from M to $\mathbb{P}^n$; in fact — as we shall prove later in this section in case M is a surface — the converse is true: any rational map $f: M \longrightarrow \mathbb{P}^n$ is induced by a holomorphic map on a (possibly multiple) blow-up $\tilde{M}$ of M.

⌈
$$\begin{tikzcd} M \arrow[r, "f \circ \pi^{-1}"] \arrow[d, "\pi^{-1}"'] & \mathbb{P}^n \\ \tilde{M} \arrow[ur, "f"'] & \end{tikzcd}$$

$f \circ \pi^{-1}: M - \{p_i\} \longrightarrow \mathbb{P}^n$ is holomorphic. $\Rightarrow$ $f \circ \pi^{-1}$ is rational.
See P510. proof of theorem ①. ⌋

3. As mentioned at the beginning of this section, the projection map

$$\pi_p : C - \{p\} \longrightarrow \mathbb{P}^{n-1}$$

of a curve $C \subset \mathbb{P}^n$ from a point p on C into a hyperplane in $\mathbb{P}^n$ extends to a holomorphic map on all of C. In general, if $V \subset \mathbb{P}^n$ is any variety, $p \in V$ any point, the projection map π_p of V from p to a hyperplane is a rational map. Indeed, π_p may always be extended to a holomorphic map on the blow-up $\tilde{V}$ of V at p by sending a point $r \in E$ in the exceptional divisor to the point of intersection of $\mathbb{P}^{n-1}$ with the tangent line to V at p corresponding to r.

⌈ Here p is a smooth point of V, since we deals with the blow-up at a smooth point. $\Rightarrow$
$$\begin{tikzcd} V \arrow[r, "\pi^{-1}"] \arrow[rr, bend right, "\pi_p"'] & \tilde{V} \arrow[r, "\tilde{\pi}_p"] & H \end{tikzcd}$$
is rational. ⌋