$\Rightarrow \pi^{k-2}(V) = \{ f_1(z_1, \cdots z_{n-k}, h(z_1 \cdots z_{n-k}), z_{n-k+2}) = f_2(\quad)$
$\cdots = f_\ell(\quad) = 0 \}$.

$$\begin{array}{c} \pi^{k-2}(V) \\ \downarrow \\ \pi^{k-1}(V) = \{ r = 0 \} \\ \downarrow \\ \pi^{k}(V) \subset \mathbb{C}^{n-k} \end{array}$$

$\Rightarrow f_1 \cdots f_\ell$ must have a common divisor since otherwise $\pi^{k-2}(V)$ is contained in the common locus of two relatively prime functions, and by assertion 2, $\pi^k(V)$ would be a proper analytic subvariety of $\mathbb{C}^{n-k}$.

$\Rightarrow$ Let k be the greatest common divisor of $f_1(z_1, \cdots z_{n-k}, h(z_1, \cdots z_{n-k}), z_{n-k+2}) \cdots\cdots f_\ell(\quad)$.

$\Rightarrow$ For generic points in $\pi^k(V)$, we have open nbds of the generic points s.t on the open nbd $\{(z_1, \cdots z_{n-k}, h(z_1, \cdots z_{n-k}), h'(z_1, \cdots z_{n-k})\}$ is open in $\pi^{k-2}(V)$.
The argument is essentially the same as before.
Now consider the following

$$\begin{array}{c} \pi^{k-3}(V) \subset \mathbb{C}^{n-k+3} \\ \downarrow \\ \pi^{k-2}(V) \subset \mathbb{C}^{n-k+2} \\ \downarrow \\ \pi^{k}(V) \subset \mathbb{C}^{n-k} \end{array}$$

On open subset $U \subset \pi^k(V)$, $\{(z_1, \cdots z_{n-k}, h(z_1 \cdots z_{n-k}), h'(z_1, \cdots z_{n-k})\}$ is open in $\pi^{k-2}(V)$.
$\Rightarrow$ Since $\pi^{k-3}(V)$ is an analytic subvariety of $\mathbb{C}^{n-k+3}$,