must be of the form $R(t)\,dt$ on $\mathbb{P}^1$ with R a rational function. Thus for $(x_0, y_0), (x, y) \in C$,

$$\int_{(x_0,y_0)}^{(x,y)} \frac{dx}{y} = \int_{t(x_0,y_0)}^{t(x,y)} R(t)\,dt,$$

and the latter integral is easy to solve.

Γ $C = \{ y^2 = x^2 + ax + b \}$ is of deg 2. $\Rightarrow g(C) = \frac{(d-1)(d-2)}{2} = 0 \Rightarrow C \cong \mathbb{P}^1$ in case $a^2 - 4b \neq 0$

$$\Rightarrow \left(\frac{Z_2}{Z_0}\right)^2 = \left(\frac{Z_1}{Z_0}\right)^2 + a\frac{Z_1}{Z_0} + b \Rightarrow Z_2^2 = Z_1^2 + aZ_1Z_0 + bZ_0^2$$

$$\Rightarrow \left(\frac{Z_2}{Z_1}\right)^2 = 1 + a\frac{Z_0}{Z_1} + b\left(\frac{Z_0}{Z_1}\right)^2$$

$\Rightarrow$ Nonsingula on $(Z_1 \neq 0)$ if $a^2 - 4b \neq 0$.

$$\Rightarrow 1 = \left(\frac{Z_1}{Z_2}\right)^2 + a\frac{Z_1}{Z_2}\frac{Z_0}{Z_2} + b\left(\frac{Z_0}{Z_2}\right)^2$$

$$\Rightarrow 1 = x^2 + axy + by^2.$$

$$\Rightarrow \begin{pmatrix} 2by + ax \\ 2x + ay \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} 2x + ay = 0 \\ 2by + ax = 0 \end{array}$$

$$\Rightarrow 2by + a\left(-\frac{a}{2}y\right) = 2by - \frac{a^2}{2}y = 0 \qquad y = 0$$

$\Rightarrow x = 1 \;\&\; 0 \Rightarrow$ Curve C is nonsingular if $a^2 \neq 4b$.

contradiction

We can give an isomorphism from C to $\mathbb{P}^1$ as follows:

$$\begin{array}{ccc} C = (Z_2^2 = Z_1^2 + aZ_1Z_0 + bZ_0^2) & \longrightarrow & \mathbb{P}^1 \\ \rotatebox{90}{$\in$} & & \\ [Z_0, Z_1, Z_2] & \longmapsto & [Z_0, Z_2 - Z_1] \end{array}$$

If we consider $\frac{Z_2 - Z_1}{Z_0}$, then it has possible zeros at $Z_0 = 0$ i.e. $[0, Z_1, Z_1]$ & $[0, Z_1, -Z_1]$. But in case