bound the $\mathcal{H}_{s+1}$-norm of u_ϵ in terms of the $\mathcal{H}_s$-norms of Qu_ϵ and u_ϵ.

┌ $\| \frac{\partial u_\epsilon}{\partial x_1} \| \leq \| P \frac{\partial u_\epsilon}{\partial x_1} \|_0 + \| \frac{\partial u_\epsilon}{\partial x_1} \|_0$

$\leq \| P u_\epsilon \|_1 + \| u_\epsilon \|_1$.

$\Rightarrow \sum_\alpha \| \frac{\partial u_\epsilon}{\partial x_\alpha} \| \leq \| P u_\epsilon \|_1 + \| u_\epsilon \|_1$, where we omit constants.

$\Rightarrow \| u_\epsilon \|_2 \leq \| P u_\epsilon \|_1 + \| u_\epsilon \|_1$.

Continue this process, then we get

$\| u_\epsilon \|_{s+1} \leq \| P u_\epsilon \|_s + \| u_\epsilon \|_s$. ┘

Inductively, we may assume that $u \in \mathcal{H}_s$, and then the s-norm of u_ϵ is bounded by the s-norm of u.

┌ $D u_\epsilon \to D u$ uniformly, $\Rightarrow \| D u_\epsilon - D u \|_0^2 \to 0$ as $\epsilon \to 0$. $\Rightarrow$ $\| u_\epsilon \|_s$ is bounded by $\| u \|_s$ for ϵ sufficiently small. ┘

It remains to bound the s-norm of Qu_ϵ. We know how to bound the s-norm of $(Qu)_\epsilon = -(Ru)_\epsilon + v_\epsilon$, and so we must bound the s-norm of the difference

(**) $$(Qu)_\epsilon - Q(u_\epsilon).$$

┌ In our situation, it remains to bound the s-norm of