Here we assumed that $s^{-1}(0) = \{p\}$ is an isolated point. ⌟

Because of the functorial property of duality we have

$$0 = \langle d_n \left(\bigoplus_{p \in Z} e_p \right), \psi \rangle$$

$$= \sum_{p \in Z} \operatorname{Res}_p \left(\frac{\psi}{s} \right),$$

which we may state formally as the

Residue Theorem for Vector Bundles. Given a rank-n holomorphic vector bundle $E \longrightarrow M$ over a compact, complex n-manifold and holomorphic section $s \in H^0(M, \mathcal{O}(E))$ having a set Z of isolated zeros, if for each $\psi \in H^0(M, \mathcal{O}(K \otimes \det E))$ and $p \in Z$ we define the residue

$$\operatorname{Res}_p \left(\frac{\psi}{s} \right)$$

by (**) above, then

$$\sum_{p \in Z} \operatorname{Res}_p \left(\frac{\psi}{s} \right) = 0.$$

⌜ By the slight variation of the Global Duality Theorem II, we have the following diagram, see P708.