i.e., A_p is determined up to conjugation.

⌜ According to the computation above,

$$\sum a_{ij}\, g_{ik}^{-1}\, \omega_k \cdot g_{\ell j} \frac{\partial}{\partial \omega_\ell} + [0] = \sum a'_{ij}\, w_i \cdot \frac{\partial}{\partial w_j} + [0]$$

$$\underset{\substack{\parallel \\ ({}^t g^{-1} A_p\, {}^t g)_{k\ell}}}{\sum_{i,j} g_{ik}^{-1}\, a_{ij}\, g_{\ell j}} \text{ is equal to } a'_{k\ell} \quad \Rightarrow \quad {}^t g^{-1} A_p\, {}^t g = A'_p.$$

⌟

The value $P(A_p)$ of any invariant polynomial P on A is therefore an invariant of v and p, and we may hope that the numbers $P(A_p)$ carry some global information. This is in fact the case: if Θ is any curvature matrix in the holomorphic tangent bundle $T'(M)$ of the compact complex manifold M, P any invariant polynomial of degree $n = \dim M$, v a global holomorphic vector field and A_p as above, we have the

Bott Residue Formula

$$\sum_{v(p)=0} \frac{P(A_p)}{\det(A_p)} = \int_M P\left(\frac{\sqrt{-1}}{2\pi}\Theta\right),$$

i.e., if we write P as a polynomial

$$P = Q(P^1, \cdots P^n)$$