

$\mathbb{P}^2$  is a smooth curve of degree  $d$ , given in  $\mathbb{P}^2$  as the locus of zeros of a homogeneous polynomial  $F(\mathbb{Z}_0, \mathbb{Z}_1, \mathbb{Z}_2)$  of degree  $d$ . In terms of Euclidean coordinates  $z_1 = \mathbb{Z}_1/\mathbb{Z}_0$ ,  $z_2 = \mathbb{Z}_2/\mathbb{Z}_0$  on  $\mathbb{C}^2 \subset \mathbb{P}^2$ , the equation is  $f(z_1, z_2) = F(1, z_1, z_2)$ .

⌈ See P 166

Choose a point  $p \in \mathbb{P}^2$  not on  $S$  and a line  $H$  not containing  $p$ ; after a linear change of coordinates we may take

$$p = [0, 0, 1], \quad H = (\mathbb{Z}_2 = 0);$$

we may also assume the line  $L$  at infinity ( $\mathbb{Z}_0 = 0$ ) is not tangent to  $S$ .

⌈ First of all, choose a line  $L$  which is not tangent to  $S$ . Choose a point  $p \in L$  s.t.  $p \notin S$ . It is possible, if not,  $L \subset S \Rightarrow L$  is tangent to  $S$ .  $\Rightarrow \exists$  a line  $H$  s.t.  $p \notin H$ .  $\Rightarrow$  By a linear change of coordinates, we can make  $p = [0, 0, 1]$ ,  $H = (\mathbb{Z}_2 = 0)$ , and  $L = (\mathbb{Z}_0 = 0)$ .

Here the line at infinity =  $(\mathbb{Z}_0 = 0)$  see p15  
~ P16

Now consider the map  $\pi_p: S \rightarrow \mathbb{P}^1$  given by projecting from  $p$  to  $H$ . Near a point  $q \in S$  with  $(\partial f / \partial z_2)(q) \neq 0$ ,  $z_1$  will serve as local coordinate on  $S$ , so the map is unramified; if  $(\partial f / \partial z_2)(q) = 0$ ,