of duality ≥ 0 means positive orientation. ⌟

Then the residue is given by

$$\mathrm{Res}_{\{0\}}\omega = \left(\int_{\Gamma}\omega\right)\left(\frac{1}{2\pi\sqrt{-1}}\right)^{n}.$$

Here are some elementary properties of the residue.

First, since $\omega \in H^0(U-D, \Omega^n)$ is holomorphic in $U-D$, the exterior derivative $d\omega = 0$. Consequently, the residue depends only on the homology class of $\Gamma \in H_n(U-D, \mathbb{Z})$ and the cohomology class $[\omega] \in H^n_{DR}(U-D)$ of ω.

Γ $\omega = \dfrac{g\, dz_1 \wedge \cdots \wedge dz_n}{f_1 \cdots f_n} \Rightarrow \partial\omega = \partial\left(\dfrac{g}{f_1 \cdots f_n}\right) \wedge dz_1 \cdots \wedge dz_n = 0$

since $U \subset \mathbb{C}^n$. Since ω is holomorphic, $\bar{\partial}\omega = 0$.
$\Rightarrow d\omega = 0$. $\displaystyle\int_{\Gamma+\partial\sigma} d\eta + \omega = \int_{\Gamma+\partial\sigma}\omega + \int_{\Gamma+\partial\sigma} d\eta$

$$= \int_{\Gamma}\omega + \int_{\partial\sigma}\omega + \int_{\Gamma} d\eta + \int_{\partial\sigma} d\eta = \int_{\Gamma}\omega$$

by Stokes' theorem. ⌟

Second, the residue is linear in g and alternating in the f_i, the latter being due to the manner in which the cycle Γ has been oriented.