

Define trivializations for N_V^* as follows:

We know that df_α is nonvanishing section of $N_V^*|_{U_\alpha}$
 $= \text{Hom}(T^{M_V}/T_V, \mathbb{C})|_{U_\alpha}$.

$$U_\alpha \cap V \times \mathbb{C} \xleftarrow{\varphi_\alpha} N_V^*|_{U_\alpha} \text{ for each } \alpha.$$

$$(z, \ell) \xleftarrow{\quad} \ell df_\alpha$$

$$\Rightarrow$$

$$V \cap U_\beta \cap U_\alpha \times \mathbb{C} \xleftarrow{\varphi_\beta} N_V^*|_{U_\alpha \cap U_\beta} \xrightarrow{\varphi_\alpha} V \cap U_\alpha \cap U_\beta \times \mathbb{C}$$

$$(z, g_\beta(z)) \xleftarrow{\quad} \ell df_\alpha \xrightarrow{\quad} (z, 1)$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$U_\alpha \cap U_\beta \cap V \quad \downarrow \quad g_{\alpha\beta} df_\beta$$

$$\downarrow$$

$$z$$

$$(z, \varphi_\alpha \circ \varphi_\beta^{-1}(z) g_{\alpha\beta}(z))$$

\Rightarrow The transition functions^V for N_V^* are given by

$$g_{\alpha\beta}^{-1}(z)'s.$$

\Rightarrow Clearly, $N_V^* \otimes [V]|_V$ is trivial since

the transition function for $[V]|_V$ are given by $g_{\alpha\beta}$'s.