$$= \int_{V_1} \phi_1 \circ \psi_1^{-1} \cdot \psi_1^{-1*}(\pi'^*(\bar\partial \varphi) \wedge \pi'^*(\beta))$$

$$+ \int_{V_2} \phi_2 \circ \psi_2^{-1} \cdot \psi_2^{-1*}(\pi'^*(\bar\partial \varphi) \wedge \pi'^*(\beta))$$

$$\psi_1^{-1*}(\pi'^*(\bar\partial\varphi))(z_1, w) = ? \qquad \psi_1^{-1*}(\pi'^*(\beta)) = ?$$

$$\begin{array}{ccccc} \tilde{\mathbb{C}}^2 & \longrightarrow & \mathbb{C}^2 & \longrightarrow & \mathbb{P}^1 \text{ on } U_0 \\ \ni ((z_1,z_2),\ell) & \longrightarrow & (z_1,z_2) & \longrightarrow & \ell \\ \psi_1 \swarrow & & & & \downarrow \\ \mathbb{C}^2 \ni (z_1, w) & & & \dfrac{z_2}{z_1} = w & \in \mathbb{C} \end{array}$$

$$\psi_1^{-1*}(\pi'^*(C_2\, \partial \log (|z_1|^2 + |z_2|^2)) \wedge \pi'^*(\partial\bar\partial \log(|z_1|^2 + |z_2|^2)))$$

$$\psi_1^{-1*}\pi'^*(C_2\, \partial \log (|z_1|^2 + |z_2|^2))$$

$$= C_2\, \partial \log (|z_1|^2 + |z_1 w|^2) = C_2 \frac{1}{|z_1|^2 + |z_1 w|^2} \Big(\bar z_1\, dz_1 + \bar z_1 |w|^2 dz_1$$

$$+ |z_1|^2 \bar w\, dw \Big)$$

$$= C_2 \frac{1}{|z_1|^2(1+|w|^2)} \left((1+|w|^2)\, \bar z_1\, dz_1 + |z_1|^2 \bar w\, dw \right)$$

$$= C_2 \frac{dz_1}{z_1} + C_2 \frac{\bar w}{1+|w|^2} dw . \quad \cdots\cdots \quad ①$$

$$\psi_1^{-1*}\pi'^*(\partial\bar\partial \log(|z_1|^2 + |z_2|^2)) = \partial\bar\partial \log(|z_1|^2 + |z_1 w|^2)$$

$$= \partial \left(\frac{1}{|z_1|^2 + |z_1 w|^2} \left(z_1\, d\bar z_1 + z_1 |w|^2 d\bar z_1 + |z_1|^2 w\, d\bar w \right) \right)$$

$$= - \frac{1}{(|z_1|^2 + |z_1 w|^2)^2} \left(\bar z_1 (1+|w|^2)\, dz_1 + |z_1|^2 \bar w\, dw \right) \wedge \Big(z_1 (1+|w|^2) d\bar z_1 +$$

$$|z_1|^2 w\, d\bar w \Big) + \frac{1}{|z_1|^2(1+|w|^2)} \Big(dz_1 \wedge d\bar z_1 + |w|^2 dz_1 \wedge d\bar z_1 + z_1\, dw \wedge d\bar z_1 \cdot \bar w + \bar z_1 w\, dz_1 \wedge d\bar w$$

$$+ |z_1|^2 dw \wedge d\bar w \Big)$$