

and $\dim \mathcal{O}_{\mathbb{A}^2, P} = \dim (\mathcal{O}_P / \mathcal{I}_P) = 4$ see p669

More explicitly, \mathcal{O}

I,

$$I = \{ xy, (x-y)(x-ry) \}$$

$$\Rightarrow \int \frac{dx dy}{xy} \wedge \frac{d(x-y)(x-ry)}{(x-y)(x-ry)} = 4 \text{ (?) } \Rightarrow \text{We will show this below.}$$

Clearly if we consider

$$\mathbb{C}^2 \xrightarrow{f} \mathbb{C}^2$$

$$(x, y) \mapsto (xy, (x-y)(x-ry))$$

$$\Rightarrow xy = a \quad (x-y)(x-ry) = b$$

$$x^2 - (1+r)xy + ry^2 = b$$

$$\Rightarrow x^2 - a(1+r) - b + r \frac{a^2}{x^2} = 0$$

$$x^4 - (a+ar+b)x^2 + ra^2 = 0$$

For general a & b , there are 4 distinct roots. $\Rightarrow \deg f = 4 = \dim (\mathcal{O}_P / \mathcal{I}_P)$ by 670.