

and observation that we may find  $\varphi \in H^0(\mathcal{O}_C(K+2D))$  with prescribed residue subject only to the residue theorem (cf. Section 2 in Chapter 2).

Consider the following map

$$\begin{array}{ccc} H^0(M, \Omega^1(2D)) & \xrightarrow{f} & \mathbb{C}^g \\ \downarrow \varphi & \longmapsto & (\text{Res}_{p_1}\varphi, \text{Res}_{p_2}\varphi, \dots, \text{Res}_{p_g}\varphi) \end{array}$$

In case  $p_1, \dots, p_g$  are distinct, the <sup>result on p460</sup> above is true. According to P233 lemma,

$\exists$  a meromorphic function  $\tilde{\varphi}$  on  $M$ , holomorphic on  $M - \{p_1, \dots, p_g\}$ , and residue  $a_\lambda$  at  $p_\lambda$ ,  $\sum a_\lambda = 0$ .

$\Rightarrow s_0^2 \tilde{\varphi} \in \Omega^1(2D)$ , where  $D = (s_0 = 0)$ .

$\Rightarrow s_0^2 \tilde{\varphi} \in H^0(M, \Omega^1(2D))$ .

Thus, dimension of  $\text{im } f = g-1$ , since  $f$  is onto the hyperplane  $\sum a_\lambda = 0$  of  $\mathbb{C}^g$ .

$\Rightarrow \dim \ker f = \dim H^0(M, \Omega^1(2D)) - \dim \text{im } f$   
 $= h^0(K_C + 2D) - (g-1) = 3g-1 - (g-1) = 2g$ .

$\Rightarrow$  Note that  $\ker f$  = the set of all meromorphic 1-forms having no residues and polar divisor  $\geq D$ .  $\square$

So the space of 1-forms of the second kind with polar divisor  $\geq D$  has dimension

$$3g-1 - (g-1) = 2g,$$

and none of these can be exact by our remark about meromorphic functions with polar divisor  $D$ . Q.E.D.