reducible, we deduce that $\dim|\mathcal{J}_{P_0}(4)| = 3$, which gives our conclusion.

Q. E. D.

⊓ $|\mathcal{J}_{P_0}(4)| \geq 2$ since $\sigma_0 s_1$, $\sigma_0 s_2$, $\sigma_0 s_3$ in $H^0(\mathbb{P}^2; \mathcal{J}_{P_0}(4))$ are linearly independent. If $|\mathcal{J}_{P_0}(4)| = 2$,
$\langle \sigma_0 s_1, \sigma_0 s_2, \sigma_0 s_3 \rangle = H^0(\mathbb{P}^2; \mathcal{J}_{P_0}(4))$.
$\Rightarrow$ All in $|\mathcal{J}_{P_0}(4)|$ are reducible. By the assumptions above, $C \in H^0(\mathbb{P}^2; \mathcal{J}_{P_0}(4))$ is not reducible, ie irreducible. $\Rightarrow$ $|\mathcal{J}_{P_0}(4)| > 2$ $\Rightarrow$ $|\mathcal{J}_{P_0}(4)| = 3$. since $|\mathcal{J}_{P_0}(4)| \leq 3$.
⌋

It is interesting to investigate the rational map

$$f : \mathbb{P}^2 \longrightarrow \mathbb{P}^3$$

defined by the linear system $|\mathcal{J}_{P_0}(4)|$ when the configuration is in special position. The image is a surface S of degree four with the property that there are ∞^2 reducible hyperplane sections.

⊓ $\dim H^0(\mathbb{P}^2; \mathcal{J}_{P_0}(4)) = \dim|\mathcal{J}_{P_0}(4)| + 1 = 4$
$\Rightarrow$ By the arguments above, $H^0(\mathbb{P}^2; \mathcal{J}_{P_0}(4)) = \langle \sigma_0 s_1, \sigma_0 s_2, \sigma_0 s_3, \tau \rangle$.
$\Rightarrow$

$$\begin{array}{ccc} \mathbb{P}^2 & \longrightarrow & \mathbb{P}^3 \\ \rotatebox{90}{$\in$} & & \\ x & \longmapsto & [(\sigma_0 s_1)(x), (\sigma_0 s_2)(x), (\sigma_0 s_3)(x), \tau(x)] \end{array}$$