

② Let $\dim M = 1 \Rightarrow M$ is a curve.

$$\Rightarrow (i) \dim(M \cap H)_s = 0.$$

$$\Rightarrow (M \cap H)_s = \{a_1, a_2, \dots, a_k\}.$$

$$\Rightarrow T_{a_i} M \subset T_{a_i} H$$

Suppose all $T_{a_i} M$'s are distinct each other.

\Rightarrow We can get the generic hyperplanes $\{H_i\}$ for each $T_{a_i} M$ s.t. $T_{a_i} M + T_{a_i} H_i = T_{a_i} \mathbb{P}^n$.

$$\Rightarrow \text{Consider } \{H_1\} \cap \{H_1\} \cap \dots \cap \{H_k\} = \{H'\}$$

$\Rightarrow \{H'\}$ is the set of generic hyperplanes s.t. $M \cap H'$ is smooth.

Suppose $T_{a_1} M = T_{a_2} M$ without loss of generality.

$$\langle a_1, a_2 \rangle \subset H \Rightarrow \langle a_1, a_2 \rangle = T_{a_1} M = T_{a_2} M = \mathbb{P}^1 \subset H$$

\Rightarrow We have to embed M differently so that M has no bitangent point.

Let's accept the statement (which circumstance we can always avoid by embedding M differently.)

$$(ii) \dim(M \cap H)_s = 1. \Rightarrow (M \cap H)_s = \mathbb{P}^1$$

$$\Rightarrow \mathbb{P}^1 \subset M \Rightarrow M = \mathbb{P}^1$$

$$\mathbb{P}^1 \xrightarrow{f} \mathbb{P}^2$$

$$[z_0, z_1] \longmapsto [z_0^2, z_0 z_1, z_1^2]$$

\Rightarrow

$$f(t) = (1, t, t^2)$$

$$f'(t) = (0, 1, 2t) \Rightarrow f'(t) \neq f'(t') \text{ if } t \neq t'$$

$\Rightarrow \exists$ no bitangent point on $f(\mathbb{P}^1)$.

□