$$x \longmapsto \begin{bmatrix} a_{11}, & \cdots & a_{1n} \\ \vdots & & \\ a_{k1}, & \cdots & a_{kn} \end{bmatrix},$$

so that ι_V is clearly holomorphic.

$\ulcorner$ $\sigma_i = \Sigma\, a_{\alpha i}\, e_\alpha$ on an open set.
If $\sigma = x_1\sigma_1 + \cdots + x_n\sigma_n = \Sigma\, x_i\sigma_i = x_i\, a_{\alpha i}\, e_\alpha$ in $G(n-k, V)$, $\sigma(x) = 0 \Rightarrow a_{\alpha i} x_i = 0$ for all α.
$\Rightarrow a_{\alpha i}(x)\, x_i = 0$.

Since the identification $G(n-k, V) \cong G(k, V^*)$ is given as follows:

$$\begin{array}{ccc} V & \longrightarrow & V^* \\ \Psi & & \\ \sigma_i & \longmapsto & \sigma_i^* \text{ defined by} \end{array}$$

$$\sigma_i^*(\sigma_j) = \delta_{ij} \quad (\text{Kronecker delta}).$$

$$G(n-k, V^*) \longrightarrow G(k, V^*)$$

$$\langle \tau_1, \tau_2 \cdots \tau_{n-k} \rangle \longmapsto \langle \tau_1^*, \cdots \tau_k^* \rangle$$

$$\begin{aligned} \tau_1 &= b_{11}\sigma_1 + \cdots + b_{n1}\sigma_n, \\ \tau_2 &= b_{12}\sigma_1 + \cdots + b_{n2}\sigma_n \\ &\vdots \\ \tau_i &= b_{1i}\sigma_1 + \cdots + b_{ni}\sigma_n \\ &\vdots \\ \tau_{n-k} &= b_{1\,n-k}\sigma_1 + \cdots + b_{n\,n-k}\sigma_n \end{aligned}$$

$b_{ij} \in \mathbb{C}$

$$\begin{aligned} \tau_1^* &= c_{11}\sigma_1^* + \cdots + c_{n1}\sigma_n^* \\ \tau_2^* &= c_{12}\sigma_1^* + \cdots + c_{n2}\sigma_n^* \\ &\vdots \\ \tau_k^* &= c_{1k}\sigma_1^* + \cdots + c_{nk}\sigma_n^* \end{aligned}$$

$$\Rightarrow \tau_i^*(\tau_j) = 0 = c_{\alpha i}\,\sigma_\alpha^*(b_{\beta j}\sigma_\beta) = \overset{\alpha=\beta}{c_{\alpha i}\, b_{\beta j}} = ({}^tC)_{i\alpha}\, b_{\beta j}$$
$$= ({}^tC)_{i\alpha}\, b_{\alpha j} = ({}^tC\,B)_{ij} = 0.$$