⌈ See $P\,216$ $b_1(S) = 2h^{1,0}(S)$. $\Rightarrow$ $h^{1,0}(S) = h^{0,1}(S)$
$= g(S)$ ⌟

In general, the number $h^{n,0}(M)$ of holomorphic forms of top degree on a compact complex n-manifold M is called the geometric genus of M and denoted $P_g(M)$.

2. An alternative generalization of the notion of genus is the number

$$P_a(M) = h^{n,0}(M) - h^{n-1,0}(M) + \cdots + (-1)^{n-1} h^{1,0}(M),$$

called the arithmetic genus of M. Using $h^{q,0}(M) = h^{0,q}(M)$ (see below) we can also write

$$P_a(M) = (-1)^n(\chi(\mathcal{O}_M) - 1).$$

⌈ $(-1)^n P_a(M) = h^{0,0}(M) - h^{1,0}(M) + h^{2,0}(M) - \cdots + (-1)^n h^{n,0}(M)$
$- h^{0,0}(M) = h^{0,0}(M) - h^{0,1}(M) + h^{0,2}(M) - \cdots + (-1)^n h^{0,n}(M) - 1$
$= \chi(\mathcal{O}_M) - 1$ $\Rightarrow$ $P_a(M) = (-1)^n(\chi(\mathcal{O}_M) - 1)$. ⌟

3. The number $h^{1,0}(M)$ of holomorphic 1-forms on a compact complex manifold M is often denoted $q(M)$ and called the irregularity of M. If M is Kähler, of course, the irregularity is simply half the first Betti number.

⌈ $h^{1,0}(M) = q(M) = h^{0,1}(M)$
$b_1(M) = \dim H^1(M) = h^{1,0}(M) + h^{0,1}(M)$. $\Rightarrow$ $q(M) = \frac{1}{2} b_1(M)$

" Since d commutes with the conjugate operation (i.e., $\varphi \mapsto \bar{\varphi}$)