$$= \lim_{\epsilon\to 0}\int_{\Delta-\Delta(\epsilon)} \partial(\log\bar z\ \phi\, d\bar z) - \lim_{\epsilon\to 0}\int_{\Delta-\Delta(\epsilon)} \underbrace{(\partial \log \bar z)}_{0}\wedge\phi\, d\bar z$$

$$= \lim_{\epsilon\to 0}\int_{\Delta-\Delta(\epsilon)} d(\log\bar z\ \phi\, d\bar z) = \lim_{\epsilon\to 0}\int_{\partial\Delta} \log\bar z\ \cancel{\phi}^{\,0}\, d\bar z$$

$$- \lim_{\epsilon\to 0}\int_{\partial\Delta(\epsilon)} \log\bar z\ \phi\, d\bar z \underset{z=\epsilon e^{i\theta}}{=} - \lim_{\epsilon\to 0}\int_0^{2\pi} (\log\epsilon - i\theta)\ \phi(\epsilon e^{i\theta})\ \underbrace{\phantom{x}}_{\epsilon(-i)e^{-i\theta}d\theta}$$

$$= - \lim_{\epsilon\to 0}\int_0^{2\pi} \epsilon(\log\epsilon - i\theta)\ \phi(\epsilon e^{i\theta})\ (-ie^{-i\theta})\, d\theta$$

$$\lim_{\epsilon\to 0}\int_0^{2\pi} \epsilon|\log\epsilon|\ |\phi(\epsilon e^{i\theta})| + \epsilon\theta\, d\theta$$

$$\le \lim_{\epsilon\to 0} \epsilon\cdot 2\pi + \lim_{\epsilon\to 0}\left(\int_0^{2\pi} \epsilon|\log\epsilon|\, d\theta\right)\cdot M$$

$$= \lim_{\epsilon\to 0} \epsilon|\log\epsilon| \cdot M \cdot 2\pi = 0$$

since $\lim_{x\to 0} x\log x = \lim_{x\to 0} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0} -x = 0$

In case $i \neq n+1$, for simplicity, $n=1$.

$$\int_{\Delta^2} \log\bar z_2\ \partial(\psi\, dz_2\wedge d\bar z_1\wedge d\bar z_2) \quad \cdots (**)$$

$$= \lim_{\epsilon\to 0}\int_{\Delta^2-\Delta\times\Delta(\epsilon)} \log\bar z_2\ \partial(\psi\wedge dz_2\wedge d\bar z_1\wedge d\bar z_2)$$