

let $\bar{e}_k: A_c^{p,q}(\mathbb{C}^n) \longrightarrow A_c^{p,q+1}(\mathbb{C}^n)$ similarly be given by
 $\bar{e}_k(\varphi) = d\bar{z}_k \wedge \varphi.$

Let \bar{i}_k and $\bar{\tau}_k$ be the adjoints of e_k and \bar{e}_k , respectively.
 Note that $e_k, \bar{e}_k, \bar{i}_k$ & $\bar{\tau}_k$ are all linear over $C^\infty(\mathbb{C}^n)$.

$$\begin{aligned} \bar{i}_k(\varphi) &= d\bar{z}_k \wedge \varphi & e_k(f\varphi) &= d\bar{z}_k \wedge f\varphi = f d\bar{z}_k \wedge \varphi \\ &= f e_k(\varphi). & \langle \varphi, \bar{i}_k(f\eta) \rangle &= \langle e_k(\varphi), f\eta \rangle = \langle \bar{f} e_k(\varphi), \eta \rangle \\ &= \langle e_k(\bar{f}\varphi), \eta \rangle = \langle \bar{f}\varphi, \bar{i}_k(\eta) \rangle = \langle \varphi, f \bar{i}_k(\eta) \rangle \\ &\Rightarrow \bar{i}_k(f\eta) = f \bar{i}_k(\eta). \quad \Rightarrow \bar{i}_k \text{ is linear over } C^\infty(\mathbb{C}^n). \end{aligned}$$

Now $\bar{i}_k(dz_J \wedge d\bar{z}_K) = 0$, if $k \in J$
 and, recalling that the length $\|d\bar{z}_k\| = 1$,
 $\bar{i}_k(dz_k \wedge dz_J \wedge d\bar{z}_K) = dz_J \wedge d\bar{z}_K$;

since in the former case, we have for any multiindexes L and M

$$(\bar{i}_k(dz_J \wedge d\bar{z}_K), dz_L \wedge d\bar{z}_M) = (dz_J \wedge d\bar{z}_K, dz_k \wedge dz_L \wedge d\bar{z}_M) = 0,$$

so $\bar{i}_k(dz_J \wedge d\bar{z}_K) = 0$, while in the latter case

$$(\bar{i}_k(dz_k \wedge dz_J \wedge d\bar{z}_K), dz_L \wedge d\bar{z}_M) = (dz_k \wedge dz_J \wedge d\bar{z}_K, dz_k \wedge dz_L \wedge d\bar{z}_M) = dz_J \wedge d\bar{z}_K, dz_L \wedge d\bar{z}_M).$$

Similarly, we see that

$$\bar{\tau}_k(dz_J \wedge d\bar{z}_K) = 0, \quad \text{if } k \in K,$$

and

$$\bar{\tau}_k(d\bar{z}_k \wedge dz_J \wedge d\bar{z}_K) = dz_J \wedge d\bar{z}_K.$$

For any multiindexes L and M ,

$$\begin{aligned} \langle \bar{\tau}_k(dz_J \wedge d\bar{z}_K), dz_L \wedge d\bar{z}_M \rangle &= \langle dz_J \wedge d\bar{z}_K, d\bar{z}_k \wedge dz_L \wedge d\bar{z}_M \rangle \\ &= 0 \quad \Rightarrow \quad \bar{\tau}_k(dz_J \wedge d\bar{z}_K) = 0. \quad \text{)} \end{aligned}$$

Note also that for any monomial $dz_J \wedge d\bar{z}_K$,