Date . .

Thus to prove assertion 1 for x and y, we have to show the map γ_E is surjective.

⌈

$$\begin{array}{ccc} \tilde{L}^k = \pi^* L^k & \longrightarrow & L_x^k \\ \downarrow & & \downarrow \\ E_x & \xrightarrow{\pi} & x \end{array}$$

$\Rightarrow$ Since $L_x^k \to x$ is trivial, the pull back $\pi^* L_x^k$ is again trivial.

$$\Rightarrow H^0(E, \mathcal{O}_E(\tilde{L}^k)) = H^0(E, \mathcal{O}_E(E_x \times L_x^k \cup E_y \times L_y^k))$$
$\cong L_x^k \oplus L_y^k$. The commutativity is clear. ⌋

Now, on $\tilde{M}$ we have the exact sheaf sequence

$$0 \longrightarrow \mathcal{O}_{\tilde{M}}(\tilde{L}^k - E) \longrightarrow \mathcal{O}_{\tilde{M}}(\tilde{L}^k) \xrightarrow{\gamma_E} \mathcal{O}_E(\tilde{L}^k) \longrightarrow 0.$$

Choose k_1 such that $L^{k_1} + K_M^*$ is positive on M.
⌈ Since L is positive line bundle and M is compact, it is possible to choose k_1 s.t $L^{k_1} + K_M^*$ is positive on M. See note P471 ⌋

By virtue of the computation on P.186, we can choose k_2 such that $\tilde{L}^k - nE$ is positive on $\tilde{M}$ for $k \geq k_2$.
⌈ Here $\tilde{L}^k = \pi^* L^k$. ⌋

By the previous lemma,
$$K_{\tilde{M}} = \underset{\underset{\tilde{K}_M}{\parallel}}{\pi^* K_M} + (n-1)E,$$
where $\tilde{K}_M = \pi^* K_M$; and so for $k \geq k_0 = k_1 + k_2$,
$$\mathcal{O}_{\tilde{M}}(\tilde{L}^k - E) = \Omega^n_{\tilde{M}}(\tilde{L}^k - E + K^*_{\tilde{M}})$$
$$= \Omega^n_{\tilde{M}}((\tilde{L}^{k_1} + \tilde{K}_M^*) \otimes (\tilde{L}^{k'} - nE)) \quad \text{with } k' \geq k_2.$$

⌈ $\mathcal{O}_{\tilde{M}}(\tilde{L}^k - E \otimes K_{\tilde{M}} \otimes K^*_{\tilde{M}}) = \mathcal{O}_{\tilde{M}}((\tilde{L}^k - E) \otimes K_{\tilde{M}}) \otimes K^*_{\tilde{M}})$
$= \Omega^n_{\tilde{M}}(\tilde{L}^k - E + K^*_{\tilde{M}}) = \Omega^n_{\tilde{M}}(\tilde{L}^{k_1} + \tilde{L}^{k'} - E + K^*_{\tilde{M}})$

$\tilde{K}_M^* + (n-1)E^* = \tilde{K}_M^* - (n-1)E$