$f(v_{ij}) = b_{ij}$.

For example, if we have $v_1 = (v_{11}, v_{12}, \dots, v_{14})$, $v_2 = (v_{21}, \dots)$, $v_3 = (v_{31}, \dots)$, then let $f(z_1, z_2, z_3, z_4) = a_1 z_1 + a_2 z_2 + a_3 z_3 + a_4 z_4$ and choose distinct b_1, b_2, b_3.

⇒ We can find a_1, a_2, a_3, a_4 satisfying the following

$$
\begin{aligned}
a_1 v_{11} + a_2 v_{12} + a_3 v_{13} + a_4 v_{14} &= b_1 \\
a_1 v_{21} + a_2 v_{22} + a_3 v_{23} + a_4 v_{24} &= b_2 \\
a_1 v_{31} + a_2 v_{32} + a_3 v_{33} + a_4 v_{34} &= b_2 .
\end{aligned}
$$

If necessary, we may take $f = \sum a_{ijkl} z_1^i z_2^j z_3^k z_4^l$, to get sufficiently many a_{ijkl}'s becaus we need more unknowns than equations. ⇒ We can find a function seperating the sheets.”

If the germ $\mathbb{V}$ is irreducible, then the inclusion ${}_n\mathcal{O} \subseteq {}_V\mathcal{O}$ extends to an inclusion ${}_n\mathcal{M} \subseteq {}_V\mathcal{M}$ of the quotient fields (since ${}_V\mathcal{O}$ is again an integral domain and ${}_n\mathcal{O}$ is a subring of ${}_V\mathcal{O}$, see P Theorem 11, on P18) and ${}_V\mathcal{M}$ is an algebraic extension of ${}_n\mathcal{M}$ of degree ν.

“

Suppose $\{1, f, \dots, f^{\nu-1}\}$ is linearly dependent. ⇒ ∃ a polynomial $P(X) \in {}_n\mathcal{O}[X]$, for, if

$$\frac{a_1}{b_1} + \frac{a_2}{b_2} f + \dots + \frac{a_{\nu-1}}{b_{\nu-1}} f^{\nu-1} = 0, \quad a_i, b_i \in {}_n\mathcal{O}.$$

then, by multiplying $b_1 \cdots b_{\nu-1}$, we get $P(X) \in {}_n\mathcal{O}[X]$ satisfying $P(f) = 0$, which is imposible unless $P(X) \equiv 0$,