generated by $f_1, \cdots f_k$, $K = \mathcal{O}f_1 \oplus \mathcal{O}f_2 \oplus \cdots \oplus \mathcal{O}_{f_k} \cong \mathcal{O}^{(k)}$.

$\Rightarrow$ By the definition of projective,

$$\begin{tikzcd} & M \arrow[d, dashed, "\gamma"'] \arrow[dr, "\text{identity}"] & & \\ & \mathcal{O}^{(k)} \arrow[r, "\alpha"] & M \arrow[r] & 0 \end{tikzcd} \qquad \alpha \circ \gamma = id$$

$$(g_1 \cdots g_k) \longmapsto \sum_i g_i f_i$$

We may assume that k is the minimal number of generators of M, or equivalently that the map $\mathbb{C}^k \xrightarrow{\alpha_0} M_0$ on fibers is an isomorphism. Then $\gamma \circ \alpha$ is surjective and γ is surjective on the fibers.

First of all, we want to show $\mathcal{O}^{(k)} / m\mathcal{O}^{(k)} \cong \mathbb{C}^k$.

For example $k = 2$. $\dfrac{\mathcal{O} \oplus \mathcal{O}}{m(\mathcal{O} \oplus \mathcal{O})} \xrightarrow{\ \phi\ } \mathcal{O}/m\mathcal{O} \oplus \mathcal{O}/m\mathcal{O}$

$$(g_1, g_2) + m(\ \ ) \longmapsto (g_1 + m\mathcal{O},\ g_2 + m\mathcal{O})$$

well-defined, since $(m_1 h_1,\ m_1 h_2) \longmapsto (h_1 m_1 + m\mathcal{O},\ m_1 h_2 + m\mathcal{O})$ $= (m\mathcal{O},\ m\mathcal{O}) = (0, 0)$.

$$\mathcal{O}/m\mathcal{O} \oplus \mathcal{O}/m\mathcal{O} \xrightarrow{\ \psi\ } \frac{\mathcal{O} \oplus \mathcal{O}}{m(\mathcal{O} \oplus \mathcal{O})}$$

$$(g_1 + m\mathcal{O},\ g_2 + m\mathcal{O}) \longmapsto (g_1, g_2) + m(\mathcal{O} \oplus \mathcal{O})$$

If $g_1 = m_1 h_1 + m_2 h_2$, $g_2 = m_1' h_1' + m_2' h_2'$, then (g_1, g_2) $= (m_1 h_1 + m_2 h_2,\ m_1' h_1' + m_2' h_2') = (m_1 h_1, 0) + (m_2 h_2, 0) + (0, m_1' h_1') + (0, m_2' h_2') = m_1(h_1, 0) + m_2(h_2, 0) + m_1'(0, h_1') + m_2'(0, h_2') \in m(\mathcal{O} \oplus \mathcal{O})$ $\Rightarrow$ ψ is well-defined.

JONG IE WAH YUN PILL