

P 217

Because f is a covering map away from B , we can take a triangulation of S whose open cells are just the connected components of the inverse images of the open cells in our triangulation of S' .

For each point $p \in S' - B'$, we have an open set U_p evenly covered by f . We have a better situation here. We can choose an open set around any point in B' which are evenly covered by f . Note that the branch points set is finite. \smile

Then if C_0, C_1, C_2 denote the number of 0-, 1-, and 2-cells in S' , respectively, we will have $n \cdot C_1$ 1-cells and $n \cdot C_2$ 2-cells in S . Since for any $p \in S'$,

$$\sum_{q \in f^{-1}(p)} v(q) = n,$$

we see also that the number of distinct points

$$\#(f^{-1}(p)) = n - \sum_{q \in f^{-1}(p)} (v(q) - 1).$$

$$\begin{array}{ccc} S & \xrightarrow{f} & S' \\ & \downarrow & \\ & p & \end{array} \quad q \in f^{-1}(p) \Rightarrow \sum_{q \in f^{-1}(p)} v(q) = n$$

$$\Rightarrow \sum_{q \in f^{-1}(p)} (v(q) - 1) = \sum_{q \in f^{-1}(p)} v(q) - \sum_{q \in f^{-1}(p)} 1$$

$$\#(f^{-1}(p)) = \# \text{ of distinct points} \quad \smile$$