p67.

$$\begin{tikzcd} \pi^{-1}(U_\alpha) \arrow[rr, "\varphi_\alpha"] \arrow[dr] & & U_\alpha \times \mathbb{C}^n \arrow[dl] \\ & U_\alpha & \end{tikzcd}$$

$$\varphi_\beta \circ \varphi_\alpha^{-1} = g_{\beta\alpha}$$

$$\begin{tikzcd} \pi^{-1}(U_\alpha^*) \arrow[rr, "\varphi_\alpha^*"] \arrow[dr] & & U_\alpha \times \mathbb{C}^{n^*} \arrow[dl] \\ & U_\alpha & \end{tikzcd}$$

$$\grave{j}_{\alpha\beta} \equiv \varphi_\alpha^* \circ \varphi_\beta^{*^{-1}}.$$

$$\bar{j}_{\alpha\beta}(\sigma) = \varphi_\alpha^* \circ \varphi_\beta^{*^{-1}}(\sigma)$$

$v \in \mathbb{C}^n \qquad \bar{j}_{\alpha\beta}(\sigma)(v)$

$$= \varphi_\alpha^* \circ \varphi_\beta^{*^{-1}}(\sigma)\ (v) = \varphi_\beta^{*^{-1}}(\sigma)\ (\varphi_\alpha^{-1}(v)) = \sigma(\varphi_\beta \circ \varphi_\alpha^{-1}(v))$$

$$= \sigma(g_{\beta\alpha}(v))$$

$$\Rightarrow \quad \bar{j}_{\alpha\beta}(\sigma)(v) = \sigma(g_{\beta\alpha}(v))$$

By using matrix notation,

$${}^t(\bar{j}_{\alpha\beta}\sigma)\, v = {}^t\sigma\, g_{\beta\alpha} v$$

$$\Rightarrow \quad {}^t(\bar{j}_{\alpha\beta}\sigma) = {}^t\sigma\, g_{\beta\alpha}$$

$$\Rightarrow \quad \bar{j}_{\alpha\beta}\sigma = {}^t g_{\beta\alpha}\, \sigma = {}^t g_{\alpha\beta}^{-1}\, \sigma \quad \text{since } g_{\alpha\beta}\, g_{\beta\alpha} = 1$$