$\Rightarrow [\varphi]$ is in the image of $H^p_{DR}(M) \longrightarrow H^p_{DR}(U)$. ⌟

Historically, differentials of the second kind for $p=1,2$ played a pivotal role in the early development of the theory of algebraic surfaces. They furnished the technique for the first proof that the irregularity q of an algebraic surface was equal to $\frac{1}{2}b_1$ — so that in particular b_1 is even — and the original proof that the Neron-Severi group defined below

$$\frac{\{\text{divisors on } S\}}{\{\text{divisors algebraically equivalent to zero}\}}$$

is finitely generated. For $p \geq 3$ the differentials of the second kind are only partially understood, and even that is fairly recent. Because of their historical importance and close tie-in with the algebraic de Rham theorem, we shall give a brief discussion of differentials of the second kind with special emphasis on the cases $p=1,2$.

We begin by amplifying the definition of second kind in two ways. Given a closed meromorphic p-form φ and divisor D such that φ is holomorphic in $U = M - D$, we define a residue to be an integral

$$\int_\gamma \varphi,$$

where $\gamma \in H_p(U, \mathbb{Z})$ is a p-cycle that is homologous to zero in M. It is clear that φ is of the second kind $\Leftrightarrow$ it has no residues in open sets