$$\begin{cases} I = \{f_1, \cdots f_k\} & \text{an ideal in } \mathcal{O}, \\ M = \mathcal{O}/\{f_1 \cdots f_k\}. \end{cases}$$

Very roughly speaking, the second of these is the local ring at the origin of the variety $f_1(z) = \cdots = f_k(z) = 0$. We shall say more about this later. Given an ideal $\{f_1 \cdots f_k\}$, where the $f_i \in \mathcal{O}_{n-1}[z_n]$ are Weierstrass polynomials, it is natural to consider $I' = I \cap \mathcal{O}_{n-1}[z_n]$ as a quotient-module of $\mathcal{O}_{n-1}^{(k)}$ over $\mathcal{O}_{n-1}$.

pf Consider the following homomorphism.

$$\mathcal{O}_{n-1}^{(k)} \xrightarrow{\phi} I' = I \cap \mathcal{O}_{n-1}[z_n] \longrightarrow 0$$

$$(a_1, \cdots a_k) \longmapsto a_1 f_1 + \cdots + a_k f_k, \quad a_i \in \mathcal{O}_{n-1}.$$

$$\ker\phi = \{(a_1 \cdots a_k) \in \mathcal{O}_{n-1}^{(k)} \mid a_1 f_1 + \cdots + a_k f_k = 0\}$$

$$\Rightarrow \quad I' = \mathcal{O}_{n-1}^{(k)} / \ker\phi$$

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Consequently, even though ideals in $\mathcal{O}_n$ may be our primary interest, more general modules arise naturally in inductive arguments.

$\mathcal{O}$-modules admit the operations of linear algebra, such as

$$M \oplus N, \quad M \otimes_{\mathcal{O}} N, \quad \mathrm{Hom}_{\mathcal{O}}(M, N).$$

Given an exact sequence of $\mathcal{O}$-modules

$$0 \to P \to Q \longrightarrow R \longrightarrow 0,$$

the resulting sequences

$$\Big\{ \; P \otimes_{\mathcal{O}} M \to \quad Q \otimes_{\mathcal{O}} M \longrightarrow R \otimes_{\mathcal{O}} M \longrightarrow 0$$