$\Rightarrow$ We only need the fact $H^1(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{K}^l - E)) = 0$.
The key point is that $\tilde{K}^l - E$ is a positive line bundle.
Thus if we let $K = L^m + F$, where F is a line bundle, by choosing sufficiently large m, we can have $\tilde{L}^m + \tilde{F} - E$ which is positive line bundle.
Actually $\tilde{L}^m$ controls $\tilde{F}$ for $m \gg 0$ so that $\tilde{L}^m + \tilde{F}$ is positive, as $L^{k_1} + K_M^*$ is positive for $k_1 \gg 0$. see p190.

Similarly, we can conclude that the same argument for immersion works.

5. Grassmannians.

Def. The cell decomposition and Schubert Cycles.

In this section, we will construct and describe the Grassmannians, a fundamental family of compact complex manifolds. Grassmannians may be thought of as a generalization of projective space; the analogy will be apparent throughout.

Let V be a complex vector space of dim. n. The Grassmannian $G(k, V)$ is defined to be the set of k-dimensional linear subspaces of V; we write $G(k,n)$ for $G(k, \mathbb{C}^n)$. Given a k-plane Λ in $\mathbb{C}^n$, we may represent Λ by a set of k row vectors in $\mathbb{C}^n$ spanning Λ, i.e., by a $k \times n$ matrix

$$\begin{bmatrix} v_{11}, & \cdots & v_{1n} \\ \vdots & & \\ v_{k1}, & \cdots & v_{kn} \end{bmatrix} \text{ of rank } k.$$