2. Next, let $E \to M$, $F \to M$ be two vector bundles with connections D, D' and curvature matrices Θ, Θ' respectively. Then the operator

$$D'' = D \oplus D' : \mathcal{a}^0(E \oplus F) \longrightarrow \mathcal{a}^1(E \oplus F)$$

is a connection for the bundle $E \oplus F$, with curvature matrix

$$\Theta'' = \begin{pmatrix} \Theta, & 0 \\ 0, & \Theta' \end{pmatrix}.$$

Then we have

$$\det(\Theta'' + tI) = \det(\Theta + tI) \cdot \det(\Theta' + tI)$$

as polynomials in t; i.e.,

$$c(E \oplus F) = c(E) \cdot c(F).$$

This is the Whitney product formula.

3. Similarly, if E is a vector bundle of rank r and L is a line bundle, we have seen that for appropriate connections on E, L, and $E \otimes L$,

$$\Theta_{E\otimes L} = \Theta_E \otimes 1 + 1 \otimes \Theta_L,$$

so that

$$c_1(E \otimes L) = \left[\operatorname{trace} \frac{\sqrt{-1}}{2\pi} \Theta_{E\otimes L} \right] = c_1(E) + r \cdot c_1(L).$$

See P74, lemma. $D_{E\otimes L} = D_E \otimes 1 + 1 \otimes D_L$

$\{e_i\}$ a frame for E. $\{\sigma\}$ a frame for L.

$D_{E\otimes L}(e_i \otimes \sigma) = D_E e_i \otimes \sigma + e_i \otimes D_L \sigma$

$= \theta_{ij} e_j \otimes \sigma + e_i \otimes \theta\sigma = \theta_{ji} e_i \otimes \sigma + \theta e_i \otimes \sigma = (\theta_{ji} + \theta) e_i \otimes \sigma.$

$D_{E\otimes L}(\theta_{ji} + \theta) \otimes (e_i \otimes \sigma) = D_E \otimes 1 \, (\theta_{ji} + \theta) \otimes (e_i \otimes \sigma)$ wrong
$+ 1 \otimes D_L (\theta_{ji} + \theta) \otimes (e_i \otimes \sigma) = d(\theta_{ji} + \theta) \otimes e_i \otimes \sigma -$