$+\ \beta\langle w, b_1\rangle = 0$ and $\langle v_2, e_{b_2}\rangle = 1 = \alpha\langle v_1, e_{b_2}\rangle + \beta\langle w, b_2\rangle$

$$\Rightarrow \quad \alpha + \beta\langle w, b_1\rangle = 0$$
$$\beta\langle w, b_2\rangle = 1. \quad \Rightarrow \beta = \frac{1}{\langle w, b_2\rangle}$$
$$\alpha = -\frac{\langle w, b_1\rangle}{\langle w, b_2\rangle}$$

Clearly, the choice of v_i at each stage is completely specified by these conditions; thus the k-plane Λ has a unique matrix representative of the form

$$\begin{bmatrix} v_1 \\ \vdots \\ v_k \end{bmatrix} = \left[\begin{array}{cccccccccccccc} * & * & * & 1 & 0 & 0 & \cdots & & \overset{n-k}{1} & & 0 & \cdots & & 0 \\ & & & 0 & * & 1 & 0 & 0 & \cdots & & & & & 0 \\ & & & 0 & * & 0 & * & * & \cdots & 1 & 0 & 0 & \cdots & \vdots \\ & & & \vdots & & 0 & & & & 0 & & & & \\ & & & & & & & & & \vdots & & & & \\ * & * & * & 0 & * & 0 & & & & 0 & & & & \end{array}\right]$$

$\longleftarrow a_1 \longrightarrow$

$\longleftarrow a_2 \longrightarrow$

$\longleftarrow a_3 \longrightarrow$

「 In the page P492 ~ P493, we may have the following case, for example,

$$M = \begin{pmatrix} a_1 & 0 & b_1 & c_1 & 0 & * & \cdots \\ a_2 & 1 & b_2 & c_2 & 0 & & \\ \vdots & 0 & b_3 & c_3 & 0 & & \\ \vdots & \vdots & \vdots & \vdots & 1 & & \\ \vdots & \vdots & \vdots & \vdots & 0 & & \\ & 0 & & & 0 & & \end{pmatrix} \quad \begin{array}{l} \text{rank } n \\ \#\text{ of } e^{i}\text{'s} = \ell \end{array}$$

Now we want M to have rank $n+k$.

$\Rightarrow$ Obviously, if M is $p \times q$ matrix, $p, q \geq n+k$.

$\Rightarrow$ Since M has rank n, the following matrix N