$$\pi(C) = \frac{K\cdot C + C\cdot C}{2} + 1$$

Now let D be any smooth, irreducible curve on the surface M, and let $L = [D]$ be its associated line bundle. From the long exact cohomology sequence associated to the exact sheaf sequence

$$0 \to \mathcal{O}_M \longrightarrow \mathcal{O}_M(L) \longrightarrow \mathcal{O}_D(L) \longrightarrow 0$$

we obtain $\chi(L) = \chi(\mathcal{O}_M) + \chi(\mathcal{O}_D(L))$.

⌈ See p139

$$\begin{aligned} 0 \to H^0(M,\mathcal{O}) &\longrightarrow H^0(M,\mathcal{O}(L)) \longrightarrow H^0(M,\mathcal{O}_D(L)) \to \\ H^1(M,\mathcal{O}) &\longrightarrow H^1(M,\mathcal{O}(L)) \longrightarrow H^1(M,\mathcal{O}_D(L)) \to \\ H^2(M,\mathcal{O}) &\longrightarrow H^2(M,\mathcal{O}(L)) \longrightarrow H^2(M,\mathcal{O}_D(L)) \to \\ &\vdots \end{aligned}$$

$$\begin{aligned} \Rightarrow 0 = {} & \dim H^0(M,\mathcal{O}) - \dim H^0(M,\mathcal{O}(L)) + \dim H^0(M,\mathcal{O}_D(L)) \\ & - \dim H^1(M,\mathcal{O}) + \dim H^1(M,\mathcal{O}(L)) - \dim H^1(M,\mathcal{O}_D(L)) \\ & \vdots \end{aligned}$$

$= \chi(\mathcal{O}_M) - \chi(L) + \chi(\mathcal{O}_D(L))$ by the def. of χ. p246.

$\Rightarrow \chi(L) = \chi(\mathcal{O}_M) + \chi(\mathcal{O}_D(L))$. ⌋

Now, by Riemann-Roch for D,

$$\begin{aligned} \chi(\mathcal{O}_D(L)) &= -\pi(D) + \deg(L|_D) + 1 \\ &= -\pi(D) + L\cdot L + 1. \end{aligned}$$

⌈ By p246, $\chi(\mathcal{O}_D(L)) = h^0(L|_D) - h^0(K_D - D|_D)$

$= \deg(L|_D) - g + 1 = \underbrace{\deg(L|_D)}_{= L\cdot L} + 1 - \pi(D)$. by p245 ⌋