NO.

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In fact, we showed that $L \subset G$ in the proof of ②, i.e, in the proof of $x'' \in G$. And by the same argument, $L \subset F \Rightarrow L \subset G \cap F$.

I think: To prove Σ is smooth, the point is that $L \subset G \cap F$. But in proving this, the authors made a mistake, I guess. $M: G \to G$ is not clear, and so on. ..

One more thing: Since $L = \overline{x x'}$ and $M(x) = x'$ & $M(x') = x''$ lie on L, $M: L \longrightarrow L$.

⌟

M must have a fixed point y somewhere on L, i.e., for some $y \in L$,

$$Qy = Q'y.$$

⌜ M is linear $\Rightarrow$ (i) $M \in PGL(2) = GL(2)/\mathbb{C}^*$, since $L = \mathbb{P}^1$ or (ii) $M \in \mathrm{End}(\mathbb{C}^2)/\mathbb{C}^*$.

$\Rightarrow$ (i) $\overset{\text{case}}{\Rightarrow}$ $M \sim id \Rightarrow L(M) =$ Lefschetz number of $M = L(id) = 2 > 0 \Rightarrow M$ has a fixed point by Lefshetz Fixed-point formula on P^1

(ii) case $\Rightarrow M \sim \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow L(M) =$ trace of $M^*|_{H^0} + \mathrm{tr}(M^*|_{H^2}) = 1 + 0 = 1$

$\Rightarrow M$ has a fixed point.

⌟

But since $L \subset F \cap G$, this implies that F and G are tangent