

3. Further Techniques.

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We return now to be the subject of general analytic varieties in order to develop some further techniques especially intended for higher-dimensional considerations. The motif of this chapter is differential forms; the theme is their wide variety of applications, cohomological and otherwise, to complex analytic geometry.

We begin in Sections 1 and 2 with the theory of currents, or differential forms with distribution coefficients. This theory, initiated by de Rham to include both the C^∞ forms and piecewise smooth chains in the same framework, is especially fruitful in the complex analytic case. A pattern for the entire chapter is established, in that first the real or C^∞ situation is discussed, and then the theory in the richer complex-analytic case developed. The topics in Section 1 are pretty much standard and well described by the table of contents. Coming to Section 2, there has recently been a flurry of research into the remarkable properties of currents associated to complex-analytic varieties. We have taken advantage of this to illustrate how the theory of currents is useful in establishing many of the foundational results required in an analytic treatment of algebraic geometry. For example, there is now an elegant method for recognizing when a current is one defined by an analytic variety, and this affords a direct method for proving such as Remmert's proper mapping theorem which, although intuitively plausible, were traditionally rather difficult to establish rigorously.

Next we turn to the theory of Chern classes. The definition by differential forms that are polynomials in the curvature matrix provides a quick and easy derivation of the functoriality properties, especially, Whitney duality, of