$\Rightarrow$ By P148. $\Theta_{[E]} = -\partial\bar{\partial}\log|\sigma|^2 = -\frac{2\pi}{i} dd^c \log|\sigma|^2$ by P109

$$\Rightarrow \Omega_{[E]} = \frac{i}{2\pi}\Theta_{[E]} = -dd^c\log|\sigma|^2 = dd^c\log\frac{1}{|\sigma|^2} \equiv 0 \text{ on } \tilde{M}_x - \tilde{U}_{2\epsilon}.$$

2. On $\tilde{U}_\epsilon - E \cong U_\epsilon - \{x\}$, let σ be given in terms of the representation (*) by $\sigma(z, l) = z$;
then

$$\Omega_{[E]} = dd^c\log\frac{1}{\|z\|^2} = -dd^c\log\|z\|^2,$$

i.e., $-\Omega_{[E]}$ is just the pullback $\pi'^*\omega$ of the associated (1,1)-form ω of the Fubini-Study metric on $\mathbb{P}^{n-1}$ under the map $\pi': \tilde{U}_\epsilon \longrightarrow \mathbb{P}^{n-1}$ given by $(z, l) \mapsto l$.
Thus

$$-\Omega_{[E]} \geq 0 \quad \text{on } \tilde{U}_\epsilon - E.$$

$\ulcorner$

$$\begin{array}{ccccc}
\sigma: \tilde{U}_\epsilon - E & \longrightarrow & [E] & \subset & \tilde{U}_\epsilon - E \times \mathbb{C}^n \\
\cup & & & & \\
\tilde{U}_{\epsilon,i} & \longrightarrow & \mathbb{C} & & \\
\psi & & \psi & & \\
(z, l) & \longmapsto & z_i. & &
\end{array}$$

$$\begin{array}{ccc}
\tilde{U}_{\epsilon,i} \times \mathbb{C}^n & \ni & ((z,l), v) \\
\downarrow & & \uparrow \\
\tilde{U}_{\epsilon,i} & \ni & (z, l)
\end{array}$$

$\Rightarrow v \in l$

$v = (v_1, \cdots v_n)$

$\Rightarrow$ The trivialization is given as follows:

$$\begin{array}{ccc}
((z, l), v) & \longmapsto & v_i. \\
\uparrow & \nearrow & \\
(z, l) & &
\end{array}$$

$\Rightarrow$ Since $\sigma_i((z, l)) = z_i$,
$z_i = v_i$.