

Thus we proved the correspondence. \square

From yet another viewpoint, we may consider a rational map $f: M \rightarrow \mathbb{P}^n$ as a subvariety of $M \times \mathbb{P}^n$. Explicitly, we define the graph $\Gamma_f \subset M \times \mathbb{P}^n$ of f to be the closure in $M \times \mathbb{P}^n$ of the graph

$$\{(p, X) : f(p) = X\}$$

of f where defined. Note that this is an analytic subvariety: if f is given locally by

$$f: p \mapsto [g_0(p), \dots, g_n(p)],$$

where g_0, \dots, g_n are holomorphic functions with no common factor, then Γ_f will be contained in the variety

$$P_0 = (g_i(p) \cdot X_j - g_j(p) X_i = 0)$$

and will agree with P_0 over the domain of definition M_0 of f in M .

$$\Gamma \quad \{(p, X) : f(p) = X\}$$

$$\Rightarrow f(p) = [g_0(p), \dots, g_n(p)] = X = [X_0, X_1, \dots, X_n] \text{ on } U \subset M$$

$$\Rightarrow g_i(p) X_j = g_j(p) X_i$$

$$\Rightarrow P_0 \subset U \times \mathbb{P}^n \Rightarrow \{(p, X) : f(p) = X\} \subset P_0 \text{ in } U \times \mathbb{P}^n$$

$$\Rightarrow \overline{\{(p, X) : f(p) = X\}} = \Gamma_f \cap U \times \mathbb{P}^n \subset P_0.$$

\square

Γ_f is thus the irreducible component of P_0 containing $P_0 \cap M_0 \times \mathbb{P}^n$.