

connected and we can set

$$\pi(s) = \int_{s_0}^s \omega$$

to obtain a holomorphic function π in Δ with $d\pi = \omega$. Consider the meromorphic differential $\pi \cdot \eta$ on Δ ; since η is smooth on the arcs δ_i , the integral of $\pi \cdot \eta$ along the boundary of Δ is well-defined, and by the same argument as used in the first reciprocity law

$$\int_{\partial \Delta} \pi \cdot \eta = \sum_{i=1}^g (\pi^{\bar{i}} N^{\theta+i} - \pi^{\theta+i} N^{\bar{i}}).$$

What's the difference? Same \Rightarrow

On the other hand, near a singular point p of η with local coordinate z as above,

$$\pi(z) = \int_{s_0}^z \omega = b_0^p z + \frac{1}{2} b_1^p z^2 + \frac{1}{3} b_2^p z^3 + \dots$$

So that

$$\int_{\partial \Delta} \pi \cdot \eta = 2\pi\sqrt{-1} \sum_p \text{Res}_p(\pi \cdot \eta) = 2\pi\sqrt{-1} \sum_p \left[\sum_{j=2}^n \frac{a_j^p \cdot b_{j-2}^p}{j-1} \right]$$

$$\begin{aligned} \Gamma \quad \pi \cdot \eta = & \dots + \frac{1}{z} (b_0^p \cdot a_{-2}^p + \frac{1}{2} b_1^p \cdot a_{-3}^p + \frac{1}{3} b_2^p \cdot a_{-4}^p + \dots \\ & + \frac{1}{n-1} b_{n-2}^p a_{-n}^p) + \dots \end{aligned}$$

$$\Rightarrow \int_p \pi \cdot \eta = \sum_{j=2}^n \frac{a_j^p b_{j-2}^p}{j-1}$$

Thus we have the reciprocity law for differentials of

