C^∞ piecewise smooth cycles A and B on M representing α and β and intersecting transversely.

$\Rightarrow$ The intersection number $\#(A \cdot B)$ is determined by the classes α, β, and so we define a bilinear ~~#~~ pairing

$$H_k(M, \mathbb{Z}) \times H_{n-k}(M, \mathbb{Z}) \longrightarrow \mathbb{Z} \quad \text{called}$$

intersection pairing, and denoted by $\#(\alpha \cdot \beta)$.

Note that from the def of the intersection index,

$$\#(\beta \cdot \alpha) = (-1)^{k(n-k)} \#(\alpha \cdot \beta)$$

$$[(v_1, \cdots v_k)\,(v_{k+1}, \cdots v_n)] = (-1)^{(n-k)k} [(v_{k+1}, \cdots v_n), (v_1 \cdots v_k)]$$

We can also define a product

$$H_{n-k_1}(M, \mathbb{Z}) \otimes \times H_{n-k_2}(M, \mathbb{Z}) \longrightarrow H_{n-k_1-k_2}(M, \mathbb{Z}).$$

on the homology of M in arbitrary dimensions; if $\alpha \in H_{n-k_1}(M)$ and $\beta \in H_{n-k_2}(M)$ are classes, we can find cycles A and B representing them and intersecting transversely almost everywhere.

The intersection C is given the orientation s.t if $v_1 \cdots v_{n-k_1-k_2}$ is an oriented basis for $T_p(C)$ at a smooth pt of C and we complete it to bases.

$w_1 \cdots w_{k_2}\ v_1 \cdots v_{n-k_1-k_2}$	oriented basis for	$T_p(A)$
$v_1 \cdots v_{n-k_1-k_2},\ u_1 \cdots u_{k_1}$	"	$T_p(B)$
$w_1 \cdots w_{k_2}\ v_1, \cdots v_{n-k_1-k_2},\ u_1, u_2 \cdots u_{k_1}$	"	$T_p(M)$

If ~~yo~~ we change $v_1, v_2 \cdots v_{n-k_1-k_2}$ into $v_2, v_1 \cdots v_{n-k_1-k_2}$,

$w_1 \cdots w_{k_2}$ should be changed into $w_2, w_1 \cdots w_{k_2}$

$u_1 \cdots u_{k_1}$ " " $u_2, u_1 \cdots u_{k_1}$ so that