$\Rightarrow$ $D_2 + D_{\xi_0} + \{(\lambda_0 f + \lambda_1 = 0)\}$ gives a family of divisors parametrized by $\mathbb{P}^1$.
$\Rightarrow$ $D_1 + D_e \equiv D_2 + D_{\xi_0}$ by the definition of algebraically equivalent in the strong sense. ⌟

In particular $D_1 + D_e \equiv D_2 + D_{\xi_0}$,
and it is clear that

$$D_e \equiv D_{\xi_0}$$

via the family of divisors $\{D_\xi\}$ $(\xi \in \mathrm{Pic}^\circ(M))$.
Thus $D_1 \equiv D_2$ and we are done. Q.E.D.

⌜ D_1, D_2 effective
Since $D_1 + D_e \equiv D_2 + D_{\xi_0}$, let $\underbrace{D_1 + D_e + \{(f_\lambda = 0)\}}_{``D'_\lambda"}$, $\lambda \in \mathbb{P}^1$.
Since $D_e \equiv D_{\xi_0}$, we have $\{D_\xi\}$, $\xi \in \mathrm{Pic}^\circ(M)$.
Consider $D'_\lambda - D_\xi$, $(\lambda, \xi) \in \mathbb{P}^1 \times \mathrm{Pic}^\circ(M)$.
$\Rightarrow$ If $(\lambda, \xi) = (0, e)$, $D'_0 - D_e = D_1 + D_e - D_e = D_1$.
If $(\lambda, \xi) = (\infty, \xi_0)$, $D'_\infty - D_{\xi_0} = D_2 + D_{\xi_0} - D_{\xi_0} = D_2$
$\Rightarrow$ $D_1 \equiv D_2$. $\Rightarrow$ $D_1 \equiv D_2$ ⌟

"Note" $D_1, D_2 \in \mathrm{Div}(M)$. $D_1 \equiv D_1'$ $D_2 \equiv D_2'$
$\Rightarrow$ By the lemma above, $\eta_{D_1} = \eta_{D'_1}$, and $\eta_{D_2} = \eta_{D'_2}$
$\Rightarrow$ $\eta_{D_1+D_2} = \eta_{D_1} + \eta_{D_2} = \eta_{D_1'} + \eta_{D_2'} = \eta_{D_1'+D_2'}$ $\Rightarrow$ By the lemma, $D_1 + D_2 \equiv D_1' + D_2'$. $\Rightarrow$ This proves that the algebraically equivalence relation is compatible with the group structure on $\mathrm{Div}(M)$.