pencils having a line in common with either L or $L'(L)$. $\Rightarrow$ Since j^*h is a divisor, $j^*h = B_L \cup B_{L'(L)}$.

$\lrcorner$

(To avoid confusion, we will here use the union symbol $\cup$ to denote addition of divisors.) Similarly, for any point $p \in S$, the pullback $j'^*(p^*)$ of the dual hyperplane $p^* \subset \mathbb{P}^{3*}$ of hyperplanes containing p will consist of pencils whose plane contains p — that is, of pencils having a line in common with either of pencils L' and $L(L')$ with focus p. Thus

$$j'^*(p^*) = B_{L'} \cup B_{L(L')}.$$

Now in general, for any two lines L and L' and any element $\lambda \in A$, the divisors

$$B_L \cup B_{L'} \quad \text{and} \quad (B_L + \lambda) \cup (B_{L'} - \lambda)$$

are linearly equivalent: the map

$$A \longrightarrow \hat{A} = \mathrm{Pic}^\circ(A)$$

defined by

$$\lambda \longmapsto [(B_L+\lambda) \cup (B_{L'} - \lambda)] - [B_L \cup B_{L'}]$$

sends the points λ and $\lambda' = L - L' - \lambda$ into the same point — but being a group homomorphism, this implies it is constant.

$\ulcorner$ For Pic°, see P313 & P328. For $\hat{A}$, see P328.