Now suppose M is compact. For every $D' \in |D|$, there exists $f \in \mathcal{L}(D)$ s.t

$$D' = D + (f), \quad \text{and}$$

conversely any two such functions f, f' differ by a non-zero constant. Thus we have additional correspondence

$$|D| \cong \mathbb{P}(\mathcal{L}(D)) \cong \mathbb{P}(H^0(M, \mathcal{O}([D])))$$

projective space

Γ $D' \in |D| \Rightarrow$ This means $D' \sim D$ and D' is effective.
$\Rightarrow$ By the definition of linearly equivalence (P134), $D' = D + (f)$.
s.t $f \in \mathcal{L}(D)$. since $D' = D + (f) \geq 0$.

Conversely, if we have two such functions f, f'.
$\Rightarrow D + (f) = D + (f') \Rightarrow D = D + (f) + (f'^{(-1)}) = D + \left(\frac{f}{f'}\right)$
$\Rightarrow \left(\frac{f}{f'}\right) = 0 \Rightarrow \frac{f}{f'}$ is holomorphic and nonzero on M (or f & f' have the same zero sets & pole sets)
$\Rightarrow \frac{f}{f'}$ must be constant on M since M is compact complex manifold. $\Rightarrow \frac{f}{f'} = c \neq 0 \Rightarrow \frac{f}{f'} = c \Rightarrow f = cf'$.

Thus

$$\begin{array}{ccccccc}
|D| & \xrightarrow{\phi \to \text{ multi-valued map}} & \mathcal{L}(D) & \xrightarrow{\text{one-valued}} & \mathcal{L}(D)/\mathbb{C}^* & = & \mathbb{P}(\mathcal{L}(D)) \\
\Downarrow & & \Downarrow & & \Downarrow & & \| \\
D' & \longmapsto & f & \longmapsto & [f] & & \mathbb{P}(H^0(M, \mathcal{O} \\
\simeq D + (f) \geq 0 & & & & & &
\end{array}$$

$[D]))$. Given $f' \in \mathcal{L}(D)$, $D + (f') \geq 0$.
Choose $D + (f') \in |D|$. $\Rightarrow \phi(D + (f')) = f'$. $\Rightarrow \phi$ is onto. If $\phi(D + (f)) = \phi(D + (f'))$,
If $D' = D + (f) = D + (f')$, then. $f = cf'$. $c \neq 0$.

$$\Rightarrow \begin{array}{ccc}
\mathcal{L}(D) & \xrightarrow{\Downarrow} & |D| \\
\Downarrow f & \longmapsto & \Downarrow D + (f)
\end{array} \qquad (f) = (cf) \quad c \neq 0.$$