Thus if we consider $f(z_1, z_2) = f(\frac{1}{w_0}, \frac{w_2}{w_0})$, since f is polynomial, f can be extended to $\mathbb{P}^2$ as easily seen. F has degree d, and F is irreducible. $\Rightarrow$ f is a polynomial of degree d. The following case can not happen:

$$F(Z_0, Z_1, Z_2) = Z_0^2 + Z_0 Z_1 \quad \Rightarrow \quad f(z_1, z_2) = 1 + z_1$$

$\Rightarrow$ But F is reducible.

Since f is a polynomial of deg d, $f(\frac{1}{w_0}, \frac{w_2}{w_0})$ has a pole of order d along $w_0 = 0$.

$$\lim_{w_0 \to 0} f(\frac{1}{w_0}, \frac{w_2}{w_0}) w_0^d = \text{finite} \neq 0.$$

It follows that g must extend to a meromorphic function with a pole of order $\leq d-3$ along L, i.e., g must be a polynomial of degree $\leq d-3$ in z_1, z_2.

If we write $\omega = h \, dz_1 \wedge dz_2$, $h = \frac{g}{f}$.

$\Rightarrow \omega = \frac{g}{f} dz_1 \wedge dz_2$ and ω is a meromorphic 2-form on $\mathbb{P}^2$

$\Rightarrow$ Since f & $dz_1 \wedge dz_2$ can be extend to $\mathbb{P}^2$, g can be extended to $\mathbb{P}^2$ as a meromorphic function.

$\Rightarrow$ By P168, since any meromorphic function on $\mathbb{P}^n$ is rational, g is a quotient of two polynomials, i.e., $g = \frac{G}{F}$ where F, G homogeneous polynomials of the same degree on $\mathbb{P}^2$.

Since $dz_1 \wedge dz_2$ & f have a pole of order 3 and d respectively, $\frac{dz_1 \wedge dz_2}{f}$ has a pole of order