finite cover of M.

$\Rightarrow \quad D^2 s = D^2 f_i e_i = f_i D^2 e_i = f_i \Theta_{ji} e_j$

$\Rightarrow \quad \langle D^2 s, D^2 s \rangle_x = \langle f_i \Theta_{ji} \otimes e_j, \; f_\alpha \Theta_{\beta\alpha} \otimes e_\beta \rangle$

$= \langle f_i \Theta_{ji}, \; f_\alpha \Theta_{j\alpha} \rangle = f_i \bar{f}_\alpha \langle \Theta_{ji}, \Theta_{j\alpha} \rangle_x$

$\leq \underbrace{\|f\|_x^2}_{= \|s\|_x^2} \times K_u$ where $K_u > 0$

Since M is compact, take $K = \max \{K_u\}$.

$\Rightarrow \quad \int \underbrace{\langle D^2 s, D^2 s \rangle_x dx}_{= \|D^2 s\|^2} \leq K \int \langle s, s \rangle_x dx$

$= K \|s\|^2$

$\Rightarrow \quad \|D^2 s\| \leq \sqrt{K} \|s\| \Rightarrow D^2$ is bounded operator which is denoted by Θ.

$\Rightarrow \quad [\Lambda, 1 \otimes \Theta_E]$ is bounded.

$\Rightarrow \quad |\langle [\Lambda, 1 \otimes \Theta_E]\eta, \eta \rangle| \leq \| [\Lambda, 1 \otimes \Theta_E] \eta \| \, \|\eta\|$

$\leq C \|\eta\| \, \|\eta\| = C \|\eta\|^2$

where $\| [\Lambda, 1 \otimes \Theta_E] \eta \| \leq C \|\eta\|$.

$\Rightarrow$ If $p < n$, $\mu > \frac{C}{2\pi} \Leftrightarrow 2\pi\mu > C$, & $\|\eta\| \neq 0$,

$i \langle [\Lambda, \Theta]\eta, \eta \rangle = i \langle [\Lambda, 1 \otimes \Theta_E]\eta, \eta \rangle$

$- 2\pi\mu(n-p) \|\eta\|^2 \geq 0.$