on the irreducible embedded Riemann surface V.

⸢ By P21, Proposition, an analytic variety V is irreducible $\Leftrightarrow$ V^* is connected.
Since V is smooth, $V^* = V$. Since $H_0(V, \mathbb{Z}) = \mathbb{Z}$, V is connected. $\Rightarrow$ By the above proposition, V is irreducible. ⸥

We may also apply it to hypersurfaces of projective space: since any effective nonzero divisor on $\mathbb{P}^n$ is positive, the theorem tells us that if V is any smooth hypersurface in $\mathbb{P}^n$, then $H^{2k-1}(V) = 0$ for $k \neq n/2$, while $H^{2k}(V)$ is generated by the class of a k-plane section of V for $k < n/2$.

⸢ For homology,

$$H_q(V, \mathbb{Z}) \longrightarrow H_q(\mathbb{P}^n, \mathbb{Z}) \qquad \text{① if } q \le n-2 \quad \text{isomo} \qquad \text{② if } q = n-1, \quad \text{onto.}$$

For cohomology

$$H^q(\mathbb{P}^n, \mathbb{Q}) \longrightarrow H^q(V, \mathbb{Q}) \qquad \text{① if } q \le n-2, \quad \text{isomo} \qquad \text{② if } q = n-1, \quad \text{one to one}$$

$\Rightarrow$ By Poincare duality,

$$H^{2n-q}(\mathbb{P}^n, \mathbb{Z}) \cong H^{2n-2-q}(V, \mathbb{Z}) \quad \text{if } q \le n-2$$

$$\Rightarrow \quad H^{\ell}(\mathbb{P}^n, \mathbb{Z}) \cong H^{\ell-2}(V, \mathbb{Z}) \quad \text{if } n \le \ell-2 \quad \cdots \text{(i)}$$

and

$$H^{\ell}(\mathbb{P}^n, \mathbb{Q}) \cong H^{\ell}(V, \mathbb{Q}) \quad \text{if } \ell \le n-2. \quad \text{(ii).}$$

By (i), $H^{2k-1}(V, \mathbb{Z}) \cong H^{2k+1}(\mathbb{P}^n, \mathbb{Z}) = 0$ if $2k+1-2 \ge n$
$\Leftrightarrow$ $2k-1 \ge n$