$\deg(\iota_E M) = c_1(L)^n$.

$\ulcorner$ $\iota_E : M \longrightarrow \mathbb{P}^N$ By the proper mapping theorem, $\iota_E M$ is an analytic subvariety of $\mathbb{P}^N$. Let $n = \dim \iota_E M$.

$\Rightarrow \deg(\iota_E M) = \#(\iota_E M \cap \mathbb{P}^{N-n}) = \#(\iota_E M \cap H_1 \cap \cdots \cap H_n)$
where H_i's are hyperplanes of $\mathbb{P}^N$.

$\Rightarrow \#(\iota_E M \cdot \mathbb{P}^{N-n}) = \int_{\mathbb{P}^N} \eta_V \wedge \sigma_n = \int_{\mathbb{P}^{N-n}} \eta_V$ where $\iota_E M = V$

$\sigma \in H^2_{DR}(\mathbb{P}^N, R)$ s.t $\int_{\mathbb{P}^1} \sigma = 1$, & $\sigma_n = \overline{\sigma \wedge \overset{n}{\cdots} \wedge \sigma} = \sigma^n$.

$\Rightarrow \int_{\mathbb{P}^{N-n}} \eta_V = \int_{\mathbb{P}^N} \eta_V \wedge \sigma_m = \int_V \sigma_n = \int_M \iota_E^*(\sigma_n) = \int_M \iota_E^*(\sigma \wedge \cdots \wedge \sigma)$

$= \int_M \iota_E^*(\sigma) \wedge \cdots \wedge \iota_E^*(\sigma) = \int_M (\iota_E^*(\sigma))^n \overset{?}{=} \int_M c_1(L)^n$

Now it remains to show that $\iota_E^*(\sigma) = c_1(L) = \eta_D$, where $[D] = L = \iota_E^*(H)$.

According to P59, $[[D]]$ (homology class) is Poincare dual to the pullback via ι_E of the Poincare dual of $[[H]]$, i.e.,

$$\begin{tikzcd}
{[[D]] \in H_{2m-2}(M, Z)} \arrow[r, "\iota_{E*}"] \arrow[d, leftrightarrow] & {H_{2N-2}(\mathbb{P}^N, Z) \ni [[H]]} \arrow[d, leftrightarrow, "\cong"] \\
{H^2_{DR}(M, R)} & {H^2_{DR}(\mathbb{P}^N, Z) \otimes R \ni \sigma} \arrow[l, "\iota_E^*"'] \\
{\eta_D} \arrow[u, phantom, "\in"] &
\end{tikzcd}$$