

$$[z_0, z_1, z_2] \longmapsto [z_0^2, z_0 z_1, z_0 z_2, z_1^2, z_1 z_2, z_2^2]$$

$\Rightarrow$  A hyperplane section of  $i_2(\mathbb{P}^2)$  is given as follows:

$$a_0 z_0^2 + a_1 z_0 z_1 + a_2 z_0 z_2 + a_3 z_1^2 + a_4 z_1 z_2 + a_5 z_2^2 = 0$$

for some  $a_0, a_1, a_2, a_3, a_4, a_5$ .

$\Rightarrow$  Clearly it is a curve of  $\deg 2$  in  $\mathbb{P}^2$ , since  $i_2$  is a smooth embedding and  $\{[z_0, z_1, z_2] : a_0 z_0^2 + \dots + a_5 z_2^2 = 0\}$  is a curve of degree 2. In general, we can see from the example above that the hyperplane sections of  $i_n(\mathbb{P}^2)$  are just the curves of degree  $n$ .

If we prove that  $\{i_n(P_U)\}$  lie on a  $\mathbb{P}^{n-2}$  in  $\mathbb{P}^N$ , consider  $\{H \cap i_n(\mathbb{P}^2) \mid H \text{ is a hyperplane containing the } \mathbb{P}^{n-2} \text{ in } \mathbb{P}^N\}$ , which is a set of curves of degree  $n$ .

$\Rightarrow$  We have only to prove that  $\dim \{H \cap i_n(\mathbb{P}^2)\}$  is equal to  $1+1$ , since  $\dim$  of linear system is <sup>one</sup> less than the  $\dim$  of sections.

$$\sigma_{a,b}([z_0, \dots, z_N]) = \{[z_0, \dots, z_N] \mid a z_{N-1} + b z_N = 0\}$$

$$\sigma_{a,b} = a \sigma_1 + b \sigma_2, \text{ where } \sigma_1 = \{[z_0, \dots, z_N] \mid z_{N-1} = 0\}$$

$$\sigma_2 = \{z_N = 0\}, \text{ when we assumed the } \mathbb{P}^{n-2} = \{z_{N-1} = z_N = 0\}.$$

$$\dim \{[a, b]\} = 1 \Rightarrow \text{Pencil see P137.} \quad \sqcup$$

Suppose we select  $N$  of the points  $P_U$ , say  $P_1, \dots, P_{\frac{n(n+3)}{2}}$  for simplicity of notation, and let  $A \subset \mathbb{P}^N$  be any hyperplane containing  $N-1$  of them, say  $P_2, \dots, P_{\frac{n(n+3)}{2}}$ .

Since

$$h^2 = \frac{n(n+3)}{2} + \frac{n(n-3)}{2}$$

and  $\dim H^0(\mathbb{P}^2, \mathcal{O}((n-3)H)) = n(n-3)/2 + 1$ , we may find a curve  $B$  of degree  $n-3$  passing through  $P_{\frac{n(n+3)}{2}+1}$ .