

Conversely, if the rank of the last  $k \times (k + a_i - i)$  minor of a matrix representative for  $\Lambda$  is exactly  $k - i$ ,

$$A = \begin{pmatrix} v_{11}, v_{12}, \dots, v_{1, n-k+i-a_i}, & \overleftarrow{k+a_i-i} \quad \overrightarrow{a_i} \quad v_{1, n-k+i-a_i+1}, \dots, v_{1, n} \\ v_{21}, v_{22}, \dots, v_{2, n-k+i-a_i}, & v_{2, n-k+i-a_i+1}, \dots, v_{2, n} \\ \vdots \\ v_{k1}, v_{k2}, \dots, v_{k, n-k+i-a_i}, & \dots, v_{k, n} \end{pmatrix}$$

$\Rightarrow$  By elementary row operations (i.e. multiplications by <sup>non-zero</sup> scalars and addition of row vectors), we may have a matrix representative as follow  $\leftarrow k+a_i-c \rightarrow$

time as follow

$$\Lambda = \begin{pmatrix} \sqrt{t_1} & \dots & \sqrt{t_{n-kr+ci}} & 0 & \dots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ \sqrt{t_1} & \dots & \sqrt{t_c} & 0 & \dots & 0 \\ * & \dots & * & * & & * \end{pmatrix} \quad \leftarrow k+ci-c \rightarrow$$

$$\Rightarrow \langle v_1, \dots, v_{\bar{i}} \rangle \text{ has rank } \bar{i}. \Rightarrow \langle v_1, \dots, v_{\bar{i}} \rangle \in V_{n-k+\bar{i}-a_{\bar{i}}}$$

$$\Rightarrow \dim(L \cap V_{n-k+\bar{i}-a_{\bar{i}}}) = \bar{i}.$$

⇒ If  $\dim(\Lambda \cap V_{n-k+i-a_i}) = i$ ,  $\Lambda$  is represented by the following matrix, for example

[illegible]

$$\Rightarrow \quad \begin{aligned} & K(n-k-a_i) + (k-i)a_i = \{ \wedge^i \dim(\Lambda \cap V_{n-k+i-a_i}) = i \} \\ \text{i.e. } & \textcircled{1} \quad K(n-k-a_1) + (k-1)a_1 \cong \{ \wedge^1 \dim(\Lambda \cap V_{n-k+1-a_1}) = 1 \} = K_1 \\ & \textcircled{2} \quad K(n-k-a_2) + (k-2)a_2 \cong \{ \wedge^2 \dim(\Lambda \cap V_{n-k+2-a_2}) = 2 \} = K_2 \\ & \textcircled{3} \quad K(n-k-a_k) + 0 \cong \{ \wedge^k \dim(\Lambda \cap V_{n-a_k}) = k \} = K_k \end{aligned}$$

which are analytic subvarieties of  $G(k, n)$ .