The expression in the Reiss relation has an interpretation encountered in the differential geometry of plane curves – in the calculus sense. Namely, suppose that $(x, y(x))$ is a parametric representation of C near the origin. Differentiating $f(x, y(x)) \equiv 0$ at the origin gives the equations

$$\begin{cases} f_x + f_y y' = 0, \\ f_{xx} + 2 f_{xy} y' + f_{yy} y'^2 + f_y y'' = 0, \end{cases}$$

and eliminating y' yields

$$y'' = -\frac{(f_{xx} f_y^2 - 2 f_{xy} f_x f_y + f_{yy} f_x^2)}{f_y^3}.$$

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$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dx} = \frac{df}{dx} = 0 = f_x + f_y y'$$

$$f_{xx} + f_{xy} y' + f_{xy} y' + f_{yy} (y')^2 + f_y y''$$

$$= f_{xx} + 2 f_{xy} y' + f_{yy} (y')^2 + f_y y'' = 0$$

$$\Rightarrow f_{xx} + 2 f_{xy} \left(-\frac{f_x}{f_y}\right) + f_{yy} \left(\frac{f_x}{f_y}\right)^2 + f_y y'' = 0$$

$$\Rightarrow y'' = -\frac{1}{f_y} \left(f_{xx} - \frac{2 f_x f_{xy}}{f_y} + \frac{f_x^2 f_{yy}}{f_y^2} \right)$$

$$= -\frac{f_{xx} f_y^2 - 2 f_{xy} f_x f_y + f_x^2 f_{yy}}{f_y^3}$$

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On the other hand, it is elementary calculus that

$$y''(0) = \frac{K}{\sin^3 \theta},$$