

Suppose that we decompose Z into two disjoint sets P_0 and P_1 ; we may think of P_0 as part of the base of the pencil $|s + \lambda s'| \subset |L|$, and shall refer to P as the residue of P_0 .

Γ s & s' are linearly independent sections of L .

$\Rightarrow \{s + \lambda s' \mid \lambda \in \mathbb{P}^1\}$ is a line in $|H^0(S, \mathcal{O}(L))| = |L|$

$\Rightarrow |s + \lambda s'|_{\lambda \in \mathbb{P}^1}$ is a pencil in $|L|$.

Any $s + \lambda s'$ vanishes on Z . \Rightarrow

Reciprocity Formula (II). With the above notations;

$$h^1(\mathcal{I}_{P_0}(L)) = h^0(\mathcal{I}_P(K+L)) - 2P_g + h^0(\mathcal{O}(K-L)).$$

In particular, if both $q = P_g = 0$, then

$$h^1(\mathcal{I}_{P_0}(L)) = h^0(\mathcal{I}_P(K+L)).$$

Γ Here $K = K_S$, and by $p \in P$. $P_g(S) =$

$$h^{2,0}(S). \quad h^0(\mathcal{O}(K-L)) = \dim H^0(S, \mathcal{O}(K-L))$$

$$= \dim H^0(S, \Omega^2(-L)) = \dim H^2(S, \Omega^0(L)) = \tilde{h}^2(\mathcal{O}(L))$$

$$= 0, \text{ for, first } H^2(S, \mathcal{O}) = 0 \quad (\because P_g = 0).$$

This implies that the divisor K_S is "some sort of" negative. Second, since L has a section, the divisor L is "some sort of" positive.