

If $H \subset W$ is the hyperplane in W consisting of conics containing a point $p \in \mathbb{P}^2$, then the pullback f^*H is the divisor of pairs (l_1, l_2) with either $p \in l_1$ or $p \in l_2$, and so it represents the class $\omega_1 + \omega_2$.

$$\mathbb{F} \quad \dim H^0(\mathbb{P}^2, \mathcal{O}(H)) = 4C_2 = 6$$

$$\Rightarrow H^0(\mathbb{P}^2, \mathcal{O}(\otimes H)) = \langle \sigma_1, \sigma_2, \dots, \sigma_6 \rangle.$$

$$\{ (a_1, \dots, a_6) \mid a_1 \sigma_1(p) + \dots + a_6 \sigma_6(p) = 0 \} = \tilde{H}$$

$\Rightarrow \tilde{H}$ is a hyperplane in $H^0(\mathbb{P}^2, \mathcal{O}(\otimes H))$. $\Rightarrow H$ is the hyperplane corresponding to \tilde{H} in $W = \mathbb{P}(H^0(\mathbb{P}^2, \mathcal{O}(\otimes H)))$.

$$f: \mathbb{P}^{2*} \times \mathbb{P}^{2*} \longrightarrow \underset{H}{W}$$

$$f^*H \ni (l_1, l_2) \Rightarrow (l_1 + l_2)(p) \Rightarrow l_1(p) l_2(p) = 0$$

where we use l_1, l_2 as polynomials representing the lines l_1, l_2 respectively.

$$\Rightarrow l_1(p) = 0 \text{ or } l_2(p) = 0 \Rightarrow p \in l_1 \text{ or } p \in l_2.$$

By the argument above, we can see that

$$\{ l \in \mathbb{P}^{2*} \mid l(p) = 0 \} \text{ is a hyperplane in } \mathbb{P}^{2*}.$$

$$\Rightarrow \text{If we let } H' = \{ l \in \mathbb{P}^{2*} \mid l(p) = 0 \},$$

$$\pi_1^* H' = \omega_1 \text{ \& } \pi_2^* H' = \omega_2. \quad \Rightarrow \quad \omega_1 + \omega_2 = f^*H. \quad \square$$

Consequently, since f is two to one,

$$\deg W_1 = (H)_{W_1}^4$$

$$= \frac{1}{2} (\omega_1 + \omega_2)_{\mathbb{P}^{2*} \times \mathbb{P}^{2*}}^4$$

$$= \frac{1}{2} \cdot 6 = 3.$$