The reason for this description of the result is this. An affine algebraic variety $\mathcal{U}$ is a complex submanifold of $\mathbb{C}^N$ defined by polynomial equations. We denote by $\Omega^*(\mathcal{U}, \mathrm{alg})$ the complex of holomorphic forms on $\mathcal{U}$ that are the restrictions of rational differential forms in $\mathbb{C}^N$. This notation is consistent, since if we take the projective closure $M_0$ of $\mathcal{U} \subset \mathbb{C}^N \subset \mathbb{P}^N$ and apply Hironaka's theorem to obtain a resolution of singularities

$$M \xrightarrow{\pi} M_0$$

that is an isomorphism on $\mathcal{U}$, then $\Omega^*(\mathcal{U}, \mathrm{alg})$ are just the meromorphic forms on $M$ that are holomorphic in $\mathcal{U}$ — cf. Section 4 of Chapter 1.

⌈ I guess, if we let $D = M_0 - \mathcal{U}$, then $\pi^{-1}(D)$ will be a diver of $M$.

⇒ As on P189,

$$\underbrace{H^0(M_0, \mathcal{O}_{M_0}(\overbrace{L}^{\Lambda^p T^* M_0}))}_{\Omega^p(\mathcal{U},\, \mathrm{alg})} \xrightarrow{\pi} \underbrace{H^0(M, \mathcal{O}_M(\overbrace{\tilde{L}}^{\Lambda^p T^* M}))}_{\Omega^p(\pi^{-1}(\mathcal{U}),\, \mathrm{alg})}$$

is an isomorphism.

⇒ We may conclude that $H^0(M, \mathcal{O}_M(\Lambda^p T^* M \otimes \tilde{L})) = \Omega^p(\pi^{-1}(\mathcal{U}), \mathrm{alg}) = \pi^*(\Omega^p(\mathcal{U}, \mathrm{alg}))$.

For a resolution of singularities, refer to P182.」

The algebraic de Rham theorem then asserts that cohomology $H^*(\mathcal{U}, \mathbb{C})$ may be computed from the complex $\Omega^*(\mathcal{U}, \mathrm{alg})$.

⌈ $H^p_{DR}(\pi^{-1}(\mathcal{U}), \mathrm{alg}) = H^p_{DR}(\mathcal{U}, \mathrm{alg})$ since $\pi$ is isomorphic.