-ned by the points p_i. Thus

$$C_q(\mathbb{P}^n) = \binom{n+1}{q} \cdot \omega^q,$$

as we computed before.

As an immediate application, we can add one more identity to those previously mentioned. If $E \longrightarrow M$ is a complex vector bundle of rank k, then the first Chern class $c_1(E)$ is dual to the cycle $D_k \subset X$ given as the locus where k generic sections $\sigma_1, \sigma_2, \cdots \sigma_k$ of E are linearly dependent. But the k sections σ_i of E together give one section

$$\sigma = \sigma_1 \wedge \cdots \wedge \sigma_k$$

of the line bundle $\Lambda^k E \longrightarrow M$, and the degeneracy set D_1 of σ is equal to D_k. Checking that the orientations are in fact the same, we have

$$c_1(\Lambda^k E) = c_1(E).$$

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$$\begin{array}{ccccc} \Lambda^k E & \quad & H^2_{DR}(M) & \longleftrightarrow & H(M) \\ \downarrow & & \Downarrow & & \Downarrow \\ M & & c_1(\Lambda^k E) & \longleftrightarrow & D_{1-1+1} = D_1 = \{x \in M \,;\, \sigma(x) = 0\} \\ & & & & \quad \| \\ & & & & \{x \in M \,;\, \sigma_1(x) \wedge \cdots \wedge \sigma_k(x) = \sigma(x) = 0\} = D_k \end{array}$$

The orientation on D_k is given as follows: For $x \in D_k - D_{k-1}$,

$$\sigma_1 = e_1 \;,\; \cdots \;,\; \sigma_{k-1} = e_{k-1}, \quad e_k.$$

$\Rightarrow \sigma_k = f_1 e_1 + \cdots + f_{k-1} e_{k-1} + f_k e_k$

$\Rightarrow \Phi_k \wedge \frac{\sqrt{-1}}{2} df_k \wedge d\bar{f_k}$ is the given orientation on M.

Consider $\sigma_1 \wedge \cdots \wedge \sigma_{k-1} \wedge e_k$ which is a local frame for $\Lambda^k E$. $\Rightarrow \sigma = f_k \sigma_1 \wedge \cdots \wedge \sigma_{k-1} \wedge e_k$