

The cohomology class defined by T_Z together with the isomorphism

$$H_{DR}^*(M) \cong H^*(\mathcal{D}^*(M), d)$$

is the fundamental class of Z .

$$\begin{aligned} \Gamma \quad 0 &\longrightarrow \mathbb{R} \longrightarrow \mathcal{D}^0 \xrightarrow{d} \mathcal{D}^1 \xrightarrow{d} \mathcal{D}^2 \longrightarrow \mathcal{D}^3 \longrightarrow \dots \\ \Rightarrow H_{DR}^q(M) &\cong \tilde{H}^q(M, \mathbb{R}) \cong \frac{H^0(M, \ker d)}{d H^0(M, \mathcal{D}^{q-1})} \\ &= \frac{\ker d(M)}{d \mathcal{D}^{q-1}(M)} = H^q(\mathcal{D}^*(M), d) \end{aligned}$$

$$H_{DR}^q(M) \longleftrightarrow H^q(\mathcal{D}^*(M), d)$$

$$\downarrow$$

$$\downarrow$$

$$\varphi$$

$$\longmapsto$$

$$T_\varphi$$

defined by

$$T_\varphi(\phi) = \int_M \varphi \wedge \phi, \quad \phi \in A_c^{n-q}(M)$$

$$\begin{aligned} T_Z \in \mathcal{D}^{p,p} &\Rightarrow dT_Z = 0 \text{ and } T_Z \text{ is positive.} \\ \Rightarrow T_Z \in H^{2p}(\mathcal{D}^*(M), d) \end{aligned}$$

$$H_{2(n-p)}(M) \longleftrightarrow H_{DR}^{2p}(M) \longleftrightarrow H^{2p}(\mathcal{D}^*(M), d)$$

$$\downarrow$$

$$\downarrow$$

$$[Z]$$

$$\longleftrightarrow$$

$$\phi_Z$$

$$\longrightarrow$$

$$T_{\phi_Z} = T_Z$$

$$\text{where } T_{\phi_Z}(\varphi) = \int_M \phi_Z \wedge \varphi, \text{ \& } T_Z = \int_{Z^*} \varphi. \quad \text{p60~p61}$$

2. A smooth (1,1) form