

If  $\psi$  is harmonic and the hermitian form  $R_{\bar{i}\bar{j}} \zeta^{\bar{i}} \bar{\zeta}^{\bar{j}}$  is positive definite, then we deduce that  $\psi \equiv 0$ .

$\square$   $\psi$  is harmonic  $\Leftrightarrow \Delta\psi = 0$ .

$$\begin{aligned} LHS = 0 &= \|\bar{\nabla}\psi\|^2 + (R\psi, \psi) \\ &= \|\bar{\nabla}\psi\|^2 + 9 \int_M \sum (R_{\bar{i}\bar{j}} \psi_{\bar{i}_1 \dots \bar{i}_{q-1} \bar{i}} \bar{\psi}_{\bar{i}_1 \dots \bar{i}_{q-1} \bar{j}}) \bar{\psi} \end{aligned}$$

Since  $R$  is positive definite,  $(R(\psi), \psi) \geq 0$ .

$$\Rightarrow \|\bar{\nabla}\psi\|^2 = 0 \quad \Rightarrow \quad \bar{\nabla}\psi = 0.$$

$$\psi = \sum \psi_I \bar{\varphi}_I$$

$$\bar{\nabla}\psi = \sum \bar{\partial}\psi_I \otimes \bar{\varphi}_I + \psi_I \bar{\partial}\bar{\varphi}_I$$

$\Rightarrow$  Since  $\Delta\psi = 0$ ,  $\bar{\partial}\psi = 0$ .  $\Rightarrow$  the 1st term is zero.  $\Rightarrow \psi_I \bar{\partial}\bar{\varphi}_I = 0$ .  $\Rightarrow$  This is not the right way to go further

$\square$  For each fixed  $\bar{i}_1, \dots, \bar{i}_{q-1}$ .

$\sum_{\bar{i}, \bar{j}} R_{\bar{i}\bar{j}} \psi_{\bar{i}_1 \dots \bar{i}_{q-1} \bar{i}} \bar{\psi}_{\bar{i}_1 \dots \bar{i}_{q-1} \bar{j}}$  is positive definite

$$\Rightarrow \sum_{\bar{i}, \bar{j}} R_{\bar{i}\bar{j}} \psi_{\bar{i}_1 \dots \bar{i}_{q-1} \bar{i}} \bar{\psi}_{\bar{i}_1 \dots \bar{i}_{q-1} \bar{j}} \geq 0. \Rightarrow \text{Equality holds}$$

$$\Leftrightarrow \psi_{\bar{i}_1 \dots \bar{i}_{q-1} \bar{i}} = 0 \text{ for all } \bar{i}.$$

Since  $\{\bar{i}_1, \dots, \bar{i}_{q-1}\}$  is arbitrary,  $\psi_I = 0$ .

$$\Rightarrow \psi = 0$$

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