

$$L_E(y) = \left\langle \begin{matrix} b_{11}(y) \sigma_1^* + \dots + b_{1n}(y) \sigma_n^* \\ \vdots \\ b_{k1}(y) \sigma_1^* + \dots + b_{kn}(y) \sigma_n^* \end{matrix} \right\rangle,$$

$$\left\langle \begin{pmatrix} b_{11} & \dots & b_{1n} \\ \vdots & & \vdots \\ b_{k1} & \dots & b_{kn} \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{k1} & \dots & a_{kn} \end{pmatrix} \right\rangle.$$

But $C = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} \perp A$, and $C \notin B$.

\Rightarrow Contradiction. \square

Similarly, we have a differential map
 $(**) \quad H^0(M, f_x(E \otimes L^m)) \longrightarrow T_x^* \otimes (E \otimes L^m)_x$

defined as for line bundles; $L_{E \otimes L^m}$ will be smooth at x if this map is surjective.

\mathbb{F} Suppose that $L_{E^*}(v) = 0$ for $v \neq 0$, $v \in T_x'$
 $\Rightarrow \exists \sigma \in H^0(M, f_x(E))$ s.t. $d_{\neq 0}^* \sigma(v) \in E_x$.

$\Rightarrow \sigma = c_1 \sigma_1 + \dots + c_n \sigma_n$, where $V = H^0(M, \mathcal{O}(E))$
 $= \langle \sigma_1, \dots, \sigma_n \rangle$

If $\sigma_i = a_{\alpha i} e_{\alpha}$, $L_E(x) = \left\langle \begin{matrix} a_{11} \sigma_1^* + \dots + a_{1n} \sigma_n^* \\ \vdots \\ a_{k1} \sigma_1^* + \dots + a_{kn} \sigma_n^* \end{matrix} \right\rangle$

Assume that $A = \begin{pmatrix} a_{11}(x) & \dots & a_{1n}(x) \\ \vdots & & \vdots \\ a_{k1} & \dots & a_{kn} \end{pmatrix}$ is non-singular.