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V, $x \in \overline{\mathbb{P}^{n-k-1}, p} \cap V$.

Conversely, if $x \in \overline{\mathbb{P}^{n-k-1}, p} \cap V$,

$$x = a(p_0, p_1, \cdots p_k, 0 \cdots 0) + b(0, 0 \cdots 0, x'_{k+1}, \cdots x'_n)$$
$$= (p_0, p_1 \cdots p_k, x_{k+1}, \cdots x_n) \quad \text{in } \mathbb{C}^{n+1}$$

$\Rightarrow x = [(p_0, p_1 \cdots p_k, x_{k+1}, \cdots x_n)]$
$\Rightarrow \pi(x) = p. \quad \Rightarrow \quad x \in \pi^{-1}(p) \cap V.$

Thus we conclude that $\pi^{-1}(p) \cap V = \overline{\mathbb{P}^{n-k-1}, p} \cap V$.

$\{\overline{\mathbb{P}^{n-k-1}, p}\}_{p \in \mathbb{P}^k}$ is parametrized by $\mathbb{P}^k$, and $\overline{\mathbb{P}^{n-k-1}, p}$ generically intersect V in $d = \deg(V)$ points. $\Rightarrow$ π expresses V as a d-sheeted branched cover of $\mathbb{P}^k$ almost every-where

In fact, π must be everywhere finite: if for any point p in $\mathbb{P}^k$ the $(n-k)$-plane $\overline{\mathbb{P}^{n-k-1}, p}$ intersected V in a curve, that curve would necessarily meet the hyperplane $\mathbb{P}^{n-k-1} \subset \overline{\mathbb{P}^{n-k-1}, p}$, contrary to the hypothesis that $\mathbb{P}^{n-k-1}$ is disjoint from V.

If $\overline{\mathbb{P}^{n-k-1}, p}$ intersects V in infinitely many points, $\overline{\mathbb{P}^{n-k-1}, p} \cap V$ contains a curve.

$\overline{\mathbb{P}^{n-k-1}, p} \cap V$ is an analytic varity of dim 0.
$\Rightarrow$ $\overline{\mathbb{P}^{n-k-1}, p} \cap V$ must be a set of discrete points. $\Rightarrow$ Since $\mathbb{P}^n$ is compact, $\overline{\mathbb{P}^{n-k-1}, p} \cap V$

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