Now consider the map $f : \mathbb{P}^2 \longrightarrow \mathbb{P}^3$. Since the linear system $|\mathcal{J}_{\Gamma_0}(4)|$ contains, as a subsystem, the triangle Δ plus the linear system of lines, $\tilde{f}$ will be one-to-one and smooth away from the inverse image in $\tilde{\mathbb{P}}^2$ of the triangle.

Γ $\tilde{f} : \tilde{\mathbb{P}}^2 \longrightarrow \mathbb{P}^3$

$$\tilde{f} : \tilde{\mathbb{P}}^2 - \pi^{-1}(\Delta) \longrightarrow \mathbb{P}^3$$
$$x \longmapsto [((\sigma_0 \cdot s_1)(x), (\sigma_0 \cdot s_2)(x), (\sigma_0 \cdot s_3)(x), \tau(x))]$$

Suppose $x_0 \neq y_0 \in \tilde{\mathbb{P}}^2 - \pi^{-1}(\Delta)$, but $\tilde{f}(x_0) = \tilde{f}(y_0)$.

$\Rightarrow$ $((\sigma_0 \cdot s_1)(x_0), (\sigma_0 \cdot s_2)(x_0), (\sigma_0 \cdot s_3)(x_0), \tau(x_0)) = \lambda((\sigma_0 \cdot s_1)(y_0), (\sigma_0 \cdot s_2)(y_0), (\sigma_0 \cdot s_3)(y_0), \tau(y_0))$.

$\Rightarrow \exists$ not all zero $a_1, a_2, a_3 \in \mathbb{C}$ s.t

$$a_1(\sigma_0 \cdot s_1) + a_2(\sigma_0 \cdot s_2) + a_3(\sigma_0 \cdot s_3) = 0 \quad \text{at } x_0, y_0.$$

$\Rightarrow$ Since $\sigma_0(x_0) \neq 0 \neq \sigma_0(y_0)$,

$$a_1 s_1 + a_2 s_2 + a_3 s_3 = 0 \quad \text{at } x_0, y_0$$

$\Rightarrow$ Since s_1, s_2, s_3 are linearly independent sections of $[H]$ over $\mathbb{P}^2$, $a_1 s_1 + a_2 s_2 + a_3 s_3$ is another section of $[H]$, and $\{a_1 s_1 + a_2 s_2 + a_3 s_3 = 0\}$ contains a line $\mathbb{P}^1 \ni x_0, y_0$.

$\Rightarrow a_1 s_1 + a_2 s_2 + a_3 s_3 = 0 \Rightarrow$ Contradiction.

$\Rightarrow \tilde{f}$ is one to one. Obviously $\tilde{f}$ is smooth away from the inverse image in $\tilde{\mathbb{P}}^2$ of the triangle. ⏌

The proper transforms $\tilde{L}_{ij}$ of the lines $L_{ij} = \overline{P_i P_j}$, on the other hand, are blown down to points: any