

as the product of  $X_0^3$  and  $X_1$ , i.e.  $\rightarrow$  triple lines

$$\Rightarrow H \in H^0(\mathbb{P}^2, \mathcal{I}_{P_1}^2 \otimes \mathcal{I}_{P_2}^2 \otimes \mathcal{I}_{P_3}^2(2)).$$

$\Rightarrow G$  may be expressed as a linear combination of  $X_1^2 X_2^2$ ,  $X_0^2 X_2^2$ ,  $X_0^2 X_1^2$ ,  $X_0 X_1 X_2^2$ ,  $X_0 X_1^2 X_2$  &  $X_0^2 X_1 X_2$ .

$$\Rightarrow |\mathcal{I}_{P_1}^2 \otimes \mathcal{I}_{P_2}^2 \otimes \mathcal{I}_{P_3}^2(2)| = 6 - 1 = 5.$$

Note:  $H^0(\mathbb{P}^2, \mathcal{I}_{P_i}(4)) =$  Space of sections of  $[4H]$  s.t. the restriction to "near" point  $P_i$  is  $\{\lambda_1 f_1 + \lambda_2 f_2\}$  where  $(f_1=0)=C$   $(f_2=0)=C'$ . So it is not a surprising fact that  $H^0(\mathbb{P}^2, \mathcal{I}_{P_i}(4)) \subsetneq H^0(\mathbb{P}^2, \mathcal{I}_{P_1}^2 \otimes \mathcal{I}_{P_2}^2 \otimes \mathcal{I}_{P_3}^2(2))$ .  $\# \{f_1, f_2\} = 2$ , but  $\# \{X_1^2 X_2^2, \dots\} = 5$ .

Choose  $\sigma_1, \sigma_2, \sigma_3, \tau$  as follows:

$$\sigma_1 = (X_0 X_1 X_2) X_0$$

$$\sigma_2 = (X_0 X_1 X_2) X_1$$

$$\sigma_3 = (X_0 X_1 X_2) X_2$$

$$\tau = X_1^2 X_2^2 \text{ (for example) in general see back.}$$

Since  $C$  and  $C'$  have equations

$$xy = 0$$

$$(x-y)(x-xy) = 0 \quad \text{respectively,}$$

$$x = \frac{X_1}{X_0} \quad y = \frac{X_2}{X_0}$$

$$X_1 X_2 = 0 \Rightarrow X_1^2 X_2^2 = 0 \text{ represents}$$

a curve in  $|C + \lambda C'|$ .