Consider the following

$$K(\varphi) = \int_V k(z,\omega)\wedge\varphi(\omega).$$

$\Rightarrow$ $K(\varphi)(z)$ is well-defined since $k(z,\omega)$ is locally integrable & φ is bounded on the compact set $\overline{V}$.

$\Rightarrow$ For a test form $\psi \in A_c^{n,n-q}(V)$,

$$\lim_{\epsilon\to 0}\int_{\mathbb{C}^n\times\mathbb{C}^n-\Delta_\epsilon} d(\psi(z)\wedge k(z,\omega)\wedge\varphi(\omega))$$

$$= \int_{\mathbb{C}^n\times\mathbb{C}^n} \bar\partial\psi(z)\wedge k(z,\omega)\wedge\varphi(\omega) + (-1)^{q+1}\int_{\mathbb{C}^n\times\mathbb{C}^n}\psi(z)\wedge k(z,\omega)\wedge\bar\partial\varphi(\omega)$$

$$= \int_{\mathbb{C}^n}\psi(z)\wedge\int_{\|\omega-z\|=\epsilon} k(z,\omega)\wedge\varphi(\omega)$$

If we let $L = \operatorname{supp}\psi$, for ϵ sufficiently small, $\{\omega \mid \|\omega-z\|=\epsilon\}\subset V$.

$\Rightarrow$ We get

$$\bar\partial K_\varphi + (-1)^q K_\varphi\bar\partial = T_\varphi.$$

$\Rightarrow$ $K(\bar\partial\varphi)(z) + \bar\partial K(\varphi)(z) = \varphi(z)$ for $z\in V$ by using the same argument on p121.

$\Rightarrow$ Since $\bar\partial\varphi = 0$, $\varphi(z) = \bar\partial K(\varphi)(z)$. ┘

Now, suppose we say that a current $T\in\mathcal{D}^{0,q}(\mathbb{C}^n)$ is compactly supported if, for some relatively compact open set $U\subset\mathbb{C}^n$, $T(\varphi)=0$ whenever $\operatorname{supp}\varphi\subset\mathbb{C}^n-U$.