⌈ $\tilde{F}$ is a quadric in V_{k-1} and $L \cap V_{k-1}$ is a line.
$\Rightarrow \#(\tilde{F} \cap L) \geq 2$, and since $L \cap \Lambda \neq \phi$ and $\Lambda \cap \tilde{F} = \phi$,
$\#(L \cap F) \geq 3$ ($\because$ $F \supset \Lambda \cup \tilde{F}$).
$\Rightarrow$ Since F is a quadric, $F \supset L$. ⌟

Conversely, if $p \in F$ is any point lying off Λ, the $(n-k+1)$-plane spanned by p and Λ must meet V_{k-1} in a point q.

⌈ It is clear, since $\langle p, \Lambda \rangle = \mathbb{P}^{n-k+1}$ and $V_{k-1} \cong \mathbb{P}^{k-1}$. ($\because$ $n-k+1 + k-1 = n$) ⌟

The line $L = \overline{pq}$ then meets F at p and twice again in Λ, and so lies in F; in particular $q \in \tilde{F}$, so p lies on a line joining Λ and $\tilde{F}$.
Consequently,

A quadric $F \subset \mathbb{P}^n$ of rank k is the cone through an $(n-k)$-plane $\Lambda \subset F \subset \mathbb{P}^n$ over a quadric of rank k in $\mathbb{P}^{k-1}$.

⌈ L is in $\langle p, \Lambda \rangle$, and $L \cap \Lambda \neq \phi$.
$\Rightarrow$ Let $p' \in L \cap \Lambda$.
By changing the coordinates, we may assume that $L = \{ X_2 = X_3 = \cdots = 0 \}$ and $p = [(0,1,0,\cdots,0)]$.
$\Rightarrow F|_L = a_0 X_0^2 + a_1 X_0 X_1 + a_2 X_1^2 = 0$.