Take Hom functor,

$$0 \longrightarrow \mathrm{Hom}\left(\frac{T_M'|_V}{T_V'}, \mathbb{C}\right) \xrightarrow{\bar{\iota}} \mathrm{Hom}(T_M', \mathbb{C}) \xrightarrow{r} \mathrm{Hom}(T_V', \mathbb{C}) \to 0.$$

If we have $\sigma \in \mathrm{Hom}(T_M', \mathbb{C})$. s.t $\sigma(v) = 0$ $\forall v \in T_V'$.
$\Rightarrow r(\sigma) = 0$. $\Rightarrow \exists$ a corresponding elat $\sigma' \in \mathrm{Hom}\left(\frac{T_M'|_V}{T_V'}, \mathbb{C}\right)$
s.t $\bar{\iota}(\sigma') = \sigma$.))

There is an easy formular for the conormal bundle of a smooth hypersurface V, which we now derive: Suppose V is given locally by functions $f_\alpha \in \mathcal{O}(U_\alpha)$; the line bundle $[V]$ on M is then given by transition functions $\{g_{\alpha\beta} = f_\alpha / f_\beta\}$. Now since $f_\alpha \equiv 0$ on $V \cap U_\alpha$, the differential df_α is a section of the conormal bundle N_V^* of V; since V is smooth, df_α is everywhere non-zero. On $U_\alpha \cap U_\beta \cap V$, moreover, we have

$$\begin{aligned} df_\alpha = d(g_{\alpha\beta} f_\beta) &= (dg_{\alpha\beta}) f_\beta + g_{\alpha\beta}\, df_\beta \\ &= 0 + g_{\alpha\beta}\, df_\beta, \quad (\because f_\beta = 0 \text{ on } V \cap U_\alpha \cap U_\beta) \end{aligned}$$

i.e., the section $df_\alpha \in \Gamma(U_\alpha, \mathcal{O}(N_V^*))$ together give a non-zero global section of $N_V^* \otimes [V]$.
Thus $N_V^* \otimes [V]$ is the trivial bundle; this is the

Adjunction Formula I

$$N_V^* = [-V]|_V,$$

$\ulcorner$ $df_\alpha(v) = 0$ for $v \in T_V'$.

$$\begin{array}{ccc} U_\alpha \cong \mathbb{C}^n & & df_\alpha : U_\alpha \longrightarrow \mathrm{Hom}(T'U_\alpha, \mathbb{C}) \\ \uparrow \qquad \uparrow & & \| \qquad\qquad \| \\ U_\alpha \cap V \cong \mathbb{C}^{n-1} & & df_\alpha : \mathbb{C}^n \longrightarrow \mathrm{Hom}(\mathbb{C}^n \times \mathbb{C}^n \to \mathbb{C}) \end{array}$$