

$\dim \bar{D}_2 = g-1$ . By the geometric Riemann-Roch theorem,  $\dim \bar{D}_1 = d-1 - h^0(D)+1 = d-1 - \frac{d+2}{2} + 1 = \frac{d}{2} - 1$ ,

$$\begin{aligned} \text{and } \dim \bar{D}_2 &= 2g-2-d - h^0(K-D)+1 \\ &= 2g-2-d - (g+1-\frac{d+2}{2})+1 = 2g-2-d-g-1+\frac{d+2}{2} \\ &\quad +1 = g-1 - \frac{d}{2} - 1 = g - \frac{d}{2} - 2. \end{aligned}$$

$\Rightarrow$  This is impossible since at least one of them is  $(g-1)$ -plane in  $\mathbb{P}^{g-1}$  unless  $d=0$

or  $\frac{d}{2} = g-1$ . Thus  $d=0$ , then  $D=\emptyset \Rightarrow D=0$   
 $d=2g-2 \Rightarrow D=K$ .

Thus we can conclude that  $2h^0(D)$  can equal  $d+2$  only if  $D=0$ ,  $D=K$  or  $K$  is not one to one.  $\square$

Summing up, then, we have

Clifford's Theorem. For any two effective divisors on the compact Riemann surface  $S$ ,

$$\dim |D| + \dim |D'| \leq \dim |D+D'|$$

and for  $D$  special

$$\dim |D| \leq \frac{d}{2}$$

with equality holding only if  $D=0$ ,  $D=K$ , or  $S$  is hyperelliptic.

$\square$  Just plug  $h^0(D)-1$  in  $\dim |D| \Rightarrow O.K. \quad \square$

Corollary. If  $C \subset \mathbb{P}^n$  is any <sup>nondegenerate</sup> curve of degree  $d \leq 2n$  and genus  $g$ ,

$$g \leq d-n$$

