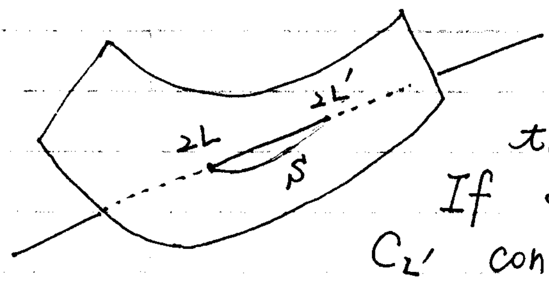
In the limiting case, then, we see that any pencil tangent to W_2 at $2L$ consists entirely of singular conics; the tangent plane $T_{2L}(W_2)$ to W_2 at a point $2L$ is therefore contained in — hence equal to — the 2-plane

$$\{L+L'\}_{L'\in \mathbb{P}^{2*}} \subset W.$$



Let $C_{L'} = \langle 2L, 2L' \rangle$ be the pencil containing $2L$ & $2L'$. If $2L' \longrightarrow 2L$ along S, $C_{L'}$ consists of entirely singular conics. $\Rightarrow$ In the limiting case, the tangent line of S is a pencil consisting of entirely singular conics, since, if not so, the limiting one is a pencil containing a smooth conic, but this is impossible (since smooth conics form an open set).

Let $\alpha(t)$ be a curve in W_2 passing $2L$ $(= X_0^2)$.

$\Rightarrow \alpha(t) = (a(t)X_0 + b(t)X_1 + c(t)X_2)^2, \; a(0)=1, \; b(0)=c(0)=0$

$\Rightarrow \alpha'(t) = 2(a(t)X_0 + b(t)X_1 + c(t)X_2)(a'(t)X_0 + b'(t)X_1 + c'(t)X_2) \Rightarrow \alpha'(0) = 2X_0(a'(0)X_0 + b'(0)X_1 + c'(0)X_2)$

$\Rightarrow X_0^2 + \lambda\alpha'(0) = X_0(X_0 + \lambda(aX_0 + bX_1 + cX_2))$

Thus $X_0(X_0 + \lambda L_0)$, L_0 is a line, is a typical tangent line at $X_0^2 = 2L$.

$\Rightarrow T_{2L}(W_2) = \{X_0(X_0 + \lambda L_0)\}_{L_0 \in \mathbb{P}^{2*}}$ $\Rightarrow$ But since