$\ulcorner$ $\sigma(p) \supset l_x, l_{x'} \Rightarrow x' \in \sigma(p) \subset G$
$\Rightarrow \sigma(p) \subset T_{x'}G = T_x F \Rightarrow \sigma(p)$ is tangent to F at $x \Rightarrow x \in \Sigma$.

$\lrcorner$

Conversely, if $\sigma(p) \subset T_x(F)$, then the quadric threefold $T_x(F) \cap G$ contains a 2-plane and so by our earlier argument must be singular; thus $T_x(F)$ must be tangent to G somewhere. Q.E.D.

$\ulcorner$ $T_x(F) \cap G \supset \sigma(p)$, since $\sigma(p) \subset T_x(F)$.
If $T_x(F) \cap G$ is a smooth quadric of dimension 3, then, by Proposition on P735, it can not contain a linear space of dimension $2 > 3/2$. $\Rightarrow$ This is not our case. $\Rightarrow$ $T_x(F) \cap G$ is singular. $\Rightarrow$ $T_x(F)$ must be tangent to G somewhere.

$\lrcorner$

This argument will become clearer if we refer back to our picture of the locus $T_x(G) \cap G$ as the cone over a quadric $Q = T_x(G) \cap G \cap H$, H a hyperplane disjoint from x. (See Figure 14.)

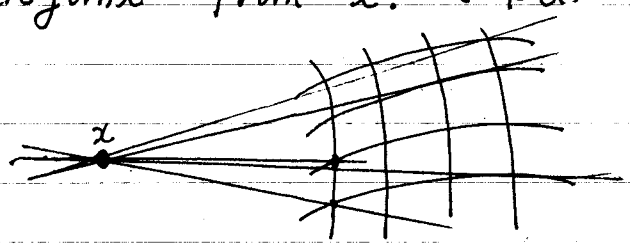

Figure 14. $T_x(G) \cap G$.