$\pi' : \Sigma \longrightarrow S^*$ $\quad \pi'^{-1}(h_i) = X_{h_i}$ $\quad \pi'^{-1}(h_j) = X_{h_j}$.
$\pi'^{-1}(h_k) = X_{h_k}$.
$p_0 \in h_i, h_j, h_k \Rightarrow X_{p_0} \cap X_{h_i} \neq \phi$ $\quad X_{p_0} \cap X_{h_j} \neq \phi$
$X_{p_0} \cap X_{h_k} \neq \phi$ by P935 note.
Let h be the hyperplane containing p_{ij}, p_{jk} and p_{ik}.
$\Rightarrow X_h = X_{p_{ij}\, p_{jk}\, p_{ik}}$.
Clearly, $X_h \cap X_{p_{ij}} \neq \phi$ since $p_{ij} \in h$, and
$X_h \cap X_{p_{ij}} \ni h \cap h'$, where $X_{p_{ij}} = \sigma(p_{ij}, h')$.

Note that these seven lines lie in the hyperplane in $\mathbb{P}^5$ spanned by the points that are circled.

$X_{p_0} \cap X_{p_{ij}} = \phi$ since $p_0 \neq p_{ij}$.
If $X_{h_j} \cap X_{p_{ij}} = X_{h_i} \cap X_{p_0}$, then $X_{p_{ij}} \cap X_{p_0} \neq \phi$. *
Thus the points circled are all distinct.
The four points on the top left coner are linearly independent since $X_{h_j} \cap X_{h_i} = \phi$, more precisely, if $l_1, l_2 \subset \mathbb{P}^5$, $l_1 \cap l_2 = \phi$,
$l_1 = \langle v_1, v_2 \rangle$ $\quad l_2 = \langle w_1, w_2 \rangle$ and $a_1 v_1 + a_2 v_2 + b_1 w_1 + b_2 w_2 = 0 \Rightarrow a_1 v_1 + a_2 v_2 = -b_1 w_1 - b_2 w_2 \in l_1 \cap l_2 = \phi \Rightarrow a_1 = a_2 = b_1 = b_2 = 0$
$\Rightarrow \langle l_1, l_2 \rangle = \mathbb{P}^3$.
Suppose X_{h_j}, X_{h_i} and X_{h_k} are linearly independent.
$\Rightarrow X_{h_k} \subset \langle X_{h_j}, X_{h_i} \rangle = \mathbb{P}^3$.
Consider $F \cap \langle X_{h_j}, X_{h_i} \rangle$ which is a quadric surface in