{P}^2$.

$\Rightarrow$ $\mathbb{P}^2 \xrightarrow{i} \mathbb{P}^2 \times \mathbb{P}^2$

$x \longmapsto (x, y_0)$. y_0 fixed point of $\mathbb{P}^2$.

$\Rightarrow$ $\pi \circ i = id$. $\Rightarrow$ As we can see easily, π is not generically one to one.

② $f: M \to N$ birational if $\exists$ an inverse rational map $g: N \to M$ s.t $f \circ g$ & $g \circ f$ are identities as rational maps.

(*) $\tilde{M} \xrightarrow{\pi} M$ is the blow-up of M at a point p.

$\Rightarrow$ $\pi^{-1}: M - \{p\} \to \tilde{M}$ is an inverse rational map, s.t $\pi \circ \pi^{-1}: M - \{p\} \to M - \{p\}$. But $\pi^{-1} \circ \pi: \tilde{M} - E \to M - \{p\} \to \tilde{M} - E$ is not a rational map since E has a codimension 1.

One possibility is the following:

A rational map $f: M \to N$ is birational if it is generically one to one. "

4. If $\varphi: M \to \mathbb{P}^n$ is any holomorphic map of a k-dimensional manifold to $\mathbb{P}^n$, the associated Gauss map

$$\mathcal{G}: M^* \longrightarrow G(k+1, n+1),$$