Local Duality Theorem II. For regular ideals $I = \{f_1, \cdots f_n\}$, the pairing

$$\text{res} : \mathcal{O}/I \otimes \text{Ext}^n_{\mathcal{O}}(\mathcal{O}/I, \Omega^n) \longrightarrow \mathbb{C}$$

defined by the isomorphism (*) in the previous proposition and residue res_f, is nondegenerate, independent of the choice of local coordinates and generators of I, and functorial in the sense that the diagram

$$\begin{array}{ccccc}
\mathcal{O}/I & \otimes & \text{Ext}^n_{\mathcal{O}}(\mathcal{O}/I, \Omega^n) & \xrightarrow{\text{res}} & \mathbb{C} \\
\uparrow \pi & & \downarrow \pi^* & & \| \\
\mathcal{O}/I' & \otimes & \text{Ext}^n_{\mathcal{O}}(\mathcal{O}/I', \Omega^n) & \xrightarrow{\text{res}} & \mathbb{C}
\end{array}$$

is commutative for regular ideals $I' \subset I$.

Proof. The independence of the pairing "res" from choice of local coordinates and generators for I, together with the functoriality, all follow from the commutative diagram (**) and transformation formula.

$$\begin{array}{ccccc}
\mathcal{O}/I & \otimes & \text{Ext}^n_{\mathcal{O}}(\mathcal{O}/I, \Omega^n) & \xrightarrow{\text{res}} & \mathbb{C} \\
\| & \downarrow & \downarrow \cong & & \| \\
\mathcal{O}/I & \otimes & \mathcal{O}/I & \xrightarrow{\text{res}_f} & \mathbb{C}
\end{array}$$

$I = \{f_1, \cdots f_n\} = \{f_1', \cdots f_n'\} = I'$, # of generators must be same by P660.