and

$$A = \sum a_{\alpha\beta k l} \tau_\alpha^k \times \tau_\beta'^l$$ $\Rightarrow$ this proves onto. is a linear combination of products of cycles of M and N. Similarly, we see that A is homologous to 0 in $M \times N$ only if $a_{\alpha\beta k l} = 0$ for each $\alpha, \beta, k l$ and, so we have established the Künneth formula:

$$H_*(M \times N, Q) \cong H_*(M.Q) \otimes H_*(N.Q)$$

$$A \sim 0 \iff \partial B = A.$$

$$B = \sum a_{\alpha\beta k l} \tau_\alpha^k \times \tau_\beta'^l + \sum b_{\alpha\beta k l} \tau_\alpha^k \times \mu_\beta'^l + \sum c_{\alpha\beta k l} \mu_\alpha^k \times \tau_\beta'^l$$
$$+ \sum d_{\alpha\beta k l} \mu_\alpha^k \times \mu_\beta'^l + \sum e_{\alpha\beta k l} \mu_\alpha^k \times \sigma_\beta'^l.$$

$\partial B = A$ and $\partial A = 0 \Rightarrow \partial B = \sum b_{\alpha\beta k l} (-1)^k \tau_\alpha^k \times \sigma_\beta'^{l-1}$

$+ \sum c_{\alpha\beta k l} \sigma_\alpha^{k-1} \times \tau_\beta'^l + \sum d_{\alpha\beta k l} \sigma_\alpha^{k-1} \times \mu_\beta'^l + \sum (-1)^k d_{\alpha\beta k l} \mu_\alpha^k \times \sigma_\beta^{l-1}$

$+ \sum e_{\alpha\beta k l} \sigma_\alpha^{k-1} \times \sigma_\beta'^l. = \sum p_{\alpha\beta k l} \tau_\alpha^k \times \tau_\beta'^l.$ by the above argument

$\Rightarrow \quad b_{\alpha\beta k l} = c_{\alpha\beta k l} = d_{\alpha\beta k l} = d_{\alpha\beta k l} = e_{\alpha\beta k l} = p_{\alpha\beta k l} = 0.$

$\Rightarrow \quad A = 0.$

$\Rightarrow$ This proves one to one.

We now relate intersections of cycles to wedge products of forms on a compact oriented n-manifold M.

σ a k-cycle on M, τ an $(n-k)$-cycle.
$\varphi \in A^{n-k}(M)$, $\psi \in A^k(M)$ closed representing the cohomology classes Poincaré dual to the classes of σ and τ, i.e. s.t for any $(n-k)$-cycle μ,

$$\int_\mu \varphi = {}^\#(\sigma \cdot \mu) \quad \text{and for any } k\text{-cycle } \nu$$

$$\int_\nu \psi = {}^\#(\tau \cdot \nu).$$