NO.
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$H^0(S, \mathcal{O}(mD)) \cong H^2(S, \Omega^2((mD)^*)) = H^2(S, \Omega^2(-mD))$
$= H^2(S, \mathcal{O}(K-mD))$ by Kodaira – Serre duality on P153 & P102. $\Rightarrow H^0(S, \mathcal{O}(K-mD)) = H^2(S, \mathcal{O}(mD))$

$h^0(K-mD) = h^2(mD) = \frac{1}{2}m^2 d - \frac{m}{2}K\cdot D + \chi(\mathcal{O}_S) + h^1(mD) - h^0(mD)$

But we know that mD can not be effective for $m \neq 0$.

Suppose $H^0(S, \mathcal{O}(mD)) \neq 0$

$\Rightarrow \exists$ a section σ over S which is holomorphic

$\Rightarrow (\sigma = 0) \sim mD \Rightarrow mD$ is linearly equivalent to an effective divisor $(\sigma=0)$. $\Rightarrow$ Contradiction

$\Rightarrow H^0(S, \mathcal{O}(mD)) = 0$ if $m \neq 0$.

$h^2(mD) = \frac{1}{2}m^2 d - \frac{m}{2}K\cdot D + \chi(\mathcal{O}_S) + h^1(mD)$ ⌟

In particular, $K + mD$ is linearly equivalent to an effective divisor E_m for all $m \gg 0$

⌜ As m goes to $-\infty$, $h^0(K-mD)$ becomes large.

$\Rightarrow$ For $m \gg 0$, $H^0(S, \mathcal{O}(K+mD)) \neq 0 \Rightarrow \exists$ a section $\sigma \in H^0(S, \mathcal{O}(K+mD)) \Rightarrow (\sigma=0)$ is linearly equivalent to $K+mD$. ⌟

But now the map

$$|K-mD| \longrightarrow |2K|$$

given by

$$G \longmapsto G + E_m$$

is injective, and so the dimension of $|K-mD|$ is bounded – a contradiction.