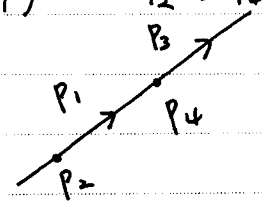
$L_{23} + L_{45} + L_{67} \ni P_1 \Rightarrow$ Contradiction, since a line can not have nonzero second derivative. Thus almost trivially $P_1, P_2, P_3, \cdots P_7$ impose linearly independent conditions on cubics. Since P_4, P_5, P_6, P_7, P_8 are noncollinear, $\exists$ three noncollinear points, say P_6, P_7, P_8. We can choose a conic τ containing P_1, P_2, P_3, P_4, P_5. Again consider $\tau + L_{68}$, $\tau + L_{67}$, $\tau + L_{78}$.
$\Rightarrow \tau \ni L_6, L_8, L_7$ by the dependence of $P_1, P_2, \cdots P_7, P_8$. ///

Continuation from P750 note (iii): Correction.

(iii) $L_{12} \ni P_4, P_3$



Consider a conic τ containing P_1, P_2, P_5, P_6, P_7.

$\Rightarrow \tau + L_{45} \ni P_1, P_2, P_4, P_5, P_6, P_7 \Rightarrow$ By the linear dependence of $P_1, P_2, P_3, P_4, P_5, P_6, P_7$, $P_3 \in \tau + L_{45}$.

$\Rightarrow$ If $P_3 \in L_{45} \Rightarrow$ done, i.e. L_{12} has five points.

If $P_3 \in \tau$, then $\#(L_{12} \cap \tau) \geq 3$. $\Rightarrow$ Since $\deg \tau = 2$, $L_{12} \subset \tau$. $\Rightarrow \tau = L_{12} + L_{567}$, since $\tau \ni P_1, P_2, P_3, P_4, P_5, P_6, P_7$, otherwise $L \ni P_5$ or P_6 or P_7.

① $P_1 P_2, P_3 P_4, P_6, P_7, P_8$ fail to impose linearly independent conditions

By the argument above, $\tau' + L^{46} \ni P_3 \Rightarrow \tau' \ni P_3$
$\Rightarrow \tau' = L_{12} + L_{678}$, where $\tau' \ni P_1, P_2, P_4, P_7, P_8$.
$\Rightarrow L_{678} = L_{567} \Rightarrow P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8$ lie on the conic $L_{12} + L_{56}$.

② $P_1, P_2, P_3, P_4, P^6 P_7, P_8$ impose linearly independent conditions. and so do $P_1, P_2, P_3, P_4, P_5, P_6, P_8$ ····.