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$\ulcorner$ $\sigma(x) = \sum \alpha_K(x)\, e_K(x)$. $\alpha_K(p_\nu) = 0$.

$\Rightarrow$ By the well-known fact that if $f: M \to \mathbb{C}$ diff. & $f(p)=0$, then $f = \sum a_i x_i$ locally, where $M \xrightarrow{x} \mathbb{R}^n$ local diffeph s.t $x(p)=0$, $\alpha_K = \sum a_{aK} x_a$.

$\Rightarrow$ $\sigma(x) = \sum (a^\nu_{aK} + \sqrt{-1}\, b^\nu_{aK}) \cdot x_a \cdot e_K(x) + [2]$. by Taylor expansion. $\lrcorner$

Let A_{p_ν} be the $2n \times 2n$ matrix (A^ν, B^ν), where $A^\nu = (a^\nu_{aK})$ and $B^\nu = (b^\nu_{aK})$. Then, if we write

$$\sigma(x) = \sum f_K(x) \cdot e_K(x)$$

as before, we have

$(df_K \wedge d\bar{f}_K)(p_\nu) = \sum_a (a^\nu_{aK} + \sqrt{-1}\, b^\nu_{aK})\, dx_a \wedge \sum_b (a^\nu_{bK} - \sqrt{-1}\, b^\nu_{bK})\, dx_b = -2\sqrt{-1} \sum a^\nu_{aK} \cdot b^\nu_{bK} \cdot dx_a \wedge dx_b$,

and so by linear algebra the sign of the point p_ν in the degeneracy cycle D_1 of σ is

$$(-1)^{n(n-1)/2} \cdot \operatorname{sgn} \det (A_{p_\nu}).$$

$\ulcorner$ $f_K = \sum_{\alpha=1}^{2n} (a^\nu_{\alpha K} + \sqrt{-1}\, b^\nu_{\alpha K})\, x_\alpha + [2]$.

If we plug $(x_\alpha) = p_\nu$, then $[2]$ does not contribute.
So assume $f_K = \sum (a^\nu_{\alpha K} + \sqrt{-1}\, b^\nu_{\alpha K})\, x_\alpha$.

$\Rightarrow$ $df_K = \sum (a^\nu_{\alpha K} + \sqrt{-1}\, b^\nu_{\alpha K})\, dx_\alpha$

$\Rightarrow$ $(df_K \wedge d\bar{f}_K)(p_\nu) = \sum (a^\nu_{\alpha K} + \sqrt{-1}\, b^\nu_{\alpha K})\, dx_\alpha \wedge \sum (a^\nu_{\beta K} - \sqrt{-1}\, b^\nu_{\beta K})\, dx_\beta = \sum a^\nu_{\alpha K} a^\nu_{\beta K}\, dx_\alpha \wedge dx_\beta + \sum b^\nu_{\alpha K} b^\nu_{\beta K}\, dx_\alpha \wedge dx_\beta - \sqrt{-1} \sum a^\nu_{\alpha K} b^\nu_{\beta K}\, dx_\alpha \wedge dx_\beta + \sqrt{-1} \sum b^\nu_{\alpha K} a^\nu_{\beta K}\, dx_\alpha \wedge dx_\beta$.

Then $\sum a^\nu_{\alpha K} a^\nu_{\beta K}\, dx_\alpha \wedge dx_\beta = 0$, since $a^\nu_{\alpha K} a^\nu_{\beta K}\, dx_\alpha \wedge dx_\beta + a^\nu_{\beta K} a^\nu_{\alpha K}\, dx_\beta \wedge dx_\alpha = 0$. Similarly, $\sum b^\nu_{\alpha K} b^\nu_{\beta K}\, dx_\alpha \wedge dx_\beta = 0$.