

$$\Rightarrow \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix}$$

$$= \frac{1}{D} \begin{pmatrix} D & 0 & a_{22}a_{13} - a_{12}a_{23} & a_{22}a_{14} - a_{12}a_{24} \\ 0 & D & -a_{21}a_{13} + a_{11}a_{23} & -a_{21}a_{14} + a_{11}a_{24} \end{pmatrix}$$

where $D = a_{11}a_{22} - a_{12}a_{21}$.

\Rightarrow By P 540 note \circledast .

$$f_1 = a + \frac{b}{D} (a_{11}a_{13} - a_{12}a_{23}) + \frac{c}{D} (a_{22}a_{14} - a_{12}a_{24})$$

$$f_2 = d + \frac{b}{D} (-a_{21}a_{13} + a_{11}a_{23}) + \frac{c}{D} (-a_{21}a_{14} + a_{11}a_{24})$$

$$\Rightarrow \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} a \\ d \end{pmatrix} + b A^{-1} \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix} + c A^{-1} \begin{pmatrix} a_{14} \\ a_{24} \end{pmatrix}$$

$$\Rightarrow A \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = A \begin{pmatrix} a \\ d \end{pmatrix} + b \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix} + c \begin{pmatrix} a_{14} \\ a_{24} \end{pmatrix}$$

Let $a = x$, $d = y$.

$$= \begin{pmatrix} a_{11}x + y a_{12} + b a_{13} + c a_{14} \\ a_{21}x + y a_{22} + b a_{23} + c a_{24} \end{pmatrix}$$

Let $b = z$, $c = w$

Consider $\sigma = x\sigma_1 + y\sigma_2 + z\sigma_3 + w\sigma_4$

$$= x(a_{11}e_1 + a_{21}e_2) + y(a_{12}e_1 + a_{22}e_2) + z(a_{13}e_1 + a_{23}e_2) + w(a_{14}e_1 + a_{24}e_2)$$

$$= (x a_{11} + y a_{12} + z a_{13} + w a_{14}) e_1 + (x a_{21} + y a_{22} + z a_{23} + w a_{24}) e_2$$