If $\varphi : \mathcal{F} \longrightarrow \mathcal{G}$ is a morphism of sheaves, we define the cokernel of φ, denoted $\operatorname{coker}\varphi$, to be the sheaf associated to the presheaf cokernel of φ.

Remark : A morphism $\varphi : \mathcal{F} \longrightarrow \mathcal{G}$ of sheaves is injective $\Leftrightarrow$ the map on the sections $\varphi(U) : \mathcal{F}(U) \longrightarrow \mathcal{G}(U)$ is injective for each U.
The corresponding statement for surjective morphisms is not true : if $\varphi : \mathcal{F} \longrightarrow \mathcal{G}$ is surjective, the maps $\varphi(U) : \mathcal{F}(U) \longrightarrow \mathcal{G}(U)$ on sections need not be surjective.

However, we can say that φ is surjective $\Leftrightarrow$ the map $\varphi_p : \mathcal{F}_p \longrightarrow \mathcal{G}_p$ on stalks are surjective for each p.
More generally, a sequence of sheaves and morphisms is exact $\Leftrightarrow$ it is exact on stalks. (Ex. 1.2)
See for more details. Hartshorne P60.~P64 Algebraic Geometry.

Example $X = \mathbb{C} - \{0\}$.

$$\begin{array}{rccc} \exp : & \mathcal{O} & \longrightarrow & \mathcal{O}^* \\ & \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ & f & \longmapsto & e^{2\pi i f} \end{array}$$

We have U, V s.t U, V contractible & $U \cup V = \mathbb{C} - \{0\}$.
$z \in \mathcal{O}^*(\mathbb{C} - \{0\})$. $\Rightarrow$ $z|_U \in \mathcal{O}^*(U) \Rightarrow \exists\ \sigma_1 \in \mathcal{O}(U)$
s.t $\exp(\sigma_1) = z|_U$. Similarly we have $\sigma_2 \in \mathcal{O}(V)$

s.t $\exp(\sigma_2) = z|_V$. $\cancel{\Rightarrow}$ $\cancel{\exp(\sigma_1)|_{U\cap V} = \exp(\sigma_2)|_{U\cap V}}$

$\cancel{\Rightarrow z|_U = 0 \text{ in } \mathcal{O}^*/\mathcal{O}\,(U), \text{ by the sheaf property}}$ (i)

~~since~~

$\Rightarrow$ $z|_U = 0$ in $(\mathcal{O}^*/e\mathcal{O})(U)$ & $z|_V = 0$ in $(\mathcal{O}^*/e\mathcal{O})(V)$

$\Rightarrow$ By the sheaf property (i). $z = 0$ $\mathcal{O}^*/e\mathcal{O}$. $\Rightarrow$ $*$
If we assume $\mathcal{O}^*/\mathcal{O}$ is a sheaf, we have a contradiction.