so that $x \longmapsto f_x$ can be considered a section of $\mathrm{Hom}(E, F)$. //

If e a frame for E, then, in terms of the frame $\{e_i^* \otimes e_j\}$ for $E^* \otimes E$, we can represent $\Theta \in A^2(E^* \otimes E)$ by a matrix Θ_e of 2-forms. - i.e. we can write

$$D^2 e_i = \sum \Theta_{ij} \otimes e_j$$ Θ_e is called the curvature matrix of D in terms of e.

$$D^2 : E \to \Lambda^2 T^* \otimes E \quad \Leftrightarrow \quad D^2 \in \Gamma(\Lambda^2 T^* \otimes E^* \otimes E)$$

$$\Rightarrow \quad D^2|_U = \sum \Theta_{ij} \otimes e_i^* \otimes e_j \quad \Rightarrow \quad D^2(e_i) = \sum \Theta_{ij}\, e_j$$ //

If $\{e_i' = \sum g_{ij} e_j\}$ is a another frame, then

$$D^2 e_i' = D^2\left(\sum g_{ij} e_j\right) = \sum g_{ij} D^2 e_j = \sum g_{ij}\, \Theta_{jk}\, e_k$$
$$= \sum g_{ij}\, \Theta_{jk}\, g_{kl}^{-1} e_l' \quad \Rightarrow \quad \Theta_{e'} = g\, \Theta_e\, g^{-1}.$$

The curvature matrix is expressed in terms of the connection matrix;

$$D^2 e_i = D\left(\sum \theta_{ij} \otimes e_j\right) = \sum \left(d\theta_{ij} - \sum \theta_{ik} \wedge \theta_{kj}\right) \otimes e_j$$

$\Rightarrow \quad \Theta_e = d\theta_e - \theta_e \wedge \theta_e$

$\Rightarrow$ This is called the Cartan structure equation.

We can say more about Θ in the holomorphic case.
$E \longrightarrow M$ hermitian & D on E compatible with the complex structure $\Rightarrow D'' = \bar{\partial} \Rightarrow D''^2 = 0. \Rightarrow \Theta^{0,2} = 0$
D compatible with the metric $\Rightarrow$ in terms of a unitary frame e, the θ_e is skew-hermitian $\Rightarrow \Theta = d\theta - \theta \wedge \theta$ is skew-hermitian, too. $\Rightarrow \Theta^{2,0} = -{}^t\overline{\Theta^{0,2}} = 0$