and the analytic variety $W+\epsilon$ — that is, W translated by ϵ — respectively; moreover, $\pi_2^{-1}(\epsilon)$ will meet the intersection $\tilde{V}\cap\tilde{W}$ transversely at a point (p,ϵ) exactly when V and $W+\epsilon$ meet transversely at p.

Γ
$$\begin{array}{ccc} \Delta'\times\Delta' & \qquad & \Delta'\times\Delta' \\ \downarrow \pi_1 & & \downarrow \pi_2 \\ \Delta' \supset V,\ W & & \Delta' \end{array}$$

$\pi_1^{-1}(V) = \{(z,w) ;\ z\in V\}$.
$\quad \overset{\shortparallel}{\tilde{V}}$

$\tilde{W} = \{(z,w) ;\ z-w\in W\}$

$\tilde{V}\cap\pi_2^{-1}(\epsilon) = \pi_2^{-1}(\epsilon)$. $\qquad \tilde{W}\cap\pi_2^{-1}(\epsilon) = \{(z,\epsilon) ;\ z\in W+\epsilon\}$

$= \{(z,\epsilon) \mid z\in V\}$

If $\pi_2^{-1}(\epsilon)\cap\tilde{V}\cap\tilde{W}\ni(p,\epsilon) \Rightarrow (p,\epsilon)\in\tilde{V} \Rightarrow p\in V$

$(p,\epsilon)\in\tilde{W} \Rightarrow p-\epsilon\in W \Rightarrow p\in W+\epsilon \Rightarrow p\in V\cap W+\epsilon$.

Conversely, $p\in V\cap W+\epsilon \Rightarrow p-\epsilon\in W \Rightarrow (p,\epsilon)\in\tilde{W}$

$(p,\epsilon)\in\tilde{V}\cap\pi_2^{-1}(\epsilon) \Rightarrow (p,\epsilon)\in\pi_2^{-1}(\epsilon)\cap\tilde{W}\cap\tilde{V}$.

Suppose V and $W+\epsilon$ meet transversely at p.

$\Rightarrow T_pV + T_p(W+\epsilon) = T_pV + T_{p-\epsilon}W = T\Delta'$

$T_{(p,\epsilon)}(\pi_2^{-1}(\epsilon)) = (T\Delta', 0) \qquad \tilde{V}\cap\tilde{W}\ni(p,\epsilon)$.

For no confusion, $\epsilon = x$. $\qquad (p,x)\in\tilde{V}\cap\tilde{W}$.

$(\alpha(t),\beta(t))\in\tilde{V}\cap\tilde{W}$, $\qquad \alpha(0)=p$, $\beta(0)=x$.

$\Rightarrow (\alpha'(0),\beta'(0))\in T_{(p,x)}(\tilde{V}\cap\tilde{W}) \qquad \alpha'(0)\in T_pV$.

$\alpha(t)-\beta(t)\in W \Rightarrow \alpha'(0)-\beta'(0)\in T_{p-x}W$

$\Rightarrow \beta'(0)\in\alpha'(0)+T_{p-x}W = \alpha'(0)+T_p(W+x)$

$= \alpha'(0)+T_p(W+\epsilon)\subset T_pV+T_p(W+\epsilon)$