

\mathbb{P} By P177, since $L: B \longrightarrow E_L \subset \mathbb{P}^3$,
 $L^* H^0(E_L, \mathcal{O}(H)) = H^0(B, \mathcal{O}(D)) = L$ hyperplane in \mathbb{P}^3 .
 Note: E_L is nondegenerate, otherwise $g(E_L) = 2$
 $= \frac{(5-1)(5-2)}{2}$ which is impossible. Here we can see
 that $\dim H^0(B, \mathcal{O}(D)) = 4 = \dim H^0(\mathbb{P}^3, \mathcal{O}(H))$.

\Rightarrow By Kodaira embedding theorem, and its application P215,
 $\deg D = 5 > \deg K_B + 2 = (2 \times g(B) - 2) + 2 = 4$,
 $L: B \longrightarrow \mathbb{P}^3$ is well-defined and an embedding, where
 $L = H^0(B, \mathcal{O}(D))$.

Second, since by Riemann-Roch

$$h^0(2D) = 10 - 2 + 1 = 9$$

and the vector space $H^0(\mathbb{P}^3, \mathcal{O}(2H))$ of quadrics on \mathbb{P}^3 has dimension 10, the restriction map

$$H^0(\mathbb{P}^3, \mathcal{O}(2H)) \longrightarrow H^0(B, \mathcal{O}(2D))$$

must have a kernel — i.e., E_D must lie on a quadric surface Q in \mathbb{P}^3 .

\mathbb{P} By Riemann-Roch formula on P245,

$$\begin{aligned}
 h^0(2D) &= \deg(2D) - g(B) + h^0(K_B - 2D) + 1 \\
 &= 10 - 2 + 0 + 1 = 9, \text{ since } \deg(K_B - 2D) \\
 &= \deg K_B - \deg(2D) = 4 - 10 = -6, \text{ and by P215,} \\
 h^0(K_B - 2D) &= 0. \quad \dim H^0(\mathbb{P}^3, \mathcal{O}(2H)) = 5C_2 = 10, \text{ P166.}
 \end{aligned}$$

$$L: B \longrightarrow \mathbb{P}^3 \Rightarrow L^*: H^0(\mathbb{P}^3, \mathcal{O}(2H)) \longrightarrow H^0(B, \mathcal{O}(2D))$$