

may be naturally identified with the abstract curve  $B$  of irreducible families of  $\lambda$ -planes in the quadrics of the pencil  $\{F_t\}$ .

To be explicit, suppose that our original pair of quadrics  $G$  and  $F$  are given as the locus of two symmetric quadric forms  $Q$  and  $Q'$ . We can, of course, take  $Q$  to be given by the identity matrix, and by standard linear algebra we may at the same time diagonalize  $Q'$ ; i.e., we may take

$$G = (\sum X_i^2 = 0) \text{ and } F = (\sum \lambda_i X_i^2 = 0).$$

$\mathbb{F}$   $Q$  symmetric matrix, nonsingular.

Consider  $\{X \in \mathbb{C}^n \mid (Q - \lambda_1 I)X = 0\} = A$ .  $\lambda_1$  is nonzero eigenvalue of  $Q$ . If we let  $B = \{X \in \mathbb{C}^n \mid {}^tXX \neq 0\}$ , then  $A \cap B \neq \emptyset$ , otherwise

$A \subset B^c = \{{}^tXX = 0\} \Rightarrow$  Since  $A$  is a hyperplane in  $\mathbb{C}^n$ , we may assume  $A = (X_1 = 0)$ .

$\Rightarrow B^c = \{X_1 f(X_2, \dots, X_n) = 0\}$  which is impossible since  $B^c$  is smooth.

Thus we have  $v_1 \in \mathbb{C}^n$  s.t.  ${}^tv_1 v_1 \neq 0$  and  $(Q - \lambda_1 I)v_1 = 0 (\Leftrightarrow Qv_1 = \lambda_1 v_1)$ . Let  $v_1^\perp = \{X \in \mathbb{C}^n \mid {}^tv_1 X = 0\} \Rightarrow QX \in v_1^\perp$  if  $X \in v_1^\perp$  since  ${}^tv_1 QX = {}^t(Qv_1)X = {}^t(\lambda_1 v_1)X = \lambda_1 {}^tv_1 X = 0$ .  $\Rightarrow$  Again apply the argument above to  $v_1^\perp$ .  $\Rightarrow$  We find  $v_2 \in v_1^\perp$  s.t.  ${}^tv_2 v_2 \neq 0$ , and