

phic to  $X_0^2$ , if  $F \in W_2$ , then  $F = l^2$ ,  $l$  line.  
 $\Rightarrow f^{-1}(W_2) = \Delta \cong \mathbb{P}^{2*}$   $\square$

Since the series  $|W_1 + W_2|$  cuts out the complete series  $|O_{\mathbb{P}^2}(2H)|$  on  $\Delta$ ,  $W_2$  is the Veronese surface  $v_{2H}(\mathbb{P}^2)$  in  $W \cong \mathbb{P}^5$ .

$\square$  To understand  $|W_1 + W_2|$ , we had better make clear on  $W_1, W_2$ .

Claim:  $W_1$  &  $W_2$  are Poincare dual of  $l \times \mathbb{P}^2$  &  $\mathbb{P}^2 \times l$  respectively,  $l$  a line in  $\mathbb{P}^2$ .

To show this, let  $l \times \mathbb{P}^2$  be the Poincare dual of  $l \times \mathbb{P}^2$  in  $H_6(\mathbb{P}^2 \times \mathbb{P}^2)$ .

$$\Rightarrow \widetilde{l \times \mathbb{P}^2} = a W_1 + b W_2 \in H^2(\mathbb{P}^2 \times \mathbb{P}^2)$$

$$\Rightarrow \widetilde{l \times \mathbb{P}^2} (l \times *) = \#((l \times \mathbb{P}^2) \cdot (l \times *)) = 1$$

$$= a W_1 (l \times *)$$

$\nwarrow$  Poincare duality theorem.

$$+ b W_2 (l \times *) = a \pi_1^* \omega (l \times *) + b \pi_2^* \omega (l \times *)$$

$$= a \omega (\pi_{1*}(l \times *)) + b \omega (\pi_{2*}(l \times *))$$

$$= a \omega(l) + b \cdot 0 = a \Rightarrow a = 1$$

$$\widetilde{l \times \mathbb{P}^2} (* \times l) = b = \#((l \times \mathbb{P}^2) \cdot (* \times l)) = 0$$

$$\Rightarrow \widetilde{l \times \mathbb{P}^2} = W_1$$

Similarly, we obtain  $\widetilde{\mathbb{P}^2 \times l} = W_2$

$\Rightarrow$  Thus we may understand  $|W_1 + W_2|$  as  $|l \times \mathbb{P}^2 + \mathbb{P}^2 \times l|$ .  $\square$

$$\dim H^0(\mathbb{P}^2, O(2H)) = 4C_2 = 6$$