tal class of V is nonzero in the homology of M.

This is easy: if M has dimension m and V dimension k, we can find a linear subspace $\mathbb{P}^{n-k}$ of $\mathbb{P}^n$ meeting V in isolated points, and setting $W = M \cap \mathbb{P}^{n-k}$,

$$\#(W \cdot V) > 0,$$

which implies $\eta_V \neq 0 \in H^{2n-2k}(M)$.

「 Suppose we can find a linear subspace $\mathbb{P}^{n-k}$ of $\mathbb{P}^n$ meeting V in isolated points. Let $W = M \cap \mathbb{P}^{n-k}$

$$W \cap V = M \cap \mathbb{P}^{n-k} \cap V = V \cap \mathbb{P}^{n-k}.$$

$$\Rightarrow \#\{W \cap V\} > 0 \Rightarrow \#(W \cdot V) > 0$$

$\Rightarrow$ Since $\displaystyle\int_M \eta_V \wedge \eta_W = \#(W \cdot V) > 0$,

$$\eta_V \neq 0 \text{ in } H^{2n-2k}(M).$$

It remains to show that $\exists$ a linear subspace $\mathbb{P}^{n-k}$ meeting V in isolated points.

Choose a smooth point $x \in V$. Consider all linear subspaces $\mathbb{P}^{n-k}$ passing x. $\Rightarrow$ $\exists$ a linear subspace $\mathbb{P}^{n-k}$ passing through x which meet with V transversely. 」

As a corollary, we see that

The even Betti numbers of M are positive,

since by the above the intersection V of M with a