$\mathbb{P}(H^{2n} \oplus \mathbb{C}_{\mathbb{P}^1}) = S_{2n}$ $\Rightarrow$ By the result on P519, since $S_{2n} \neq \mathbb{P}^1 \times \mathbb{P}^1$, $n \neq 0$. $\Rightarrow$ $n = 0$ $\Rightarrow$ $N_{L/X}$ is trivial, for L nonspecial.

$\mathbb{P}(H^m \oplus H^{m+2}) = \mathbb{P}(H^{m+2} \oplus H^m) = \mathbb{P}((H^{m+2} \otimes H^{-m}) \oplus \mathbb{C}_{\mathbb{P}^1})$ by P517 $= \mathbb{P}(H^2 \oplus \mathbb{C}_{\mathbb{P}^1}) = S_2$ $\Rightarrow$ Once we proved $\mathbb{P}(N_{L/X}) = S_2$, then $n = 1$. $\Rightarrow$ $N_{L/X} = H \oplus H^{-1}$ for L special.

$$\mathbb{P}(N) \xrightarrow{\tilde{f}} Q \subset \mathbb{P}^3$$

$$\tilde{f}^* : \quad H^2(Q) \longrightarrow H^2(\mathbb{P}(N))$$

$$\begin{matrix} \cup\!\!\!\!\!\!\text{\small $\in$} \\ [Q \cap H] \end{matrix} \longmapsto [\tilde{f}^{-1}(Q \cap H)], \quad H \text{ hyperplane.}$$

$\Rightarrow$ ① $[\tilde{f}^{-1}(Q \cap H)] = \tilde{f}^{-1}(p) + \pi^{-1}(x) + \pi^{-1}(x')$ where p is the singular point of Q, and $\pi : \pi^{-1}(L) \longrightarrow L$, $x \neq x' \in L$. And H is a hyperplane passing p.

② $[\tilde{f}^{-1}(Q \cap H)] = \tilde{f}^{-1}(Q \cap H)$ is a curve since $\tilde{f}$ is one to one.

$\Rightarrow$ $(\tilde{f}^{-1}(p) + \pi^{-1}(x) + \pi^{-1}(x')) \cdot (\tilde{f}^{-1}(p) + \pi^{-1}(x) + \pi^{-1}(x'))$

$= \tilde{f}^{-1}(p) \cdot \tilde{f}^{-1}(p) + 2\tilde{f}^{-1}(p) \cdot \pi^{-1}(x) + 2\tilde{f}^{-1}(p) \cdot \pi^{-1}(x')$

$\quad + \pi^{-1}(x) \cdot \pi^{-1}(x) + \pi^{-1}(x') \cdot \pi^{-1}(x') = \tilde{f}^{-1}(p) \cdot \tilde{f}^{-1}(p) + 4 + 0$

$= \tilde{f}^{-1}(Q \cap H) \cdot \tilde{f}^{-1}(Q \cap H) = (Q \cap H) \cdot (Q \cap H) = 2$

$\Rightarrow$ $\tilde{f}^{-1}(p) \cdot \tilde{f}^{-1}(p) = -2$. Here, since $\tilde{f}^{-1}(p)$ is a non-vanishing section of $\mathbb{P}(N)$ by the result on P798, and $\pi^{-1}(x)$ is a fiber of $\mathbb{P}(N)$, $\pi^{-1}(x) \cdot \tilde{f}^{-1}(p) = 1$.

$\pi^{-1}(x) \cdot \pi^{-1}(x') = 0 = \pi^{-1}(x) \cdot \pi^{-1}(x) = \pi^{-1}(x') \cdot \pi^{-1}(x')$.

$\Rightarrow$ Since $\mathbb{P}(N)$ is a $\mathbb{P}^1$-bundle over $\mathbb{P}^1$, by the