If we choose another frame $\{v_i'\}$, then we get different tensors $\{\tau_i'\}$. $\Rightarrow$ $\tilde{\tau}' = \sum \tau_i' \otimes v_i'$

$$= \sum g^*_{ij}\tau_j \otimes g_{il} v_l$$
$$= \sum {}^t g^*_{ji}\, g_{il}\ \tau_j \otimes v_l$$
$$= \sum ({}^t g^* g)_{jl}\ \tau_j \otimes v_l$$
$$= \sum (g^{-1} g)_{jl}\ \tau_j \otimes v_l$$
$$= \sum \tau_i \otimes v_i$$

$\Rightarrow$ $\tilde{\tau} = \sum \tau_i \otimes v_i$ is well-defined.

Suppose we have another metric $\langle\ \rangle_2$ on $T'M$, i.e. on $M$. And we have a unitary frame $\{v_i'\}$ w.r.t the new metric, and the dual frame $\{\varphi_i'\}$.

$\Rightarrow$
$$v_i' = \sum g_{ij} v_j$$
$$\varphi' = g^* \varphi$$

Then we have the metric connection. $D$

Let $\{e_1, e_2, \cdots e_n\}$ be a holomorphic frame. and

$e_i = \sum A_{ij} v_j = \sum B_{ij} v_j'$ $\Rightarrow$ $A = Bg$.

$\langle e_i, e_j\rangle_1 = (h_1)_{ij} = \langle A_{ik} v_k, A_{jl} v_l\rangle_1 = (A\,{}^t\bar{A})_{ij}$

$\Rightarrow$ $h_1 = A\,{}^t\bar{A}$, here $\langle\ \rangle_1$ is the metric given in the beginning, and $\langle\ \rangle_2$ is a new metric.

Similarly, $h_2 = B\,{}^t\bar{B}$.

$D' e_i = (\theta_e^1)_{ij} e_j$

$\Rightarrow$ By 72,
$$\theta_e^1 = dA\cdot A^{-1} + A\cdot\theta_v^1 A^{-1}$$
$$\theta_e^2 = dB\cdot B^{-1} + B\ \theta_{v'}^2 B^{-1},$$
where $v = \{v_1 \cdots v_n\}$ & $v' = \{v_1', \cdots v_n'\}$.

By lemma on P73,