

$$A'_{23} \neq \tilde{w}_1 A_1 + \dots + \tilde{w}_{21} A_{21}$$

$$A'_{24} \neq \tilde{w}_1 A_1 + \dots + \tilde{w}_{21} A_{21}, \text{ where } (A_{24}, B'_{24}, C'''_{24}) = \tilde{w}_1 (A_1, B_1, C'_1) + \tilde{w}_2 (A_2, B_2, C'_2) + \dots + \tilde{w}_{21} (A_{21}, B_{21}, C'_{21})$$

①, ② & ③ are key points. (See also P494~P495)

To see that $\overline{W_{a_1, a_k}} = \{ \Lambda : \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i \}$ is analytic subvariety of $G(k, n)$, notice the following simple fact.

Given the set of $k \times n$ matrices

$$\left\{ \begin{pmatrix} v_{11}, \dots, v_{1n} \\ v_{21}, \dots, v_{2n} \\ \vdots \\ v_{k1}, \dots, v_{kn} \end{pmatrix} \mid v_{ij} \in \mathbb{C} \right\},$$

then rank $\leq m$ matrices form an analytic subvariety, $m \leq k, n$.

For each I th $m \times m$ minor A_I ,

$\det(A_I) = 0 \Rightarrow \bigcap \{ \det A_I = 0 \}$ is the set of all $k \times n$ matrices with rank $\leq m$.

Thus $\overline{W} \cap U_J = \bigcap \{ \det A_L = 0 \}$, where A_L is any $(k-i) \times (k-i)$ minor of the last $k \times (k+a_i-i)$ minor, $i = 1, \dots, k$. :))

We can choose a special basis for a k -plane $\Lambda \in W_{a_1, a_k}$ as follows: let v_i be a generator for the line $\Lambda \cap V_{n-k+i-a_i}$, normalized so that $\langle v_i, e_{n-k+i-a_i} \rangle = 1$; i.e.,