Conversely,
$$\begin{aligned} D_1 &= b_{11}(x)\, v_1 + b_{21}(x)\, v_2 + \cdots + b_{n1}(x)\, v_n \\ D_2 &= b_{12}(x)\, v_1 + b_{22}(x)\, v_2 + \cdots + b_{n2}(x)\, v_n \\ &\;\vdots \\ D_n &= b_{1n}(x)\, v_1 + b_{2n}(x)\, v_2 + \cdots + b_{nn}(x)\, v_n \end{aligned}$$

$b_{ij}(x)$'s are C^∞.

$n=2$,
$$D_1\varphi = b_{11}(x)\, v_1\cdot\varphi + b_{21}(x)\, v_2\varphi$$
$$D_2\varphi = b_{12}(x)\, v_1\cdot\varphi + b_{22}(x)\, v_2\varphi$$

$|b_{ij}(x)| < K'$ for all $x \in \overline{V}$ ($\overline{V} \supset V$).

$$\Rightarrow \quad |D_1\varphi| \le |b_{11}(x)|\,|v_1\varphi| + |b_{21}(x)|\,|v_2\varphi| \le K'(|v_1\varphi| + |v_2\varphi|)$$
$$|D_2\varphi| \le K'(|v_1\varphi| + |v_2\varphi|)$$
$$\Rightarrow \quad \sum |D_i\varphi|^2 \le (2K')^2 (2|v_1\varphi|^2 + |v_2\varphi|^2)$$
$$\Rightarrow \quad |\varphi|^2 + \sum_{[\alpha]=1} |D^\alpha\varphi|^2 \le (8K'^2+1)\,(|\varphi|^2 + \sum_i |v_i\varphi|^2)$$
$$\Rightarrow \quad \int_V \rho(x)\{|\varphi|^2 + \sum_i |v_i\varphi|^2\}\,dx \ge \int_V \rho(x)\frac{1}{(8K'^2+1)}\{|\varphi|^2 + \sum_{[\alpha]=1} |D^\alpha\varphi|^2\}\,dx$$
$$\ge mK' \int_V |\varphi|^2 + \sum_{[\alpha]=1} |D^\alpha\varphi|^2\,dx = mK' \int_V \sum_{[\alpha]\le 1} |D^\alpha\varphi|^2\,dx .$$
$$K' = \frac{1}{8K'^2+1}$$

$\Rightarrow \exists \; C_1, C_2 > 0$ s.t

$$C_1 \int_V \sum_{[\alpha]\le 1} |D^\alpha\varphi|^2\,dx < \int_V \rho(x)\{|\varphi|^2 + \sum_i |v_i\cdot\varphi|^2\}\,dx$$
$$< C_2 \int_V \sum_{[\alpha]\le 1} |D^\alpha\varphi|^2\,dx .$$

$\Rightarrow$ This proves the equivalence.