

$$v(z) = \sum a_{ij} z_i \frac{\partial}{\partial z_j} + [\partial],$$

then the Jacobian of  $f_t(z)$  at  $p$  is given by

$$B_p = e^{t \cdot A_p},$$

where  $A_p = (a_{ij})$ .

For simplicity,  $n=1$ .

$$\begin{aligned} v(z) &= v(x, y) = a z \frac{\partial}{\partial z} + [\partial] \\ &= a(x + \sqrt{-1}y) \frac{\partial}{\partial z} + [\partial] \\ &= a x \frac{\partial}{\partial x} + a y \frac{\partial}{\partial y} + [\partial] \quad \text{as the corresponding real vector field.} \end{aligned}$$

$\Rightarrow$  We claim  $f(t, x, y) = (f_1(t, x, y), f_2(t, x, y))$ , where

$$f_1(t, x, y) = x e^{at} + [\partial]$$

$$f_2(t, x, y) = y e^{at} + [\partial],$$

is a solution.

$$f_1(t, x, y) - x e^{at} \stackrel{?}{=} [\partial].$$

$$f_1(0, x, y) = x \quad \text{at } t=0. \text{ by the}$$

assumption  $f(t, x, y) = (x, y)$ .

$\Rightarrow$  By the well-known fact (see P107 Spivak 2. lemma),

$$f_1(t, x, y) = x e^{at} + t g_1(t, x, y).$$

If  $\frac{\partial g_1}{\partial x} \neq 0$  at  $x=0, y=0$ , then

we get the following