

$$\begin{aligned} \Gamma \quad a_1 &= 2, \quad k=1. \\ a_2^* &= a_{a_1}^* \geq 1 & a_1^* &\geq a_2^* \geq 1 \end{aligned}$$

\Rightarrow The smallest sequence a^* is 1, 1.

$$\Rightarrow \sigma_{1,1} = * \sigma_2$$

$$a_1=2, a_2=1, a_3=1. \Rightarrow a_{a_1}^* \geq 1 \quad a_{a_2}^* = a_{a_3}^* = a_1^* \geq 3$$

$$\Rightarrow a_1^* = 3 > a_2^* = 1 \Rightarrow *(\sigma_{2,1,1}) = \sigma_{3,1} \quad \sqcup$$

In general, we will have.

$\delta(a, b; c) = \delta(a^*, b^*; c^*)$, and so we may expect that any formula for the intersection of Schubert cycles σ_a, σ_b gives a dual formula, when applied to σ_a^*, σ_b^* .

$$\Gamma \quad \text{Let } * = f: G(k, n) \longrightarrow G(n-k, n).$$

$$\sigma_a \cdot \sigma_b = \sum n_c \cdot \sigma_c = \sum \delta(a, b, c) \cdot \sigma_c$$

$$\begin{array}{ccc} \sigma_a \in H_\ell(G(k, n), \mathbb{Z}) & \xrightarrow{f_*} & H_\ell(G(n-k, n), \mathbb{Z}) \\ \uparrow \text{P.D.} & & \uparrow \text{P.D.} \\ \tilde{\sigma}_a \in H^{k(n-k)-\ell}(G(k, n), \mathbb{Z}) & \xleftarrow{f^*} & H^{k(n-k)-\ell}(G(n-k, n), \mathbb{Z}) \end{array}$$

$$\Rightarrow \tilde{\sigma}_a \wedge \tilde{\sigma}_b = \sum \delta(a, b; c) \tilde{\sigma}_c$$

$$\Rightarrow f^*(\tilde{\sigma}_a \wedge \tilde{\sigma}_b) = f^*(\tilde{\sigma}_a) \wedge f^*(\tilde{\sigma}_b) = \sum \delta(a, b; c) f^*(\tilde{\sigma}_c)$$