We are going to do it for $n=2$.

$$\widetilde{\mathbb{C}}^2 \xrightarrow{\pi'} \mathbb{C}^2 \xrightarrow{P} \mathbb{P}^1$$

$$\int_{\mathring{B}[r]} \bar\partial\varphi \wedge \beta = \int_{\pi'^{-1}(\mathring{B}[r])} \pi'^*(\bar\partial\varphi \wedge \beta) = \int_{\widetilde{\Delta} \cap (\mathring{B}[r] \times \mathbb{P}^1)} \pi'^*(\bar\partial\varphi) \wedge \pi'^*(\beta)$$

where $\widetilde{\Delta} = \{((z_1, z_2), \ell) ; \quad (z_1, z_2) \in \ell\} \subset \Delta \times \mathbb{P}^1$

$\Delta = \{(z_1, z_2) \in \mathbb{C}^2 ; \quad |z_i| < 2r\}$

$\Rightarrow$ $\widetilde{\Delta} \cap (\mathring{B}[r] \times \mathbb{P}^1)$ is covered by a cover

$\{\widetilde{\Delta} \cap (\mathring{B}[r] \times U_0), \ \widetilde{\Delta} \cap (\mathring{B}[r] \times U_1)\}$, where $U_0 = (z_1 \neq 0)$.
$U_1 = (z_2 \neq 0)$.

$\Rightarrow$
$$\begin{array}{ccl}
\widetilde{\Delta} \cap (\mathring{B}[r] \times U_0) & \xrightarrow{\psi_1} & V_1 \subset \mathbb{C}^2 \\
((z_1, z_2), \ell) & \longmapsto & (z_1, \frac{z_2}{z_1}) = (z_1, w), \quad w = \frac{z_2}{z_1} \\
\widetilde{\Delta} \cap (\mathring{B}[r] \times U_1) & \xrightarrow{\psi_2} & V_2 \subset \mathbb{C}^2 \\
((z_1, z_2), \ell) & \longmapsto & (z_2, \frac{z_1}{z_2}) = (z_2, \eta), \quad \eta = \frac{z_1}{z_2}
\end{array}$$

$\Rightarrow$ By using the partition of unity, we can find ϕ_1, ϕ_2 s.t $\phi_1 + \phi_2 = 1$. $\operatorname{supp} \phi_i \subset \widetilde{\Delta} \cap (\mathring{B}[r] \times U_{i-1})$, where $\operatorname{supp} \phi_i \subset U_{i-1}$. and $\underset{(z, \ \ell)}{\mathring{B}[r] \times U_{i-1}} \xrightarrow{\phi_i} \mathbb{R}$ same notation $\longmapsto \phi_i(\ell)$.

$$\int_{\widetilde{\Delta} \cap (\mathring{B}[r] \times \mathbb{P}^1)} \pi'^*(\bar\partial\varphi) \wedge \pi'^*(\beta) = \int_{\widetilde{\Delta} \cap (\mathring{B}[r] \times \mathbb{P}^1)} (\phi_1 + \phi_2)(\pi'^*(\bar\partial\varphi)) \wedge \pi'^*(\beta)$$

$$= \int_{\widetilde{\Delta} \cap (\mathring{B}[r] \times \mathbb{P}^1)} \phi_1 \ \pi'^*(\bar\partial\varphi) \wedge \pi'^*(\beta) + \int_{\widetilde{\Delta} \cap (\mathring{B}[r] \times \mathbb{P}^1)} \phi_2 \ \pi'^*(\bar\partial\varphi) \wedge \pi'^*(\beta)$$

$$= \int_{\mathbb{C}^2 \cap V_1} \phi_1 \circ \psi_1^{-1} \cdot \psi_1^{-1*}(\pi'^*(\bar\partial\varphi) \wedge \pi'^*(\beta)) + \int_{\mathbb{C}^2 \cap V_2} \phi_2 \circ \psi_2^{-1} \cdot \psi_2^{-1*}(\pi'^*(\bar\partial\varphi) \wedge \pi'^*(\beta))$$