

Since the Schubert cycle $\sigma_{2,1}(p,h) \subset G$ has intersection number 1 with the hyperplane class σ_1 , it follows that

Every Schubert cycle $\sigma_{2,1}(p,h) \subset G$ is a line in \mathbb{P}^5 .

By P746, $\sigma_{2,1} \cdot \sigma_1 = 1$.

$$\#(\sigma_{2,1} \cap \sigma_1) = \#(\sigma_{2,1} \cap (G \cap H)) = \#(\sigma_{2,1} \cap H) = 1$$

$\Rightarrow \deg \sigma_{2,1} = 1$ in $\mathbb{P}^5 \Rightarrow \sigma_{2,1}$ is a line in \mathbb{P}^5 ,
since $\text{codim of } \sigma_{2,1} = 3$ in $G(2,4)$. $\quad \sqcup$

Similarly, since

$$\sigma_1^2 \cdot \sigma_{1,1} = \sigma_1^2 \cdot \sigma_2 = 1,$$

Every Schubert cycle $\sigma_2(p)$ or $\sigma_{1,1}(h) \subset G$ is a 2-plane in \mathbb{P}^5 .

$$\sigma_1^2 \cdot \sigma_{1,1} = \sigma_1 \cdot (\sigma_1 \cdot \sigma_{1,1}) = \sigma_1 \cdot (\sigma_{2,1}) = 1$$

$$\sigma_1^2 \cdot \sigma_2 = \sigma_1 \cdot (\sigma_1 \cdot \sigma_2) = \sigma_1 \cdot \sigma_{2,1} = 1.$$

$\sigma_1^2 \cdot \sigma_{1,1} = \#((G \cap H) \cap (G \cap H') \cap \sigma_{1,1}) = \#(H \cap H' \cap \sigma_{1,1})$
 $= \#(\mathbb{P}^3 \cap \sigma_{1,1}) = 1 \Rightarrow \deg \sigma_{1,1} = 1 \Rightarrow \sigma_{1,1}$ is a linear subspace of \mathbb{P}^5 by the result on P74. \Rightarrow
 Since $\dim \sigma_{1,1} = 2$, $\sigma_{1,1} = \mathbb{P}^2$. Similarly, $\sigma_2 = \mathbb{P}^2$. $\quad \sqcup$

To prove the converse of the last two statements,