NO.

DATE . . ( )

But since B_L has genus 2,

$$H_1(B_L, \mathbb{Z}) \cong H_1(A, \mathbb{Z}) \cong \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z};$$

so the kenel of i_* must have rank zero; since $H_1(B_L, \mathbb{Z})$ has no torsion, this implies that the map i_* is an isomorphism.

∵ $g(B_L) = 2 = \frac{b_1}{2} \Rightarrow b_1 = \dim H_1(B_L, \mathbb{Z}) = 4$
$\Rightarrow H_1(B_L, \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$. Since $A = \mathbb{C}^2/\Lambda$,
$A \cong S^1 \times S^1 \times S^1 \times S^1$ topologically. $\Rightarrow H_1(A, \mathbb{Z}) = \mathbb{Z}^4$.

」

Likewise, by Lefschetz the restriction map

$$H^{1,0}(A) \longrightarrow H^{1,0}(B_L)$$

is an isomorphism, and so we have

$$A \cong \frac{(H^{1,0}(A))^*}{H_1(A, \mathbb{Z})} = \frac{(H^{1,0}(B_L))^*}{H_1(B_L, \mathbb{Z})} = \mathcal{J}(B_L)$$

i.e.,

The Abelian variety A is the Jacobian of the curve B_L.

∵ $i^* : H^1(A) \longrightarrow H^1(B_L)$ is isomorphism.
$\Rightarrow$ Since i is holomorphic, $i^* : H^{1,0}(A) \longrightarrow H^{0,1}(B_L)$ is isomorphic. By the note on P327,