

the generic point. $\Rightarrow |D|$ is of $\dim 0 \Rightarrow |D|$ is just a one-point set. \Rightarrow The generic fiber of $\mu^{(g)}$ is one point. $\Rightarrow \mu^{(g)}$ is one to one generically.

Here we used Sard's theorem. More precisely, $\mu^{(g)}: S^{(d)} \rightarrow f(S)$ is nonsingular for the generic points $\Rightarrow \exists$ a set $K \subset S^{(d)}$ s.t. $\mu^{(g)}$ is singular on K and the measure of K is zero. \Rightarrow By lemma 1.1 P 68 (Hirsch) $\mu^{(g)}(K)$ is a set of measure zero. $\Rightarrow \mu^{(g)}: S^{(d)} - \mu^{(g)^{-1}(\mu^{(g)}(K))} \rightarrow f(S) - \mu^{(g)}(K)$ is a covering map.

(Hirsch)
By P 69, Morse-Sard Theorem, the set of regular values of $\mu^{(g)}$ is dense, i.e. $A = \{\lambda \in f(S) \mid |f(\mu^{(g)})| \neq 0 \text{ at all } q \text{ s.t. } \mu^{(g)}(q) = \lambda\}$ is dense in $f(S)$. \Rightarrow Since $S^{(d)}$ is compact, by the inverse function theorem, A is open. $\Rightarrow \mu^{(g)^{-1}(A)$ is open and dense in $S^{(d)}$. For, if it is not, $\exists x_0 \in S^{(d)}$ s.t. $x_0 \notin \overline{\mu^{(g)^{-1}(A)}}$. $\Rightarrow \exists U_{x_0}$ open and $x_0 \in U_{x_0}$ s.t. $U_{x_0} \cap \overline{\mu^{(g)^{-1}(A)}} = \emptyset$, and open $V \subset U_{x_0}$ s.t. $\mu^{(g)}(V)$ open in $f(S)$. $\Rightarrow \mu^{(g)}(V) \cap A \neq \emptyset \Rightarrow$ Contradiction since $\mu^{(g)^{-1}(A) \cap V = \emptyset$. \Rightarrow

Note that as a corollary to Jacobi inversion, we see that every divisor of degree $\geq g$ on a Riemann surface of genus g is linearly equivalent to an effective divisor.

Γ Given a divisor D of degree d , consider $D - dp_0$.

$\Rightarrow \deg(D - dp_0) = 0 \Rightarrow D - dp_0 \in \text{Div}^0(S)$.

$\Rightarrow \mu(D - dp_0) \in f(S) \Rightarrow$ By Jacobi inversion; \exists

$p_1 + p_2 + \dots + p_g$ s.t. $\mu(p_1 + \dots + p_g - gp_0) = \mu(D - dp_0)$. \Rightarrow