$$+ e_i \otimes D'^2 e_j' + \theta_{ki} \wedge \theta'_{\ell j} \otimes e_k \otimes e_\ell' - \theta_{ki} \wedge \theta'_{\ell j} \otimes e_k \otimes e_\ell'$$

$$= D^2 e_i \otimes e_j + e_i \otimes D'^2 e_j = \Theta_{ki}\, e_k \otimes e_j + e_i \otimes \Theta'_{\ell j}\, e_\ell'.$$

$$\Rightarrow \quad \Theta_{E \otimes E'} = \Theta_E \otimes 1 + 1 \otimes \Theta_{E'}. \quad \lrcorner$$

In particular, for L and E as above with any metric on E,

$$\Theta_{L^\mu \otimes E} = \frac{2\pi\mu}{i} \omega \otimes 1_E + \Theta_E .$$

$\ulcorner$

$$\Theta_{L^\mu \otimes E} = \Theta_{L^\mu} \otimes 1 + 1 \otimes \Theta_E$$

$$= \frac{\mu i}{2\pi} \Theta_L \otimes 1 + 1 \otimes \Theta_E$$

$$= \frac{2\pi\mu}{i} \omega \otimes 1_E + 1 \otimes \Theta_E \quad \lrcorner$$

Let $\eta \in \mathcal{H}^{0,p}(L^{-\mu} \otimes E)$ be harmonic. Writing Θ for $\Theta_{L^{-\mu} \otimes E}$, D for $D_{L^{-\mu} \otimes E}$, we have

$$\Theta = D^2 = D'\bar\partial + \bar\partial D',$$

so $\quad \Theta\eta = \bar\partial D'\eta,$ and by the Kähler identity

$[\Lambda, \bar\partial] = -\frac{i}{2} D'^*$ we see that

$$2i\,(\Lambda\Theta\eta, \eta\rangle = 2i\,\langle \Lambda\bar\partial D'\eta, \eta\rangle$$

$$= 2i\,\left(\left(\bar\partial\Lambda + \frac{1}{2i} D'^*\right) D'\eta, \eta\right\rangle = \langle D'^* D'\eta, \eta\rangle$$