

$p \in N$ ,  $f^{-1}(p)$  is a single point, then the graph  $P_f \subset M \times N$  of  $f$  has intersection number 1 with the fibers  $M \times \{p\}$ , and so defines an inverse rational map.

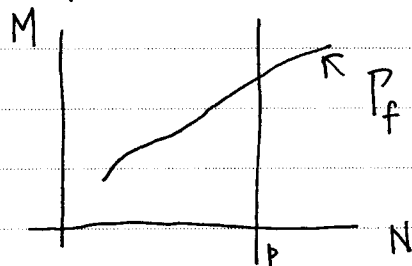
$\Upsilon (\Rightarrow)$  By the above.  $\exists g: N \rightarrow M$  rational s.t

$$\begin{array}{ccccc} M-V & \xrightarrow{f} & N-W & \xrightarrow{g} & M-V & \xrightarrow{f} & N-W \\ & & \searrow & \nearrow & & & \\ & & \text{id} & & \text{id} & & \end{array}$$

$\Rightarrow M-V \xrightarrow{f} N-W$  is biholomorphic, where  $V, W$  are subvarieties of  $M$  &  $N$  respectively.

$\Rightarrow f$  is generically one to one.

$(\Leftarrow) \#(P_f \cap M \times \{p\}) = 1$  if  $f^{-1}(p)$  is a single point.



Define  $g: N \rightarrow M$  by

$$\check{p} \mapsto \check{g}(p), \quad f(\check{g}(p)) = p$$

$\Rightarrow$  By the inverse function theorem, for generic  $p \in N$ ,  $g$  is holomorphic.  $\Rightarrow g$  is biholomorphic since  $g$  has an inverse.  $\Rightarrow g: N-V \rightarrow g(N-V) \subset \mathbb{P}^n$ , since

$N-V \subset \mathbb{P}^n$ . Question: If  $M$  is a complex manifold, and

$\exists$  an open dense subset  $U$  in  $M$ , which is embedded in  $\mathbb{P}^n$ , then  $M$  is embedded into a projective space?