If f is written locally as g/h, we take the divisor of zeros $(f)_0$ of f to be

$$(f)_0 = \sum \text{ord}_V(g)\cdot V$$ and the divisor of poles $(f)_\infty$

to be $$(f)_\infty = \sum \text{ord}_V(h)\cdot V.$$

Clearly these are well-defined as long as we require g and h to be relatively prime and

$$(f) = (f)_0 - (f)_\infty .$$

Divisors can also be described in sheaf-theoretical terms, as follows: Let $\mathcal{M}^*$ denote the multiplicative sheaf of meromorphic functions on M not identically zero, and $\mathcal{O}^*$ the subsheaf of non-zero holomorphic functions. Then a divisor D on M is simply a global section of the quotient sheaf $\mathcal{M}^*/\mathcal{O}^*$.
On the one hand, a global section $\{f\}$ of $\mathcal{M}^*/\mathcal{O}^*$ is given by an open cover $\{U_\alpha\}$ of M, and meromorphic functions $f_\alpha \not\equiv 0$ in U_α with

$$\frac{f_\alpha}{f_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta);$$

for any $V \subset M$, then $\text{ord}_V(f_\alpha) = \text{ord}_V(f_\beta)$, and we can associate to $\{f\}$ the divisor

$$D = \sum_V \text{ord}_V(f_\alpha)\cdot V$$ where for each V

we choose α such that $V \cap U_\alpha \neq \phi$.

Γ Given a global section $\{f\}$ of $\mathcal{M}^*/\mathcal{O}^*$, f is a map from $M \longrightarrow \bigcup_{p\in M} (\mathcal{M}^*/\mathcal{O}^*)_p$.