Assume that $Z \times W$ intersects with Δ at (p,p).
Assume that the intersection is not transversal.
$\Rightarrow$ At p, Z or W is singular. If not, so what? I wonder if $Z \times W$ must intersect Δ transversally.
Consider the following example.
In $\mathbb{C}^4$, $N_1 = \{ (0, \alpha, \alpha^2, \beta) ; \alpha, \beta \in \mathbb{C} \}$.
$N_2 = \{ (\alpha, \beta, 0, 0) \mid \alpha, \beta \in \mathbb{C} \}$.
At the origin $TN_2 = \langle (1,0,0,0), (0,1,0,0) \rangle$
$TN_1 = \langle (0,1,0,0), (0,0,0,1) \rangle$
$\Rightarrow TN_1 + TN_2 \neq T\mathbb{C}^4$, since $TN_1 \cap TN_2 \supset \langle (0,1,0,0) \rangle$

$\Rightarrow T(N_1 \times N_2) = \langle (0,1,0,0,\ 1,0,0,0),\ (0,1,0,0;\ 0,1,0,0)$
$(0,1,0,0,\ 1,0,0,0),\ (0,0,0,1,\ 1,0,0,0), (0,0,0,1,\ 0,1,0,0) \rangle$
$\Rightarrow \Delta = \{ (a_1, a_2, a_3, a_4,\ a_1, a_2, a_3, a_4) \}$.
$\Rightarrow T\Delta = \langle (1,0,0,0,\ 1,0,0,0),\ (0,1,0,0,\ 0,1,0,0), \cdots \rangle$.
$\Rightarrow T\Delta \cap T(N_1 \times N_2) \neq 0$.

Now it is clear why we can reduce to the case when W is smooth. Assume that we prove the theorem for W smooth. By Künneth & Poincare formulas,
$Z \cdot W = (Z \times W) \cdot \Delta$, where Z, W are not necessarily smooth.
$\Rightarrow (Z \times W) \cdot \Delta = \sum_{(p,p) \in Z \times W} m_{(p,p)}(Z \times W, \Delta)$ by the