

$$= 2\pi\sqrt{-1} \sum \text{ord}_{p_i}(f) \log g(p_i), \text{ since by P222, } \sum_{p_i} \text{ord}_{p_i}(f) \\ = \sum_i \text{Res}_{p_i}(d \log f) = 0. \quad \Rightarrow$$

In sum, we have

$$2\pi\sqrt{-1} \left(\sum \text{ord}_{q_i}(g) \cdot \log f(q_i) - \sum \text{ord}_{p_i}(f) \cdot \log g(p_i) \right) \\ = \sum_{i=1}^g \left(\left(\int_{\delta_i} d \log f \right) \left(\int_{\delta_{g+i}} d \log g \right) - \left(\int_{\delta_i} d \log g \right) \left(\int_{\delta_{g+i}} d \log f \right) \right).$$

By considering the meromorphic differential

$$\psi = \log g \cdot d \log f.$$

as we did for φ , (of course, we have to draw arcs β_i from s_i to q_i as we did for α_i). We get

$$\sum_i \int_{\beta_i + \beta_i^{-1}} \psi = 2\pi\sqrt{-1} \sum \text{ord}_{q_i}(g) \cdot \log f(q_i) \text{ and}$$

so on.

$$2\pi\sqrt{-1} \left(\sum \text{ord}_{q_i}(g) \cdot \log f(q_i) - \sum \text{ord}_{p_i}(f) \cdot \log g(p_i) \right)$$

=?

$$0 = \sum \int_{\delta_i + \delta_i^{-1}} \varphi + \int_{\delta_{g+i} + \delta_{g+i}^{-1}} \varphi + \int_{\alpha_i + \alpha_i^{-1}} \varphi$$

$$= - \sum \left(\int_{\delta_i} d \log g \right) \left(\int_{\delta_{g+i}} d \log f \right) + \sum \left(\int_{\delta_{g+i}} d \log g \right) \left(\int_{\delta_i} d \log f \right)$$

$$+ 2\pi\sqrt{-1} \cdot \sum \text{ord}_{p_i}(f) \cdot \log g(p_i) - \dots \quad \textcircled{1}$$