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the hard Lefschetz theorem

$$L^k : R_\pi^{n-k}(\mathbb{C}) \longrightarrow R_\pi^{n+k}(\mathbb{C})$$

is valid, simply because each stalk

$$R_\pi^q(\mathbb{C})_x \cong H^q(F_x, \mathbb{C})$$

and we may apply the usual hard Lefschetz theorem.

$\ulcorner$

$$\begin{array}{ccc} L^k : R_\pi^{n-k}(\mathbb{C})_x & \longrightarrow & R_\pi^{n+k}(\mathbb{C})_x \\ \wr\Vert & & \wr\Vert \\ H^{n-k}(F_x, \mathbb{C}) & & H^{n+k}(F_x, \mathbb{C}) \end{array}$$

is isomorphic by the usual hard Lefschetz theorem, see P122. $\Rightarrow$ By P63 Proposition 1.1 Algebraic geometry (by Hartshorne), $L^k : R_\pi^{n-k}(\mathbb{C}) \longrightarrow R_\pi^{n+k}(\mathbb{C})$ is isomorphic. $\lrcorner$

Continuing this line of thought, if we define the primitive Leray sheaf by

$$P^{n-k} = \ker\{ L^{k+1} : R^{n-k} \longrightarrow R^{n+k+2} \}, \quad R^q = R_\pi^q(\mathbb{C}),$$

then for the same reasons the Lefschetz decomposition

$$R^q \cong \bigoplus_k L^k P^{q-2k} \qquad (q \leq n)$$

is valid.

$\ulcorner$ Again at each stalk, by applying the usual hard Lefschetz theorem,

$$R_x^q \cong \bigoplus_k L^k P_x^{q-2k}.$$

$\Rightarrow$ Again by P63 Proposition 1.1, by Hartshorne's book,

JONG IE WAH YUN PILL