

$$\#(\tilde{\phi}(\sigma_{2,2,2,1}) \cdot H) = 1 = \#(\sigma_{2,2,2,1} \cdot \sigma_1).$$

In $\sigma_{2,2,2,1}(V_3) = \{V_3 \subset \mathbb{P}^5 : V_2 \subset V_3 \subset V_4\}$, we may assume $V_2 = \langle e_0, e_1, e_2 \rangle$ and $V_4 = \langle e_0, e_1, e_2, e_3, e_4 \rangle$.
 $\Rightarrow V_3 = \langle e_0, e_1, e_2, a e_3 + b e_4 \rangle$.

$$\Rightarrow \sigma_1(V_3') = \{ \omega \in \Lambda^2 \mathbb{C}^6 \mid \omega \wedge (e_0 \wedge e_1 \wedge e_2 \wedge (a' e_3 + b' e_4)) = 0 \}$$

$$\text{and } \sigma_1(V_3'') = \{ \omega \in \Lambda^2 \mathbb{C}^6 \mid \omega \wedge (e_0 \wedge e_1 \wedge e_2 \wedge (a'' e_3 + b'' e_4)) = 0 \}$$

$$\Rightarrow \sigma_1(V_3') + \sigma_1(V_3'') = \sigma_1(V_3''') = \{ \omega \in \Lambda^2 \mathbb{C}^6 \mid \omega \wedge (e_0 \wedge e_1 \wedge e_2 \wedge ((a' + a'') e_3 + (b' + b'') e_4)) = 0 \}$$

$\Rightarrow \tilde{\sigma}_{2,2,1}(V_3)$ is a linear system \Rightarrow Since $\tilde{\phi}(\sigma_{2,2,2,1}(V_3))$ is of degree 1, it is a pencil. \square

Now, as we have seen, through a generic $x \in X$ there pass four lines in X , comprising the locus $X \cap T_x(X)$.

\mathbb{P}^1 $K =$ The set of all lines (in X) passing through x . $\Rightarrow K \subset X$ and $K \subset T_x X \cap X \Rightarrow$ By P. 264, $T_x X \cap X = K$. \square

Let V_2 be a generic α -plane, meeting X in four distinct such points, and let V_4 be a generic hyperplane containing V_2 , not containing any line on X meeting V_2 ; consider the pencil $\{P_V\}$ $V: V_2 \subset V \subset V_4$ on A .