$$
\begin{tikzcd}
U_0 \cap U_i \times \mathbb{C} & J|_{U_0 \cap U_i} \arrow[l, "\varphi_{U_0}"'] \arrow[r, "\varphi_{U_i}"] \arrow[d] & U_i \cap U_0 \times \mathbb{C} \arrow[dl] \\
& U_0 \cap U_i \arrow[from=ul] &
\end{tikzcd}
$$

$$([z],\ 1) \leftarrow e_0 \qquad e_i \mapsto ([z],\ 1)$$

$$([z],\ 1) \longleftrightarrow \frac{z_i}{z_0} e_i \longleftrightarrow \left([z],\ \frac{z_i}{z_0}\right)$$

$$\frac{z_i}{z_0} e_i = e_0$$

Since
$$e_0([z]) = \left(1,\ \frac{z_1}{z_0},\ \cdots \frac{z_i}{z_0},\ \cdots \frac{z_n}{z_0}\right)$$
$$= \frac{z_i}{z_0}\left(\frac{z_0}{z_i},\ \frac{z_1}{z_i}\ \cdots\ 1,\ \cdots\ \frac{z_n}{z_i}\right)$$
$$= \frac{z_i}{z_0}\, e_i([z]) \quad \Rightarrow \quad e_i = \frac{z_0}{z_i} e_0$$

Thus we can see that the transition function g_{i0} is given by
$$g_{i0}([z]) = \frac{z_i}{z_0}.$$

$$\Rightarrow \quad g_{i0} f_0 = \frac{z_i}{z_0} = f_i$$

$$g_{ij} f_j = \frac{z_i}{z_j} \cdot \frac{z_j}{z_0} = \frac{z_i}{z_0} = f_i$$

$\Rightarrow$ $\{(U_i, f_i)\}$ defines a meromorphic section of J with a single pole along $(z_0 = 0) = \mathbb{P}^{n-1}$.