$\Rightarrow \{ H_1(\omega) = H_2(\omega) = 0 \} \ni (\frac{1}{4}, \frac{1}{3}) = \omega_0$ since
$h_1(\sqrt{\frac{1}{4}}, \frac{1}{3}) = 0 = h_2(-\sqrt{\frac{1}{4}}, \frac{1}{3})$, $z_1(\omega_0) = (\frac{1}{2}, \frac{1}{3})$, $z_2(\omega_0)$
$= (-\frac{1}{2}, \frac{1}{3})$.
$\Rightarrow \{ H_1(\omega) = H_2(\omega) = 0 \} \neq f\{ h_1 = h_2 = 0 \} = \phi$
ϕ ※

Actually $H_1(\omega) = -\omega_1 + \frac{1}{4}$ $H_2(\omega) = -\omega_1 + \frac{1}{4}$.
$\Rightarrow \{ H_1(\omega) = 0 = H_2(\omega) \} = \{ (\frac{1}{4}, \omega_2) : |\omega_2| < 1 \}$.
$\Rightarrow$ The claim is not correct, for $n \neq 1$.
If V is defined by $\{ h(z) = 0 \}$, $f(V) = \{ H(\omega) = 0$, $H(\omega) = h(z_1(\omega)) \cdots h(z_d(\omega)) \}$.
Specially for $\omega = (\frac{1}{4}, \frac{1}{3})$, $f(\frac{1}{2}, \frac{1}{3}) = \omega$. $(\frac{1}{2}, \frac{1}{3}) \in \{ h_1 = 0 \}$.
$f(-\frac{1}{2}, \frac{1}{3}) = \omega$, $(-\frac{1}{2}, \frac{1}{3}) \in \{ h_2 = 0 \}$ but $(\frac{1}{2}, \frac{1}{3}) \neq (-\frac{1}{2}, \frac{1}{3})$.
Point : $f(V_1 \cap V_2) \neq f(V_1) \cap f(V_2)$.

Given a general finite surjective mapping $f : U \to W$, for any point $f(p) \in W$, by choosing a small nbd W' of $f(p)$, by the finiteness of f, $f^{-1}(W') \ni p \xrightarrow{f} W'$ is a finite holomorphic mapping. $\Rightarrow$ If V is an algebraic subvariety of U, $f(V \cap f^{-1}(W'))$ is an algebraic subvariety of W'.
Let $f^{-1}(f(p)) = \{ p = P_1, P_2 \}$. Consider the possibilities such as below.