Consider $H - \lambda G$.

Unless $H-\lambda G = 0$, $H - \lambda G = X_0 h$, $h \neq 0$.

$\Rightarrow$ Again $X_0 h$ is <u>tangent to</u> G at p & p'. $\Rightarrow$

$h = \delta X_0$, $\delta \in \mathbb{C}$. (multiplicity > 1)

$\Rightarrow$ $H = F + \lambda G$, $\lambda \in \mathbb{P}^1$.

$\Rightarrow$ L consists of all conics passing p & p' and tangent to G at those two points.

$\lrcorner$

The only singular conic of L other than F is thus the sum of the tangent lines to G at p and p'; so $m_F(L, W_1) = 2$ and F is a double point of W_1.

$\ulcorner$ By a proper coordinate change, we may assume that

$$L = \left\{ (X_0, X_1, X_2) \,\middle|\, \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \lambda Q \right) \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} = 0 \right\}$$

Q symmetric 3×3 matrix of rank 3.

$$\left| \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \lambda Q \right| = 0.$$