

Thus Q is singular — it is the cone over a conic curve, with vertex p .

$$\begin{aligned} \sqcap \quad & \text{By the result on P197, } Q = V_3 \cap \left(\bigcup_{x \in L} T_x(X) \right) \\ & = \bigcup_{x \in L} (V_3 \cap T_x(X)) = \bigcup_{x \in L} \tilde{f}_L(\pi^{-1}(x)) \end{aligned}$$

$$= \text{The set of lines passing } p = (X_0^2 + X_1^2 + X_2^2 = 0)$$

$$= \text{The cone over a smooth conic curve.}$$

□

Away from the inverse image $\tilde{f}_L^{-1}(p)$, \tilde{f}_L is as before one-to-one except on the locus of lines in X meeting L and as we shall see later, the vertex p of Q lies in the closure E_L of the image of this locus.

$$\begin{aligned} \sqcap \quad & \tilde{f}_L|_{\pi^{-1}(x)} : \pi^{-1}(x) \longrightarrow \tilde{f}_L(\pi^{-1}(x)) = V_3 \cap T_x(X) \text{ is isomorphic} \\ & \text{since } \pi^{-1}(x) \text{ \& } V_3 \cap T_x(X) \text{ are isomorphic to } \mathbb{P}^1, \text{ and} \\ & L = \{ [* * 0 \dots 0] \} \quad V_3 = \{ [0, 0, * * \dots *] \} \Rightarrow \tilde{f}_L(\eta_1) \neq \\ & \tilde{f}_L(\eta_2) \text{ if } \eta_1 \neq \eta_2 \in \pi^{-1}(x). \quad \tilde{f}_L(\pi^{-1}(x)) \cap \tilde{f}_L(\pi^{-1}(x')) = \\ & V_3 \cap T_x(X) \cap V_3 \cap T_{x'}(X) = V_3 \cap T_x(X) \cap T_{x'}(X) = V_3 \cap \left(\bigcap_{x \in L} T_x(X) \right) \end{aligned}$$

$$= p, \text{ and since } \tilde{f}_L(L') \text{ is } \overline{L, L'} \cap V_3, \quad \tilde{f}_L \text{ is one-to-one except on } \bigcup_{L' \in B_L} L' \cup \tilde{f}_L^{-1}(p).$$

□