

$$\Rightarrow \bar{\partial} k = \pm T_{\Delta}^0$$

Actually we obtain the following

$$\begin{aligned}\bar{\partial} k(\pi_1^* \psi \wedge \pi_2^* \varphi) &= (-1)^0 T_{\Delta}^0(\pi_1^* \psi \wedge \pi_2^* \varphi) = (\pm) T_{\varphi}(\psi) \\ \bar{\partial} k(\pi_1^* \psi \wedge \pi_2^* \varphi) &= -K_{\varphi}(\bar{\partial} \psi) + (-1)^{q-1} K_{\bar{\partial} \varphi}(\psi) \\ &= \pm T_{\varphi}(\psi)\end{aligned}$$

“Comment on bitype $(0, *)$, $(n, n-*)$.”

The current $T_{\Delta}^0(\varphi) = \int_{\Delta} \sum_q \varphi^{(n, n-q), (0, q)}$ is well-defined.

Suppose we have $T(\varphi) = \int_{\Delta} \varphi^{(n, 0), (0, q)}$

$$= \int_{M \times M} (-1)^n \pi_1^* \psi_{0, n, \mu} \wedge \pi_2^* \psi_{n, 0, \nu}^* \wedge \varphi \quad \text{--- ①}$$

Consider $\bar{\partial} T(\varphi) = \pm T(\bar{\partial} \varphi)$

$$= \int_{M \times M} (-1)^n \pi_1^* \psi_{0, n, \mu} \wedge \pi_2^* \psi_{n, 0, \nu}^* \wedge \bar{\partial} \varphi \quad \text{--- ②}$$

$$= \pm \int_{M \times M} \bar{\partial} (\pi_1^* \psi_{0, n, \mu} \wedge \pi_2^* \psi_{n, 0, \nu}^*) \wedge \varphi = 0 \quad \text{--- ③}$$

$[\pi_1^* \psi_{0, n, \mu} \wedge \pi_2^* \psi_{n, 0, \nu}^*]$ is a cohomology class in $H_{\bar{\partial}}^{n, n}(M \times M)$

Ah! When we compute ①, ② & ③, we have to use a representative of $[\pi_1^* \psi_{0, n, \mu} \wedge \pi_2^* \psi_{n, 0, \nu}^*]$, i.e. a form representing the form. Thus we have a lot of choice and we can not have a representative form only from $A^{(0, n), (n, 0)}$ because the current must be independent of the choice of forms representing $[\pi_1^* \psi_{0, n, \mu} \wedge \pi_2^* \psi_{n, 0, \nu}^*]$.
Not so satisfactory!