for all $h \supset l_x$, $X_h = \sigma(h) \cap F$ = union of two distinct lines or a smooth conic curve. $\Rightarrow$ $\{X_h\}$ is Lefschetz. $\Rightarrow$ By the general formula, $\chi(U) = 2\chi(X_h) - n + \mu = 4 - 0 + \mu = 4 + \mu \Rightarrow \mu = \chi(U) - 4$ is the number of singular fibers.

$\lrcorner$

But while the singular fibers of the pencil $\{X_p\}$ correspond to points of intersection of l_x with S, singular fibers of $\{X_h\}$ correspond to points of intersection of the dual line $l_x^* \subset \mathbb{P}^{3*}$ of hyperplanes containing l_x with S^*.

$\ulcorner$ As on P765, $l_x \cap S \overset{1-1}{\longleftrightarrow} \{X_p : X_p \text{ is singular}\}$
$l_x^* \cap S^* \longleftrightarrow \{X_h : X_h \text{ is singular}\}$

See and refer to P908 note. $\lrcorner$

In particular, $\#(l_x \cap S) < 4 \Leftrightarrow \#(l_x^* \cap S^*) < 4$, i.e., l_x is tangent to $S \Leftrightarrow l_x^*$ is tangent to S^*.

$\ulcorner$ $\#(l_x \cap S) < 4 \Leftrightarrow l_x$ is tangent to S
$\Rightarrow$ Since $l_x \cap S$ corresponds to $\{X_p : X_p \text{ singular}\}_{p \in l_x}$, $\#\{X_p : X_p \text{ singular}\} < 4$. $\Rightarrow$ Since $X_p = X_h$ for some h, $\#\{X_h : X_h \text{ singular}\} < 4 \Leftrightarrow l_x^*$ is tangent to S^*. Conversely, it works that way. $\lrcorner$