

$$\frac{1}{z^6} + \frac{c}{z^2} + [-1] + \frac{c'}{z^3} + [1] - \frac{1}{z^6} + (-\frac{c}{z^3}) - \frac{c''}{z^2} - [-1] + (c'' - c)(\frac{1}{z^2} + [1])$$

$= [-1] \Rightarrow$ holomorphic away from p with at most a single pole at p . \Rightarrow By the argument for genus 0 above, $F'(z)^2 + c'F'(z) - F(z)^3 + (c'' - c)F(z)$ must be constant, see P.222. \Rightarrow

single

The image of S under the embedding ι_L is accordingly the locus of the polynomial

$$y^2 = x^3 + ax + b - c'y,$$

where $x = z/z_0$, $y = w/z_0$ are Euclidean coordinates on \mathbb{P}^2 .

$$\Gamma \quad y^2 + c'y - x^3 + (c'' - c)x = b \text{ (constant)}$$

if we let $F' = y$, $F = x$.

$$\Rightarrow y^2 + c'y = x^3 + ax + b, \quad a = c - c''. \quad \Rightarrow$$

After a linear change of the coordinate y , we may take this polynomial of the form

$$(*) \quad y^2 = x^3 + ax + b,$$

and finally, after a linear change in the x -coordinate, taking two of the roots of the polynomial $x^3 + ax + b$ to 0 and 1, we see that any curve of genus 1 is the zero locus in \mathbb{P}^2 of a cubic polynomial

$$y^2 = x(x-1)(x-\lambda) \quad \text{for some } \lambda \in \mathbb{C}.$$

