

$$G(X_0, X_1, X_2, X_3) = (X_0, X_1, X_2, X_3) \begin{pmatrix} q'_{00} & q'_{01} & q'_{02} & q'_{03} \\ q'_{10} & q'_{11} & q'_{12} & q'_{13} \\ q'_{20} & q'_{21} & q'_{22} + \lambda & q'_{23} \\ q'_{30} & q'_{31} & q'_{32} & q'_{33} + \lambda \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{pmatrix} = 0$$

$$\cap \left\{ \frac{\partial F}{\partial X_0} = 0, \frac{\partial F}{\partial X_1} = 0, \frac{\partial F}{\partial X_2} = 2X_2 = 0, \frac{\partial F}{\partial X_3} = 2X_3 = 0 \right\} \\ (= \{ [X_0, X_1, 0, 0] \})$$

$= G(X_0, X_1, 0, 0)$ is singular quadric in \mathbb{P}^1
i.e. G is tangent to the singular line $\{[X_0, X_1, 0, 0]\}$ of F .

\Rightarrow Thus the tangent cone of W_1 at a point $F \in W_2$ is the set of all quadrics tangent to the singular line of F ($= \{[X_0, X_1, 0, 0]\}$), for given any quadric H tangent to $\{[*, *, 0, 0]\}$,

then $\begin{vmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{vmatrix} = 0$. Consider the pencil

$$L = \{ \lambda Q + H \} \subset T_F(W_1).$$

□

To find the degree of W_2 , we proceed as in 4 above: W_2 is the image in W of $\mathbb{P}^{3*} \times \mathbb{P}^{3*}$ by the map f sending a pair (H_1, H_2) of hyperplanes in \mathbb{P}^{3*} to the quadric $H_1 + H_2$.

$\mathbb{P} \quad f : \mathbb{P}^{3*} \times \mathbb{P}^{3*} \longrightarrow \mathbb{P}^{3*} \times W$ is defined by $(H_1, H_2) \mapsto$