$E_x \oplus E_y$ is surjective, $\exists$ $\sigma \in H^0(M, \mathcal{O}(E))$ s.t $\sigma(x) = 0$ and $\sigma(y) \neq 0$.

$\sigma = c_1\sigma_1 + \cdots + c_n\sigma_n$, $H^0(M, \mathcal{O}(E)) = V = \langle \sigma_1 \cdots \sigma_n \rangle$

$$\iota_E(x) = \left\langle \begin{matrix} a_{11}(x)\sigma_1^* + \cdots + a_{1n}(x)\sigma_n^* \\ \vdots \qquad\qquad \vdots \\ a_{k1}(x)\sigma_1^* + \cdots + a_{kn}(x)\sigma_n^* \end{matrix} \right\rangle$$

$\Rightarrow \langle A(x) \rangle = \langle A(y) \rangle$.

$\sigma = c_i\sigma_i = c_i a_{\alpha i} e_\alpha$ where c_i's constant

$\Rightarrow \sigma(x) = a_{\alpha i} c_i e_\alpha = 0$

$$\Rightarrow \begin{pmatrix} a_{11}(x) & \cdots & a_{1n}(x) \\ \vdots & & \\ a_{k1}(x), & \cdots & a_{kn}(x) \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} = 0 \text{ , but}$$

$$\begin{pmatrix} a_{11}(y), & \cdots & a_{1n}(y) \\ \vdots & & \\ a_{k1}(y) & \cdots & a_{kn}(y) \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} \neq 0 \text{ since } \sigma(y) = 0.$$

This contradicts to the fact that $\left\langle \begin{matrix} a_{11}(x), \cdots a_{1n}(x) \\ \vdots \\ a_{k1}(x), \cdots a_{kn}(x) \end{matrix} \right\rangle =$

$$\left\langle \begin{matrix} a_{11}(y), \cdots a_{1n}(y) \\ \vdots \qquad \vdots \\ a_{k1}(y), \cdots a_{kn}(y) \end{matrix} \right\rangle .$$

In general, if $\iota_E(x) = \iota_E(y)$. and

$$\iota_E(x) = \left\langle \begin{matrix} a_{11}(x)\sigma_1^* + \cdots + a_{1n}(x)\sigma_n^* \\ \vdots \qquad\qquad \vdots \\ a_{k1}(x)\sigma_1^* + \cdots + a_{kn}(x)\sigma_n^* \end{matrix} \right\rangle$$