

$$\frac{V}{\mathbb{Z}\{\lambda_1, \dots, \lambda_n\}} \cong (\mathbb{C}^*)^n,$$

and we can factor our projection map  $\pi: V \rightarrow M$  by

$$V \rightarrow \frac{V}{\mathbb{Z}\{\lambda_1, \dots, \lambda_n\}} \xrightarrow{\pi_1} M.$$

$\mathbb{F}$   $(1, 0)$ ,  $(3, 3)$  and  $(3, 2)$  form a basis for  $\mathbb{Z}^2$  but they are <sup>not</sup> linearly independent.

$$\frac{V}{\mathbb{Z}\{\lambda_1, \dots, \lambda_n\}} \cong \frac{\mathbb{C}^n}{\mathbb{Z}\{\lambda_1, \dots, \lambda_n\}} \cong \frac{\mathbb{C}^n}{\mathbb{Z}^n = \mathbb{Z}\left\{\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix}\right\}} \xrightarrow{\psi} \frac{\mathbb{C}^n}{\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \mapsto \begin{pmatrix} \lambda_1 & \dots & \lambda_n \end{pmatrix}^{-1} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}}$$

$$\Rightarrow \frac{\mathbb{C}^n}{\mathbb{Z}^n} \xrightarrow{\phi} (\mathbb{C}^*)^n$$

$$\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \mapsto \begin{pmatrix} e^{2\pi i \alpha_1} \\ \vdots \\ e^{2\pi i \alpha_n} \end{pmatrix} \Rightarrow \phi \text{ is isomorph.}$$

$$\Rightarrow \frac{V}{\mathbb{Z}\{\lambda_1, \dots, \lambda_n\}} \cong (\mathbb{C}^*)^n.$$

What is  $\pi_1$ ?  $\pi_1: \frac{V}{\mathbb{Z}\{\lambda_1, \dots, \lambda_n\}} \rightarrow$

Now we have also seen on P. 27 that

$$H^1((\mathbb{C}^*)^n, \mathcal{O}) = H^2((\mathbb{C}^*)^n, \mathcal{O}) = 0,$$

and hence