

⌈ I already mentioned this case, see P516.

Thus we may use the formula in all circumstances, if we adopt the convention that  $\sigma_b$  is null for  $b$  not a nonincreasing sequence.

⌈ All right ! . I see.

As a sample calculation, we compute the coefficient  $\delta(311, 21; 521)$  of  $\sigma_{521}$  in the expression for  $(\sigma_{311} \cdot \sigma_{21})$  as a linear combination of Schubert cycles. By (\*) and the reductions we have

$$\begin{aligned} \delta(311, 21; 521) &= \#(\sigma_{311} \cdot \sigma_{21} \cdot \sigma_{43}) && \text{in } G(3, 8) \\ &= \#(\sigma_2 \cdot \sigma_{21} \cdot \sigma_{43}) && \text{in } G(3, 7) \\ &= \#(\sigma_2 \cdot \sigma_{21} \cdot \sigma_3) && \text{in } G(2, 6) \\ &= \#(\sigma_2 \cdot \sigma_1 \cdot \sigma_3) && \text{in } G(2, 5) \\ &= \#(\sigma_2 \cdot \sigma_1) && \text{in } G(1, 4) = \mathbb{P}^3 \\ &= 1. \end{aligned}$$

⌈ In  $G(3, 8)$ ,  $k=3$ ,  $n=8$ .

$$\begin{aligned} \delta(311, 21; 521) &= \#(\sigma_{311} \cdot \sigma_{21} \cdot \sigma_{p-3-c_3, 5-c_2, 5-c_1}) \\ &= \#(\sigma_{311} \cdot \sigma_{21} \cdot \sigma_{4,3,0}) = \#(\sigma_{311} \cdot \sigma_{21} \cdot \sigma_{43}) && \text{in } G(3, 8) \\ &\nwarrow \text{by P198.} \end{aligned}$$

$$\begin{aligned} \Rightarrow a_3 + b_0 + c_0 &= 1 + 5 + 5 \geq 2 \cdot 5 + 1 \text{ and } \alpha = 3 \\ &= 3 \Rightarrow \#(\sigma_{311} \cdot \sigma_{21} \cdot \sigma_{43})_{G(3, 8)} = \#(\sigma_{a_1-1, a_2-1, a_3-1} \cdot \sigma_{b_1, b_2, b_3} \cdot \sigma_{c_1, c_2, c_3}) \\ &= \#(\sigma_2 \cdot \sigma_{21} \cdot \sigma_{43})_{G(3, 7)} \end{aligned}$$