Lemma. The decomposable forms are L^2-dense in all the forms on $M \times N$.

pf) We will do this in the case of functions; the modifications necessary to treat general forms will be clear.

It must be proved that a function $\varphi(z,w)$ that satisfies
$$\int_{M\times N} \varphi(z,w)\,(\overline{\psi(z)\,\eta(w)}) = 0 \quad \text{for all } \psi \,\&\, \eta$$
is zero.

Suppose $\mathrm{Re}\,\varphi(z_0,w_0) > 0$, choose $\psi(z)$, $\eta(w)$ to have compact support near z_0, w_0, respectively, and satisfy
$$\mathrm{Re}\,(\varphi(\overline{\psi\eta})) \geq 0 \quad , \quad \mathrm{Re}\,(\varphi(z_0,w_0)\,\overline{\psi(z_0)\,\eta(w_0)}) > 0 .$$

This is easy to accomplish using a real nonnegative bump function. Then the above integral is non-zero. Q.E.D.

Forms on $M \times N$ are locally written
$$\varphi(z,w) = \sum \varphi_{II'JJ'}\, dz_I \wedge dw_{I'} \wedge d\bar{z}_J \wedge d\bar{w}_{J'},$$

and then $\quad \bar\partial_{M\times N} = \bar\partial_M + \bar\partial_N$

where $\bar\partial_M$ is exterior derivative with respect to the $\bar{z}_j$'s and similarly for $\bar\partial_N$.

Since the metric is a product, we may choose an orthonormal coframe for $M\times N$ of the form
$$\{\varphi_1(z), \cdots \varphi_m(z);\ \psi_1(w) \cdots \psi_n(w)\},$$
where the $\varphi_i(z)$ are an orthonormal coframe for M and