

$\subset M - f^{-1}(V) - W$. For any $x \in \mathbb{P}^n - V - g^{-1}(W)$, $g(x) \in M$.
 $g(x) \notin W \Rightarrow g(x) \in M - W$. Suppose $g(x) \in f^{-1}(V) \Rightarrow f(g(x)) = x \in V \Rightarrow$ Contradiction to the fact $x \notin V$. We cannot go further.
 $\mathbb{C}^2 \xrightarrow{\pi} \mathbb{C}$ and $\mathbb{C} \xrightarrow{g} \mathbb{C}^2$
 $\downarrow \quad \downarrow$
 $z \longmapsto (z, 1)$

$\Rightarrow \mathbb{C} \xrightarrow{\pi \circ g} \mathbb{C}$ is identity. $\Rightarrow \mathbb{C}^2$ is birational to \mathbb{C} according to the definition above. Nonsense!!! (or $\mathbb{P}^1 \times \mathbb{P}^1 \xrightarrow{\pi} \mathbb{P}^1$ is birational)

We have to correct as follows:

We say that a rational map $f: M \rightarrow N$ is birational if there exists a rational map $g: N \rightarrow M$ s.t. $f \circ g$ & $g \circ f$ are the identities as rational maps.

$M \xrightarrow{f} \mathbb{P}^n$ birational $\Rightarrow \mathbb{P}^n \xrightarrow{g} M$ rational s.t.
 $g \circ f$ & $f \circ g = \text{id}$ as rational maps.

$\Rightarrow g \circ f: M \rightarrow \mathbb{P}^n \rightarrow M$.

$\Rightarrow g \circ f: M - W \rightarrow \mathbb{P}^n - g^{-1}(W) \rightarrow M - W$ is holomorphic

Similarly, $f \circ g = \text{id}: \mathbb{P}^n - V \rightarrow M - f^{-1}(V) \rightarrow \mathbb{P}^n - V$.

Consider the following maps

$$g \circ f: M - W - f^{-1}(V) \xrightarrow[\text{id}]{f} \mathbb{P}^n - V - g^{-1}(W) \xrightarrow{g} M - W - f^{-1}(V)$$

$$f \circ g: \mathbb{P}^n - V - g^{-1}(W) \xrightarrow[\text{id}]{g} M - W - f^{-1}(V) \xrightarrow{f} \mathbb{P}^n - V - g^{-1}(W)$$

We have only to prove that $f(M - W - f^{-1}(V)) \subset \mathbb{P}^n - V - g^{-1}(W)$,
 $(x \in M - W - f^{-1}(V) \Rightarrow f(x) \notin V, \text{ and } f(x) \notin g^{-1}(W), \text{ since } g(f(x)) = x \in W.$