By a standard transversality argument, we may take the chain C to meet B transversely almost everywhere, so that the intersection $C \cap B$ will consist of a collection $\{\gamma_\alpha\}$ of piecewise smooth arcs. The endpoints of these arcs will, of course, constitute the points of intersection of A and B: we claim that for each γ, the two endpoints $\gamma(0), \gamma(1) \in A \cap B$ will have opposite intersection index for A and B.

We can find C^∞ vector fields $\{v_i(t) \in T_{\gamma(t)}(C)\}_{i=1,2,\cdots k}$ to C along γ and $\{v_i(t) \in T_{\gamma(t)}(B)\}_{i=k+2,\cdots n}$ to B along γ,

s.t ① $v_1(t), v_2(t) \cdots v_k(t), \gamma'(t)$ is an oriented basis for $T_{\gamma(t)}(C)$.

② $\gamma'(t), v_{k+2}(t) \cdots v_n(t)$ is an oriented basis for $T_{\gamma(t)}(B)$,

③ $v_1(t) \cdots v_k(t), \gamma'(t), v_{k+2}(t) \cdots v_n(t)$ is an oriented basis for $T_{\gamma(t)}(M)$

and such that

$v_1(0), \cdots v_k(0)$ is an oriented basis for $T_{\gamma(0)}(A)$,

$v_1(1) \cdots v_k(1)$ is a basis for $T_{\gamma(1)}(A)$.

Then since $\gamma'(1)$ is outward normal to C and $v_1(1), \cdots v_k(1), \gamma'(1)$ is positively oriented for C, the basis $v_1(1), v_2(1), \cdots v_i(1)$ for $T_{\gamma(1)}(A)$ must be negatively oriented.

Thus $i_{\gamma(0)}(A \cdot B) = +1$ & $i_{\gamma(1)}(A \cdot B) = -1$

These two cancel each other out. $\Rightarrow$ $\#(A \cdot B) = 0$ $\Rightarrow$ We are done.

Now if $\alpha \in H_k(M, \mathbb{Z})$, $\beta \in H_{n-k}(M, \mathbb{Z})$, we may find