But $\mathrm{Ext}^2_{\mathcal{O}}(\mathcal{O}/I,\ \Omega^2) \cong \mathrm{Ext}^2_{\mathcal{O}}(\mathcal{O}/I,\ \mathcal{O}) \longrightarrow \mathcal{O}/I$

$$\varphi \longrightarrow \varphi'$$

$$\varphi \longmapsto h\, dz_1 \wedge dz_2 = h'\, dw_1 \wedge dw_2 = h' \left|\frac{\partial w}{\partial z}\right| dz_1 \wedge dz_2$$

$\Rightarrow \quad h = h' \left|\frac{\partial w}{\partial z}\right|$

$$\Rightarrow \quad res_f(g, h')_w = Res_{\{0\}}\left(\frac{g h'\, dw_1 \wedge dw_2}{f_1 f_2}\right) = Res_{\{0\}}\left(\frac{g h\, dz_1 \wedge dz_2}{f_1 f_2}\right) = res_f(g, h)_z = res(g, \varphi)$$

$$\begin{tikzcd}
\mathcal{O}/I \otimes \mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I, \Omega^n) \arrow[r, "res"] \arrow[d, "\pi^*"] & \mathbb{C} \arrow[d, equal] \\
\mathcal{O}/I' \otimes \mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I', \Omega^n) \arrow[u, "\pi"'] \arrow[r, "res"] & \mathbb{C}
\end{tikzcd}$$

$$res(g+I, \varphi) = res_f(g, h)$$
$$= res_{f'}(g, \Delta h) = res(g+I', \pi^*\varphi)$$

$$\begin{tikzcd}
\varphi \arrow[r, leftrightarrow] \arrow[d] & h+I \arrow[d] \\
\pi^*\varphi \arrow[r, leftrightarrow] & \Delta h + I' \\
\mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I', \Omega^n) \arrow[r, leftrightarrow] & \mathcal{O}/I' \arrow[u]
\end{tikzcd}$$

⌋

To show that the pairing $\mathcal{O}/I \otimes \mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I, \Omega^n) \xrightarrow{res} \mathbb{C}$ is nondegenerate, it will suffice to find a regular ideal $I' \subset I$ for which the pairing is nondegenerate and then use the functoriality together with the facts that $\mathcal{O}/I' \xrightarrow{\pi} \mathcal{O}/I$ is surjective (obvious) and $\mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I, \Omega^n) \xrightarrow{\pi^*} \mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I', \Omega^n)$ is injective which was proved in the previous proposition. Appealing to nullstellensatz in Section 1, we may take $I' = \{z_i^d$