Here is one more illustration of the reciprocity formula.

Let Γ_0 be a set of 12 points in $\mathbb{P}^2$ that fails to impose independent conditions on $|\mathcal{O}_{\mathbb{P}^2}(4)|$. Then either

$\Gamma_0 = C_4 \cdot C_3$ is a complete intersection, or
10 points from Γ_0 are on a conic, or
six points from Γ_0 are collinear.

Proof. We assume that $\dim |\mathcal{J}_{\Gamma_0}(4)| \geq 3$.

$\ulcorner$ By P713, $\dim |\mathcal{J}_{\Gamma_0}(4)| = \frac{4(4+3)}{2} - 12 + \omega = 2 + \omega$
$\Rightarrow$ Since $\omega = 0 \Leftrightarrow$. Γ_0 imposes independent conditions on $|\mathcal{O}_{\mathbb{P}^2}(4)|$, $\omega \geq 1$. $\Rightarrow$ $\dim |\mathcal{J}_{\Gamma_0}(4)| \geq 3$. $\lrcorner$

Recalling that $\dim |\mathcal{O}_{\mathbb{P}^2}(4)| = 14$, if there are two curves C, C' from this linear system having no common component, then

$$C \cdot C' = \Gamma_0 + \Gamma.$$

$\ulcorner$ By the application on P714. $\lrcorner$

By the reciprocity formula, Γ consists of four collinear points, and this implies that $\Gamma_0 = C_4 \cdot C_4$.

$\ulcorner$ By the reciprocity formula I,