

The number  $v$  is called the ramification index of the map  $f$  at  $p$ ;  $p$  is called a branch point if  $v(p) > 1$ . The branch locus of the map  $f$  is taken to be either the divisor

$$B = \sum_{p \in S} (v(p) - 1) \cdot p$$

on  $S$  or its image

$$B' = \sum_{p \in S} (v(p) - 1) \cdot f(p) \quad \text{on } S'.$$

For any point  $p \in S'$ , we can write

$$f^*(p) = \sum_{q \in f^{-1}(p)} v(q) \cdot q,$$

$$\deg f^*(p) = n = \sum_{q \in f^{-1}(p)} v(q),$$

where the summation is over distinct points.

⌈ We have to count multiplicities.  $\hookrightarrow$

This then gives us a picture of the map  $f$ : away from the branch locus of  $f$  in  $S'$ ,  $f$  is a covering map; at a branch point  $p \in S$  of ramification index  $k$ ,  $k$  sheets of the covering come together.

⌈ Away from the branch locus of  $f$  in  $S'$ ,  $f$  is locally one to one.  $\Rightarrow$   $f$  is a covering map.  $\hookrightarrow$

We can, in terms of the sheet number and ramification of  $f$ , relate the genus of  $S$  to the genus of  $S'$ . Take a triangulation of  $S'$  in which every point of the branch locus appears as a vertex.