$V_1 = (f_1' = \cdots = f_e' = 0)$ One of them (f_i's) must not identically zero on V_2; if so, then $V_1 \supset V_2 \Rightarrow$ Contradiction. $\Rightarrow$ We have $f_1, f_2 \in \mathcal{O}_n$. $f_1 \equiv 0$ on V_1 $f_1 \not\equiv 0$ on V_2, $f_2 \not\equiv$ on V_1. $f_2 \equiv 0$ on V_2. ⌟

By the Nullstellensatz, f must divide the product $f_1 \cdot f_2$; since f is irreducible, it follows that f must divide either f_1 or f_2; i.e., either $V_1 \supset V$ or $V_2 \supset V$, a contradiction.

⌜ Since $\mathcal{O}_n$ is a UFD, P 10, f must divide either f_1 or f_2. $f_1 \cdot f_2 = g_1 \cdots g_e$, g_i's irreducible elts. $\Rightarrow$ Some g_i must be equal to f up to unit. $\Rightarrow$ If g_i comes from f_1, then f divides f_1. Similarly we get a conclusion on f_2. ⌟

In addition to the basic picture of an analytic hypersurface (p. 9) we see that

1. Suppose $V \subset U \subset \mathbb{C}^n$ is an analytic hypersurface, given by $V = \{f(z) = 0\}$ in a nbd of $0 \in V$. Since $\mathcal{O}_n$ is a UFD, we can write

$$f = f_1 \cdot f_2 \cdots f_n$$

with f_i irreducible in $\mathcal{O}_n$: if we set $V_i = \{f_i(z) = 0\}$ then we have

$$V = V_1 \cup \cdots \cup V_k$$