for every pair $(a, b) \in \mathbb{C}^2$. 」

The Plücker Embedding

We close this section by describing the classical Plücker embedding of the Grassmannian $G(k,n)$ in projective space; this will illustrate both the Kodaira embedding theorem and Chow's theorem. The embedding line bundle over $G(k,n)$ will be $L = \det S^* = \det Q$. L may be seen to be positive by introducing a suitable metric with a positive curvature form in a similar manner to the Fubini-Study metric on projective space; rather than do this, however, we shall give the Plücker embedding directly.

「See P30 & P150. & P148 」

The Plücker map

$$p : G(k,n) \longrightarrow \mathbb{P}(\Lambda^k \mathbb{C}^n) = \mathbb{P}^{\binom{n}{k}-1}$$

simply sends a k-plane $\Lambda = \mathbb{C}\{v_1, v_2, \cdots v_k\} \subset \mathbb{C}^n$ to the multivector $v_1 \wedge \cdots \wedge v_k$. Explictly, in terms of the basis $\{e_I = e_{i_1} \wedge \cdots \wedge e_{i_k}\}_{\#I=k}$ for $\Lambda^k\mathbb{C}^n$, this map is given by

$$\Lambda \longmapsto [\cdots, |\Lambda_I|, \cdots],$$

i.e., the homogeneous coordinates of the map are