the only possibility is $F \cap \sigma(q) =$ double line.

Refer to P762 ~ P763.

Thus since $l_x \cap R = \phi$ is assumed, l_x intersects with S in distinct points.

Note that all three of these arguments apply as well to show that the dual Kummer surface S^* is a quartic surface smooth away from the locus R^*.

Sometime later in detail. At least let's follow up the arguments below.

In the first argument, we observe that the points of intersection of S^* with the pencil $l_x^* \subset \mathbb{P}^{3*}$ of hyperplanes in $\mathbb{P}^3$ containing a line l_x of our complex again correspond to the pencils in X containing l_x, and hence to the lines L on X containing x.

$$\begin{array}{ccc} S^* \cap l_x^* & \longleftrightarrow & \text{Set of pencils in } X \text{ containing } l_x. \\ \Psi & & \Psi \\ h & \longleftrightarrow & \sigma(h) \cap X \end{array}$$

A pencil in X containing l_x is of form $\sigma(p, h)$ which is a line (in X) passing through x.