

$$K^{p,q-1} \xrightarrow{\delta^{p,q-1}} K^{p,q} \xrightarrow{\delta^{p,q}} K^{p,q+1}$$

$$K^{p,q} \xrightarrow{d^{p,q}} K^{p+1,q} \xrightarrow{d^{p+1,q}} K^{p+2,q}$$

Since  $d^{p,q} \delta^{p,q-1} + \delta^{p+1,q-1} d^{p,q-1} = 0,$

$$\begin{array}{ccc} H_{\delta}^q(K^{p,*}) & \xrightarrow{d^{p,q}} & H_{\delta}^q(K^{p+1,*}) \\ \parallel & & \parallel \\ \ker \delta^{p,q} & & \ker \delta^{p+1,q} \\ \hline \text{Im } \delta^{p,q-1} & & \text{Im } \delta^{p+1,q-1} \end{array}$$

$$\begin{aligned} & d^{p,q} \delta^{p,q-1}(a) + \delta^{p+1,q-1} d^{p,q-1}(a) = 0 \\ \Rightarrow & d^{p,q} \delta^{p,q-1}(a) = -\delta^{p+1,q-1} d^{p,q-1}(a) \in \text{Im } \delta^{p+1,q-1} \\ \Rightarrow & d^{p,q} \text{ is well-defined. This is it, that is,} \end{aligned}$$

$$\rightarrow H_{\delta}^q(K^{p,*}) \xrightarrow{d} H_{\delta}^q(K^{p+1,*}) \xrightarrow{d} \dots$$

has meaning.  $\Rightarrow$

There is one point to be careful of here. A class  $[a] \in 'E_1^{p,q}$  is given by  $a \in K^{p,q}$  satisfying  $\delta a = 0$  and taken modulo  $\delta K^{p,q-1}$ . Then a class  $[a] \in 'E_2^{p,q}$  is given by  $a \in K^{p,q}$  satisfying

$$\left\{ \begin{array}{l} \delta a = 0 \\ da \in \delta K^{p+1,q-1} \end{array} \right.$$

and taken modulo

$$\delta K^{p,q-1} + dK^{p-1,q} \cap \ker \delta;$$

we can not assume  $da=0$ , but only that  $[da] = 0$  in  $H_{\delta}^q(K^{p+1,*})$ .

$$\mathbb{F} H_{\delta}^q(K^{p,*}) = \frac{\ker \delta^{p,q}}{\delta K^{p,q-1}} \xrightarrow{d} \frac{\ker \delta^{p+1,q}}{\delta K^{p+1,q-1}} = H_{\delta}^q(K^{p+1,*})$$