$$\Rightarrow \sum_{[\alpha]\le s} \| D^\alpha \varphi \|_0^2 \le \| \varphi \|_s^2 \le C_s \sum_{[\alpha]\le s} \| D^\alpha \varphi \|_0^2 .$$

$\Rightarrow$ $\| \ \|_s^2$ is equivalent to $\sum_{[\alpha]\le s} \| D^\alpha \_ \|_0^2$. ⌟⌟

Indeed, H_s is the completion of $C^\infty(T)$ in this norm.

F $H_s \subset \mathcal{F}$. $u \in H_s$

$$\Rightarrow u = \sum_{\xi \in \mathbb{Z}^n} u_\xi e^{i\langle \xi, x\rangle} \quad \text{s.t} \quad \|u\|_s^2 < \infty .$$

Let $u_R = \sum_{\|\xi\|<R} u_\xi e^{i\langle \xi, x\rangle} \in C^\infty(T)$.

Question? $u_R \longrightarrow u$ in the sense of $\| \ \|_s$?
Obviously, $\|u\|_s^2 = \sum_\xi (1+\|\xi\|^2)^s |u_\xi|^2 < \infty$.

$\xi \longmapsto (1+\|\xi\|^2)^s |u_\xi|^2$ is a sequence which has the absolutely convergent sum. $\Rightarrow$ Any arrangement of this series has the same limit value. ⌟⌟

There is a partial converse to this, the important

Sobolev Lemma. $H_{s+[n/2]+1} \subset C^s(T)$: that is, every $u \in H_{s+[n/2]+1}$ is the Fourier series of a function $\varphi \in C^s(T)$, and this series converges uniformly to φ.

pf) First, consider the case $s=0$: let $u = \sum_\xi u_\xi e^{i\langle \xi, x\rangle}$ with $\|u\|_{[n/2]+1}^2 = \sum_\xi (1+\|\xi\|^2)^{[n/2]+1} |u_\xi|^2 < \infty$.

The partial sums $S_R = \sum_{\|\xi\|<R} u_\xi e^{i\langle \xi, x\rangle}$, are continuous, and for $R \le R'$,