B : the base locus of D_1, D_2

$$\begin{array}{ccc} M - B & \xrightarrow{f} & \mathbb{P}^1 \\ \rotatebox{90}{$\in$} & & \\ p & \longmapsto & [(s_{\alpha_1}(p), s_{\alpha_2}(p))] \end{array}, \qquad \begin{tikzcd} L_{U_\alpha} \arrow[r] \arrow[d, shift left] & U_\alpha \times \mathbb{C} \arrow[dl, "{(x,\, s_{\alpha 1}(x))}"] \\ U_\alpha \arrow[u, shift left, "s_1"] & \end{tikzcd}$$

① $\{ s_{\alpha_1}(p) = 0 \}$ is a measure zero set in $M-B$.

②

$$\begin{tikzcd} M - B \arrow[r, "f"] \arrow[dr, "f"] & U_0 \ (\subset \mathbb{P}^1) \arrow[d] \\ & \mathbb{C} \end{tikzcd} \qquad (z_0 \neq 0)$$

$$p \longmapsto \frac{s_{\alpha_2}(p)}{s_{\alpha_1}(p)}$$

$\Rightarrow \left\{ \dfrac{\partial \frac{s_{\alpha_2}(p)}{s_{\alpha_1}(p)}}{\partial z_i} = 0 \right\}$ is measure zero. Similarly, for $U_1 (z_1 \neq 0)$ $m\left\{ \dfrac{\partial \frac{s_{\alpha_1}}{s_{\alpha_2}}}{\partial z_i} = 0 \text{ at } p \right\} = 0$ since M is compact. In other words, $\{ \operatorname{rank} J(f) < 0 \}$ is measure zero.

$V = \{ p \in M-B \mid$ rank of Jacobian of f at $p < 0 \}$

$\Rightarrow$ By Sard's theorem, see P40. Differential Topology by V. Guillemin & A. Pollack. $f(V)$ has measure zero

For a fixed $\lambda = [(\lambda_1, \lambda_2)] \in \mathbb{P}^1$,

$f^{-1}(\lambda) = \{ p \in M-B \mid s_{\alpha_1}(p) = \alpha \lambda_1, \ s_{\alpha_2}(p) = \alpha \lambda_2 \}$