$$\Rightarrow g_{\alpha\beta}\, s_\beta = s_\alpha. \quad \Rightarrow g_{\alpha\beta}\left(\frac{f_\alpha}{f_\beta}\right) = 1 \quad \Rightarrow g_{\alpha\beta} = \frac{f_\beta}{f_\alpha}.$$

P135

$$\Rightarrow s_\alpha = \frac{1}{f_\alpha} \;\&\; s_\beta = \frac{1}{f_\beta}. \quad \Rightarrow \frac{1}{f_\alpha} = \frac{f_\beta}{f_\alpha}\,\frac{1}{f_\beta}.$$

$\Rightarrow \{(U_\alpha, \frac{1}{f_\alpha})\}$ represents the locally free sheaf I of rank 1. $\Rightarrow I = [(\frac{1}{f_\alpha})]. = [-D]$, where $D = (f)$, $f = \{f_\alpha\}$.

The sheaf $\mathcal{O}(D)$ is then the sheaf of meromorphic functions g with $(g) + D \geq 0$. In general, sheaves $\mathcal{O}(D)$ for a not necessarily effective divisor D are said to be invertible.

In this context, $\mathcal{O}(D)$ is the set of all sections of $[D]$ over M, i.e. $H^0(M, \mathcal{O}([D]))$. No! According to P130, we have a divisor for any function g defined locally.
$\Rightarrow$ By the result on P136, the multiplication by $f = \{f_\alpha\}$ gives an identification

$$\{g :\ (g) + D \geq 0\} \xleftrightarrow{\otimes f} H^0(M, \mathcal{O}(D)).$$
g meromorphic

See P616 note for correction

The multiplicative group of invertible sheaves on an algebraic variety M is just $H^1(M, \mathcal{O}^*) = \mathrm{Pic}(M)$.
See P134

An ideal sheaf I is a sheaf of regular ideals if locally $I = \{f_1, \cdots f_r\}$ where the f_i define a regular sequence in the local ring $\mathcal{O}_z$. For sheaves of regular ideals the Koszul complex from the preceding section provides