$\Rightarrow \quad r_{x,y}(s) = s_2(x) \otimes s_1(y) \neq 0 \quad \Rightarrow \quad r_{x,y}$ is surjective.

($\Leftarrow$) Here is a counterexample.
$L = M \times \mathbb{C} \quad \Rightarrow \quad H^0(M, \mathcal{O}(L)) = \langle s \rangle$ where
$s(x) = 1$ for all x.

$\Rightarrow$ $L_x \otimes L_y$ should be corrected to $L_x \oplus L_y$.

($\Rightarrow$) Clearly. ($\Leftarrow$) $\exists$ $s(x) = 0$ but $s(y) \neq 0$ &
$\exists$ $\sigma(x) \neq 0$ but $\sigma(y) = 0$. ⌟

Note that if L satisfies this condition, then $|L|$ must be base-point-free.

⌜ $H^0(M, \mathcal{O}(L)) \xrightarrow{r_{x,y}} L_x \oplus L_y$ is surjective.
$\Rightarrow \exists$ $s \in H^0(M, \mathcal{O}(L))$ s.t. $s(x) \neq 0$. ⌟

2. i_L has nonzero differential everywhere. If φ_α is a trivialization of L near x, then this is the case $\Leftrightarrow$ for all $v^* \in T_x^*(M)$, there exists $s \in H^0(M, \mathcal{O}(L))$ with $s_\alpha(x) = 0$ and $ds_\alpha(x) = v^*$ where $s_\alpha = \varphi_\alpha^* s$.

⌜($\Rightarrow$)
$$\begin{tikzcd} L|_U \arrow[rr, "\varphi_\alpha"] \arrow[dr] & & U \times \mathbb{C} \arrow[dl] \\ & U & \end{tikzcd}$$

$$\begin{tikzcd} i_L : U \arrow[r] \arrow[dr] & \mathbb{P}^N \arrow[d, "\text{if } s_0 \neq 0 \text{ on } U"] \\ x \arrow[r, mapsto] & {[s_0(x), \cdots s_N(x)]} \\ & \mathbb{C}^N \end{tikzcd}$$

$$\left(\frac{s_1}{s_0}, \frac{s_2}{s_0}, \cdots \frac{s_N}{s_0} \right)$$
$$\quad \| \qquad \| \qquad \|$$
$$\quad w_1 \quad\; w_2 \quad\; w_N$$

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