The definition of intersection of cycles on a general oriented manifold differs from this special case only in the difficulty of verifying the transversality statements made.

Suppose ~~A~~ M is an oriented n-manifold.
A, B two piecewise smooth cycles on M of dim k & $n-k$ respectively.
$p \in A \cap B$. point of transverse intersection of A and B
$v_1, \dots v_k \in T_p(A) \subset T_p(M)$ oriented basis for $T_p(A)$
$w_1, \dots w_{n-k} \in T_p(B) \subset T_p(M)$ " for $T_p(B)$

We define the intersection index $i_p(A \cdot B)$ of A with B at p to be $+1$ if $v_1 \cdots v_k, w_1 \cdots w_{n-k}$ is an oriented basis for $T_p(M) = T_p(A) \oplus T_p(B)$ and -1 if not.

If A and B intersect transversely everywhere, we define the intersection $\#(A \cdot B)$ to be

$$\#(A \cdot B) = \sum_{p \in A \cap B} i_p(A \cdot B).$$

Since A and B are compactly supported and $A \cap B$ is discrete, this sum is finite.

Claim: The intersection # $\#(A \cdot B)$ depends only on the homology class of A and B. i.e $A \sim 0 \Rightarrow \#(A \cdot B) = 0$.

Sketch of Proof) We may take $A = \partial C$ to be the sum of boundaries of piecewise smooth $(k+1)$-manifolds C_i, so that at each smooth pt $p \in A$, an oriented basis $v_1 \cdots v_k$ for $T_p(A)$ together with an inward normal vector to C_i gives the orientation on C_i.