$\mathbb{P}^3$ cut out in $G(2,4) \subset \mathbb{P}^5$ by hypersurfaces in $\mathbb{P}^5$. In particular, we define

Definition. A line complex of degree d in $\mathbb{P}^3$ is the three-parameter family of lines in $\mathbb{P}^3$ corresponding to the intersection of the Grassmannian $G(2,4) \subset \mathbb{P}^5$ with a hypersurface of degree d in $\mathbb{P}^5$. ⌜ $\dim G(2,4) = 4 \Rightarrow \dim(\text{line complex}) = 3$ ⌟

We consider first linear line complexes, that is, line complexes $X = G \cap H$ given as the intersection of G with a hyperplane $H \subset \mathbb{P}^5$. If X is singular – i.e., if $H = T_x(G)$ is the tangent plane to G at some point x – then, as we have seen, the complex X is the Schubert cycle $\sigma(l_x)$ of lines in $\mathbb{P}^3$ meeting l_x:

⌜ $G \cap H \ni {}^\forall x,\quad T_x(G) \neq H \Leftrightarrow H$ meets G (at x) transversely. The rest follows from p757 ⌟

Suppose on the other hand that X is smooth. For each $p \in \mathbb{P}^3$, then, the set

$$X_p = \sigma(p) \cap H$$

of lines of the complex X passing through p is