$$\frac{\partial}{\partial t} P(\Theta_t) = \frac{\partial}{\partial t} \tilde{P}(\Theta_t, \cdots, \Theta_t)$$
$$= k \cdot \tilde{P}(\tfrac{\partial}{\partial t}\Theta_t, \Theta_t, \cdots \Theta_t)$$
$$= k\,\tilde{P}(d\eta, \Theta_t, \cdots \Theta_t) - k\tilde{P}(\tilde{\theta}\wedge\eta + \eta\wedge\tilde{\theta}, \Theta_t, \cdots \Theta_t)$$
$$-2kt\,\tilde{P}(\eta\wedge\eta, \Theta_t, \cdots \Theta_t).$$

Γ

$$\Theta_t = d(\tilde{\theta}+t\eta) - (\tilde{\theta}+t\eta)\wedge(\tilde{\theta}+t\eta)$$
$$= d\tilde{\theta} + t\,d\eta - t\,\eta\wedge\tilde{\theta} - t\,\tilde{\theta}\wedge\eta - \tilde{\theta}\wedge\tilde{\theta} - t^2\eta\wedge\eta$$

$$\Rightarrow \frac{\partial}{\partial t}\Theta_t = d\eta - (\eta\wedge\tilde{\theta} + \tilde{\theta}\wedge\eta) - 2t\,\eta\wedge\eta.$$

$\frac{\partial}{\partial t} P(\Theta_t) = \frac{\partial}{\partial t} \tilde{P}(\Theta_t \cdots \Theta_t)$, $\tilde{P}$ is the polarization of P.

$= \sum_i \tilde{P}(\Theta_t, \cdots, \frac{\partial}{\partial t}\Theta_t, \cdots \Theta_t)$ since $\tilde{P}$ is linear

$= \sum_i \tilde{P}(\frac{\partial}{\partial t}\Theta_t, \Theta_t, \cdots \Theta_t)$, since $\tilde{P}$ is symmetric and $\frac{\partial}{\partial t}\Theta_t$, Θ's are 2-forms.

$= k\,\tilde{P}(\frac{\partial}{\partial t}\Theta_t, \Theta_t, \cdots \Theta_t) = k\,\tilde{P}(d\eta - (\eta\wedge\tilde{\theta}+\tilde{\theta}\wedge\eta) - 2t\,\eta\wedge\eta, \cdots \Theta_t)$

$= k\,\tilde{P}(d\eta, \Theta_t, \cdots \Theta_t) - k\,\tilde{P}((\eta\wedge\tilde{\theta}+\tilde{\theta}\wedge\eta), \Theta_t, \cdots \Theta_t)$
$- 2kt\,\tilde{P}(\eta\wedge\eta, \Theta_t, \cdots \Theta_t).$ ⅃

Applying the identity (*) with $\theta = \eta$,

$\tilde{P}(\eta\wedge\eta, \Theta_t, \cdots \Theta_t) - (k-1)\,\tilde{P}(\eta, \eta\wedge\Theta_t, \Theta_t, \cdots \Theta_t)$
$= -\tilde{P}(\eta\wedge\eta, \Theta_t, \cdots \Theta_t) - (k-1)\,\tilde{P}(\eta, \Theta_t\wedge\eta, \Theta_t, \cdots \Theta_t),$

so that