which is equivalent to Riemann's relation $g = \frac{1}{2} b_1(M)$.

「 $$c_1(M) = 2(1-g) = \underset{\dim H^0(M)}{\underset{\|}{1}} - b_1(M) + \underset{\dim H^2(M).}{\underset{\|}{1}} = 2 - b_1(M)$$
$$\Rightarrow \quad g = \frac{1}{2} b_1(M)$$ 」

This we proved by harmonic theory. 「 See p124. 」

For a surface, we have Noether's formula

$$\chi(\mathcal{O}_M) = \frac{c_1(M)^2 + c_2(M)}{12},$$

which we will prove and use extensively in the next chapter.

「 Since $n = 2$, $$Td_2 = \frac{P^1(A)^2 + P^2(A)}{12},$$

by Hirzebruch - Riemann - Roch formula,

$$Td_2 = \frac{c_1(M)^2 + c_2(M)}{12} = \chi(\mathcal{O}_M).$$ 」

Unfortunately, our analogy between the Gauss - Bonnet III and Riemann - Roch formulas fails in one crucial aspect: while any differentiable manifold has many C^∞ vector fields to use as pros in the proof of Gauss - Bonnet III, relatively few compact manifolds have any global holomorphic vector fields. (Cf. the theorem of Carrell and Liebermann proved in Section 4 of Chapter 5.) Of course, since the Riemann - Roch formula itself has nothing to do with the vector field v used to obtain it,