Next, suppose that $p(w) = w^n + a_1 w^{n-1} + \cdots + a_0$ is a polynomial in one complex variable, possibly with repeated roots. Then we have the distribution equation

$$\bar{\partial}\left(\frac{1}{2\pi\sqrt{-1}} \frac{p'(w)\,dw}{p(w)}\right) = \sum_{p(w_\nu)=0} \delta_{w_\nu}.$$

「 $$\frac{p'(w)}{p(w)} = \frac{\left(\prod(w-w_\nu)\right)'}{\prod(w-w_\nu)} = \sum_{p(w_\nu)=0} \frac{1}{w-w_\nu}$$

$$\bar{\partial}\left(\frac{1}{2\pi\sqrt{-1}} \frac{dw}{w-w_\nu}\right) = \delta_{w_\nu} \Rightarrow \bar{\partial}\left(\frac{1}{2\pi\sqrt{-1}} \frac{p'(w)\,dw}{p(w)}\right) = \sum_{p(w_\nu)=0} \delta_{w_\nu}.$$ 」

This means that the 1-form $\partial \log p(w)$ is integrable, and moreover for $\alpha \in C_c^\infty(\mathbb{C})$,

$$\frac{1}{2\pi\sqrt{-1}} \iint \frac{\partial \alpha(w)}{\partial \bar{w}} \frac{p'(w)}{p(w)}\, dw \wedge d\bar{w} = \sum_{p(w_\nu)=0} \alpha(w_\nu).$$

The formula follows by writing $p(w) = \prod_{\nu=1}^{n}(w-w_\nu)$ and using the Cauchy formula on each factor.

「 $$|\partial \log p(w)| \leq \sum \frac{1}{|w-w_\nu|}$$

$$\Rightarrow \left|\int_K \frac{1}{|w-w_\nu|}\, dw\wedge d\bar{w}\right| < \infty \Rightarrow \partial \log p(w) \text{ is locally integrable}$$

Choose a disk Δ containing all w_ν's.

$\Rightarrow$ By the Cauchy formula.

$$\frac{1}{2\pi i}\int_\Delta \frac{\partial \alpha(w)}{\partial \bar{w}} \frac{p'(w)}{p(w)}\, dw\wedge \bar{w} = \frac{1}{2\pi i}\sum \int_\Delta \frac{\partial \alpha(w)}{\partial \bar{w}} \frac{1}{w-w_\nu}\, dw\wedge d\bar{w}$$