$$\tilde{f}(z) = \sum \tilde{f}_m(z)$$

and

$$\tilde{f}_m(z) = \sum_{|a|=m} c_a \cdot (z_i \cdot z(i)_1)^{a_1} \cdots z_i^{a_i} \cdots (z_i \cdot z(i)_n)^{a_n}$$

$$= z_i^m \cdot \sum c_a \cdot z(i)_1^{a_1} \cdots \hat{z}_i^{a_i} \cdots z(i)_n^{a_n}$$

Γ $\tilde{f}_m(z) = (\pi^* f_m)(z) = (\pi^* f_m)(z(i)_1, \cdots z(i)_i \cdots z(i)_n)$

$= f_m(\pi(z(i)_1, \cdots z(i)_n)) = f_m(z(i)_i\, z(i)_1, \cdots, z(i)_i, \cdots z(i)_i\, z(i)_n)$

$= f_m(z_i\, z(i)_1, \cdots, z_i, \cdots z_i z(i)_n)$

$= \sum_{|a|=m} C_a \cdot (z_i\, z(i)_1)^{a_1} \cdots z_i^{a_i} \cdots (z_i \cdot z(i)_n)^{a_n}$

$= z_i^m \sum_{|a|=m} C_a\; z(i)_1^{a_1} \cdots \hat{z}_i^{a_i} \cdots z(i)_n^{a_n}$ ⅃

Consequently, if f vanishes to order m_0 at p — i.e., if $f_0 = f_1 = \cdots = f_{m_0-1} = 0$ — then $\tilde{f}$ vanishes to order m_0 along E, and

$$\tilde{V} = \pi^* V - \mathrm{mult}_p(V) \cdot E$$
$$= \pi^* V - \mathrm{ord}_E(\pi^* V) \cdot E.$$

Γ $p \iff z = 0 \Rightarrow f(z) = 0$, and $f_0 = \cdots = f_{m_0-1} \approx$ $\Rightarrow$ for $|a| \le m_0 - 1$, $C_a = 0$. $\Rightarrow$ $\tilde{f}(z) = 0$ at $z \in \pi^{-1}(0) = E$.

and $|a| \le m_0 - 1$, $C_a = 0 \Rightarrow \tilde{f}$ vanishes to order m_0.

By the Proposition on P&I, $\overline{\pi^{-1}(V - \{p\})}$ is an algebraic variety, compare P396.

$(\tilde{f} = 0)$ contains $E = \pi^{-1}(p)$ with some multiplicity, which is the order to which $\tilde{f}$ vanishes at every point of E.

As we saw above, the vanishing order of f is the sa-