~~of~~ on M is exact; in other words, the sequences

$$0 \longrightarrow \mathbb{R} \longrightarrow \mathcal{A}^0 \xrightarrow{d} \mathcal{Z}^1 \longrightarrow 0$$

$$0 \longrightarrow \mathcal{Z}^1 \longrightarrow \mathcal{A}^1 \longrightarrow \mathcal{Z}^2 \longrightarrow 0$$

$$\vdots$$

$$0 \longrightarrow \mathcal{Z}^p \longrightarrow \mathcal{A}^p \longrightarrow \mathcal{Z}^{p+1} \longrightarrow 0$$

are all exact. We have seen that

$$H^q(M, \mathcal{A}^p) = 0 \quad \text{for } q > 0 \text{. and all } p. \text{ (see p 42).}$$

By the exact cohomology sequences associated to the short exact sheaf sequences, above,

$$\check{H}^p(M, \mathbb{R}) \cong H^{p-1}(M, \mathcal{Z}^1) \quad (\quad H^{p-1}(M, \mathbb{R}) \to H^{p-1}(M, \mathcal{A}^0) \to$$

$$\cong H^{p-2}(M, \mathcal{Z}^2) \cong H^{p-3}(M, \mathcal{Z}^3) \quad \cdot H^{p-1}(M, \mathcal{Z}^1) \longrightarrow H^p(M, \mathbb{R}) \to H^p(M, \mathcal{A}^0)$$

$$\cong \cdots \cong H^1(M, \mathcal{Z}^{p-1})$$

$$\Rightarrow \quad 0 \to H^0(M, \mathcal{Z}^{p-1}) \longrightarrow H^0(M, \mathcal{A}^{p-1}) \xrightarrow{d^*} H^0(M, \mathcal{Z}^p) \xrightarrow{\delta^*} H^1(M, \mathcal{Z}^{p-1}) \to 0$$

$$\Rightarrow \quad H^1(M, \mathcal{Z}^{p-1}) \cong \frac{H^0(M, \mathcal{Z}^p)}{d^*(H^0(M, \mathcal{A}^{p-1}))} = \frac{Z^p(M)}{d^*(\mathcal{A}^{p-1}(M))}$$

$$= \frac{Z^p(M)}{d\, A^{p-1}(M)} = H^p_{DR}(M).$$

Note that the de Rham isomorphism is functorial: if $f: M \longrightarrow N$ is a differentiable map of C^∞ manifolds, φ closed p-form on N representing $[\varphi] \in H^p_{sing}(N, \mathbb{R})$ under the de Rham map, and

$\sigma = \sum a_i f_i$ a piecewise smooth p-cycle on M,