$\ulcorner$ $\pi^{-1}(C') = \{(C_1, C_2, \cdots C_5)\}$
$\Rightarrow$ $C' \in V_{C_i}$ $\Rightarrow$ $C_i$ is tangent to $C'$
$\Rightarrow$ $C_i \in V_{C'}$ $\Rightarrow$ $\pi^{-1}(C') \subset (V_{C'})^5$
Given any element $(D_1, \cdots D_5) \in (V_{C'})^5$, $\Rightarrow$ $C' \in V_{D_i}$
$\Rightarrow$ $(V_{C'})^5 \subset \pi^{-1}(C')$ $\Rightarrow$ $\pi^{-1}(C')$ is isomorphic to $(V_{C'})^5$.
$\Rightarrow$ Since $V_{C'}$ is irreducible, $(V_{C'})^5$ is irreducible.
We may assume (or choose $U$ irreducible) that $U$ is irreducible. $\Rightarrow$ $I$ is irreducible.

$\lrcorner$

Since the map $\pi_1 : I \longrightarrow (W)^5$ is generically finite-to-one, then, we see that assertion 1 can fail to hold - i.e., $J$ can map surjectively onto $(W)^5$ - only if $J = I$.

$\ulcorner$ Suppose, for generic smooth $C_1, C_2 \cdots C_5$,
$\dim(V_{C_1} \cap \cdots \cap V_{C_5} \cap U) \geq 1$.
$\Rightarrow$ $\dim(V_{C_1} \cap \cdots \cap V_{C_4} \cap U) \geq 2$, for, otherwise,
(i) $\dim(V_{C_1} \cap \cdots \cap V_{C_4} \cap U) \leq 0$ . O. K.
(ii) $\dim(V_{C_1} \cap \cdots \cap V_{C_4} \cap U) = 1$.
$\Rightarrow$ $V_{C_1} \cap \cdots \cap V_{C_4} \cap U = W_1 \cup \cdots \cup W_\ell$, where $W_i$'s irreducible components.
If $\dim W_i = 1$, $\exists$ $C_i' \in W_i$.
$\Rightarrow$ Choose a smooth conic $C_5$ s.t $C_5$ is not tangent to $C_i'$. $\Rightarrow$ $\dim V_{C_5} \cap W_i = 0$, since $V_{C_5} \not\supset W_i$. $\Rightarrow$ $\dim V_{C_1} \cap \cdots \cap V_{C_4} \cap U \cap V_{C_5} = 0$