If the D_i meet transversely at $d_1 \cdots d_n$ distinct points,

$$\operatorname{Res}_{P_\nu}\left\{\frac{g(x)\,dx_1\wedge\cdots\wedge dx_n}{f_1\cdots f_n(x)}\right\} = \left(\frac{1}{2\pi\sqrt{-1}}\right)^n \int_{\Gamma} \frac{g(x)\,dx_1\wedge\cdots\wedge dx_n}{f_1\cdots f_n(x)}$$

$$= \frac{1}{(2\pi\sqrt{-1})^n}\int_{|f_i|=\epsilon} g(x)\,\frac{df_1}{f_1}\wedge\cdots\wedge\frac{df_n}{f_n}\,\frac{1}{J_f(x)}$$

$$= \frac{g(P_\nu)}{J_f(P_\nu)}$$ since $\{f_1 \cdots f_n\}$ forms a new coordinates around P_ν.

On the global Bezout theorem

$f = (f_1, \cdots f_n) : U \longrightarrow \mathbb{C}^n - \{0\}$, where $\deg f_i = d_i$, f_i polynomial.

Since $\cap D_i \subset U$ may be assumed,

$$\deg(D_1 \cdots D_n) = \sum_\nu m_\nu = \sum (D_1 \cdots D_n)_{P_\nu}$$

$$= (D_1 \cdots D_n)_U = \int_{\partial U} f^*\beta$$

According to P172, $\deg(D_1 \cap D_2) = \deg D_1 \cdot \deg D_2 = d_1 d_2$

$\cdots \Rightarrow \deg(D_1 \cap \cdots \cap D_n) = d_1 \cdots d_n = \sum m_\nu$

On $\dim_{\mathbb{C}}(\mathcal{O}/I) = d = (D_1, \cdots D_n)_{\{0\}} = \deg(f)$.

We know that $\mathcal{O}/I$ is a finite dimensional complex vector space over $\mathbb{C}$. Consider the following accending vector spaces.