$\operatorname{sgn} \det(\Delta) = 1$ ($\because C \det(A)^2 = \det(\Delta)$) $\Rightarrow$ See p411

$\Rightarrow$ Always, for a holomorphic vector field v,
$\iota_v(p) = 1$.

$\Rightarrow$ $RHS = \sum_{v(p_j)=0} \iota_v(p_j) = \#$ of zeros of v.

$\Rightarrow$ By Hopf index theorem, $C_n = 1$. $\Rightarrow$
We completes the proof of Bott Residue Formula. $\lrcorner$

The General Hirzebruch – Riemann – Roch Formula

Consider now how we arrived at the identity

$$C_n(M) = \chi(M)$$

for a compact complex manifold M of dimension n: On the one hand, the general Gauss–Bonnet formula tells us that we can realize $C_n(M)$ as the number of zeros, properly counted, of a generic C^∞ vector field on M; on the other hand, the Lefschetz fixed point formula tells us that the Euler characteristic of M is equal to the number of fixed points, properly counted, of the map $\varphi_v : M \longrightarrow M$ obtained by integrating v — that is, again the number of zeros of v. Now we have obtained refinements of both the Gauss–Bonnet and the Lefschetz fixed-point formulas in the holomorphic case, and we may try to apply them in the same way to arrive at a formula for the holomorphic Euler characteristic of a complex manifold.