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$\mathbb{P}^3$. Let $S = F \cap \langle X_{hj}, X_{hi} \rangle$.

(i) S smooth.

$\Rightarrow$ By the result on P478 ~ P479, we can not have three disjoint lines X_{hj}, X_{hi} and X_{hk}.

(ii) S singular

① singular set is a point.

$\Rightarrow$ S is the cone through a point over a smooth conic curve. $\Rightarrow$ Again we can't have disjoint 3 lines.

② singular set is a line

S is $X_0^2 + X_1^2 = 0$ $\Rightarrow$ S is the union of two planes $\Rightarrow$ By the same reason above, contradiction.

③ singular set is a plane

Same as above.

$\lrcorner$

Now in that hyperplane, X_{pik} and X_{pjk} span a 3-plane, which must then meet X_{pij} in a point; thus there is a line $L \subset \mathbb{P}^5$ meeting X_{pij}, X_{pik} and X_{pjk}.

$\ulcorner$ By the same argument above, $\langle X_{pik}, X_{pjk} \rangle = \mathbb{P}^3$.

$\Rightarrow$ The hyperplane $H = \mathbb{P}^4$ contains $\mathbb{P}^3$ and X_{pij}.

$\Rightarrow$ $\mathbb{P}^3 \cap X_{pij}$ is a point since $X_{pij} \not\subset \mathbb{P}^3$.

Let p be the point. $\Rightarrow$ $p \notin X_{pik}$ and $p \notin X_{pjk}$

Consider $\overline{p, X_{pik}}$.