

⌈ C, D, E cubics \Rightarrow They are represented by homogeneous polynomials of degree 3, in P^2 . $\Rightarrow \#(C \cdot D) = 9$.

$$m+n-3=3. \text{ see P672 } \Rightarrow$$

Proof. Let F, G, H be homogeneous cubic polynomials defining C, D, E , respectively. Suppose L is a linear form vanishing at q and at two points R_1, R_2 on C but not on D .

⌈ We can find a hyperplane H passing through q .
 $\Rightarrow \#(H \cap C) = \{q, R_1, R_2\}$, $R_1, R_2 \notin D$. $\Rightarrow H$ represents a linear form L .

Applying the lemma and Max Noether theorem to HL , we have

$$HL = AF + BG.$$

⌈ Note here that $r \notin C \cap D$. $\Rightarrow F(r) \neq 0$ or $G(r) \neq 0$
 $\Rightarrow I_r = \mathcal{O}_r \Rightarrow (HL)_r \in \mathcal{O}_r$ obviously.

So we have only to worry about points in $C \cap D$.

$\forall p \in C \cap D$. $C = (F=0)$.

$\text{Ord}_{C,p} HL \geq \text{Ord}_{C,p} G$ by the assumption $\text{Ord}_{C,p}(H) \geq$

$\text{Ord}_{C,p}(G)$ for all $p \in C \cap D - \{q\}$ and $\text{Ord}_{C,q}(H) \geq \text{Ord}_{C,q}(G)$,

since L passes q . \Rightarrow By the lemma above,