$$\Rightarrow \; 0 \to \mathrm{Hom}(E,\mathcal{O}) \to \overset{0}{\mathrm{Hom}(\mathcal{O}\oplus(\mathcal{O}\oplus\mathcal{O}),\mathcal{O})} \xrightarrow{\delta} \overset{1}{\mathrm{Hom}(\mathcal{O},\mathcal{O})} \to \overset{2}{0} \to 0 \cdots$$

$\Rightarrow$ $\mathrm{Ext}^n_{\mathcal{O}}(E,\mathcal{O}) = 0$ if $n \geq 2$.

Note: δ is surjective. $\Rightarrow$ $\mathrm{Ext}^1_{\mathcal{O}}(E,\mathcal{O}) = 0$.

Anyway, the point is to get a proper projective resolution for E. ∎

We use this to prove that $\mathrm{Ext}^q_{\mathcal{O}}(E, N) = 0$ for any $\mathcal{O}$-module N and $q \geq 1$. The argument is by induction on the length of a projective resolution of N. Thus we may assume given

$$0 \to R \to F \to N \to 0,$$

where F is free and $\mathrm{Ext}^q_{\mathcal{O}}(E, R) = 0$ for $q \geq 1$. The exact sequence of $\mathrm{Ext}^*_{\mathcal{O}}(E, \cdot)$ then gives the result.

Γ Let $0 \to E_n \to E_{n-1} \to \cdots \to E_0 \xrightarrow{\sigma} N \to 0$ be a projective resolution. $\Rightarrow$ $0 \to \ker\sigma \to E_0 \to N \to 0$.

$\Rightarrow$ $0 \to E_n \to E_{n-1} \to \cdots E_1 \to \ker\sigma \to 0$ → projective resolution of $\ker\sigma$

$\Rightarrow$ length of a projective resolution of $\ker\sigma$ < length of a projective resolution of N.

$\Rightarrow$ By induction, $\mathrm{Ext}^q_{\mathcal{O}}(E, R) = 0$ for $q \geq 1$. $R = \ker\sigma$.

$$\underset{\substack{\| \\ 0}}{\mathrm{Ext}^n_{\mathcal{O}}(E, R)} \to \mathrm{Ext}^n_{\mathcal{O}}(E, F) \to \mathrm{Ext}^n_{\mathcal{O}}(E, N) \to \underset{\substack{\| \\ 0}}{\mathrm{Ext}^{n+1}_{\mathcal{O}}(E, R)}$$

$n \geq 1$ $\Rightarrow$

$\Rightarrow$ $\mathrm{Ext}^n_{\mathcal{O}}(E, F) \cong \mathrm{Ext}^n_{\mathcal{O}}(E, N) \cong \mathrm{Ext}^n_{\mathcal{O}}(E, \mathcal{O}\oplus\cdots\oplus\mathcal{O})$

$\cong$ $\mathrm{Ext}^n_{\mathcal{O}}(E, \mathcal{O}) \oplus \cdots \oplus \mathrm{Ext}^n_{\mathcal{O}}(E, \mathcal{O}) = 0$ ∎