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$x \in G = G(2,4)$ and x means l_x, see P197.

By Pieri's formula on P203, since, if $a = a, 0, 0 \cdots$

$$\sigma_a \cdot \sigma_b = \sum_{\substack{b_i \le c_i \le b_{i-1} \\ \sum c_i = a + \sum b_i}} \sigma_c ,$$

① $\sigma_1 \cdot \sigma_1 = ?$ $a = 1, 0$ $b = \underset{b_1}{\overset{}{1}}, \underset{b_2}{0}$. and, by P202, $b_0 = n-k = 4-2=2$.

$\Rightarrow c_1 + c_2 = 2$

$\Rightarrow \sigma_1 \cdot \sigma_1 = \sigma_2 + \sigma_{1,1}$.

② $\sigma_1 \cdot \sigma_2 = ?$

$a = 1, 0$ $b = 2, 0$ $\Rightarrow c_1 + c_2 = 3$ $\Rightarrow$ $2 \le c_1 \le 2 \Rightarrow c_1 = 2$

$\Rightarrow c_2 = 1$ $\Rightarrow$ $\sigma_1 \cdot \sigma_2 = \sigma_{2,1}$

③ $\sigma_1 \cdot \sigma_{1,1} = ?$

$a = 1, 0$, $b = 1, 1$ $\Rightarrow$ $c_1 + c_2 = 3$ $1 \le c_1 \le 2$ $\Rightarrow$ If $c_1 = 2$,

$\Rightarrow c_2 = 1$. Otherwise, if $c_1 = 1$, then $c_2 = 2$

$\Rightarrow$ Contradiction to the fact that $c_1 \ge c_2$.

$\Rightarrow$ Thus $\sigma_1 \cdot \sigma_{1,1} = \sigma_{2,1}$

④ $\sigma_2 \cdot \sigma_2 = ?$

$a = \underset{a_1}{2}, 0$. $b = 2, 0$ $\Rightarrow$ $c_1 + c_2 = 4$, $2 \le c_1 \le 2 \Rightarrow c_1 = 2$

$\Rightarrow c_2 = 2$. $\Rightarrow$ $\sigma_2 \cdot \sigma_2 = \sigma_{2,2} = 1$

⑤ $\sigma_{1,1} \cdot \sigma_{1,1} = ?$

$a = 1, 1$ $b = 1, 1$ $\Rightarrow$ $\overset{\not= 1}{a_1} = 2 - \overset{=1}{b_2}$ $\overset{\not= 1}{a_2} = 1 = 2 - \overset{1}{b_1}$

$\Rightarrow$ By the result on P198, $\sigma_{1,1} \cdot \sigma_{1,1} = 1$

⑥ $\sigma_1 \cdot \sigma_{2,1} = ?$

$a = 1, 0$ $b = 2, 1$ $c_1 + c_2 = 4$ $2 \le c_1 \le 2$ $c_1 = 2$