$\ulcorner$ $E \cdot E = \int_M \eta \wedge \eta$, where $[\eta] \in H^2_{DR}(M)$, η represents the E, which means that η is Poincare-dual to $E = [\mathbb{P}^1]$.

$= \int_{\mathbb{P}^1} \eta = -1$.

Claim: $E \cdot E = \deg(\text{normal bundle}^{H^*} \text{ of } E)$
$= \langle c_1(H^*), E \rangle$ by P144.

Proof. $E \cdot E$ = self-intersection number of E = $\#(s(E) \cdot E)$, where s is a nonzero section of H^* over E. Let's think of what a divisor means. It expresses a set of points which intersect with the base manifold, roughly. $\Rightarrow$ Since $\langle c_1(H^*), E \rangle = -1$, $E \cdot E = 1$. If $\exists$ a holomorphic section, then the explanation above is valid for our case. The first Chern class may be interpreted as an obstruction for a nonvanishing section of a line bundle over a curve. Let's leave it as it is right now. $\lrcorner$

If T_ϵ is a smoothing of T_E, then

$$E \cdot E = \int_E T_\epsilon$$

shows that we can not take T_ϵ to be a positive $(1,1)$-form.