Then by P175 (tangent cone),

$$T_{p_i}D = \left\{ \left(\frac{\partial^2 f}{\partial x^2} + \lambda\frac{\partial^2 g}{\partial x^2}\right)x^2 + 2\left(\frac{\partial^2 f}{\partial x\partial y} + \lambda\frac{\partial^2 g}{\partial x\partial y}\right)xy + \left(\frac{\partial^2 f}{\partial y^2} + \lambda\frac{\partial^2 g}{\partial y^2}\right)y^2 = 0 \right\}.$$

$\Rightarrow$ $T_{p_i}D = \{\alpha\, xy + (x-y)(x-\gamma y) = 0\}$

where γ, α are on P718.

$\Rightarrow$ $(x-y)(x-\gamma y) + \alpha xy = (x-\mu y)(x-\lambda y) = 0$

$\lambda\mu = \gamma$, $\lambda + \mu = 1 + \gamma - \alpha$.

$\Rightarrow$ If we fix μ, then clearly λ is determined as $\frac{\gamma}{\mu}$. $\Rightarrow$ Fixing one tangent line to a curve $D \in |\mathcal{J}_{P_0}(4)|$ at p_i determines the other.

$$\tilde{f} : \widetilde{\mathbb{P}}^2 \longrightarrow \mathbb{P}^3$$
$$x \longmapsto [(\tilde{\sigma}_1(x), \tilde{\sigma}_2(x), \tilde{\sigma}_3(x), \tilde{\tau}(x))]$$

$\tilde{\sigma}_1 = \pi^*\sigma / E_i$ $\cdots$ $\tilde{\tau} = \pi^*\sigma / E_i$.

If $x \in E_1$, $\pi(x) = P_1$, $\pi : \widetilde{\mathbb{P}}^2 \longrightarrow \mathbb{P}^2$.

Suppose $a_1\tilde{\sigma}_1(x) + a_2\tilde{\sigma}_2(x) + a_3\tilde{\sigma}_3(x) + a_4\tilde{\tau}(x) = 0$, for $a_1, a_2, a_3, a_4 \in \mathbb{C}$ not all zeros.

$\Rightarrow$ x determines one of tangent lines of $a_1\sigma_1 + a_2\sigma_2 + a_3\sigma_3 + a_4\tau$ at P_1.

$\Rightarrow$ As we saw above, the other tangent line of $a_1\sigma_1 + a_2\sigma_2 + a_3\sigma_3 + a_4\tau$ at P_1 is determined.

$\Rightarrow$ By the def. of blowing-up, $\exists$ a unique $x' \in E_1$