For example,

$$\begin{array}{ccc} |D| & \longleftrightarrow & \mathbb{P}(H^0(M, \mathcal{O}(D))) \\ \cup & & \cup \\ V & \longleftrightarrow & \mathbb{P}\langle \sigma_0, \sigma_1, \sigma_2 \rangle \end{array}$$

Assume the following

① The generic element of the linear system $V$ is smooth away from $D_0 \cap D_1$

② The generic element of the linear system $V$ is smooth away from $D_1 \cap D_2$

③ The generic element of the linear system $V$ is smooth away from $D_{i+1} \cap D_0$. ( abundant )

Let $V_{01}$ be the set of elements in $V$ which is smooth away from $D_0 \cap D_1$

" $V_{12}$ " " $D_1 \cap D_2$

" $V_{02}$ " " $D_0 \cap D_2$

$\Rightarrow$ Any element of $V_{01} \cap V_{12} \cap V_{02}$ is smooth away from $D_0 \cap D_1 \cap D_2$ and $V - (V_{01} \cap V_{12} \cap V_{02})$ is open & dense, which implies that the generic element in $V$ is smooth away from $D_0 \cap D_1 \cap D_2$.

It remains to show ①, ② and ③.

But it is not easy to get a proof following the argument along the proof on P137. So I guess that the authors misunderstood the conclusion. ( Proof. … thus it suffices to prove Bertini for a pencil. ).