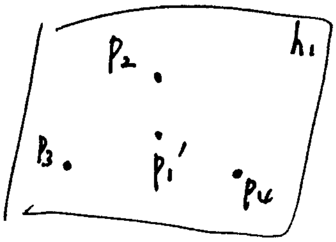
96 6 5

First, we want to show that $p_i' \in h_i \cap h_j'$.
for simplicity, let $i = 1$ and $j = 2$.
Since p_1' is the focus of the pencil $\tilde{\Phi}^{-1}(X_{h_1}) = \tilde{\Phi}^{-1}(\tilde{\Phi}(\sigma(h_1)) \cap H)$ of lines of X lying in h_1, $p_1' \in h_1$.
So to show $p_1' \in h_1 \cap h_2'$, it remains to show that $p_1' \in h_2'$. To do this, what is h_2'?

$$G(2,4) \xrightarrow{\tilde{\Phi}} \mathbb{P}^5 \supset G .$$

$\Rightarrow$ ⊛ : $\tilde{\Phi}^{-1}(\tilde{\Phi}(\sigma(p_2)) \cap H) \ni l \iff l \ni p_2$ and $l \subset h_2'$
Consider the line $\overline{p_1' p_2}$.



by the construction.

$\Rightarrow$ Clearly $\overline{p_1' p_2} \subset h_1$ and $\overline{p_1' p_2} \ni p_1'$
$\Rightarrow$ $\overline{p_1' p_2} \in \tilde{\Phi}^{-1}(\tilde{\Phi}(\sigma(h_1)) \cap H)$
$\Rightarrow$ $\tilde{\Phi}(\overline{p_1' p_2}) \in H$, and $\overline{p_1' p_2} \ni p_2$ $\Rightarrow$
$\tilde{\Phi}(\overline{p_1' p_2}) \in \tilde{\Phi}(\sigma(p_2)) \Rightarrow \tilde{\Phi}(\overline{p_1' p_2}) \in \tilde{\Phi}(\sigma(p_2)) \cap H$
$\Rightarrow$ $\overline{p_1' p_2} \in \tilde{\Phi}^{-1}(\tilde{\Phi}(\sigma(p_2)) \cap H) \Rightarrow \overline{p_1' p_2} \subset h_2'$ by ⊛
$\Rightarrow$ $p_1' \in h_2'$ //
The claim that $h_1 \cap h_2'$ is a line is wrong, I think.
Fix p_2, and let h_2' be the plane of the