⫍ $dT_P(\varphi) = (-1)^{q+1}\, T_P(d\varphi) = (-1)^{q+1} \int_P d\varphi \underset{\substack{\Uparrow \\ \text{Stokes' theorem}}}{=} (-1)^{q+1} \int_{\partial P} d\varphi$

$= (-1)^{q+1}\, T_{\partial P}(\varphi)$. ⫎

Thus, d on the currents induces the usual exterior derivative on the smooth forms and $\pm\partial$ on the piecewise smooth chains.

Here is an example that interpolates between these two.

3. Suppose that $\psi \in L^q(\mathbb{R}^n, loc)$ is C^∞ outside a closed set S, and assume moreover that $d\psi$ on $\mathbb{R}^n - S$ extends to a locally L'-form on $\mathbb{R}^n$. We define the residue $R(\psi)$ by the equation of currents

$$dT_\psi = T_{d\psi} + R(\psi). \qquad (*)$$

It is clear that the support $\operatorname{supp} R(\psi) \subset S$.

⫍ $(dT_\psi - T_{d\psi})(\varphi) = (dT_\psi)(\varphi) - T_{d\psi}(\varphi) = 0$ for all $\varphi \in A_c^{n-q}(\mathbb{R}^n - S) \Rightarrow \operatorname{supp}(dT_\psi - T_{d\psi}) = \operatorname{supp}(R(\psi)) \subset S$. ⫎

For example, suppose that we consider the Cauchy kernel

$$\kappa = \frac{1}{2\pi\sqrt{-1}} \frac{dz}{z}$$

on $\mathbb{C}$. Then $\kappa \in L^{1,0}(\mathbb{C}, loc)$ and is C^∞ on $\mathbb{C} - \{0\}$; moreover, $d\kappa = \bar\partial\kappa = 0$ there.

⫍ κ is $(1,0)$-form and its coefficient $\frac{1}{z}$ is locally L' in $\mathbb{C} - \{0\}$. By P185, Rudin's F.A.,

A complex measurable function f, defined in an open set $\Omega \subset \mathbb{R}^n$, is said to be locally L^2 in Ω if $\int_K |f|^2 dm_n < \infty$