Such an automorphism is given as multiplication by a nonzero complex number; if we denote this number by $e_\lambda(z)$, we obtain a collection of functions

$$\{ e_\lambda \in \mathcal{O}^*(V) \}_{\lambda \in \Lambda}$$

called a set of multipliers for L. The functions e_λ necessarily satisfy the compatibility relation

$$e_{\lambda'}(z+\lambda)\, e_\lambda(z) = e_\lambda(z+\lambda')\, e_{\lambda'}(z) = e_{\lambda+\lambda'}(z)$$

for all $\lambda, \lambda' \in \Lambda$.

$\ulcorner$ $e_\lambda(z) = \varphi_{z+\lambda} \circ \varphi_z^{-1}$

$$\Rightarrow \quad e_{\lambda+\lambda'}(z) = \varphi_{z+\lambda+\lambda'} \circ \varphi_z^{-1} = \varphi_{z+\lambda+\lambda'} \circ \varphi_{z+\lambda}^{-1} \circ \varphi_{z+\lambda} \circ \varphi_z^{-1}$$

$$= e_{\lambda'}(z+\lambda) \cdot e_\lambda(z) = \varphi_{z+\lambda+\lambda'} \circ \varphi_{z+\lambda'}^{-1} \circ \varphi_{z+\lambda'} \circ \varphi_z^{-1}$$

$$= e_\lambda(z+\lambda') \cdot e_{\lambda'}(z).$$

$\lrcorner$

Conversely, given any collection of entire nonzero holomorphic functions $\{e_\lambda\}_{\lambda \in \Lambda}$ satisfying these relations we can construct a line bundle $L \longrightarrow M$ having multipliers $\{e_\lambda\}$: we take L to be the quotient space of $V \times \mathbb{C}$ under the identifications

$$(z, \xi) \sim (z+\lambda, e_\lambda(z) \cdot \xi).$$

$\ulcorner$

$$L = V \times \mathbb{C} \Big/ (z, \xi) \sim (z+\lambda, e_\lambda(z) \cdot \xi).$$

$\lrcorner$