$$\Rightarrow C^\circ(\{U, V\}, \mathcal{O}) = \{(f, g) : f \in \mathcal{O}(U),\ g \in \mathcal{O}(V)\}$$
$$\| \quad \prod_{U} \mathcal{O}(U) = \mathcal{O}(U) \times \mathcal{O}(V)$$

and $C'(\{U, V\}, \mathcal{O}) = \mathcal{O}(U \cap V) = \{h \in \mathcal{O}(U \cap V)\}$

Given $(f, g) \in C^\circ(\{U, V\}, \mathcal{O})$, we can write.

$$f = \sum_{n=0}^{\infty} a_n u^n, \qquad g = \sum_{n=0}^{\infty} b_n v^n = \sum_{n=0}^{\infty} b_n u^{-n}.$$

$$\Rightarrow \delta((f, g)) = -f + g \in \mathcal{O}(U \cap V) = 0 \iff \sum_{n=0}^{\infty} a_n u^n$$
$$= \sum_{n=0}^{\infty} b_n u^{-n}, \iff a_n = b_n \text{ for all } n > 0, \text{ and } a_0 = b_0.$$
$$\Rightarrow Z^\circ(\{U, V\}, \mathcal{O}) = \{a_0 = b_0 \mid a_0 \in \mathbb{C}\} = H^\circ(\mathbb{P}^1, \mathcal{O}) \cong \mathbb{C}$$

In other words, the only global holomorphic functions on $\mathbb{P}^1$ are constants.

Remark: From the maximum principle that $H^\circ(M, \mathcal{O}) \cong \mathbb{C}$ for any compact, connected complex manifold, it is a special case.

On the other hand, given any

$$h = \sum_{n=-\infty}^{\infty} a_n u^n = \sum_{n=-\infty}^{\infty} a_n v^{-n} \in C'(\{U, V\}, \mathcal{O})$$
$$\| \quad \mathcal{O}^*(U \cap V)$$

we can write $h = \delta((f, g))$ where $f = -\sum_{n=0}^{\infty} a_n u^n$,

$$g = \sum_{n=1}^{\infty} a_{-n} v^n = \sum_{n=1}^{\infty} a_{-n} u^{-n}$$
$$= \sum_{n=-1}^{-\infty} a_n u^n$$

$\Rightarrow$ It follows that $\delta C^\circ = C' \Rightarrow H'(\mathbb{P}^1, \mathcal{O}) = 0$