$\Rightarrow$ If $\Lambda \in \sigma_a(V)$, then $e_i \in \Lambda$ since $e_i \in V_{k-r+1}$ and $\Lambda \supset V_{k-r+1}$. $\Rightarrow$ This implies that over $\sigma_a(V)$, $\sigma_a(V) \times V_{k-r+1} \subset S|_{\sigma_a(V)}$. $\lrcorner$

The sections e_i of $S|_{\sigma_a(V)}$ then extend to give $k-r+1$ everywhere linearly independent sections $\tilde{e}_i$ of S over an open set $\mathcal{U} \subset G(k,n)$ containing $\sigma_a(V)$.

$\ulcorner$ For each point $x \in \sigma_a(V)$, $\exists$ open subsets U_x, V_x of $G(k,n)$ s.t $\overline{V_x} \subset U_x$ and

$$\begin{tikzcd} S|U_x \arrow[rr, "\cong"] \arrow[dr] & & U_x \times \mathbb{C}^k \arrow[dl] \\ & U_x & \end{tikzcd}$$

local trivialty.

$\Rightarrow$ $\{V_x\}_{x \in \sigma_a(V)}$ is an open cover of $\sigma_a(V)$ and since $\sigma_a(V)$ is compact, $\exists$ a finite # of V_x's s.t

$\bigcup_{i=1}^{m} V_{x_i} \supset \sigma_a(V)$. For simplicity, let $V_{x_i} = V_i$ and $U_{x_i} = U_i$, & $\sigma_a(V) = K$.

We are going to show that, given a section τ on K, then we can extend τ to $\bigcup V_i$.

First, $K \cap \overline{V_1}$ is closed in $G(k,n)$ (in $\overline{V_1}$).

Consider $\tau|_{K\cap\overline{V_1}}$, which can be considered a map from $K \cap \overline{V_1}$ to $\mathbb{C}^k$. $\Rightarrow$ By the Tietze extension theorem, $\tau|_{K\cap\overline{V_1}}$ can be extended to $\overline{V_1}$. Second Consider $\tau|_{(K\cap\overline{V_2})\cup(\overline{V_1}\cap\overline{V_2})}$ which can be considered a map from $(K\cap\overline{V_2})\cup(\overline{V_1}\cap\overline{V_2})$ to $\mathbb{C}^k$ again. Again, $(K\cap\overline{V_2})\cup(\overline{V_1}\cap\overline{V_2})$ is closed in $\overline{V_2}$ (which is normal).

$\overset{\shortparallel}{(K\cup\overline{V_1})\cap\overline{V_2}}$