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p_ν — just take C to be a union of lines. It is equally easy to prescribe the tangent lines T_ν that C is to have at p_ν. However, if we assign second-order elements of arc C_ν passing through p_ν and with tangent T_ν, then it is not always possible to find an algebraic curve C having the prescribed second-order behavior C_ν around p_ν. There is one condition here, the Reiss relation, which we proceed to derive.

Suppose that C has affine equation $f(x,y)=0$, that L is the line $\{x=0\}$, and that the n points of intersection of C with L are distinct finite points on the y-axis. We shall prove the

Reiss Relation. With the notations $f_y = (\partial f/\partial y)(x,y)$, etc.

$$\sum \frac{(f_{xx} f_y^2 - 2 f_{xy} f_x f_y + f_{yy} f_x^2)}{f_y^3} = 0,$$

the terms in the sum being evaluated at the points $L \cdot C$.

Proof. In a general vein, the mth-order behavior of C near the points of intersection $C \cap L$ will be reflected in the residues of

$$\omega = \frac{p(x,y)\, dx \wedge dy}{x f(x,y)^m}.$$

If $f(x,y)$ has degree n, then ω will not have the line at infinity as a component of its polar divisor provided that $\deg(p) \le mn-2$.