Suppose that P_0 is a set of distinct points in $\mathbb{P}^2$ and $|C_0| = |\mathcal{J}_{P_0}(n)|$ is the linear system of curves of degree n passing through P_0. Let S be the proper transform of $\mathbb{P}^2$ along P_0 and $|C|$ the linear system on S of proper transforms of curves $C \in |\mathcal{J}_{P_0}(n)|$.

⌈ I think S is the blow-ups of $\mathbb{P}^2$ along P_0 and $|C|$ the linear system on S of proper transforms of curves $C \in |\mathcal{J}_{P_0}(n)|$. See P474 ~ P475 ⌋

Denote by $\pi : S \longrightarrow \mathbb{P}^2$ the projection map and $E = \pi^{-1}(P_0)$ the exceptional curve. Then $|C|$ is the complete system $|L|$ where

$$L = \pi^* H^n - E$$

for $H \to \mathbb{P}^2$ the hyperplane bundle.

⌈ According to P476, if $|C| = \{\pi^* C - E\}$, then $|C|$ is the complete system $|L|$ where $L = \pi^* H^n - E$, as follows:

Given an effective divisor $C' \in |\pi^* H^n - E|$,

$C' \sim \pi^* H^n - E$. $\Rightarrow$ By P475, (*), $C' \in \pi^* \mathrm{Div}\, M \oplus \mathbb{Z}\{E_1\} \oplus \cdots \oplus \mathbb{Z}\{E_d\}$, $d = {}^{\#}P_0 = {}^{\#}\{P_1, \cdots, P_d\}$.

$\Rightarrow$ $C' = \pi^* C + n_1 E_1 + \cdots + n_d E_d$, where C is a curve on M. $\Rightarrow$ Since C is a curve in $\mathbb{P}^2$, $C \sim H^n$.

$\Rightarrow$ $C' = \pi^* C + n_1 E_1 + \cdots + n_d E_d \sim \pi^* H^n + n_1 E_1 + \cdots + n_d E_d \sim \pi^* H^n - E_1 - \cdots - E_d$. $\Rightarrow$ $n_1 E_1 + \cdots + n_d E_d \sim E_1 + \cdots + E_d$

$\Rightarrow$ $n_1 = \cdots = n_d = 1$, for $n_1 E_1 + \cdots + n_d E_d$ is homologous to