~ P6 something. $K(U) \ni \sigma$.

$\Rightarrow \sigma : U \longrightarrow \bigcup_{x\in U} \mathrm{Ext}^q(\mathcal{F}_x, \mathcal{G}_x)$. s.t for each point $q \in U$, $\exists$ (S, U_q) s.t. S is defind on U_q, here we need a presheaf, we don't know even if K is a presheaf or not. In some sense, we need something near each point. $\Rightarrow$ We desperately need the coherency. For each point, we have a nbd which makes $\mathrm{Hom}(\mathcal{F}(U), \mathcal{G}(U))$ sense. This enables us to define $\mathrm{Ext}^q(\mathcal{F}(U), \mathcal{G}(U))$. See P6 ?? $\lrcorner$

3. This procedure globalizes to give a pairing

$$\mathrm{Ext}^p(M; \mathcal{F}, \mathcal{G}) \otimes \mathrm{Ext}^q(M; \mathcal{G}, \mathcal{H}) \longrightarrow \mathrm{Ext}^{p+q}(M; \mathcal{F}, \mathcal{H}).$$

Taking $q = n-p$,

$\mathcal{F} = \mathcal{O}$, $\mathcal{G} = \mathcal{F}$, $\mathcal{H} = \Omega^n$.

this pairing is

(*) $$H^p(M, \mathcal{F}) \otimes \mathrm{Ext}^{n-p}(M; \mathcal{F}, \Omega^n) \to H^n(M, \Omega^n),$$

since $\mathrm{Ext}^*(M; \mathcal{O}, \mathcal{F}) \cong H^*(M, \mathcal{F})$.

Γ $\mathrm{Ext}^q(M; \mathcal{O}, \mathcal{F}) \cong H^q(M, \mathcal{O}^* \otimes_{\mathcal{O}} \mathcal{F}) \equiv H^q(M, \mathcal{O} \otimes_{\mathcal{O}} \mathcal{F}) = H^q(M, \mathcal{F})$

by the first property on P706.

My question ! $\mathrm{Ext}^q(M; \mathcal{F}, \mathcal{G}) = \underline{\mathrm{Ext}}^q_{\mathcal{O}}(\mathcal{F}, \mathcal{G})(M)$?

$$\mathrm{Ext}^p(M; \mathcal{F}, \mathcal{G}) = \mathbb{H}^p(M, \mathrm{Hom}(E.(\mathcal{F}), \mathcal{G}))$$
$$= \lim_{\underline{\mathcal{U}}} H^p(\underline{\mathcal{U}}, \mathrm{Hom}(E.(\mathcal{F}), \mathcal{G}))$$