classical Plücker formula on P280. $\Rightarrow$ The # of branch points of $r \circ f$ is $2 \cdot 2 + 2 \cdot 1 - 2 = 4$ by the formula on P255. Clearly, $r(p_i)$ is the branch point of r, but it is not a branch point of $r \circ f$. Thus the branch locus of $r \circ f$ on $\tilde{\tilde{C}}_h$ descends on $\tilde{C}_h$.
Thus r can be branched at at most four points other than $r(p_i)$.

The generic line $L \subset \mathbb{P}^2$ through $r(p_i)$ thus meets F at most four times away from $r(p_i)$, and so we see that the images $\bar{p}_i = r(p_i)$ of the double points of S are double points of F.

$L = h \cap \mathbb{P}^2$, where h is a generic hyperplane passing through p_0 and p_i. $\Rightarrow$ By the argument above,

$$\#(L \cap F - r(p_i)) \le 4.$$

Now, suppose the curve F has irreducible components F_i of degree d_i. Singular points of F then arise in two ways: either as points of intersection of components F_i, F_j or as singular points of a component F_i. There are, of course, at most $\sum_{i \neq j} d_i d_j$ singular points of F of the former kind and, by the result of