

For each  $x \in L$ , the set of lines  $r$  passing through  $x$  is  $T_x(X) \cap X$  by P764.  $\Rightarrow$  The branch locus of  $r$  is  $\{x \mid T_x(X) \cap X \text{ consists of fewer than four lines}\}$ .  $\Rightarrow$  By P793, the branch locus of  $r$  in  $L$  is  $\Delta \cap L$ .  $\Rightarrow$  By P793,  $\#(\Delta \cdot L)_x = 8$ , and the eight nonsingular tangent lines in  $\Delta \cap L$  form the branch points of  $r$ .

We can locate the two singular lines in the pencil  $L$  readily enough: first, the common line of the pencil  $L$  and its confocal pencil  $\iota(L)$ , i.e., the tangent line to  $C_L$  at  $P_L$ .

$\Gamma L = \sigma(P_L, h_L) \Rightarrow \sigma(P_L) \cap X = \sigma(P_L, h_L) \cup \sigma(P_L, h')$   
 $\Rightarrow h' \neq h_L$ , for, otherwise,  $L$  is a multiple component of  $T_x(X) \cap X$  for  $x \in L$ .  $\Rightarrow$  Contradiction to the assumption.  $\Rightarrow P_L \in S - R \Rightarrow$  By P765 & P775,  $h' \cap h_L = l_x$  is tangent line to  $C_L$  at  $P_L$ , and  $x \in \sigma(P_L, h_L) \cap \sigma(P_L, h') = L \cap \iota(L)$ .

Second, we have seen that a line  $l_x$  of the complex lies in two confocal pencils if and only if it lies on two other coplanar pencils, so the line  $\overline{P_L P_L'}$  held in common by  $L$  and its coplanar pencil  $L'$  must be the second singular line of the pencil  $L$ .