$$\Rightarrow (-1)^{j-1} \frac{dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n}{\frac{\partial f}{\partial z_i}} = (-1)^{i-1} \frac{dz_1 \wedge \cdots \wedge \widehat{dz_j} \wedge \cdots dz_n}{\frac{\partial f}{\partial z_j}}$$

$$\Rightarrow (-1)^{i-1} \frac{dz_1 \wedge \cdots \wedge \widehat{dz_i} \wedge \cdots \wedge dz_n}{\frac{\partial f}{\partial z_i}} = (-1)^{j-1} \frac{dz_1 \wedge \cdots \wedge \widehat{dz_j} \wedge \cdots \wedge dz_n}{\frac{\partial f}{\partial z_j}}.$$ 」

Note that the kernel of the Poincaré residue map consists only of the holomorphic n-forms on M.

「 $P.R.(\omega) = 0 \Rightarrow g(z) = 0.$ on $\{f=0\}$

$\Rightarrow$ This implies. g is divided by f.

$\Rightarrow$ ω is clearly holomorphic. 」

The exact sheaf sequence

$$\begin{tikzcd}
0 \arrow[r] & \Omega^n_M \arrow[r] \arrow[dr] & \Omega^n_M(V) \arrow[r, "P.R."] \arrow[d, leftrightarrow] & \Omega^{n-1}_V \arrow[r] & 0 \\
& & K_M(-V) & &
\end{tikzcd}$$

then gives us, in part, the exact sequence

$$H^0(M, \Omega^n_M(V)) \xrightarrow{P.R.} H^0(V, \Omega^{n-1}_V) \xrightarrow{\delta} H^1(M, \Omega^n_M),$$

i.e the Poincaré residue map is surjective on global sections if $H^1(M, \Omega^n_M) = H^{n,1}(M) = 0$. For example, since $H^{n,1}(\mathbb{P}^n) = 0$ for $n > 1$, every holomorphic form of top degree on hypersurface V in $\mathbb{P}^n$ is the Poincaré residue of a meromorphic form on $\mathbb{P}^n$.