But this contradicts the regular sequence property of $\{f_1', f_2, \cdots f_n\}$. Q.E.D.

Step Three. The theorem now follows easily. Given $f = (f_1, \cdots, f_n)$, we inductively choose a coordinate system so that

$$F_i = (z_1, \cdots z_i, f_{i+1}, \cdots f_n)$$

has an isolated zero at the origin. Appealing to the nullstellensatz, we may take k_i sufficiently large so that $z_i^{k_i} \in I(F_{i-1})$. Then res_{F_n} is nondegenerate by step one, and by the lemma

$$\begin{aligned} \mathrm{res}_{F_n} \text{ nondegenerate} &\Rightarrow \mathrm{res}_{F_{n-1}} \text{ nondegenerate} \\ &\;\vdots \\ &\Rightarrow \mathrm{res}_{F_1} \text{ nondegenerate} \\ &\Rightarrow \mathrm{res}_{F_0} = \mathrm{res}_f \text{ nondegenerate}. \end{aligned}$$

Pf Choose a coordinate system so that $F_i = (z_1, \cdots z_i, f_{i+1}, \cdots f_n)$ has an isolated zero at the origin. This is possible since $\operatorname{codim}\{f_{i_1} = \cdots = f_{i_k} = 0\} = k$.

$\vdots$

$\{z_1^{k_1}, f_2, \cdots f_n\} \subset \{f_1, \cdots f_n\} \Rightarrow$ Again by the nullstellensatz, since $z_1^{k_1}, f_2, \cdots f_n$ is a regular sequence, $z_2^{k_2} \in \{z_1^{k_1}, f_2 \cdots f_n\}$. $\Rightarrow \{z_1^{k_1}, z_2^{k_2}, f_3, \cdots f_n\} \subset \{z_1^{k_1}, f_2 \cdots f_n\} \subset \{f_1 \cdots f_n\} \Rightarrow$ Continue this process, then we get the following:

$$\underbrace{\{z_1^{k_1}, z_2^{k_2}, \cdots z_n^{k_n}\}}_{= I_n} \subset \underbrace{\{z_1^{k_1}, \cdots z_{n-1}^{k_{n-1}}, f_n\}}_{= I_{n-1}} \subset \{z_1^{k_1}, \cdots z_{n-2}^{k_{n-2}}, f_{n-1}, f_n\}$$