is described in the

Proposition. A smooth quadric F of dimension m contains no linear spaces of dimension strictly greater than $m/2$; on the other hand

1. If $m = 2n+1$ is odd then F contains an irreducible $(n+1)(n+2)/2$-dimensional family of n-planes; while
2. If $m = 2n$ is even, then F contains two irreducible, $n(n+1)/2$-dimensional families of n-planes and moreover for any two n-planes $\Lambda, \Lambda' \subset F$,

$$\dim(\Lambda \cap \Lambda') \equiv n \ (2)$$

if and only if Λ and Λ' belong to the same family.

Before we prove this, note that we have already observed this phenomenon in the case $m=2$: on a quadric surface in $\mathbb{P}^3$ there are two one-dimensional families of lines; and two lines of opposite families always meet in a point, while lines of the same family are either disjoint or meet in a line.

$\ulcorner$ See P478 ~ P480. $\lrcorner$

The reader is also referred to the discussion in