for all $\eta \in A^{p,q}(M)$. The mapping
$\psi = T(\varphi)$ from $\mathcal{H}_0^{p,q}(M)$ to $\mathcal{H}_1^{p,q}(M)$ is bounded, and therefore the mapping

$$T : \mathcal{H}_0^{p,q}(M) \longrightarrow \mathcal{H}_0^{p,q}(M) \text{ is compact and self-adjoint.}$$

pf) The Garding inequality says that the Dirichlet norm is equivalent to the Sobolev 1-norm on $\mathcal{H}_1^{p,q}(M)$. (We will discuss this later.)

The linear functional $\eta \longmapsto (\varphi, \eta)$ $(\eta \in A^{p,q}(M))$ extends to a bounded linear form on $\mathcal{H}_1^{p,q}(M)$ with the Dirichlet norm, by virtue of

$$|\langle \varphi, \eta \rangle| \leq \|\varphi\|_0 \|\eta\|_0 \lesssim \|\varphi\|_0 \mathcal{D}(\eta).$$ by the equivalence of Diri & $\| \|_1$

$\ulcorner$
$$\|\varphi\|_0 \|\eta\|_0 \leq \|\varphi\|_0 \|\eta\|_1 \leq C\|\varphi\|_0 \mathcal{D}(\eta)$$

The above inequalities prove that the linear functional is uniformly continuous on $A^{p,q}(M)$, which guarante that L can be extended to the completion of $A^{p,q}(M)$ w.r.t $\| \|_1 = \mathcal{D}$, i.e $\mathcal{H}_1^{p,q}(M)$. $\lrcorner$

Thus the equation
$$(\varphi, \eta) = \mathcal{D}(\psi, \eta)$$ has a unique solution $\psi = T(\varphi)$ characterized by

$$\langle \varphi, \eta \rangle = (T\varphi, (I+\Delta)\eta) \qquad \eta \in A^{p,q}(M).$$

$\ulcorner$ $L_1(\eta) = \langle \varphi, \eta \rangle$ is a continuous linear functional on $\mathcal{H}_1^{p,q}(M)$ and $\mathcal{D}$ is an inner product on $\mathcal{H}_1^{p,q}(M)$ which is equivalent to $\| \|_1$.
$\Rightarrow$ By P85. Th 4.12 Rudin R&C.A, $\exists\ \psi$ s.t