The relation between the line bundle $\mathcal{L}$ and vector bundle $\mathcal{E}$ is

$$c_1(\mathcal{L}) = -c_1(\mathcal{E}).$$

Proof. On $S^* = S - Z$ we have

$$0 \longrightarrow \mathcal{L}|S^* \longrightarrow \mathcal{E}^*|S^* \longrightarrow \mathcal{O}_{S^*} \longrightarrow 0$$

$$\Rightarrow c_1(\mathcal{L}) = c_1(\mathcal{E}^*) \quad \text{in } H^2(S^*; \mathbb{Z})$$
$$\Rightarrow c_1(\mathcal{L}) = c_1(\mathcal{E}^*) \quad \text{in } H^2(S, \mathbb{Z}),$$

since in the exact cohomology sequence

$$H^2(S, S^*; \mathbb{Z}) \longrightarrow H^2(S, \mathbb{Z}) \longrightarrow H^2(S^*, \mathbb{Z})$$

we have by excision

$$H^2(S, S^*, \mathbb{Z}) \cong \sum_{p \in Z} H^2(B_p, B_p^*; \mathbb{Z})$$
$$= 0,$$

where B_p is a ball around p. Q.E.D.

⸢ From $0 \longrightarrow \mathcal{L} \longrightarrow \mathcal{E}^* \longrightarrow I \longrightarrow 0$,
we have

$$0 \longrightarrow \mathcal{L}|_{S^*} \longrightarrow \mathcal{E}^*|_{S^*} \longrightarrow I|_{S^*} \longrightarrow 0$$

Note that $I|_{S^*} = \mathcal{O}|_{S^*}$, since $\operatorname{supp}(\mathcal{O}/I) = Z$.

$\Rightarrow$

$$\mathcal{E}^*|S^* = \mathcal{L}|S^* \oplus \mathcal{O}_{S^*} \quad \text{since } \mathcal{O}_{S^*} \text{ is free.}$$