form on $\tilde{M}$. For any $p \in M$, we have isomorphisms of a nbd U of p in M with nbds U_i of the points $q_i \in \pi^{-1}(p)$; we can define a $(1,1)$-form ω' on M by

$$\omega'(p) = \sum_{q \in \pi^{-1}(p)} \omega(q).$$

┌ Suppose $\tilde{M}$ is algebraic. ⇒ By P191. Kodaira Embedding Theorem, $\tilde{M}$ has a closed, positive $(1,1)$-form ω which is integral. ω' is well-defined. ┘

Then ω' is closed and of type $(1,1)$, and if $\eta \in H_{DR}^{2n-2}(M)$ is any integral cohomology class, then

$$\int_M \omega' \wedge \eta = \frac{1}{m} \int_{\tilde{M}} \omega \wedge \pi^* \eta \in \mathbb{Q},$$

where m is the number of sheets of the cover. Thus $[\omega']$ is rational.

┌ Clearly, ω' is closed and of type $(1,1)$.
For simplicity, assume that M is evenly covered by two coordinate open sets diffeomorphic to R^n.
Let $M = U_1 \cup U_2$. and $\rho_1 + \rho_2 = 1$, $\operatorname{supp} \rho_1 \subset U_1$, $\operatorname{supp} \rho_2 \subset U_2$. $\rho_1, \rho_2 \geq 0$
⇒ $\pi^*\rho_1$, $\pi^*\rho_2$ are a partition of unity of $\tilde{M}$.

$$\int_{\tilde{M}} \omega \wedge \pi^* \eta$$

$$= \int_{\pi^{-1}(U_1) \cup \pi^{-1}(U_2)} (\pi^*\rho_1 + \pi^*\rho_2) \cdot (\omega \wedge \pi^* \eta)$$

$$= \int_{\pi^{-1}(U_1)} \pi^*\rho_1 \cdot (\omega \wedge \pi^*\eta) + \int_{\pi^{-1}(U_2)} \pi^*\rho_2 \cdot (\omega \wedge \pi^*\eta)$$

, note that $\operatorname{supp} \pi^*\rho_1 \subset \pi^{-1}(U_1)$. $\pi^{-1}(U_2) \supset \operatorname{supp} \pi^*\rho_2$.