ways; we mention two here.

1. The complement of a hyperplane $H \subset \mathbb{P}^n$ is isomorphic to $\mathbb{C}^n$ via Euclidean coordinates; we may take the tangent space to $V \subset \mathbb{P}^n$ at p to be the closure in $\mathbb{P}^n$ of the usual tangent subspace $T_p(V) \subset T_p(\mathbb{C}^n)$. Explicitly, if $x_1, x_2, \cdots x_n$ are Euclidean coordinates on $\mathbb{P}^n$ in a nbd of $p = (\alpha_1, \cdots \alpha_n)$ and V is cut out by functions $\{f_\alpha(x_1, x_2, \cdots x_n)\}$, this is just the linear subspace of $\mathbb{P}^n$ defined by

$$\sum_{i=1}^{n} \frac{\partial f_\alpha}{\partial x_i}(p) \cdot (x_i - \alpha_i) = 0.$$

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2. Alternatively, if V is given in terms of homogeneous coordinates $X_0, X_1, \cdots X_n$ as the locus of polynomials $\{F_\alpha(X_0, X_1, \cdots X_n)\}$, this is the linear subspace

$$\sum_{i=0}^{n} \frac{\partial F_\alpha}{\partial X_i}(p) \cdot X_i = 0,$$

where the differentiation is formal: if f_α is the inhomogeneous form of F_α, then $\partial f_\alpha / \partial x_i$ is the inhomogeneous form of $\partial F_\alpha / \partial X_i$, and by virtue of the relation

$$\frac{1}{d} \sum_{i=0}^{n} \frac{\partial F}{\partial X_i} = F,$$

where $d = \deg(F)$, we can write

$$\sum_{i=0}^{n} \frac{\partial F_\alpha}{\partial X_i}(p) \cdot X_i = X_0 \sum_{i=0}^{n} \frac{\partial F_\alpha}{\partial X_i}(p) \cdot x_i$$