reason. $C\cdot C = (\pi^* H^n - E)\cdot(\pi^* H^n - E) = n^2 + E\cdot E = n^2 - d.$
$C\cdot K_S = C\cdot(\pi^* H^{-3} + E) = (\pi^* H^n - E)\cdot(\pi^* H^{-3} + E) = -3n - E\cdot E$
$= -3n + d.$ ⅃

Consequently, by the Riemann-Roch formula,

$$r = \dim|L| = \frac{1}{2}(C\cdot C - C\cdot K_S) + \omega$$
$$= \frac{n(n+3)}{2} - d + \omega.$$

┌ $\dim|L| + h^0(K_S - L) = \frac{1}{2}(L\cdot L - K_S\cdot L) + p_g - q + \omega$

$\Rightarrow \dim|L| = r = \frac{1}{2}(n^2 - d + 3n - d) + \omega = \frac{n(n+3)}{2} - d + \omega$ ⅃

On the other hand, from the exact cohomology sequence of

$$0 \longrightarrow \mathcal{J}_{P_0}(n) \longrightarrow \mathcal{O}_{\mathbb{P}^2}(n) \longrightarrow \mathcal{O}_{P_0}(n) \longrightarrow 0$$

and $h^1(\mathcal{O}_{\mathbb{P}^2}(n)) = 0$, we obtain

$$r = \dim|\mathcal{J}_{P_0}(n)|$$
$$= \frac{1}{2}(n+3)n - d + h^1(\mathcal{O}_{\mathbb{P}^2}(n)),$$

so that the superabundance

$$\omega = h^1(\mathcal{O}_S(L)) = h^1(\mathcal{J}_{P_0}(n)).$$

┌ $\mathcal{J}_{P_0}(n) = \mathcal{O}([nH - C_0]) = \mathcal{O}([nH]\otimes[-C_0])$ = sheaf of sections of $[nH]$ vanishing to order ≥ 1 along C_0, where C_0 is a curve of