ions $\mathbb{P}(E)^* \cong \mathbb{P}^N$ corresponding to the choice of basis $s_0, s_1, \cdots s_N$, then, the map i_E is given by

$$i_E(p) = [s_0(p), \cdots s_N(p)].$$

We see from this representation that i_E is holomorphic.

Γ

$$\begin{tikzcd} U_\alpha \times \mathbb{C} = U_\beta \times \mathbb{C} \arrow[dr] & L|U_\alpha \arrow[l, "\varphi_\beta"'] \arrow[r, "\varphi_\alpha"] \arrow[d, shift right] & U_\alpha \times \mathbb{C} \arrow[dl] \\ & U_\alpha \arrow[u, shift right, "s"'] & \end{tikzcd}$$

$(x, \varphi_\alpha^* s) = (x, \varphi_\alpha \circ s)$

$$\varphi_\alpha \circ s(x) = g_{\alpha\beta}(x)\, \varphi_\beta \circ s(x)$$

$$(\varphi_\alpha^* s)(x) = g_{\alpha\beta}(x)\, (\varphi_\beta^* s)(x)$$

For any other trivialization φ_β,

$$[s_{0,\beta}(p), \cdots s_{N\beta}(p)] = [g_{\alpha\beta}^{-1}(p)\, s_{0,\alpha}(p), \cdots g_{\alpha\beta}^{-1}(p)\, s_{N,\alpha}(p)]$$
$$= [s_{0,\alpha}(p), \cdots s_{N,\alpha}(p)]$$

Now let H be the hyperplane bundle on $\mathbb{P}^N$. The pullback bundle $i_E^*(H)$ on M is given by the divisor (s_i) – that is,

$$L = i_E^*(H).$$

Γ $i_E^*(H) = \{(p, v) \in M \times H \mid i_E(p) = \pi(v),\ \pi : H \longrightarrow \mathbb{P}^N\}$

Define a section $\sigma : M \longrightarrow i_E^*(H)$ by

$\sigma(p) = (p, \tau \circ i_E(p))$, where $\tau : \mathbb{P}^N \longrightarrow H$ is a section.
$\Rightarrow$ σ is holomorphic clearly.
Let $(\tau = 0) = \{[X_0, X_1, \cdots X_n] \in \mathbb{P}^N \mid a_0 X_0 + \cdots + a_N X_n = 0\}$. (specially chosen).