a diffeomorphism $\varphi_U : \pi^{-1}(U) \longrightarrow U \times \mathbb{C}^k$ taking the vector space E_x is isomorphically onto $\{x\} \times \mathbb{C}^k$, for each $x \in U$; φ_U is called a trivialization of E over U.

The dimension of the fibers E_x of E is called the rank of E; in particular, a vector bundle of rank 1 is called a line bundle.

Note that for any pair of trivializations φ_U and φ_V, the map

$$g_{UV} : U \cap V \longrightarrow GL_k \quad \text{given by } g_{UV}(x) = (\varphi_U \circ \varphi_V^{-1})\big|_{\{x\} \times \mathbb{C}^k}$$

is C^∞; the maps g_{UV} are called transition functions for E relative to the trivializations φ_U, φ_V.

The transition functions of E necessarily satisfy the identities

$$g_{UV}(x) \cdot g_{VU}(x) = 1 \quad \text{for all } x \in U \cap V$$

$$g_{UV}(x) \cdot g_{VW}(x) \cdot g_{WU}(x) = 1 \quad \text{for all } x \in U \cap V \cap W.$$

Conversely, given an open cover $\underline{U} = \{U_\alpha\}$ of M and C^∞ maps $g_{\alpha\beta} : U_\alpha \cap U_\beta \longrightarrow GL_k$ satisfying these identities, there is a unique complex vector bundle $E \longrightarrow M$ with transition functions $\{g_{\alpha\beta}\}$; it is not hard to check that

$$E = \bigcup_\alpha U_\alpha \times \mathbb{C}^k \Big/ (x, \lambda) \sim (x, g_{\alpha\beta}(x)(\lambda)) \in U_\beta \times \mathbb{C}^k, \ (x, \lambda) \in U_\alpha \times \mathbb{C}^k$$

as a set, and the manifold structure is induced by the inclusions $U_\alpha \times \mathbb{C}^k \hookrightarrow E$.

As a general rule, operations on vector spaces induce operations on vector bundles. For example, if $E \longrightarrow M$ is a complex vector bundle, we take the dual bundle $E^* \longrightarrow M$ to be the complex vector bundle with fiber $E_x^* = (E_x)^*$; trivializations