Now let ds^2 be a metric with $(1,1)$-form ω, multiplied by a constant so that $\int_S \omega = 1$.

ω is a 2-form. $\Rightarrow$ By p28, ω is a volume form. $\Rightarrow$ $\int_S \omega$ = some constant $\Rightarrow$ We can normalize ω so that $\int_S \omega = 1$.

$[\omega] \in H^2_{DR}(S)$ is an integral cohomology class, and by the Kodaira embedding theorem S can be embedded in projective space $\mathbb{P}^N$.

Since ω is a volume form, $\omega = \alpha\, dz \wedge d\bar{z}$.
$\Rightarrow$ ω is a positive definite form $\Rightarrow$ By p191, Kodaira embedding theorem, S can be embedded in projective space $\mathbb{P}^N$.

In fact, as suggested in the discussion of the embedding theorem, a sharper statement and a simpler proof of the theorem are possible for Riemann surfaces, and we give these here. (See p181)

Let $L \longrightarrow S$ be a holomorphic line bundle. Recall that the degree of L is defined to be its first Chern class $c_1(L) \in H^2(S, \mathbb{Z})$ under the identification $H^2(S, \mathbb{Z}) = \mathbb{Z}$ given by the natural orientation of S. (See p144) If $L = [D]$ for

$$D = \sum a_i p_i \in \mathrm{Div}(S), \quad \text{then } \deg L = \sum a_i.$$

$\deg L = \langle c_1(L), [S] \rangle = \#(D \cdot S) = \sum a_i \#(p_i \cdot S)$
$= \sum a_i \cdot 1 = \sum a_i$

As we have seen, L has a nontrivial global