$\Rightarrow$ By the argument above, α' and β' are relatively prime.
$\Rightarrow$ α' & β' have no common components. But if we assume that $(\gamma'=0) \neq \pi(W)$, then α' & β' have a common component, as follows:
For some $z' \in \mathbb{C}^{n-1}$, assume $\gamma'(z')=0$, and f and g have no common zeros along the line $\pi^{-1}(z')$. $\Rightarrow$ α' vanishes at all the zeros of g in $\pi^{-1}(z')$; since $\deg \alpha' < \deg g$, this implies that α', and hence β', vanish identically on $\pi^{-1}(z')$. Thus α' and β' both are zero on the inverse image of any component of the zero locus of γ' other than $\pi(W)$. Remember $(\gamma'=0) - \pi(W)$ is open in $(\gamma'=0)$. ⅃

We see then that $\pi(W)$ is an analytic hypersurface in a nbd of the origin in $\mathbb{C}^{n-1}$, and, reiterating our basic description of analytic hypersurfaces, that projection of W onto a suitable chosen $(n-2)$-plane $\mathbb{C}^{n-2} \subset \mathbb{C}^n$ expresses W locally as a finit-sheeted branched cover of a nbd of the origin in $\mathbb{C}^{n-2}$.

⅂

$$W \longrightarrow \mathbb{C}^{n-1} \longrightarrow \mathbb{C}^{n-2}$$

$$\pi(W) = (\gamma'=0) \xrightarrow{\pi} \pi(\gamma'=0) = \text{nbd of } 0$$

if we assume that $(\gamma'=0)$ does not contain the z_{n-1}-axis. See P9, the statement above the Riemann Extension Theorem. ⅃