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The Reciprocity Formula II gives

$$h^1(\mathcal{J}_{P_0}(4)) = h^0(\mathcal{J}_P(1)) \leq 1,$$

where $\mathcal{J}_{P_0}(4) = \mathcal{J}_{P_0} \otimes \mathcal{O}_{\mathbb{P}^2}(4)$, and where equality holds on the right if and only if the points of Γ are distinct and collinear.

$\ulcorner$ $P_g(\mathbb{P}^2) = h^{2,0}(\mathbb{P}^2) = \dim H^{2,0}(\mathbb{P}^2) = 0$

$q(\mathbb{P}^2) = h^{1,0}(\mathbb{P}^2) = \dim H^{1,0}(\mathbb{P}^2) = 0.$

$\Rightarrow$ By the Reciprocity Formula II, if $P_g = q = 0$, then

$$h^1(\mathcal{J}_{P_0}(4)) = h^0(\mathcal{J}_P(K+4H))$$
$$= h^0(\mathcal{J}_P(-3H+4H)) = h^0(\mathcal{J}_P(H)) = h^0(\mathcal{J}_P(1))$$

$$\dim H^0(\mathbb{P}^2, \mathcal{O}(H)) = \binom{2+1}{2} = 3$$

$$= \dim \{ a_0 Z_0 + a_1 Z_1 + a_2 Z_2 = 0 \mid [(a_0, a_1, a_2)] \in \mathbb{P}^2 \}$$

Any hyperplane can not pass through a point with multiplicity ≥ 2.

$\Rightarrow$ Unless the points of Γ are distinct and collinear (i.e., lie on a hyplane),

$\dim H^0(\mathbb{P}^2, \mathcal{J}_P(H)) = 0$, i.e $\exists$ no hyplane containing Γ.

Furthermore, since the points of Γ are distinct $\exists$ at most one hyperplane containing Γ. $\lrcorner$

Since $h^0(\mathcal{O}_{\mathbb{P}^2}(4)) = 15$ and $h^1(\mathcal{O}_{\mathbb{P}^2}(4)) = 0$, it follows that: The linear system of plane quartics through Γ_0

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