Date

Fix z_1, z_2. Then since any two such forms differ by a holomorphic 1-form on S, we see moreover that there exists a unique such differential φ_a with all A-periods zero. Let $W \cong \mathbb{C}^3$ denote the vector space of such forms, and consider the linear map

$$\psi : W \longrightarrow \mathbb{C}^g$$

obtained by integration over the B-cycles of S:

$$\psi : \varphi_a \longmapsto \left(\int_{\delta_{g+1}} \varphi_a, \cdots \int_{\delta_{2g}} \varphi_a \right).$$

Clearly, the vector space V above is just the kernel of the map ψ. " $\psi(\varphi_a) = 0$. $\Rightarrow$ φ_a has a pole of order ≤ 3 at p and a pole of order ≤ 2 at q. φ_a has no residues. and if $\psi(\varphi_a) = 0$, φ_a has no periods $\Rightarrow$ $\varphi_a \in V$."

To describe ψ explicitly, let $\omega_1, \omega_2, \cdots \omega_g$ be a normalized basis for $H^0(S, \Omega^1)$. By the reciprocity law for differentials of the first and second kinds,

let $\omega = \omega_i$ $\quad \eta = (a_1 z_1^{-3} + a_2 z_1^{-2} + [0])\, dz_1$

$(b_0^p + b_1^p z_1 + \cdots)\, dz_1 \quad \varphi_a = (a_3 z_2^{-2} + [0])\, dz_2$

$(b_0^q + b_1^q z_2 + \cdots)\, dz_2$

$$\int_{\delta_{g+i}} \varphi_a = 2\pi\sqrt{-1}\left(\frac{a_1 b_1^p}{2} + a_2 b_0^p \right) + 2\pi\sqrt{-1}\left(\frac{a_3 b_0^q}{1} \right)$$

$$= 2\pi\sqrt{-1}\left(a_3 \frac{\omega}{dz_2}(q) \right) + 2\pi\sqrt{-1}\left(a_2 \frac{\omega}{dz_1}(p) + \frac{1}{2} a_1 \left(\frac{\omega}{dz_1}\right)'(p_1) \right)$$