Universal Bundles.

Let $\mathbb{C}^n \times G(k,n)$ denote the trivial vector bundle of rank n over $G(k,n)$. We define the universal subbundle $S \longrightarrow G(k,n)$ to be the subbundle of $\mathbb{C}^n \times G(k,n)$ whose fiber at each point $\Lambda \in G(k,n)$ is just the subspace $\Lambda \subset \mathbb{C}^n$.

$$\begin{tikzcd} S \arrow[d] & \subset \mathbb{C}^n \times G(k,n) \\ G(k,n) & \end{tikzcd}$$

$$S = \{ (v, \Lambda) \in \mathbb{C}^n \times G(k,n) \mid v \in \Lambda \}.$$

S is clearly a holomorphic subbundle of $\mathbb{C}^n \times G(k,n)$ — explicitly, in each open $U_I \subset G(k,n)$ the row vectors of the normalized matrix representatives for $\Lambda \in U_I$ give a frame for S over U_I; transition functions relative to these frames are given in $U_I \cap U_{I'}$ by $g_{U_I U_{I'}} = \Lambda_I \cdot \Lambda_{I'}^{-1}$.

$$\begin{tikzcd} S \arrow[d] & \subset G(k,n) \times \mathbb{C}^n \\ G(k,n) & \end{tikzcd}$$

Let $U_I = \{ \Lambda \in G(k,n) \mid \Lambda \cap V_{I^\circ} = \{0\} \}$.

$$U_I \xrightarrow{\varphi_I} \mathbb{C}^{k \times n}$$

$$\Lambda \longmapsto \begin{pmatrix} * \cdots & 1 & * \cdots & 0 & \cdots & \vdots \\ \vdots & 0 & & 0 & & \vdots \\ * & 0 \cdots & & 0 & \cdots & \end{pmatrix} = \Lambda^I$$