

⌈ Note that the canonical curve is nondegenerate.

If not, $\exists a_1, a_2, \dots, a_g$ s.t. $a_1 \omega_1(p) + \dots + a_g \omega_g(p) = 0$ for all p , since $\bar{L}_K(S) \subset H$, H a hyperplane of \mathbb{P}^{g-1} .

$\Rightarrow a_1 \omega_1 + \dots + a_g \omega_g = 0 \Rightarrow \{\omega_1, \dots, \omega_g\}$ is linearly dependent \Rightarrow Contradiction.

\Rightarrow Given a hyperplane H in \mathbb{P}^{g-1} , $H \cap \bar{L}_K(S)$ is a set of points, which spans the whole hyperplane. If it does not span, we have some other hyperplane H' s.t. $\#(H' \cdot \bar{L}_K(S)) > \#(H \cdot \bar{L}_K(S))$ which is impossible.

Note that $H \cap \bar{L}_K(S)$ represents an effective divisor K_S on S , see P177

$$\begin{array}{ccc} & K_S & [H] \\ & \downarrow & \downarrow \\ S & \xrightarrow{L_K} & \bar{L}_K(S) \subset \mathbb{P}^{g-1} \end{array}$$

$$\Rightarrow L_K^*[H] = K_S = L_K^*([H]|_S).$$

\Rightarrow If we have a hyperplane H in \mathbb{P}^{g-1} containing D , then $H \cap \bar{L}_K(S) - D \sim K - D$ and it spans the space which makes \mathbb{P}^{g-1} with \bar{D} . $\Rightarrow \dim(H \cap \bar{L}_K(S) - D) \geq l$, where $\dim \bar{D} + 1 + l + 1 = g$.

$$\Rightarrow l = g - 1 - \dim \bar{D} - 1 = g - 1 - (d - s - 1) - 1 = g - d + s - 1.$$

$$\text{Take } H \cap \bar{L}_K(S) - \bar{D} \Rightarrow \dim(H \cap \bar{L}_K(S) - \bar{D})$$

$$= l = g - d + s - 1. \Rightarrow H \cap \bar{L}_K(S) - \bar{D} \text{ is a subseries of an effective divisor linearly equivalent to } K - D. \Rightarrow$$

Applying (*) to a divisor $E \in |K - D|$, then, we see that.