ϕ is onto $\Rightarrow$ $\mathbb{C}^2 \cong \mathcal{O}^{(2)} / m\,\mathcal{O}^{(2)}$.

We have proved the finite dimensionality and also Theorems A and B in case $\mathcal{F} = \mathcal{O}(E)$ is locally free so that potential-theoretic methods may be used.

$\ulcorner$ $\dim H^q(M, \mathcal{O}(E)) < \infty$ see p151 ~ p153.
$H^q(M, \mathcal{O}(E \otimes [kH])) = 0$ see p159. Theorem B
$H^0(M, \mathcal{O}(E \otimes [kH])) \xrightarrow{r_x} \mathcal{O}(E \otimes [kH])_x / m_x \mathcal{O}$ is onto

see p180 & p208.

Now we prove the assertions:

1. Theorem A $\Rightarrow$ Theorem B.
2. Theorem B $\Rightarrow$ Theorem A.
3. $\dim H^1(M, \mathcal{F}) < \infty$ for all coherent sheaves $\mathcal{F} \Rightarrow$ Theorem A.

Proof of 1. Assuming Theorem A, we have

$$0 \to \mathcal{G}' \to \mathcal{O}^{(p)} \to \mathcal{F}(k) \to 0$$

for some large k.

$\ulcorner$ We assumed here that $\dim H^0(M, \mathcal{F}(k)) < \infty$. $\Rightarrow$
By Theorem A,

$$H^0(M, \mathcal{F}(k)) \longrightarrow \mathcal{F}(k) \longrightarrow 0 \quad \text{globally}$$

$$\Rightarrow \quad \mathcal{O}^{(p)} \xrightarrow{\alpha} \mathcal{F}(k) \to 0, \quad p = \dim H^0(M, \mathcal{F}(k)).$$

$$\Rightarrow \quad 0 \to \ker\alpha \to \mathcal{O}^{(p)} \longrightarrow \mathcal{F}(k) \to 0. \quad \text{Let } \ker\alpha = \mathcal{G}'.$$