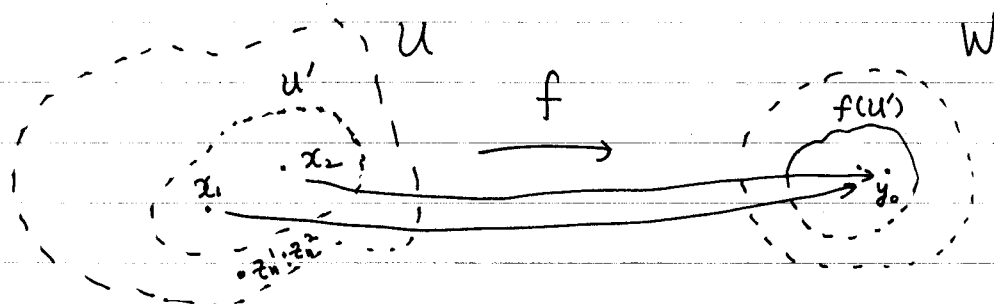


finite. The first of these is by definition. The second means that open sets map onto open sets, which is clear, as is the third property.

$f: U \rightarrow W$ . open  $U' \subset U$ .

$y_0 \in f(U') \subset W \Rightarrow f^{-1}(y_0) = \{x_1, x_2, \dots, x_d\}$ .



Suppose we can not find an open set containing  $y_0$  which is contained in  $f(U')$ .  $\Rightarrow \exists$ , for each  $n$ ,  $y_n$  s.t.  $f(U') \not\ni y_n$   $\|y_0 - y_n\| < \frac{1}{n}$ .  $f^{-1}(y_n) = \{z_n^1, z_n^2\} \subset U$ .  
 $\Rightarrow \exists$  a limit point  $z^0$  in  $U - U'$ .  $\Rightarrow f(z^0) = y_0$ .  
 $\Rightarrow f(x_1) = f(x_2) = f(z^0) = y_0 \Rightarrow$  Contradiction to the fact that  $\# f^{-1}(y_0) = 2$ . Here we assumed  $d=2$ .

Since  $f^{-1}(\bar{W})$  compact and  $f^{-1}(W)$  open in an open ball, we may assume that  $\exists$  a limit point  $z^0$  in  $U - U'$ .  
 If necessary, we may choose  $W' \subset \bar{W}' \subset W$ .  $f(\bar{W}')$  compact, for  $f(\bar{W}')$  in the original  $U$ , and  $f(\bar{W}') \cap \partial U = \emptyset \Rightarrow f$  is proper.  $\square$

These finite mappings have quite differently from, for example, blowing-down mappings such as