Note that Pieri's formula together with the formula (**) give an algorithm for evaluating an arbitrary intersection of Schubert cycles.

The Schubert calculus will appear frequently in the remainder of the book, in a variety of contexts; for the time being we give some applications of our formulas to elementary problems in enumerative geometry. Perhaps the simplest nontrivial such problem is the question: given four lines L_1, L_2, L_3, L_4 in $\mathbb{P}^3$ in general position, how many lines meet all four? The answer is easily obtained: since the set of lines meeting L_i is just the Schubert cycle $\sigma_1(L_i)$, the answer is just the fourfold self-intersection number of σ_1 in $G(2,4)$; this is

$$\sigma_1^4 = \sigma_1^2 \cdot (\sigma_{1,1} + \sigma_2) = \sigma_1 \cdot (2\,\sigma_{2,1}) = 2.$$

$$\sigma_1(L_i) = \{ \Lambda \in G(2,4) : \dim(\Lambda \cap \underbrace{V_{4-2+1-1}}_{=\text{2-plane, i.e. line } L_i}) \geq 1 \}$$
$$= \{ l \in G(2,4) \mid l \cap L_i \neq \phi \}$$

$$\sigma_1^2 = \sigma_1 \cdot \sigma_1 = \sum_{\substack{\sum c_i = 1+1=2 \\ b_i \leq c_i \leq b_{i-1}}} \sigma_c \qquad \text{by Pieri's formula.}$$

$$\Rightarrow \begin{array}{l} 1 \leq c_1 \leq b_0 = 4-2 = 2 \\ 0 \leq c_2 \leq 1 \end{array} \quad \Rightarrow \quad c = 1,1 \text{ or } 2.$$

$$\Rightarrow \sigma_1^2 = \sigma_{1,1} + \sigma_2$$

$$\sigma_1 \cdot \sigma_{1,1} = \sum_{\substack{\sum c_i = 3 \\ b_i \leq c_i \leq b_{i-1}}} \sigma_c \qquad c = 2,1,\ 1,1,1 \text{ or } 3.$$

$$1 \leq c_1 \leq 2, \quad 0 \leq c_2 \leq 1, \qquad \sigma_1 \cdot \sigma_{1,1} = \sigma_{2,1}$$