In particular if we take $\omega' = \omega$, then since $\omega \wedge \overline{\omega}$ is positive we find that

$$(**) \qquad 0 < \sqrt{-1} \int_S \omega \wedge \overline{\omega} = \sqrt{-1} \sum_{i=1}^{g} \left(\Pi^i \, \overline{\Pi^{g+i}} - \Pi^{g+i} \, \overline{\Pi^i} \right)$$

for $\omega \not\equiv 0$.

$\ulcorner$ $\omega \wedge \overline{\omega}$ positive $\Rightarrow$ $\omega \wedge \overline{\omega} = f \sigma$. σ volume form f positive function. $\lrcorner$

It follows from this that any holomorphic 1-form ω whose A-periods all vanish must be identically zero, i.e., the first $g \times g$ minor of the period matrix Ω is nonsingular.

$\ulcorner$ If ω is such a holomorphic 1-form, then $\Pi^i = 0$.
$\Rightarrow$ In $(**)$, RHS $= 0$ $\Rightarrow$ $\omega = 0$, since $\int_S \omega \wedge \overline{\omega} = 0$. $\Rightarrow$ If $\omega \not\equiv 0$, at least one of its A period does not vanish. If the first $g \times g$ minor of the period matrix Ω is singular, $\exists$ $a_1, a_2 \cdots a_g$ s.t

$$a_1 \begin{pmatrix} \int_{\delta_1} \omega_1 \\ \vdots \\ \int_{\delta_1} \omega_g \end{pmatrix} + a_2 \begin{pmatrix} \int_{\delta_2} \omega_1 \\ \vdots \\ \int_{\delta_2} \omega_g \end{pmatrix} + \cdots + a_g \begin{pmatrix} \int_{\delta_g} \omega_1 \\ \vdots \\ \int_{\delta_g} \omega_g \end{pmatrix} = 0 .$$

$$\Rightarrow \int_{a_1\delta_1 + \cdots + a_g\delta_g} \omega_1 = 0 \quad \cdots \quad = \int_{a_1\delta_1 + \cdots + a_g\delta_g} \omega_g$$

or $a_1' \left(\int_{\delta_1} \omega_1 , \cdots \int_{\delta_g} \omega_1 \right) + a_2' \left(\int_{\delta_1} \omega_2 , \cdots \int_{\delta_g} \omega_2 \right) + \cdots + a_g' \left(\int_{\delta_1} \omega_g , \cdots \int_{\delta_g} \omega_g \right) = 0$ $\Rightarrow$ $\int_{\delta_i} a_1' \omega_1 + a_2' \omega_2 + \cdots + a_g' \omega_g = 0$ for all i.