$$\mathrm{Res}_{\{0\}}\left(\frac{h\det A\, dz_1\wedge\cdots\wedge dz_n}{g_1\cdots g_n}\right) = \lim_{t\to 0}\left(\sum \mathrm{Res}_{P_t}\left(\frac{h\det A_t\, dz_1\wedge\cdots\wedge dz_n}{g_{t,1}\cdots g_{t,n}}\right)\right)$$

$$= \lim_{t\to 0}\left(\mathrm{Res}_{(0)}\left(\frac{h\ \det A_t\, dz_1\wedge\cdots\wedge dz_n}{g_{t,1}\cdots g_{t,n}}\right)\right)$$

$$= \lim_{t\to 0} \mathrm{Res}_{\{0\}}\left(\frac{h\, dz_1\wedge\cdots dz_n}{f_1\cdots f_n}\right)$$

$$= \mathrm{Res}_{\{0\}}\left(\frac{h\, dz_1\wedge\cdots\wedge dz_n}{f_1\cdots f_n}\right). \qquad \text{Q.E.D.}$$

$$\mathrm{Res}_{\{0\}}\left(\frac{h\det A\, dz_1\wedge\cdots\wedge dz_n}{g_1\cdots g_n}\right) = \lim_{t\to 0}\left(\sum \mathrm{Res}_{P_t}\left(\frac{h\det A_t\, dz_1\wedge\cdots\wedge dz_n}{(g_t)_1\cdots(g_t)_n}\right)\right)$$

$$\text{"} \sum_{P\in g^{-1}(0)} \mathrm{Res}_P\,\omega \qquad\qquad \text{"} \lim_{t\to 0}\sum_{P_t\in g_t^{-1}(0)} \mathrm{Res}_{P_t}\,\omega_t$$

by continuity

$$= \lim_{t\to 0} \mathrm{Res}_{\{0\}}\left(\frac{h\det A_t\, dz_1\wedge\cdots\wedge dz_n}{(g_t)_1\cdots(g_t)_n}\right) \quad \text{by the result above}$$

$$= \lim_{t\to 0} \mathrm{Res}_{\{0\}}\left(\frac{h\ dz_1\wedge\cdots\wedge dz_n}{(f_t)_1\cdots(f_t)_n}\right) \quad \text{by case 2 with } \det A_t(0)\neq 0$$

and f_t possibly degenerate

$$\underset{\text{Continuity}}{=} \mathrm{Res}_{\{0\}}\left(\frac{h\, dz_1\wedge\cdots\wedge dz_n}{f_1\cdots f_n}\right)$$

"Implicitly, the authors were thinking $U_i = U - D_i \cong \Delta^*\times\Delta^{n-1}$, so that $P_i = \{z : |f_j| = \epsilon_j$ for $j\neq i$, $|f_i(z)|\leq\epsilon_i\} \cong (\partial\Delta)^{n-1}\times\Delta$

Maybe my concern is related to the statement on P453.
" Using the resolution of singularities theorem. ......
... holds with no assumption on the singularities of D"