index j when $I \cup \{j\}$ is ordered in the usual manner.

For example, $n=2$. $I=\{1\}$.
$P_I = \{(z_1, z_2) \mid |z_1| = \epsilon_1, \ |z_2| \le \epsilon_2\}$
$\Rightarrow \partial P_I = (-1)^0 P_{\{1,2\}}$. $P_I = \{(z_1, z_2) \mid |z_2| = \epsilon_2, \ |z_1| \le \epsilon_1\}$
$\Rightarrow \partial P_I = (-1)^1 P_{\{2,1\}}$ Let's see a little later what it means really.

Now, since $d\xi_I = \partial \xi_I$ we may apply Stokes' theorem to obtain

$$\sum_{\#I=p} \int_{P_I} \omega_{p-1,I} = \sum_{\#I=p} \int_{P_I} d\xi_{p,I}$$

$$= \sum_{\#I=p} \int_{\partial P_I} \xi_{p,I}$$

$$= \sum_{\#I=p} \left(\sum_{j \notin I} \int_{P_{I\cup\{j\}}} (-1)^{(j,\, I\cup\{j\})} \xi_{p,I} \right)$$

$$= \sum_{\#J=p+1} \int_{P_J} (-1)^{(j,J)} \xi_{p,\, J-\{j\}}$$

$$= \sum_{\#J=p+1} \int_{P_J} (\delta \xi_p)_J$$

$$= \sum_{\#J=p+1} \int_{P_J} \omega_{p,J}$$

by the combinatoral definition of δ. Consequently, the total sum

$$\sum_{\#I=p+1} \int_{P_I} \omega_{p,I}$$

is the same for all p.