Given a regular sequence, the Koszul complex will give a particularly nice projective resolution of the $\mathcal{O}$-module I. It is modeled on the well-known fact from linear algebra that, for an n-dimensional vector space V and nonzero vector $v^* \in V^*$, the contraction operator

$$i(v^*) : \Lambda^k V \longrightarrow \Lambda^{k-1} V$$

induces an exact sequence of vector spaces

$$0 \longrightarrow \Lambda^n V \longrightarrow \Lambda^{n-1} V \longrightarrow \cdots \longrightarrow \Lambda^2 V \longrightarrow V \longrightarrow \mathbb{C} \longrightarrow 0$$

$(\mathbb{C} = \Lambda^0 V)$.

Pf $\quad {}_nC_n - {}_nC_{n-1} + {}_nC_{n-2} - \cdots \pm {}_nC_0 = (1+x)^n|_{x=-1} = 0.$

$$\begin{array}{ccccc} \Lambda^k V & \xrightarrow{i(v^*)} & \Lambda^{k-1} V & \xrightarrow{i(v^*)} & \Lambda^{k-2} V \\ \Lambda & \longmapsto & i(v^*)\Lambda & \longmapsto & i(v^*)(i(v^*)\Lambda) \end{array}$$

$\langle i(v^*)(i(v^*)\Lambda), \xi \rangle = \langle i(v^*)\Lambda, v^* \wedge \xi \rangle = \langle \Lambda, v^* \wedge v^* \wedge \xi \rangle = 0$ for all $\xi \in \Lambda^{k-2} V^* = (\Lambda^{k-2} V)^*$ by P.210.

$\Rightarrow \ker i(v^*) \supset \operatorname{im} i(v^*)$.

Given $P \in \Lambda^{k-1} V$ s.t $i(v^*) P = 0$, then $\langle i(v^*) P, \xi \rangle = 0$ for all $\xi \in (\Lambda^{k-2} V)^* = \Lambda^{k-2} V^*$.

We have to find $\eta \in \Lambda^k V$ s.t $i(v^*)\eta = P$.

By linearity of i i.e., $i(\alpha v^*) = \alpha\, i(v^*)$, $i(v_1^* + v_2^*) = i(v_1^*) + i(v_2^*)$, we may assume that v is a unit vector.

$\Rightarrow$ We may assume that $v = e_1$, $V = \langle e_1, \cdots e_n \rangle$, $e_1 \cdots e_n$ orthonormal basis.