$g_{\alpha\beta} = \psi_\alpha \circ \psi_\beta^{-1} = f \circ \varphi_\alpha^* \circ \varphi_\beta^{*-1} \circ f^{-1}$.

$v \in \mathbb{C}^n \Rightarrow f^{-1}(v) \in \mathbb{C}^{n*}$. $\Rightarrow$ What is $\varphi_\alpha^* \circ \varphi_\beta^{*-1} \circ f^{-1}(v)$?

$\omega \in \mathbb{C}^n$. $\varphi_\alpha^* \circ \varphi_\beta^{*-1} \circ f^{-1}(v) \in \mathbb{C}^{n*}$.

Evaluate

$$(\varphi_\alpha^* \circ \varphi_\beta^{*-1} \circ f^{-1}(v))(\omega) = (\varphi_\alpha^*(\varphi_\beta^{*-1} \circ f^{-1}(v)))(\omega)$$
$$= (\varphi_\beta^{*-1} \circ f^{-1}(v))(\varphi_\alpha^{-1}(\omega)) = f^{-1}(v)(\varphi_\beta \circ \varphi_\alpha^{-1}(\omega))$$
$$= f^{-1}(v)(g_{\beta\alpha}(\omega)) = f^{-1}(v) \circ g_{\beta\alpha}(\omega).$$

$f^{-1}(v) \in \mathbb{C}^{n*}$ & $g_{\beta\alpha} \in \mathrm{Hom}(\mathbb{C}^n, \mathbb{C}^n) \Rightarrow f^{-1}(v) \circ g_{\beta\alpha} \in \mathbb{C}^{n*}$.

$$\begin{array}{ccc} \mathbb{C}^{n*} & \xleftrightarrow{\ f\ } & \mathbb{C}^n \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ f^{-1}(v) & \longleftrightarrow & v \end{array}$$

$\Rightarrow$ What is $f(f^{-1}(v) \circ g_{\beta\alpha})$?

Let $e_1, \dots e_n$ be the column standard basis for $\mathbb{C}^n$. $\Rightarrow v = x_1 e_1 + \dots + x_n e_n$.

$\Rightarrow f$ is expressed as follows.

$$\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \quad \text{w.r.t } \{e_i\} \ \& \ \{e_i^*\}$$

Let $f(f^{-1}(v) \circ g_{\beta\alpha}) = w$.

$f^{-1}(v) = x_1 e_1^* + \dots + x_n e_n^*$.

$g_{\beta\alpha} = (a_{ij})$.

$\Rightarrow f^{-1}(v) \circ g_{\beta\alpha}(e_i) = f^{-1}(v)(a_{1i} e_1 + a_{2i} e_2 + \dots + a_{ni} e_n)$

$= x_1 a_{1i} + x_2 a_{2i} + \dots + x_n a_{ni} \Rightarrow$

$f^{-1}(v) \circ g_{\beta\alpha}$ is represented by ${}^t({}^t v\, g_{\beta\alpha}) =$