Date

By P232. $\int_{\delta_{g+i}} \omega_j = \int_{\delta_{g+j}} \omega_i$.

$\Rightarrow \quad N'^{g+i} = m_i$.

Summarizing, we have proved

Abel's Theorem (Second Version). Given $D = \sum(p_\lambda - q_\lambda) \in \mathrm{Div}(S)$ and $\omega_1, \omega_2, \cdots \omega_g$ a basis for the space of holomorphic 1-forms on S, then $D = (f)$ for some meromorphic function f on S if and only if

$$\mu(D) = \left(\sum_\lambda \int_{q_\lambda}^{p_\lambda} \omega_1, \cdots \sum_\lambda \int_{q_\lambda}^{p_\lambda} \omega_g \right) \equiv 0 \ (\Lambda).$$

In fancier language: recalling that $\mathrm{Pic}^\circ(S)$ is the group of divisors of degree zero on S modulo linear equivalence, the map

$$\mu : \mathrm{Div}^\circ(S) \longrightarrow \mathcal{J}(S)$$

factors

$$\begin{tikzcd}
\mathrm{Div}^\circ(S) \arrow[rr, "\mu"] \arrow[dr] & & \mathcal{J}(S) \\
& \mathrm{Pic}^\circ(S) \arrow[ur, "\tilde{\mu}"'] &
\end{tikzcd}$$

to give an injection $\tilde{\mu} : \mathrm{Pic}^\circ(S) \longrightarrow \mathcal{J}(S)$.

Pf. $\mathrm{Pic}^\circ(S) = \{ L$ line bundles of degree $0 \}$

$\ni L \Rightarrow L = [D]$ by p161 proposition.

$\Rightarrow$ Define $\tilde{\mu}(L) = \mu(D)$. $\Rightarrow \tilde{\mu}$ is well-defined for if

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