Question: $\sigma_a(V) = i_2^{-1}(\sigma_a(V'))$

$\Lambda \in \sigma_a(V) \Rightarrow \dim(\Lambda \cap V_{n+k-2k+i-a_i}) \geq i$,
$i_2(\Lambda) = \Lambda \oplus \{e_{n+1}\} \Rightarrow (\Lambda \oplus \{e_{n+1}\}) \cap V'_{n+1-k+i-a_i-1} =$
$(\Lambda \cap V_{n-k+i-1-a_i}) \oplus \{e_{n+1}\} \Rightarrow \dim((\Lambda \oplus \{e_{n+1}\}) \cap V'_{n-k+i-a_i}))$
$= \dim(\Lambda \cap V_{n-k+i-1-a_i}) + 1 \geq i \Rightarrow \Lambda \oplus \{e_{n+1}\} \in i_2^{-1}(\sigma_a(V'))$

Conversely, if $\Lambda \oplus \{e_{n+1}\} = i_2(\Lambda) \in \sigma_a(V')$,
$\Lambda \in \sigma_a(V) \Rightarrow i_2^{-1}(\sigma_a(V')) \subset \sigma_a(V)$

According to P59. $i_1^{-1}(\sigma_a)$ is Poincare dual to $i_1^*\tilde{\sigma}_a$.
$i_2^{-1}(\sigma_a)$ " to $i_2^*\tilde{\sigma}_a$.

Thus any formula $(\sigma_a \cdot \sigma_b) = \sum n_c \cdot \sigma_c$ for the intersection of Schubert cycles in $G(k, n+1)$ or $G(k+1, n+1)$ holds as well in $G(k, n)$, and we can define the universal Schubert coefficients $\delta(a, b; c)$ to be such that the formula
$$(\sigma_a \cdot \sigma_b) = \sum \delta(a, b; c) \cdot \sigma_c \text{ holds in}$$
all $G(k, n)$.

Pf $n_c = \#(\sigma_a \cdot \sigma_b \cdot \sigma_c)$.

$$G(k, n) \xrightarrow{i_1} G(k, n+1)$$
$$\sigma_a(V) \longrightarrow \sigma_a(V')$$
$$\#(\sigma_a(V) \cdot \sigma_b(V) \cdot \sigma_c(V)) = \int_{G(k,n)} \tilde{\sigma}_a(V) \wedge \tilde{\sigma}_b(V) \wedge \tilde{\sigma}_c(V)$$
$$= \int_{G(k,n)} i_1^*\tilde{\sigma}_a(V') \wedge i_1^*\tilde{\sigma}_b(V') \wedge i_1^*\tilde{\sigma}_c(V') \overset{?}{=} \int_{G(k,n+1)} \tilde{\sigma}_a(V') \wedge \tilde{\sigma}_b(V') \wedge \tilde{\sigma}_c(V')$$
$$= \#(\tilde{\sigma}_a(V') \cdot \tilde{\sigma}_b(V') \cdot \tilde{\sigma}_c(V'))$$