$$= r(z) - r(z_0) + i \int_0^t \frac{\partial r}{\partial x} \alpha_2'(t) - \frac{\partial r}{\partial y} \alpha_1'(t)\, dt$$

$\Rightarrow \quad r(z) - r(z_0) = \operatorname{Re} f.$ Easily generalized. ⌟

If $T = T_\omega$ is the current associated to a Kähler metric, then its potential function φ is smooth. For instance, $\varphi(z) = \|z\|^2$ is a global potential function for the Euclidean metric on $\mathbb{C}^n$.

⌜ $\bar\partial \|z\|^2 = z_i\, d\bar z_i \quad \Rightarrow \quad \partial(z_i\, d\bar z_i) = dz_i \wedge d\bar z_i$ is the current associated to the standard metric

$$ds^2 = \sum dz_i \otimes d\bar z_i$$ P30. Example 1.

Let $\Lambda(\phi) = \int \varphi\, \phi. \quad \phi \in C_c^\infty.$

$$\frac{\partial \Lambda}{\partial z_1}(\phi) = -\Lambda\left(\frac{\partial \phi}{\partial z_1}\right) = -\int \varphi \frac{\partial \phi}{\partial z_1} = \int h\, \phi \quad \text{for}$$

some $h \in C^\infty$.

$$\Rightarrow \quad \int h\, \phi = \int \left(\int h\, dz_1\right) \frac{\partial \phi}{\partial z_1} \quad \Rightarrow \quad \varphi = \int h\, dz_1$$

$\Rightarrow \quad \varphi$ is smooth. $\quad \Rightarrow \quad \Lambda$ is smooth.

The argument above is not rigorous, if we use regularity theorem on P380, we can make it clear.

Let $T_\omega = h_{11}\, dz_1 \wedge d\bar z_1 + h_{12}\, dz_1 \wedge d\bar z_2 + h_{21}\, dz_2 \wedge d\bar z_1 + h_{22}\, dz_2 \wedge d\bar z_2.$

$\Rightarrow$ Since $dT_\omega = 0$, $dh_{11} \wedge dz_1 \wedge d\bar z_1 + dh_{12} \wedge dz_1 \wedge d\bar z_2 + dh_{21} \wedge dz_2 \wedge d\bar z_1 + dh_{22} \wedge dz_2 \wedge d\bar z_2 = 0.$