

By Stokes' theorem, for $r < R$ (remembering that p' is the origin)

$$\begin{aligned} & \mathbb{H}(T_\psi, p', R) - \mathbb{H}(T_\psi, p', r) \\ &= \int_{\partial B(r, R)} \psi \wedge \left\{ \left(\frac{\sqrt{-1}}{2} \right)^{n-p} \bar{\partial} \log \|z\|^2 \wedge (\partial \bar{\partial} \log \|z\|^2)^{n-p-1} \right\} \\ &= \pi^{n-p} \int_{B(r, R)} \psi \wedge \Omega^{n-p}, \end{aligned}$$

where $\Omega = \frac{\sqrt{-1}}{2\pi} \partial \bar{\partial} \log \|z\|^2$

is the pullback to \mathbb{C}^n of the Fubini-Study metric on \mathbb{P}^{n-1} . Since $\psi \wedge \Omega^{n-p} \geq 0$, we have proved the lemma. Q.E.D.

$$\begin{aligned} & \Gamma \mathbb{H}(T_\psi, p', R) - \mathbb{H}(T_\psi, p', r) \\ &= \left(\frac{\sqrt{-1}}{2} \right)^{n-p} \int_{\|z\|=R} \psi \wedge \bar{\partial} \log \|z\|^2 \wedge (\partial \bar{\partial} \log \|z\|^2)^{n-p-1} \\ &= \left(\frac{\sqrt{-1}}{2} \right)^{n-p} \int_{\|z\|=r} \psi \wedge \bar{\partial} \log \|z\|^2 \wedge (\partial \bar{\partial} \log \|z\|^2)^{n-p-1} \\ &= \left(\frac{\sqrt{-1}}{2} \right)^{n-p} \int_{\partial B(r, R)} \psi \wedge \left\{ \bar{\partial} \log \|z\|^2 \wedge (\partial \bar{\partial} \log \|z\|^2)^{n-p-1} \right\} \\ &= \int_{\partial B(r, R)} \psi \wedge \left\{ \left(\frac{\sqrt{-1}}{2} \right)^{n-p} \bar{\partial} \log \|z\|^2 \wedge (\partial \bar{\partial} \log \|z\|^2)^{n-p-1} \right\} \\ &= \frac{\sqrt{-1}}{2} \int_{\partial B(r, R)} \psi \wedge \left\{ \bar{\partial} \log \|z\|^2 \wedge \left(\frac{\sqrt{-1}}{2\pi} \partial \bar{\partial} \log \|z\|^2 \right)^{n-p-1} \right\} \cdot \pi^{n-p-1} \\ &= \int_{\partial B(r, R)} \psi \wedge \frac{\sqrt{-1}}{2\pi} \bar{\partial} \log \|z\|^2 \wedge \Omega^{n-p-1} = \pi^{n-p} \int_{B(r, R)} \psi \wedge \Omega^{n-p} \end{aligned}$$