$\Rightarrow$ $\pi \circ \varphi^{-1}$ is holomorphic and onto. $\{ J(\pi\circ\varphi^{-1}) = 0 \}$ is an analytic subvariety of $\mathbb{C}^3$. Clearly it is proper subvariety. $\Rightarrow$ Locally, $\pi\circ\varphi^{-1}$ is biholomorphic around almost all points. $\Rightarrow$ $\mathbb{C}^3 \longrightarrow \Gamma$ is locally biholomorphic. $\Rightarrow$ Γ & $\{p\}\times\mathbb{P}^n$ intersect each other transversely for generic point p. ⌟

Indeed, the locus V of points $p \in M$ where the latter case occurs must have codimension at least 2: if V were of dimension $k-1$, the inverse image of V in Γ would have dimension k, and so would form a component of the irreducible variety Γ. Γ thus defines a rational map $f: M \longrightarrow \mathbb{P}^n$.

⌜ For each $p\in V$, $\pi^{-1}(p)$ has dimension ≥ 1.
$\Rightarrow$ $\pi^{-1}(V)$ is of dimension $\geq \dim V + 1 = k$.
$\pi^{-1}(V)$ is an analytic subvariety, for if $U\cap V = \{f_1 = \cdots = f_\ell = 0\}$ then $\pi^{-1}(V)\cap\pi^{-1}(U) = \{\pi^* f_1 = \cdots = \pi^* f_\ell = 0\}$.
$\Rightarrow$ $\pi^{-1}(V)$ is a component of Γ. $\Rightarrow$ Contradiction, since π(any compnt) contains an open set of M.
$\Rightarrow$ V is of $\dim \leq k-2$. Thus we have a rational map $f: M \longrightarrow \mathbb{P}^n$, for, for each $p\in M-V$, $\exists$ an open set $U_p \subset M-V$ s.t $U_p \longrightarrow U_p \times \mathbb{P}^n \cap \Gamma$ is biholomorphic. $\Rightarrow$ $\exists$ biholomorphic map from $M-V$ to $\Gamma \cap (M-V)\times\mathbb{P}^n$. $\Rightarrow$ $\exists$ a holomorphic map $f: M-V \longrightarrow \mathbb{P}^n$, which is a rational map. ⌟