

Let $\deg f = d_1$ & $\deg g = d_2 \Rightarrow$ By Bezout's theorem, f, g have at most $d_1 d_2$ simultaneous solutions, i.e. \exists at most $d_1 d_2$ points satisfying $f=0$ & $g=0$, which means $\#(\{f=0\} = C \cap \{g=0\} = D) \leq d_1 d_2$. \square

The degree also behaves well with respect to the geometric operations of projection and coning. Clearly, if $V \subset \mathbb{P}^n$ is any variety, $p \in \mathbb{P}^n$ any point not on V , and $\pi_p: V \rightarrow \mathbb{P}^{n-1}$ the projection onto a hyperplane, then $\deg(V) = \deg(\pi_p(V))$:

the number of points of intersection of $\pi(V)$ with a generic $(n-k-1)$ -plane \mathbb{P}^{n-k-1} in \mathbb{P}^{n-1} is just the number of points of intersection of V with the $(n-k)$ -plane $\mathbb{P}^{n-k} = \overline{\mathbb{P}^{n-k-1} p}$ in \mathbb{P}^n ; since by Bertini the generic \mathbb{P}^{n-k} through p meets V transversely, this is just the degree of V .

∇ I don't think that $\deg(V) = \deg(\pi_p(V))$ is correct.

Here is a trivial counterexample.

$$V = \{[1, 1, 2], [1, 1, 3]\} \subset \mathbb{P}^2$$

$$p = [0, 0, 1] \Rightarrow \text{Obviously, } p \notin V.$$

$$\pi_p: V \rightarrow \mathbb{P}^1$$

$$[1, 1, 2] \mapsto [1, 1, 0] \Rightarrow \pi_p(V) = \{[1, 1, 0]\}$$

$$[1, 1, 3] \mapsto [1, 1, 0]$$

\Rightarrow The number of points of intersection of $\pi_p(V)$ with a generic 1-plane \mathbb{P}^1 in \mathbb{P}^1 is 1, while the # of points of intersection of V with 2-plane $\mathbb{P}^2 = \overline{\mathbb{P}^1 p}$ in \mathbb{P}^2 is 2.

(See Mumford, Algebraic Geometry I, Complex Projective Geometry p12, Proposition (5.5))

The set S of all hyperplanes through p in \mathbb{P}^n forms a