Suppose $\#(V \cap W+\epsilon)=\mu$., and V and W meet transversely. $V' = f(V)$. $W' = f(W)$.
Suppose V' and $W'+\epsilon'$ meet transversely in μ' points.
Claim: $\mu = \mu'$.
Consider

$$\mathbb{C}^n \times \mathbb{C}^n \xrightarrow{F} \mathbb{C}^n \qquad \mathbb{C}^n \times \mathbb{C}^n \xrightarrow{G} \mathbb{C}^n$$
$$(t, z) \longmapsto f(z)+t. \qquad (t, z) \longmapsto f(z+t).$$

$\Rightarrow$ $F(\epsilon', z)$ is homotopic to $G(\epsilon, z)$

$\Rightarrow$ By the similar argument on P51 ~ P52, (or see P108 Proposition , Differential Topology by Guillemin & Pollack), $\#(V' \cap W'+\epsilon') = \#(V' \cap f(W+\epsilon)) = \#(V \cap W+\epsilon) = \mu = \mu'$.

This proves the independence of the choice of coordinates. ⌟

We now check that if V and W are analytic subvarieties of complementary dimension on a compact complex manifold M, then

$$\#(V \cdot W) = \sum_{p \in V \cap W} m_p(V \cdot W).$$

To do this, let z be local coordinates around a point $p \in V \cap W$, with $p = (0,0)$ the only point of intersection of V with W in the ball Δ of radius 1. Let $\rho(r)$ be a C^∞ bump function, identically 1 on the ball Δ'' of radius $\frac{1}{4}$ and identically zero outside the ball Δ' of radius $\frac{1}{2}$. Then for ϵ generic and sufficiently small, the locus