Existence.

$s \in \mathcal{F}^+(U)$, $\tau \in \mathcal{F}^+(V)$. s.t $\mathcal{F}^+(U\cap V) \ni s|_{U\cap V} = \tau|_{U\cap V}$.

$\Rightarrow$ define $\rho : U \cup V \longrightarrow \bigcup_{p \in U\cup V} \mathcal{F}_p$ by

$$\rho(p) = s(p). \quad \text{if } p \in U$$
$$\rho(p) = \tau(p). \quad \text{if } q \in V$$

$\Rightarrow$ ρ is well-defined since $s(p) = \tau(p)$ if $p \in U\cap V$.
ρ satisfies the first condition. & second condition.

Uniqueness.
$\sigma \in \mathcal{F}^+(U \cup V)$. s.t $\sigma|_U = 0$ $\sigma|_V = 0$.

$$\sigma : U \cup V \longrightarrow \bigcup_{p\in U\cup V} \mathcal{F}_p.$$

$\sigma(p) = 0$ for $p \in U$. $\sigma(p) = 0$ for $p \in V \Rightarrow \sigma(p) = 0$ for all $p \in U \cup V$.

Define $\theta : \mathcal{F} \longrightarrow \mathcal{F}^+$ by

$$\theta(U) : \mathcal{F}(U) \longrightarrow \mathcal{F}^+(U)$$
$$\sigma \longmapsto s_\sigma : U \longrightarrow \bigcup_{p\in U} \mathcal{F}_p$$
$$p \longmapsto \sigma(p) \in \mathcal{F}_p.$$

$\Rightarrow$

$$\begin{array}{ccc} \mathcal{F}(U) & \xrightarrow{\theta_U} & \mathcal{F}^+(U) \\ \rho_{UV} \downarrow & & \downarrow \rho'_{UV} \\ \mathcal{F}(V) & \xrightarrow{\theta_V} & \mathcal{F}^+(V) \end{array}$$