and P697 note. Let $\mathcal{E}_\alpha = \dfrac{\mathcal{F} \oplus \mathcal{E}^\circ}{\operatorname{Im}\mu}$.

$\Rightarrow$ We have an extension sheaf of $\mathcal{G}$ by $\mathcal{F}$, i.e.,

$$0 \longrightarrow \mathcal{F}|_{U_\alpha} \longrightarrow \mathcal{E}_\alpha \longrightarrow \mathcal{G}|_{U_\alpha} \longrightarrow 0,$$

where $\mathcal{F}|_{U_\alpha}$ and $\mathcal{G}|_{U_\alpha}$ mean sheaves over U_α, not groups.

$$\begin{tikzcd}
0 \arrow[r] & \mathcal{F}|_{U_\alpha} \arrow[r] \arrow[d, equal] & \mathcal{E}_\alpha \arrow[r] \arrow[d, "?"] & \mathcal{G}|_{U_\alpha} \arrow[r] \arrow[d, equal] & 0 \\
0 \arrow[r] & \mathcal{F}|_{U_\beta} \arrow[r] & \mathcal{E}_\beta \arrow[r] & \mathcal{G}|_{U_\beta} \arrow[r] & 0
\end{tikzcd}$$

Define a sheaf map $\underset{\text{over } U_\alpha \cap U_\beta}{\wedge}$ from $\mathcal{E}_\alpha|_{U_\alpha \cap U_\beta}$ to $\mathcal{E}_\beta|_{U_\alpha \cap U_\beta}$ as follows:

$$\frac{\mathcal{F} \oplus \mathcal{E}_0}{\mu_\alpha(\mathcal{E}_1/\partial\mathcal{E}_2)} \xrightarrow{(id - \eta_{\alpha\beta}) \oplus id \,=\, \varphi_{\alpha\beta}} \frac{\mathcal{F} \oplus \mathcal{E}_0}{\mu_\beta(\mathcal{E}_1/\partial\mathcal{E}_2)}$$

It is well-defined, since

$$(\varphi_\alpha(e_1), \ \partial(e_1)) \xmapsto{(id - \eta_{\alpha\beta}) \oplus id} (\varphi_\alpha(e_1) - \eta_{\alpha\beta}(\partial(e_1)), \ \partial(e_1))$$
$$= (\varphi_\beta(e_1), \ \partial(e_1))$$

by the relation $\varphi_\alpha - \varphi_\beta = \partial^* \eta_{\alpha\beta}$. ($\because$

$$\prod \underline{\mathrm{Hom}}_{\mathcal{O}}(\mathcal{E}_1/\partial\mathcal{E}_2, \mathcal{F})(U_\alpha) \longrightarrow \prod_{\alpha \neq \beta} \underline{\mathrm{Hom}}_{\mathcal{O}}(\mathcal{E}_1/\partial\mathcal{E}_2, \mathcal{F})(U_\alpha \cap U_\beta)$$