holomorphic

$\Rightarrow \quad \sigma_\alpha = \sum \phi_{\alpha.i} \otimes h_{\alpha.i} \in \mathbb{C}^n \otimes \mathbb{C}$. through trivializations.

$\mathrm{ord}_{V_j}(\phi_{\alpha.i}) + \mathrm{ord}_{V_j}(h_{\alpha.i}) \geq 0$.

Let $S_{0\alpha}$ be the meromorphic function on U_α through the same trivialization as above.

$$\sigma_\alpha = \sum \phi_{\alpha.i} \otimes h_{\alpha.i} = \sum \phi_{\alpha.i} \cdot \frac{h_{\alpha.i}}{S_{0\alpha}} \otimes S_{0,\alpha}$$

$$= \left(\sum \phi_{\alpha,i} \frac{h_{\alpha.i}}{S_{0,\alpha}} \right) \otimes S_{0,\alpha}$$ since $S_{0,\alpha}$ is holomorphic.

$\Rightarrow \quad \sum \phi_{\alpha.i} \dfrac{h_{\alpha.i}}{S_{0.\alpha}}$ is well-defined global section on E since. $g_{\alpha\beta}\left(\sum \phi_{\beta.i} \dfrac{h_{\beta.i}}{S_{0.}} \right)$

$$= \sum g_{\alpha\beta} \phi_{\beta.i} \frac{h_{\beta.i}}{S_{0.\beta}} = \sum \phi_{\alpha.i} \frac{h_{\beta.i}}{S_{0.\beta}}$$

$\left(\because \quad \dfrac{h_{\beta.i}}{S_{0,\beta}} = \dfrac{h_{\alpha.i}}{S_{0,\alpha}} \right.$

Do this over again, because it is doubtful.

Given $\sigma \in \mathcal{O}(E \otimes [D])$, on U_α, $\sigma_\alpha : U_\alpha \to \mathbb{C}^n \otimes \mathbb{C} = \mathbb{C}^n$. is a holomorphic function. s.t

$g'_{\alpha\beta}$ transition for E

$g_{\alpha\beta}$ " [D]

$$g'_{\alpha\beta} \cdot g_{\alpha\beta} = \varphi_{\alpha\beta} \qquad \varphi_{\alpha\beta}\, \sigma_\beta = \sigma_\alpha.$$

$\Rightarrow$ For S_0, we have $(S_0)_\alpha$ on U_α s.t

$$g_{\alpha\beta} (S_0)_\beta = (S_0)_\alpha.$$

Consider $\dfrac{\sigma_\alpha}{(S_0)_\alpha}$ which is meromorphic on U_α.

with poles of order $\leq a_i$. on V_i,