P 195

488

93 3 8

Let $b_i = n-k+i-a_i$. $\Rightarrow b_{i+1}-b_i = 1-(a_{i+1}-a_i) \geq 1$ since $\{a_i\}$ is nonincreasing sequence.

Let $I = \{b_1, b_2, \cdots b_k\} \subset \{1, 2, 3, \cdots n\}$.

Define $W_{a_1, \cdots a_k} = \{ \Lambda \in U_I : \dim(\Lambda \cap V_{b_1}) = 1, \dim(\Lambda \cap V_{b_2}) = 2 \cdots \dim(\Lambda \cap V_{b_k}) = k \}$.

As we showed above, we know that U_I is open and holomorphic to $\mathbb{C}^{k(n-k)}$.

We want to show that $\overline{U_I} = G(k,n)$ by computing an example.

proof).

Let $n=5$. $k=3$. If $\Lambda \notin U_{\{1,2,3\}}$, then $\Lambda \in U_{\{1,4,5\}}$ or $\cdots$ $U_{\{3,4,5\}}$. Suppose $\Lambda \in U_{\{1,4,5\}}$.

$\Rightarrow \dim(\Lambda \cap V_{\{4,5\}}) = \dim(\Lambda \cap \langle e_4, e_5 \rangle) = 1$ or 2.

As above, Λ can be expressed uniquely as follows.

$$\left\langle \begin{pmatrix} 1, & a_1, & a_2 & 0 & 0 \\ 0 & b_1, & b_2 & 1 & 0 \\ 0 & b_3, & b_4, & 0 & 1 \end{pmatrix} \right\rangle = \Lambda.$$

Since $\dim(\Lambda \cap \langle e_4, e_5 \rangle) = 1$ or 2. $b_1 b_4 - b_2 b_3 = 0$ is the necessary and sufficient condition.

$\Rightarrow \dim\{\Lambda \in U_{\{1,4,5\}} \mid \dim(\Lambda \cap \langle e_4, e_5 \rangle) = 1 \text{ or } 2\} < 6$.

Similarly, $\dim\{\Lambda \in U_{\{2,4,5\}} \mid \dim(\Lambda \cap \langle e_4, e_5 \rangle) = 1 \text{ or } 2\} < 6$.

$\Rightarrow \dim\{\Lambda \notin U_{\{1,2,3\}}\} < 6$.

$\Rightarrow \overline{U_{\{1,2,3\}}} = G(3,5)$.

Proof of $\overline{W_{a_1 \cdots a_k}} = \{\Lambda \in G(k,n) : \dim(\Lambda \cap V_{b_i}) \geq i\}$.

Suppose $\Lambda_0 \in \overline{W} = \overline{W_{a_1 \cdots a_k}}$.