$$\mathbb{C}^{n+1} - \{0\} \xrightarrow{\pi} \mathbb{P}^n$$

$\frac{\sigma}{\sigma_F}$ is a meromorphic function on $\mathbb{P}^n$.

$\Rightarrow$ $\pi^*\left(\frac{\sigma}{\sigma_F}\right)$ is a meromorphic function on $\mathbb{C}^{n+1} - \{0\}$, defined by $\pi^*\left(\frac{\sigma}{\sigma_F}\right)(X) = \frac{\sigma}{\sigma_F}(\pi(X))$

$$= \frac{\sigma(\pi(X))}{\sigma_F(\pi(X))}.$$

$G'$ has a simple pole along the divisor $F = 0$ in $\mathbb{C}^{n+1} - \{0\}$ and is holomorphic elsewhere, so the function

$$G = G' \cdot F$$

is holomorphic everywhere in $\mathbb{C}^{n+1} - \{0\}$ and hence by Hartogs's theorem extends to an entire holomorphic function on $\mathbb{C}^{n+1}$.

Comment; "simple pole" is not a right word, I think.

Now since $G'(\lambda X) = G'(X)$ for all $X \in \mathbb{C}^{n+1}$ and $\lambda \in \mathbb{C}$, and $F(\lambda X) = \lambda^d F(X)$,

$$G(\lambda X) = \lambda^d G(X),$$

i.e., $G$ is homogeneous of degree $d$.

$$G'(\lambda X) = \pi^*\left(\frac{\sigma}{\sigma_F}\right)(\lambda X) = \frac{\sigma}{\sigma_F}(\pi(\lambda X))$$