On the other hand, we have

$$\omega = \sum \alpha_i \wedge \beta_i ,$$

so that the n th exterior power

$$\omega^n = n! \cdot \alpha_1 \wedge \beta_1 \wedge \alpha_2 \wedge \beta_2 \wedge \cdots \wedge \alpha_n \wedge \beta_n$$
$$= n! \ d\mu .$$

$\ulcorner$
$$ds^2 = \sum \varphi_i \otimes \overline{\varphi_i} = \sum (\alpha_i + \sqrt{-1}\beta_i) \otimes (\alpha_i - \sqrt{-1}\beta_i)$$
$$= \sum \alpha_i \otimes \alpha_i + \beta_i \otimes \beta_i - \sqrt{-1}(\alpha_i \otimes \beta_i - \beta_i \otimes \alpha_i)$$
$$= Re(ds^2) - \sqrt{-1} \sum (\alpha_i \wedge \beta_i) 2 .$$
$$\Rightarrow Im(ds^2) = -2 \sum \alpha_i \wedge \beta_i$$
$$\Rightarrow -\frac{1}{2} Im(ds^2) = \sum \alpha_i \wedge \beta_i = \omega$$
$\lrcorner$

Now let $S \subset M$ be a complex submanifold of dimension d. As we have observed, the $(1,1)$-form associated to the metric induced on S by ds^2 is just $\omega|_S$, and applying the above to the induced metric on S, we have the

Wirtinger Theorem

$$vol(S) = \frac{1}{d!} \int_S \omega^d .$$

$\ulcorner$ By p29, the $(1,1)$-form associated to the metric induced on S by ds^2 is $\omega|_S$.
$\Rightarrow$ By the argument above, $(\omega|_S)^d = d! \, d\mu$, where $d\mu$ is the volume element of S.

$$\Rightarrow vol(S) = \frac{1}{d!} \int_S \omega^d$$
$\lrcorner$