$$\Rightarrow \frac{\omega_3}{dz_1}(p_1) = \frac{c_1\omega_1+c_2\omega_2}{dz_1}(p_1), \text{ and } \frac{c_1\omega_1+c_2\omega_2}{dz_2}(p_2) = \frac{\omega_3}{dz_2}(p_2).$$

$$\Rightarrow \omega_3(p_1) = (c_1\omega_1+c_2\omega_2)(p_1). \quad \omega_3(p_2) = (c_1\omega_1+c_2\omega_2)(p_2).$$

$\Rightarrow \{\omega_1, \omega_2, \omega_3 - c_1\omega_1 - c_2\omega_2\}$ is linearly independat set. $(\omega_3 - c_1\omega_1 - c_2\omega_2)(p_i) = 0$ for $i = 1.2$.

$\Rightarrow$ In general, we can see that the number of independent relations along the row vectors of the metrix $\left(\frac{\omega_i}{dz_j}(p_j)\right)$ = the # of linearly independent holomorphic differentials vanishing at p_λ for all λ. More explanation: If ω is a holomorphic differential vanishing at p_λ for $\lambda = 1.2$, since $\omega = a\omega_1 + b\omega_2 + c\omega_3$. we have

$$a\omega_1(p_1) + b\omega_2(p_1) + c\omega_3(p_1) = 0$$
$$a\omega_1(p_2) + b\omega_2(p_2) + c\omega_3(p_2) = 0.$$

If $c = 0$, then. since $(\omega_1(p_1), \omega_2(p_1))$ & $(\omega_1(p_2), \omega_2(p_2))$ are lin. indep., $a = b = 0$.

So $c \neq 0$. $\Rightarrow$ $\omega_3(p_1) = -\frac{a}{c}\omega_1(p_1) - \frac{b}{c}\omega_2(p_1)$

$$\omega_3(p_2) = -\frac{a}{c}\omega_1(p_2) - \frac{b}{c}\omega_2(p_2).$$

"In fact", $a\frac{\omega_1}{dz_1}(p_1) + b\frac{\omega_2}{dz_1}(p_1) + c\frac{\omega_3}{dz_1}(p_1) = 0 = \frac{\omega}{dz_1}(p_1)$

$$a\frac{\omega_1}{dz_2}(p_2) + b\frac{\omega_2}{dz_2}(p_2) + c\frac{\omega_3}{dz_2}(p_2) = 0 = \frac{\omega}{dz_2}(p_2)$$

$\Rightarrow \left(\frac{\omega_3}{dz_1}(p_1), \frac{\omega_3}{dz_2}(p_2)\right) = \frac{-a}{c}\left(\frac{\omega_1}{dz_1}(p_1), \frac{\omega_1}{dz_2}(p_2)\right) - \frac{b}{c}\left(\frac{\omega_2}{dz_1}(p_1), \frac{d\omega_2}{dz_2}(p_2)\right)$ $\Rightarrow$ Again, by the independence,

$-\frac{a}{c} = c_1 \quad -\frac{b}{c} = c_2 \quad \Rightarrow \omega \Rightarrow c(-c_1\omega_1 - c_2\omega_2 + \omega_3)$

For the general case, it is true. Do it.