with V_i irreducible at 0. Thus if p is any point on any analytic hypersurface $V \subset U \subset \mathbb{C}^n$, V can be expressed uniquely in some nbd U' of p as the union of a finite number of analytic hypersurfaces irreducible at p.

⌈ $V = (f(z)=0) \qquad f(p)=0.$
Let $g(z) = f(z-p)$, i.e., translation.
!! $[f] = [f_1][f_2] = [f_1 f_2]$
$\Rightarrow \exists$ open $U \ni 0$, s.t $f = f_1 f_2$. ⌋

2. Let $W \subset U \subset \mathbb{C}^n$ be an analytic variety given in a nbd Δ of $0 \in W$ as the zero locus of two functions $f, g \in \mathcal{O}_n$. If W contains no analytic hypersurface through 0, then f and g are necessarily relatively prime in $\mathcal{O}_n$;

⌈ f and g are not relatively prime $\Rightarrow \exists\ h \in \mathcal{O}_n$ s.t h divides f & g $\Rightarrow (h=0) \subset (f=0) \cap (g=0)$.
$\Rightarrow$ The hypersurface $(h=0)$ is in W. $\Rightarrow$ Contradiction to the assumption. ⌋

If W does not contain the line $\{z'=0\}$, then by taking linear combinations we may assume that neither $\{f(z)=0\}$ or $\{g(z)=0\}$ contains $\{z'=0\}$, and