It follows that D meets each vertical disc $\{z_1 = C, |z_2| \leq \epsilon, 0 < |C| \leq \delta\}$ the same number d of times.

⌈ The integral $\frac{1}{2\pi\sqrt{-1}} \int_{|z_2|=\epsilon} \frac{dh}{h}$ represents the number of zeros of h with multiplicities in $\{(z_1, z_2) ; |z_2| \leq \epsilon\}$ with $0<|z_1| \leq \delta$, for each fixed z_1, in other words, $\#(D \cap \{(z_1, z_2) ; |z_2| \leq \epsilon\}) = d$, for every z_1, $0<|z_1| \leq \delta$. ⌋

Thus, projecting $\overline{D}$ on the z_1-axis gives a proper mapping $\pi : \overline{D} \longrightarrow \Delta$ that restricts to a d-sheeted covering $\pi : D^* \longrightarrow \Delta^*$ over the punctured disc.

⌈ Compactness is not changed by subspace topology. $\Rightarrow$ That $\pi : \overline{D} \longrightarrow \Delta$ is proper, is not correct. Any way $\pi : D^* \longrightarrow \Delta^*$ is a d-sheeted branched covering map. Proper is correct if we consider on $\{(z_1, z_2) ; |z_1| < \delta, |z_2| < \epsilon\}$. ⌋

If $d = 1$, we have the graph of a bounded holomorphic function, and our result follows from the Riemann extension theorem.

⌈ On $\{(z_1, z_2) ; 0<|z_1| \leq \delta, |z_2| \leq \epsilon\}$, we have a graph of bounded holomorphic function as below: