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$\Rightarrow \quad \phi'_z \circ \psi'^{-1}_z = \begin{pmatrix} a_{11}(z), & a_{12}(z) \\ a_{21}(z), & a_{22}(z) \end{pmatrix}$ where each $a_{ij}(z)$ is holomorphic.

as ⊛. $\Rightarrow$ Thus locally free sheaves of rank r are exactly the sheaves $\mathcal{O}(E)$, rank $E = r$, $E \to M$ holomorphic vector bundle. ⌟

A subsheaf $I \subset \mathcal{O}$ that is locally finitely generated is called an ideal sheaf or sheaf of ideals.

⌈ I should be considered as a sheaf of $\mathcal{O}$-modules. ⌟

By Oka's lemma, these are always coherent, and because of the exact sequence

$$0 \longrightarrow I \to \mathcal{O} \longrightarrow \mathcal{O}/I \longrightarrow 0$$

the same is true for $\mathcal{O}/I$.

⌈ Consider the sheaf map $F: \mathcal{O}^{(p)} \longrightarrow \mathcal{O}$ defined by $e_i \longmapsto f_i$, where $I = \{f_1, \cdots f_p\}$

$\Rightarrow$ By Oka's lemma, $\ker F = \mathcal{R}$ is finitely generated locally as a sheaf of $\mathcal{O}$-modules. $\Rightarrow \exists\ \mathcal{O}^{(k)}$ s.t

$$\begin{tikzcd} \mathcal{O}^{(k)} \arrow[r] \arrow[dr] & \mathcal{O}^{(p)} \arrow[r] & \mathcal{O} \\ & \ker F \arrow[u] & \end{tikzcd}$$

, in particular,

$\mathcal{O}^{(k)} \longrightarrow \mathcal{O}^{(p)} \longrightarrow I \longrightarrow 0$, which implies that I is coherent. $0 \longrightarrow \mathcal{O} \longrightarrow \mathcal{O} \longrightarrow 0$ says that $\mathcal{O}$ is coherent. $\Rightarrow$ By ⊛ above, $\mathcal{O}/I$ is coherent. ⌟