

$$\begin{aligned} q=1 & \quad ('E_{\mathbb{R}^*})_2^{p,1} = 0 \Rightarrow ('E_{\mathbb{R}^*})_3^{p,1} = 0 = \dots = ('E_{\mathbb{R}^*})_{\infty}^{p,1} = 0 \\ q > 1 & \quad ('E_{\mathbb{R}^*})_2^{p,q} = \dots = ('E_{\mathbb{R}^*})_{\infty}^{p,q} = 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} ('E_{\mathbb{R}^*})_2^{p,0} &= \dots = H^p(M, \mathbb{R}) \\ ('E_{\mathbb{R}^*})_2^{p,q} &= \dots = 0 \quad q > 0 \end{aligned}$$

$$\begin{aligned} F^0 H^q &= E_r^{0,q} \oplus F^1 H^q = E_r^{0,q} \oplus E_r^{1,q-1} \oplus F^2 H^q \\ H^q &= E_r^{0,q} \oplus E_r^{1,q-1} \oplus \dots \oplus E_r^{q,0} \end{aligned}$$

$$\Rightarrow H^q(X, \mathbb{R}^*) = E_r^{0,q} \oplus E_r^{1,q-1} \oplus \dots \oplus E_r^{q,0}$$

$$\text{If } q=0, \quad H^0(X, \mathbb{R}^*) = E_r^{0,0} = E_2^{0,0} = ('E_{\mathbb{R}^*})_2^{0,0} = H^0(M, \mathbb{R})$$

$$\text{If } q > 0, \quad H^q(X, \mathbb{R}^*) = E_r^{q,0} = ('E_{\mathbb{R}^*})_2^{q,0} = H^q(M, \mathbb{R})$$

$$\text{Thus } H^q(X, \mathbb{R}^*) = H^q(M, \mathbb{R})$$

$$\text{Here we put } E_r^{p,q} = ('E_{\mathbb{R}^*})_2^{p,q} \text{ sometimes.} \quad \Rightarrow$$

On the other hand, by the partition of unity argument $H^q(M, \mathbb{Q}^*) = 0$ for $q > 0$, and so

$$('E_{\mathbb{Q}^*})_2^{p,q} = \begin{cases} H_{DR}^p(M), & q=0 \\ 0, & q > 0. \end{cases}$$

See p42. Put $'E_{\mathbb{Q}^*} = 'E$.

$X=M$

$$('E)_{i+1}^{p,q} = H_d^q(H^p(X, \mathbb{Q}^*)) = \frac{\ker d}{\text{im } d}$$

$$H^q(X, \mathbb{Q}^{p-1}) \xrightarrow{d} H^q(X, \mathbb{Q}^p) \xrightarrow{d} H^q(X, \mathbb{Q}^{p+1}) \rightarrow \text{See note p439 back.}$$

$$q > 0 \Rightarrow 'E_2^{p,q} = 0$$

since $H^p(X, \mathbb{Q}^q) = 0$ for $q > 0$.