

the above exact sequence is the top row in the diagram

$$\begin{array}{ccccc}
 \text{Ext}^1(S; I, \mathcal{O}) & \longrightarrow & \bigoplus_{p \in Z} \Lambda^2 T'_p(S) & \longrightarrow & H^1(S, \mathcal{O}) \\
 \updownarrow & & \updownarrow & & \updownarrow \\
 \text{Ext}^1(S; I, \mathcal{O})^* & \longleftarrow & \bigoplus_{p \in Z} \Lambda^2 T_p^*(S) & \xleftarrow{p} & H^0(S, \Omega^2)
 \end{array}$$

(***)

$$\Gamma \quad \underline{\text{Ext}}_{\mathcal{O}}^q(\mathcal{O}_Z, \mathcal{O})_p \cong \text{Ext}_{\mathcal{O}}^q(\mathcal{O}_{Z,p}, \mathcal{O}_p) = 0 \quad \text{if } q \neq 2.$$

$$\begin{aligned}
 \underline{\text{Ext}}_{\mathcal{O}}^2(\mathcal{O}_Z, \mathcal{O})_p &\cong \mathcal{O}_{Z,p} = \frac{\mathcal{O}_p}{I_p} \cong \mathbb{C} \cong \Lambda^2 T'_p(S) \\
 \text{since } T'_p(S) &\cong \mathbb{C}^2.
 \end{aligned}$$

The bottom row are the dual vector spaces, where the dual of $\text{Ext}^2(S; \mathcal{O}_Z, \mathcal{O})$ is $H^0(S, \mathcal{O}_Z \otimes \Omega^2)$ by the duality theorem (I). By functoriality, the mapping p is simply the restriction of a global holomorphic 2-form on S to each point $p \in Z$.

$$\Gamma \quad \text{Ext}^2(S; \mathcal{O}_Z, \mathcal{O}) = H^0(S, \underline{\text{Ext}}_{\mathcal{O}}^2(\mathcal{O}_Z, \mathcal{O})) \quad \text{since } \underline{\text{Ext}}_{\mathcal{O}}^q(\mathcal{O}_Z, \mathcal{O}) = 0, \quad 0 \leq q < 2.$$

$$\Rightarrow \underline{\text{Ext}}_{\mathcal{O}}^2(\mathcal{O}_Z, \mathcal{O}) = \bigoplus_{p \in Z} \mathcal{O}_{Z,p} = \bigoplus_{p \in Z} \Lambda^2 T'_p(S)$$

$$\begin{aligned}
 \Rightarrow \text{Ext}^2(S; \mathcal{O}_Z, \mathcal{O}) &= H^0(S, \underline{\text{Ext}}_{\mathcal{O}}^2(\mathcal{O}_Z, \mathcal{O})) \\
 &= H^0(S, \bigoplus_{p \in Z} \mathcal{O}_{Z,p}) \cong H^0(S, \bigoplus_{p \in Z} \Lambda^2 T'_p(S)) \\
 &= \bigoplus_{p \in Z} \Lambda^2 T'_p(S).
 \end{aligned}$$