Date

Comparing the two expressions for $\int_{\partial\Delta} \pi\cdot\eta$, we find the

Reciprocity Law I

$$\sum_{i=1}^{g} \left(\Pi^{i} N^{g+i} - \Pi^{g+i} N^{i} \right) = 2\pi\sqrt{-1} \sum_{\lambda} \operatorname{Res}_{s_\lambda}(\eta)\cdot \int_{s_0}^{s_\lambda} \omega,$$

where the integrals on the right are taken in the interior of Δ.

Γ

$$\int_{\partial\Delta^*} \pi\cdot\eta = 2\pi i \sum \operatorname{Res}_{s_\lambda}(\eta)\cdot\int_{s_0}^{s_\lambda}\omega$$

$$\sum_{i} \int_{\delta_i+\delta_i^{-1}} \pi\cdot\eta = \int_{\partial\Delta} \pi\cdot\eta = \sum_{i=1}^{g} \Pi^{i}\cdot N^{g+i} - \sum_{i=1}^{g} \Pi^{g+i} N^{i}$$

$$= \sum_{i=1}^{g} \left(\Pi^{i} N^{g+i} - \Pi^{g+i} N^{i} \right).$$

$\lrcorner$

This is classically known as the reciprocity law for differentials of the first and third kinds. In classical terminology, a differential of the first kind on a Riemann surface S is a holomorphic 1-form; a differential of the second kind is a meromorphic 1-form with no residues, and a differential of the third kind is a meromorphic form with only single poles. Clearly a differential is of the first kind if and only if it is both of the second kind and of the third kind; we shall see shortly that any meromorphic 1-form is the sum of differentials of the second and third kinds. Later on we will prove a reciprocity law for differentials of the first and second kinds.