Suppose φ & φ' real plurisubharmonic s.t $\sqrt{-1}\,\partial\bar\partial\varphi = \sqrt{-1}\,\partial\bar\partial\varphi'$ $\Rightarrow$ $\varphi - \varphi' = \gamma$ s.t $\sqrt{-1}\,\partial\bar\partial\gamma = 0$.

If γ is any such function then $\partial\gamma$ is a $\bar\partial$-closed current of type $(1,0)$, and so again by regularity is a closed holomorphic 1-form.

$$\partial\gamma = \sum T_i\, dz_i \Rightarrow \bar\partial\partial\gamma = 0 = \sum \bar\partial T_i \wedge dz_i$$
$$= \sum \frac{\partial T_i}{\partial \bar z_j}\, d\bar z_j \wedge dz_i \Rightarrow \frac{\partial T_i}{\partial \bar z_j} = 0 \Rightarrow \bar\partial T_i = 0$$

$\Rightarrow \exists\ f_i$ s.t f_i holomorphic and $T_i = T_{f_i}$. by regularity theorem, P380. $\Rightarrow$ $\partial\gamma$ is a closed holomorphic 1-form, since $\bar\partial\partial\gamma = \partial\partial\gamma = 0 = d\partial\gamma$.

Setting $f(z) = \int_{z_0}^{z} \partial\gamma$, then, we have $\gamma = \mathrm{Re} f$.

Q.E.D.

$n = 1$, $\quad \partial\gamma = \frac{\partial\gamma}{\partial z}\, dz = \left(\frac{\partial\gamma}{\partial x} - i\frac{\partial\gamma}{\partial y}\right) dz$

$$\Rightarrow \int_{z_0}^{z} \partial\gamma = \int_{z_0}^{z}\left(\frac{\partial\gamma}{\partial x} - i\frac{\partial\gamma}{\partial y}\right) dz = \int_0^t \left(\frac{\partial\gamma}{\partial x} - i\frac{\partial\gamma}{\partial y}\right)\cdot \alpha'(t)\, dt$$

$$= \int_0^t \frac{\partial\gamma}{\partial x}(\alpha(t))\,\alpha'(t)\,dt - i\int_0^t \frac{\partial\gamma}{\partial y}(\alpha(t))\,\alpha'(t)\,dt$$

$$= \int_0^t \frac{\partial\gamma}{\partial x}\alpha_1'(t) + \frac{\partial\gamma}{\partial y}\alpha_2'(t)\,dt + i\int_0^t \frac{\partial\gamma}{\partial x}\alpha_2'(t) - \frac{\partial\gamma}{\partial y}\alpha_1'(t)\,dt$$

$$= \int_0^t \frac{d\gamma(\alpha_1(t), \alpha_2(t))}{dt}\,dt + i\int_0^t \boxed{}\,dt$$