The explanation above is not enough. Since the checking the holomorphism is a local problem, and S is a Riemann surface, we have only to consider $f = \frac{h}{k}$ where h, k holomorphic in z.

$$\lambda_0 h - \lambda_1 k = 0$$

$$\mathbb{C}^3 \xrightarrow{F} \mathbb{C}^3$$

$$(z, \lambda_0, \lambda_1) \longmapsto (\lambda_0 h - \lambda_1 k, \lambda_0, \lambda_1).$$

If $\left.\frac{\partial(\lambda_0 h - \lambda_1 k)}{\partial z}\right|_{z=z_0} \neq 0$, by the inverse function theorem,

$$\exists \quad G(\lambda_0 h - \lambda_1 k, \lambda_0, \lambda_1) = (z, \lambda_0, \lambda_1).$$

On $\lambda_0 h = \lambda_1 k$, $G(0, \lambda_0, \lambda_1) = (z, \lambda_0, \lambda_1)$

$\Rightarrow$ $z = g(\lambda_0, \lambda_1)$, where g is holomorphic.

$\Rightarrow$ If g is bounded, then by the Riemann extension theorem, we can extend g to the set

$\frac{\partial(\lambda_0 h - \lambda_1 k)}{\partial z} = 0$. I guess that g must be bounded locally 95 % sure. (100 % 6.22)

Conversely, we will now show that if $D = \sum (p_\lambda - q_\lambda)$ is any divisor on S of degree 0 and $\mu(D) = 0$, then D is the divisor of the meromorphic function.

The problem, which may at first seem difficult, becomes straightforward once we transpose it from a question about the existence of a meromorphic function to one about the existence of a certain meromorphic form. Note that if f is a meromorphic function with $(f) = \sum (p_i - q_i)$, then the differential

$$\eta = \frac{1}{2\pi\sqrt{-1}} \, d \log f = \frac{1}{2\pi\sqrt{-1}} \frac{df}{f}$$