$\begin{pmatrix} v_1 - w_1 \\ \vdots \\ v_n - w_n \end{pmatrix}$ is the matrix having $v_i - w_i$ as the i-th row vector.

Thus since v_i's are linearly independent,
$\dim K = \dim(\Lambda_B \cap \Lambda_{B'}) = n - \operatorname{rank}(B - B')$. ///

By the lemma above,
$\dim(\Lambda_B \cap \Lambda_{B'}) = n+1 - \dim(B-B') - 1 = n - \dim(B-B')$, since Λ_B is an n-plane. ⅃

More generally, if $I = \{i_1, \cdots i_m\}$ is any subset of $\{0, \cdots n\}$, then the automorphism φ_I of F defined by

$$\varphi_I[X] = [X'], \quad X_i' = \begin{cases} X_{n+i+1}, & i \in I \\ X_i, & i \notin I, \end{cases}$$

$$X'_{n+1+i} = \begin{cases} X_i, & i \in I \\ X_{n+i+1}, & i \notin I, \end{cases}$$

carries the set $\Gamma_0 = \{\Lambda_B\}$ of n-planes into another set Γ_I; Γ_I will be of the same family as Γ_0 if and only if $m = \#I$ is even. In this way, we represent all n-planes on F.

⅂ We can see the general case by the following example. For example, $n=2$, and $I = \{0\}$.
We will show that $\dim(\Lambda_B \cap \Lambda_{B'}) = n-1$, and
$n-1 \not\equiv n \ (2) \Rightarrow \Lambda_B$ & $\Lambda_{B'}$ belong to opposite families.