

If $\varphi: \mathcal{F} \rightarrow \mathcal{G}$ is a morphism of sheaves, we define the image of φ , denoted by $\text{im } \varphi$, to be the sheaf \mathcal{F} associated to the presheaf image of φ . By the universal property of the sheaf associated to a presheaf, there is a natural map $\text{im } \varphi \rightarrow \mathcal{G}$.

In fact, the map is injective and thus $\text{im } \varphi$ can be identified with a subsheaf of \mathcal{G} .

We say that a morphism $\varphi: \mathcal{F} \rightarrow \mathcal{G}$ of sheaves is surjective if $\text{im } \varphi = \mathcal{G}$.

We say that a sequence $\dots \rightarrow \mathcal{F}^{i-1} \xrightarrow{\varphi^{i-1}} \mathcal{F}^i \xrightarrow{\varphi^i} \mathcal{F}^{i+1} \rightarrow \dots$ of sheaves and morphism is exact if at each stage

$\ker \varphi^i = \text{im } \varphi^{i-1}$.
Thus $0 \rightarrow \mathcal{F} \xrightarrow{\varphi} \mathcal{G}$ is exact $\Leftrightarrow \varphi$ is injective
and $\mathcal{F} \xrightarrow{\varphi} \mathcal{G} \rightarrow 0$ is exact $\Leftrightarrow \varphi$ is surjective.

Now let \mathcal{F}' be a subsheaf of a sheaf \mathcal{F} .

We define the quotient sheaf \mathcal{F}/\mathcal{F}' to be the sheaf associated to the presheaf $U \mapsto \mathcal{F}(U)/\mathcal{F}'(U)$.

It follows that for any point p , the stalk $(\mathcal{F}/\mathcal{F}')_p =$

$$\frac{\mathcal{F}_p}{\mathcal{F}'_p}. \quad \left(\frac{\mathcal{F}}{\mathcal{F}'} \right)_p = \varinjlim_U \frac{\mathcal{F}(U)}{\mathcal{F}'(U)} \quad U \ni p.$$

$$\begin{array}{ccc} [(U, \sigma)] \in \left(\frac{\mathcal{F}}{\mathcal{F}'} \right)_p & \longrightarrow & \frac{\mathcal{F}_p}{\mathcal{F}'_p} \\ \parallel & & \\ [(V, \tau)] & \searrow & \\ \sigma \in \mathcal{F}(U) & \longrightarrow & [(U, \sigma)] + \mathcal{F}'_p \\ \Downarrow \sigma|_W - \tau|_W \in \mathcal{F}'(W) & \Leftrightarrow & [(V, \tau)] + \mathcal{F}'_p \\ & & [(U, \sigma)] - [(V, \tau)] \in \mathcal{F}'_p \end{array}$$