We have to decompose $\bar\partial\sigma^{(p_1,q_1)(p_2,q_2)}$ again, i.e. $\bar\partial\sigma^{(p_1,q_1)(p_2,q_2)} \notin$ any component.

$\bar\partial\sigma$ can be decomposed. I think "decomposition does not commute with $\bar\partial$" means that $\bar\partial\sigma^{(p_1,q_1)(p_2,q_2)}$ is not decomposed fully still.

But if we decompose $A^{p,q}(M\times M)$ into

$$\bigoplus_{p_1} A^{(p_1,*),(p-p_1,q-*)}(M\times M),$$

given $\sigma \in A^{p,q}(M\times M) = \bigoplus_{\substack{p_1,q_1\\ 0\le q_1\le q}} A^{(p_1,q_1)(p-p_1,q-q_1)}(M\times M)$,

$$\sigma = \bigoplus\Big(\bigoplus_{q_1}\sigma^{(p_1,q_1)(p-p_1,q-q_1)}\Big)$$

$$\bar\partial\sigma = \bigoplus\Big(\bigoplus\bar\partial\sigma^{(p_1,q_1)(p-p_1,q-q_1)}\Big)$$

$$\Rightarrow \quad \bar\partial\bigoplus_{0\le q_1\le q}\sigma^{(p_1,q_1)(p-p_1,q-q_1)} \in \bigoplus_{0\le q_1\le q}\Big(A^{(p_1,q_1+1),(p-p_1,q-q_1)} \oplus A^{(p_1,q_1),(p-p_1,q-q_1+1)}\Big)$$

$$\subset \bigoplus_{0\le q_1\le q+1} A^{(p_1,q_1)(p-p_1,q+1-q_1)} = A^{(p_1,*)(p-p_1,q+1-*)}$$

where $*$ represents an index running from zero to $q+1$.

I guess, the definition of a current T_Δ of bitype $(0,*),(n,n-*)$ is a functional defined on $(n,*),(0,n-*)$ forms.

$$\varphi \in \bigoplus_{0\le q\le n} A^{(n,q)(0,n-q)}(M\times M)$$