We will interpret the relation at the end of the preceding section as a residue theorem for general vector bundles and will then put our conclusions in a more geometric form.

Suppose that M is a compact, complex manifold of dimension n and

$$\mathbb{C}^n \longrightarrow E \longrightarrow M$$

is a rank-n vector bundle having a holomorphic section $s \in H^0(M, \mathcal{O}(E))$ with a set Z of isolated zeros. Using the notations

$$\begin{cases} \Lambda^q \mathcal{E}^* = \mathcal{O}(\Lambda^q E^*) \\ \mathcal{J}_Z = \text{ideal sheaf of } Z \text{ and } \mathcal{O}_Z = \mathcal{O}/\mathcal{J}_Z, \end{cases}$$

the sequence

$$(*) \qquad 0 \longrightarrow \Lambda^n \mathcal{E}^* \xrightarrow{s} \cdots \longrightarrow \Lambda^2 \mathcal{E}^* \xrightarrow{s} \mathcal{E}^* \longrightarrow \mathcal{J}_Z \longrightarrow 0$$

localizes to the Koszul complex, and therefore gives a global projective resolution of both $\mathcal{O}_Z$ and $\mathcal{J}_Z$.

$$\begin{tikzcd}
0 \arrow[r] & (\Lambda^n \mathcal{E}^*)_p \arrow[r] \arrow[d, equal] & \cdots \arrow[r] & (\Lambda^2 \mathcal{E}^*)_p \arrow[r] \arrow[d, equal] & \mathcal{E}^*_p \arrow[r] \arrow[d, equal] & (\mathcal{J}_Z)_p \arrow[r] \arrow[d, equal] & 0 \\
& \mathcal{O}_p \otimes \Lambda^n \mathbb{C}^{n*} \arrow[d, equal] & & \mathcal{O}_p \otimes \Lambda^2 \mathbb{C}^{n*} \arrow[d, equal] & \mathcal{O}_p \otimes \Lambda \mathbb{C}^{n*} \arrow[d, equal] & I & \\
& \mathcal{O}_p \otimes \Lambda^n \mathbb{C}^n & & \mathcal{O}_p \otimes \Lambda^2 \mathbb{C}^n & \mathcal{O}_p \otimes \Lambda \mathbb{C}^n & &
\end{tikzcd}$$

$$\begin{tikzcd}
0 \arrow[r] & \Lambda^n \mathcal{E}^* \arrow[r] \arrow[d, equal] & \cdots \arrow[r] & \Lambda^2 \mathcal{E}^* \arrow[r] \arrow[d, equal] & \mathcal{E}^* \arrow[r] \arrow[d, equal] & \mathcal{O} \arrow[r] & \mathcal{O}_Z \arrow[r] & 0. \\
& \Lambda^n \mathcal{E} & & \Lambda^2 \mathcal{E} & \mathcal{E} & & &
\end{tikzcd}$$

In particular $(*)$ gives an element