Fix $\eta_\cdot = \eta_\alpha \otimes e_\alpha$ fixed on U.

$$\sum_{\alpha,\beta} \eta_\alpha \wedge \theta_{\alpha\beta} \wedge * \tau_\beta = f(\tau)\,\Phi, \quad f: A^{p,q}(E)(U) \longrightarrow \mathbb{C}$$

$$\Rightarrow \quad \eta_\alpha \wedge \theta_{\alpha\beta} \wedge *(\tau'_\beta + \tau_\beta) = \eta_\alpha \wedge \theta_{\alpha\beta} \wedge * \tau'_\beta + \eta_\alpha \wedge \theta_{\alpha\beta} \wedge * \tau_\beta$$

$$= f(\tau)\,\Phi + f(\tau')\,\Phi = f(\tau+\tau')\,\Phi$$

$$\Rightarrow \quad f(\tau+\tau') = f(\tau) + f(\tau')$$

$$\Rightarrow \quad \eta_\alpha \wedge \theta_{\alpha\beta} \wedge *(k\,\tau_\beta) = f(k\tau)\,\Phi$$
$$= \bar{k}\,\eta_\alpha \wedge \theta_{\alpha\beta} \wedge * \tau_\beta = \bar{k}\, f(\tau)\,\Phi \quad \Rightarrow \quad \bar{k} f(\tau) = f(k\tau).$$

Consider $f(\tau) = \langle \eta, \theta^* \tau \rangle$

$$f(\tau+\tau') = \langle \eta, \theta^*(\tau+\tau') \rangle = \langle \eta, \theta^*\tau \rangle + \langle \eta, \theta^* \tau' \rangle = f(\tau) + f(\tau')$$
$$\Rightarrow \quad \theta^*(\tau+\tau') = \theta^*\tau + \theta^*\tau'$$
$$f(k\tau) = \langle \eta, \theta^*(k\tau) \rangle = \bar{k}\, f(\tau) = \bar{k} \langle \eta, \theta^*\tau \rangle = \langle \eta, k\theta^*\tau \rangle.$$

$\Rightarrow \quad \theta^*(k\tau) = k\,\theta^*\tau$. $\qquad$ θ^* is algebraic operator.

$\Rightarrow$ If $\theta_{\alpha\beta}(z_0) = 0$, $\qquad \theta^*(z_0) = 0$.

$\Rightarrow$ We conclude that

$$D'^*\tau = \partial^*\tau_\alpha \otimes e_\alpha + \theta^*\tau,$$ where θ^* is defined as above.

The difference

$$[\Lambda, \bar\partial] + i\,D'^* = [\Lambda, \theta''] + i\,\theta'^*$$

is consequently an intrinsically defined algebraic operator; since we can choose at each $z_0 \in M$