

$$
\begin{tikzcd}
& & & H^{n+k+2}(M) \\
& H^{n+k}(M) \arrow[urr, "L"] \arrow[drr, "P.D"] & & 0 \cap H_{n-k-2}(M) \\
H^{n-k}(M) \supset P^{n-k}(M) \arrow[ur, "L^k"] \arrow[dr, "P.D"'] & P^{n+k}(M) \arrow[r, phantom, "\cong"] \arrow[l, phantom, "\cong"] & P^{n-k}(M) \subset H_{n-k}(M) \arrow[ur, "(\cap H)"] & \nwarrow \text{hyperplane} \\
& H_{n+k}(M) \arrow[ur, "(\cap P^{N-k})"'] & &
\end{tikzcd}
$$

to the subgroup of $(n-k)$-cycles that do not intersect a hyperplane, i.e, the image of the map

$$H_{n-k}(M-V) \xrightarrow{\cap H} H_{n-k}(M).$$

$\ulcorner$

$$
\begin{tikzcd}
H^{n+k}(M) \arrow[r, "L"] & H^{n+k+2}(M) \\
P^{n-k}(M) \arrow[u, phantom, "\cup"] \arrow[r] & 0 \arrow[u, phantom, "\cup"]
\end{tikzcd}
$$

$$\Rightarrow \quad \underset{\substack{\cup \\ P^{n-k}(M)}}{H_{n-k}(M)} \xrightarrow[\cap H]{} H_{n-k-2}(M) \quad \text{corresponds to}$$

This explains the above claim. $\lrcorner$

Such cycles are called finite cycles since $M-V$ is the "finite part" $M \cap \mathbb{C}^N$ of M;
(See p16)
their importance will be more apparent when we prove the Lefschetz theorem on hyperplane sections.

As another application of the Hodge and Lefschetz decompositions, we will now describe the Hodge-Riemann bilinear relations. We define a bilinear form

$$Q: H^{n-k}(M) \otimes H^{n-k}(M) \longrightarrow \mathbb{C}$$

by setting $$Q(\xi, \eta) = \int_M \xi \wedge \eta \wedge \omega^k.$$