Conversely, of course, any pencil of lines in X containing l_x has its focus on l_x, and hence in $l_x \cap S$.

Let L be a pencil in X containing l_x.
$\Rightarrow$ $L = \sigma(p, h)$, $p \in l_x \subset h$.
$\Rightarrow$ $L \subset X \cap \sigma(p)$ $\Rightarrow$ $X_p = X \cap \sigma(p)$ is singular
$\Rightarrow$ $p \in S$ $\Rightarrow$ $l_x \cap S \ni p$

Thus, if we make the assumption that the generic line l_x does not lie on two confocal pencils of X, the points of intersection of l_x with S correspond exactly to the lines of L on X through x.

$$
\begin{array}{ccc}
l_x \cap S & \xrightarrow{\quad\quad} & \text{Set of lines in } X \text{ passing through } x \\
\cup & \varphi_1 & \cup \\
p & \longmapsto & L(p). \\
 & \varphi_2 & \\
q & \longleftarrow\!\shortmid & K
\end{array}
\qquad
\begin{array}{l}
F \cap \sigma(p) = L(p) \cup l_2 \\
L(p) \ni x.
\end{array}
$$

Given a line K in X passing through x, by the argument above, the focus q of $K \in l_x \cap S$.
$\Rightarrow$ $\varphi_1 \circ \varphi_2$ & $\varphi_2 \circ \varphi_1$ are identities. $\Rightarrow$
$l_x \cap S \xleftrightarrow{1-1}$ Set of lines in X passing through x.