Also we can describe C as a map from $\mathbb{C}$ to $\mathbb{P}^n$ i.e., locally $z \longmapsto [b_0(z), \cdots b_n(z)]$, where $b_i(z)$'s are holomorphic and $[b_0(0), \cdots b_n(0)] = p_i$.

Then finding the point $p_i(H)$ is the solution of
$a_0 \; b_0(z) + \cdots + b_n(z)\, a_n = 0$.

Consider $f(a_0, a_1, \cdots a_n, z) = (a_0, a_1, \cdots a_n, a_0 \; b_0(z) + \cdots + \cdots + a_n \; b_n(z))$, $f : \mathbb{C}^{n+2} \longrightarrow \mathbb{C}^{n+2}$.

$$\Rightarrow J(f) = \begin{pmatrix} 1, & 0 & \cdots & 0 & \cdots 0 \\ 0 & 1 & \cdots & 0 & \vdots \\ & & & 0 & \\ \vdots & & & \vdots & 0 \\ 0 & 0 & \cdots\cdots & a_0 & b_0'(z) + a_1 b_1'(z) + \cdots + a_n b_n'(z) \end{pmatrix}$$

At $z=0$, $a_0' \; b_0(0) + \cdots + b_n(0)\, a_n' = 0$ and $a_0' \, b_0'(0) + \cdots + a_n' \; b_n'(0) \neq 0$ since, at $z=0$, $g(a_1, \cdots a_n, z) = b_0(z)\, a_0 + \cdots + b_n(z) a_n$ does not have a double zero.

$\Rightarrow$ By the inverse function theorem, $\exists \; G$ s.t

$$G(a_0, a_1 \cdots a_n, \; g(a_0 \cdots a_n, z)) = (a_0, a_1, \cdots a_n, z).$$

On $g(a_0, a_1, \cdots z) = 0$, $\quad G(a_0, a_1, \cdots a_n, 0) = (a_0, \cdots a_n, z)$.

$\Rightarrow z = h(a_0, a_1, \cdots a_n)$, h is homorphic in variables $a_0, a_1, \cdots a_n$, which means that h is holomorphic in H determined by $a_0, a_1 \cdots a_n$. $\lrcorner$

Accordingly, for every multiindex $I = \{ i_1, \cdots i_n \} \subset \{1, 2, \cdots d\}$, we get a map

$$\pi_I : U \longrightarrow C^n = C \times \cdots \times C,$$
$$H \longmapsto (p_{i_1}(H), \cdots p_{i_n}(H));$$