$$\varphi = \sum \frac{\varphi_{JK}}{z_J} \frac{dz_{I-J}}{z_{I-J}} \wedge dz_K.$$

$$d\varphi = \sum d\left(\frac{\varphi_{JK}}{z_J}\right) \wedge \frac{dz_{I-J}}{z_{I-J}} \wedge dz_K$$

$$= \sum \frac{\frac{\partial \varphi_{JK}}{\partial z_j} z_J - \frac{\partial z_J}{\partial z_j} \varphi_{JK}}{z_J^2} dz_j \wedge \frac{dz_{I-J}}{z_{I-J}} \wedge dz_K$$

$$= \sum \frac{\frac{\partial \varphi_{JK}}{\partial z_j}}{z_J} dz_j \wedge \frac{dz_{I-J}}{z_{I-J}} \wedge dz_K$$

$$- \sum \frac{\partial z_J}{\partial z_j} \varphi_{JK} \frac{1}{z_J^2} dz_j \wedge \frac{dz_{I-J}}{z_{I-J}} \wedge dz_K$$

$$\equiv - \sum_{j \in J} z_{J-\{j\}} \varphi_{JK} \frac{1}{z_J^2} dz_j \wedge \frac{dz_{I-J}}{z_{I-J}} \wedge dz_K$$

$$= - \sum_{j \in J} \frac{\varphi_{JK}}{z_J} \frac{dz_j}{z_j} \wedge \frac{dz_{I-J}}{z_{I-J}} \wedge dz_K$$

$$= - \sum_{j \in J} \frac{\varphi_{JK}}{z_J} \left(\pm \frac{dz_{(I-J)\cup\{j\}}}{z_{(I-J)\cup\{j\}}} \wedge dz_K \right)$$

$$= \sum_{L \subset I} \psi_{LK} \frac{dz_L}{z_L} \wedge dz_K$$

$$\psi_{LK} = \sum \pm \frac{\varphi_{(I-L)\cup\{j\},K}}{z_{(I-L)\cup\{j\}}}.$$

$L = (I-J)\cup\{j\} \Rightarrow J = (I-L)\cup\{j\}$ since $J \ni j$ and $L \ni j$.

$$\Rightarrow \quad z_{I-L} \psi_{LK} = \pm \sum_{j \in L.} \frac{\varphi_{(I-L)\cup\{j\},K}}{z_j}$$