Divide by f both sides, then we obtain

$$\frac{df}{f} \wedge \omega' = \frac{g(z)\, dz_1 \wedge \cdots \wedge dz_n}{f(z)} = \omega \quad (\text{identification image})$$

If we let $\omega = \sigma \otimes \eta$, and $df \otimes \sigma = 1$, then $df \otimes (\sigma \otimes \eta) = df \wedge (\sigma \otimes \eta) = \eta$. Everything fits with each other.

$$\begin{tikzcd}
U_\beta \times \mathbb{C} & {[V]|_{U_\alpha}} \arrow[l, "\varphi_\beta"'] \arrow[r, "\varphi_\alpha"] & U_\alpha \times \mathbb{C} \\
(x, 1) \arrow[rr, bend right, "U_\alpha"'] & \sigma_\alpha \arrow[d] & (x, 1) \arrow[l] \arrow[d] \\
& \sigma_\beta = h^{-1}\sigma_\alpha \arrow[ul] \arrow[r, "U_\alpha"] & (x, h^{-1}),\ h^{-1} = g_{\alpha\beta}
\end{tikzcd}$$

$V \cap U_\alpha = (f_\alpha = 0)$

Consider $\{f_\alpha\}$ which defines a global section.
Let $\{f_\alpha \sigma_\alpha\}$. $\Rightarrow$

$$\begin{aligned} f_\alpha \sigma_\alpha = g_{\alpha\beta} f_\beta \sigma_\alpha &= f_\beta\, g_{\alpha\beta} \sigma_\alpha \\ &= f_\beta\, h\, g_{\alpha\beta} \sigma_\beta \\ &= f_\beta \sigma_\beta . \end{aligned}$$

$\sigma_\beta = h^{-1} \sigma_\alpha$

If we let $U = U_\alpha$, O.K.