$L \cap V_C - C'$ is smooth, i.e. $L \cap V_C - C'$ is a set of distinct points, by Bertini's theorem.
$\Rightarrow$ The branched cover has branch points less than $5+1$, since it has one simple branch point at C' and has simple branch points (other than C') of less than 5. $\Rightarrow$ It contradicts to the result that the 4-sheeted cover of $\mathbb{P}^1$ has 6 branch points by Riemann-Hurwitz formula.

Next, we compute the multiplicity of the locus W_2 of double lines in the generic divisor V_C. This is not hard: for C a conic, $2L$ a generic double line, and $\{C_\lambda\}$ a generic pencil of conics containing the double line $2L$ as an element, $\{C_\lambda\}$ again cuts out on C a pencil of degree 4, without base points. The corresponding map then has six branch points as before — but two of these are just the points of intersection of L with C.

Counting multiplicity, we have six branch points. $\#(2L \cap C) = 4$, counting multiplicity.
$L \cap C$ is a set of two distinct points, since L is a generic line. $\Rightarrow$ Two of six branch points are $L \cap C$.