$(\varphi^{-1})^*(x_i)\ (\varphi(H_i)) = x_i\ (\varphi^{-1}\cdot\varphi(H_i)) = 0.$

Consider the function

$$f(\cdots) = \frac{\varphi^{-1*}(x_i)}{a_{0,i} + a_{1,i}x_1 + \cdots + a_{n,i}x_n}.$$

$\Rightarrow$ Since $a_{0,i} + a_{1,i}x_1 + \cdots + a_{n,i}x_n$ is a function on $\mathbb{P}^n$ with a simple pole along H and a zero along $\varphi(H_i)$, f is a holomorphic function on $\mathbb{P}^n$.

$\Rightarrow$ f is constant, since $\mathbb{P}^n$ is compact.

$\Rightarrow$

$$\varphi^{-1*}(x_i) = (a_{0,i} + a_{1,i}x_1 + \cdots + a_{n,i}x_n)\cdot C$$
$$= a'_{0,i} + a'_{1,i}x_1 + \cdots + a'_{n,i}x_n$$

where $a'_{\ell,i} = a_{\ell,i}\,C$.

$\Rightarrow$ φ^{-1} is linear $\Rightarrow$ φ is linear. 끝

Note that the group of automorphisms of $\mathbb{P}^n$ is thus the quotient PGL_{n+1} of the general linear group GL_{n+1} by the one-dimensional subgroup of scalar matrices $\{\lambda I\}$.

즉 $PGL_{n+1} = GL_{n+1}/\mathbb{C}^*$.

$n = 3-1$, $\varphi^{-1} = \psi$

$$\psi^*(x_1) = a_{01} + a_{11}x_1 + a_{21}x_2$$
$$\psi^*(x_2) = a_{02} + a_{12}x_1 + a_{32}x_2$$