$$\psi(\lambda) = \sum_i \int_{q_i(\lambda)}^{p_i(\lambda)} \omega \quad (\text{modulo } \Lambda).$$

Γ $f : C \longrightarrow \mathbb{P}^1$

$$\lambda = [\lambda_0, \lambda_1].$$

$D_\lambda = (\lambda_0 f - \lambda_1) \Rightarrow$ zeros correspond to points where value λ, and poles are unchanged.

ψ is thus a holomorphic map $\mathbb{P}^1 \longrightarrow \mathbb{C}/\Lambda$; by the same argument as before we see

$$\psi^* dz \in H^0(\mathbb{P}^1, \Omega^1) = 0$$

$\Rightarrow$ ψ constant and since, as $\lambda_0 \longrightarrow 0$, $\{p_i(\lambda)\} \longrightarrow \{q_i(\lambda)\}$, we have $\psi \equiv 0$ (modulo Λ). Q.E.D.

Following some preliminaries concerning the reciprocity formulas, we will give the converse to this version of Abel's theorem for Riemann surfaces of arbitrary genus. Together with the Riemann-Roch formula, these constitute the fundamental tools in the study of algebraic curves.

Let S now be a compact Riemann surface of genus g, and let $\delta_1, \cdots \delta_{2g}$ be 1-cycles in S forming a basis for $H_1(S, \mathbb{Z})$. We may take $\delta_1, \cdots \delta_{2g}$ to be a canonical basis, i.e., such that δ_i intersects δ_{i+g} once positively, and does not intersect any other δ_j. In such a canonical basis, the cycles $\delta_1 \cdots \delta_g$ are called the A-cycles, $\delta_{g+1}, \delta_{g+2}, \cdots \delta_{2g}$ the B-cycles.