of sections of M over U s.t $\{\sigma_1(x), \dots \sigma_k(x)\}$ is a basis for E_x for all $x \in U$. A frame for E over U is essentially the same thing as a trivialization of E over U: given

$\varphi_U : E_U \longrightarrow U \times \mathbb{C}^k$ a trivialization, the sections

$\sigma_i(x) = \varphi_U^{-1}(x, e_i)$. form a frame, and ~~cover~~

conversely, given $\sigma_1, \cdots \sigma_k$ a frame, we can find a trivialization φ_U by

$$\varphi_U(\lambda) = (x, (\lambda_1, \lambda_2 \cdots \lambda_k)) \text{ for}$$

$\lambda = \sum \lambda_i \sigma_i(x)$ in E_x. Note that given a trivialization φ_U of E over U, we can represent every section σ of E over U uniquely as a C^∞ vector-valued function $f = (f_1, f_2, \cdots f_k)$ by writing

$\sigma(x) = \sum f_i(x) \cdot \varphi_U^{-1}(x, e_i)$; if φ_V is a trivialization of E over V and $f' = (f_1', \cdots f_k')$ the corresponding representation of $\sigma|V \cap U$, then

$$\sum f_i(x) \cdot \varphi_U^{-1}(x, e_i) = \sum f_i'(x) \varphi_V^{-1}(x, e_i),$$

so $\quad \sum f_i(x)\, e_i = \sum f_i'(x)\, \varphi_U \varphi_V^{-1}(x, e_i) \Rightarrow f = g_{UV} f'$.

Thus, in terms of trivializations $\{\varphi_\alpha : E_{U_\alpha} \rightarrow U_\alpha \times \mathbb{C}^k\}$, sections of E over $\cup U_\alpha$ corresponds exactly to collections $\{f_\alpha = (f_{\alpha_1}, \cdots f_{\alpha_k})\}_\alpha$ of vector-valued C^∞ functions s.t $\quad f_\alpha = g_{\alpha\beta} \cdot f_\beta \quad$ for all α, β, where the $g_{\alpha\beta}$ are transition functions of E relative to $\{\varphi_\alpha\}$.

Let M be a complex manifold. A holomorphic vector bundle