$$\begin{array}{ccc} {}'E_2^{2,0} & \longrightarrow & H^2(M, \mathbb{C}) \\ \Psi & & \Psi \\ [a] & \longmapsto & [a] \end{array}$$

$$\begin{array}{ccc} {}'E_2^{0,1} & \longrightarrow & {}'E_2^{2,0} \\ \| & & \\ \dfrac{\{ a \in C^0(\underline{U}, \Omega^1) + C^1(\underline{U}, \Omega^0) : (d+\delta)(a) \in C^2(\underline{U}, \Omega^0) \}}{C^1(\underline{U}, \Omega^0) \cap \{ \cdots \}} & \xrightarrow{d+\delta} & \dfrac{\{ a \in C^2(\underline{U}, \Omega^0) : (d+\delta)(a) = 0 \}}{(d+\delta) C^{1,0} \cap \{ a \in C^{2,0} \mid (d+\delta)(a) = 0 \}} \\ \Psi & & \\ a & \longmapsto & \delta a^{1,0} \\ \| & & \\ a^{0,1} + a^{1,0} & & \end{array}$$

We omit the notation [ ].

<u>Note</u> that $\delta a^{1,0}$ need not be in $(d+\delta) C^1(\underline{U}, \Omega^0(*))$,

suppose $\delta a^{1,0} = (d+\delta) b^{1,0}$

$\Rightarrow$ $\delta a^{1,0} - \delta b^{1,0} = d b^{1,0} = 0$ $\Rightarrow$ $\delta(a^{1,0} - b^{1,0}) \in \ker \delta$.

and $d b^{1,0} = 0$. We can not conclude that

$\exists$ such $b^{1,0} \in C^1(\underline{U}, \Omega^0(*))$.

From $d a^{1,0} = -\delta a^{0,1}$, we see that

$(d\, \delta a^{1,0})_{\alpha\beta\gamma} = -\delta(\delta a^{0,1})_{\alpha\beta\gamma} = 0$, and $(\delta a^{1,0})_{\alpha\beta\gamma}$ is constant.

As in the proof of Proposition on P141, we may assume that each U_α in $\underline{U}$ is simply connected, and set

$$h_{\alpha\beta} = \frac{1}{2\pi\sqrt{-1}} \log g_{\alpha\beta}. \qquad g_{\alpha\beta} = \frac{\prod f_{\alpha i}}{\prod f_{\beta j}}.$$

$(h_{\alpha\beta}) \in C^1(\underline{U}, \Omega^0(*))$.

$\Rightarrow$ If we set $z_{\alpha\beta\gamma} = h_{\alpha\beta} + h_{\beta\gamma} - h_{\alpha\gamma}$

$$= \frac{1}{2\pi\sqrt{-1}} (\log g_{\alpha\beta} + \log g_{\beta\gamma} - \log g_{\alpha\gamma}),$$