⌈ Since $\sigma_F([X])(X) = F(X)$.

$$G(X) = \pi^*\left(\frac{\sigma}{\sigma_F}\right)(X)\; F(X)$$

$$= \frac{\sigma([X])(X)}{\sigma_F([X])(X)}\; F(X) = \sigma([X])(X).$$

$$\Rightarrow \sigma = \sigma_G.$$

⌋

We note in passing that there is a useful formula for the dimension $h^0(\mathbb{P}^n, \mathcal{O}(H^d))$ of the space of global sections of H^d, that is, the number of monomials $X_0^{i_0} \cdots X_n^{i_n}$ of degree d in $(n+1)$ variables. We associate to any sequence $i_0, \cdots i_n$ of integers with $\Sigma i_k = d$ the set of n integers

$$\{i_0+1,\ i_0+i_1+2,\ \cdots\cdots\ i_0+\cdots+i_{n-1}+n\} \subset \{1, \cdots, d+n\}.$$

This subset of $\{1, 2, \cdots d+n\}$ determines the sequence i_k, and conversely any subset of n distinct numbers between 1 and $d+n$ corresponds to such a sequence.

⌈

$$\begin{array}{ccc} (i_0 \cdots i_n) & \xmapsto{\ \phi\ } & \{i_0+1,\ i_0+i_2+2,\ \cdots\ i_0+\cdots+i_{n-1}+n\} \\ & & \cap \\ & & \{1, 2, \cdots d+n\} \\ & & \cup \\ (j_1-1,\ j_2-2-(j_1-1),\ \cdots) & \xleftarrow{\ \psi\ } & \{j_1 \cdots\cdots j_n\} \end{array}$$

$$j_n = i_0 + \cdots + i_{n-1} + n.$$