On the other hand,

$$\partial\eta = \partial(C\,((\bar\nabla\psi,\psi)\wedge\omega^{n-1}) = \partial(-\sum_{I,J,K}\psi_{I\bar J.\bar K.K}\,\overline{\psi_{I\bar J}})\,\Phi$$

$$-(\sum_{I,J,K}\psi_{I\bar J.\bar K}\,\overline{\psi_{I\bar J.\bar K}})\Phi + \langle A'(\psi),\psi\rangle\,\Phi.$$

$\ulcorner$ $\partial(-\sum_{I,J,K}(-1)^{K-1}\psi_{I\bar J.\bar K}\,\overline{\psi_{I\bar J}}\,\varphi_1\wedge\cdots\wedge\hat\varphi_K\cdots\wedge\varphi_n)\wedge\overline{\Phi'} + \partial A^0(\psi)\,\Box$ —

$$= -\sum_{IJK}\{(-1)^{K-1}(\psi_{I\bar J.\bar K.K}\,\overline{\psi_{I\bar J}})\varphi_K + (-1)^{K-1}\psi_{I\bar J\bar K}\,(\overline{\psi_{I\bar J}})_K\,\varphi_K\,)\varphi_1\wedge\cdots\wedge\hat\varphi_K\wedge\varphi_n)$$

$$\wedge\,\overline{\Phi'} + \langle A'(\psi),\ \psi\rangle\,\Phi$$

$$= (-\sum_{I,J,K}\psi_{I\bar J.\bar K.K}\,\overline{\psi_{I\bar J}})\,\Phi - (\sum_{IJK}\psi_{I\bar J\bar K}\,\overline{\psi_{I\bar J.\bar K}})\,\Phi + \langle A'(\psi),\ \psi\rangle\,\Phi \ \lrcorner$$

Thus, by the Weitzenböck formula.

$$\langle\Delta\psi,\psi\rangle = \|\bar\nabla\psi\|^2 + \langle A'(\psi),\psi\rangle,$$

where $\|\bar\nabla\psi\|^2 = \int_M\langle\bar\nabla\psi,\bar\nabla\psi\rangle\,\Phi$. is the L^2-norm of the

$\ulcorner$
$$\bar\partial\eta = \langle\Delta\psi,\psi\rangle\Phi - \langle\overset{\nabla''}{\bar\nabla}\psi,\ \overset{\nabla''}{\bar\nabla}\psi\rangle\Phi + \langle A'(\psi),\psi\rangle\,\Phi.$$

since $\bar\nabla\psi = \bar\nabla(\sum\psi_{I\bar J}\,\varphi_I\wedge\overline{\varphi_J}) = \sum_{IJ.K}\psi_{I\bar J.\bar K}\,\overline{\varphi_K}\otimes\varphi_I\wedge\overline{\varphi_J}$

$$0 = \int_M\bar\partial\eta = \int_M\langle\Delta\psi,\psi\rangle\Phi - \int_M\langle\bar\nabla\psi,\bar\nabla\psi\rangle\Phi + \int_M\langle A'(\psi),\psi\rangle\Phi$$

$$= \langle\Delta\psi,\psi\rangle - \langle\bar\nabla\psi,\bar\nabla\psi\rangle + \langle A'(\psi),\psi\rangle$$

$$\Rightarrow\quad \langle\Delta\psi,\psi\rangle = \|\bar\nabla\psi\|^2 + \langle A'(\psi),\psi\rangle$$