$l \in B_L \Rightarrow l = \sigma(p, h) \Rightarrow \overline{p, p_L} \in l \cap L$ if $p \neq p_L$.
$\Rightarrow \pi(p) = \pi \circ j(l) = \overline{p, p_L} = l \cap L$. In case $p = p_L$, $\overline{p_L, p_L}$ is the tangent line of C_L at p_L, since h_L is not tangent plane of S at p_L, and $h_L \cap S$ is smooth at p_L.
$\Rightarrow l = \sigma(p_L, h) \Rightarrow \pi \circ j(l) = \pi(p_L) = \overline{p_L, p_L} = h \cap h_L$ by P265, P775. $h \cap h_L$ is tangent to C_L in case $p_L \notin S - R$.
If $p_L \in R$, then $\sigma(p_L, h_L) = L$ is a multiple component of $T_x(X) \cap X$, $x \in L$. $\Rightarrow$ Since L is not special, contradiction. $\Rightarrow \gamma = \pi \circ j|_{B_L}$

To start, we note that the pencil $\{l_x\}_{x \in L}$ contains 10 tangent lines to S: two singular lines corresponding to points of intersection of L with $\Sigma = X \cap H$, and eight nonsingular tangents, corresponding to the eight points of intersection of L and Δ. (In fact, only eight of these lines — namely the eight nonsingular tangent lines — correspond to honest branch points of the map $\gamma : B_L \longrightarrow L$.)

$L \cap \Sigma = L \cap (X \cap H) = L \cap H$ since $L \subset X$
$\Rightarrow$ Since $\deg H = 2$, $\#(L \cap \Sigma) = \#(L \cap H) = 2$.
$\Rightarrow$ Again by P265 & P775, if $\pi(x) = p \in S - R$, $l_x = h_1 \cap h_2$ is tangent to S at p, $\sigma(p) \cap F = \sigma(p, h_1) \cup \sigma(p, h_2)$. If $\pi(x) \in R$, since it is a double point, automatically, l_x is tangent to S at p.