and P207 Theorem. $\Rightarrow$ By the observations on P176 & P177, $\iota_{[D]}^*(H) = [D]$ and $\iota_{[L+D]}^*(H) = [L+D]$, where H represents the hyperplane bundle over $\mathbb{P}^N$ & $\mathbb{P}^M$. $\Rightarrow$ By Bertini's theorem $\exists$ a smooth hyperplane section $V = H \cap M$.
$\Rightarrow$ V is linearly equivalent to D. (similarly we get a smooth hyperplane section which is linearly equivalent to $D+L$.). More precisely, $H = (S=0)$ is a hyperplane in $\mathbb{P}^N$, and consider $S|_D$. $\Rightarrow$ $(S|_D = 0) = V$ is linearly equivalent to $D|_D$. $\iota_{[D]}^* S$
Similarly for $L+D$. $\Rightarrow$ Do the whole thing above to V.

But

$$\chi(L'|_D) = -\pi(D) + \deg L'|_D + 1$$
$$= -\frac{D\cdot D + D\cdot K}{2} + L'\cdot D;$$

so

$$\chi(L) = \chi(\mathcal{O}_M) + \frac{L'\cdot L' - L'\cdot K}{2} + \frac{D\cdot D + D\cdot K - 2(L'\cdot D)}{2}$$
$$= \chi(\mathcal{O}_M) + \frac{(L'\cdot L' - 2L'\cdot D + D\cdot D) - (L'\cdot K - D\cdot K)}{2}$$
$$= \chi(\mathcal{O}_M) + \frac{L\cdot L - L\cdot K}{2};$$

this is the Riemann-Roch formula for line bundles on a surface.

Pf $L' = L+D \sim V'$ (smooth divisor lin. eq. to $L+D$) and $D \sim V$.