⇒ The fomulas are absolutely correct.

Our final result on Schubert cycles is a formula that expresses the general Schubert cycle as a polynomial in the special Schubert cycles $\sigma_{b,0,\dots}$.

We proceed as follows: for $\sigma_{a_1,\dots,a_d}$ any Schubert cycle, we consider the cycle.

$$(*)\qquad \tilde{\sigma}_a = \sum_{j=1}^{d} (-1)^j \sigma_{a_1,\dots a_{j-1},\, a_{j+1}-1,\, a_d-1} \cdot \sigma_{a_j+d-j}.$$

Note that $\tilde{\sigma}_a$ has the same dimension as σ_a.

$$\begin{aligned}
&k(n-k) - (a_1+\dots+a_{j-1}+(a_{j+1}-1)+\dots+(a_d-1)) + \\
&k(n-k) - (a_j+d-j) - k(n-k) \\
&= 2k(n-k) - \{ a_1+\dots+a_{j-1}+a_j+a_{j+1}+\dots+a_d - \underbrace{(1+\dots1)}^{(d-j)} \\
&\quad + (d-j)\} = 2k(n-k) - \Sigma a_i - k(n-k) \\
&= k(n-k) - \Sigma a_i \text{ is the dimension of } \tilde{\sigma}_a,
\end{aligned}$$

which is the same as that of σ_a.

Now we can by Pieri's formula write out each of the intersections in the sum (*) as a sum of Schubert cycles. Let $\sigma_{c_1,\dots c_d}$ be any Schubert cycle: if σ_c appears in this expression, consider the sequence of integers

$$c_1-1,\ c_2-2,\ \cdots\ c_d-d.$$

By Pieri, at most one of these numbers will lie in each of the $(d+1)$ closed intervals

$$[a_1-1,\ n-k],\ [a_2-2,\ a_1-2],\cdots\ [a_d-d,\ a_{d-1}-d],$$