

$\phi(E_i) = \tau_i$, $\phi(G_j) = l_j$, where l_j is the unique line intersecting with five lines $\tau_1, \dots, \tau_j, \dots, \tau_6$ by the first observation.

$\phi(H_{ij}) = \eta_{ij}$ where η_{ij} is the unique line intersecting with τ_i & τ_j , $i \neq j$, by the second observation.

Let \oplus be the intersection operation, i.e.

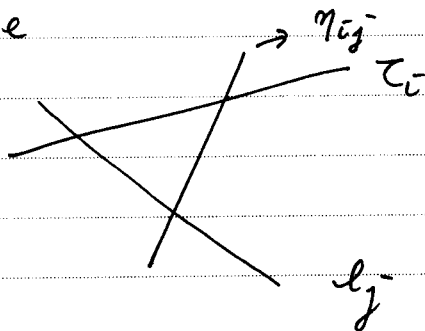
$E_i \oplus G_j = H_{ij}$ the unique line meeting with E_i & G_j , $i \neq j$.

What we have to show is that ϕ is homomorphism, i.e. $\phi(E_i \oplus H_{ij}) = \phi(E_i) \oplus \phi(H_{ij})$, etc.

$$(i) \phi(E_i \oplus H_{ij}) = \phi(G_j) = l_j$$

$$\phi(E_i) \oplus \phi(H_{ij}) = \tau_i \oplus (\tau_i \oplus l_j) = \tau_i \oplus \eta_{ij} = l_j$$

since



$$(ii) \phi(G_i \oplus H_{ij}) = \phi(E_j) = \tau_j \text{ and } \phi(G_i) = l_i, \phi(H_{ij}) = \eta_{ij} \Rightarrow \tau_j = l_i \oplus \eta_{ij}$$

$$(iii) \phi(H_{12} \oplus H_{34}) = \phi(H_{56}) = \eta_{56} \Rightarrow \phi(H_{12}) \oplus \phi(H_{34}) = \eta_{12} \oplus \eta_{34} = \eta_{12} \oplus (\tau_3 \oplus l_4) = \textcircled{?} = \eta_{56}. \text{ To show this, we need the third observation}$$

③ η_{ij} does not meet with η_{ik} , $j \neq k$
 η_{ij} " τ_k and l_k , $k \neq i, j$.