「 $\mathbb{P}^1 \longrightarrow \mathbb{P}^n$

$[Z_0, Z_1] \longmapsto [Z_0^n, Z_0^{n-1}Z_1, Z_0^{n-2}Z_1^2, \cdots Z_1^n]$

$\qquad\qquad\qquad\qquad\qquad\qquad \|$

$t \longmapsto [1, \frac{Z_1}{Z_0}, (\frac{Z_1}{Z_0})^2, \cdots (\frac{Z_1}{Z_0})^n]$ 」

Its image is a nondegenerate curve of deg n, called the rational normal curve.

「 $t=0, \{[1, 0 \cdots 0], [1, 2, 2^2, \cdots 2^n] \cdots [1, 1, \cdots 1]$
$[1, 3, 3^2, \cdots 3^n] \cdots [1, n, n^2, \cdots n^n] \}$.

$$\begin{array}{c|c} & \overbrace{\qquad\qquad\qquad\qquad}^{(n+1)} \\ (n+1) & \begin{vmatrix} 1, 0 & \cdots & & 0 \\ 1, 1, 1^2, & \cdots & & 1^n \\ 1, 2, 2^2, & \cdots & & 2^n \\ 1, 3, 3^2 & \cdots & & 3^n \\ \vdots & & & \\ 1, n, n^2 & \cdots & & n^n \end{vmatrix} \end{array} = \begin{vmatrix} 1, 0 & \cdots & 0 \\ 0, 1, 1 & \cdots & 1^n \\ 0, 2, 2^2, & \cdots & 2^n \\ 0, 3, 3^2, & \cdots & 3^n \\ \vdots & & \\ 1, n, n^2, & \cdots & n^n \end{vmatrix} = 1\cdot 2\cdot 3 \cdots n \begin{vmatrix} 1, 0 & \cdots & 0 \\ 0, 1, & \cdots & 1^{n-1} \\ 0, 1, 2, & \cdots & 2^{n-1} \\ 0, 1, 3 & \cdots & 3^{n-1} \\ \vdots & & \\ 1, n, & & n^{n-1} \end{vmatrix}$$

$= n! \prod_{j>i} (j-i) \neq 0$ See Lang P179. Ex.5 (linear algebra)

Vandermonde determinant.

$\Rightarrow$ Its image contains $(n+1)$ - linearly independent points, so that it can not be contained in any hyperplane. $\Rightarrow$ The image is nondegenerate.

Consider the intersection $i_{nH}(\mathbb{P}^1) \cap (\mathbb{P}^{n-1} = (X_0 = 0))$.
$\Rightarrow$ It is $\{ Z_0^n = 0 \}$, which is $[0, 1]$ with multiplicities n. $\Rightarrow$ The image degree is n. (Conglish) 」