$\Rightarrow \quad g(x,y) = zw\,(b_{11} + \cdots) \qquad b_{11} \neq 0$

$\qquad\qquad = (x-y)(x-ry)(b_{11} + \cdots)$

Thus we may say that $xy = 0$ & $(x-y)(x-ry) = 0$ represent C and C' respectives respectively. See P683 note back

So in fact r is the cross-ratio associated to these four tangent lines in some order. Functions $f(x,y)$ in the ideal have the form

$$f(x,y) = \alpha xy + \beta(x-y)(x-ry).$$

In general β will be a unit; then we may assume $\beta = 1$ and

$$f(x,y) = (x-\mu y)(x-\lambda y) + (\text{higher-order terms})$$

where

$$\mu\lambda = r, \quad \lambda + \mu = 1 + r - \alpha.$$

「 I think, the authors wanted to consider the zero set, for the general case. $\Rightarrow$ β will be a unit.

$$\Rightarrow \quad \frac{f}{\beta} = \frac{\alpha}{\beta} xy + (x-y)(x-ry)$$

Let $f/\beta = f$. $\alpha/\beta = \alpha$.

$$\begin{aligned}
\Rightarrow \quad f &= \alpha xy + (x-y)(x-ry) \\
&= x^2 - (1+r)xy + ry^2 + \alpha xy \\
&= x^2 - (1+r-\alpha)xy + ry^2 \\
&= x^2 - (\lambda+\mu)xy + \mu\lambda y^2 = (x-\mu y)(x-\lambda y)
\end{aligned}$$

where $\mu\lambda = r$, $\lambda + \mu = 1 + r - \alpha$.

In case β is a unit, f of the type above cont-