$z_1]) = \phi([z_0, z_1], [w_0, w_1]) \Rightarrow \exists$ a continuous function from $\mathbb{P}^{1(2)}$ to $\mathbb{P}^2$. But on $U_0 \times U_0 \subset \mathbb{P}^1 \times \mathbb{P}^1$,
$U_0 = (z_0 \neq 0) = (w_0 \neq 0)$.

$$\begin{tikzcd} U_0 \times U_0 \arrow[r, "\phi"] \arrow[d, "\mathbb{C}^2"] & U_0 = (\cancel{z_0} \neq 0) \subset \mathbb{P}^2 \arrow[d, "\mathbb{C}^2"] \\ (z, w) \arrow[r, mapsto] & (z+w, zw) \end{tikzcd}$$

$$z = \frac{z_1}{z_0}, \quad w = \frac{w_1}{w_0}$$

$\Rightarrow$ The induced continuous function ϕ is holomorphic. Thus we have only to check that ϕ is one to one by P19. Proposition.

Given $[a, b, c] \in \mathbb{P}^2$, solve $z_0 w_0 = a$, $z_1 w_0 + z_0 w_1 = b$, $z_1 w_1 = b$. If $a \neq 0$ or $b \neq 0$, we can find easily, since

$$\frac{z_1 w_0 + z_0 w_1}{z_0 w_0} = \frac{b}{a} = \frac{z_1}{z_0} + \frac{w_1}{w_0} = z + w.$$

$$\frac{z_1 w_1}{z_0 w_0} = zw = \frac{c}{a} \quad \Rightarrow \quad t^2 - \frac{b}{a} t + \frac{c}{a} = 0 \cdots (*)$$

z, w are the roots of $(*)$ $\Rightarrow$ There are two possibilities for $[z_0, z_1]$, $[w_0, w_1]$, for exaple $2, 3 \Rightarrow$
$[1, 3]$ & $[1, 2]$ or $[1, 2]$ & $[1, 3]$.

If $a = 0$, assume $z_0 = 0 \Rightarrow z_1 w_0 = b$
$z_1 w_1 = c \Rightarrow w_1 = \frac{c}{z_1}$, $w_0 = \frac{b}{z_1}$

$$\Rightarrow \underset{[0, 1]}{[0, z_1]} \quad \left[\frac{b}{z_1}, \frac{c}{z_1}\right] = [b, c]$$

If $w_0 = 0$, we have $[b, c]$, $[0, 1]$.
From the argument above, we can conclude that ϕ is one to one.