

respectively. Then, in terms of dual coordinates  $x^*$  on  $\mathbb{P}^{5*}$ , the Gauss maps of  $G$  and  $F$  are given by

$$x^* = Qx \quad \text{and} \quad x^* = Q'x;$$

the dual hypersurfaces  $G^*$  and  $F^* \subset \mathbb{P}^5$  of tangent hyperplanes to  $G$  and  $F$  are thus

$$G^* = ((x^*, Q^{-1}x^*) = 0)$$

and  $F^* = ((x^*, Q'^{-1}x^*) = 0.)$

$$\begin{array}{ccc} \Gamma & & \\ G & \longrightarrow & \mathbb{P}^{5*} \\ \downarrow & & \downarrow \\ x & \longmapsto & T_x G. \end{array}$$

$${}^t x Q x = 0 = x_i g_{ij} x_j$$

$$\Rightarrow \frac{\partial {}^t x Q x}{\partial x_i} = g_{ij} x_j$$

$$\Rightarrow T_x G = (Qx, z) = 0 \Rightarrow Qx = x^* \in \mathbb{P}^{5*}$$

Recall that, given a hyperplane  $H = (a_0 X_0 + \dots + a_n X_n = 0)$ , it corresponds to the element  $[a_0, \dots, a_n] \in \mathbb{P}^{n*}$ .

$$\Rightarrow (Qx, x) = (x^*, Q^{-1}x^*) = 0 \Rightarrow G^* = ((x^*, Q^{-1}x^*) = 0)$$

Similarly, we get  $F^* = ((x^*, Q'^{-1}x^*) = 0)$

□

We see from this that for  $x \in F$ ,  $T_x(F)$  will be tangent to  $G$  if and only if

$$G_F(x) \in G^*,$$