By the lemma on p 73, D_S is equal to the composition of the operator

$$D_E|_{a^0(S)} : \quad a^0(S) \longrightarrow a^1(E)$$ with the projection

$a^1(E) \longrightarrow a^1(S)$; thus the operator

$$A = D_E|_{a^0(S)} - D_S \quad \text{maps} \quad a^0(S) \text{ to } a^1(Q).$$

A is called the second fundamental form of S in E.

$\Rightarrow$ A is of type $(1,0)$ & linear over C^∞ functions. i.e.

⌈

$$\begin{tikzcd} a^0(S) \arrow[r, "D_E|_{a^0(S)}"] \arrow[rr, bend right] & a^1(E) \arrow[r, "\pi_1"] & a^1(S) \oplus a^1(S^\perp) \end{tikzcd}$$

$$a^1(S^\perp) = a^1(Q)$$

$$A = D_E|_{a^0(S)} - D_S$$
$$= \pi_2 \circ D_E|_{a^0(S)}$$

$\cancel{\pi_1(D_E|a^0(S))'' = (\pi_1 \circ D_E|_{a^0(S)})'' = \pi_1 \circ D_E|_{a^0(S)}}$

$\cancel{(D_S)''}$ Obviously, $D_S''\xi = \bar\partial \xi$, $\quad \xi \in a^0(S)$.

$$\left(D_E|_{a^0(S)}\right)''(\xi) = \bar\partial \xi \quad \Rightarrow \quad D_S''\xi = \left(D_E|_{a^0(S)}\right)''$$

$$\Rightarrow \quad A'' = \left(D_E|_{a^0(S)} - D_S\right)'' = 0 \quad \Rightarrow \quad A \text{ is of type } (1,0).$$

$$A(f\sigma) = \left(D_E|_{a^0(S)}(f\sigma)\right) - D_S(f\sigma)$$
$$= df \otimes (\sigma) + f\, D_E|_{a^0(S)}(\sigma) - df \otimes \sigma - f\, D_S\sigma = f\left(D_E|_{a^0(S)} - D_S\right)(\sigma)$$

$\Rightarrow$ A is linear over C^∞-functions. ⌋ i.e

$$A \in a^{1,0}(\mathrm{Hom}(S, Q)).$$