Γ By the definition of locally finite, $\exists$ an open cover $\{U_\alpha\}$ of M. s.t each U_α intersects only finite # of V_i's appearing in D.

For a fixed open set U_α, suppose that $V_1, V_2 \cdots V_k$ are the members of V_i's meeting with U_α.

Since V_1 is closed in M, $\exists$ an open cover $\{W_{\beta,1}\}$ of M s.t $V_1 \cap W_{\beta,1}$ has a local defining function.

$\Rightarrow$ $\{U_\alpha \cap W_{\beta,1}\}$ is an open covering of U_α and each of them has a local defining function.

We can do the above process to $V_2, V_3 \cdots V_K$.

Consider $\{U_\alpha \cap W_{\beta,1} \cap W_{\beta,2} \cap \cdots \cap W_{\beta,K}\}$

$\Rightarrow$ It is an open covering of U_α. and. each open set $O_\alpha = U_\alpha \cap W_{\beta,1} \cap W_{\beta,2} \cap \cdots \cap W_{\beta,K}$ has a local defining function.

Now if we vary α, we get an open cover $\{O_\alpha\}_\alpha$ s.t in each O_α, every V_i appearing in D has a local defining function $g_{i\alpha} \in \mathcal{O}(U_\alpha)$. $\lrcorner$

We can then set $\quad f_\alpha = \prod_i g_{i\alpha}^{a_i} \in \mathcal{M}^*(U_\alpha)$. to obtain a global section of $\mathcal{M}^*/\mathcal{O}^*$.

Γ $\dfrac{f_\alpha}{f_\beta} = \dfrac{\prod_i g_{i\alpha}^{a_i}}{\prod g_{i\beta}^{a_i}}$ $\Rightarrow$ For each i, $g_{i\alpha} \in \mathcal{O}(U_\alpha)$ and $g_{i\beta} \in \mathcal{O}(U_\beta)$,

since a local defining function is unique up multiplication by a non-zero at any point $p \in U_\alpha \cap U_\beta$,
by the second sheaf property, $\left.\dfrac{g_{i\alpha}}{g_{i\beta}}\right|_{U_\alpha \cap U_\beta}$ is non-zero.

$\Rightarrow$ $\left.\dfrac{g_{i\alpha}}{g_{i\beta}}\right|_{U_\alpha \cap U_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta)$ $\Rightarrow$ $\dfrac{f_\alpha}{f_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta)$ $\lrcorner$