

is a restriction and has a kernel, i.e., $\exists Q \in H^0(\mathbb{P}^3, \mathcal{O}(2H))$ s.t. $L^*Q = 0 \in H^0(B, \mathcal{O}(2D))$, which implies that $Q \supset B$. \Rightarrow In other expression, $Q \supset E_D$, since E_D is the embedded image of B .

\Rightarrow

Since E_D is nondegenerate of degree 5, moreover, we see that the quadric Q is uniquely determined by E_D .

$\mathbb{F}_2(E_D) = 2 \neq \frac{(5-1)(5-2)}{2} \Rightarrow E_D$ is nondegenerate.

Suppose $E_D \subset Q_1$ and $E_D \subset Q_2$. \Rightarrow Since E_D is nondegenerate, Q_i can not be singular^{of rank ≤ 2} , otherwise, since E_D is smooth, and E_D lies in a hyperplane in \mathbb{P}^3 ($\because Q_i = H_1 \cup H_2$ or H in case Q_i is singular^{of rank ≤ 2}).

The only remaining possibility is E_D is the intersection of two smooth quadrics, \Rightarrow By the adjunction

formula, $\deg(Q_1 \cap Q_2) = d$, and $d(d-3) = -2$.

$\Rightarrow d^2 - 3d + 2 = 0$ $d = 1$ or 2 . Here we assumed that Q_1 intersects with Q_2 transversely. See p550. *

If Q_1 meets with Q_2 nontransversely, ~~then~~

$E_D \subset Q_1 \cap Q_2$ is a curve. $Q_1 \cap Q_2$ is homologous to $2H \cap 2H \sim 4H^2 \sim 4\ell$, while $E_D \sim 5H$. \Rightarrow Contradiction.

Thus Q is uniquely determined by E_D .

\rightarrow since $Q_1 \cap Q_2$ can not contain E_D .

\Rightarrow

Suppose first that Q is a singular quadric, i.e., Q is