⌈ D, D' effective divisors $\Rightarrow$ By the fact above, $\exists$ $h^\circ(D)-1 = \dim|D|$ points $p_1, \dots, p_{h^\circ(D)-1}$, s.t. $p_1 + \dots + p_{h^\circ(D)-1}$ is a linear subseries of $E \in |D|$. Similarly for D'. $\Rightarrow \exists\ E \sim D+D'$ containing $h^\circ(D)-1 + h^\circ(D')-1$ points of S.

$\Rightarrow \dim|E| \geq h^\circ(D)-1 + h^\circ(D')-1 \Rightarrow h^\circ(E) \geq h^\circ(D)-1+h^\circ(D')-1+1 = h^\circ(D)+h^\circ(D')-1.$ ⌋

In particular, suppose D is special, so that $h^\circ(K-D) \neq 0$ and we can take $D' = K-D$. Then $h^\circ(D+D') = h^\circ(K) = g$, and we have

$$\begin{aligned} h^\circ(D) + h^\circ(K-D) &\leq g+1 \\ h^\circ(D) - h^\circ(K-D) &= d-g+1 \\ \hline 2h^\circ(D) &\leq d+2 \end{aligned}$$

⌈ $h^\circ(D) + h^\circ(K-D) \leq h(\overset{D+K-D}{\overset{\shortparallel}{K}})+1$ by the above
$h^\circ(D) - h^\circ(K-D) = d-g+1$ by Riemann-Roch. ⌋

Note as well that equality holds in the last line if and only if every divisor in the canonical series $|D+D'| = |K|$ is the sum of a divisor in the linear system $|D|$ and a divisor from $|D'|$.

⌈($\Rightarrow$) $\{D_1+D_2 : D_1 \in |D|,\ D_2 \in |D'|\} \subset \{K' : K' \in |K|\}$ since $D_1+D_2 \sim D+D' = D+K-D = K$.
If $\{D_1+D_2\} \neq \{K'\}$, then $\dim\{D_1+D_2\} \neq \dim\{K'\}$.
$\Rightarrow \dim\{D_1+D_2\} \geq \dim|D| + \dim|D'|$, for if we consider $\{D_1\} \times \{D_2\} \xrightarrow{\phi} \{D_1+D_2\}$, $\Rightarrow$ then for a fixed