in the global theory of coherent sheaf cohomology on a compact manifold.

A second application of the Hodge theorem is to Kodaira-Serre duality. From the formula $\bar\partial^* = -*\bar\partial*$, we see that

$$*\Delta = \Delta * .$$

「 $\underbrace{-*\bar\partial*\bar\partial*}_{*\Delta} - **\bar\partial*\bar\partial = -*\bar\partial*\bar\partial - \underbrace{\bar\partial*\bar\partial**}_{\Delta*}$. 」

This implies that the star operator induces an isomorphism

$$* : \mathcal{H}^{p,q}(M) \longrightarrow \mathcal{H}^{n-p,\, n-q}(M).$$

In particular,

$$\mathcal{H}^{n,n}(M) \cong \mathbb{C}\cdot\Phi$$

where $\Phi = *1$ is the volume form of the metric

「 $\mathcal{H}^{n,n}(M) \overset{*}{\cong} \mathcal{H}^{0,0}(M) = H^0(M, \Omega^0) = H^0(M, \mathcal{O}) \cong \mathbb{C}$ 」

To put this isomorphism in intrinsic form not depending on the choice of a metric, we remark in a general fashion that, given sheaves $\mathcal{F}$, $\mathcal{G}$, and $\mathcal{H}$ over a space X, and a sheaf mapping

$$\mathcal{F}\otimes\mathcal{G} \longrightarrow \mathcal{H},$$

there is an induced cup product

$$H^*(X, \mathcal{F}) \otimes H^*(X, \mathcal{G}) \longrightarrow H^*(X, \mathcal{H})$$ given by the cochain formula at the end of the discussion of