orphic to $\mathcal{O}$ in such a way that $i(h) = (-f_2 h, f_1 h)$.
Q.E.D.

Γ $\mathcal{E}^* \cong \mathcal{O}^* \oplus \mathcal{O}^*$ locally
$\cong \mathcal{O} \oplus \mathcal{O}$.

$$\begin{array}{ccc} \mathcal{O}^* & \longleftrightarrow & \mathcal{O} \\ \Downarrow & & \Downarrow \\ \phi_g & \longleftrightarrow & g. \end{array}$$

$\Rightarrow$ $\mathcal{E}^* \xrightarrow{s} \mathcal{O}$ locally

$(g_1, g_2) \longmapsto g_1 f_1 + g_2 f_2$

$(\phi_{g_1}, \phi_{g_2}) : (f_1, f_2) \longmapsto \phi_{g_1}(f_1) + \phi_{g_2}(f_2)$

I think: The statement " divisor (s) is Z idealtheoretically" means that $\{f_1, f_2\} = I$.
(or implies).

Let $I = \{z_1, z_2\}$. $J = \{z_1^2 + z_2, \ z_2^2 + z_1\}$.
$\Rightarrow \mathrm{supp}(\mathcal{O}/I) = \mathrm{supp}(\mathcal{O}/J)$. but $I \neq J$.

$0 \longrightarrow \ker s \longrightarrow \mathcal{E}^* \xrightarrow{s} I \longrightarrow 0$ exact sequence of sheaves

$f_1 g_1 + f_2 g_2 = 0$ $\Rightarrow$ Since $I = \{f_1, f_2\}$ is regular,
$g_2 \in \{f_1\}$. $\Rightarrow$ $g_2 = h f_1$ $\Rightarrow$ $f_1 g_1 + h f_1 f_2 = 0$
$= f_1 (g_1 + h f_2) \Rightarrow$ $g_1 + h f_2 = 0$ by regularity of I.
$\Rightarrow$ $g_1 = -h f_2$ $\Rightarrow$ $\ker s = \{(-h f_2, h f_1)\}$.
$\Rightarrow$ $\mathcal{O} \longrightarrow \ker s$ isomorphic locally.

$h \longmapsto (-h f_2, h f_1)$

$\Rightarrow$ $\mathcal{L}$ is locally isomorphic to $\mathcal{O}$. ■