Thus if we let $A^{\alpha} = (a_{ij}^{\alpha})$

$$\langle A \wedge {}^t\bar{A} \mid \tfrac{\partial}{\partial z_\alpha}, \tfrac{\partial}{\partial \bar{z}_\alpha} \rangle = \sum a_{ik}^{\alpha} \, \overline{a_{jk}^{\alpha}} \, e_i \otimes e_j^*$$

$$= A^{\alpha} \, {}^t\bar{A}^{\alpha} \geq 0.$$

i.e $\langle \sum a_{ik}^{\alpha} \, \overline{a_{jk}^{\alpha}} \, e_i \otimes e_j^* (e_j), \; e_j \rangle$

$$= \sum a_{jk}^{\alpha} \, \overline{a_{jk}^{\alpha}} \geq 0. \quad \Leftarrow \text{ positive definite.}$$

$$\langle {}^t\bar{A} \wedge A ; \tfrac{\partial}{\partial z_\alpha}, \tfrac{\partial}{\partial \bar{z}_\alpha} \rangle = \langle \sum \bar{a}_{ik}^{\alpha} \, a_{jk}^{\beta} \, d\bar{z}_\alpha \wedge dz_\beta \otimes e_i \otimes e_j^*, \tfrac{\partial}{\partial z_\alpha} \tfrac{\partial}{\partial \bar{z}_\alpha} \rangle$$

$$= - \sum \bar{a}_{ik} \, a_{jk}^{\alpha} \, e_i \otimes e_j^* \leq 0.$$

which implies

$$\Theta_S \leq \Theta_E|_S,$$
$$\Theta_Q \geq \Theta_E|_Q.$$

with equality holding $\Leftrightarrow A \equiv 0$. The principle that curvature decreases in holomorphic subbundles and increases in holomorphic quotient bundles is in marked contrast to the real case.

For example, if $M \subset \mathbb{C}^n$ complex submanifold with metric induced from the Euclidean metric on $\mathbb{C}^n$, we see that

$$T'(M) \subset T'(\mathbb{C}^n)|_M = \Theta_M \leq \Theta_{\mathbb{C}^n}|_M = 0$$

If M is Riemann surface, by the calculations on p77, this just means that its Gaussian curvature $K \leq 0$.

$$\Theta\left(\tfrac{\partial}{\partial z}, \tfrac{\partial}{\partial \bar{z}}\right) = -\tfrac{1}{2} \Delta \log h \leq 0$$