

$$\int_{\mathbb{C}^n \times \mathbb{C}^n} \bar{\partial} \phi \wedge k(z, w) \wedge \varphi(w)$$

$$\begin{aligned} K_{\varphi} \circ \bar{\partial}(\phi) &= K_{\varphi}(\bar{\partial} \phi) = \int_{\mathbb{C}^n} K(\varphi)(z) \wedge \bar{\partial} \phi(z) \\ &= \int_{\mathbb{C}^n} \int_{\mathbb{C}^n} \overset{2n-1}{\underset{\uparrow}{k(z, w)}} \wedge \overset{q}{\underset{\uparrow}{\bar{\partial} \varphi(w)}} \wedge \overset{2n-q+1}{\underset{\uparrow}{\bar{\partial} \phi}} \\ &= (-1)^{q-1} \int_{\mathbb{C}^n \times \mathbb{C}^n} \bar{\partial} \phi \wedge k(z, w) \wedge \varphi(w). \end{aligned}$$

$$\begin{aligned} K_{\bar{\partial} \varphi}(\phi) &= \int_{\mathbb{C}^n} K(\bar{\partial} \varphi)(z) \wedge \phi(z) \\ &= \int_{\mathbb{C}^n} \left(\int_{\mathbb{C}^n} \overset{2n-1}{\underset{\uparrow}{k(z, w)}} \wedge \overset{q+1}{\underset{\uparrow}{\bar{\partial} \varphi(w)}} \right) \wedge \overset{2n-q}{\underset{\uparrow}{\phi(z)}} \\ &= \int_{\mathbb{C}^n \times \mathbb{C}^n} \overset{2n-1}{\underset{\uparrow}{k(z, w)}} \wedge \overset{q+1}{\underset{\uparrow}{\bar{\partial} \varphi(w)}} \wedge \overset{2n-q}{\underset{\uparrow}{\phi(z)}} = (-1)^q \int_{\mathbb{C}^n \times \mathbb{C}^n} \phi(z) \wedge k(z, w) \wedge \bar{\partial} \varphi(w) \end{aligned}$$

\Rightarrow The first term is $(-1)^{q-1} K_{\varphi} \circ \bar{\partial}(\phi)$ and the second term is $(-1)^q K_{\bar{\partial} \varphi}(\phi)$.

$$\Rightarrow (-1)^{q-1} K_{\varphi} \circ \bar{\partial}(\phi) + (-1)^{q+1} (-1)^q K_{\bar{\partial} \varphi}(\phi) = 0$$

$$\Rightarrow K_{\bar{\partial} \varphi}(\phi) + (-1)^q K_{\varphi} \circ \bar{\partial}(\phi) = 0$$

$$\Rightarrow K_{\bar{\partial} \varphi} + (-1)^q K_{\varphi} \circ \bar{\partial} = 0$$

$$\Rightarrow \text{By } \S 3.6, \quad \bar{\partial} K_{\varphi} = K_{\bar{\partial} \varphi} \Rightarrow \bar{\partial} K_{\varphi} + (-1)^q K_{\varphi} \circ \bar{\partial} = 0. \quad \square$$