

where $\text{Gr}^p K^* = \frac{F^p K^*}{F^{p+1} K^*}$

and the differential is the obvious one. The filtration $F^p K^*$ on K^* also induces a filtration $F^p H^*(K^*)$ on the cohomology by $F^p H^q(K^*) = \frac{F^p Z^q}{F^p B^q}$.

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$$\begin{array}{ccccccc} K^0 & \xrightarrow{d} & K^1 & \xrightarrow{d} & K^2 & \xrightarrow{d} & K^3 \rightarrow \dots \rightarrow K^q \\ \cup & & \cup & & \cup & & \cup \\ F^1 K^0 & \xrightarrow{d} & F^1 K^1 & \xrightarrow{d} & F^1 K^2 & \xrightarrow{d} & F^1 K^3 \rightarrow \dots \rightarrow F^1 K^q \\ \cup & & \cup & & \cup & & \cup \\ F^2 K^0 & \xrightarrow{d} & F^2 K^1 & \xrightarrow{d} & F^2 K^2 & \xrightarrow{d} & F^2 K^3 \rightarrow \dots \rightarrow F^2 K^q \end{array}$$

Let $F^1 Z^q = \ker d$, $d: F^1 K^q \rightarrow F^1 K^{q+1}$
 $F^1 B^q = \text{im } d$, $d: F^1 K^{q-1} \rightarrow F^1 K^q$.

Define $F^1 H^q(K^*) = \frac{F^1 Z^q}{F^1 B^q}$, similarly,

$$F^{1+} H^q(K^*) = \frac{F^1 Z^q}{F^1 B^q}.$$

$\Rightarrow F^{1+} H^q(K^*) \longrightarrow F^1 H^q(K^*)$ is well-defined
 $\alpha + F^{1+} B^q \longmapsto \alpha + F^1 B^q$

Since, for any $y \in F^{1+} B^q$,

$$y \in F^2 B^q = \text{im } d$$

$$F^1 B^q, (\because F^{1+} K^q \subset F^1 K^q).$$

$$\begin{array}{c} y = d \alpha^{q-1}, \alpha^{q-1} \in F^{1+} K^{q-1} \\ \downarrow \\ \boxed{F^{1+} H^q(K^*) \longrightarrow H^q(K^*)} \end{array}$$

The associated graded cohomology is