$\{f_i\}$ a sequence of one variable x. $f_i(x) \longrightarrow g_0(x)$ uniformly on closed bounded balls. $f_i'(x) \longrightarrow g_1(x)$ uniformly on closed balls. Then $g_1(x) = g_0'(x)$.

$$\left| \frac{g_0(x+h) - g_0(x)}{h} - g_1(x) \right| \leq \left| \frac{f_i(x+h) - f_i(x)}{h} - g_1(x) \right| + \epsilon$$

$$\leq \left| \frac{f_i(x+h) - f_i(x)}{h} - f_i'(x) \right| + \left| f_i'(x) - g_1(x) \right| \cdots \quad \circledast$$

$\Rightarrow$ Choose i sufficiently large and $|h|$ small enough.
$\Rightarrow$ We can make $\circledast$ less than $< 3\epsilon$, which implies that

$$g_1(x) = g_0'(x).$$

$\lrcorner$

Thus $C^\infty(\Omega)$ is a Fréchet space. $\ulcorner$ Since we proved that $C^\infty(\Omega)$ is complete, it satisfies all the conditions for Fréchet space. $\lrcorner$

The same is true of each of its closed subspaces $\mathcal{D}_K$.

Suppose next that $E \subset C^\infty(\Omega)$ is closed and bounded. By Th 1.37, (P26 ~ P27), the boundedness of E is equivalent to the existence of numbers $M_N < \infty$ such that $p_N(f) \leq M_N$ for $N = 1, 2, 3, \cdots$ and for all $f \in E$. The inequalities $|D^\alpha f| \leq M_N$, valid on K_N when $|\alpha| \leq N$, imply the equicontinuity of $\{D^\beta f : f \in E\}$ on K_{N-1}, if $|\beta| \leq N-1$.

$\ulcorner$ If $|f(x)'| < M$, $|f(x+h) - f(x)| = |h f'(x+\theta h)|$ by Mean value theorem. $\Rightarrow$ $|f(x+h) - f(x)| \leq |h| M$ $\Rightarrow$ This proves the claim above. $\lrcorner$

It now follows from Ascoli's theorem and Cantor's diagonal