$\Rightarrow$ Set of all meromorphic 1-forms on S with a simple pole at each point P_λ. $= K \longrightarrow \oplus\, \mathbb{C}_{P_\lambda}$

$$\frac{\sigma}{s_0} \longmapsto \left(\mathrm{Res}_{P_\lambda} \frac{\sigma}{s_0}\right)$$

Still there is some difficulty.

We should check the commutativity.

$$\begin{tikzcd} K \arrow[r] \arrow[d, leftrightarrow] & \mathbb{C}_{P_\lambda} \arrow[d, leftrightarrow] \\ H^0(S, \Omega^1(\Sigma P_\lambda)) \arrow[r] & \mathbb{C}_{P_\lambda} \end{tikzcd}$$

We should define this

Consider the exact sheaf sequence on P 147

$$0 \longrightarrow \Omega^1 \longrightarrow \Omega^1(\Sigma P_\lambda) \xrightarrow{P.R} \mathcal{O}_{\Sigma P_\lambda} \longrightarrow 0$$

$$\mathcal{O}_{\Sigma P_\lambda} = \oplus\, \mathbb{C}_{P_\lambda}$$

$$\Rightarrow \quad \begin{tikzcd} H^0(S, \Omega^1(\Sigma P_\lambda)) \arrow[d, leftrightarrow, "1\text{-}1"'] \arrow[r] & \oplus\,\mathbb{C}_{P_\lambda} \\ K \arrow[ur, "\text{residue map}"'] & \end{tikzcd}$$

$$K \xrightarrow{\phi} \oplus\, \mathbb{C}_{P_\lambda}$$

$$\frac{\sigma}{s_0} \longmapsto \left(\mathrm{Res}_{P_\lambda} \frac{\sigma}{s_0}\right).$$

$\Rightarrow \quad \phi(K) \subset \{ (a_\lambda) \in \oplus\, \mathbb{C}_{P_\lambda} \mid \Sigma a_\lambda = 0 \} = H$

But since $\operatorname{cod} \phi(K) \le 1$, and $\operatorname{cod} H = 1$,

$H = \phi(K)$. $\Rightarrow$ Given any $(a_\lambda) \in \oplus\, \mathbb{C}_{P_\lambda}$ s.t $\Sigma a_\lambda = 0$, $\exists$ a meromorphic 1-form on S whose residue is a_λ at each point P_λ.

Now choose cycles $\delta_1, \dots, \delta_{2g}$ representing a canonical basis.