given by intersection of cycles: the fact that the polarization is principal is a reflection of Poincare duality.

ω represents an intersection of cycles, and ω principal proves Poincare duality.

Note that the polarizing class $[\omega]$ does not depend on the choice of basis $\delta_1, \cdots \delta_{2g}$ for $H_1(S, \mathbb{Z})$.

Since ω is given by intersection of cycles, ω is independent of the choice of basis $\delta_1, \cdots \delta_{2g}$.

A note: Up to now, we have indexed our complex basis $\{e_\alpha\}$ and dual complex coordinates $\{z_\alpha\}$ for V by $\alpha = 1, 2, \cdots n$; the integral basis $\{\lambda_i\}$ and dual real coordinates $\{x_i\}$ by $i = 1, \cdots 2n$. Once we have normalized our basis, however, we can no longer maintain the notational distinction; we will instead denote the integral basis by

$$\{\lambda_\alpha, \lambda_{n+\alpha}\}_{\alpha=1,\cdots,n} \quad \text{and} \quad \{x_\alpha, x_{n+\alpha}\}_{\alpha=1,\cdots n}.$$