a manifold, $M-D$ is path-connected.
$\Rightarrow$ We may choose the γ_{D_i} to lie in $M-D$. ⅃

Indeed, by assumption $\gamma = \partial \Delta$, where Δ is a 2-chain in M. Since the singularities of D are in real codimension 4, we may assume that Δ meets D transversely at simple points.

⌜ By the proposition on p21, D_s has the codimension ≤ 2.
$\Rightarrow$ The singularities of D are in real codimension ≤ 4.
$\Rightarrow$ We may choose Δ so that Δ intersects D transversely at simple points by transversality. ⅃
" $[\gamma] = 0$ in $H_1(M, \mathbb{Z})$ "

If $x_0 \in D_i$ is such an intersection point, then near x_0 we may picture the part Δ_ϵ of Δ lying within distance ϵ of D_i as a normal disc at x_0 (Figure 5), and so $\partial \Delta_\epsilon = \gamma_{D_i}$.

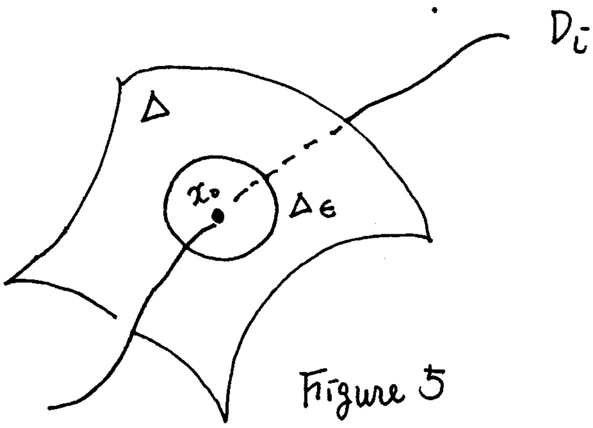

Figure 5

Consequently $\gamma - \gamma_{D_i}$ has one less intersection point with D, and repeating the argument gives a homology

$$\gamma \sim \sum m_i \gamma_{D_i}.$$