sume that $\det\begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & & \\ x_{k1}, & \cdots & x_{kk} \end{pmatrix} \neq 0$.

$$\left(\begin{array}{cccccc|ccc} x_{11}, & x_{12}, & \cdots & , x_{1k}, & \cdots & x_{1l} & x_{1,(l+1)}, & \cdots & x_{1n} \\ x_{21}, & x_{22}, & \cdots & , x_{2k}, & \cdots & x_{2l} & x_{2,(l+1)} & \cdots\cdots & x_{2n} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots & & \vdots \\ x_{k1}, & x_{k2}, & \cdots & , x_{kk}, & \cdots & x_{kl} & x_{k,(l+1)}, & \cdots & x_{kn} \end{array}\right)$$

$$v_1 \wedge v_2 \wedge \cdots \wedge v_k = \det\begin{pmatrix} x_{11} & & \\ & \ddots & \\ & & x_{kk} \end{pmatrix} e_1 \wedge e_2 \wedge \cdots \wedge e_k + \det(\quad)$$

$$+ \cdots + \det(\quad) e_{n-k+1} \cdots \wedge e_n .$$

Except the term $e_1 \wedge e_2 \wedge \cdots \wedge e_k$, all $\det(\quad)$ are zero. For example,

$$\det\begin{pmatrix} x_{11}, & \cdots & x_{1,(k-1)}, & x_{1,l+1} \\ \vdots & & \vdots & \vdots \\ x_{k1}, & \cdots & x_{k,(k-1)}, & x_{k,(l+1)} \end{pmatrix} = 0$$

$\Rightarrow$ If $\begin{pmatrix} x_{1,l+1} \\ \vdots \\ x_{k,l+1} \end{pmatrix} \neq 0$, $\exists$ $a_i \neq 0$ for some i s.t

$$a_1 \begin{pmatrix} x_{11} \\ \vdots \\ x_{k1} \end{pmatrix} + a_2 \begin{pmatrix} x_{12} \\ \vdots \\ x_{k2} \end{pmatrix} + \cdots + a_i \begin{pmatrix} x_{1i} \\ \vdots \\ x_{ki} \end{pmatrix} + \cdots + a_n \begin{pmatrix} x_{1n} \\ \vdots \\ x_{kn} \end{pmatrix}$$

$= \begin{pmatrix} x_{1,l+1} \\ \vdots \\ x_{k,l+1} \end{pmatrix}$ $\Rightarrow$ We can replace $\begin{pmatrix} x_{1i} \\ \vdots \\ x_{ki} \end{pmatrix}$ by