

assmannian $G(2, 4)$ of 2-planes in \mathbb{C}^4 , viewed primarily as the set of lines in \mathbb{P}^3 . Recall from Section 5 of Chapter 1 that the Plücker embedding

$$G(2, 4) \longrightarrow \mathbb{P}(\Lambda^2 \mathbb{C}^4) = \mathbb{P}^5$$

is given by mapping the 2-plane Λ spanned by vectors $v_1, v_2 \in \mathbb{C}^4$ into the wedge product $v_1 \wedge v_2 \in \Lambda^2 \mathbb{C}^4$.

⌈ See P209

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As was proved there, a general multivector w will be decomposable — that is, of the form $v_1 \wedge v_2$ — exactly when

$$w \wedge w = 0.$$

⌈ P209 ~ P211, especially, P211.

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This is a quadratic relation: the image of $G(2, 4)$ under the Plücker embedding is therefore a quadric hypersurface in \mathbb{P}^5 , which we will henceforth denote by G .

⌈ See P211.

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