

at

$$\Delta \cdot I \subset I',$$

so that multiplication by Δ induces a map $\mathcal{O}_I \xrightarrow{\Delta} \mathcal{O}_{I'}$ going in the non obvious direction. By Cramer's rule

$$\Delta \delta_{\bar{i}\bar{j}} = \sum_k a_{k\bar{j}} A_{ik},$$

where

A_{ik} is \pm the (i, k) th cofactor of $(a_{\bar{i}\bar{j}})$.

$$\mathbb{F} \quad BA = I = AB.$$

$$\Rightarrow b_{\bar{i}\bar{j}}^1 A^1 + b_{\bar{i}\bar{j}}^2 A^2 + \dots + b_{\bar{i}\bar{j}}^{\bar{j}} A^{\bar{j}} + \dots + b_{\bar{i}\bar{j}}^n A^n = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \Big| \begin{matrix} 1 \\ i \end{matrix}$$

$$\left| \begin{array}{c} \overline{\phantom{A^1 \dots A^{\bar{j}}}} \\ \bar{i} \end{array} \begin{array}{c} A^1 \dots A^{\bar{j}-1} \end{array} \begin{array}{c} \bar{j} \\ \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \end{array} \begin{array}{c} A^{\bar{j}+1} \dots A^n \end{array} \right| = \begin{matrix} \text{the } (\bar{i}, \bar{j}) \text{th} \\ \pm \text{cofactor of } (a_{\bar{i}\bar{j}}) \end{matrix}$$

$$= b_{\bar{i}\bar{j}}^{\bar{j}} \Delta \Rightarrow b_{\bar{i}\bar{j}} = \frac{\pm (\bar{i}, \bar{j}) \text{th cofactor of } (a_{\bar{i}\bar{j}})}{\Delta} = \frac{A_{\bar{i}\bar{j}}}{\Delta}$$

$$\Rightarrow \begin{pmatrix} \frac{A_{\bar{i}\bar{j}}}{\Delta} \end{pmatrix} \begin{pmatrix} a_{\bar{i}\bar{j}} \end{pmatrix} = I = \begin{pmatrix} a_{\bar{i}\bar{j}} \end{pmatrix} \begin{pmatrix} \frac{A_{\bar{i}\bar{j}}}{\Delta} \end{pmatrix}$$

$$\Rightarrow \delta_{\bar{i}\bar{j}} = \sum_k \frac{A_{ik}}{\Delta} a_{k\bar{j}} \Rightarrow \Delta \delta_{\bar{i}\bar{j}} = \sum_k a_{k\bar{j}} A_{ik}$$