If we denote by $\mathcal{D}(T)$ the space of distributions, then we conclude that

$$\mathcal{D}(T) = H_{-\infty}.$$

The derivatives of a distribution are defined by

$$(D^\alpha \lambda)(\varphi) = \lambda(D^\alpha \varphi).$$

The Fourier coefficients of $D^\alpha \lambda$ are $(D^\alpha \lambda)_\zeta$

$$\overline{(D^\alpha \lambda)_\zeta} = D^\alpha \lambda(e^{i\langle \zeta, x\rangle}) = \lambda(D^\alpha e^{i\langle \zeta, x\rangle})$$

$$= \lambda(\mp \zeta^\alpha e^{i\langle \zeta, x\rangle}) = \zeta^\alpha \lambda(e^{i\langle \zeta, x\rangle}) = \zeta^\alpha \overline{\lambda_\zeta}$$

$\Rightarrow (D^\alpha \lambda)_\zeta = \zeta^\alpha \lambda_\zeta$. With this definition, a distribution is obtained by taking a finite # of derivatives of a continuous function.

A final piece of useful terminology is this: A distribution λ is said to be in L^2 in case $\lambda \in H_0 \subset H_{-\infty}$. Then we may describe the Sobolev spaces by

H_s consists of all distributions λ s.t the distributional derivatives $D^\alpha \lambda$ are in L^2 for $[\alpha] \leq s$.

(See p 86. for equivalent norms)

An example of an interesting distribution is the delta function defined by

$$\delta(\varphi) = \varphi(0).$$

It has formal Fourier series

$$\delta = \sum_\zeta e^{i\langle \zeta, x\rangle} = \sum \delta(e^{\overline{i\langle \zeta, x\rangle}}) e^{i\langle \lambda, x\rangle}.$$