$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$\sigma_i = v_{1i}\,\omega_1 + \cdots + v_{ki}\,\omega_k = v'_{1i}\,\omega'_1 + \cdots + v'_{ki}\,\omega'_k$$

$$\vdots \qquad\qquad\qquad\qquad = v'_{ji}\,\omega'_j$$

Let $\omega_i = g_{ji}\,\omega'_j$.

$$\Rightarrow \quad \sigma_i = v_{ji}\,\omega_j = v_{ji}\,g_{\alpha j}\,\omega'_\alpha = g_{\alpha j}\,v_{ji}\,\omega'_\alpha$$

$$= (g\,v)_{\alpha i}\,\omega'_\alpha.$$

$$\overset{\shortparallel}{v'_{\alpha i}\,\omega'_\alpha}$$

$$\Rightarrow \quad \iota(x) = \left\langle \begin{matrix} (v_{11}, & v_{12}, & \cdots & v_{1n}) \\ (v_{21}, & v_{22}, & \cdots & v_{2n}) \\ & \vdots & & \\ (v_{k1}, & v_{k2}, & \cdots & v_{kn}) \end{matrix} \right\rangle$$

$$\left\langle \begin{pmatrix} v'_{11}, & \cdots & v'_{1n} \\ \vdots & & \\ v'_{k1} & \cdots & v'_{kn} \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} (gv)_{11}, & \cdots & (gv)_{1n} \\ & & \\ (gv)_{k1}, & \cdots & (gv)_{kn} \end{pmatrix} \right\rangle$$

For example, $k = 2.\ n = 3$

$$\begin{pmatrix} g_{11}, & g_{12} \\ g_{21}. & g_{22} \end{pmatrix} \begin{pmatrix} v_{11}, & v_{12}, & v_{13} \\ v_{21}, & v_{22}, & v_{23} \end{pmatrix}$$

$$= \begin{pmatrix} g_{11}v_{11} + g_{12}v_{21}, & g_{11}v_{12} + g_{12}v_{22}, & g_{11}v_{13} + g_{12}v_{23} \\ & & \\ g_{21}v_{11} + g_{22}v_{21}, & g_{21}v_{12} + g_{22}v_{22}, & g_{21}v_{13} + g_{22}v_{23} \end{pmatrix}$$

$$\Rightarrow \alpha\big(\, g_{11}(v_{11}, v_{12}, v_{13}) + g_{12}(v_{21}, v_{22}, v_{23})\big) + \beta\,(\, g_{21}(v_{11},$$