

$$= \frac{1}{q!} C_q(\Theta). \quad (\text{Here we use } (\sqrt{-1})^{q^2} = (\sqrt{-1})^{q^2-q} \cdot (\sqrt{-1})^q)$$

$$= \left((\sqrt{-1})^{\frac{q^2-q}{2}} \right)^2 (\sqrt{-1})^q = \left((\sqrt{-1})^2 \right)^{\frac{q^2-q}{2}} (\sqrt{-1})^q = (-1)^{\frac{q(q-1)}{2}} (\sqrt{-1})^q.$$

“Comment on $P^k(A) = \sum_{\#I=k} \det(A_{I,I})$

$$\det(I + tA) = 1 + \dots + t^k P^k(A) + \dots$$

$\det(A_{I,I})$ is the sum of coefficients of t^k in
 $\in (1 + t a_{j_1 j_1}) (1 + t a_{j_2 j_2}) \dots (1 + t a_{j_{n-k} j_{n-k}}) t a_{i_1 \sigma(i_1)} \dots t a_{i_k \sigma(i_k)}$
 where $\{i_1 \dots i_k\} = I$

$$\{j_1 \dots j_{n-k}\} = \{1, \dots, n\} - I.$$

Thus every term of $\det(A_{I,I})$ is in $P^k(A)$ and vice versa.

$$\int_Z C_q(\Theta) = q! \sum_{\mu} (\sqrt{-1})^{q^2} \int_Z \eta_{\mu} \wedge \bar{\eta}_{\mu}$$

$$(\sqrt{-1})^{q^2} \int_Z \eta_{\mu} \wedge \bar{\eta}_{\mu} = ?$$

$$\begin{aligned} dz_1 \wedge \dots \wedge dz_q \wedge d\bar{z}_1 \wedge \dots \wedge d\bar{z}_q &= dz_1 \wedge d\bar{z}_1 \wedge \dots \wedge dz_q \wedge d\bar{z}_q (-1)^{\frac{q(q-1)}{2}} \\ &= (-2i \, dx_1 \wedge dy_1) \wedge \dots \wedge (-2i \, dx_q \wedge dy_q) (-1)^{\frac{q(q-1)}{2}} \\ &= (-1)^q i^q \, dx_1 \wedge dy_1 \wedge \dots \wedge (dx_q \wedge dy_q) (-1)^{\frac{q(q-1)}{2}} \\ &= (-1)^q \sqrt{-1}^q \, dx_1 \wedge \dots \wedge dx_q \wedge dy_1 \wedge \dots \wedge dy_q. \\ &= (-1)^q (-1)^{\frac{q(q-1)}{2}} \sqrt{-1}^q \text{ volume form.} \end{aligned}$$

$$\begin{aligned} \Rightarrow (\sqrt{-1})^{q^2} \int_Z \eta_{\mu} \wedge \bar{\eta}_{\mu} &= (\sqrt{-1})^{q^2} \int |\varphi|^2 (-1)^q (-1)^{\frac{q(q-1)}{2}} (\sqrt{-1})^q \text{ volume form} \\ &= (\sqrt{-1})^{q^2+q} (-1)^{\frac{q(q+1)}{2}} \int |\varphi|^2 \text{ vol} = (-1)^{\frac{q^2+q}{2}} (-1)^{\frac{q^2+q}{2}} \int |\varphi|^2 \text{ vol} \geq 0. \end{aligned}$$