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Note: The integration value is independent of the choices of coordinates.

By the note above, we have only to prove that the residue is independent of the choices of holomorphic local frames. Let $e_1', \cdots e_n'$ be another local holomorphic frame for E. Then

$$e_i' = g_{ij} e_j .$$

$$\Rightarrow \psi = h(z)\, dz_1 \wedge \cdots \wedge dz_n \otimes e_1 \wedge \cdots \wedge e_n$$
$$= h'(z)\, dz_1 \wedge \cdots \wedge dz_n \otimes e_1' \wedge \cdots \wedge e_n' = h'(z) \det(g)\, dz_1 \wedge \otimes e_1 \wedge \wedge e_n$$
$$s = s_i e_i = s_i' e_i' = s_i' g_{ij} e_j = {}^t g_{ij} \; s_j' e_i$$
$$\Rightarrow \quad s_i = ({}^t g)_{ij} \; s_j' \quad \Leftrightarrow \quad s_i' = ({}^t g^{-1})_{ij} s_j .$$

$\Rightarrow$

$$\mathrm{Res}_p\left(\frac{\psi}{s}\right) = \mathrm{Res}_p \left\{ \frac{h(z)\, dz_1 \wedge \cdots \wedge dz_n}{s_1(z) \cdots s_n(z)} \right\}$$

$$= \mathrm{Res}_p \left\{ \frac{h(z) \det({}^t g^{-1})\, dz_1 \wedge \cdots \wedge dz_n}{s_1'(z) \cdots s_n'(z)} \right\}$$

by Transformation Law on p.657 ~ p658

$$= \mathrm{Res}_p \left\{ \frac{h(z)\, dz_1 \wedge \cdots \wedge dz_n}{\det(g)\, s_1'(z) \cdots s_n'(z)} \right\}$$

$$= \mathrm{Res}_p \left\{ \frac{h'(z)\, dz_1 \wedge \cdots \wedge dz_n}{s_1'(z) \cdots s_n'(z)} \right\}$$

since $h(z) = h'(z) \det(g)$.

This proves what we wanted.

MOOKEUK CO