Then we have an open chart U' s.t $\exists$ a diffeomph $F : U' \longrightarrow W \subset R^n$ al $F|_{f^{-1}(y)} : f^{-1}(y) \to \{0\} \times R^{n-1}$. diffeomorphm.

② If y is a regular value of $f : M \longrightarrow N$, then the preimage $f^{-1}(y)$ is a submanifold of M with $\dim f^{-1}(y) = \dim M - \dim N$, i.e. codimension is same.

pf)

$$\begin{array}{ccc} U' & \xrightarrow{f} & V' \ni y \\ \downarrow \varphi & & \downarrow \psi \\ U \subset R^m & \xrightarrow{\psi \circ f \circ \varphi^{-1}} & V \subset R^n \ni y_0 \end{array}$$

$\dim M = m \geq n = \dim N$.

$\Rightarrow \psi \circ f \circ \varphi^{-1} : U \longrightarrow V \ni y_0$.

Choose $x_0 \in U$ s.t $\psi \circ f \circ \varphi^{-1}(x_0) = y_0$.

$\mathrm{rank}(D(\psi \circ f \circ \varphi^{-1})(x_0)) = m$.

Assume that $\underset{\text{matrix}}{m \times n} \left(\dfrac{\partial \psi_i f}{\partial \varphi_j} \right)(x_0)_{1 \leq i, j \leq m}$ is non-singular.

Now define $F : U \longrightarrow V \times R^{m-n}$. by

$(x_1, x_2 \cdots x_m) \longmapsto (\psi \circ f \circ \varphi^{-1}(x_1 \cdots x_m), x_{n+1}, \cdots x_m)$

$$\Rightarrow DF(x_0) = \left(\begin{array}{c|c} \left(\dfrac{\partial \psi_i f}{\partial \varphi_i} \cdots \right) & O \\ \hline O & \begin{matrix} 1 & 0 & & \\ 0 & 1 & & 0 \\ & & \ddots & \\ & 0 & & 1 \end{matrix} \end{array} \right).$$

$\Rightarrow$ Again, by the inverse function theorem, $\exists$ open sets $A \ni x_0$, and $B \ni y_0$. s.t $\exists$ F-inverse. C^∞. $G : B \to A$.