$\Rightarrow$ By Prop. (P 21.), the set of $\mathbb{P}^k$ in $\mathbb{P}^n$ is "very small" (which doesn't intersect transversally with V)

Comment: I think that I can find a holomorphic map on $G(k,n)$ locally whose zero locus is a set of a $\mathbb{P}^k$ in $\mathbb{P}^n$ which does not intersect transversally with V.

Wait until we read Grassmannians.

Let π denote the projection from $\mathbb{P}^{n-k-2}$ onto a complementary $(k+1)$-plane $\mathbb{P}^{k+1}$; choose coordinates $X_0, \cdots X_n$ on $\mathbb{P}^n$ so that

$$\mathbb{P}^{k+1} = (X_{k+2} = \cdots = X_n = 0)$$
$$\mathbb{P}^{n-k-2} = (X_0 = X_1 = \cdots = X_{k+1} = 0)$$

and $\pi([X_0, X_1, \cdots X_n]) = [X_0, X_1, \cdots, X_{k+1}]$.

By the proper mapping theorem, the image $\pi(V)$ of V in $\mathbb{P}^{k+1}$ is an analytic hypersurface in $\mathbb{P}^{k+1}$, and by the hypothesis that $\mathbb{P}^{n-k-1} = \overline{\mathbb{P}^{n-k-2}, p}$ misses V, $\pi(p)$ will lie outside $\pi(V)$.

Since $\exists$ a $(n-k)$-plane $\mathbb{P}^{n-k} \supset \mathbb{P}^{n-k-1}$ which meet with V transversally, $\pi(V)$ is k-dimensional