

for the following reason:

Given a global section of  $H^0(M, \underline{\text{Ext}}_O^1(\mathcal{G}, \mathcal{H}))$ ,  
choose a sufficiently fine covering  $\underline{U} = \{U_\alpha\}$  and  
corresponding local extensions

$$(E_\alpha) \quad 0 \longrightarrow \mathcal{H}|_{U_\alpha} \longrightarrow E_\alpha \longrightarrow \mathcal{G}|_{U_\alpha} \longrightarrow 0.$$

In  $U_\alpha \cap U_\beta$  there will be a commutative diagram

$$\begin{array}{ccccccc} 0 & \longrightarrow & \mathcal{H}|_{U_\alpha \cap U_\beta} & \longrightarrow & E_\alpha|_{U_\alpha \cap U_\beta} & \longrightarrow & \mathcal{G}|_{U_\alpha \cap U_\beta} \longrightarrow 0 \\ & & \parallel & & \downarrow \varphi_{\alpha\beta} & & \parallel \\ 0 & \longrightarrow & \mathcal{H}|_{U_\alpha \cap U_\beta} & \longrightarrow & E_\beta|_{U_\alpha \cap U_\beta} & \longrightarrow & \mathcal{G}|_{U_\alpha \cap U_\beta} \longrightarrow 0, \end{array}$$

but it need not be the case that in  $U_\alpha \cap U_\beta \cap U_\gamma$   
the triangle

$$\begin{array}{ccc} & E_\alpha & \\ \varphi_{\alpha\beta} \swarrow & & \searrow \varphi_{\alpha\gamma} \\ E_\beta & \xrightarrow{\varphi_{\beta\gamma}} & E_\gamma \end{array}$$

is commutative — i.e., the “transition functions” for  
gluing the local extensions  $(E_\alpha)$  may not satisfy  
the cocycle rule, and thus may not patch together  
to give a global extension.

“Definition of Sheaf of  $\mathcal{O}$ -modules”