

By the result on P.141, $\Sigma_{1,2} \sim 2^2 \cdot \sigma_{2,1}$, since $k=1, n=2$. \square

The base locus of the pencil L , being a smooth intersection of two quadrics, is an elliptic curve C of degree 4, and in fact it is not hard to see that $V_0(L)$ is just the set of chords to C : on the one hand, if $l \subset F_\lambda$ for some λ , then $C \cap l = F_\lambda \cap F_{\lambda'} \cap l = F_{\lambda'} \cap l$ consists of two points; on the other hand, if l meets C in two points p and q , then for any third point $r \in l$ some quadric $F_\lambda \in L$ contains r , and so contains l .

Observation:

① If we let F, G be smooth quadrics, then since $V(F) \& V(G)$ are homologous to $4\sigma_{2,1}$ and $4\sigma_{2,1} \cdot 4\sigma_{2,1} = 0$ in $G(2,4)$ ($\because a_1=2 \neq 2-b_2=1 \nexists$ see P.198), $F \cap G$ does not contain \mathbb{P}^1 if F, G are generic quadrics.

② If we let $F \cap G = C$, C smooth curve, by the adjunction formula on P.147, refer to P.550,

$$K_F = K_{\mathbb{P}^3} + F|_F = -4H + 2H|_F = -2H|_F$$

$$K_C = K_F + G|_C = K_F + C|_C, \text{ since } C = F \cap G$$

and if $(\sigma=0)=G$, $\sigma \in H^0(F, \mathcal{O}(2H))$ and $\sigma|_C \in H^0(C, \mathcal{O}(2H|_C))$