

Similarly, we get,

$$|v_2(f_1)|^2 \leq C' (|v_2(f_1) + f_0 \theta_{\rho_1}(v_2)|^2 + \|f\|^2)$$

$$|v_1(f_2)|^2 \leq C' (|v_1(f_2) + f_0 \theta_{\rho_2}(v_1)|^2 + \|f\|^2)$$

$$|v_2(f_2)|^2 \leq C' (|v_2(f_2) + f_0 \theta_{\rho_2}(v_2)|^2 + \|f\|^2)$$

$$\Rightarrow \|v_1(f_1, f_2)\|^2 + \|v_2(f_1, f_2)\|^2 < C (\|\nabla f\|^2 + \|f\|^2) \quad C > 0.$$

$$\Rightarrow \sum_{|k| \leq 1} \int_U |D^k(f_1, f_2)|^2 dx < C \sum_{|k| \leq 1} \int_U \|\nabla^k f\|^2 dx.$$

$$\begin{aligned} \nabla^2 f &= \nabla_{v_1} \nabla_{v_2} f = d_{v_1} (d_{v_2} f) \otimes e_1 + d_{v_2} (d_{v_1} f) \otimes e_2 + d_{v_1}^2 f \otimes e_1 + d_{v_2}^2 f \otimes e_2 \\ &= d_{v_1} (v_2(f_1) e_1 + f_1 \nabla_{v_2} e_1 + v_2(f_2) e_2 + f_2 \nabla_{v_2} e_2) \otimes e_1 + d_{v_2} (v_1(f_2) e_1 + f_1 \nabla_{v_1} e_1 + v_2(f_2) e_2 + f_2 \nabla_{v_2} e_2) \otimes e_2 \\ &= d_{v_1}^2 f \otimes e_1 + d_{v_2}^2 f \otimes e_2 + d_{v_1} d_{v_2} f \otimes e_1 + d_{v_2} d_{v_1} f \otimes e_2 \end{aligned}$$

$$(\nabla^2 f)(v_1, v_2) = \nabla_{v_1} (\nabla_{v_2} f)$$

$$\begin{aligned} \nabla_{v_2} f &= \nabla_{v_2} (f_1 e_1 + f_2 e_2) = d_{v_2} f_1 \otimes e_1 + f_1 \nabla_{v_2} e_1 \\ &\quad + d_{v_2} f_2 \otimes e_2 + f_2 \nabla_{v_2} e_2 \\ &= v_2(f_1) e_1 + f_1 \nabla_{v_2} e_1 + v_2(f_2) e_2 + f_2 \nabla_{v_2} e_2 \\ &= v_2(f_1) e_1 + v_2(f_2) e_2 + A^0(f) e_1 + A^1(f) e_2 \end{aligned}$$

$$\begin{aligned} \nabla_{v_1} (\nabla_{v_2} f) &= d_{v_1} v_2(f_1) e_1 + v_1 v_2(f_2) e_2 + A^1(f) e_1 + A^1(f) e_2 \\ &= v_1 v_2(f_1) e_1 + v_1 v_2(f_2) e_2 + A^1(f) e_1 + A^1(f) e_2 \end{aligned}$$

$$\nabla_{v_2} (\nabla_{v_1} f) = v_2 v_1(f_1) e_1 + v_2 v_1(f_2) e_2 + A^1(f) e_1 + A^1(f) e_2$$

$$\Rightarrow \nabla_{v_1} \nabla_{v_2} - \nabla_{v_2} \nabla_{v_1} - \nabla_{[v_1, v_2]} = \boxed{\nabla^2(v_1, v_2)} \rightarrow \text{zero order}$$

\downarrow
 1-order