for $A \cap \overline{U} =$ closure of $A \cap U$ in A by Munkres, Topology a first course p95. Th 6.4

$A \cap U =$ closure of $A \cap U = A \cap \overline{U}$.

$\Rightarrow H_\lambda \cap V = H_\lambda \cap \overline{V'} = H_\lambda \cap V' =$ irreducible component of $H_\lambda \cap V \Rightarrow H_\lambda \cap V$ is irreducible

Now the original lemma follows readily from the curve case: if $V \subset \mathbb{P}^n$ is any irreducible nondegenerate k-dimensional variety of degree d, then the generic intersection of V with hyperplanes is an irreducible, nondegenerate curve of degree d in $\mathbb{P}^{n-k+1}$, and so,

$$d \geq n-k+1.$$

Γ $V \subset \mathbb{P}^n$ irreducible, nondegenerate, k-dimensional variety of degree d. $\Rightarrow V \cap H_1 \cap \cdots \cap H_{k-1}$ is an irreducible nondegenerate curve of degree d in $\mathbb{P}^{n-k+1}$.
$\dim(V \cap H_1 \cap \cdots \cap H_{k-1}) = k-(k-1) = 1 \Rightarrow d \geq n-k+1$, since we proved above that $d \geq n$ for a curve $V \subset \mathbb{P}^n$.

We can restate the lemma as follows: any irreducible k-dimensional variety $V \subset \mathbb{P}^n$ of degree d must lie in a linear space of dimension $d+k-1$; as a corollary, then we see again that any variety of degree one in $\mathbb{P}^n$ is a linear subspace.

Γ $(\Rightarrow)$ $V \subset \mathbb{P}^n$ irreducible
(i) nondegenerate V
$\Rightarrow d \geq n-k+1 \Rightarrow d+k-1 \geq n \Rightarrow V \subset \mathbb{P}^n \subset \mathbb{P}^{d+k-1}$