We get $y^2 = x(x-1)(x-\lambda)$.

$y^2 = x(x-1)(x-\lambda)$ represents a curve in $\mathbb{P}^2$.

$\Rightarrow$ Since S is irreducible, $S = \{ y^2 = x(x-1)(x-\lambda) \}$.

Note that by the above a Riemann surface of genus 1 is determined by the one parameter λ in the polynomial (*) above; since the quotient $\mathbb{C}/\Lambda$ of $\mathbb{C}$ by any rank 2 lattice $\Lambda \subset \mathbb{C}$ is a Riemann surface of genus 1, and since one complex parameter is required to specify a lattice $\Lambda \subset \mathbb{C}$ of rank 2 up to an automorphism of $\mathbb{C}$, we might expect that in fact all curves of genus 1 may be realized as $\mathbb{C}/\Lambda$. This is in fact the case, as we shall see in the next section.

⌈ A Riemann surface of genus 1 is determined by the one parameter λ in the polynomial $x(x-\lambda)(x-1) = y^2$. For the definition of a lattice, see Husemöller (Elliptic Curve) P 163. $L = \mathbb{Z}\omega_1 + \mathbb{Z}\omega_2$ $\omega_1/\omega_2 \in \mathbb{C} - \mathbb{R}$.

$\Rightarrow$ $\frac{\omega_1}{\omega_2}$ is a complex parameter.

In closing we note that meromorphic functions on $S = \mathbb{C}/\Lambda$ are the same as entire meromorphic functions on $\mathbb{C}$, which are periodic for the lattice Λ.

2. Abel's Theorem

Abel's Theorem — First Version.

The definite integrals of the form (*) $\int \frac{dx}{\sqrt{x^2 + ax + b}}$ are