Given any $\Lambda \in G(k,n)$, $\exists\ I$ s.t $U_I \ni \Lambda$.
Suppose $I = \{1 \le i_1 < i_2 < \cdots < i_k \le n\}$

Let $n-k+j-a_j = i_j \Rightarrow n-k+j-i_j = a_j$
$\Rightarrow a_j = n-k+j-i_j \le n-k$.
$a_{j+1} - a_j = n-k+j+1-i_{j+1} - n+k-j+i_j = 1-(i_{j+1}-i_j)$
≤ 0 since $i_{j+1} \ge i_j + 1$.
$\Rightarrow \{a_j\}$ is nonincreasing sequence between 0 and $n-k$.

$\Rightarrow$ Consider $W_{a_1 \cdots a_k} = \{\Lambda \in U_{\{b_1 \cdots b_k\}} \mid \dim(\Lambda \cap V_{b_i}) = i\}$
where $b_i = n-k+i-a_i = i_i$
If $\Lambda \in U_{\{b_1 \cdots b_k\}}$, then $\dim(\Lambda \cap V_{b_i}) \le i$
since Λ is expressed as follows.

$$
\begin{array}{c}
\qquad\quad b_1 \qquad\qquad b_2 \qquad\qquad b_3 \qquad\quad b_4 \,|\leftarrow n-b_4 \rightarrow| \\
\begin{pmatrix}
* & * & 1 & *\cdots* & 0 & *\cdots* & 0 & *\cdots* & 0 & \\
* & * & 0 & *\cdots* & 1 & *\cdots* & 0 & *\cdots* & 0 & \\
* & * & 0 & *\cdots* & 0 & *\cdots* & 1 & *\cdots* & 0 & \\
\vdots & & 0 & & 0 & *\cdots* & 0 & *\cdots* & 1 & \\
\vdots & & 0 & & 0 & \vdots & 0 & *\cdots* & 0 & \\
\vdots & & \vdots & & \vdots & \vdots & \vdots & & \vdots & \\
* & * & 0 & *\cdots* & 0 & * \quad * & 0 & *\cdots* & 0 &
\end{pmatrix}
\end{array}
$$

the last $k \times (n-b_4)$ minor has rank $\ge k-i$

We can find $1 \le c_1 < c_2 < c_3 < \cdots < c_k \le n$ so that the last $k \times (n-c_i)$ has rank $k-i$ exactly.
$\Rightarrow$ Let $n-c_i = k+a_i-i$.
$\Rightarrow a_i = n-k+i-c_i \le n-k$ and $a_{i+1}-a_i = 1-(c_{i+1}-c_i) \le 0$. $\Rightarrow \{a_i\}$ nonincreasing.
We need one more condition on $c_1, c_2 \cdots c_k$.
so that $\Lambda \cap V_{c_i} \ni (\underline{*, * \cdots}\ \underset{c_i}{1}, \underline{0 \cdots 0})$.