

$$T. \Rightarrow dT_\epsilon = 0 \text{ since } T(\varphi) = \int_M T_\epsilon \wedge \varphi.$$

$$\begin{aligned} \bar{\partial}T=0 &\Rightarrow (\bar{\partial}T)(\varphi) = \pm T(\bar{\partial}\varphi) = \pm \int_M T_\epsilon \wedge \bar{\partial}\varphi = \pm \int_M \bar{\partial}T_\epsilon \wedge \varphi \Rightarrow \\ &\Rightarrow \bar{\partial}T_\epsilon = 0. \Rightarrow \text{This implies that for a given real (p,p)} \\ &\text{-current } T, \exists \text{ only one cohomology class rep-} \\ &\text{resented by } T_\epsilon \text{ satisfying } dT_\epsilon = \bar{\partial}T_\epsilon = 0. \quad \square \end{aligned}$$

"Comment": For a real (p,p) current T , if η_1, η_2 forms s.t. $\eta_1 \in H_{\bar{\partial}}^{p,p}(M)$, $\eta_2 \in H_{\text{DR}}^{2n-2p}(M) \Rightarrow [\eta_1] = [\eta_2]$.

(Now suppose that T is a closed, positive (p,p)-current.

T is real (p,p)-current.

$$\Rightarrow \exists \eta \in H_{\bar{\partial}}^{p,p}(M) \text{ s.t. } T(\varphi) = \int_M \eta \wedge \varphi, \quad \varphi \in A^{n-p,n-p}$$

$$\Rightarrow \exists \sigma \in H_{\text{DR}}^{2p}(M) \text{ s.t. } T(\phi) = \int_M \sigma \wedge \phi, \quad \phi \in A^{2n-2p}$$

$$\Rightarrow \text{Note that } T=0 \text{ on } A^{l,q}, \quad l+q=2n-2p, \quad l \neq q.$$

$$\Rightarrow [\eta] = [\sigma] \text{ in } H_{\text{DR}}^{2p}(M). \quad \square$$

Then by the smoothing of cohomology there is a closed, real smooth (p,p) form T_ϵ in the same cohomology class as T . With some care we could insure that

$$\lim_{\epsilon \rightarrow 0} T_\epsilon = T$$

□ The smoothing of cohomology means the isomorphisms