「 By P191 (Kodaira Embedding Theorem), and P303 & P304 Riemann Conditions II,

$$\Omega \cdot Q_\delta^{-1}\, {}^t\Omega = 0 \implies Z - {}^tZ = 0 \implies \omega \text{ is of type}$$

(1, 1), and

$$-\sqrt{-1}\,\Omega \cdot Q_\delta^{-1} \cdot {}^t\overline{\Omega} = -\sqrt{-1}\,(Z - {}^t\overline{Z}) \overset{Z - {}^tZ = 0}{=} -\sqrt{-1}\,(Z - \overline{Z}) =$$

$$2\,\mathrm{Im}\, Z > 0 \implies \omega \text{ is positive.}$$

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Thus we have

Riemann Conditions III. $M = V/\Lambda$ is an Abelian variety if and only if there exists an integral basis $\lambda_1, \cdots \lambda_{2n}$ for Λ and complex basis $e_1, \cdots e_n$ for V such that

$$\Omega = (\Delta_\delta,\ Z)$$

with Z symmetric and $\mathrm{Im}\, Z$ positive definite.

Note that the matrix Π above likewise takes a relatively simple form in terms of the bases $\{\lambda_1, \cdots \lambda_{2n}\}$ and $\{e_1, \cdots e_n\}$: solving

$$(\Pi,\ \overline{\Pi}) \cdot \begin{pmatrix} \Omega \\ \overline{\Omega} \end{pmatrix} = I_{2n},$$

we see that
$$\Pi = \begin{pmatrix} \Pi_1 \\ \Pi_2 \end{pmatrix}$$