condition explicitly; this alternate approach illustrates the use of nilpotents.

⊓ $0 \to H^0(\mathbb{P}^2, \mathcal{O}((n-3)H)) \to H^0(\mathbb{P}^2, \mathcal{O}(nH)) \longrightarrow H^0(L_{(2)}, \mathcal{L}_{(2)}) \to 0$

$$0 \to H^1(\mathbb{P}^2, \mathcal{O}^*) \to H^1(\mathbb{P}^2, \mathcal{O}^*_{(2)}) \xrightarrow{\psi} \mathbb{C} \to 0$$

$$\begin{array}{cc} \psi & \psi \\ \mathcal{L} & \mathcal{L}_2 \end{array}$$

To get the lift $\mathcal{L}$ for $\mathcal{L}_2$, we need the necessary and sufficient condition $\psi(\mathcal{L}_2) = 0$. If we have $\mathcal{L}$,

since $\mathcal{L} = [nH]$, $\exists\ \sigma \in H^0(\mathbb{P}^2, \mathcal{O}(nH))$. Consider $\sigma|_{L_2} \in H^0(L_{(2)}, \mathcal{L}_{(2)})$. $\Rightarrow \exists$ a curve $C = \{\sigma = 0\}$ of degree n. $\Rightarrow \#(C \cap L) = n$. we have n. second-order arc C_i.

We used the nilpotents of $\mathcal{O}_{(2)}$ to get the exact sequence

$$0 \to \mathcal{O}_{\mathbb{P}^2}((n-3)H) \to \mathcal{O}_{\mathbb{P}^2}((nH)) \longrightarrow \mathcal{O}_{L_{(2)}}(\mathcal{L}_{(2)}) \to 0.$$

More unstandable for Reiss relation, but not so clear to me yet.

As another example of nilpotents, we assume that $I \subset \mathcal{O}$ is a sheaf of regular ideals whose support Z consists of a finite set of points.

⊓ $Z = \{ z \in M \mid I_z \neq \mathcal{O}_z \} = \{ f_1 = \cdots = f_n(z) = 0 \}$

$\Rightarrow$ Since I is regular, $\operatorname{codim}\{f_1 = \cdots = f_n = 0\} = n \Rightarrow$