is just the Plücker embedding of the dual Grassmannian $G(4,6)$ of 3-planes in $\mathbb{P}^5$.

「 ① $G \cap H$ singular.

If $G \cap H = G \cap H'$, then $H = T_x G = H'$ by the result on P135. and P159

② $G \cap H$ smooth

$\Rightarrow$ $G \cap H$ is a smooth quadric in H

$\Rightarrow$ $G \cap H = (X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = 0)$ where we may assume $H = (X_5 = 0)$.

$[1, i, 0, 0, 0]$, $[0, 1, i, 0, 0]$ $[0, 0, 0, 1, i]$ $[0, 0, 1, i, 0]$ $[1, 0, 0, 0, i]$ are linearly independent in $G \cap H$, and they span H.

$\Rightarrow$ If $G \cap H = G \cap H'$, then $H = H'$.

I misunderstood. !!!

$\sigma_1(V_3) = \{ L \subset \mathbb{P}^5 \mid L \cap V_3 \neq \phi \}$.

$$\begin{array}{ccc} G(2,6) & \xrightarrow{\tilde{\phi}} & \mathbb{P}(\Lambda^2 \mathbb{C}^6). \\ \cup\!\! \text{\tiny I} & & \cup\!\! \text{\tiny I} \\ L = \overline{v_1 v_2} & \longmapsto & v_1 \wedge v_2 \end{array}$$

Let $v_1 = x_0 e_0 + \cdots + x_5 e_5$. and $v_2 = y_0 e_0 + \cdots + y_5 e_5$.

$\Rightarrow$ We may assume $V_3 = \langle e_0, e_1, e_2, e_3 \rangle$, and since $L \cap V_3 \neq \phi$, $\{ v_1, v_2, e_0, e_1, e_2, e_3 \}$ is linearly