$V_0(N) \cap \sigma_{2,1}$ = The set of all lines contained in H, containing p, contained in some quadric of N.
$\overline{P_1P_2}$, $\overline{P_1P_3}$ & $\overline{PP_4}$ are contained in H, and contain p. Remember they are in L, since $\overline{PP_2}+\overline{P_3P_4}$, $\overline{PP_3}+\overline{P_2P_4}$, & $\overline{PP_4}+\overline{P_2P_3}$ are members of L, see P742.

$\Rightarrow \ \#(V_0(N)\cdot\sigma_{2,1}) = 3$

Let $V_0(N) \sim a\,\sigma_1$. $\Rightarrow$ $\sigma_1\cdot\sigma_{2,1} = 1$

$\Rightarrow$ $V_0(N) \sim 3\,\sigma_1$ ⅃

Finally let W be a generic web of quadrics. Since in this case the set $V_0(W)$ of all lines on W is all of $G(2,4)$, we will be concerned with the variety $V_1(W) \subset G(2,4)$ of lines lying on a pencil of quadrics from W.

⅂ Suppose $W = \{x_0\sigma_0 + x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3$, σ_i generic quadric$\}$. Given an arbitrary line $l \in G(2,4)$, choose distinct three points $p_1, p_2, p_3 \in l$.
Consider the following equations:

$$\left.\begin{aligned} x_0\sigma_0(p_1) + x_1\sigma_1(p_1) + x_2\sigma_2(p_1) + x_3\sigma_3(p_1) &= 0\\ x_0\sigma_0(p_2) + x_1\sigma_1(p_2) + x_2\sigma_2(p_2) + x_3\sigma_3(p_2) &= 0\\ x_0\sigma_0(p_3) + x_1\sigma_1(p_3) + x_2\sigma_2(p_3) + x_3\sigma_3(p_3) &= 0\end{aligned}\right\} \cdots (*)$$

$\Rightarrow \exists$ nontrivial solutions, i.e. $\exists$ not all zero solutions x_0, x_1, x_2, x_3 satisfying $(*)$.
$\Rightarrow$ Since $x_0\sigma_0 + \cdots + x_3\sigma_3$ is a quadric, $l \subset \{x_0\sigma_0 + \cdots + x_3\sigma_3 = 0\}$. $\Rightarrow$ $V_0(W) = G(2,4)$. ⅃