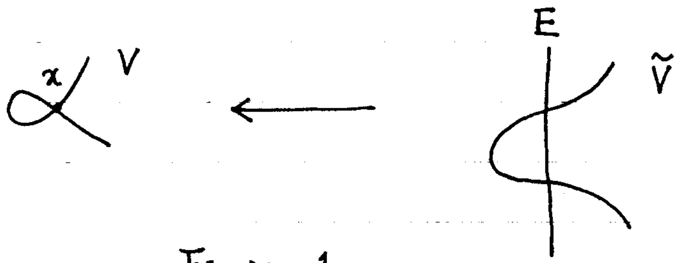

Figure 1.

Note finally that if p is a smooth point of the subvariety $V \subset M$, then the proper transform $\tilde V$ of V under the blow-up of M at p is just the blow-up of V at p. Accordingly, we sometimes refer to the proper transform $\tilde V \subset \tilde M$ of a subvariety $V \subset M$ as the blow-up of V at p, even when p is a singular point of V.

Pf

$$\begin{array}{c} \Delta^n \times \mathbb{P}^{n-1}, \\ \big\downarrow \pi \\ \Delta^n \supset \Delta^k \ni 0, \end{array}$$

since V is a submanifold of M near p.

$$\pi^{-1}(\Delta^k - \{0\}) \subset \{(z, l) \mid z \in l\} \subset \Delta^n \times \mathbb{P}^{n-1}$$

$$[\{(z_1, \cdots z_k, 0 \cdots 0), l\} \mid (z_1 \cdots z_k, 0 \cdots 0) \in l\}] \subset \Delta^n \times \mathbb{P}^{n-1}$$

$$\Rightarrow \quad \overline{\pi^{-1}(\Delta^k - \{0\})} = \{(z, l) \mid z \in l\} \subset \Delta^k \times \mathbb{P}^{k-1}.$$

$\Rightarrow$ $\tilde V$. proper transform, is the blow-up of V at p. //

We now consider the case of a surface M and its blow-up $\tilde M \xrightarrow{\pi} M$ at $p \in M$. First, we see that if C is any curve on $\tilde M$ not containing the exceptional divisor E, then C is the proper transfor-