of the family of trichords to E_L. Wait a minute. We need to show that if there is a line ℓ in $\mathbb{P}^3$ s.t $\#(\ell \cap E_L) = 3$, then $\ell = \tilde{f}_L(\pi^{-1}(x))$ for some $x \in L$.

proof) ℓ line s.t $\#(E_L \cap \ell) = 3$

$\Rightarrow$ Since $E_L \cap Q$, $\#(Q \cap \ell) \geq 3$. $\Rightarrow$ Since Q is a quadric, $Q \supset \ell$. $\Rightarrow$ ℓ lies in one of two families of Q.

$\Rightarrow$ Since $\#(\ell \cap E_L) = 3$, by the note on P 800, $\ell \in \{\tilde{f}_L(\pi^{-1}(x))\}$.

② L special

As in the case of nonspecial, see P 1001 note, $\tilde{f}_L(\pi^{-1}(x)) \cap E_L$ corresponds to $T_x(X) \cap X - L$, for if, given a line ℓ passing through x in X, $\ell \cap V_3 = q$. $\Rightarrow$ $\ell = \overline{q, x}$. and $\overline{L, q} \cap V_3 = q \in E_L \cap T_x(X) \cap X$. In sum,

$$\begin{array}{ccc} \tilde{f}_L(\pi^{-1}(x)) \cap E_L & \longleftrightarrow & T_x(X) \cap X - L \\ \Psi & & \Psi \\ q & \longleftrightarrow & \overline{q, x}. \end{array}$$

$\Rightarrow$ Since $\#(T_x(X) \cap X - L) = 3$, counting multiplicity, $\#((\tilde{f}_L(\pi^{-1}(x))) \cap E_L) = 3$, and each $\tilde{f}_L(\pi^{-1}(x))$ meets E_L 3 times, counting multiplicity.

Thus $\pi^{-1}(x)$ is the proper transform of $\tilde{f}_L(\pi^{-1}(x))$.

"Note: Given $X \subset \mathbb{P}^5$, X is constructed independent of the choice of L. For 'abstract variety', see Hartshorne.

One question that arises in this context is: what is the hyperplane bundle on the curve E_L? Explicitly, we have seen that for every line $L \in A$