of Riemann surfaces. Whereas curves, having the genus as their sole discrete invariant, fall into an orderly sequence of families, surfaces possess a variety of numerical invariants and are not so readily classified. Conversely, while curves have a natural continuous invariant — their periods, realized geometrically by the Jacobian — no fully satisfactory continuous invariant has been found for surfaces. As a result, the theory of algebraic surfaces does not possess the natural cohesiveness of the theory of curves; it tends to concentrate more on the study of special classes of surfaces. This is reflected in our treatment: with the exception of the basic tools presented in Sections 1 and 2 and the proof of Noether's formula, virtually all our results either describe or characterize specific families of surfaces.

Sections 1 and 2 contain all the techniques used in our study. For the most part, these results are special cases of general phenomena discussed before; the one new idea introduced here is the notion of a rational map. This is an important aspect of the theory of varieties in dimension two or more; in the case of surfaces we are able to give a complete description of birational maps.

In Section 3 we describe the general rational surface, and obtain in consequence the answer to some pos-