

$$\bar{c} \langle [\Lambda, \oplus] \eta, \eta \rangle + 2\pi\mu(n-p)\|\eta\|^2 = \bar{c} \langle [\Lambda, 1 \otimes \oplus_E] \eta, \eta \rangle$$

$$\Rightarrow \text{left-hand side} \geq 2\pi\mu(n-p)\|\eta\|^2 > C(n-p)\|\eta\|^2 > C\|\eta\|^2$$

$$\text{right-hand side} \leq C\|\eta\|^2$$

\Rightarrow Thus only chance for equality is $\eta = 0$.

The Lefschetz Theorem on $(1,1)$ -classes.

As an application of Theorem B, we will complete our picture of the correspondences among divisors, line bundles, and Chern classes on a complex submanifold of projective space. First, we have the

Proposition. Let $M \subset \mathbb{P}^N$ be a submanifold. Then every line bundle on M is of the form $L = [D]$ for some divisor D ; i.e.,

$$\text{Pic}(M) \cong \frac{\text{Div}(M)}{\text{linear equivalence.}}$$

pf) To prove this, we have to show that every line bundle on M has a global meromorphic section.

See P136, if s is a global meromorphic section of L , $L = [s]$. \rightarrow

To find such a section, let H denote the restriction to M of the hyperplane bundle on \mathbb{P}^N .