cohomology greatly facilitates the study of divisors on a variety – a case where the local theory is relativ-ely simple – the introduction of some algebraic

machinery will clarify some of the preceding discussion concerning the local properties of a set of analytic equations $f_1(z_1, \cdots z_n) = \cdots = f_n(z_1, \cdots z_n) = 0$ having the origin as isolated common zero. This will be especially true of the transformation law and local duality theorem associated to our analytically defined residues; these two results will eventually achieve a very symmetric form.

We use the notation

$$\mathcal{O} = \lim_{\{0\} \in U} \mathcal{O}(U).$$

for the germs of analytic functions defined in some nbd U of the origin in $\mathbb{C}^n$. Clearly, $\mathcal{O} = \mathbb{C}\{z_1, \cdots z_n\}$ is the ring of convergent power series. When involved in inductive arguments we shall write $\mathcal{O}_n$ for $\mathcal{O}$. Recall that a local ring is a ring having a unique maximal ideal. $\mathcal{O}$ is such a local ring with maximal ideal $m = \{z_1, \cdots z_n\}$ the ideal of functions $f \in \mathcal{O}$ with $f(0) = 0$. The units are just $\mathcal{O}^* = \mathcal{O} - m$.

In Section 1 of Chapter 0 we proved that, given $f \neq 0$ in $\mathcal{O}_n$, there is a linear coordinate system $(z_1, z_2, \cdots z_n) = (z', z_n)$ and unique Weierstrass polynomial

$$W(z) = z_n^d + a_1(z') z_n^{d-1} + \cdots + a_d(z') \in \mathcal{O}_{n-1}[z_n],$$

where $a_i(z') \in \mathcal{O}_{n-1}$ are nonunits such that