We will show that

$$0 \longrightarrow \mathcal{J}(L) \longrightarrow \mathcal{J}_{P_0}(L) \longrightarrow \mathcal{O}_P(L) \longrightarrow 0$$

$$\mathcal{O}_P(L) = \mathcal{O}(L)/\mathcal{J}_P(L).$$

is exact.

Given $\sigma \in \mathcal{J}_{P_0}(L)(U)$ s.t $\sigma = 0$ in $\mathcal{O}(L)/\mathcal{J}_P(L)\,(U)$,

we will prove that $\sigma \in \mathcal{J}(L)(U')$, $U' \subset U$.

$$\sigma = 0 \text{ in } \left(\mathcal{O}(L)/\mathcal{J}_P(L)\right)_p = \mathcal{O}(L)_p / \mathcal{J}_P(L)_p.$$

① $p \in P$.

$\sigma(p) = 0$.

Assume $p$ is the origin, by way of coordinatization.

$$\sigma : U \longrightarrow \mathbb{C}, \quad U \cong \mathbb{C}^2$$

$$s : U \longrightarrow \mathbb{C}, \quad U \cong \mathbb{C}^2 \qquad s' : U \longrightarrow \mathbb{C}$$

$\Rightarrow$ $s(z_1, z_2) = z_1^{m_1} z_2^{m_2} f_1(z_1, z_2)$. where $f_1(0,0) \neq 0$

$s'(z_1, z_2) = z_1^{n_1} z_2^{n_2} f_2(z_1, z_2)$. " $f_2(0,0) \neq 0$

$\Rightarrow$ Since $s$ and $s'$ are relatively prime,