⌈ By the argument above, we saw that $C(W_2) \subset W_1$, see note p 836. $\Rightarrow$ The chords to W_2 lie in W_1.

We may assume that the two distinct double lines are X_0^2 and X_1^2. $\Rightarrow$ The pencil is $\lambda X_0^2 + X_1^2$, $\lambda \in \mathbb{P}^1$. $\Rightarrow$ The base locus is $p = \{[0, 0, 1]\}$. Let α be a number s.t $-\alpha^2 = \lambda$, for $\lambda \in \mathbb{C}$.

$\Rightarrow \quad \lambda X_0^2 + X_1^2 = (X_1 + \alpha X_0)(X_1 - \alpha X_0)$

$\Rightarrow \quad (X_1 + \alpha X_0 = 0)$ in $\mathbb{P}^2$ is the pullback of $[1, -\alpha] \in \mathbb{P}^1$, via the projection $\pi_p : \mathbb{P}^2 \longrightarrow \mathbb{P}^1$ defined by $[X_0, X_1, X_2] \longmapsto [X_0, X_1]$.

Similarly, $(X_1 - \alpha X_0 = 0)$ in $\mathbb{P}^2$ is the pullback of $[1, \alpha] \in \mathbb{P}^1$, via the projection $\pi_p : \mathbb{P}^2 \longrightarrow \mathbb{P}^1$ defined by $[X_0, X_1, X_2] \longmapsto [X_0, X_1]$.

Thus, $(\lambda X_0^2 + X_1^2 = 0)$ in $\mathbb{P}^2$ is the pullback of $\{[1, -\alpha], [1, \alpha]\} \in \mathbb{P}^1$, via π_p: The pencil $\{(\lambda X_0^2 + X_1^2 = 0)\}_{\lambda \in \mathbb{P}^1}$ is the pullback of $\{[1, -\alpha], [1, \alpha]\}_{\alpha \in \mathbb{P}^1}$ which is a pencil of $\mathbb{P}^1$. ///

Choosing such a pencil is parametrized by $W_2 \times W_2$, and the family of such pencils is of dim. 4.

Similarly, choosing a fixed line and a <u>pencil</u> is parametrized by $\mathbb{P}^{2*} \times \mathbb{P}^{2*}$ ($\because$ $l_\lambda = \{[a_0, a_1, a_2] \mid a_0(\lambda) X_0 + a_1(\lambda) X_1 + a_2(\lambda) X_2 = 0$, see note p 837).

$\Rightarrow$ Again the dimension is 4.

⌋