Given a holomorphic map $\pi : M \longrightarrow N$ of complex manifolds we define a map

$$\pi^* : Div(N) \longrightarrow Div(M)$$

by associating to every divisor $D = (\{U_\alpha\}, \{f_\alpha\})$ on M the pullback divisor. $\pi^* D = (\{\pi^{-1} U_\alpha\}, \{\pi^* f_\alpha\})$ on M; this is well-defined as long as $\pi(M) \not\subset D$.

See P261 note.

$$\pi^* : Div(N) \longrightarrow Div(M)$$

$$(\{U_\alpha\}, \{f_\alpha\}) \longmapsto (\{\pi^{-1}(U_\alpha)\}, \{\pi^* f_\alpha\})$$

① Is $(\{\pi^{-1}(U_\alpha)\} \{\pi^* f_\alpha\})$ a divisor on M?
In other words, $(\{\pi^{-1}(U_\alpha), \pi^* f_\alpha\})$ defines a global section of $\mathcal{M}^*/\mathcal{O}^*$? On $\pi^{-1}(U_\alpha) \cap \pi^{-1}(U_\beta) = \pi^{-1}(U_\alpha \cap U_\beta)$,

$$\frac{\pi^* f_\alpha}{\pi^* f_\beta}(x) = \frac{\pi^* f_\alpha(x)}{\pi^* f_\beta(x)} = \frac{f_\alpha(\pi(x))}{f_\beta(\pi(x))} \in \mathbb{C}^*. \text{ since } \pi(x) \in U_\alpha \cap U_\beta \text{ and } \frac{f_\alpha}{f_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta). \Rightarrow \frac{\pi^* f_\alpha}{\pi^* f_\beta} \in \mathcal{O}^*(\pi^{-1}(U_\alpha \cap U_\beta))$$

$\Rightarrow$ Yes. (Suppose. $\pi : U_\alpha \longrightarrow a V$, $a > 0$, $\Rightarrow f_\alpha(\pi(x))$ for all $x \in U_\alpha$. $\Rightarrow \frac{\pi^* f_\alpha}{\pi^* f_\beta}$ is not defined. If $\pi(M) \not\subset D$, $\exists$ at least one pt which does not make zero)

② Given a different expression for $(\{U_\alpha\}, \{f_\alpha\})$, i.e. $(\{V_\alpha\}, \{g_\alpha\})$,

On $\{V_\alpha \cap U_\beta\}$, $g_\alpha | \mathcal{O}^*(V_\alpha \cap U_\beta) = \bar{g}_\alpha$, & $f_\beta | \mathcal{O}^*(V_\alpha \cap U_\beta) = \bar{f}_\beta$ have to have the same stalk at any point in $V_\alpha \cap U_\beta$. $\Rightarrow g_\alpha = f_\beta h_{\alpha\beta}$ where $h_{\alpha\beta} \in \mathcal{O}^*(V_\alpha \cap U_\beta)$.

Thus on $\{\pi^{-1}(V_\alpha \cap U_\beta)\}$, $\pi^* h_{\alpha\beta} \pi^* f_\beta = \pi^* g_\alpha$

$\Rightarrow (\{\pi^* g_\alpha\}, \{\pi^{-1} V_\alpha\})$ define the same global section as