Claim: $\sum_{\#I=k} \det((gAg^{-1})^{\tau}_{I}) = \sum_{\#I=k} \det(A^{\tau}_{I})$

We will prove this claim by giving an example.
Let $n=3$, $k=2$, and $\tau(1)=1$. $\tau(2)=2$.

$$A_1 = \begin{pmatrix} a_{11} & a_{12}, & a_{13} \\ a_{21}, & a_{22}, & a_{23} \\ a_{31}, & a_{32}, & a_{33} \end{pmatrix} \qquad A_2 = \begin{pmatrix} b_{11}, & b_{12}, & b_{13} \\ b_{21}, & b_{22}, & b_{23} \\ b_{31}, & b_{32}, & b_{33} \end{pmatrix}$$

$$\Rightarrow \sum_{\#I=2} \det(A^{id}_{I}) = \det \underbrace{\begin{pmatrix} a_{11}, & b_{12} \\ a_{21}, & b_{22} \end{pmatrix}}_{A^{id}_{\{1,2\}}} + \det \underbrace{\begin{pmatrix} a_{11}, & b_{13} \\ a_{13} & b_{33} \end{pmatrix}}_{A^{id}_{\{1,3\}}}$$

$$+ \det \underbrace{\begin{pmatrix} a_{22}, & b_{23} \\ a_{32}, & b_{33} \end{pmatrix}}_{A^{id}_{\{2,3\}}} \quad \cdots \text{①}$$

$$\sum_{\#I=2} \det(A_1)_I = \det \begin{pmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \end{pmatrix} + \det \begin{pmatrix} a_{11}, & a_{13} \\ a_{13}, & a_{33} \end{pmatrix} + \det \begin{pmatrix} a_{22}, & a_{23} \\ a_{32}, & a_{33} \end{pmatrix} \quad \text{②}$$

$\Rightarrow$ Observe that the differences between ① & ② are

$b_{12} \longleftrightarrow a_{12}$ $\quad b_{13} \longleftrightarrow a_{13}$ $\quad b_{23} \longleftrightarrow a_{23}$

$b_{22} \longleftrightarrow a_{22}$ $\quad b_{33} \longleftrightarrow a_{33}$ $\quad b_{33} \longleftrightarrow a_{33}$.

We already know that $\sum_{\#I=2} \det(A_1)_I = \sum_{\#I=2} \det(gA_1g^{-1})_I$

for all A_1 and all g, from note P 31 9. back.