$\Rightarrow \quad \chi(C_\lambda) = \chi(\tilde{C}_\lambda) = -2.$

┘

In the latter case — when $H_\lambda$ is simply tangent to $S$ — $C_\lambda$ is isomorphic to a plane quartic with one ordinary double point and

$$\chi(C_\lambda) = -3.$$

┌ $S \cap H_\lambda$ is a quartic with one double point since $\{H_\lambda \cap S\}$ is Lefschetz. $\Rightarrow$ Since $\pi$ is one to one over $S - R$, $\pi^{-1}(S \cap H_\lambda) = C_\lambda$ is isomorphic to a quartic with one ordinary double point. $\Rightarrow$ Again by P508

$\chi(C_\lambda) = \chi(C_0) + 1$. $C_0$ is smooth quartic.

Since $\chi(C_0) = -4$, $\chi(C_\lambda) = -3$.

┘

Thus if $\nu$ is the number of tangent hyperplanes to $S$ in a pencil, the pencil $\{C_\lambda\}$ on $\Sigma$ has exactly

$$\mu = \nu + \#R$$

singular elements.

┌ $\nu = \mu - \#R$, since $C_\lambda$ is singular $\Leftrightarrow$ $H_\lambda$ is tangent to $S$ or $H_\lambda \cap R \neq \phi$, $\#(H_\lambda \cap R) = 1$.

┘

By the formula of P509, then we see that