

$$\begin{aligned} \Lambda \cap \mathbb{P}_{ij}^{n-3} \cap \overline{Pq} &\neq \emptyset \Rightarrow \exists y \in \Lambda \cap \mathbb{P}_{ij}^{n-3} \cap \overline{Pq}. \\ \Rightarrow \overline{Py} &\ni q, \Rightarrow q \in \overline{\Lambda \cap \mathbb{P}_{ij}^{n-3}, P_i, P_j}. \\ \Rightarrow \Lambda &= \overline{\Lambda \cap \mathbb{P}_{ij}^{n-3}, P_i, P_j}. \end{aligned}$$

(\Rightarrow) If any $\Lambda \ni P_i, P_j$ meets $\overline{V_6}$ in a 2-plane,

Since $\Lambda \cap \overline{V_6} \supset \overline{P_i, P_j}$, $\exists q \in \overline{P_i, P_j}$ and $q \in \Lambda \cap \overline{V_6}$.

\Rightarrow Since $T_{P_i}(H) \cap T_{P_j}(H) \cap H$ is the cone through $\overline{P_i, P_j}$ over $H \cap \mathbb{P}_{ij}^{n-3}$, $\overline{P_i, q} \cap \mathbb{P}_{ij}^{n-3} \cap H \neq \emptyset$. ($\because q \in T_{P_i}(H) \cap T_{P_j}(H) \cap H$).

$\Rightarrow \exists x \in \mathbb{P}_{ij}^{n-3} \cap H$ s.t. $\overline{P_i, x} = \overline{P_i, q}$.

$$\begin{aligned} \overline{P_i, x} \subset \overline{V_6} &\Rightarrow \overline{P_i, x} \cap H_{ij} = \overline{P_i, x} \cap (\mathbb{P}_{ij}^{n-3} \cap H) \\ &\subset \overline{V_6} \cap H_{ij} = \{P_{ij1}, P_{ij2}\}. \end{aligned}$$

$\Rightarrow x$ must be P_{ij1} or $P_{ij2} \Rightarrow$ Since $\overline{P_i, x} \subset \Lambda$,

Λ contains P_{ij1} or P_{ij2} .

(\Leftarrow) If $\Lambda \ni P_{ij1}$ or P_{ij2} , $\Lambda \supset \overline{P_i, P_j, P_{ij1}} = \mathbb{P}^2$ since

P_i, P_j, P_{ij1} are linearly independent by the construction. $\overline{V_6} \supset \overline{P_i, P_j, P_{ij1}} = \mathbb{P}^2 \Rightarrow \Lambda \cap \overline{V_6} \supset \mathbb{P}^2$.

Suppose $\Lambda \cap \overline{V_6} \supset \mathbb{P}^3 \Rightarrow \overline{V_6} \cap H \supset \mathbb{P}^3$.

$\Rightarrow \overline{V_6} \cap H = H_4$ contains \mathbb{P}^3 , which is impossible by the result on P139. $3 > \frac{4-2}{2} = 2$. \Rightarrow

The process is now clear. Defining inductively a collection of points $P_i, P_{i1}, \dots, P_{i, \dots}, i_m$ by letting $P_i, \dots, i_{m-1,1}$ and $P_i, \dots, i_{m-1,2}$ be the two points of intersection of $\overline{V_{2m}}$ with a chosen $(n-2m+1)$ -plane in the intersection of the tan-