

exactly six of the lines $\{X_h\}$, and vice versa.

By the result above, $p \in h_1, h_2, \dots, h_6$, $h_i \in R^*$.

We have to show that $X_{h_1} \cap X_p \neq \emptyset$.

$$\begin{aligned} X_p \cap X_{h_1} &= (\sigma(p) \cap F) \cap (\sigma(h_1) \cap F) = \sigma(p, h_1) \cap \sigma(p', h_1) \\ &= \sigma(p) \cap \sigma(h_1) \cap F = \sigma(p, h_1) \cap F \neq \emptyset \text{ since } \deg F = 2 \\ &\text{and } \sigma(p, h_1) \text{ is a line.} \end{aligned}$$

If $p \notin h_1$, $\sigma(p) \cap \sigma(h_1) \cap F = X_p \cap X_{h_1} = \emptyset$ since $\sigma(p) \cap \sigma(h_1) = \emptyset$.

□

Note that these are all the lines on Σ : if $L \subset \Sigma$ is any line, $\sigma(p, h)$ the corresponding pencil, then by definition every line $l \in \sigma(p, h)$ belongs to two confocal pencils of X .

If $l \in \sigma(p, h) \subset \Sigma \Rightarrow \sigma(\pi(l)) \cap F$ is a union of two pencils with the focus $\pi(l)$. □

If the common focus of these two pencils is p for every l , then clearly $\sigma(p, h) = X_p$, while if for generic $l \in \sigma(p, h)$ the common focus of the pencils containing l is a point $q \neq p \in C_h$, then clearly h can not contain two pencils, and so $\sigma(p, h) = X_h$.

For every $l \in L$, $\pi(l) = p \Rightarrow$ Since $\Sigma \subset X$,