$p+q=n-1$. Q.E.D.

Γ From $0 \to \Omega_M^p(-V) \to \Omega_M^p \to \Omega_M^p|_V \to 0$,

$$H^q(M, \Omega_M^p(-V)) \to H^q(M, \Omega_M^p) \to H^q(M, \Omega_M^p|_V) \to H^{q+1}(M, \Omega_M^p(-V)) \to H^{q+1}(M, \Omega_M^p) \to$$

$H^q(M, \Omega_M^p(-V)) = 0$ ($p+q<n$), $H^{q+1}(M, \Omega_M^p(-V)) = 0$ ($p+q+1<n$)

$$\Rightarrow \quad H^q(M, \Omega_M^p) \cong H^q(M, \Omega_M^p|_V) \quad \text{if } p+q+1<n$$
$$H^q(M, \Omega_M^p) \longrightarrow H^q(M, \Omega_M^p|_V) \quad \text{if } p+q=n-1$$
injective

From $0 \to \Omega_V^{p-1}(-V) \to \Omega_M^p|_V \to \Omega_V^p \to 0$,

$$H^q(V, \Omega_V^{p-1}(-V)) \to H^q(V, \Omega_M^p|_V) \to H^q(V, \Omega_V^p) \to H^{q+1}(V, \Omega_V^{p-1}(-V)) \to$$

$H^q(V, \Omega_V^{p-1}(-V)) = 0$ if $p+q-1<n-1$, $H^{q+1}(V, \Omega_V^{p-1}(-V)) = 0$ if $p-1+q+1<n-1 \Leftrightarrow p+q<n-1$

if $p+q<n-1$ $\quad H^q(V, \Omega_M^p|_V) \cong H^q(V, \Omega_V^p)$
if $p+q=n-1$. $\quad H^q(V, \Omega_M^p|_V) \longrightarrow H^q(V, \Omega_V^p)$ is injective.

Thus, if $p+q \le n-2$,

$$H^q(M, \Omega_M^p) \cong \underbrace{H^q(M, \Omega_M^p|_V)}_{\overset{?}{=}\, H^q(V, \Omega_M^p|_V)} \cong H^q(V, \Omega_V^p)$$

if $p+q=n-1$ $\quad H^q(M, \Omega_M^p) \longrightarrow H^q(M, \Omega_M^p|_V)$ is injective
$\quad H^q(V, \Omega_M^p|_V) \longrightarrow H^q(V, \Omega_V^p)$ is injective.

$$\Rightarrow \quad H^q(M, \Omega_M^p) \longrightarrow H^q(V, \Omega_V^p) \text{ is injective.}$$

It remains to convince myself that $H^*(M, \Omega_M^p|_V) \cong H^*(V, \Omega_M^p|_V)$.