Let l and $p = l^2$ denote the classes of a line and a point in $W_2 \cong \mathbb{P}^2$; let $\tilde{p} = \pi^* p$ and $\tilde{l} = \pi^* l$ be the pullback classes in $E \subset \tilde{W}$.
We have

$$\omega|_{W_2} = 2l,$$

and so

$$\omega^2|_{W_2} = (2l)^2 = 4p.$$

Γ Let $f = \iota_{2H} : \mathbb{P}^2 \longrightarrow \mathbb{P}^5 = W$.
$\Rightarrow f(\mathbb{P}^2) = W_2 \quad \Rightarrow \quad l = f(\mathbb{P}^1). \quad \Rightarrow l^2 = p$ point in W_2.

$\omega|_{W_2} = H \cdot W_2 = a\,\mathbb{P}^1$ in $\mathbb{P}^5$

$\Rightarrow \#(H \cap H \cap W_2) = \#(\mathbb{P}^3 \cap W_2) = a\#(\mathbb{P}^1 \cap \mathbb{P}^3) = a = 4$

since W_2 is of degree 4, see P179

It remains to show that $l = 2\,\mathbb{P}^1$ in W.

$\#(l \cap H) = \#(f(\mathbb{P}^1) \cap H) = \#(\mathbb{P}^1 \cap f^{-1}(H)) = c_1(f^* H)$
$= 2$ by the result on P179.

Thus $\omega^2|_{W_2} = (\omega|_{W_2})^2 \qquad (\because \#(H \cap H \cap W_2) = 4)$
$= (2l)^2 = 4l^2 = 4p$

$\lrcorner$

Now by the computation for the Chern classes of projective space,

$$c(T(W)|_{W_2}) = (1 + 6\omega + 15\omega^2)|_{W_2}$$
$$= 1 + 12l + 60l^2$$