$A = (\alpha_{ij})$. s.t $\alpha_{0j} = 1$ for all j ⋯ ①

② ⋯ all $(\ell+1)\times(\ell+1)$ minor determinants are nonzero, i.e., $\ell \geq 1$

$$A = \begin{pmatrix} 1, & 1, & 1 & \cdots & 1 \\ \alpha_{10} & \alpha_{11}, & \alpha_{12}, & \cdots & \alpha_{1n} \\ & , & \cdots & \cdots & \\ \vdots & & & & \\ & & , \cdots & & \\ \alpha_{n0} & \alpha_{n1}, & \cdots & & \alpha_{nn} \end{pmatrix}$$

$\Rightarrow$ For $n = 2$

v_1 & v_2 are linearly independent if $X_0 \neq 0$.

If $X_0 = 0$, v_1 & v_2 are linearly dependent.

$$\lambda_1 \pi_*(\alpha_{1j} X_j \frac{\partial}{\partial x_j}) + \lambda_2 \pi_*(\alpha_{2j} X_j \frac{\partial}{\partial x_j}) = 0$$

$$= \pi_*((\lambda_1 \alpha_{1j} X_j + \lambda_2 \alpha_{2j} X_j)\frac{\partial}{\partial x_j})$$

$$\Rightarrow (\lambda_1 \alpha_{1j} + \lambda_2 \alpha_{2j}) X_j \frac{\partial}{\partial x_j} = k X_j \frac{\partial}{\partial x_j}$$

$$\Rightarrow [(\lambda_1 \alpha_{10} + \lambda_2 \alpha_{20}) X_0, (\lambda_1 \alpha_{11} + \lambda_2 \alpha_{21}) X_1, \cdots, (\lambda_1 \alpha_{1n} + \lambda_2 \alpha_{2n}) X_n]$$
$$= [X_0, X_1, \cdots X_n]$$

$\Rightarrow$ Since $X_0 = 0$ & $n = 2$,

$$⊛ \cdots \begin{cases} \lambda_1 \alpha_{11} + \lambda_2 \alpha_{21} = k \\ \lambda_1 \alpha_{12} + \lambda_2 \alpha_{22} = k \end{cases}$$ has non trivial solution.

$$\Rightarrow [0, (\lambda_1 \alpha_{11} + \lambda_2 \alpha_{21}) X_1, (\lambda_1 \alpha_{12} + \lambda_2 \alpha_{22}) X_2]$$
$$= [0, X_1, X_2]$$ for all X_1, X_2. if