If we let $\tilde{S}$ be the blow-up of S at p, however, we can extend the map π_p continuously over the exceptional divisor $E \subset \tilde{S}$, obtaining a holomorphic map $\tilde{\pi} : \tilde{S} \longrightarrow H$: for a point $r \in E$ corresponding via the identification $E \cong \mathbb{P}(T_p'(S))$ to the line $\bar{r} \subset T_p(S)$, take $\tilde{\pi}(r)$ to be the point of intersection of H with the line $L_r \subset \mathbb{P}^3$ through p with tangent line $\bar{r}$.

$\ulcorner$ Once we extend the map π_p continuously over the exceptional divisor $E \subset \tilde{S}$, then we can say that π_p is bounded in $\tilde{S} - E$ and holomorphic. $\Rightarrow$ Locally, E is given as the zero locus of some holomorphic function f_{pq} on $\mathbb{C}^2$. $\Rightarrow$ By the Riemann Extension Theorem, π_p can be extended over $\tilde{S}$, and since E has measure zero in $\tilde{S}$, the continuous extension of π_p is equal to the holomorphic extension of π_p.

$$\begin{tikzcd}
S - \{p\} \arrow[r, "\phi"] \arrow[d, "\pi"'] & \phi(S) - [(0,0,0,1)] \arrow[d, "\pi"'] \\
H \arrow[r] & \mathbb{P}^2 = \{[(*,*,*,0)]\}
\end{tikzcd}$$

$[(0,0,0,1)] = P_\infty$

Consider the curve $[(0,0,x,1)]$.
$\Rightarrow$ By the isomorphism to $\mathbb{C}^3$, $[(0,0,x,1)] \longmapsto (0,0,x)$.
$\Rightarrow$ The tangent line of the curve passing through P_∞ is $[(0,0,*,*)]$. $\Rightarrow$ $L_{P_\infty} \cap \mathbb{P}^2 = [(0,0,1,0)]$.