

Let $W_1 = f^{-1}(V_2)$ open in \mathbb{C}^n . Let W_2 open in \mathbb{C}^n s.t.
 $W_2 \cap \mathbb{C}^k = V_1 \Rightarrow$ Let $W = W_1 \cap W_2$.
 $\Rightarrow W \cap \mathbb{C}^k = V_1$ and $f(W) = f(V_1) = V_2 = f(W \cap \mathbb{C}^k)$.

(ii) $N > K$.

Assume that $|(\frac{\partial f_i}{\partial z_j})| \neq 0$ for $1 \leq i, j \leq k$, where

$$\mathbb{C}^n \xrightarrow{f} \mathbb{C}^N \xrightarrow{\pi} \mathbb{C}^k$$

$$(z_1, \dots, z_n) \mapsto (f_1, \dots, f_N) \mapsto (f_1, f_2, \dots, f_k)$$

\Rightarrow Applying the argument above to $\pi \circ f$, we have
 an open set W in \mathbb{C}^n s.t. $\pi \circ f(W) = \pi \circ f(W \cap \mathbb{C}^k)$.
 $\pi \circ f|_{\mathbb{C}^k}$ is biholomorphic locally.

We have only to show that $\pi : f(W) \rightarrow \pi \circ f(W)$
 is one to one, since this implies $\pi \circ f(W \cap \mathbb{C}^k) = \pi \circ f(W)$
 and $f(W \cap \mathbb{C}^k) = f(W)$. Let's do in a different way.

For example,

$$\mathbb{C}^3 \xrightarrow{f} \mathbb{C}^4$$

has the Jacobian with

$$(z_1, z_2, z_3) \mapsto (f_1, f_2, f_3, f_4)$$

rank 2 at the origin.

\Rightarrow Consider the following map

$$F : \mathbb{C}^3 \times \mathbb{C}^2 \rightarrow \mathbb{C}^4$$

$$(z_1, z_2, z_3) \times (w_1, w_2) \mapsto (f_1, f_2, f_3 + w_1, f_4 + w_2)$$

Of course, as we assume before that $(\frac{\partial f_i}{\partial z_j})$ is
 non singular $1 \leq i, j \leq 2$.