P196

$\Lambda \in W_{a_1 \cdots a_k} = \{ \Lambda \in U_I : \dim(\Lambda \cap V_{c_i}) = i \}$
Thus, $G(k,n) = \bigcup W_{a_1 \cdots a_k}$ where $\{a_1 \cdots a_k\}$ is a set of nonincreasing numbers. ⌋

⌈It remains to show that $W_{a_1 \cdots a_k} \cap W_{b_1 \cdots b_k} = \phi$ if $\{a_1 \cdots a_k\} \neq \{b_1, \cdots b_k\}$.
Suppose $\Lambda \in W_{a_1 \cdots a_k} \cap W_{b_1 \cdots b_k}$.
Assume that i is the number s.t $a_i \neq b_i$. and assume $a_i > b_i$.
$\Rightarrow$ $\dim(\Lambda \cap V_{a_i}) = i = \dim(\Lambda \cap V_{b_i}) = i$.
$\Rightarrow$ Since $V_{a_i} \supsetneq V_{b_i}$, we can not find $v \in \Lambda \cap V_{a_i}$ s.t $\langle v, e_{a_i} \rangle = 1$. If not, $\dim(\Lambda \cap V_{a_i}) > i$. See P195. ⌋

$\Rightarrow$ We conclude. $\bigcup W_{a_1 \cdots a_k} = G(n,k)$, where $W_{a_1 \cdots a_k} \cong \mathbb{C}^{k(n-k) - \sum a_i}$ and $\{W_{a_1 \cdots a_k}\}$ is a set of disjoint disks. See Milnor. P263. Th. A.4 (for CW complex homol.)

In general, for any flag $V = (V_1 \subset V_2 \subset \cdots \subset V_{n-1} \subset V_n)$ in $\mathbb{C}^n$, we let

$$\sigma_a(V) = \{ \Lambda : \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i \}.$$

Clearly the homology class of the subvariety $\sigma_a(V)$ is independent of the flag chosen, since we can find a continuous family of linear automorphisms of $\mathbb{C}^n$ taking any flag into any other.

⌈ $V_1 = \langle (v_{11}, v_{12} \cdots v_{1n}) \rangle$ $V_2 = \langle (v_{11} \cdots v_{1n}), (v_{21}, \cdots v_{2n}) \rangle$
$\cdots$ $V_{n-1} = \langle v_1 \cdots v_{n-1} \rangle$, $V_n = \langle v_1 \cdots v_n \rangle$. $v_i = (v_{i1} \cdots v_{in})$