

The zero cycle is effective in case all $m_j \geq 0$. The degree of a zero cycle is given by

$$\deg(P) = \sum m_j.$$

Suppose now that $f: U \rightarrow \mathbb{C}^n$ is a holomorphic mapping with $f^{-1}(0) = \{0\}$. We define the multiplicity of f at the origin to be the topological degree of $f: U^* \rightarrow \mathbb{C}^n - \{0\}$.

We then say that the equation

$$f(z) = 0$$

has the origin as a solution of multiplicity d .

Now, according to the continuity property (C) of the local intersection numbers, for $\|W\| < \varepsilon$ the equation

$$f(z) = W$$

will have exactly d solutions $z_j(W)$ close to the origin. Of course, some of the $z_j(W)$ may be repeated.

Consider $g(z, W) = f(z) - W$. Since $\|f(z)\| \geq \varepsilon$ on ∂U , for $\|W\| < \delta$, δ sufficiently small, $\|f(z) - W\| \geq \frac{\varepsilon}{2}$ on ∂U .
 \Rightarrow For each W with $\|W\| < \delta$, $\{f(z) = W\}$ is a finite set of isolated points. \Rightarrow By continuity principle, $\# \{f(z) = W\} = d$.
 $\# \{f(z) = W\} = \text{constant}$. In our case, $\# \{f(z) = W\} = d$.

Using the zero-cycle notation, we write

$$f^{-1}(W) = \sum z_j(W).$$

Let $W = \{ \|W\| < \varepsilon \}$ and redefine $U = f^{-1}(W)$. Then we claim that the holomorphic mapping

$$f: U \rightarrow W$$

has the following properties: It is surjective, open and