$$\mathrm{Res}_{\alpha_i}(\varphi) = \frac{-\alpha_i^{n-r} \sum_{\# I = r} \prod_{j \in I} (\alpha_j - \alpha_i)}{\prod_{j \neq i} (\alpha_j - \alpha_i)}$$

$$\mathrm{Res}_{\infty}(\varphi) = (-1)^{n-r} \binom{n+1}{r}$$

and the residue theorem together with $c_1(\mathbb{P}^n) = (n+1)\omega$ imply

$$c_r(\mathbb{P}^n) = \binom{n+1}{r} \omega^r.$$

Pf

$$\mathrm{Res}_{\alpha_i}(\varphi) \cdot 2\pi\sqrt{-1} = \int_{B_\epsilon(\alpha_i)} \frac{g(z)}{\prod_{k=0}^{n} (\alpha_k - z)} dz$$

$$= \int_{B_\epsilon(\alpha_i)} \frac{z^{n-r} \sum_{\# I = r, i \notin I} \prod_{k \in I} (\alpha_k - z)}{\prod_{j=0}^{n} (\alpha_j - z)} dz$$ (since if $i \in I$, $(\alpha_i - z)$'s cancel each other)

$$= -\int_{B_\epsilon(\alpha_i)} \frac{1}{z - \alpha_i} \frac{z^{n-r} \sum_{\# I = r, i \notin I} \prod_{k \in I} (\alpha_k - z)}{\prod_{j \neq i} (\alpha_j - z)} dz$$

again, by Cauchy formula

$$\overset{\Downarrow}{=} (-1) \frac{\alpha_i^{n-r} \sum_{\# I = r, i \notin I} \prod_{k \in I} (\alpha_k - \alpha_i)}{\prod_{j \neq i} (\alpha_j - \alpha_i)}$$

Replace z by $\frac{1}{z}$, then $-\frac{g(\frac{1}{z})}{f(z)} \frac{1}{z^2} dz =$