The entries in the Maurer-Cartan matrix $dg\cdot g^{-1}$ are right-invariant holomorphic forms on G and hence descend to M.

$$g=\begin{pmatrix}1, & a, & b\\ 0, & 1, & c\\ 0, & 0, & 1\end{pmatrix}\qquad dg=\begin{pmatrix}0, & da, & db\\ 0, & 0, & dc\\ 0, & 0, & 0\end{pmatrix}$$

$$dg\cdot g^{-1}=\begin{pmatrix}0, & da, & db\\ 0, & 0, & dc\\ 0, & 0, & 0\end{pmatrix}\begin{pmatrix}1, & -a, & -b\\ 0 & 1, & -c\\ 0, & 0, & 1\end{pmatrix}=\begin{pmatrix}0, & da, & -c\,da+db\\ 0, & 0, & dc\\ 0, & 0, & 0\end{pmatrix}$$

$$A=\begin{pmatrix}1 & x, & y\\ 0 & 1 & z\\ 0, & 0, & 1\end{pmatrix}\qquad x, y, z \text{ constants in } \mathbb{C}.$$

$$R_A^*(dg\cdot g^{-1}) = d(gA)\cdot(gA)^{-1} = (dg)A\,A^{-1}g^{-1}$$
$$= dg\cdot g^{-1}, \quad \text{since } dA=0.$$

$$gA=\begin{pmatrix}1, & x+a, & y+az+b\\ 0 & 1 & z+c\\ 0 & 0 & 1\end{pmatrix}$$

$d(x+c) = da$, since $dx=0$

$d(z+c) = dc$ • $dz=0$

$-(c+z)\,d(x+a)+d(y+az+b) = -c\,da - z\,\cancel{dx} - \cancel{z\,da}$

$+ \cancel{dy} + a\,\cancel{dz} + \cancel{z\,da} + db = -c\,da+db.$

Point: We don't need $\omega(\tilde{X})=0$ for any vertical vector $\tilde{X}$, for $G \longrightarrow G/P$ is a discrete P a principal bundle, and we have a unique lift at a given point.