Global Duality and Superabundance of Points on a Surface.

Let $L \to S$ be a holomorphic line bundle over an algebraic surface. In Riemann-Roch theorem for surfaces

$$\chi(\mathcal{O}_S(L)) = \frac{1}{2}(L\cdot L - K\cdot L) + \chi(\mathcal{O}_S)$$

the terms

$$h^0(L),\quad h^2(L) = h^0(K-L),\quad p_g,\quad q,\quad L\cdot L,\quad K\cdot L$$

all have immediate geometric interpretations, at least in case $L = [D]$ for some effective divisor D on S.

Γ See P 470 for the definition of an algebraic surface. See P 472 for Riemann-Roch theorem.
$h^2(L) = \dim H^2(M, \mathcal{O}(L)) = \dim H^0(M, \Omega^2(L^*)) = \dim H^0(M, \mathcal{O}(K-L)) = h^0(K-L)$. What are P_g & q, ω? $\lrcorner$

The Italian algebraic geometers first wrote this formula as

$$\dim|L| + h^0(K-L) = \frac{1}{2}(L\cdot L - K\cdot L) + P_g - q + \omega,$$

and then proved directly that the quantity ω defined by this equation was nonnegative, which they then called the superabundance. The reader should keep in mind that the dual of $H^1(\mathcal{O}_S(L))$ is $H^1(\mathcal{O}_S(K-L))$, and sheaf cohomology was 50 years in the future. Working backward historically, we shall use our global duality theorem for coherent sheaves to geometrically interpret the superabundance in some cases. We begin with an example; the final result is the Reciprocity Formula II on page 716.