

$C'$ .

Let  $C \cap C_\lambda = (\sigma'_\lambda = 0)$  and  $(\tau = 0) = p$ , where  $\sigma'_\lambda$  is a section of some bundle of  $C$ .

$\Rightarrow \{\sigma'_\lambda \otimes \tau^{-1}\}$  is a pencil on  $C$  of degree 3.

$\Rightarrow$  It gives a 3-sheeted cover  $C$  of  $\mathbb{P}^1$ .

$\Rightarrow$  By the Riemann-Hurwitz formula, by P255

$$b = 2g(C) - 2 + 3 \cdot \chi(\mathbb{P}^1)$$

$$= -2 + 6 = 4 = \# \text{ of branch points.}$$

Note, here:  $\{\sigma'_\lambda \otimes \tau^{-1} = 0\}$  has no base point for,

since  $\{C'_\lambda\}$  is a generic pencil, and choose

$C'_\lambda \cap C = \{p, q_1, q_2, q_3\}$  where  $C' \cap C = \{p, p_1, p_2, p_3\}$   
 $p_i \neq q_j$ .

Since, if  $\lambda_0$  is a branch point, then  $C'_{\lambda_0}$  is tangent to  $C$ ,

the pencil  $\{C'_\lambda\}$  can contain at most  $b$  conics tangent to  $C$  other than  $C'$ .

$\Rightarrow$

It follows that  $H_p$  is the tangent plane to  $V_C$  at  $C'$ , and conversely if  $C'$  is simply tangent to  $C$  at only one point, then  $C'$  is a smooth point of  $V_C$ .

Since  $C$  is a smooth point of  $V_C$ , by Bertini's theorem applied to the smooth locus  $V_C^*$  of  $V_C$ ,