

\mathcal{O}_M -modules. Γ By the property 4 on P100, \mathcal{G} is coherent. \Rightarrow

Multiplying a section of \mathcal{G} by σ gives zero, since $\sigma \cdot \mathcal{O}_H = 0$, and so $\mathcal{G} = \mathcal{G}_H$ is a coherent sheaf of \mathcal{O}_H -modules.

Γ \mathcal{O}_H is not a \mathcal{O}_M -free module, but a \mathcal{O}_M -module. From the exact sequence

$$0 \rightarrow \underline{\text{Tor}}_1^{\mathcal{O}_M}(\mathcal{O}_H, \mathcal{F}) \rightarrow \mathcal{F}(-1) \xrightarrow{\otimes \sigma} \mathcal{F} \rightarrow \mathcal{F}_H \rightarrow 0$$

$\sigma \cdot \underline{\text{Tor}}_1^{\mathcal{O}_M}(\mathcal{O}_H, \mathcal{F}) = 0$. Given a section $\tau \in \underline{\text{Tor}}_1^{\mathcal{O}_M}(\mathcal{O}_H, \mathcal{F})$, $\sigma \cdot \tau = 0 \Rightarrow$ For any $x \notin H$, $\sigma_x \tau_x = 0 \Rightarrow$ Since σ_x is a unit in \mathcal{O}_x ($\because x \notin H$), $\tau_x = 0 \Rightarrow \tau = 0$ on $M-H \Rightarrow \tau$ can be considered as a section defined on $H \Rightarrow \underline{\text{Tor}}_1^{\mathcal{O}_M}(\mathcal{O}_H, \mathcal{F})$ may be considered as a sheaf of \mathcal{O}_H -modules, since $\mathcal{O}_H \hookrightarrow \mathcal{O}_M$ at stalk level, (remember H is a smooth hyperplane section of M). $\Rightarrow \underline{\text{Tor}}_1^{\mathcal{O}_M}(\mathcal{O}_H, \mathcal{F})$ sheaf on $H \Rightarrow$ Let $\mathcal{G}_H = \underline{\text{Tor}}_1^{\mathcal{O}_M}(\mathcal{O}_H, \mathcal{F}) \xrightarrow{\text{see p37}}$ " $\underline{\text{Tor}}_1^{\mathcal{O}_M}(\mathcal{O}_H, \mathcal{F})$ may be considered as a sheaf extended by zero from H ."

Now apply $\otimes^{\mathbb{L}} \mathcal{L}$ to this sequence. Since locally $\mathcal{L} \cong \mathcal{O}$, exactness is preserved and we obtain

$$(**) \quad 0 \rightarrow \mathcal{G}_H(k) \rightarrow \mathcal{F}(k-1) \rightarrow \mathcal{F}(k) \rightarrow \mathcal{F}_H(k) \rightarrow 0.$$

$$\Gamma \quad 0 \rightarrow \mathcal{G}_H \otimes \mathcal{L}^k \rightarrow \mathcal{F}(k-1) \rightarrow \mathcal{F}(k) \rightarrow \mathcal{F}_H(k) \rightarrow 0 \quad \Rightarrow$$