$\sum_i \varphi \wedge P(A_1, \cdots, gA_i, \cdots A_k)$. $A_1, \cdots A_k$ must appear in that order. ⅃

In general, if θ is any matrix of 1-forms, θ can be written $\sum \theta_\alpha g_\alpha$, where θ_α is a 1-form and g_α is a matrix of functions; by linearity again.

$$(*) \quad \sum_i (-1)^{d_1+\cdots+d_{i-1}} P(A_1, \cdots, \theta \wedge A_i, \cdots A_k) = \sum_i (-1)^{d_1+\cdots+d_i} P(A_1, \cdots, A_i \wedge \theta, \cdots A_k).$$

Pf

$$\sum_i (-1)^{d_1+\cdots+d_{i-1}} P(A_1, \cdots, \theta \wedge A_i \cdots A_k)$$

$$= \sum_i (-1)^{d_1+\cdots+d_{i-1}} P(A_1, \cdots, \sum_\alpha \theta_\alpha g_\alpha \wedge A_i, \cdots A_k)$$

$$= \sum_i (-1)^{d_1+\cdots+d_{i-1}} \sum_\alpha P(A_1, \cdots, \theta_\alpha g_\alpha \wedge A_i, \cdots A_k)$$

$$= \sum_i (-1)^{d_1+\cdots+d_{i-1}} \sum_\alpha P(A_1, \cdots, g_\alpha \theta_\alpha \wedge A_i, \cdots A_k)$$

$$= \sum_i (-1)^{d_1+\cdots+d_{i-1}} \sum_\alpha P(A_1, \cdots \theta_\alpha \wedge A_i g_\alpha, \cdots A_k)$$

$$= \sum_i \sum_\alpha \theta_\alpha \wedge P(A_1, \cdots, A_i g_\alpha, \cdots A_k)$$

$$= \sum_i (-1)^{d_1+\cdots+d_{i-1}+d_i} \sum_\alpha P(A_1, \cdots, A_i \wedge \theta_\alpha g_\alpha, \cdots A_k)$$

$$= \sum_i (-1)^{d_1+\cdots+d_i} P(A_1, \cdots, A_i \wedge \sum_\alpha \theta_\alpha g_\alpha, \cdots A_k)$$

$$= \sum_i (-1)^{d_1+\cdots+d_i} P(A_1, \cdots, A_i \wedge \theta, \cdots A_k).$$

⅃