the Gauss maps

$$\mathcal{G}_F : F \longrightarrow \mathbb{P}^{5*} \quad \text{and} \quad \mathcal{G}_G : G \longrightarrow \mathbb{P}^{5*}$$

would each map $\sigma(p,h)$ isomorphically onto the set $\langle \sigma(p), \sigma(h) \rangle^*$ of hyperplanes containing $\langle \sigma(p), \sigma(h) \rangle$ ($\langle \sigma(p), \sigma(h) \rangle^* \cong \mathbb{P}^1$). But then

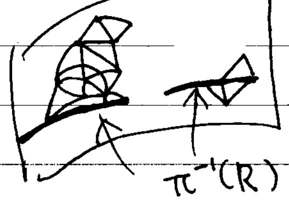
$$\mathcal{G}_F^{-1} \circ \mathcal{G}_G : \underset{\cong \mathbb{P}^1}{\sigma(p,h)} \longrightarrow \underset{\| \mathbb{P}^1}{\sigma(p,h)}$$

would have a fixed point. $\Rightarrow$ For some $y \in \sigma(p,h)$ $T_y(F) = T_y(G)$. $\Rightarrow$ Contradiction

$\lrcorner\!\lrcorner$

For later use, we compute the Euler characteristic $\chi(S)$, as follows. Take a triangulation of Σ that extends a triangulation of $\pi^{-1}(R) = \bigcup X_p$. Then the images of the simplices in Σ not in $\pi^{-1}(R)$, together with the points $p \in R$ as vertices form a cell decomposition of S.

$\ulcorner\!\ulcorner$ Any algebraic variety is triangulable, see p447 Hartshorne, Algebraic Geometry.



$\Sigma \Rightarrow R \cup \{\text{simplices in } \Sigma \text{ not in } \pi^{-1}(R)\}$ covers S. $\Rightarrow$ The form a cell