It remains to show $\dim_{\mathbb{C}}(\mathcal{O}_z/I) = \dim_{\mathbb{C}}(\mathcal{O}_{z'}/I')$.

$$\phi : \mathcal{O}_z/I \longrightarrow \mathcal{O}_{z'}/I'$$

$$g + I \longmapsto g|_{D_1} + I'.$$

Given $h \in \mathcal{O}_{z'}$, consider $\bar{h}(z_1, \cdots z_n) = h(z_2, \cdots z_n)$.

$\Rightarrow$ ϕ is onto.

Suppose $g|_{D_1} \in I'$.

$\Rightarrow$ $g(0, z') = b_2(z')\, f_2(0, z') + \cdots + b_n(z')\, f_n(0, z')$.

Consider $F(z_1, z') = g(z_1, z') - h(z_1, z')$ where $h(z_1, z') = b_2(z')\, f_2(z_1, z_2) + \cdots + b_n(z')\, f_n(z_1, z')$

$\Rightarrow$ $F(0, z') = 0$ $\Rightarrow$ Since z_1 is irreducible, z_1 divides $F(z_1, z')$ by Weak Nullstellensatz on P11. $\Rightarrow$ $F(z_1, z_2) = z_1\, k(z_1, z_2)$. $\Rightarrow$ $g(z_1, z') = h(z_1, z') + z_1\, k(z_1, z_2) = z_1\, k(z_1, z_2) + b_2(z') f_2 + \cdots + b_n f_n \in \{z_1, f_2, \cdots f_n\} = I$

$\Rightarrow$ $g + I = I$ $\Rightarrow$ ϕ is one to one.

$\Rightarrow$ ϕ is isomorphic $\Rightarrow$ $\dim_{\mathbb{C}}(\mathcal{O}_z/I) = \dim_{\mathbb{C}}(\mathcal{O}_{z'}/I')$.

For $n = 1$. $I = \{f\}$. $\Rightarrow$ $f(0) = 0$ $\Rightarrow$ $\exists$ m s.t.

$$\lim_{z \to 0} \frac{f(z)}{z^m} \neq 0 \quad \Rightarrow \quad f(z) = z^m h(z), \quad h(z) \neq 0 \text{ at } z = 0.$$

$$\Rightarrow \{f\} = \{z^m\} \quad \Rightarrow \quad \mathcal{O}_z/\{f\} = \mathcal{O}_z/\{z^m\}$$

$$\Rightarrow \dim_{\mathbb{C}} \mathcal{O}_z/\{z^m\} = m = (D)_{\{0\}} \quad \text{where } D = (z^m = 0)$$

$$\frac{1}{2\pi\sqrt{-1}} \int_{\|z\| = \epsilon} \frac{dz^m}{z^m} = \frac{1}{2\pi\sqrt{-1}} \int_{\|z\| = \epsilon} \frac{m\, dz}{z} = m.$$

Maybe the following argument is right, I hope.