Clearly,

$$d\,\Omega^p(\log D) \subset \Omega^{p+1}(\log D),$$

and the resulting complex $(\Omega^*(\log D), d)$ is called the log complex.

⌈ $d\left(\frac{dz_i}{z_i}\right) = -\frac{1}{z_i^2}\, dz_i \wedge dz_i = 0.$ and $d($ holomorphic forms $)$ = holomorphic forms $\Rightarrow d\,\Omega^p(\log D) \subset \Omega^{p+1}(\log D).$ ⌋

An intrinsic characterization is given by the following

Lemma. If f is a local defining equation for D, then $\Omega^p(\log D)$ is given by those meromorphic forms φ s.t both

$$f\varphi \text{ and } f d\varphi$$

are holomorphic.

Proof. Obviously we may take $f = z_1 \cdots z_k$, and then the necessary condition is clear.

⌈ $\varphi = \sum f_I \frac{dz_I}{z_I}$.

$\Rightarrow f \frac{dz_I}{z_I} = z_1 \cdots z_k \frac{dz_I}{z_I}$ is holomorphic

$\Rightarrow f\varphi$ is holomorphic. Similarly, $f d\varphi$ is holomorphic. ⌋

Suppose, conversely, that $f\varphi$ and $fd\varphi$ are holomorphic. Using the notations