which implies the lemma. Q.E.D.

⌈ $\eta = \bar\partial\bar\partial^* G_{\bar\partial}\eta$

$\Rightarrow$ Since $\bar\partial^* G_{\bar\partial}\eta = \partial\partial^* G_\partial(\bar\partial^* G_{\bar\partial}\eta)$,

$$\eta = \bar\partial\bar\partial^* G_{\bar\partial}\eta = \bar\partial\partial\partial^* G_\partial(\bar\partial^* G_{\bar\partial}\eta) = \bar\partial\partial\partial^*\bar\partial^* G_{\bar\partial}^2\eta$$

⌈ If η is real, $\partial\eta = 0 \Leftrightarrow \bar\partial\eta = 0, \Leftrightarrow d\eta = 0$, ⌋

$\eta = \partial\bar\partial\gamma$.

Suppose η is real, and $\gamma = a + ib$, a, b real.

$$\Rightarrow \eta = \partial\bar\partial(a+ib) = \partial\bar\partial a + i\partial\bar\partial b.$$

$$\bar\eta = \bar\partial\partial a - i\bar\partial\partial b = -\partial\bar\partial a + i\partial\bar\partial b = \eta$$

$$\Rightarrow 2\eta = 2i\,\partial\bar\partial b \Rightarrow \eta = i\partial\bar\partial b = \partial\bar\partial(ib)$$

$\Rightarrow \eta = \partial\bar\partial\varphi$. $\varphi = ib \Rightarrow i\varphi = -b$, which is real. ⌋

The basic example of a positive line bundle is the hyperplane bundle $[H]$ on $\mathbb{P}^n$. Recall that the dual of the hyperplane bundle is the bundle J whose fibre at $Z \in \mathbb{P}^n$ is the line $\{\lambda Z\} \subset \mathbb{C}^{n+1}$; we can put a metric on J by setting

$|(Z_0, \cdots Z_n)|^2 = \Sigma |Z_i|^2$. If Z is any non-zero section of J — i.e., a local lifting $U \subset \mathbb{P}^n \to \mathbb{C}^{n+1} - \{0\}$ — then the curvature form in J is given by