$$\Lambda = \Lambda_{123}\, e_1 \wedge e_2 \wedge e_3 + \Lambda_{124}\, e_1 \wedge e_2 \wedge e_4 + \Lambda_{234}\, e_2 \wedge e_3 \wedge e_4 + \Lambda_{134}\, e_1 \wedge e_3 \wedge e_4$$

$$\Rightarrow i(z)\Lambda = \Lambda_{123}\,(a_{123.1}\, e_1^* + a_{123.2}\, e_2^* + a_{123.3}\, e_3^* + a_{123.4}\, e_4^*) + \Lambda_{124}\,(a_{124.1}\, e_1^* + a_{124.2}\, e_2^* + a_{124.3}\, e_3^* + \cdots) + \cdots + \Lambda_{134}\,(a_{134.1}\, e_1^* + \cdots + a_{134.4}\, e_4^*)$$

$$\Rightarrow i(i(z)\Lambda)\Lambda' = \Lambda_{123}\, a_{123.1}\, i(e_1^*)\Lambda' + \Lambda_{123}\, a_{123.2}\, i(e_2^*)\Lambda' + \cdots + \Lambda_{134}\, a_{134.4}\, i(e_4^*)\Lambda'$$
$$= \Lambda_{123}\, a_{123.1}\, \Lambda'_{123}\, b_{123.12}\, e_1 \wedge e_2 + \cdots + \cdots + \Lambda_{134}\, a_{134.4}\, \Lambda'_{134}\, b_{134.34}\, e_3 \wedge e_4$$

$\Rightarrow$ Since $a_{ij\cdots}$ $b_{ij\cdots}$ are some constants depending on z, we have a bilinear form of Λ & Λ'. $\Rightarrow$ If $\Lambda = \Lambda'$, we get a quadratic for $\Lambda (= (\Lambda_{ijk})$ in the basis $e_i \wedge e_j \wedge e_k$'s.

In sum,

the image of the Grassmannian under the Plücker embedding $p: G(k, V) \longrightarrow \mathbb{P}(\Lambda^k V)$ is cut out by the linear system of quadrics given by (*).

Alternatively, we may characterize W as being the image of $\Lambda^{k-1} V^* \longrightarrow V$ under the map

$$\bar{z} \longrightarrow i(\bar{z})\Lambda, \quad \bar{z} \in \Lambda^{k-1} V^*.$$

Then the condition $W' = W$ is equivelent to

(**) $\quad (i(\bar{z})\Lambda) \wedge \Lambda = 0$ for all $\bar{z} \in \Lambda^{k-1} V^*$.