$$\pi^* f \otimes \sigma_i^{-1}|_E : \mathcal{O}_i \longrightarrow \mathbb{C}$$

$$\left(\frac{l_1}{l_i}, \cdots \frac{l_n}{l_i}\right) \longmapsto \frac{l_1}{l_i}\frac{\partial f}{\partial z_1} + \cdots + \frac{l_i}{l_i}\frac{\partial f}{\partial z_i} + \cdots \frac{l_n}{l_i}\frac{\partial f}{\partial z_n}$$

$\Rightarrow \{(\pi^* f \otimes \sigma_i^{-1}|_E, \mathcal{O}_i)\}$ represents a section of $[-E]$ over E whose divisor is $l_1 \frac{\partial f}{\partial z_1} + l_2 \frac{\partial f}{\partial z_2} + \cdots + l_n \frac{\partial f}{\partial z_n} = 0$ in E. ---- ⊛

Given $f \in H^0(U, \mathcal{J}_x)$. $d_x f = \sum \frac{\partial f}{\partial z_i} dz_i$, we need to know what is the corresponding element in $T_x^{*\prime}(M)$.
To know this, we had better look closely the identification between $H^0(E, \mathcal{O}(-E))$ and $T_x^{*\prime}(M)$.

Given $\bar{\sigma} \in H^0(E, \mathcal{O}(-E))$, $\bar{\sigma}_i : \mathcal{O}_i = (l_i \neq 0) \longrightarrow \mathbb{C}$ is given by $\bar{\sigma}_i([l_1, l_2, \cdots l_n]) = a_1 \frac{l_1}{l_i} + a_2 \frac{l_2}{l_i} + \cdots + \frac{l_n}{l_i} a_n$.

$$\Rightarrow \begin{array}{ccc} H^0(E, \mathcal{O}(-E)) & \longrightarrow & T_x^{*\prime}(M) \\ \Downarrow & & \Downarrow \\ \bar{\sigma} & \longmapsto & \sum a_j dz_j \end{array}$$

$\Rightarrow$ For $d_x f = \sum \frac{\partial f}{\partial z_i} dz_i \in T_x^{*\prime}(M)$, the corresponding element τ in $H^0(E, \mathcal{O}(-E))$ is given by

$$\tau_i : \mathcal{O}_i \longrightarrow \mathbb{C}$$

$$[(l_1 \cdots l_n)] \longmapsto \frac{l_1}{l_i}\frac{\partial f}{\partial z_1} + \cdots + \frac{l_i}{l_i}\frac{\partial f}{\partial z_i} + \cdots + \frac{l_n}{l_i}\frac{\partial f}{\partial z_n}.$$

$\Rightarrow$ By ⊛. $\tau = \pi^* f \otimes \sigma^{-1}|_E$ $\Rightarrow$ The diagram is commutative. //