Date

Now we are ready to prove that given F a multilinear form, F can be expressed as follows.

$$F : \overbrace{V \times V \times \cdots \times V}^{n} \longrightarrow \mathbb{C}.$$

Let $Q(v) = F(v, \cdots v)$.

$$F(v_1, v_2, \cdots v_n) = \frac{1}{n!} \{ Q(v_1 + \cdots + v_n) - \sum_{i_1 < \cdots < i_{n-1}} Q(v_{i_1} + \cdots + v_{i_{n-1}})$$

$$+ \sum_{i_1 < \cdots < i_{n-2}} Q(v_{i_1} + \cdots + v_{i_{n-2}}) - \cdots + (-1)^{n+1} \sum_{i=1}^{n} Q(v_i) \}$$

Observe that we can make 1-1 correspondence as follows.

$$F(v_1, v_2, \cdots v_n) \longleftrightarrow x_1 x_2 \cdots x_n.$$

$$\Rightarrow \quad Q(v_1 + \cdots v_n) = F(v_1 + \cdots + v_n, v_1 + \cdots + v_n, \cdots v_1 + \cdots + v_n)$$

$$= \sum F(v_{i_1}, v_{i_2} \cdots v_{i_n}) \longleftrightarrow (x_1 + \cdots + x_n)^n.$$

and so on.

Thus we can reduce this problem to the lemma. Q.E.D.

$$\Rightarrow \quad \{ F : V \times \cdots \times V \longrightarrow \mathbb{C} \text{ n-linear symmetric forms} \} \xrightarrow{\phi} \{ Q : V \longrightarrow \mathbb{C} \quad Q(\alpha v) = \alpha^n Q(v) \}$$

defined by $\phi(F) = Q$ where $Q(v) = F(v, \cdots v)$.

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