Note that $\sum \int_{q_\lambda}^{p_\lambda} \omega_i$ is independent of the order of points; in other word, for example, if $p_1 + p_2 - q_1 - q_2$ is a divisor of degree 0 on S,

$$\int_{q_1}^{p_1} \omega_i + \int_{q_2}^{p_2} \omega_i \equiv \int_{q_1}^{p_2} \omega_i + \int_{q_2}^{p_1} \omega_i \quad (\text{modulo } \Lambda)$$

, since $\displaystyle\int_{q_1}^{p_1} \omega_i - \int_{q_1}^{p_2} \omega_i + \int_{q_2}^{p_2} \omega_i - \int_{q_2}^{p_1} \omega_i$

$$\equiv \int_{p_1}^{p_1} \omega_i \quad (\text{modulo } \Lambda).$$

To study this map, we need to learn something about the relations among the periods of the ω_i. These are expressed in the reciprocity laws, one of which we now derive.

The First Reciprocity Law and Corollaries.

To begin with, we may assume that all of the cycles δ_i on the Riemann surface S issue from a common point $s_0 \in S$. The complement of the δ_i's is then a simply connected region Δ on S; the boundary $\partial\Delta$ contains each δ_i twice with opposite orientation, and may be pictured as in Figure 2. What we are doing is making the familiar topological representation of a surface of genus g as a polygon with $4g$ sides, which are identified in pairs.