The proof is just tedious checking of details and will be omitted.

⌈ See, for the proof, Introduction to Holomorphic Functions of Several Variables. 15. Theorem. P20 ⌋

Now we come to some examples. The simplest are those $\mathcal{F}$ such that locally $\mathcal{F} \cong \mathcal{O}^{(r)}$. Then $\mathcal{F}$ is said to be locally free of rank r, and such $\mathcal{F}$ are exactly the sheaves $\mathcal{O}(E)$, where $E \longrightarrow M$ is a holomorphic vector bundle with fiber $\mathbb{C}^r$.

⌈ $$\mathcal{F}(U) \overset{\phi}{\cong} \mathcal{O}(U) \oplus \mathcal{O}(U), \quad \mathcal{F}(V) \overset{\psi}{\cong} \mathcal{O}(V) \oplus \mathcal{O}(V)$$

$$\mathcal{O}(U\cap V) \oplus \mathcal{O}(U\cap V) \xleftarrow{\psi|} \mathcal{F}(U\cap V) = \mathcal{F}(U\cap V) \xrightarrow{\phi|} \mathcal{O}(U\cap V) \oplus \mathcal{O}(U\cap V)$$

$$(f_1, \ f_2) \longleftarrow \sigma = \sigma \longmapsto (g_1 \ \ g_2)$$

$$\phi \circ \psi^{-1} : \mathcal{O}(U\cap V) \oplus \mathcal{O}(U\cap V) \longrightarrow \mathcal{O}(U\cap V) \oplus \mathcal{O}(U\cap V)$$

is $\mathcal{O}$-module isomorphism. $\cdots$ ⊛

rank $E = 2$

$\Rightarrow$ $E|U \cong U \times \mathbb{C}^2$. $\Rightarrow$ $\mathcal{O}(E|U) \cong \mathcal{O}(U \times \mathbb{C}^2) \cong \mathcal{O}(U) \oplus \mathcal{O}(U)$.

$E|V \cong V \times \mathbb{C}^2$

& $\mathcal{O}(E|V) \cong \mathcal{O}(V \times \mathbb{C}^2) = \mathcal{O}(V) \oplus \mathcal{O}(V)$.

$$U\cap V \times \mathbb{C}^2 \xleftarrow{\psi'} E|_{U\cap V} \overset{\phi'}{\cong} U\cap V \times \mathbb{C}^2$$

$$\mathcal{O}(U\cap V \times \mathbb{C}^2) \longleftarrow \mathcal{O}(E|_{U\cap V}) \longrightarrow \mathcal{O}(U\cap V \times \mathbb{C}^2) \cong \mathcal{O}^{(2)}(U\cap V)$$

$\overset{\shortparallel}{\mathcal{O}^{(2)}(U\cap V)}$

$\Rightarrow$ $\phi'_z \circ \psi'^{-1}_z$ induces a morphism from $\mathcal{O}^{(2)}(U\cap V)$ to itself.