form $\Lambda = v_1 \wedge \cdots \wedge v_k$. For this we pose the more general problem of determining the minimal linear subspace $W \subset V$ such that Λ is in the image of

$$\Lambda^k W \longrightarrow \Lambda^k V.$$

If $\dim W = l$, then $l \geq k$ with equality holding $\Leftrightarrow \Lambda$ is decomposable.

Γ If $l < k$, then $\Lambda^k W = 0$. If $l = k$, $\Lambda^k W = \langle w_1 \wedge \cdots \wedge w_k \rangle$ where $W = \langle w_1, \cdots w_k \rangle$.
$\Rightarrow$ Obviously, Λ is decomposable if Λ is in the image of $\Lambda^k W \longrightarrow \Lambda^k V$.
Conversely, Λ is decomposable $\Rightarrow \Lambda = v_1 \wedge \cdots \wedge v_k$.
$\Rightarrow \{v_1, v_2, \; v_k\}$ linearly independent set $\Rightarrow$
Let $W = \langle v_1, v_2 \cdots v_k \rangle$. $\Rightarrow$ $\dim W = k$ and the image of $\Lambda^k W \longrightarrow \Lambda^k V$ contains Λ. Clearly, W is the minimal such space. $\lrcorner$

" Comment : Let $V = \underbrace{\langle e_1, e_2, \cdots e_l \rangle}_{= W} \oplus \underbrace{\langle e_{l+1}, \cdots e_n \rangle}_{= V'}$

Suppose $v_1 \wedge v_2 \wedge \cdots \wedge v_k \in \Lambda^k V$ and, in paticular, $v_1 \wedge v_2 \wedge \cdots \wedge v_k \in \Lambda^k W$.
Assume that $\{v_1, \cdots, v_k\}$ is linearly independent set.

$$\Rightarrow \begin{aligned} v_1 &= x_{11} e_1 + x_{12} e_2 + \cdots + x_{1n} e_n \\ v_2 &= x_{21} e_1 + x_{22} e_2 + \cdots + x_{2n} e_n \\ &\vdots \\ v_k &= x_{k1} e_1 + x_{k2} e_2 + \cdots + x_{kn} e_n \end{aligned}$$

Without loss of generality, we may as-