The remaining three products of $\tilde{I}_p$ and $\tilde{I}_\ell$ are dual to the ones above – e.g., a conic $C \subset \mathbb{P}^2$ will be tangent to five lines $l_1, \dots, l_5$ if the dual conic $C^* \subset \mathbb{P}^{2*}$ of tangent lines to C contains the five points $l_1, \dots l_5 \in \mathbb{P}^{2*}$ – so we have

$$\tilde{I}_p^2 \tilde{I}_\ell^3 = \tilde{I}_p^3 \tilde{I}_\ell^2 = 4, \quad \tilde{I}_p \tilde{I}_\ell^4 = \tilde{I}_p^4 \tilde{I}_\ell = 2, \quad \tilde{I}_\ell^5 = \tilde{I}_p^5 = 1.$$

For example, a conic C is tangent to l_1, l_2 and passes p_1, p_2, p_3 if C^* contains $l_1, l_2 \in \mathbb{P}^{2*}$, and C^* is tangent to $p_1, p_2, p_3 \subset \mathbb{P}^{2*}$. See p 278. The dual of the dual is the original curve.

The answer to the problem – modulo the checking of transversality assumptions – is then

$$\begin{aligned} \tilde{V}_c^5 &= 32\,(1 + 5\cdot 2 + 10\cdot 4 + 10\cdot 4 + 5\cdot 2 + 1) \\ &= 32 \cdot 102 = 3264. \end{aligned}$$

2. The Quadric Line Complex: Introduction

Geometry of the Grassmannian $G(2,4)$

First we will discuss the geometry of the Gr–