nce of $0 \longrightarrow \mathbb{Z} \longrightarrow \mathcal{O}_{\mathbb{P}^2} \longrightarrow \mathcal{O}^*_{\mathbb{P}^2} \longrightarrow 0^{(=1)}$.

Γ $H^1(\mathcal{O}_{\mathbb{P}^2}(-3L)) = H^1(\mathbb{P}^2, \mathcal{O}(-3L)) = 0$, since $1+0<2$ and $[-3L] \longrightarrow \mathbb{P}^2$ is a negative line bundle, by P155 & P 156.

$H^2(\mathcal{O}_{\mathbb{P}^2}(-3L)) = H^2(\mathbb{P}^2, \mathcal{O}(-3L)) \cong H^0(\mathbb{P}^2, \Omega^2(3L)) = H^0(\mathbb{P}^2, \Omega^2(3H))$
$= H^0(\mathbb{P}^2, \mathcal{O}(K+3H)) = H^0(\mathbb{P}^2, \mathcal{O}(-(2+1)H+3H)) = H^0(\mathbb{P}^2, \mathcal{O}) \cong \mathbb{C}$
by P146. $K_{\mathbb{P}^n} \cong [-(n+1)H]$. P153. Kodaira-Serre duality.

$H^2(\mathcal{O}^*_{\mathbb{P}^2}) = H^2(\mathbb{P}^2, \mathcal{O}^*) = ?$

$$0 \longrightarrow \mathbb{Z} \longrightarrow \mathcal{O} \longrightarrow \mathcal{O}^* \longrightarrow 0$$

$$\Rightarrow \quad \underset{\mathbb{Z}}{\underset{\|}{H^2(\mathbb{P}^2, \mathbb{Z})}} \longrightarrow \underset{0 \text{ by P49 \& P118}}{\underset{\|}{H^2(\mathbb{P}^2, \mathcal{O})}} \longrightarrow \underset{0}{\underset{\|}{H^2(\mathbb{P}^2, \mathcal{O}^*)}} \longrightarrow \underset{0}{\underset{\|}{H^3(\mathbb{P}^2, \mathbb{Z})}}$$

$\Rightarrow H^2(\mathcal{O}^*_{\mathbb{P}^2}) = 0$ $\lrcorner$

Thus we obtain

$$0 \longrightarrow H^1(\mathcal{O}^*_{\mathbb{P}^2}) \longrightarrow H^1(\mathcal{O}^*_{(2)}) \longrightarrow \mathbb{C} \longrightarrow 0,$$

and consequently: There is one condition on $\mathcal{L}_{(2)} \in H^1(\mathcal{O}^*_{(2)})$ to be the restriction of some $\mathcal{L} \in H^1(\mathcal{O}^*_{\mathbb{P}^2})$.

Γ $0 \longrightarrow 1+I^3 \longrightarrow \mathcal{O}^* \longrightarrow \mathcal{O}^*/1+I^3 \longrightarrow 0$

Note that $\mathcal{O}^*/1+I^3 \cong \mathcal{O}^*_{(2)} = (\mathcal{O}/I^3)^*$.

$$\begin{array}{ccc} (\mathcal{O}/I^3)^* & \xrightarrow{\phi} & \mathcal{O}^*/1+I^3 \\ \rotatebox{90}{$\in$} & & \\ g+I^3 & \longmapsto & g\,(1+I^3) \end{array}$$

$\Rightarrow$ Given any $g \in \mathcal{O}^*$, i.e nonzero holomorphic function