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points of intersection would be singular points of $H \cap V$, and so this can not happen. 「 O.K. see P21 」

To prove the assertion in the general case requires a different approach. We argue as follows: let $p \in V$ be any smooth point, and let $\mathbb{P}^{n-2} \subset \mathbb{P}^n$ be an $(n-2)$-plane meeting V transversely at p; let Z be the irreducible component of $V \cap \mathbb{P}^{n-2}$ containing p. Now consider the pencil $\{H_\lambda\}$ of hyperplanes in $\mathbb{P}^n$ containing $\mathbb{P}^{n-2}$.

「 If H is a hyperplane containing $\mathbb{P}^{n-2}$, H can be expressed as $\langle e_0, \cdots e_{n-2}, a e_{n-1} + b e_n \rangle$ where $e_0, \cdots e_n$ are orthonormal basis for $\mathbb{C}^{n+1}$ and $|a|^2 + |b|^2 = 1$. 」

Each hyperplane section $H_\lambda \cap V$ of V contains Z, but since each H_λ intersects V transversely at p, p — being a smooth point of $H_\lambda \cap V$ — can lie on at most one of the irreducible componenents of $H_\lambda \cap V$ for each λ.

「 I think Z is the connected component of $(V \cap \mathbb{P}^{n-2})^*$ containing p. $\Rightarrow$ Z is open in $V \cap \mathbb{P}^{n-2}$. see P21 Proposition.

Since $\mathbb{P}^{n-2}$ meets V transversely at p, each H_λ containing $\mathbb{P}^{n-2}$ meets V transversely at p.

To show that p is a smooth point of $H_\lambda \cap V$, we will show that $\begin{pmatrix} \nabla F_1 \\ \vdots \\ \nabla F_{n-k} \\ 0, 0, \cdots 1 \end{pmatrix}$ has rank $n-k+1$, where $V = \{F_1 = F_2 = \cdots = F_{n-k} = 0\}$, $H_\lambda = \{X_n = 0\}$.

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