$\Rightarrow$ This implies that $J = \{(p, q, r) \in \mathbb{P}^n \times \mathbb{P}^n \times \mathbb{P}^n \mid p \wedge q \wedge r = 0\}$ is an analytic subvariety of $\mathbb{P}^n \times \mathbb{P}^n \times \mathbb{P}^n$.

$\Rightarrow$ $I = J - \{(p, p, r) \in \mathbb{P}^n \times \mathbb{P}^n \times \mathbb{P}^n\}$ is open since $\{(p, p, r)\}$ is closed. $\quad \bar{I} = J$.

$\Rightarrow$ By the proper mapping theorem, $C(V) = \pi_3(\bar{I})$ is an analytic subvariety in $\mathbb{P}^n$.

Note that since projection on the first factor maps I onto V with $(\dim V + 1)$-dimensional fibers, I has dimension $2 \cdot \dim V + 1$.

「 I think that $C(V)$ is the union of all lines meeting V more than twice, or in the limiting case, tangent to V.

$$\begin{array}{c} I = \{(p, q, r) \,;\, p \neq q \in V,\ p \wedge q \wedge r = 0\} \\ \pi \big\downarrow \\ V \end{array}$$

Fix $p \in V$, we can choose any point $q \in V$, $q \neq p$.
$\Rightarrow$ $r = [p' + t q']$ where $p = [p']$, $q = [q']$, $t \in \mathbb{C}$.
$\dim \{(q', t)\} = \dim V + 1$

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$C(V)$ will thus have dimension at most $2 \cdot \dim V + 1$; generally, this will be exact. In particular, since the projection π_p of a smooth variety into a hyperplane will be imbedding if and only if $p \notin C(V)$, we see that if $n > 2 \cdot \dim V + 1$, then V may be smoothly projected into a hyperplane.

「 π_p is embedding $\Longleftrightarrow$ $p \in C(V)$.