Consequently, the inclusion $i : \mathbb{R}^* \longrightarrow \mathcal{A}^*$ is a quasi-isomorphism, and by the lemma

$$\mathbb{H}^*(M, \mathbb{R}^*) \cong \mathbb{H}^*(M, \mathcal{A}^*).$$

⌈

$$\begin{array}{ccccccc}
\mathbb{R} & \to & 0 & \to & \cdots & 0 & \to \cdots \\
\downarrow & & \downarrow & & & \downarrow & \\
\mathcal{A}^0 & \to & \mathcal{A}^1 & \to & \cdots & \mathcal{A}^p & \to
\end{array}$$

$$\mathcal{H}^q(\mathbb{R}^*) = 0 \text{ for } q > 0 \qquad \mathcal{H}^0(\mathbb{R}) = \mathbb{R}$$

$\Rightarrow \quad \iota_* : \mathcal{H}^q(\mathbb{R}^*) \longrightarrow \mathcal{H}^q(\mathcal{A}^*)$ is isomorphic. ⌋

Evidently

$$({}'E_{\mathbb{R}^*})_2^{p,q} = \begin{cases} H^p(M, \mathbb{R}), & q = 0 \\ 0, & q > 0, \end{cases}$$

so the first spectral sequence for $\mathbb{R}^*$ is trivial and

$$H^*(M, \mathbb{R}) \cong \mathbb{H}^*(M, \mathbb{R}^*).$$

⌈ By the results on p446, ${}'E_2^{p,q} = H^p(X, \mathcal{H}^q(K^*))$

$\Rightarrow$ Since $K^* = \mathbb{R}^*$, $({}'E_{\mathbb{R}^*})_2^{p,q} = H^p(X, \mathcal{H}^q(\mathbb{R}^*))$

$$= \begin{cases} H^p(M, \mathbb{R}), & q = 0 \\ 0, & q > 0. \end{cases}$$

$$({}'E_{\mathbb{R}^*})_2^{p-2,q+1} \xrightarrow{D} ({}'E_{\mathbb{R}^*})_2^{p,q} \xrightarrow{D} ({}'E_{\mathbb{R}^*})_2^{p+2,q-1}$$

If $q = 0$, $({}'E_{\mathbb{R}^*})_2^{p+2,q-1} = 0$ and $({}'E_{\mathbb{R}^*})_2^{p-2,q+1} = 0$

$$\Rightarrow ({}'E_{\mathbb{R}^*})_3^{p,q} = ({}'E_{\mathbb{R}^*})_2^{p,q} = \cdots = ({}'E_{\mathbb{R}^*})_{2\pm 1}^{p,0} \cong ({}'E_{\mathbb{R}^*})_\infty^{p,0}$$