6 distinct branch points.

$\Rightarrow$ $\#(V_c \cap \{C_\lambda\}) \geq 6$, counting multiplicities.

③ Assume $C: X_0^2 + X_1^2 + X_2^2 = 0$.

Let L be a generic pencil. $\Rightarrow$ We may express L as follows:

$$\{ f(X_1, X_2) + \lambda g(X_1, X_2) X_0 = 0 \}$$

f is a quadratic, g is of degree 1, since if $\exists$ a term X_0^2, plug in $X_0^2 = -X_1^2 - X_2^2$.

To solve the equation,

$$\lambda g X_0 = -f \Rightarrow X_0 = -\frac{f}{g\lambda}$$

Plug it in $X_0^2 + X_1^2 + X_2^2 = 0$, then

$$\frac{f^2}{g^2\lambda^2} + X_1^2 + X_2^2 = 0$$

$$\Rightarrow f^2 + \lambda^2 g^2 X_1^2 + g^2\lambda^2 X_2^2 = 0$$

$\Rightarrow$ This is of the following form:

$$\circledast \cdots a_1(\lambda) X_1^4 + a_2(\lambda) X_1^3 X_2 + a_3(\lambda) X_1^2 X_2^2 + a_4(\lambda) X_1 X_2^3 + a_5(\lambda) X_2^4 = 0,$$

where $a_i(\lambda)$ is a polynomial in λ of degree ≤ 2.

To get λ's for which $\circledast$ has a double root, we consider the discriminant of $\circledast$, i.e.,

$$\Delta = (\alpha_1-\alpha_2)^2(\alpha_1-\alpha_3)^2(\alpha_1-\alpha_4)^2(\alpha_2-\alpha_3)^2(\alpha_2-\alpha_4)^2(\alpha_3-\alpha_4)^2.$$