The local intersection numbers $m_p(Z.W)$ will be discussed once again and in greater detail in case Z and W are locally complete intersections in Section 2 of Chapter 5.

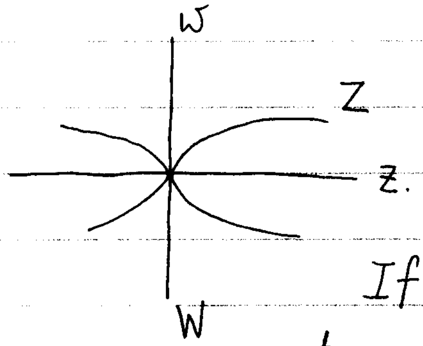
In case W is smooth of dimension 1, so that Z is an analytic hypersurface locally defined by a single holomorphic function f, the above proof gives the formula

$$Z \cdot W = \sum_{p \in Z \cap W} \operatorname{ord}_p(f|_W).$$

⌈

$$\begin{array}{ccc} U & \xrightarrow{f} & \mathbb{C} \\ \downarrow \varphi & & \\ \mathbb{C}^n & & \end{array}$$

We can find a local holomorphic coordinate $(z_1, \cdots z_{n-1}, w)$ s.t W is $\mathbb{C}^n \ni (z_1, \cdots z_{n-1}, w)$, given by $z = 0$ and the projection $(z, w) \to z$ is a finitely sheeted branched covering mapping on Z, by Weierstrass Preparation Theorem P8 ~ P9.



$$f(z, w) = (w^l + g_1(z) w^{l-1} + \cdots + g_l(z))\, h(z,w)$$
where $g_1(0) = \cdots = g_l(0) = 0$, $h(0,0) \neq 0$.

If we denote # of sheets of the projection by d, $l = d$.

If $\operatorname{ord}_p(f|_W) = k$, $\overset{\text{by P130} \sim \text{P131}}{\Rightarrow}$ $f(0, w) = w^k k(w)$, $k(0) \neq 0$.

$\Rightarrow$ $d = k$. $\Rightarrow$ $m_p(Z.W) = d = \operatorname{ord}_p(f|_W)$

$\Rightarrow$ $Z \cdot W = \sum_{p \in Z \cap W} \operatorname{ord}_p(f|_W)$. ⌋