is easy to compute: we let $|F| \cong \mathbb{P}^{(n+2)(n+3)/2-1}$ denote the linear system of all quadrics in $\mathbb{P}^{n+1}$ and consider the incidence correspondence

$$I \subset |F| \times G(k+1, n+2)$$

given by

$$I = \{(F, \Lambda) : \Lambda \subset F\}.$$

Pf $|F|$ = Set of effective divisors linearly equivelet to F
$\Rightarrow |F| = |2H|$ since $\deg F = \deg(2H)$ and, by p145, every divisor on $\mathbb{P}^{n+1}$ is linearly equivalent to a multiple of the hyperplane divisor $H \cong \mathbb{P}^n$.
$1 + \dim |F| = \dim H^0(\mathbb{P}^{n+1}, \mathcal{O}(F)) = \dim H^0(\mathbb{P}^{n+1}, \mathcal{O}(2H))$, by p166,
$= {}_{n+3}C_2 = (n+3)(n+2)/2$
$\Rightarrow |F| = \mathbb{P}^{(n+3)(n+2)/2-1}$ $\square$

The linear system $|F|$ cuts out on any k-plane Λ the complete $(k+1)(k+2)/2 - 1$ dimensional linear series of quadrics in Λ, so the fibers of the projection map $\pi_2 : I \to G(k+1, n+2)$ have dimension

$$\frac{(n+2)(n+3)}{2} - \frac{(k+1)(k+2)}{2} - 1,$$

and I has dimension

$$(k+1)(n-k+1) + \frac{(n+2)(n+3)}{2} - \frac{(k+1)(k+2)}{2} - 1;$$