NO.

DATE . . ( )

$$B_L \cdot B_L = \frac{1}{16} D_V \cdot D_V = 2$$

and hence the virtual genus

$$\pi(B_L) = \frac{B_L \cdot B_L}{2} + 1 = 2.$$

$\ulcorner$ $B_L \cdot B_L = \frac{1}{4} D_V \cdot \frac{1}{4} D_V = \frac{1}{16} \cdot 32 = 2$

$$\pi(B_L) = \frac{K_A \cdot B_L + B_L \cdot B_L}{2} + 1 = \frac{B_L \cdot B_L}{2} + 1 = 2$$

$\lrcorner$

Note also that since D_V is positive, so is B_L.

$\ulcorner$ Since $D_V = A \cap \sigma_1(V_3)$, D_V is the hyperplane section of A. $\Rightarrow$ D_V is positive by p150.
$\Rightarrow$ By the same reason, B_L is positive since $4B_L \sim D_V$, and $c_1(D_V) = 4\, c_1(B_L)$, see p148.

$\lrcorner$

We claim now that for any $L \subset X$, the curve $B_L \subset A$ is smooth. To see this, we observe that if two lines L and L' in X meet — i.e., if the corresponding pencils have a line l in common — then the focus $p_{L'}$ of the second pencil must lie on the line l, and hence on the plane h_L of the first pencil.

$\ulcorner$ $L = \sigma(p_L, h_L)$, $L' = \sigma(p_{L'}, h_{L'})$.