We note one final version:

If $\varphi : M \longrightarrow N$ is a homomorphism of $\mathcal{O}$-modules such that $\varphi_0 : M_0 \longrightarrow N_0$ is surjective, then φ is surjective.

⌈

$$\begin{tikzcd} M \arrow[r, "\varphi"] \arrow[d] \arrow[dr, phantom, "\circlearrowright"] & N \arrow[d] \\ M/\mathfrak{m}M \arrow[r, "\varphi_0"] & N/\mathfrak{m}N \end{tikzcd}$$

since $\varphi_0(\mathfrak{m}M) = \mathfrak{m}\,\varphi_0(M) \subset \mathfrak{m}N$
$\qquad\qquad\qquad\qquad\quad \overset{\shortparallel}{\mathfrak{m}\,\varphi(M)}$

Suppose $m_1, \cdots m_k$ generate M. $\Rightarrow$ $m_1 + \mathfrak{m}M, \cdots m_k + \mathfrak{m}M$ generate $M_0 = M/\mathfrak{m}M$. $\Rightarrow$ Since φ_0 is surjective, $\varphi_0(m_1+\mathfrak{m}M) \cdots \varphi_0(m_k+\mathfrak{m}M)$ generate $N/\mathfrak{m}N = N_0$ $\Rightarrow$ By the Nakayama lemma, $\varphi(m_1), \cdots \varphi(m_k)$ generate N. $\Rightarrow$ φ is surjective.⌋

Now we come to a main standard definition. An $\mathcal{O}$-module M is projective if the following diagram holds:

$$\begin{tikzcd} M \arrow[d, dashed, "\gamma"'] \arrow[dr, "\beta"] & & \\ K \arrow[r, "\alpha"'] & L \arrow[r] & 0 \end{tikzcd}$$

This diagram is to be interpreted as follows: K and L are given $\mathcal{O}$-modules, and α and β are given $\mathcal{O}$-module homomorphisms with α being surjective. Then there exists γ s. t the diagram is commutative. Briefly, the solid arrows are given and the dotted arrows can be filled in.. This notation will be consistently used.