$d^c \log(f_\alpha \overline{f_\alpha} h_\alpha)$

$= \frac{i}{4\pi}(\bar\partial - \partial) \log(f_\alpha \overline{f_\alpha} h_\alpha)$

$\Rightarrow \quad \bar\partial(\log f_\alpha \overline{f_\alpha} h_\alpha) = \bar\partial \log \overline{f_\alpha} + \bar\partial \log h_\alpha$

$\quad\quad \partial(\log f_\alpha \overline{f_\alpha} h_\alpha) = \partial \log f_\alpha + \partial \log h_\alpha.$

$\Rightarrow \frac{i}{4\pi}(\bar\partial - \partial) \log(f_\alpha \overline{f_\alpha} h_\alpha) = (\bar\partial \log \overline{f_\alpha} - \partial \log f_\alpha + (\bar\partial - \partial)\log h_\alpha) \times \frac{i}{4\pi}$

Since $d^c \log h_\alpha$ is bounded and, as we have seen in the proof of Stoke's theorem for analytic varieties,

$\mathrm{Vol}(\partial D(\epsilon)) \to 0$ as $\epsilon \to 0$, we deduce that

$$\lim_{\epsilon \to 0} \int_{\partial D(\epsilon)} d^c \log h_\alpha \wedge \psi = 0.$$

$\ulcorner$ $S_\alpha = f_\alpha \overline{s_\alpha} \qquad S_\beta = f_\beta \overline{s_\beta}$

On $U_\alpha \cap U_\beta$,

$$S_\alpha = S_\beta = f_\beta \overline{s_\beta} = f_\alpha \overline{s_\alpha} = g_{\alpha\beta} f_\beta \overline{s}_\beta$$

$\Rightarrow \langle f_\alpha \overline{s_\alpha}, f_\alpha \overline{s_\alpha} \rangle = \langle g_{\alpha\beta} f_\beta \overline{s}, g_{\alpha\beta} f_\beta \overline{s_\beta} \rangle$

$\quad //$

$|f_\alpha|^2 h_\alpha = |g_{\alpha\beta}|^2 |f_\beta|^2 h_\beta$

$\Rightarrow \frac{h_\alpha}{h_\beta} = \frac{|f_\beta|^2}{|f_\alpha|^2} |g_{\alpha\beta}|^2 = \frac{1}{|g_{\alpha\beta}|^2} |g_{\alpha\beta}|^2 = 1$

$\Rightarrow$ (h_α) defines a global non-vanishing C^∞ function on M.

$\Rightarrow$ Since M is compact, $(h_\alpha) = h$ is bounded.

On P33, $\mathrm{vol}(\partial D(\epsilon)) \to 0$ as $\epsilon \to 0$

$\lrcorner$