The image is just the quadric hypersurface $(X_0X_3 = X_1X_2)$ in $\mathbb{P}^3$. ㄷ $X_0 = z_0w_0$, $X_1 = z_0w_1$, $X_2 = z_1w_0$, $X_3 = z_1w_1 \Rightarrow X_0X_3 = X_1X_2$.
Suppose $z_0 \neq 0 \neq w_0$. $\Rightarrow$ Given (X_0, X_1, X_2, X_3), fix z_0.
$\Rightarrow w_0$ is given by X_0/z_0. $w_1 = \frac{X_1}{z_0}$, $\cdot X_2 = z_1 \frac{X_0}{z_0}$
$\Rightarrow z_1 = X_2z_0/X_0$ $\Rightarrow$ $X_3 = (X_2z_0/X_0)w_1 = \frac{X_2z_0}{X_0}\cdot\frac{X_1}{z_0}$. ⌋

Corollary. If M is an algebraic variety, $\tilde{M} \xrightarrow{\pi} M$ the blows-up of M at a point x, then $\tilde{M}$ is algebraic.

pf). We have seen in the course of the proof of the embedding theorem that if $L \longrightarrow M$ is positive and $E = \pi^{-1}(x)$, then $\pi^*L^k - E$ is positive for $k \gg 0$.

ㄷ See P187 ⊛, $\pi^*L^k - E$ is positive line bundle on $\tilde{M}$ for $k \gg 0$. By Kodaira Embedding theorem, $\exists\ k_0$ s.t for $k' > k_0$, the map

$$\iota_{(\pi^*L^k - E)^{k'}} : \tilde{M} \longrightarrow \mathbb{P}^N$$

is well-defined and is an embedding of M.

We have to clarify two following things.
① What is the positive line bundle $L \to M$?
This is easy, since $M \subset \mathbb{P}^\ell$. take $L = [H]|_M$.

② Is $\tilde{M}$ a Kähler? Yes.

$$\tilde{U} = \pi^{-1}(U) = \{(z, l) \in U \times \mathbb{P}^{n-1} \mid z \in l\}$$

$$\tilde{U}_i \longrightarrow \mathbb{C}^n$$

$$(z, l) \longmapsto \left(\frac{z_1}{z_i}, \frac{z_2}{z_i}, \cdots z_i, \frac{z_n}{z_i}\right) = (z(i)_1, z(i)_2, \cdots z(i)_i, z(i)_n)$$
$$= \left(\frac{l_1}{l_i}, \frac{l_2}{l_i}, \cdots z_i, \frac{l_n}{l_i}\right)$$