Date . .

$= \langle D'\eta, D'\eta\rangle \geq 0$, since $\langle \bar\partial \Lambda D'\eta, \eta\rangle = \langle \Lambda D'\eta, \bar\partial^*\eta\rangle = 0$.

Γ

$$\Theta = D^2 = (D'+\bar\partial)(D'+\bar\partial) = \bar\partial D' + D'\bar\partial$$

$$\Rightarrow \Theta\eta = D^2\eta = (\bar\partial D' + D'\bar\partial)\eta = \bar\partial D'\eta, \text{ since } \bar\partial\eta = \partial\eta = 0.$$

By the Kähler identity $[\Lambda, \bar\partial] = -i D'^*$,

$$i\langle \Lambda\Theta\eta, \eta\rangle = i\langle \Lambda\bar\partial D'\eta, \eta\rangle = i\langle (\bar\partial\Lambda D'\eta - i D'^* D'\eta), \eta\rangle = i\underbrace{\langle \bar\partial \Lambda D'\eta, \eta\rangle}_{= i\langle \Lambda D'\eta, \bar\partial^*\eta\rangle = 0} + \langle D'^* D'\eta, \eta\rangle$$

$$= \langle D'^* D'\eta, \eta\rangle = \langle D'\eta, D'\eta\rangle \geq 0.$$

On the other hand,

$$i\langle \Theta\Lambda\eta, \eta\rangle = i\langle D'\bar\partial\Lambda\eta, \eta\rangle$$
$$= i\langle (\Lambda\bar\partial - \tfrac{1}{\partial i} D'^*)\eta, D'^*\eta\rangle$$
$$= -\langle D'^*\eta, D'^*\eta\rangle \leq 0.$$

Γ " $i\langle \Theta\Lambda\eta, \eta\rangle = i\langle \bar\partial D'\Lambda\eta, \eta\rangle$ since $[\Delta, \Lambda] = 0$, $\Lambda\eta$ is again harmonic. (We don't know whether $[\Delta, \Lambda] = 0$ is true or not for E.) "

$$i\langle \Theta\Lambda\eta, \eta\rangle = i\langle (\bar\partial D' + D'\bar\partial)\Lambda\eta, \eta\rangle$$
$$= i\langle \bar\partial D'\Lambda\eta, \eta\rangle + i\langle D'\bar\partial\Lambda\eta, \eta\rangle = i\langle D'\Lambda\eta, \bar\partial^*\eta\rangle$$
$$+ i\langle D'\bar\partial\Lambda\eta, \eta\rangle = i\langle D'\bar\partial\Lambda\eta, \eta\rangle$$
$$= i\langle D'(\Lambda\bar\partial + iD'^*)\eta, \eta\rangle = i\langle \Lambda\bar\partial\eta, D'^*\eta\rangle$$
$$+ (i)^2\langle D'^*\eta, D'^*\eta\rangle = i\langle 0, D'^*\eta\rangle$$
$$- \langle D'^*\eta, D'^*\eta\rangle \leq 0, \text{ since } \Lambda\bar\partial\eta = 0.$$