

since f is one to one. \square

We thus obtain maps

$$f_* : \pi_1(M) \longrightarrow \pi_1(N)$$

and

$$f_*^{-1} : \pi_1(N) \longrightarrow \pi_1(M)$$

inverse to one another; and so $\pi_1(M) \cong \pi_1(N)$.

\square

$$f_* : \pi_1(M) \longrightarrow \pi_1(N)$$

$$\downarrow$$

$$\parallel$$

$$[r'] \longmapsto [f(r')]$$

$$\wedge r' \cap U = \emptyset, \quad r \sim r' \text{ homotopic.}$$

$$r' \cap V = \emptyset$$

\square

Another way in which a birational map carries structure is this: if $f: M \rightarrow N$ is a birational map, we may define two maps

$$f_* : \text{Div}(M) \longrightarrow \text{Div}(N),$$

called the proper transform and the total transform.

The proper transform of a divisor D in M is defined to be the closure in N of the image of D under f where defined, while the total transform is defined to be the image in N of the inverse image of D in the graph $\Gamma \subset M \times N$ of f .

\square

If D is an irreducible hypersurface in M ,

$$f_*(D) = \overline{D - V}, \quad \text{where } f \text{ is defined away from } V \subset M.$$