The product

$$A \times B = \sum a_\alpha b_\beta \sigma_\alpha^k \times \sigma_\beta'^l \text{ of two cycles.}$$

$A = \sum a_\alpha \sigma_\alpha^k$, $B = \sum b_\beta \sigma_\beta'^l$ in M and N is a cycle. and the homology class of $A \times B$ depends only on the homology classes of A and B, since

$$(A + \partial C) \times B = A \times B + \partial(C \times B).$$

Thus we have a map

$$H_*(M, \mathbb{Z}) \otimes H_*(N, \mathbb{Z}) \longrightarrow H_*(M \times N, \mathbb{Z}).$$

we claim that it is, modulo torsion, an isomorphism. This is readily seen once we express the chains of the complexes M and N in terms of canonical bases, ones in terms of which the boundary operators are diagonal.

We may construct such a basis for the chains in M as follows. Suppose M has dim. m;
let $\{\tau_\alpha^m\}$ be a rational basis for the m-cycles in M
coefficient $\mathbb{Q}$ (no torsion).

i.e. $\operatorname{Ker}\partial = \bigoplus \mathbb{Q}(\tau_\alpha^m) = Z_m(M) \Rightarrow C_m(M) = Z_m(M) \oplus Z_m(M)^\perp$

Complete $\{\tau_\alpha^m\}$ to a rational basis for the m-chains of M; call the additional basis element $\{\mu_\alpha^m\}$.

Set $\sigma_\alpha^{m-1} = \partial \mu_\alpha^m$. so that $\{\sigma_\alpha^{m-1}\}$ is a basis for the boundaries of M in dimension $m-1$; complete $\{\sigma_\alpha^{m-1}\}$ to a rational basis $\{\sigma_\alpha^{m-1}, \tau_\beta^{m-1}\}$ for the (m-1) cycles of M and complete $\{\sigma_\alpha^{m-1}, \tau_\beta^{m-1}, \mu_\gamma^{m-1}\}$ for all (m-1) chains on M. Set $\sigma_\alpha^{m-2} = \partial\mu_\gamma^{m-1}$, continuing in this way, we obtain a rational basis $\{\sigma_\alpha^k, \tau_\alpha^k, \mu_\alpha^k\}$ for the chains ~~for~~ of M, with $\{\sigma_\alpha^k\}$ a basis for the