

nce of arguments $= 2\pi i \cdot \text{some integer}$ ($\because p = p'$ on S
 $\Rightarrow f(p) = f(p')$). Everything of the rest follows easily.)

The Riemann-Roch Formula

The starting point for our discussion of linear systems is the natural question: given a divisor D on a Riemann surface S of genus g , determine the dimension of $H^0(S, \mathcal{O}(D))$, i.e., the number of meromorphic functions f on S with

$$(f) + D \geq 0.$$

Let $H^0(S, \mathcal{O}(D)) = \langle s_1, s_2, \dots, s_n \rangle$. Let $(s_0 = 1) = D$.

$\Rightarrow \frac{s_i}{s_0}$ is a meromorphic function f_i on S s.t.

$$(f_i) + D \geq 0.$$

Given any meromorphic function f on S with $(f) + D \geq 0$,

$f \cdot s_0$ is a section of $\mathcal{O}(D)$. $\Rightarrow f \cdot s_0 \in H^0(S, \mathcal{O}(D))$.

See P136 ~ P137. \Rightarrow

We will try to answer the question first for an effective divisor $D = \sum P_i$ of degree d on S . We will assume moreover that the points P_i are distinct - the only difference in the following computation if D has the multiple points is a much more cumbersome notation.

As with Abel's theorem, the problem becomes tractable when expressed in terms of differentials. Now if $f \in M(S)$ with $(f) + D \geq 0$, then df is a meromorphic 1-form on S .