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and $Q(v_i, v_i) \neq 0$ for $i = 1, 2, 3, 4$.

Let $v_i = a_{ji} e_j$.

$$Q(v_i, v_j) = Q(a_{ki} e_k, a_{\ell j} e_\ell) = a_{ki} a_{\ell j} Q(e_k, e_\ell)$$

$$= a_{ki} Q(e_k, e_\ell) a_{\ell j} \Rightarrow {}^tAQA = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{pmatrix}$$

$\Rightarrow$ By the argument on P731 back page, we may assume

$${}^tAQA = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = I$$

$\Rightarrow$ Q is isomorphic to I $\Rightarrow$ Any two nondegenerate quadratic symmetric forms are isomorphic.

For example, $n = 2$

$$\begin{array}{ccc} \mathbb{C}^2 & \xrightarrow{A} & \mathbb{C}^2 \\ & & \rotatebox{90}{$\in$} \\ e_i & \longmapsto & v_i \end{array}$$

$$\begin{pmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} = a_{11} e_1 + a_{21} e_2$$

$$\begin{pmatrix} a_{11}, & a_{12} \\ a_{21}. & a_{22} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} = a_{12} e_1 + a_{22} e_2$$

$$\Rightarrow \quad Q(x_i v_i, x_j v_j) = x_i x_j Q(v_i, v_j)$$

$$= x_i x_j Q(a_{ki} e_i, a_{\ell j} e_j) = x_i x_j a_{ki} a_{\ell j} Q(e_i, e_j) =$$