Consider the following map ϕ defined as below:

$$\phi : \varinjlim_{\underline{U}} H_\delta(H(C^p(\underline{U}, K^q))) \longrightarrow H^p(X, \mathcal{H}^q)$$

$$\langle \sigma + \mathrm{im}\, d^{p,q-1} + \frac{\delta(\ker d^{p-1,q}) + \mathrm{im}\, d^{p,q-1}}{\mathrm{im}\, d^{p,q-1}}, \underline{U} \rangle \mapsto \langle \sigma + \mathrm{im}\, d^{p,q-1}, \underline{U} \rangle$$

representing the limits.

ϕ is well-defined, for d & δ commute with φ, $\delta d = d \delta$, and
$\langle \sigma + \mathrm{im}\, d^{p,q-1}, \underline{U} \rangle \in H^p(X, \mathcal{H}^q)$. See note p435 back.

Suppose $\langle \sigma + \mathrm{im}\, d^{p,q-1}, \underline{U} \rangle = 0$ in $H^p(X, \mathcal{H}^q)$.
Perhaps after refining the covering $\underline{U}$, say $\underline{U}'$,
$\sigma + \mathrm{im}\, d^{p,q-1}(\underline{U}') = 0$
$\Rightarrow$ $\sigma = d\eta$, more precisely, since $\sigma = \{\sigma_\alpha \in K^q(U'_\alpha)\}$, $\sigma_\alpha = d\eta_\alpha$, $\eta_\alpha \in K^{q-1}(U'_\alpha)$.
$\Rightarrow$ $\langle \sigma + \mathrm{im}\, d^{p,q-1} + \bigcirc, \underline{U}' \rangle = 0$ in $\varinjlim_{\underline{U}} H_\delta(H(C^p(\underline{U}, K^q)))$. $\Rightarrow$ ϕ is one to one.
Clearly ϕ is onto. Thus we can conclude that

$$\varinjlim_{\underline{U}} H_\delta(H(C^p(\underline{U}, K^q))) = H^p(X, \mathcal{H}^q). \quad //$$

$$K^n = \bigoplus_{p+q} C^p(\underline{U}, K^q) \qquad {}''F^q K^n = \bigoplus_{p+q''=n} C^p(\underline{U}, K^{q''})$$

$$''E_0^{p,q} = \frac{F^p K^{p+q}}{F^{p+1} K^{p+q}} = \frac{C^0(\underline{U}, K^{p+q}) \oplus \cdots \overset{q'' \geq q}{\oplus} C^q(\underline{U}, K^p)}{C^0(\underline{U}, K^{p+q}) \oplus \cdots \oplus C^{q-1}(\underline{U}, K^{p+1})} \cong C^q(\underline{U}, K^p)$$