$$v_i = \sum a_{ji} e_j , \quad e_j = \begin{array}{c} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ 0 \\ 0 \end{pmatrix} \end{array} \begin{array}{c} \\ \\ \\ j \\ \\ \\ \end{array}$$

We may assume that ${}^t v_i v_i = 1 \Leftrightarrow a_{1i}^2 + a_{2i}^2 + a_{3i}^2 + a_{4i}^2 = 1$. No!!! We may have ${}^t v_i v_i = 0$.
Thus we can say that this is wrong approach.
$\exists\ v_1 \neq 0$ s.t $Q(v_1, v_1) \neq 0$, where $Q(v_1, v_1) = {}^t v_1 Q v_1$, for, if not so, ${}^t e_i Q e_j = Q_{ij} = 0$. $\Rightarrow$ Q is a zero matrix. This is a contradiction to the fact that Q is nonsingular.
Consider v_1, $e_2 - \dfrac{Q(v_1, e_2)}{Q(v_1, v_1)} v_1$, $e_3 - \dfrac{Q(v_1, e_3)}{Q(v_1, v_1)} v_1$, $e_4 - \dfrac{Q(v_1, e_4)}{Q(v_1, v_1)} v_1$. $\Rightarrow$ (See P90, 42:1. O'Meara Introduction to Quadratic Forms)

$$Q\left(v_1,\ e_i - \frac{Q(v_1, e_i)}{Q(v_1, v_1)} v_1\right) = Q(v_1, e_i) - \frac{Q(v_1, e_i)}{Q(v_1, v_1)} Q(v_1, v_1)$$

$= 0$, and those four vectors form a basis for $\mathbb{C}^4$.
Let $W = \left\langle e_i - \dfrac{Q(v_1, e_i)}{Q(v_1, v_1)} v_1 \right\rangle_{i=2}^{4}$

$\Rightarrow \exists\ v_2 \neq 0 \in W$ s.t $Q(v_2, v_2) \neq 0$. Otherwise

$$Q = \begin{pmatrix} * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad * .$$

In this way, we have v_1, v_2, v_3, v_4 s.t $\langle v_1, v_2, v_3, v_4 \rangle = \mathbb{C}^4$, $Q(v_i, v_j) = 0$ if $i \neq j$,