Now, Q has two families $\{L_\lambda\}_{\lambda\in\mathbb{P}^1}$ and $\{L'_\lambda\}_{\lambda\in\mathbb{P}^1}$ of lines on it, with two lines meeting $\Leftrightarrow$ they are of different families.

⌜ See p478 ~ p480. ⌟

Let L be any line of the first family. Then the 2-plane $\overline{x, L}$ spanned by x and L lies in G, and so must be of the form

$$\sigma(p), \quad \text{for some } p \in l_x,$$

or

$$\sigma(h), \quad \text{for some } h \supset l_x.$$

⌜ Since $G \cap T_x(G)$ is the cone through x over Q, and $L \subset Q$, $\overline{x, L} \subset G \cap T_x(G)$. $\Rightarrow$ $\overline{x, L} \subset G$. $\Rightarrow$
By the result above, $\overline{x, L}$ is $\sigma(p)$ or $\sigma(h)$.
I.e., $\overline{x, L} = \tilde{\Phi}(\sigma(p))$ or $\tilde{\Phi}(\sigma(h))$.

$x \in \overline{x, L} = \tilde{\Phi}(\sigma(p)) \Rightarrow \tilde{\Phi}^{-1}(x) = l_x \in \sigma(p) \Rightarrow p \in l_x$.

or $x \in \overline{x, L} = \tilde{\Phi}(\sigma(h)) \Rightarrow \tilde{\Phi}^{-1}(x) = l_x \in \sigma(h) \Rightarrow l_x \subset h$. ⌟

Indeed, since two Schubert cycles $\sigma(p)$, $\sigma(p')$ intersect only in one point, while for $p \in l_x \subset h$ the Schubert cycles $\sigma(p)$ and $\sigma(h)$ intersect in a line, we see that the 2-planes $\{\overline{x, L_\lambda}\}_{\lambda\in\mathbb{P}^1}$ spanned by x and the lines of one ruling must