

so that we have  $n.c. = \sum_{q \in S} (v(q)-1)$  as

the number of vertices.

$\Rightarrow$  By the definition  $g(S_0) = \frac{-\chi(S)+2}{2}$   
(or relation)

$$2 - 2g(S) = n(2 - 2g(S')) - \sum_{q \in S} (v(q)-1)$$

$$2g(S) - 2 = 2n(g(S') - 1) + \sum_{q \in S} (v(q)-1)$$

$$\Rightarrow g(S) - 1 = n \cdot (g(S') - 1) + \frac{1}{2} \sum_{q \in S} (v(q)-1)$$

$$\Rightarrow g(S) = n \cdot (g(S') - 1) + 1 + \frac{1}{2} \sum_{q \in S} (v(q)-1). \quad \square$$

We can also relate the canonical bundle of  $S$  to that of  $S'$ . Let  $\omega$  be a global meromorphic 1-form on  $S'$ , written locally as

$$\omega = \frac{g(w)}{h(w)} dw.$$

For any point  $p \in S$  of ramification index  $v$  we can find a coordinate  $z$  on  $S$  centered around  $p$ , with  $f$  given by

$$w = z^v.$$

Then

$$f^*\omega = \frac{g(z^v)}{h(z^v)} dz^v$$

$$= v \cdot z^{v-1} \frac{g(z^v)}{h(z^v)} \cdot dz,$$

$$\text{so } \text{ord}_p(f^*\omega) = v \cdot \text{ord}_{f(p)}(\omega) + (v-1).$$