Date . .

We can express this requirement more intrinsically as follows: let $\mathcal{J}_x \subset \mathcal{O}$ denote the sheaf of holomorphic functions on M vanishing at x, and let $\mathcal{J}_x(L)$ be the sheaf of sections of L vanishing x.

If s is any section of $\mathcal{J}_x(L)$ defined near x, and φ_α, φ_β are trivializations of L in a nbd U of x, then writing $s_\alpha = \varphi_\alpha^* s$, $s_\beta = \varphi_\beta^* s$, $s_\alpha = g_{\alpha\beta} s_\beta$, we have

$$d(s_\alpha) = d(s_\beta)\cdot g_{\alpha\beta} + dg_{\alpha\beta}\cdot s_\beta = d(s_\beta)\cdot g_{\alpha\beta} \text{ at } x.$$

⸢ Since $s_\beta(x) = 0 = s_\alpha(x)$, $\cdots$. See P36. for sheaves.⸥

Thus we have a well-defined sheaf map

$$d_x : \mathcal{J}_x(L) \longrightarrow T_x^{*\prime} \otimes L_x$$

and condition 2 can be stated as requiring that the map

$$(**) \qquad H^0(M, \mathcal{J}_x(L)) \xrightarrow{d_x} T_x^{*\prime} \otimes L_x$$

be surjective for all $x \in M$.

⸢ $(T_x^{*\prime} \otimes L_x)(U) = T_x^{*\prime} \otimes L_x$ is a group.

Define $d_x^U : \mathcal{J}_x(L)(U) \longrightarrow T_x^{*\prime} \otimes L_x(U)$ by

(i) $U \ni x$,

$d_x(s) = ds_\alpha$, where $s_\alpha = \varphi_\alpha^* s$.