

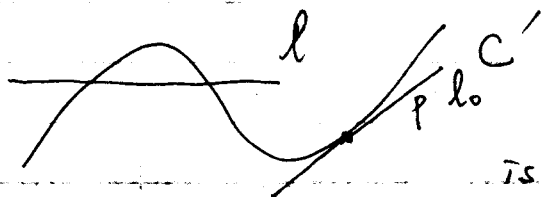
$V_C$ 's meet transversely at  $C'$ . //

Thus it remains to prove that  $\exists$  a smooth conic  $C$  s.t.  $C$  is tangent to  $C'$  at only a given arbitrary point  $p$  of  $C'$ .

To prove this, consider all conics tangent to  $C'$  at  $p$ .  $\Rightarrow$  Such conics form a linear system since the sum of <sup>such</sup> two conics is tangent to  $C'$  at  $p$ . Let  $\{C_\lambda\}$  be the linear system.

$\Rightarrow \{C_\lambda \cap C'\}$  is a linear system on  $C'$ .

$\Rightarrow$  By Bertini's theorem, a generic element of  $\{C_\lambda \cap C'\}$  is smooth away from the base locus  $P$ , since



$l + l_0$  is a conic which is tangent to  $C'$  at  $p$ , and

, by varying  $l$ , we have a lot of conics with th base locus  $P$ .

$\Rightarrow$  A generic element of  $\{C_\lambda \cap C'\}$  is  $\{p_1, p_2, 2p\}$ , where  $p_1$  and  $p_2$  are distinct &  $p_i \neq p$ .

$$\dim \{l + l_0\} = \dim l = 2.$$

$$\dim \{C_\lambda\} = ?$$

To compute the dimension, we may assume that  $l_0 = (X_0 = 0)$  and  $p = [0, 0, 1]$ .

$$a_{00}X_0^2 + \dots + a_{12}X_1X_2 = 0$$

Conic must pass through  $p \Rightarrow a_{22} = 0$ .

$$\Rightarrow a_{00}X_0^2 + a_{11}X_1^2 + a_{01}X_0X_1 + a_{02}X_0X_2 + a_{12}X_1X_2 = 0$$