ed in curve theory. Section 4 is complementary to 3: its main result is a characterization of rational surfaces by numerical invariants.

Section 5 discusses the classification theorem for surfaces; this essentially amounts to a description, in varying detail, of all surfaces except those of general type.

It remains in Section 6 to prove Noether's formula. To do this, we introduce another technique of general interest: the blow-up of a complex manifold along any submanifold. Using this construction together with some remarks on singularities of surfaces in $\mathbb{P}^3$ we represent a general surface as a smooth divisor in a blow-up of $\mathbb{P}^3$, and obtain formulas for the numerical characters of a surface in terms of the projective invariants of a birational embedding in 3-space. Noether's formula is an immediate consequence of these.

1. Preliminaries

Intersection Numbers, the Adjunction Formula, and Riemann - Roch

Let M be an algebraic surface, i.e., a compact complex manifold of dimension 2 that may be embedded in projective space. Since M is an oriented real 4-manifold, the intersection pairing