where V is a smooth hypersurface in M.

$\Rightarrow$ Let $M = \mathbb{P}^2$, $V = S$.

$$\Rightarrow \quad K_S = (K_{\mathbb{P}^2} \otimes [S])|_S = (K_{\mathbb{P}^2} + S)|_S$$

From $K_M|_V = K_V \otimes N_V^*$, $\quad K_M|_V \otimes N_V = K_V$.

$$\Rightarrow \quad K_{\mathbb{P}^2}|_S \otimes N_S = K_S \qquad \lrcorner$$

Now from that section $K_{\mathbb{P}^2} = -3H$ and $S = dH$, so $K_{\mathbb{P}^2} + S = (d-3)H$ on $\mathbb{P}^2$.

$\ulcorner$ $K_{\mathbb{P}^n} = [-(n+1)H]$ by p146

$\Rightarrow \quad K_{\mathbb{P}^2} = -3H. \quad \Rightarrow \quad K_{\mathbb{P}^2} + S = (d-3)H$ $\lrcorner$

Thus $\quad \chi(S) = -\deg K_S = -\#(S \cdot (d-3)H) = -d(d-3)$

and $\quad g(S) = \dfrac{2-\chi(S)}{2} = \dfrac{(d-1)(d-2)}{2}$.

$\ulcorner$ $K_S = (K_{\mathbb{P}^2} + S)|_S \Rightarrow$ Since $\deg K_S = \langle c_1(K_S), S \rangle$

$= \#(\text{divisor of } K_S \cdot S)$ since $\eta(\text{divisor of } K_S) = c_1(K_S)$.

See p141, p58 & p61. $\lrcorner$

The third way to compute $g(S)$ is by the Poincare residue map. Recall (p.147) that for a meromorphic 2-form ω on $\mathbb{P}^2$ holomorphic on $\mathbb{P}^2 - S$ and with a single pole along S, and written locally as

$$\omega = g(z_1, z_2)\,\frac{dz_1 \wedge dz_2}{f(z_1, z_2)},$$

the Poincare residue $R(\omega)$ is given by

$$R(\omega) = -g(z_1, z_2)\,\frac{dz_1}{(\frac{\partial f}{\partial z_2})(z_1, z_2)} = g(z_1, z_2)\,\frac{dz_2}{(\partial f/\partial z_1)(z_1, z_2)}.$$