where $\sigma = a_1(t)\sigma_1 + \cdots + a_{20}(t)\sigma_{20}$. We may assume that
$\det\begin{pmatrix} \frac{\partial\sigma}{\partial y}, & \frac{\partial\sigma}{\partial z} \\ b_2, & b_3 \end{pmatrix} \neq 0$ at a point $p$.

$\Rightarrow$ Consider the following map

$$(t, x, y, z) \longmapsto (t, x, \sigma(t,x,y,z), b_0 + b_1 x + b_2 y + b_3 z)$$

$\Rightarrow$ It has nonzero Jacobian at $p$.

$\Rightarrow$ By the inverse function theorem, locally,

$$G(t, x, \sigma(t,x,y,z), b_0 + b_1 x + b_2 y + b_3 z) = (t, x, y, z).$$

$\Rightarrow$ On $\{\sigma(t,x,y,z) = b_0 + b_1 x + b_2 y + b_3 z = 0\}$,
$z = g(t,x)$, $y = h(t,x)$, for some smooth functions $g$ & $h$. $\Rightarrow$ $X' \cap (W \times H)$ is a smooth submanifold of $X'$. Here we assumed $x, y, z$ are real for simplicity.

Take $v'$ a vector field on $Y'$ lifting $-\partial/\partial t$ and choose $v$ to extend $v'$; the diffeomorphism $\varphi = \varphi_1 : S \to S_0$ will then carry the hyperplane section $H \cap S$ to $H \cap S_0$. This argument shows in general that any two smooth hypersurfaces of degree $d$ in $\mathbb{P}^n$ are diffeomorphic via a map carrying a hyperplane section of one to a hyperplane section of the other.

$\ulcorner$ Locally, $\exists$ open set $U \subset X'$ s.t

$$\begin{array}{ccc} U \cap Y' & \xrightarrow[\cong]{\varphi_1} & \mathbb{R}^3 \\ \downarrow & & \downarrow \\ U & \xrightarrow[\cong]{\varphi} & \mathbb{R}^5 \end{array} \quad \text{and} \quad \begin{array}{ccc} U & \equiv & \mathbb{R}^5 \\ {\scriptstyle \sigma^{-1}\circ\pi}\downarrow & & \downarrow {\scriptstyle \text{natural projection}} \\ I & \longrightarrow & \mathbb{R} \end{array}$$