in cohomology. But $\iota(v)\psi \in H^{n-k-1}(M)$, and so $L^{k+1}\iota(v)\psi = 0 \Rightarrow \iota(v)\psi = 0$ by hard Lefschetz. Thus $d_1' = 0$

The argument for $d_2' = d_3' = \cdots = 0$ is the same. Q.E.D.

Pf $\quad \psi \in H^{n-k}(M) \Rightarrow \iota(v)\psi \in H^{n-k-1}(M)$

$$\begin{tikzcd}
H^{n-k-1}(M) \arrow[r, "L^{k+1}", "\cong"'] & H^{n+k+1}(M) \\
\iota(v)\psi \arrow[u, phantom, "\rotatebox{90}{$\in$}"] \arrow[r, mapsto] & L^{k+1}\iota(v)\psi = \omega^{k+1}\wedge\iota(v)\psi = 0 .
\end{tikzcd}$$

$\Rightarrow \quad \iota(v)\psi = 0$

For $\psi \in P^{n-q-k,q}(M) \subset H^{n-q-k,q}(M) \subset {}'E_1^{q+k,q}$

$$\begin{tikzcd}
H^{n-k}(M) \supset P^{n-q-k,q}(M) \arrow[r, "d_1'"] \arrow[dd, "0 = L^{k+1}"'] & H^{n-q-k-1,q}(M) \subset H^{n-k-1}(M) \arrow[dd, "\cong L^{k+1}"] \\
\ni \psi \arrow[r, mapsto] \arrow[d, mapsto] & \iota(v)\psi \arrow[d, mapsto] \\
H^{n-q+1,q+k+1}(M) \arrow[r, "d_1'"] & H^{n-q,q+k+1}(M) \subset H^{n+k+1}(M) \\
\omega^{k+1}\psi \arrow[r, mapsto] & 0
\end{tikzcd}$$

$H^{n-q+1,q+k+1}(M) \subset H^{n+k+2}(M)$, $\quad \omega^{k+1}\psi = 0$

$\Rightarrow \quad \iota(v)\psi = 0$ since L^{k+1} is