

Let $B_\epsilon(p_\alpha, p_\alpha)$ be a ball of radius ϵ around (p_α, p_α) in $M \times M$, and let ρ_α be a bump function with

$$\rho_\alpha \equiv 1 \quad \text{in } B_\epsilon(p_\alpha, p_\alpha)$$

$$\rho_\alpha \equiv 0 \quad \text{in } M \times M - B_{2\epsilon}(p_\alpha, p_\alpha);$$

let k be the current on $M \times M$ given by

$$k = \sum_\alpha \rho_\alpha \cdot k(z_\alpha, \bar{z}_\alpha),$$

where $k(z_\alpha, \bar{z}_\alpha)$ is the Bochner-Martinelli kernel. Then in $B_\epsilon(p_\alpha, p_\alpha)$ we have

$$\bar{\partial} k = \bar{\partial} k(z_\alpha, \bar{z}_\alpha) = T_\Delta^0.$$

Comment on "homotopy formula is equivalent to the distributional equation $\bar{\partial} k = T_\Delta^0$."

According to P384,

$$\psi \in A_c^{n, n-1}(\mathbb{C}^n), \quad \varphi \in A_c^{1,0}(\mathbb{C}^n)$$

$$\int_{\mathbb{C}^n \times \mathbb{C}^{n-\Delta}} d(k(z, w) \wedge \psi(z) \wedge \varphi(w)) = \int_{\mathbb{C}^n \times \mathbb{C}^{n-\Delta}} \bar{\partial}(k \wedge \psi \wedge \varphi)$$

$$(*) = - \int_{\mathbb{C}^n \times \mathbb{C}^{n-\Delta}} k \wedge \bar{\partial}(\psi \wedge \varphi) = - \int_{\mathbb{C}^n \times \mathbb{C}^{n-\Delta}} k \wedge \bar{\partial} \psi \wedge \varphi - (-1)^1 \int_{\mathbb{C}^n \times \mathbb{C}^{n-\Delta}} k \wedge \psi \wedge \bar{\partial} \varphi$$

$$= - (-1)^{(2n-1)(2n-1+1)} \int_{\mathbb{C}^n \times \mathbb{C}^{n-\Delta}} \bar{\partial} \psi(z) \wedge k(z, w) \wedge \varphi(w)$$

$$+ (-1)^{1+1} (-1)^{(2n-1)(2n-1)} \int_{\mathbb{C}^n \times \mathbb{C}^{n-\Delta}} \psi \wedge k(z, w) \wedge \bar{\partial} \varphi$$

$$= (-1)^1 \int_{\mathbb{C}^n \times \mathbb{C}^{n-\Delta}} \bar{\partial} \psi(z) \wedge k(z, w) \wedge \varphi(w) + (-1) \int_{\mathbb{C}^n \times \mathbb{C}^{n-\Delta}} \psi \wedge k(z, w) \wedge \bar{\partial} \varphi$$

$$= (-1)^1 \int_{\mathbb{C}^n \times \mathbb{C}^n} \bar{\partial} \psi(z) \wedge k(z, w) \wedge \varphi(w) + (-1) \int_{\mathbb{C}^n \times \mathbb{C}^n} \psi(z) \wedge k(z, w) \wedge \bar{\partial} \varphi(w)$$

Since $k(z, w)$ is locally integrable.