In general, we define the degree of a line bundle on M by

$$\deg(L) = \langle c_1(L), [M] \rangle,$$ or in other words

$\deg L = c_1(L)$ under the isomorphism $H^2(M, \mathbb{Z}) = \mathbb{Z}$. given by the natural orientation on M.

Note that by the relation proved on P?? between the curvature form Θ of a metric connection on the tangent bundle of a Riemann surface and the ordinary Gaussian curvature K_M the classical Gauss-Bonnet theorem gives

$$\deg T'(M) = \frac{1}{4\pi} \int_M K_M \cdot \Phi = \chi(M).$$

$\ulcorner$ $i\,\Theta = K \cdot \Phi$.

$$\deg T'(M) = \frac{i}{2\pi} \int_M \Theta = \frac{1}{2\pi} \int_M K \cdot \Phi = \chi(M).$$

$$\Phi = \frac{i}{2} h^2 \, dz \wedge d\bar{z}$$ $\lrcorner$

2. By the exact cohomology sequence

$$H^1(\mathbb{P}^n, \mathcal{O}) \longrightarrow H^1(\mathbb{P}^n, \mathcal{O}^*) \xrightarrow{c_1} H^2(\mathbb{P}^n, \mathbb{Z})$$

arising from the exponential sheaf sequence on $\mathbb{P}^n$ and by the vanishing of $H^1(\mathbb{P}^n, \mathcal{O})$ (Section 3 of Chapter 0), (See also P118. P49), we see that every line bundle on $\mathbb{P}^n$ is determined by its Chern class, i.e

$$\operatorname{Pic}(\mathbb{P}^n) \cong H^2(\mathbb{P}^n, \mathbb{Z}) \cong \mathbb{Z}.$$

In other words, every divisor on $\mathbb{P}^n$ is linearly equivalent to a multiple of the hyperplane divisor $H = \mathbb{P}^{n-1} \subset \mathbb{P}^n$.