0 with all Schubert cycles of complementary dimension except $\sigma_{n-k,\, n-k-1,\, n-k-2, \cdots}$.

$\ulcorner$ We have

$$(n-k+1)(k+1) - \frac{(k+1)(k+2)}{2} = \sum b_i \le (n-k+1)(k+1) - \frac{(k+1)(k+2)}{2}$$

$\Rightarrow$ Since $b_i \le n-k-i+1$, to be valid above, $b_i = n-k-i+1$. $\lrcorner$

To compute the intersection number of $\Sigma_{k,n}$ with

$$\sigma_{n-k,\, n-k-1, \cdots} = \{ \Lambda_{k+1} : \dim(\Lambda_{k+1} \cap V_{2i}) \ge i \}$$

we read off the defining conditions for $\sigma_{n-k,\, n-k-1, \cdots}$ one by one. The first condition says that any $\Lambda \in \Sigma_{k,n} \cap \sigma_{n-k,\, n-k-1, \cdots}$ must meet a line $\overline{V}_2 \subset \mathbb{P}^{n+1}$ in a point; since $\Lambda \subset F$, this point must be one of the two points p_1, p_2 of intersection of $\overline{V}_2$ with F.

$\ulcorner$ $\Lambda \in \Sigma_{k,n} \cap \sigma_{n-k,\, n-k-1, \cdots} \Rightarrow \dim(\Lambda_{k+1} \cap V_2) \ge 1$

$\Rightarrow$ $\overline{V}_2 = \mathbb{P}^1$ $\quad \Lambda_k \cap \mathbb{P}^1 \ne \phi$ $\quad$ If $\Lambda_k \supset \mathbb{P}^1$, then $\dim(\Lambda_{k+1} \cap V_2) \ge 2$, which is impossible since Λ_k can not contain any projective space of $\dim \ge \frac{2-2}{2} = 0$. See P739.

$\Rightarrow$ Λ meet a line $\overline{V}_2$ in a point.

Since we chose $\overline{V}_2 \subset \mathbb{P}^{n+1}$ so that $\overline{V}_2$ meets F transversely, $\#(\overline{V}_2 \cap F) = 2$ ($\because$ F is of degree 2 and $\overline{V}_2$ is a line.). $\Rightarrow$ $\overline{V}_2 \cap F = \{p_1, p_2\}$. $\lrcorner$