$$0 \longrightarrow \mathcal{F}_x \longrightarrow \mathcal{E}_x \longrightarrow \mathcal{G}_x \longrightarrow 0.$$

Define $\mathcal{E}(\mathcal{U})$ by the set of all sections from $\mathcal{U}$ to the union $\bigcup_{x \in \mathcal{U}} \mathcal{E}_x$, for sufficiently small open set $\mathcal{U} \subset M$. Still not satisfactory !

For each $x \in M$, choose a sufficiently small open set $\mathcal{U}$ so that

$$\mathcal{O}^{(k_2)}|_{\mathcal{U}} \xrightarrow{\partial} \mathcal{O}^{(k_1)}|_{\mathcal{U}} \xrightarrow{\partial} \mathcal{O}^{(k_0)}|_{\mathcal{U}} \longrightarrow \mathcal{G}|_{\mathcal{U}} \to 0 \quad \text{exact.}$$

$$\Rightarrow 0 \to \mathrm{Hom}(\mathcal{G}|_{\mathcal{U}}, \mathcal{F}|_{\mathcal{U}}) \to \mathrm{Hom}(\mathcal{O}^{(k_0)}|_{\mathcal{U}}, \mathcal{F}|_{\mathcal{U}}) \xrightarrow{\delta} \mathrm{Hom}(\mathcal{O}^{(k_1)}|_{\mathcal{U}}, \mathcal{F}|_{\mathcal{U}}) \xrightarrow{\delta} \mathrm{Hom}(\mathcal{O}^{(k_2)}|_{\mathcal{U}}, \mathcal{F}|_{\mathcal{U}})$$

$$\Rightarrow \frac{\ker\delta}{\mathrm{Im}\,\delta} = \underline{\mathrm{Ext}}^1_{\mathcal{O}}(\mathcal{G}|_{\mathcal{U}}, \mathcal{F}|_{\mathcal{U}}) \underset{\text{"Definition"}}{\equiv} \underline{\mathrm{Ext}}^1_{\mathcal{O}}(\mathcal{G}, \mathcal{F})|_{\mathcal{U}}.$$

$\Rightarrow$ Given $\sigma \in H^0(M, \underline{\mathrm{Ext}}^1_{\mathcal{O}}(\mathcal{G}, \mathcal{F}))$,

$\sigma|_{\mathcal{U}} \in \underline{\mathrm{Ext}}^1_{\mathcal{O}}(\mathcal{G}(\mathcal{U}), \mathcal{F}(\mathcal{U}))$, may be considered as a sheaf map over $\mathcal{U}$.

We have the data

① $0 \longrightarrow \dfrac{\mathcal{O}^{(k_1)}}{\partial\mathcal{O}^{(k_2)}} \longrightarrow \mathcal{O}^{(k_0)} \longrightarrow \mathcal{G} \to 0$ is exact sequence of sheaves over $\mathcal{U}$

② $\dfrac{\mathcal{O}^{(k_1)}}{\partial\mathcal{O}^{(k_2)}} \xrightarrow{\sigma} \mathcal{F}$