

$$\{p + D_0 + D_\lambda \mid D_\lambda \sim D_0\}$$

are all hyperplane sections of  $E_0 \subset \mathbb{P}^3$ , and so  $p$  must be collinear with the points of  $D_0$ .

Clearly,  $p + D_0 + D_\lambda \sim p + 2D_0$ . Since  $L^* H^0(\mathbb{P}^3, \mathcal{O}(H)) = H^0(B, \mathcal{O}(D))$  by P177,  $\exists$  a hyperplane in  $\mathbb{P}^3$  s.t. its intersection with  $E_0$  is  $p + D_0 + D_\lambda$  ( $\because \exists$  a section  $\sigma$  on  $E_0$  s.t.  $(\sigma=0) = p + D_0 + D_\lambda$ ).  
 $\Rightarrow$  Each  $p + D_0 + D_\lambda$ ,  $D_\lambda \sim D_0$ , is a hyperplane section of  $E_0 \subset \mathbb{P}^3$ . Suppose  $H_1 \cdot E_0 = p + D_0 + D_{\lambda_1}$  and  $H_2 \cdot E_0 = p + D_0 + D_{\lambda_2} \Rightarrow H_1 \cap H_2$  contains  $p$  and points in  $D_0$ .  $\deg D_0 = 2$ .

The lines  $\{L_\lambda = \overline{p, D_\lambda} \mid D_\lambda \sim D_0\}$ , containing three points of  $E_0$ , must lie on the quadric  $Q$ ; it follows that

$$Q = \bigcup L_\lambda$$

is a singular quadric with singular point  $p$ .

Since  $D_\lambda \sim D_0$ , by the same argument, i.e., by considering  $2D_\lambda + p$ , we can conclude that  $\overline{p, D_\lambda}$  is a line.  $\Rightarrow L_\lambda = \overline{p, D_\lambda} \Rightarrow \#(L_\lambda \cdot Q) \geq 3 \Rightarrow$   
 Since  $Q$  is quadric,  $Q \supset L_\lambda \Rightarrow Q \supset \bigcup_{D_\lambda \sim D_0} L_\lambda$ .

$$\Rightarrow \dim |D_0| = \dim \{D_\lambda \mid D_\lambda \sim D_0\} = h^0(D_0) - 1 = 2 - 1 = 1$$