(Note: It is convenient to define an operator d^c by

$$d^c = \frac{i}{4\pi}(\bar\partial - \partial).$$

d^c and d are both real differential operators, and

$$dd^c = -d^c d = \frac{i}{2\pi}\partial\bar\partial.$$

We can consequently write $\omega = dd^c \log \|Z\|^2$)

$\sqrt{\Gamma}$ $\overline{d^c} = d^c \Rightarrow d^c$ is real.

D is real $\Leftrightarrow \bar D = D$.

$\bar D(f) = \overline{D(\bar f)}$ $\lrcorner\!\!\rfloor$

Note by the above that any compact complex manifold that can be embedded in projective space $\mathbb{P}^n$ is Kähler.

We give some immediate topological consequences of the Kähler condition: For M a compact Kähler manifold,

1. The even Betti number $b_{2q}(M)$ are positive;
2. The holomorphic q-forms $H^0(M, \Omega^q)$ inject into the cohomology $H^q_{DR}(M)$, i.e., every such η is closed, and is never exact; and
3. The fundamental class η_V of any analytic subvariety $V \subset M$ is non-zero.

proofs). 2. Let η be a holomorphic $(q,0)$-form; we want to show $d\eta = 0$, and that $\eta = d\psi$ only if $\eta \equiv 0$.

Let $\varphi_1, \varphi_2 \cdots \varphi_n$ be a local unitary coframe;

if $\eta = \sum_I \eta_I \varphi_I$,

then
$$\eta \wedge \bar\eta = \sum_{I\bar J} \eta_I \bar\eta_J \varphi_I \wedge \bar\varphi_J.$$