

\Rightarrow By the inverse function theorem, $\exists G$ s.t

$$G(z_1, z_2, f, g) = (z_1, z_2, z_3, z_4).$$

$$\Rightarrow G(z_1, z_2, 0, 0) = (z_1, z_2, z_3, z_4).$$

$$\Rightarrow z_3 = h(z_1, z_2), \quad z_4 = k(z_1, z_2) \quad \text{locally.}$$

$$\Rightarrow \text{This proves } \{f = g = 0\} \xrightarrow{\pi} \pi(V) \xrightarrow{\pi} \mathbb{C} \quad \text{is}$$

a branched finite-sheeted covering map.

In case $(\frac{\partial f}{\partial z_4}, \frac{\partial g}{\partial z_4}) = 0$, we will deal a little bit later.

$$\textcircled{2} \quad D(r) \equiv 0.$$

\Rightarrow Since r is a Weierstrass polynomial, r is of form $(z_3^2 + \dots)^2 (z_3 + \dots) \dots$.

$$\text{For example, } r = (z_3^2 + \dots)^2 (z_3^4 + \dots).$$

$$\text{Since } \{r = 0\} = \{(z_3^2 + \dots)^2 (z_3^4 + \dots) = 0\},$$

$$\pi\{f = g = 0\} = \{(z_3^2 + \dots)^2 (z_4^4 + \dots) = 0\}.$$

\Rightarrow We may assume r has $D(r) \neq 0$.

\Rightarrow For $(z_1^0, z_2^0) \in \{D(r) \neq 0\}$, locally

$$\exists z_3 = h(z_1, z_2) \quad \text{s.t.} \quad r(z_1^0, z_2^0, z_3) = 0 \quad \text{and} \\ z_3^0 = h(z_1^0, z_2^0).$$

$$\Rightarrow f(z_4) = z_4^4 + a_1(z_1, z_2, h(z_1, z_2)) z_4^3 + a_2(z_1, z_2, h(z_1, z_2)) z_4^2 + a_3(z_1, z_2, h(z_1, z_2)) z_4 + a_4(z_1, z_2, h(z_1, z_2))$$

$$g(z_4) = z_4^3 + b_1(z_1, z_2, h(z_1, z_2)) z_4^2 + b_2(z_1, z_2, h(z_1, z_2)) z_4 + b_3(z_1, z_2, h(z_1, z_2))$$

around $(z_1^0, z_2^0) \in \{D(r) \neq 0\} \subset \mathbb{C}^2$.