

F

Since  $H^n(M, \mathbb{C}) = \bigoplus_{p+q=n} H^{p,q}(M),$

$$H^n(M, \mathbb{C}) + \overline{H^n(M, \mathbb{C})} = 2 H^n(M, \mathbb{R})$$

$$\Rightarrow \bigoplus_{p+q=n} H^{p,q}(M) + \overline{\bigoplus_{p+q=n} H^{p,q}(M)} = \bigoplus_{p+q=n} H^{p,q}(M) + \bigoplus_{p+q=n} H^{q,p}(M)$$

$$= \bigoplus_{p+q=n} (H^{p,q}(M) + H^{q,p}(M)) = 2 H^n(M, \mathbb{R})$$

$$= 2 \left\{ \bigoplus_{\substack{p+q=n \\ p < q}} (H^{p,q}(M) \oplus H^{q,p}(M)) \oplus \bigoplus_{p=q=\frac{n}{2}} H^{p,p}(M) \right\}$$

$$\Rightarrow \bigoplus_{\substack{p+q=n \\ p < q}} (H^{p,q}(M) \oplus H^{q,p}(M)) \oplus \bigoplus_{p=q} H^{p,p}(M) = H^n(M, \mathbb{R})$$

This is nonsense!

$$H^n(M, \mathbb{C}) = \bigoplus_{p+q=n} H^{p,q}(M).$$

$$H^n(M, \mathbb{C}) \cap H^n(M, \mathbb{R}) = \left( \bigoplus_{p+q=n} H^{p,q}(M) \right) \cap H^n(M, \mathbb{R})$$

$$= \left( \bigoplus_{\substack{p+q=n \\ p < q}} (H^{p,q}(M) \oplus H^{q,p}(M)) \oplus \bigoplus_{p=q} H^{p,p}(M) \right) \cap H^n(M, \mathbb{R})$$