Note that in case V is a quadric hypersurface the map π_p is a birational isomorphism. We have already seen this in the case of Q a quadric surface in $\mathbb{P}^3$, where the map π_p consists of the blow-up of the point p, followed by the blowing down of the two lines of Q through p.

F By P479 & P480

the blow-up at p ← $\tilde{Q} \supset E$ → the blow-up of q_1 & q_2

$$\begin{tikzcd}
\tilde{Q} \arrow[d, "\pi"'] \arrow[dr, "\tilde{\pi}"] & \\
Q \arrow[r, "\pi_p"] & H \, (\cong \mathbb{P}^2)
\end{tikzcd}$$

π → birational, $\tilde{\pi}$ → birational

$Q \setminus \{p\}$, $\; L_1, L_2 \subset Q$

$\pi_p(L_1) = q_1 \qquad \pi_p(L_2) = q_2$

$\Rightarrow$ π_p is birational, since
$\exists$ an inverse rational map $\pi \circ \tilde{\pi}^{-1}$: $H \setminus \{q_1, q_2\} \longrightarrow \tilde{Q} \longrightarrow Q$.

$\pi_p \circ \pi = \tilde{\pi}$.

$\pi_p \circ \pi \circ \tilde{\pi}^{-1} = id$

For V a quadric hypersurface in $\mathbb{P}^n$, $p \in V$.
$\Rightarrow$ for generic point $q \in V$, $\#(\overline{pq} \cap V) = 1$, since V is quadric. $\Rightarrow$ $\pi_p(q) = \overline{pq} \cap H$, and π_p is generically one to one $\Rightarrow$ π_p is birational by P493.
More precisely, for any generic point $r \in H$, $\#(\overline{pr} \cap V) = 1$, since V is quadric. ∎