

In general, we set

$$\Theta(T, p', r) = \frac{1}{r^{2n-2p}} T(\chi(r) \omega^{n-p})$$

and shall prove the

Lemma. $\Theta(T, p', r)$ is an increasing function of r .

Proof. The smoothing T_ϵ of a closed, positive current is again a closed, positive current.

$$\Gamma \quad T = \sum T_{i\bar{j}} dz_i \wedge d\bar{z}_j \Rightarrow T_\epsilon = \sum (T_{i\bar{j}})_\epsilon dz_i \wedge d\bar{z}_j$$

$$\Rightarrow d(T_\epsilon) = \sum d(T_{i\bar{j}})_\epsilon \wedge dz_i \wedge d\bar{z}_j$$

$$= \sum \left\{ \frac{\partial (T_{i\bar{j}})_\epsilon}{\partial z_i} dz_i + \frac{\partial (T_{i\bar{j}})_\epsilon}{\partial \bar{z}_j} d\bar{z}_j \right\} \wedge dz_i \wedge d\bar{z}_j$$

by P374.3

$$\Rightarrow \sum \left\{ \left(\frac{\partial T_{i\bar{j}}}{\partial z_i} \right)_\epsilon dz_i + \left(\frac{\partial T_{i\bar{j}}}{\partial \bar{z}_j} \right)_\epsilon d\bar{z}_j \right\} \wedge dz_i \wedge d\bar{z}_j$$

$$= \sum (dT_{i\bar{j}})_\epsilon \wedge dz_i \wedge d\bar{z}_j$$

$$= (dT)_\epsilon \Rightarrow \text{If } dT=0, dT_\epsilon=0.$$

For positiveness, it is more delicate.

In case $p=1$, we proved that $T = \sum T_{i\bar{j}} dz_i \wedge d\bar{z}_j$

is positive $\Leftrightarrow T(\lambda)(\alpha) = C_n \sum_{i,j} T_{i\bar{j}}(\alpha) \lambda_i \bar{\lambda}_j \geq 0$

by P386.

for all $\alpha \geq 0$, where C_n is some constant depending on n .