Likewise, from the sequence

$$0 \to \mathcal{O}_M(E\otimes L^{m-1}) \longrightarrow \mathcal{J}_{x,M}(E\otimes L^m) \longrightarrow \mathcal{J}_{x,V_\alpha}(E\otimes L^m) \to 0,$$

we see that for $m > m_1$ as before,

$$H^0(M, \mathcal{J}_x(E\otimes L^m)) \longrightarrow H^0(V_\alpha, \mathcal{J}_x(E\otimes L^m))$$

is surjective.

⸢ Let $s_0 \in H^0(\mathbb{P}^N, \mathcal{O}(H))$ s.t $(s_0 = 0) = H'$, $H' \cap M = V_\alpha$. $\Rightarrow$ By p138, $\mathcal{O}(E\otimes L^m\otimes L^{-1}) \xrightarrow{\otimes s_0} \mathcal{E}(L^{-1})$ is an identification.

$\mathcal{E}(L^{-1}) \subset \mathcal{O}_M(E\otimes L^m)$ and $\mathcal{E}(L^{-1})$ = the sheaf of sections of $E\otimes L^m$ vanishing to order ≥ 1 along $H'\cap M = V_\alpha$.

Thus we get the exact sequence

$$0 \to \mathcal{O}_M(E\otimes L^{m-1}) \xrightarrow{\otimes s_0} \mathcal{O}_M(E\otimes L^m) \longrightarrow \mathcal{O}_{V_\alpha}(E\otimes L^m) \to 0$$

But as we saw above. $\otimes s_0(\mathcal{O}_M(E\otimes L^{m-1})) \cap \mathcal{J}_{x,M}(E\otimes L^{m-1})$

i.e the image of $\otimes s_0$ is in $\mathcal{J}_{x,M}(E\otimes L^{m-1})$.

$$\begin{tikzcd}
0 \arrow[r] & \mathcal{O}_M(E\otimes L^{m-1}) \arrow[r, "\otimes s_0"] & \mathcal{O}_M(E\otimes L^m) \arrow[r] \arrow[dr, phantom, "\circlearrowright"] & \mathcal{O}_{V_\alpha}(E\otimes L^m) \arrow[r] & 0 \\
0 \arrow[r] & \mathcal{O}_M(E\otimes L^{m-1}) \arrow[u, equal] \arrow[r, "\otimes s_0"] & \mathcal{J}_{x,M}(E\otimes L^m) \arrow[u, hook] \arrow[r] & \mathcal{J}_{x,V_\alpha}(E\otimes L^m) \arrow[u] \arrow[r] & 0
\end{tikzcd}$$