$\#(T_x(X) \cap X) = \#($ The set of lines passing $x)$,
$\#(\tilde{f}_L(\pi^{-1}(x)) \cap E_L) = 3$ which is the number of lines passing x except L. $\#(\overline{\tilde{f}_L(\pi^{-1}(x)),\ V_3 \cap \bigcap_{x \in L} T_x(F)} \cap E_L) = 5$, since E_L is a quintic curve.
$\Rightarrow$ Let $V_3 \cap \bigcap_{x \in L} T_x(F) = l_2$, and $\tilde{f}_L(\pi^{-1}(x)) = l_1$.

$\Rightarrow \quad E_L \cap \overline{l_1, l_2} = E_L \cap \overline{l_1, l_2} \cap Q$ since $Q \supset E_L$
$= E_L \cap (l_1 \cup l_2) = (E_L \cap l_1) \cup (E_L \cap l_2)$
$\Rightarrow \#(E_L \cap \overline{l_1, l_2}) = 5 = \#(E_L \cap l_1) + \#(E_L \cap l_2) = 3 + \#(E_L \cap l_2)$
$\Rightarrow \#(E_L \cap l_2) = 2$ counting multiplicity.

┘

The situation is slightly different in case L is a special line of X. Now all the lines $\{\tilde{f}_L(\pi^{-1}(x))\} = V_3 \cap T_x(X)\}_{x \in L}$ on the quadric Q have in common the point

$$p = V_3 \cap \bigcap_{x \in L} T_x(X)$$

corresponding to the α-plane tangent to X everywhere along L.

┌ $\bigcap_{x \in L} T_x(X)$ is a 2-dimensional plane, and contains L.
$\Rightarrow$ Since $L \cap V_3 = \phi$, $V_3 \cap \bigcap_{x \in L} T_x(X)$ is a point p.

$\bigcap_{x \in L} T_x(X)$ is the α-plane tangent to X everywhere along L. See note P985 back. ┘