

Therefore we will give a purely algebraic definition of degree, using the Hilbert polynomial of a projective variety. This definition is less geometrically motivated, but it has the advantage of being precise.

"Comment on P64. "if M has dimension m and V dimension k , we can find a linear subspace \mathbb{P}^{n-k} of \mathbb{P}^n meeting V in isolated points.

To show the above, consider the following

$V \subset \mathbb{P}^n$ is an analytic subvariety \Rightarrow If H is a hyperplane in \mathbb{P}^n , then $H \cap V$ is an analytic subvariety of \mathbb{P}^n of dimension $k-1$, where $\dim V = k$, and V is irreducible.

pf). Suppose $\dim H \cap V = k$.

$\Rightarrow \exists$ an open $U \subset V^*$ s.t. $U \subset H$.

Let $H = (a_0 X_0 + \dots + a_n X_n = 0)$. Without loss of generality, $H = (X_0 = 0)$.

On $U_1 = (X_1 \neq 0)$, consider the holomorphic function

$$\begin{array}{ccc} U_1 & \xrightarrow{f_1} & \mathbb{C} \\ [X_0, \dots, X_n] & \longmapsto & \frac{X_0}{X_1} \end{array}$$

① $U_1 \cap U \neq \emptyset$.

$$\begin{array}{ccc} U_1 \cap V^* & \xrightarrow{f_1} & \mathbb{C} \\ [X_0, \dots, X_n] & \longmapsto & \frac{X_0}{X_1} \end{array}$$