Again $H \cdot \iota_{\tilde{C}}(F_{ij}) = \#((H \cap \iota_{\tilde{C}}(\tilde{\mathbb{P}}^2)) \cap \iota_{\tilde{C}}(F_{ij}))$

$$= \#(\iota_{\tilde{C}}(\tilde{C}) \cap \iota_{\tilde{C}}(F_{ij}))$$

$$= \#(\tilde{C} \cap F_{ij}) = \tilde{C} \cdot F_{ij},$$

since $[H \cap \iota_{\tilde{C}}(\tilde{\mathbb{P}}^2)] = [\tilde{C}]$ in homology group.

${}_6C_2 = \frac{6 \cdot 5}{2} = 15 \quad \#\{p_1 \cdots p_6\} = 6.$

Note that

$$F_{ij} \cdot F_{ij} = (\pi^* L_{ij} - E_i - E_j) \cdot (\pi^* L_{ij} - E_i - E_j)$$
$$= L_{ij} \cdot L_{ij} - 2 = 1 - 2 = -1$$

so that the lines F_{ij} are exceptional divisors of the first kind on S.

See P478, and $L \cdot L = 1$ in $\mathbb{P}^2$.

Also if G_i is the proper transform in $\tilde{\mathbb{P}}^2$ of the conic C_i in $\mathbb{P}^2$ through the five points $p_1 \cdots \hat{p}_i \cdots p_6$,

$$G_i \cdot \tilde{C} = (\pi^* C_i - \sum_{j \neq i} E_j)(\pi^* 3H - \sum E_i)$$

$$= C_i \cdot 3H - 5 = 6 - 5 = 1.$$

$H^0(\mathbb{P}^2, \mathcal{O}(2H)) = \langle \tau_1, \cdots \tau_6 \rangle$

$\Rightarrow \quad a_1 \tau_1 + \cdots + a_6 \tau_6 = \tau$

$\tau(p_1) = \cdots = \widehat{\tau(p_i)} = \cdots = \tau(p_6) = 0$

$\Rightarrow \exists$ a nontrivial solution a_i's. $\Rightarrow$ Actually, $\exists$ a unique conic curve.