$\Rightarrow \overline{p.X_{p_{ik}}}$ is a 2-plane in $\langle X_{p_{ik}}, X_{p_{jk}} \rangle$.
$\Rightarrow \overline{p.X_{p_{ik}}} \cap X_{p_{jk}} \ni q$
$\Rightarrow \overline{pq}$ is a line in $\overline{p.X_{p_{ik}}} \Rightarrow \overline{pq} \cap X_{p_{ik}} \neq \phi$
$\Rightarrow L = \overline{pq}$ is a line meeting $X_{p_{ij}}, X_{p_{ik}}$ and $X_{p_{jk}}$.

But since $\Sigma \subset \mathbb{P}^5$ is cut out by quadrics, the line L, meeting Σ in three points, must lie in Σ; since L meets lines of the form X_p on Σ, we must have $L = X_h$ for some $h \in R^*$; and since L meets $X_{p_{ij}}, X_{p_{jk}}$ and $X_{p_{ik}}$, we must have $h = \overline{p_{ij}. p_{jk}. p_{ik}}$.

Γ $\Sigma = F \cap G \cap H$, F, G, H quadrics, by P769 ~ P770
$X_{p_{ij}}, X_{p_{jk}}, X_{p_{ik}} \subset \Sigma$ and L meets with those three disjoint lines $\Rightarrow \#(L \cap \Sigma) \geq 3 \Rightarrow \#(L \cap F) \geq 3$, $\#(L \cap G) \geq 3$, and $\#(L \cap H) \geq 3$
$\Rightarrow L \subset G \cap F \cap H = \Sigma$
$\Rightarrow$ Since L meets lines of the form X_p on Σ, and L is of form X_p or X_h, $L = X_h$ ($\because$ X_p's are disjoint each other.)
$\phi \neq X_h \cap X_{p_{ij}} = \sigma(h) \cap \sigma(p_{ij}) \cap F \Rightarrow p_{ij} \in h$
Similarly, $p_{jk}, p_{ik} \in h \Rightarrow h = \overline{p_{ij}. p_{jk}, p_{ik}}$.

Thus all four faces of the tetrahedron in $\mathbb{P}^3$ with