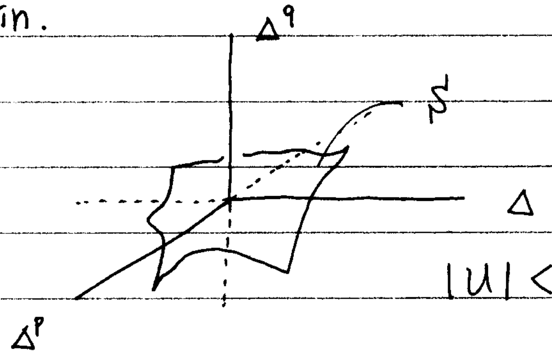
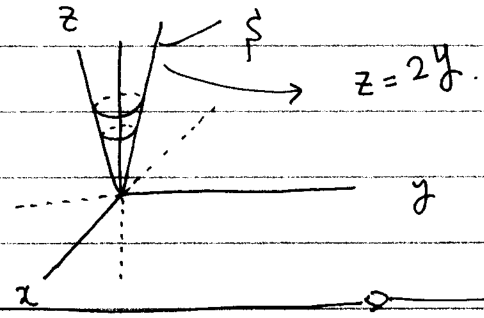
"We have to assume that $S \ni 0$, i.e., S contains the origin.



$\pi^{-1}(0) \cap S$ compact

$\Rightarrow$ By the tube lemma, $\exists\ \delta$ s.t $\{(u, v, w);$ $|u| < \delta,\ |v_i| < \delta,$

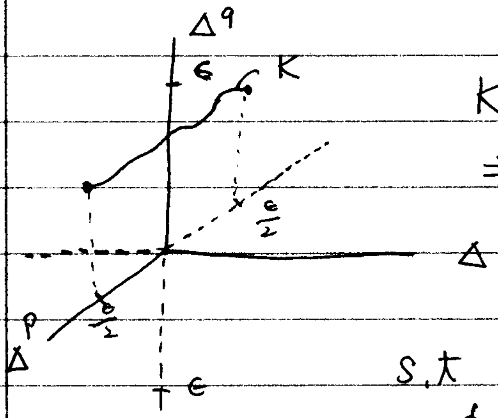
Wrong, for



"

I am going to show that $\exists\ \delta_1, \delta_2$ s.t $S \longrightarrow \Delta \times \Delta^p$ induced by $(u, v, w) \rightarrow (u, v)$ will be proper, where $\Delta = \{|u| < \delta_1\}$, $\Delta^p = \{|v_i| < \delta_2\}$.

Since $\underbrace{\pi^{-1}(\{0\} \times |v_i| \le \tfrac{\epsilon}{2}) \cap S}_{=K}$ is compact,



$K \cap \{(0, v_i, w_\alpha)\ ;\ |v_i| \le \tfrac{\epsilon}{2},\ |w_\alpha| = \epsilon\} = \phi$

$\Rightarrow \exists\ \delta$ s.t $\{(0, v_i, w_\alpha);\ |w_\alpha| = \delta,\ |v_i| \le \tfrac{\epsilon}{2}\} \cap K = \phi$, $0 < \delta < \epsilon$.

$\Rightarrow$ By the tube lemma, $\exists\ \delta_1 > 0$ s.t $\{(u, v_i, w_\alpha);\ |u| < \delta_1,\ |w_\alpha| = \delta,\ |v_i| \le \tfrac{\epsilon}{2}\} \cap S = \phi$.