that $\psi \in \mathcal{H}_0^{p,q}(E)$ is a weak solution of the equation

$\Delta \psi = \varphi$ in the sense that

$$\langle \psi, \Delta\eta \rangle = \langle \varphi, \eta \rangle \quad \text{for all } \eta \in A^{p,q}(E).$$

Then $\psi \in \mathcal{H}_{s+2}^{p,q}(E)$.

Here. $\mathcal{H}_s^{p,q}(E)$ is the completion of $A^{p,q}(E)$ in the Sobolev s-norm, $\| \ \| = \| \ \|_0$. and define the Dirichlet inner product and Dirichlet norm, respectively, by

$$\mathcal{D}(\varphi, \psi) = (\varphi, \psi) + (\bar\partial\varphi, \bar\partial\psi\rangle + \langle \bar\partial^*\varphi, \bar\partial^*\psi\rangle = \langle \varphi, (I+\Delta)\psi\rangle$$

The basic estimate in the theory is provided by

Garding's Inequality. For $\varphi \in A^{p,q}(E)$,

$$\|\varphi\|_1^2 \le c\, \mathcal{D}(\varphi) \qquad (c > 0)$$

where $\|f\|_s^2 = \sum_{k\le s} \int_M \|\nabla^k f\|_0^2 \, dx.$

$\| \ \|_0^2$ is the metric on $\Lambda^p T^{*\prime}M \otimes \Lambda^q T^{*\prime\prime}M \otimes E$ induced by the metrics on T^*M and E.

Let $\psi \in A^{p,q}(E) =$ Set of all E-valued (p,q)-type forms.

$\psi = f\, \bar\varphi_1 \wedge \cdots \wedge \bar\varphi_q \otimes e$, where e is holomorphic section of E.

$$\Rightarrow \bar\partial\psi = \bar\partial(f\,\bar\varphi_1 \wedge \cdots \wedge \bar\varphi_q) \otimes e.$$
$$* \psi = *(f\,\bar\varphi_1 \wedge \cdots \wedge \bar\varphi_q) \otimes e.$$