$$\Rightarrow \quad H^r_{DR}(M) \cong E_\infty^{0,r} \oplus F^1 H^r_{DR}(M)$$
$$\cong E_\infty^{0,r} \oplus E_\infty^{1,r-1} \oplus F^2 H^r_{DR}(M)$$
$$\cdots \cong E_\infty^{0,r} \oplus E_\infty^{1,r-1} \oplus E_\infty^{2,r-2} \oplus \cdots \oplus E_\infty^{r,0} \oplus F^{r+1} H^r_{DR}(M), \quad F^{r+1} H^r_{DR}(M) = 0$$

by the same reason as P430 note.

$$\Rightarrow \dim H^r_{DR}(M) = b_r = \sum_{p+q=r} \dim E_\infty^{p,q} \le \sum_{p+q} \dim E_1^{p,q}$$

$$= \sum_{p+q=r} \dim H^{p,q}_{\bar\partial}(M) = \sum_{p+q=r} h^{p,q}$$

$$\Rightarrow \sum_{p+q=r} h^{p,q} \ge b_r.$$

Consider the following complex

$$E_r^{p-r,q+r-1} \xrightarrow{d^{p-r,q+r-1}} E_r^{p,q} \xrightarrow{d^{p,q}} E_r^{p+r,q-r+1}$$

$$\| \qquad\qquad \| \qquad\qquad \|$$

$$\operatorname{im} d^{p-2r,q+2r-2} \oplus H^{p-r,q+r-1} \oplus V^{p-r,q+r-1} \qquad \operatorname{im} d^{p,q} \oplus H^{p+r,q-r+1} \oplus V^{p+r,q-r+1}$$

$$\| \qquad\qquad \| \qquad\qquad \|$$

$$\operatorname{im} d^{p-r,q+r-1} \oplus H^{p,q}(E_r) \oplus V^{p,q}$$

where $V^{p-r,q+r-1} \xrightarrow{d^{p-r,q+r-1}} \operatorname{im} d^{p-r,q+r-1}$

$V^{p,q} \xrightarrow{d^{p,q}} \operatorname{im} d^{p,q}$

$\vdots$

isomorphisms,

and $\ker d^{p,q} = \operatorname{im} d^{p-r,q+r-1} \oplus H^{p,q}(E_r)$

$\vdots$

$$\Rightarrow \cdots + (-1)^{p+q-1} \dim E_r^{p-r,q+r-1} + (-1)^{p+q} \dim E_r^{p,q} + (-1)^{p+q+1} \dim E_r^{p+r,q-r+1} + \cdots$$

$$= \cdots (-1)^{p+q-1} \left( \dim \operatorname{im} d^{p-2r,q+2r-2} + \dim H^{p-r,q+r-1} + \dim V^{p-r,q+r-1} \right)$$
$$+ (-1)^{p+q} \left( \dim \operatorname{im} d^{p-r,q+r-1} + \dim H^{p,q} + \dim V^{p,q} \right)$$
$$+ (-1)^{p+q+1} \left( \dim \operatorname{im} d^{p,q} + \dim H^{p+r,q-r+1} + \dim V^{p+r,q-r+1} \right) + \cdots$$