$\Rightarrow \quad \sum \bar{S}_{\alpha i} f_{\alpha i} \in \overset{s_\alpha}{\overset{\shortparallel}{\mathcal{M}(U_\alpha)}}$ & $\overset{s_\beta}{\overset{\shortparallel}{\sum \bar{S}_{\beta i} f_{\beta i}}} \in \mathcal{M}(U_\beta)$. and they have the relation $g_{\alpha\beta} s_\beta = s_\alpha$.

Given two meromorphic sections $s, s' \not\equiv$ of L,
$s'|_{U_\alpha}, s|_{U_\alpha} \in \mathcal{O}(L)(U_\alpha) \otimes_{\mathcal{O}} \mathcal{M}(U_\alpha)$

$s'|_{U_\alpha} \longleftrightarrow f'|_{U_\alpha} \in \mathcal{M}(U_\alpha)$ s.t $f'|_{U_\alpha} = g_{\alpha\beta} \cdot f'|_{U_\beta}$

$s|_{U_\alpha} \longleftrightarrow f|_{U_\alpha} \in \mathcal{M}(U_\alpha)$. s.t $f|_{U_\alpha} = g_{\alpha\beta} \cdot f|_{U_\beta}$,

$\Rightarrow$ Define $\frac{s'}{s}\Big|_{U_\alpha}$ by $\frac{f'}{f}\Big|_{U_\alpha} = \frac{f'|_{U_\alpha}}{f|_{U_\alpha}}$.

$\Rightarrow$ It is a well-defined meromorphic function since
$\frac{s'}{s}\Big|_{U_\beta} = 1 \cdot \overset{\text{transition factor is } 1}{\overrightarrow{\frac{s'}{s}\Big|_{U_\alpha}}}$. $\Rightarrow$ This implies that any two meromorphic sections differ by multiplication of a meromorphic function.

If s is a global meromorphic section of L, $s_\alpha/s_\beta \in \mathcal{O}^*(U_\alpha \cap U_\beta)$, and so for any irreducible hypersurface $V \subset M$,

$$\mathrm{ord}_V(s_\alpha) = \mathrm{ord}_V(s_\beta). \quad (\because \ s_\alpha/s_\beta \in \mathcal{O}^*(U_\alpha \cap U_\beta))$$

$\ulcorner$ If S is a global meromorphic section of L,
we can identify s with a collection of meromorphic functions s_α satisfying. $s_\alpha = g_{\alpha\beta} \cdot s_\beta$. $\quad g_{\alpha\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta)$.

$\Rightarrow \mathrm{ord}_V(s|_{U_\alpha}) \equiv \mathrm{ord}_V(s_\alpha) \Rightarrow$ It is well-defined since $g_{\alpha\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta)$. $\lrcorner$

Thus we define the order of s along V by
$\mathrm{ord}_V(s) = \mathrm{ord}_V(s_\alpha)$ for any α s.t