Consider the following map

$$\mathbb{C}^3 \times \mathbb{C}^2 \xrightarrow{f'} \mathbb{C}^4$$

$$(z_1, z_2, z_3) \times (a_1, a_2) \longmapsto (f_1(z_1,z_2,z_3) - f_1(a_1,a_2,0),\ f_2(z_1,z_2,z_3) - f_2(a_1,a_2,0),$$
$$f_3(z_1,z_2,z_3) - f_3(a_1,a_2,0),\ f_4(z_1,z_2,z_3) - f_4(a_1,a_2,0))$$

$\Rightarrow$ Since $\dfrac{\partial f_i}{\partial z_j} = \dfrac{\partial f_i}{\partial a_j}$ at the origin, $1 \le i \le 4$, $1 \le j \le 2$

$$\begin{pmatrix} \frac{\partial f_1}{\partial z_1}, & \frac{\partial f_1}{\partial z_2}, & \frac{\partial f_1}{\partial z_3}, & -\frac{\partial f_1}{\partial a_1}, & -\frac{\partial f_1}{\partial a_2} \\ \frac{\partial f_2}{\partial z_1}, & \frac{\partial f_2}{\partial z_2}, & \frac{\partial f_2}{\partial z_3}, & -\frac{\partial f_2}{\partial a_1}, & -\frac{\partial f_2}{\partial a_2} \\ \frac{\partial f_3}{\partial z_1}, & \frac{\partial f_3}{\partial z_2}, & \frac{\partial f_3}{\partial z_3}, & -\frac{\partial f_3}{\partial a_1}, & -\frac{\partial f_3}{\partial a_2} \\ \frac{\partial f_4}{\partial z_1}, & \frac{\partial f_4}{\partial z_2}, & \frac{\partial f_4}{\partial z_3}, & -\frac{\partial f_4}{\partial a_1}, & -\frac{\partial f_4}{\partial a_2} \end{pmatrix} = J(f') \text{ has rank } 2 \text{ at the origin.}$$

Let $F : \mathbb{C}^3 \times \mathbb{C}^2 \times \mathbb{C}^2 \longrightarrow \mathbb{C}^4 \times \mathbb{C}^3$

$$(z_1, z_2, z_3) \times (a_1, a_2) \times (w_1, w_2) \longmapsto (f_1', f_2', f_3' + w_1, f_4' + w_2, a_1, a_2, z_3)$$

$$\Rightarrow \begin{pmatrix} \frac{\partial f_1}{\partial z_1}, & \frac{\partial f_1}{\partial z_2}, & \frac{\partial f_1}{\partial z_3}, & -\frac{\partial f_1}{\partial a_1}, & -\frac{\partial f_1}{\partial a_2}, & 0, & 0 \\ \frac{\partial f_2}{\partial z_1}, & \frac{\partial f_2}{\partial z_2}, & \frac{\partial f_2}{\partial z_3}, & -\frac{\partial f_2}{\partial a_1}, & -\frac{\partial f_2}{\partial a_2}, & 0, & 0 \\ \frac{\partial f_3}{\partial z_1}, & \frac{\partial f_3}{\partial z_2}, & \frac{\partial f_3}{\partial z_3}, & -\frac{\partial f_3}{\partial a_1}, & -\frac{\partial f_3}{\partial a_{1+1}}, & 1, & 0 \\ \frac{\partial f_4}{\partial z_1}, & \frac{\partial f_4}{\partial z_2}, & \frac{\partial f_4}{\partial z_3}, & -\frac{\partial f_4}{\partial a_1}, & -\frac{\partial f_4}{\partial a_2}, & 0, & 1 \\ 0 & 0 & 0 & 1, & 0 & 0. & 0 \\ 0 & 0 & 0 & 0 & 1, & 0. & 0 \\ 0, & 0 & 1, & 0 & 0. & 0. & 0 \end{pmatrix} = J(F).$$