

$V' \subset U$. \Rightarrow Since f is irreducible, f can not divide in some nbd V'' of q , $\therefore V'' \subset V' \subset U$.

$\Rightarrow f^a h = g$ in some nbd of p .

\Rightarrow This prove K is open.

\Rightarrow Outside K , by using the same argument above, we can show K is closed. K is closed and open

in the connected hypersurface V . $\Rightarrow K = V$. \rightarrow

Note that for g, h any holomorphic functions, V any irreducible hyper surface,

$$\text{ord}_V(gh) = \text{ord}_V(g) + \text{ord}_V(h).$$

\square $p \in V$, in the local ring $\mathcal{O}_{m,p}$.

$$g = f^a k_1$$

$$h = f^b k_2$$

$$f \nmid k_1, f \nmid k_2 \Rightarrow f \nmid k_1 k_2$$

$$\Rightarrow gh = f^a k_1 f^b k_2 = f^{a+b} k_1 k_2$$

\Rightarrow This implies the $\text{ord}_V(gh) = a+b$. \square

Now let f be a meromorphic function on M , written locally as

$$f = \frac{g}{h} \quad \text{with } g, h \text{ holomorphic and relatively prime.}$$

For V an irreducible hyper surface, we define

$$\text{ord}_V(f) = \text{ord}_V(g) - \text{ord}_V(h).$$

We usually say that f has a zero of order a along V if $\text{ord}_V(f) = a > 0$, and f has a pole of order a along V if $\text{ord}_V(f) = -a < 0$.

We define the divisor (f) of the meromorphic function f by

$$(f) = \sum_V \text{ord}_V(f) \cdot V.$$