$$\int_V \varphi = \int_M \varphi \wedge \psi = \#(\bar{V} \cdot B)$$

φ is Poincare-dual to B-$(2n-2k)$ cycle
↑
$2k$ form

ψ is Poincare-dual to $\bar{V}$ ($2k$ cycle)
↑
$(2n-2k)$-form

Usually it will either be clear from the context or unimportant which of these we are referring to.

We now make a very simple observation. Suppose V and W are analytic varieties of dimension k and $n-k$ intersecting transversely at a point p on the complex manifold M. We may take holomorphic coordinates $z = (z_1, \cdots z_n)$ on M near p such that V and W are given by

$$V = (z_{k+1} = \cdots = z_n = 0)$$

and

$$W = (z_1 = \cdots = z_k = 0).$$

Pf Since V intersects W transversely at p, and $\dim V_s \leq k-1$, p must be a smooth point of both V and W. $\Rightarrow$ It is possible to choose holomorphic coordinates $z = (z_1, \cdots z_n)$ on M near p s.t $V = (z_{k+1} = \cdots = z_n = 0)$ $W = (z_1 = \cdots = z_k = 0)$. by the inverse function theorem.