homology class of degree $2p$ is of pure type (p,p), see p163 and see p118). In case $p=q$,

$$H^{p,p}(\tilde{M}) = H^{p,p}(\mathbb{P}^{n-1}) \oplus H^{p,p}(M) \Rightarrow \text{By } H^{p,p}(\mathbb{P}^{n-1}) = \mathbb{C}$$

for $2p \le 2(n-1)$, see p118, $H^{p,p}(\tilde{M}) = \mathbb{C} \oplus H^{p,p}(M)$.

$\Rightarrow \quad h^{p,p}(\tilde{M}) = h^{p,p}(M) + 1, \quad p > 0.$

(*) is valid since the Mayer-Vietoris sequence is natural. ⌟

We make here one new definition. Let p, M, $\tilde{M}$ and π be as above, and let $V \subset M$ be any analytic subvariety of M. Then we define the proper transform $\tilde{V} \subset \tilde{M}$ of V to be the closure in $\tilde{M}$ of the inverse image

$$\tilde{V} = \overline{\pi^{-1}(V - \{p\})} = \overline{\pi^{-1}(V) - E}$$

of V away from p.

$\ulcorner$ $\pi^{-1}(V-\{p\}) \overset{?}{=} \pi^{-1}(V) - E$

$$\pi^{-1}(V) \cap \pi^{-1}(M-\{p\}) = \pi^{-1}(V) \cap (\tilde{M} - \pi^{-1}(p)) = \pi^{-1}(V) \cap (\pi^{-1}(p))^c$$

$$= \pi^{-1}(V) \cap E^c = \pi^{-1}(V) - E$$

$$\pi^{-1}(M-\{p\}) \overset{?}{=} \tilde{M} - \pi^{-1}(p)$$

$$x \in \pi^{-1}(M-\{p\}) \Leftrightarrow \pi(x) \in M - \{p\} \Leftrightarrow \pi(x) \notin \{p\} \Leftrightarrow x \notin \pi^{-1}(p)$$

$$\Leftrightarrow x \in \tilde{M} - \pi^{-1}(p)$$ ⌟

Clearly π maps $\tilde{V} - E = \pi^{-1}(V - \{p\})$ isomorphically onto $V - \{p\}$. To get a picture of $\tilde{V}$ near the exceptional divisor, let $z = (z_1, \cdots z_n)$ be holomorphic coordinates around $p \in M$, $\tilde{\mathcal{U}}_i$ the open set $(l_i \neq 0)$ in