an especially nice local syzygy. Later on we shall be especially concerned with the codimension -2 case. If locally $I = \{f_1, f_2\}$, then the Koszul complex gives the local syzygy

$$0 \longrightarrow \mathcal{O} \xrightarrow{\lambda} \mathcal{O} \oplus \mathcal{O} \xrightarrow{\eta} \mathcal{O} \longrightarrow \mathcal{O}/I \longrightarrow 0,$$

where

$$\lambda(g) = (-f_2\, g) \oplus (f_1 g),$$
$$\eta(g_1 \oplus g_2) = f_1 g_1 + f_2 g_2$$

$\sqcap$ Consider $E_k = \mathcal{O} \otimes_{\mathbb{C}} \Lambda^k \mathbb{C}^2 \cong \mathcal{O}^{\binom{2}{k}}$, $k = 0, 1, 2$.

$$\partial : E_k \longrightarrow E_{k-1}$$
$$e_{i_1} \wedge \cdots \wedge e_{i_k} \longmapsto \sum (-1)^{\nu-1} f_{i_\nu}\, e_{i_1} \wedge \cdots \wedge \hat{e}_{i_\nu} \wedge \cdots \wedge e_{i_k}$$

We have the following exact sequence

$$0 \longrightarrow E_2 \longrightarrow E_1 \longrightarrow E_0 \longrightarrow \mathcal{O}/I \longrightarrow 0$$

since, at each stalk, by the regularity of $\{f_1, f_2\}$,

$$0 \to (E_2)_z \longrightarrow (E_1)_z \longrightarrow (E_0)_z \longrightarrow \mathcal{O}_z/I_z \longrightarrow 0 \text{ is exact.}$$

$$E_2 \cong \mathcal{O} \xrightarrow{\partial} E_1 = \mathcal{O} \oplus \mathcal{O}$$
$$e_1 \wedge e_2 \longmapsto f_1 e_2 - f_2 e_1$$
$$g \longmapsto (-g f_2,\ g f_1)$$

$$E_1 \xrightarrow{\partial} E_0 \qquad \mathcal{O} \oplus \mathcal{O} \longrightarrow \mathcal{O}$$
$$e_1 \longmapsto f_1 \qquad (g_1, g_2) \longmapsto g_1 f_1 + g_2 f_2$$
$$e_2 \longmapsto f_2$$

$$0 \longrightarrow \mathcal{O} \xrightarrow{\lambda} \mathcal{O} \oplus \mathcal{O} \xrightarrow{\eta} \mathcal{O} \longrightarrow \mathcal{O}/I \longrightarrow 0$$
$$g \longmapsto (-f_2 g,\ f_1 g)$$
$$(g_1, g_2) \longmapsto f_1 g_1 + f_2 g_2$$