

C satisfying $(**)$ satisfies all the requirements of ① & ②.

The Schubert cycle σ_c appears twice in the k th and $(k-1)$ st term.

Note that the k in $[a_{i-1}, n-k]$ is different from the k in $[a_{k-k}, a_{k+1-k}]$.

$$a_0 = n - (k-1)$$

Actually it does not matter whether we take $a_0 = n-k$ or $n-k-1$.

Originally, $\sigma_{a_1 \dots a_d} \in G(k, n)$

$$\Rightarrow \sigma_{a_1, \dots, a_{j-1}, a_{j+n-1}, \dots, a_d-1} \in G(k, n).$$

Since these two have opposite sign; σ_c will not appear in the final expression for $\tilde{\sigma}_a$.

3. If the interval $[a_{i-1}, n-k]$ is occupied, we have

$$c_i - i \in [a_{i+1} - i - 1, a_i - i - 1] \text{ for each } i -$$

but then $c_i \leq a_i - 1$, and hence $\sum c_i < \sum a_i$, so σ_c can not appear in $(*)$.

$$\Gamma \quad c_i - i \geq a_{i+1} - i - 1 \Rightarrow c_{i+1} \geq a_{i+1}$$

$$c_i - i \leq a_i - i - 1 \Rightarrow c_i \leq a_i - 1 \text{ for all } i.$$

$$\Rightarrow \sum c_i \leq \sum (a_i - 1) = \sum a_i - d < \sum a_i$$

There is no C satisfying two conditions, i.e.

$$\sum c_i = \sum a_i \text{ and } 0 \leq c_i \leq 0.$$

We have, then, the formula

$$(**) \quad (-1)^d \sigma_{a_1 \dots a_d} = \sum_{j=1}^d (-1)^j \sigma_{a_1, \dots, a_{j-1}, a_{j+1}-1, \dots, a_d-1} \cdot \sigma_{a_j+d-j}.$$