

In general, we can compute the canonical bundle  $K_V$  of a smooth analytic hypersurface  $V$  in a manifold  $M$  in terms of  $K_M$  as follows. We have an exact sequence of vector bundles on  $V$

$$0 \rightarrow N_V^* \rightarrow T_M^*|_V \rightarrow T_V^* \rightarrow 0.$$

$$\square \quad 0 \rightarrow T_V \rightarrow T_M|_V \rightarrow \frac{T_M|_V}{T_V} \cong N_V \rightarrow 0$$

Take Hom functor

$$0 \rightarrow \text{Hom}(N_V, \mathbb{C}) \rightarrow \text{Hom}(T_M|_V, \mathbb{C}) \rightarrow \text{Hom}(T_V, \mathbb{C}) \rightarrow 0$$

$$\quad \quad \quad \cong \quad \quad \quad \cong \quad \quad \quad \cong$$

$$\quad \quad \quad N_V^* \quad \quad \quad T_M^*|_V \quad \quad \quad T_V^* \quad \quad \quad \sqcup$$

By simple linear algebra, If  $g_{\alpha\beta}$  transition fn of  $T_M|_V$  then  $g_{\alpha\beta} = \begin{pmatrix} h_{\alpha\beta} & * \\ 0 & j_{\alpha\beta} \end{pmatrix}$   $j_{\alpha\beta}$  transition fn for  $T_V$

$$\square \quad (\wedge^n T_M^*)|_V \cong \wedge^{n-1} T_V^* \otimes N_V^* \Rightarrow \wedge^{n-1} g_{\alpha\beta}^{-1} = \det \begin{pmatrix} h_{\alpha\beta}^{-1} & * \\ 0 & j_{\alpha\beta}^{-1} \end{pmatrix} = \det(h_{\alpha\beta}^{-1}) \cdot \det(j_{\alpha\beta}^{-1})$$

$\mathbb{C}^\infty$  isomorphism (not holomorphically).

$$\wedge^n (T_M^*|_V) \cong \wedge^n (T_V^* \oplus N_V^*) = \bigoplus_{k=0}^n \wedge^k T_V^* \otimes \wedge^{n-k} N_V^*$$

Since  $\wedge^{n-k} N_V^* = 0$  if  $n-k \geq 2$ , and  $\wedge^n T_V^* = 0$  ( $\because \wedge^n \mathbb{C}^{n-1} = 0$ ),

$$\wedge^n (T_M^*|_V) \cong \wedge^{n-1} T_V^* \otimes N_V^*.$$

$$\quad \quad \quad \cong \quad \quad \quad \cong$$

$$\quad \quad \quad (\wedge^n T_M^*)|_V \quad \quad \quad \sqcup$$

In other words,  $K_M|_V = K_V \otimes N_V^*$ .

Combining this with the adjunction formula I above, we have the

Adjunction Formula II.  $(K_M \otimes [V])|_V = K_V$ .

since  $K_M|_V = K_V \otimes N_V^* = K_V \otimes [-V]|_V$ ,  $K_M|_V \otimes [V]|_V = K_V \otimes N_V^* \otimes [V]|_V$