Let $0 \neq f \in I$. We may assume that $f \in \mathcal{O}_{n-1}[z_n]$ is a Weierstrass polynomial. Set $I' = I \cap \mathcal{O}_{n-1}[z_n]$.

If $\{f=0\}$ contains z_n-axis, by changing coordinate system properly, we may assume that $\{f=0\} \not\supset z_n$-axis, and $f \in \mathcal{O}_{n-1}[z_n]$ is a Weierstrass polynomial.

By induction hypothesis $\mathcal{O}_{n-1}$ is Noetherian, and then the Hilbert basis theorem implies that I' has a finite set $f_1 \cdots f_k \in \mathcal{O}_{n-1}[z_n]$ of $\mathcal{O}_{n-1}$ generators.

$I' = \mathcal{O}_{n-1}[z_n] \cap I$ is an ideal of $\mathcal{O}_{n-1}[z_n]$

Th. 4.9 (Hilbert Basis Theorem) If R is a commutative Noetherian ring with identity, then $R[x_1 \cdots x_n]$ is Noetherian. Hungerford. P 391.

$\Rightarrow$ By the Hilbert basis theorem, $\mathcal{O}_{n-1}[z_n]$ is Noetherian. $\Rightarrow$ I' is finitely generated. $\Rightarrow$ $\exists$ a finite set $f_1, \cdots f_k \in \mathcal{O}_{n-1}[z_n]$. generating I'.

We claim that

$$I = \{f, f_1, \cdots f_k\}.$$

To see this, let $g \in I$ and apply the division theorem to obtain

$$g = hf + r.$$

Then $r \in I \cap \mathcal{O}_{n-1}[z_n] = I'$. and may be expressed in terms of $f_1, \cdots f_k$. Q.E.D.