$$\overline{\int \overline{\phi}\, d\mu_{12}} = \int \phi \, d\overline{\mu}_{12} = t_{21}(\phi) = \int \phi \, d\mu_{21}$$

$\Rightarrow \quad \overline{\mu}_{21} = \mu_{12}$.

Claim! $\quad |\lambda_1|^2 \mu_{11} + \lambda_1 \overline{\lambda}_2 \mu_{12} + \overline{\lambda}_1 \lambda_2 \mu_{21} + |\lambda_2|^2 \mu_{22}$

is nonnegative measure for any $(\lambda_1, \lambda_2) \in \mathbb{C}^2$.

If not, for some (λ_1,λ_2), $\exists$ a measurable set $E \subset \mathbb{C}^n$ s.t

$(|\lambda_1|^2 \mu_{11} + \lambda_1 \overline{\lambda}_2 \mu_{12} + \overline{\lambda}_1 \lambda_2 \mu_{21} + |\lambda_2|^2 \mu_{22})(E) < 0$.

Let $\mu = |\lambda_1|^2 \mu_{11} + \cdots + |\lambda_2|^2 \mu_{22}$.

$\Rightarrow \exists$ a measurable subset $E' \subset E$ s.t $\mu(E') < -\epsilon$

for some $\epsilon > 0$.

by P50, Rudin RCA

$\Rightarrow \exists$ open set $U \supset E'$ s.t $\mu(U - E') < \delta$ and

$\exists$ a positive function ρ s.t $\rho = 1$ on E'

$\operatorname{supp} \rho \subset U$, $\quad 0 \le \rho \le 1$, $\quad \rho \ C^\infty$.

$\Rightarrow \left| \int \rho \, d\mu - \int_{E'} d\mu \right| < |\mu(U) - \mu(E')| < \frac{\epsilon}{4}$ if we choose small enough δ.

$\Rightarrow T_{\lambda\lambda}(\rho) = \int \rho \, d\mu < 0 \quad \Rightarrow \quad$ Contradiction.

Comment: In the claim, μ may not be real, so we have to claim μ is not positive, i.e for some (λ_1, λ_2), $\exists$ a measurable set $E \subset \mathbb{C}^2$ s.t $\mathrm{Re}(\mu(E)) < 0$ or $\mathrm{Im}(\mu(E)) \neq 0$. But, it does not matter, since we can apply the argument above to these situations. We can use P50. Rudin's RCA since Radon measure is Borel measure. ↲