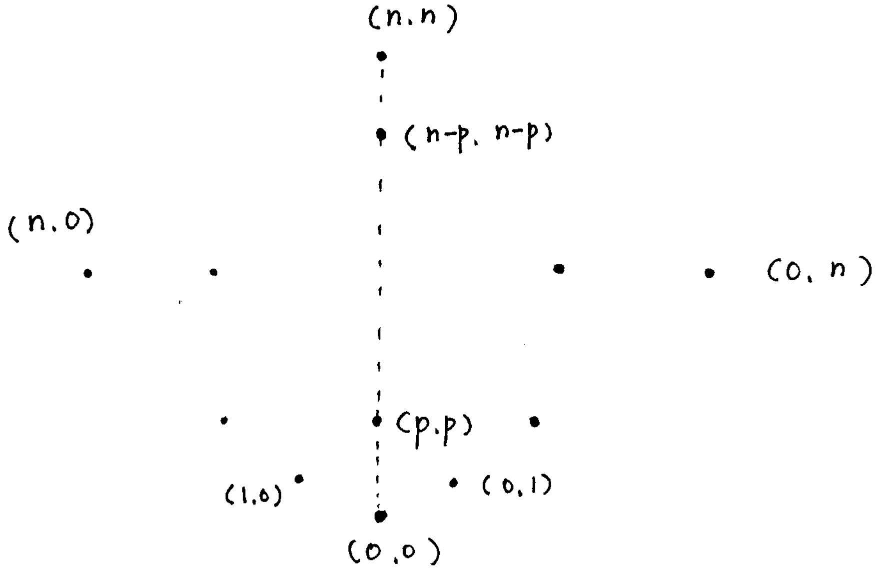
$$b_{2q+1}(M) = 2\left[\sum_{p=0}^{q} h^{p,\,2q+1-p}(M)\right].$$

We can put the cohomology groups of a compact Kähler manifold diagrammatically in the Hodge diamond (Fig. 6), so that the k-th cohomology group of M can be read off as the sum of the groups in the k-th horizontal row. The star operator gives a symmetry about the center of the diamond; conjugation gives a symmetry about the center vertical line.



As an immediate application of the Hodge decomposition, we have the

Corollary: $H^q(\mathbb{P}^n, \Omega^p) = H_{\bar\partial}^{p,q}(\mathbb{P}^n) = \begin{cases} 0 & \text{if } p \neq q \\ \mathbb{C} & \text{if } p = q \leq n. \end{cases}$

pf). This is clear: since $H^{2k+1}(\mathbb{P}^n, \mathbb{Z}) = 0$, we have $H_{\bar\partial}^{p,q}(\mathbb{P}^n) = 0$ for $p+q$ odd; since $H^{2k}(\mathbb{P}^n, \mathbb{Z}) = \mathbb{Z}$ we have for $p \neq k$,

$$1 = b_{2k}(\mathbb{P}^n) \geq h^{p,\,2k-p}(\mathbb{P}^n) + h^{2k-p,\,p}(\mathbb{P}^n)$$
$$= 2\,h^{p,2k-p} \quad\Rightarrow\quad h^{p,2k-p} = 0$$

and hence $1 = b_{2k}(\mathbb{P}^n) = h^{p,p}(\mathbb{P}^n) \;\Rightarrow\; H_{\bar\partial}^{p,p}(\mathbb{P}^n) \cong H_{DR}^{2p}(\mathbb{P}^n) \cong \mathbb{C}.$

Q.E.D.