Date

Consider functions defined as follows.

$$s_0 : f^{-1}(U_0) \longrightarrow \mathbb{C} \quad x \longmapsto 1$$
$$s_1 : f^{-1}(U_1) \longrightarrow \mathbb{C} \quad x \longmapsto f_{11}$$
$$s_2 : f^{-1}(U_2) \longrightarrow \mathbb{C} \quad x \longmapsto f_{21}$$
$$s_3 : f^{-1}(U_3) \longrightarrow \mathbb{C} \quad x \longmapsto f_{31}$$

$\Rightarrow$
$$s_0 = \frac{1}{f_{11}} \cdot s_1 \quad \text{on } f^{-1}(U_0 \cap U_1)$$
$$s_1 = \frac{f_{11}}{f_{21}} s_2 \quad \text{on } f^{-1}(U_1 \cap U_2)$$
$$s_3 = \frac{f_{31}}{f_{21}} s_2 \quad \text{on } f^{-1}(U_2 \cap U_3)$$

$\Rightarrow \{s_i, f^{-1}(U_i)\}$ defines a section of f^*H over M
since $\frac{1}{f_{11}} = f^* g_{01}$ $\quad \frac{f_{11}}{f_{21}} = f^* g_{12}$ $\cdots\cdots$
where g_{ij}'s are transition functions of $[H]$ over $\mathbb{P}^N$.

Similarly, we can get sections of $f^*H = L$ over M which defines the embedding f. $\lrcorner$

Now from the proof of the Kodaira embedding theorem, we see

Corollary. If $E \longrightarrow M$ is any line bundle and $L \to M$ a positive line bundle, then for $k \gg 0$, the bundle $L^k + E$ is very ample.

$\ulcorner$ If we look at the proof of the Kodaira embedding theorem, for any line bundle K,

$$\begin{array}{ccc} H^0(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{K}^l)) & \xrightarrow{r_E} & H^0(E, \mathcal{O}_E(\tilde{K}^l)) \\ \uparrow & & \| \\ H^0(M, \mathcal{O}(K^l)) & \xrightarrow{r_{x,y}} & K_x^l \oplus K_y^l \end{array}$$

commutes.

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