$T_p(F) \cap \mathbb{P}^{2n-1}$. Set

$$\tilde{\Lambda} = \Lambda \cap \mathbb{P}^{2n-1} \quad \text{and} \quad \tilde{\Lambda}' = \Lambda' \cap \mathbb{P}^{2n-1};$$

$\tilde{\Lambda}$ and $\tilde{\Lambda}'$ are then $(n-1)$-planes in $\tilde{F}$, and by our previous argument Λ and Λ' belong to the same family on F if and only if $\tilde{\Lambda}$ and $\tilde{\Lambda}'$ belong to the same family on $\tilde{F}$.

① The n-planes in F through p are exactly the n-planes spanned by p together with $(n-1)$-planes in $\tilde{F}$.
② Σ_{n-1} & Σ_n are the disjoint unions of two irreducible varieties.

But the intersection $\Lambda \cap \Lambda'$ is just the plane spanned by the intersection $\tilde{\Lambda} \cap \tilde{\Lambda}'$ together with p.

Suppose $\tilde{\Lambda} = \mathbb{P}^{m_1}$, $\tilde{\Lambda}' = \mathbb{P}^{m_2}$ and $\tilde{\Lambda} \cap \tilde{\Lambda}' = \mathbb{P}^{k}$.
$\Rightarrow$ $\Lambda = \mathbb{P}^{m_1+1}$ $\Lambda' = \mathbb{P}^{m_2+1}$ $\langle p, \tilde{\Lambda} \cap \tilde{\Lambda}' \rangle = \mathbb{P}^{k+1}$.
$\Rightarrow$ $\Lambda \cap \Lambda' = \mathbb{P}^{k+1}$, and clearly since $\langle p, \tilde{\Lambda} \cap \tilde{\Lambda}' \rangle \subset \langle p, \tilde{\Lambda} \rangle \cap \langle p, \tilde{\Lambda}' \rangle = \Lambda \cap \Lambda'$, $\Lambda \cap \Lambda' = \langle p, \tilde{\Lambda} \cap \tilde{\Lambda}' \rangle$.
$\Rightarrow$ $\Lambda \cap \Lambda'$ is the plane spanned by $\tilde{\Lambda} \cap \tilde{\Lambda}'$ together with p.

By the induction hypothesis, we have
Λ, Λ' belong to the same family of n-planes on F $\Leftrightarrow$ $\tilde{\Lambda}, \tilde{\Lambda}'$ belong to the same family of