$\pi^{-1}(H_\lambda) \xrightarrow{\pi} H_\lambda \Rightarrow$ The situation is as follows:

$$\Rightarrow \begin{array}{ccccc} \mathbb{C}^2 & \xrightarrow{\pi} & \mathbb{C}^2 & \xrightarrow{f} & \mathbb{C}. \\ \Psi & & \Psi & & \\ x & & p & & \end{array} \qquad (f=0) = S \cap H_\lambda.$$

(H_λ either contains a point $p \in R$ or is tangent to S)

$$\Rightarrow \frac{\partial f \circ \pi}{\partial x_i} = \sum_j \frac{\partial f}{\partial p_j}\frac{\partial p_j}{\partial x_i} = 0 \text{ since } \frac{\partial f}{\partial p_j} = 0 \ (\downarrow).$$

$\Rightarrow (f \circ \pi = 0)$ is singular at x.

⌟

In the first case, we can write

$$C_\lambda = \tilde{C}_\lambda + X_p.$$

with – by taking H_λ generic – $\tilde{C}_\lambda$ a smooth curve meeting X_p in two distinct points.

⌜ For generic H_λ, $H_\lambda \cap S$ is singular with a double point as the only singularity. Clearly $C_\lambda = X_p + \overline{\pi^{-1}(H_\lambda \cap S) - p} \Rightarrow \tilde{C}_\lambda = \overline{\pi^{-1}(H_\lambda \cap S) - p}$.

I guess that a desingularization is a generalization of blow-ups. $\Rightarrow$ Then I can accept $\tilde{C}_\lambda$ is smooth.

And $\tilde{C}_\lambda \cap X_p = \{$ distinct two points $\}$ ⌟

Now $\tilde{C}_\lambda$ is the desingularization of the plane quartic $H_\lambda \cap S$ having one double point at p, and so has genus 2; since X_p is a line and meets $\tilde{C}_\lambda$ in two points,