Date

Now we are essentially done. By hypothesis,

$$\mu(D) = \left(\sum_{\lambda'} \int_{\alpha_{\lambda'}} \omega_1, \cdots, \sum_{\lambda'} \int_{\alpha_{\lambda'}} \omega_g \right) \in \Lambda,$$

i.e., there exists a cycle

$$\gamma \sim \sum_{k=1}^{2g} m_k \cdot \delta_k, \quad m_k \in \mathbb{Z}$$

such that for each i,

$$\sum_{\lambda'} \int_{\alpha_{\lambda'}} \omega_i = \int_\gamma \omega_i,$$

and so $\quad N^{g+i} = \int_\gamma \omega_i \quad$ for all i.

$\ulcorner$ By hypothesis $\mu(D) = 0$ in $J(S) = \mathbb{C}^g/\Lambda$.

$$\Rightarrow \mu(D) = \left(\underset{\Lambda}{\sum_{\lambda'} \int_{\alpha_{\lambda'}} \omega_1, \cdots \sum_{\lambda'} \int_{\alpha_{\lambda'}} \omega_g} \right)$$

$$\Lambda = \{ m_1 \Pi_1 + \cdots + m_{2g} \Pi_{2g}, \ m_i \in \mathbb{Z} \}$$

$$\Pi_i = \begin{pmatrix} \int_{\delta_i} \omega_1 \\ \vdots \\ \int_{\delta_i} \omega_g \end{pmatrix} \in \mathbb{C}^g.$$

$$\Rightarrow \exists \ \gamma \sim \sum_{k=1}^{2g} m_k \cdot \delta_k \quad \text{s.t}$$

$$\mu(D) = m_1 \Pi_1 + \cdots + m_{2g} \Pi_{2g} = m_1 \begin{pmatrix} \int_{\delta_1} \omega_1 \\ \vdots \\ \int_{\delta_1} \omega_g \end{pmatrix} + m_2 \begin{pmatrix} \int_{\delta_2} \omega_1 \\ \vdots \\ \int_{\delta_2} \omega_g \end{pmatrix} + \cdots$$