Set $\Lambda' = \mathbb{Z}\{\lambda_1, \lambda_{n+1}\}^\perp$; we can repeat this process to obtain two elements $\lambda_2, \lambda_{n+2} \in \Lambda'$ with

$$\Lambda' = \mathbb{Z}\{\lambda_2, \lambda_{n+2}\} \oplus \mathbb{Z}\{\lambda_2, \lambda_{n+2}\}^\perp.$$

⌜ For each $\lambda \in \Lambda'$, $\{Q(\lambda, \lambda'), \lambda' \in \Lambda'\}$ forms a principal ideal $d_\lambda \mathbb{Z}$ in $\mathbb{Z}$, $d_\lambda \geq 0$. Let $\delta_2 = \min\{d_\lambda : \lambda \in \Lambda', d_\lambda \neq 0\}$, and take λ_2 and λ_{n+2} s.t. $Q(\lambda_2, \lambda_{n+2}) = \delta_2$. $\Rightarrow$ For every $\lambda \in \Lambda'$, δ_2 divides $Q(\lambda, \lambda_2)$ and $Q(\lambda, \lambda_{n+2})$, and we can write

$$\underbrace{\lambda + \frac{Q(\lambda, \lambda_2)}{\delta_2} \lambda_{n+2} - \frac{Q(\lambda, \lambda_{n+2})}{\delta_2} \lambda_2}_{*} \in \mathbb{Z}\{\lambda_2, \lambda_{n+2}\}^\perp.$$

for,
$$Q\left(\lambda_2, \lambda + \frac{Q(\lambda, \lambda_2)}{\delta_2} \lambda_{n+2} - \frac{Q(\lambda, \lambda_{n+2})}{\delta_2} \lambda_2\right)$$

$$= Q(\lambda_2, \lambda) + \frac{Q(\lambda, \lambda_2)}{\delta_2} Q(\lambda_2, \lambda_{n+2}) - 0$$

$$= Q(\lambda_2, \lambda) + Q(\lambda, \lambda_2) = 0.$$ Similarly, $Q(\lambda_{n+2}, *) = 0$.

$\Rightarrow \Lambda' \subset \mathbb{Z}\{\lambda_2, \lambda_{n+2}\} \oplus \mathbb{Z}\{\lambda_2, \lambda_{n+2}\}^\perp$.

$\Rightarrow$ Since we choose $\mathbb{Z}\{\lambda_2, \lambda_{n+2}\}^\perp \subset \Lambda'$, and $\lambda_2, \lambda_{n+2} \in \Lambda'$, $\Lambda' = \mathbb{Z}\{\lambda_2, \lambda_{n+2}\} \oplus \mathbb{Z}\{\lambda_2, \lambda_{n+2}\}^\perp$.

Continue this process, we have

$$\Lambda = \mathbb{Z}\{\lambda_1, \lambda_{n+1}\} \oplus \mathbb{Z}\{\lambda_2, \lambda_{n+2}\} \oplus \cdots \oplus \mathbb{Z}\{\lambda_n, \lambda_{2n}\}.$$ ⌟