

be a single valued function  $\log^* z$  s.t.  $e^{\log^* z} = z$ ,  
for all nonzero complex numbers  $z$ .  $\Rightarrow$

First, since  $d\vartheta/g$  has a single pole with residue  $\text{ord}_{q_i}(g)$  at each  $q_i$ , we have by the residue theorem

$$\begin{aligned} \int_{\partial \Delta'} \varphi &= 2\pi \sqrt{-1} \sum_{q_i} \text{Res}_{q_i}(\varphi) \\ &= 2\pi \sqrt{-1} \sum_{q_i} \text{ord}_{q_i}(g) \cdot \log f(q_i). \end{aligned}$$

Now, for points  $p \in \delta_i$ ,  $p' \in \delta_i^{-1}$  on  $\partial \Delta'$  identified on  $S$ ,

$$\log f(p') = \log f(p) + \int_{\delta_{g+i}} d \log f,$$

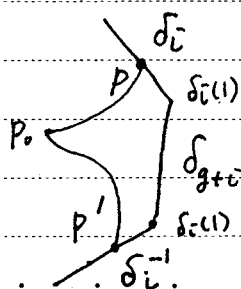
and so

$$\int_{\delta_i + \delta_i^{-1}} \varphi = \left( \int_{\delta_i} d \log g \right) \left( - \int_{\delta_{g+i}} d \log f \right)$$

and similarly

$$\int_{\delta_{g+i} + \delta_{g+i}^{-1}} \varphi = \left( \int_{\delta_{g+i}} d \log g \right) \left( \int_{\delta_i} d \log f \right).$$

$\mathbb{F}$



$d \log f$  is holomorphic in  $\Delta'$

$$\begin{aligned} \Rightarrow \int_{p_0}^p d \log f + \int_p^{p'} d \log f + \int_{p'}^{p_0} d \log f \\ + \int_{p_0}^{p'} d \log f + \int_{p'}^{p_0} d \log f = 0. \end{aligned}$$