I think the statement saying that $H^{2k}(V)$ is generated by the class of a k-plane section of V for $k < n/2$ is not correct.

In case $n = 2l-1$, $k = l-1 \Rightarrow 2k = 2l-2 = n-1$.

$\Rightarrow H^{n-1}(V, Q) \cong H^{n-1}(\mathbb{P}^n, Q) \Rightarrow H_n(\mathbb{P}^n, V, Z) = 0$ which I doubt. ⌟

In particular any smooth hypersurface of dim 2 or more is connected and simply connected.

⌜ $V \subset \mathbb{P}^n$. $\dim V \geq 2 \Rightarrow n \geq 3$.

$\Rightarrow H^1(V) = 0$ since $1 \neq n/2 \geq \frac{3}{2}$ $\Rightarrow H_1(V) = 0$

$H^0(V) = Q$ since $0 \leq \frac{n}{2} - 1$. $\Rightarrow V$ is path-connected.

According to Bott's proof, $\pi_q(V) \xrightarrow{\cong} \pi_q(\mathbb{P}^n)$ if $q \leq n-2$

$n \geq 3 \Rightarrow q = 0, 1$

$\pi_0(V) \cong \pi_0(\mathbb{P}^n) = 0$

$\pi_1(V) \cong \pi_1(\mathbb{P}^n) = 0 \Rightarrow V$ is simply connected ⌟

The same results apply, for an appropriate range of k, to any submanifold of projective space given as the transverse intersection of hypersurfaces.

⌜ V, W hypersurfaces of $\mathbb{P}^n$. $V \cap W$ submfd of $\mathbb{P}^n$.

$\Rightarrow$ Is $V \cap W$ a submanifold of V?

pf) Yes. Consider the following lemma.

Lemma. $L^k \subset N^{n-1} \subset M^n$, where N is a submfd of M

" L " of M

Prove that L is a submfd of N.