By our commutation relations, every term in the first sum is zero; for the second, we have

$$e_k \bar{e}_k \bar{\iota}_k \iota_k = 2 e_k \iota_k - e_k \bar{\iota}_k \bar{e}_k \iota_k \quad (e_k \iota_k + \iota_k e_k = 2)$$
$$\bar{\iota}_k \iota_k e_k \bar{e}_k = 2 \bar{\iota}_k \bar{e}_k - \bar{\iota}_k e_k \iota_k \bar{e}_k$$

and, since $e_k \bar{\iota}_k \bar{e}_k \iota_k = \bar{\iota}_k e_k \iota_k \bar{e}_k$, this yields

$$\begin{aligned}[L, \Lambda] &= \frac{1}{2} \sum_k (e_k \iota_k - \bar{\iota}_k \bar{e}_k) \\ &= \frac{1}{2} \sum_k (2 - \iota_k e_k - \bar{\iota}_k \bar{e}_k) \\ &= n - \frac{1}{2} (\sum (\iota_k e_k + \bar{\iota}_k \bar{e}_k)).\end{aligned}$$

To evaluate this, note that $\iota_k e_k (d z_J \wedge d \bar{z}_K)$ is zero if $k \in J$, and $2 z_J \wedge d \bar{z}_K$ otherwise; $\bar{\iota}_k \bar{e}_k (d z_J \wedge d \bar{z}_K)$ is zero if $k \in K$ and $2 z_J \wedge d \bar{z}_K$ if not. Thus

$$\sum_k (\iota_k e_k + \bar{\iota}_k \bar{e}_k) (d z_J \wedge d \bar{z}_K) = 2 \sum_{k \notin J} d z_J \wedge d \bar{z}_K + 2 \sum_{k \notin K} d z_J \wedge d \bar{z}_K$$

$$= (2 (n - \# J)) + 2 (n - \# K)) (d z_J \wedge d \bar{z}_K),$$

and so on $A_c^{p.q} (\mathbb{C}^n)$.

$$[L . \Lambda] = p+q - n \quad , \text{ where } \begin{array}{l} p = \# J \\ q = \# K \end{array} .$$

Since L and Λ are both algebraic operators, this identity will hold on any Kähler manifold.
「 See P202 of this note 」

Now set $h = \sum_{p=0}^{2n} (n-p) \Pi^p;$