$$\Rightarrow \log f(p) - \log f(p_0) + \underline{\log f(\delta_i(1)) - \log f(p)} + \int_{\delta_{g+i}} d\log f$$

cancel each other

$$+ \underline{\log f(p') - \log f(\delta_i^{-1}(1))} + \log f(p_0) - \log f(p') = 0$$

Since $\log f(\delta_i(1)) - \log f(p) = \log f(\delta_i^{-1}(1)) - \log f(p')$,

$$\log f(p) - \log f(p') = -\int_{\delta_{g+i}} d\log f.$$

$$\Rightarrow \log f(p') = \log f(p) + \int_{\delta_{g+i}} d\log f.$$

"Comment": Cutting S into Δ makes us to take a canonical way of choosing paths when we have an integral.

$$\int_{\delta_i+\delta_i^{-1}} \varphi = \int_{\delta_i+\delta_i^{-1}} \log f \cdot d\log g = \int_{\delta_i} \log f \cdot d\log g + \int_{\delta_i^{-1}} \log f \cdot d\log g$$

$$= \int_{\delta_i} \log f \cdot d\log g - \int_{-\delta_i^{-1}} \log f \cdot d\log g = \int_{\delta_i} (\log f - \log f)\, d\log g$$

$$= -\int_{\delta_{g+i}} d\log f \int_{\delta_i} d\log g$$ see p.513, note.

We also see that for points $p \in \alpha_i$, $p' \in \alpha_i^{-1}$ on $\partial\Delta'$ identified on S,

$$\log f(p') - \log f(p) = -2\pi\sqrt{-1} \cdot \mathrm{ord}_{p_i}(f),$$

and hence

$$\int_{\alpha_i+\alpha_i^{-1}} \varphi = 2\pi\sqrt{-1}\,.\mathrm{ord}_{p_i}(f) \int_{S_0}^{p_i} d\log g.$$