

conic

containing 10 points of P_0 .

$$\textcircled{2} \quad \#(C_0 \cap P_0) = 4$$

$$\Rightarrow \# P_0' = 8.$$

Suppose $|f_{P_0'}(3)|$ has a fixed conic. $\Rightarrow \dim |f_{P_0'}(3)| = \dim |f_{P_0''}(1)| = \dim |f_{P_0}(4)| \geq 3$ Impossible, since $\dim H^0(P^2, \mathcal{O}(4)) = 3$

Suppose $|f_{P_0'}(3)|$ has a fixed line l_1 which contains 5 points of P_0' . Suppose $\#(l_1 \cap P_0') = 5$. Let $P_0'' = P_0' - l_1 \cap P_0'$.

$$\Rightarrow$$

$$3 \leq \dim |f_{P_0}(4)| = \dim |f_{P_0'}(3)| = \dim |f_{P_0''}(2)|$$

(i) Case 2,

$$\dim |f_{P_0''}(2)| = 5 - d = \#P_0'' = 2 \Rightarrow \text{Contradiction}$$

(ii) Case 1

$|f_{P_0''}(2)|$ has a fixed line.

$$\Rightarrow \dim |f_{P_0''}(2)| = \dim |f_{P_0'''}(1)| \leq 2$$

Since $\dim H^0(P^2, \mathcal{O}(H)) = 3$, and $\dim |f_{P_0'''}(1)| \leq \dim H^0(P^2, \mathcal{O}(H)) - 1$, \Rightarrow contradiction. $\Rightarrow \#(l_1 \cap P_0') \geq 6$
 \Rightarrow Again $l_1 + l_0$ is a conic containing 10 points.

$$\textcircled{3} \quad \#(C_0 \cap P_0) \leq 3$$

$\Rightarrow \# P_0' = 9, 10, 11$. Let Λ_0 be a subset of P_0' s.t. $\# \Lambda_0 = 8$. $\Rightarrow \Lambda_0$ can not impose independent conditions

on $|O_{P^2}(3)|$, for, if so, then by the application on P^2 .

$$(i) \text{ Case 2 } \Rightarrow \dim |f_{\Lambda_0}(3)| = \frac{3(3+3)}{2} - 8 + 5 \Rightarrow$$