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$, \cdots z_n^{d}\}$, where $d = \dim_{\mathbb{C}}(\mathcal{O}/I)$. Q.E.D.

$\ulcorner$ Fix φ. for all $g+I$,

$$\mathrm{res}(g+I, \varphi) = 0 = \mathrm{res}(g+I', \pi^*\varphi) = 0$$

$\Rightarrow$ By the nondegenercey of res on $\mathcal{O}/I' \otimes \mathrm{Ext}^n_{\mathcal{O}}(\mathcal{O}/I', \Omega^n)$,
$\pi^*\varphi = 0$. $\Rightarrow$ By the injectivity of π^*, $\varphi = 0$. //

If the proof on the injective of π^* is not correct, we can prove the injective by using the Local duality theorem,

$\mathrm{res}_f(g, h) = \mathrm{res}_{f'}(g, \Delta h)$ $\Rightarrow$ If $\Delta h = 0$, for all g, $\mathrm{res}_f(g, h) = 0$ $\Rightarrow$ $h = 0$ by the nondegeneracy.

$I' = \{z_1^d, z_2^d\}$ $d = \dim(\mathcal{O}/I) = \deg(f)$ $\Rightarrow$ $I' \subset I = \{f_1, f_2\}$ by P 669

Suppose $\mathrm{res}_{f'}(g, h) = 0$ for all g.

$h \equiv \sum_{0 \le n_1, n_2 \le d} z_1^{n_1} z_2^{n_2} a_{n_1 n_2} (\mathrm{mod}\ z_1^d, z_2^d)$. We need not worry about $z_1^d \square + z_2^d \square$ terms since $\mathrm{res}_{f'}(g, z_1^d \square + z_2^d \square) = 0$ by the result on P650.

If $a_{n_1, n_2} \neq 0$, then for a proper g.

$$\frac{a_{n_1, n_2} z_1^{n_1} z_2^{n_2} g}{z_1^d z_2^d} = a_{n_1, n_2} \frac{1}{z_1 z_2}$$

$$\Rightarrow C \int_{|z_i| = \epsilon} a_{n_1 n_2} \frac{dz_1 \wedge dz_2}{z_1 z_2} = a_{n_1 n_2} \neq 0 = \mathrm{res}_{f'}(g, h)$$

$\Rightarrow$ Contradiction. $\Rightarrow$ res is nondegenerate for $I' = \{z_1^d, z_2^d\}$. $\lrcorner$