

$$\begin{aligned}
& (\text{Since } \omega_{u_1, n} \cdots u_{i-1, n} \cdot u_{i+1, n} \cdots u_n = 0) \\
& = P_i (-1)^{i-1} \omega_{u_1, n} \cdots u_{i-1, n} \cdot \hat{u}_j \cdot u_n + P_j (-1)^{j-2} \omega_{u_1, n} \cdots \hat{u}_i \cdot u_n \\
& = (-1)^{i-1} P_i \omega_{i, j} + (-1)^{j-2} P_j \omega_{i, j} \\
& = (-1)^{i-1} (-1)^{j-1} P_i \bar{\partial} P_j \omega + (-1)^{j-2} P_j (-1)^{i-1} \bar{\partial} P_i \omega \\
& = (-1)^i (-1)^j P_i \bar{\partial} P_j \omega - (-1)^{i+j} P_j \bar{\partial} P_i \omega \\
& = (-1)^{i+j} (P_i \bar{\partial} P_j \omega - P_j \bar{\partial} P_i \omega) \\
& \Rightarrow \bar{\partial} \zeta_{i, j} = \omega_{i, j} = (-1)^{i+j} \omega \bar{\partial} P_i \wedge \bar{\partial} P_j \wedge \omega. //
\end{aligned}$$

$$\begin{aligned}
& \zeta_{i, j, k} = \zeta_{u_1, n} \cdots \hat{u}_i \cdots \hat{u}_j \cdots \hat{u}_k \cdots u_n \\
& = \sum P_\ell \omega_{u_\ell, n} \cdots \hat{u}_i \cdots \hat{u}_j \cdots \hat{u}_k \cdots u_n \\
& = P_i \omega_{u_i, n} \cdots \hat{u}_i \cdots \hat{u}_j \cdots \hat{u}_k \cdots u_n + P_j \omega_{u_j, n} \cdots \hat{u}_i \cdots \hat{u}_j \cdots \hat{u}_k \cdots u_n \\
& \quad + P_k \omega_{u_k, n} \cdots \hat{u}_i \cdots \hat{u}_j \cdots \hat{u}_k \cdots u_n \\
& = (-1)^{i-1} P_i \omega_{u_i, n} \cdots \hat{u}_j \cdots \hat{u}_k \cdots u_n + (-1)^{j-2} P_j \omega_{u_j, n} \cdots \hat{u}_i \cdots \hat{u}_k \cdots u_n \\
& \quad + (-1)^{k-3} P_k \omega_{u_k, n} \cdots \hat{u}_i \cdots \hat{u}_j \cdots u_n \\
& = (-1)^{i-1} P_i \omega_{i, j, k} + (-1)^{j-2} P_j \omega_{i, j, k} + (-1)^{k-3} P_k \omega_{i, j, k} \\
& = (-1)^{i-1} P_i (-1)^{j+k} \omega \bar{\partial} P_j \wedge \bar{\partial} P_k \wedge \omega \\
& \quad + (-1)^{j-2} P_j (-1)^{i+k} \omega \bar{\partial} P_i \wedge \bar{\partial} P_k \wedge \omega \\
& \quad + (-1)^{k-3} P_k (-1)^{i+j} \omega \bar{\partial} P_i \wedge \bar{\partial} P_j \wedge \omega \\
& = (-1)^{i+j+k-1} \omega (P_i \bar{\partial} P_j \wedge \bar{\partial} P_k \wedge \omega - P_j \bar{\partial} P_i \wedge \bar{\partial} P_k \wedge \omega \\
& \quad + P_k \bar{\partial} P_i \wedge \bar{\partial} P_j \wedge \omega)
\end{aligned}$$

$$\begin{aligned}
& \Rightarrow \bar{\partial} \zeta_{i, j, k} = \omega_{i, j, k} = (-1)^{i+j+k+1} \omega (\bar{\partial} P_i \wedge \bar{\partial} P_j \wedge \bar{\partial} P_k \wedge \omega \\
& \quad - \bar{\partial} P_j \wedge \bar{\partial} P_i \wedge \bar{\partial} P_k \wedge \omega + \bar{\partial} P_k \wedge \bar{\partial} P_i \wedge \bar{\partial} P_j \wedge \omega) \\
& = (-1)^{i+j+k-1} 3! \bar{\partial} P_i \wedge \bar{\partial} P_j \wedge \bar{\partial} P_k \wedge \omega.
\end{aligned}$$

Continue this process, then we obtain.

$$\begin{aligned}
& \bar{\partial} \zeta_{i_1, i_2, \dots, i_{n-1}} = \bar{\partial} \zeta_{i_1} = (-1)^{i_1 + \dots + i_{n-1} + n-1} n! \bar{\partial} P_{i_1} \wedge \bar{\partial} P_{i_2} \wedge \dots \wedge \bar{\partial} P_{i_{n-1}} \wedge \omega \\
& = (-1)^{i_1 + 2 + \dots + n-1 + n-1} n! \bar{\partial} P_{i_1} \wedge \dots \wedge \hat{\bar{\partial} P_i} \wedge \dots \wedge \bar{\partial} P_{i_{n-1}} \wedge \omega \\
& = (-1)^{i_1} (-1)^{\frac{(n+1)(n-1)}{2}} n! \bar{\partial} P_{i_1} \wedge \dots \wedge \hat{\bar{\partial} P_i} \wedge \dots \wedge \bar{\partial} P_{i_{n-1}} \wedge \omega = \omega_{i_1} \in H^{n, n-1}
\end{aligned}$$