strictly greater than $m/2 = n$. Thus it remains to prove that $\overline{p.\Lambda'} \subset F$.

$\forall q \in \overline{p\Lambda'}$ (i) $q = p$ or $q \in \Lambda'$ $\Rightarrow$ done

(ii) $q \neq p$ and $q \notin \Lambda'$ $\Rightarrow$ $\overline{pq} = \overline{pq'}$, $q' \in \Lambda'$ and $q' \in \overline{pr}$, $r \in \tilde{F} \subset T_p(F) \cap F$ since $T_p(F) \cap F$ is the cone through p over $\tilde{F}$. $\Rightarrow$ $q \in T_p(F) \cap F$

$\Rightarrow$ $\overline{p\Lambda'} \subset F$

$\dim(\Lambda' \cap T_p(F)) = 2n + n - (2n+1) = 3n - 2n - 1 = n-1$. since $T_p(F)$ can not contain Λ' and $T_p(F)$ meets Λ' transversely. $\Rightarrow \dim \overline{\Lambda' \cap T_p(F), p} = \dim \Lambda'' = n-1+1 = n$.

$\Lambda'' \supset \Lambda' \cap T_p(F)$

$\Lambda'' \cap \Lambda' \supset \Lambda' \cap T_p(F) \Rightarrow \dim(\Lambda' \cap \Lambda'') \geq n-1$. but since $\Lambda' \cap \Lambda''$ can not contain p, $\dim(\Lambda' \cap \Lambda'') = n-1$.

By our first argument, in case Λ' and Λ'' intersect, $\dim(\Lambda' \cap \Lambda'') = n-1 \not\equiv n \ (2)$ $\Rightarrow$ Λ' and Λ'' can not belong to the same family. $\Rightarrow$ Λ' and Λ'' belong to opposite families. ⌟

We also see that Λ meets Λ'' only in the point p — if $\Lambda \cap \Lambda''$ contained a line, Λ would necessarily meet the hyperplane $\Lambda' \cap T_p(F) \subset \Lambda''$.

⌜ If $\Lambda \cap \Lambda''$ contained a line l, $l \subset \Lambda''$ and $\Lambda' \cap T_p(F) \subset \Lambda''$ which is a hyperplane in Λ'' since $\dim(\Lambda' \cap T_p(F)) = n-1$ by the argument above. $\Rightarrow$ $l \cap (\Lambda' \cap T_p(F)) \neq \phi$

$\Rightarrow$ Since $l \subset \Lambda$, $\Lambda \cap \Lambda' \neq \phi$. Contradiction. $\Rightarrow$ Λ meets