$(A_7, B_7), \cdots (A_{10}, B_{10})\}$ has the rank 10.

If $(A_{11}, B_{11}) = x_1 (A_1, B_1) + \cdots + x_5 (A_5, B_5) + x_6 (A_6, B_6) + \cdots + x_{10} (A_{10}, B_{10})$, change (A_{11}, B_{11}) into $x_1 (A_1, B_1') + \cdots x_5 (A_5, B_5') + x_6 (A_6, B_6) + \cdots + x_{10} (A_{10}, B_{10})$.

Similarly for (A_i, B_i), $i = 12 \cdots 100$.

$\Rightarrow \{(A_1, B_1'), (A_2, B_2'), \cdots (A_5, B_5'), (A_6, B_6), \cdots (A_{10}, B_{10}), (A_{11}, B_{11}') \cdots (A_{100}, B_{100}')\}$ are of rank 10 exactly.

Again by adding small vectors to $(A_{11}, B_{11}') \cdots (A_{20}, B_{20}')$, we can make $\{(A_1, B_1'), \cdots (A_6, B_6), \cdots (A_{10}, B_{10}), (A_{11}', B_{11}'') \cdots (A_{20}', B_{20}''), (A_{21}, B_{21}'), \cdots (A_{100}, B_{100}')\}$ have rank 20.

We can do the above process for the following case.

$$\begin{pmatrix} A_1, & B_1, & C_1 \\ \vdots & \vdots & \vdots \\ A_5, & B_5, & C_5 \\ A_6 & B_6 & C_6 \\ \vdots & \vdots & \vdots \\ A_{10} & B_{10} & C_{10} \\ \vdots & & \\ A_{20}, & B_{20} & C_{20} \\ \vdots & \vdots & \\ A_{100}, & B_{100} & C_{100} \end{pmatrix}$$

$\{(A_1, B_1, C_1) \cdots (A_{20}, B_{20}, C_{20})\}$ linear indept

$\{(B_1, C_1), \cdots (B_{10}, C_{10})\}$ span the $\{(B_1, C_1) \cdots (B_{100}, C_{100})\}$

$\{C_1, \cdots C_5\}$ span the rest.

We want
$$5 \longrightarrow 20 \quad C$$
$$10 \longrightarrow 21 \quad (B, C)$$
$$20 \longrightarrow 22+2 \quad (A, B, C)$$

Make $C_1', \cdots C_5', \cdots C_{20}'$ linear independent by adding small vectors, and change $(B_{21}, C_{21}) \cdots (B_{100}, C_{100})$ into $(B_{21}, C_{21}'') \cdots (B_{100}, C_{100}'')$ as follows.

If $(B_{21}, C_{21}) = x_1 (B_1, C_1) + \cdots + x_{10} (B_{10}, C_{10})$ — ②

then $(B_{21}, C_{21}'') = x_1 (B_1, C_1') + \cdots + x_{10} (B_{10}, C_{10}')$