

⌈ If $\#(\Sigma_{k,n} \cdot \sigma_b) \neq 0$, $\exists \Lambda_{k+1} \subset \mathbb{P}^{n+2}$ s.t. $\Lambda_{k+1} \in \Sigma_{k,n} \cap \sigma_b$. To $\Lambda_{k+1} \in \sigma_b = \{ \dim(\Lambda_{k+1} \cap V_{n-k+1+\bar{c}-b\bar{c}}) \geq \bar{c} \}$,

Since

$$\dim \Lambda_{k+1} \cap V_\alpha \leq \frac{\alpha-2}{2} + 1 \text{ by the result above,}$$

$$(\because \dim(\underbrace{\Lambda}_{\text{projective planes}} \cap \underbrace{\bar{V}_\alpha}_{\text{projective planes}}) \leq \frac{\alpha-2}{2})$$

$$\bar{c} \leq \dim(\Lambda_{k+1} \cap V_{n-k+1+\bar{c}-b\bar{c}}) \leq \frac{n-k+1+\bar{c}-b\bar{c}}{2}.$$

$$\Rightarrow 2\bar{c} \leq n-k+1+\bar{c}-b\bar{c} \Rightarrow b\bar{c} \leq n-k+1-\bar{c} \quad \Rightarrow$$

But from

$$(n-k+1)(k+1) - \frac{(k+1)(k+2)}{2}$$

$$= \sum_{\bar{c}=1}^{k+1} b\bar{c}$$

$$\leq \sum_{\bar{c}=1}^{k+1} n-k-\bar{c}+1$$

$$= (n-k+1)(k+1) - \sum_{\bar{c}=1}^{k+1} \bar{c}$$

$$= (n-k+1)(k+1) - \frac{(k+1)(k+2)}{2}$$

we deduce that $b\bar{c} = n-k-\bar{c}+1$, i.e.,

The cycle $\Sigma_{k,n} \subset G(k+1, n+2)$ has intersection number