linear subspace $\mathbb{P}^{n-m+k}$ in $\mathbb{P}^n$ is an analytic subvariety of dimension k in M, and so represents a nonzero element of $H_{2k}(M)$.

「 Let $V = M \cap \mathbb{P}^{n-m+k}$. $\Rightarrow$ $\cancel{m} + \cancel{n} + k - \cancel{m} - \cancel{n} = k$.
$\Rightarrow$ V is an analytic subvariety of dim. k in M.
$\Rightarrow$ By the above, $\eta_V \neq 0 \in H^{2n-2k}(M)$. $\Rightarrow$ V represents a nonzero element of $H_{2k}(M)$. 」

Similarly,

Any analytic subvariety of $\mathbb{P}^n$ homologous to a hyperplane is a hyperplane.

To see this, we note that if V is homologous to a hyperplane it has intersection number 1 with a line. Then if p_1, p_2 are any two points of V, the line $L = \overline{p_1 p_2}$, having two points in common with V, must have a curve in common with V; that is, L must be contained in V. V thus contains the line joining any two of its points, and so is a linear subspace of $\mathbb{P}^n$.

「 See the 5-th line of P64. 」

From this it follows that

Any holomorphic automorphism of $\mathbb{P}^n$ is induced by a line