on P 766. $\Rightarrow$ By Bertini's theorem, generic elements of the linear system are smooth, since X_p's are disjoint. But, by the defition of S, all elements of the system are singular. $\Rightarrow$ r is a 2-sheeted cover and $\exists$ no three of p_i's which are collinear.
$\Rightarrow$ Since $\overline{P_{ij}}, \overline{P_{jk}}, \cdots,$ lie on a conic, $\overline{q_1}, \overline{q_2}$ and $\overline{q_3}$ can not lie on L_i, L_j. and L_k.
Relate this a result to P773 and P929 note. ⌟

These three lines, however, account for 12 of the 15 points $\{P_{ij}\}$; consequently the points q_1, q_2, and q_3 can only be the points $P_{\ell m}$, P_{mn} and $P_{\ell n}$.

⌈ Each of L_i, L_j and L_k contains 5 $\overline{P_{ij}}$'s
$\Rightarrow 5 \times 3 - \#\{\overline{P_{ij}}, \overline{P_{ik}}, \overline{P_{jk}}\} = 12.$
$\Rightarrow$ Since $\overline{q_1}, \overline{q_2}, \overline{q_3}$ can not lie on L_i, L_j, L_k, $\overline{q_1}, \overline{q_2}, \overline{q_3}$ must be on L_m, L_n, L_ℓ.
$\Rightarrow \{q_1, q_2, q_3\} = \{P_{mn}, P_{m\ell}, P_{n\ell}\}.$ ⌟

Thus, if we label the 16 double points of S by $\{P_0, P_{ij}\}$ and the 16 double points of S^* as $\{h_i, h_{ijk} = h_{\ell mn}\}$, the incidence relations are

$$h_i \supset \{P_0, P_{ij}\}, \quad j \neq i,$$
$$h_{ijk} \supset \{P_{ij}, P_{ik}, P_{jk}, P_{\ell m}, P_{mn}, P_{n\ell}\},$$