Thus
$$I(M) = \sum_{\substack{p+q\equiv 0(2)\\ \le 2n\\ p\le q}} (-1)^p h^{p,q} + \sum_{\substack{p+q\equiv 0(2)\\ p+q>2n\\ p\le q}} (-1)^p h^{p,q}$$

$$= \sum_{\substack{p+q\equiv 0(2)\\ p\le q}} (-1)^p h^{p,q}.$$

))

Note in particular that on a Kähler surface M the cup product Q on $H^{1,1}(M)$ has exactly one positive eigenvalue; this fact is frequently called the index theorem for surfaces.

Γ $$H^2(M) = H^{2,0}(M) \oplus H^{1,1}(M) \oplus H^{0,2}(M)$$

$$H^{1,1}(M) = P^{1,1}(M) \oplus L P^{0,0}(M). \qquad P^{0,0}(M) = H^{0,0}(M) = H^0(M) = \mathbb{C}.$$

$i^{1-1}(-1)^{\frac{(1+1)(1+1-1)}{2}} Q$ is positive definite on $P^{1,1}(M)$

$\Rightarrow$ $-Q$ is positive defite $\Rightarrow$ Q is negative defite.

Q on $P^{0,0}(M)$ is positive defite since $i^{0-0}(-1)^{\frac{0\cdot(0+0-1)}{2}} = 0.$

$\Rightarrow$ Since $P^{0,0}(M)$ has one-dimension, Q has exactly one positive eigenvalue on $P^{0,0}(M)$ $\Rightarrow$ on $H^{1,1}(M)$, Q has only one positive eigenvalue))

Note, finally, one distinction between the Hodge and Lefschetz theorems of this section: the Lefschetz theorems are essentially topological, while the Hodge decomposition reflects the analytic structure of the particular manifold M. For instance, if we take a real manifold and give it