let $q(\lambda)$ be a straight line segment lying on ℓ with $q(0) = p_i$, then $\lim_{\lambda\to 0} \overline{p_i q(\lambda)} = \ell$.

$\Rightarrow \ell \in W'$.

$\tau(W) \cap \sigma_{n-2,n-3} = \{\ell_1, \cdots \ell_m,\ \ell_i\text{'s are lines}\}$.
If $\ell \in \tau(W) \cap \sigma_{n-2,n-3}$, and ℓ passes through p_1,
$\ell \subset W \cap S = W'$. $\Rightarrow \ell \subset T_{p_1}W' \cap W$.
Thus once we know $T_{p_1}W' \cap W$ consists of two lines, $\tau(W) \cap \sigma_{n-2,n-3} = 4$ lines., since ℓ can not pass p_1 & p_2. For, if so, $\ell = \ell_0 \subset W'$ contradicts to the fact $W' \cap \ell_0 = \{p_1, p_2\}$.

Thus it remains to show that $T_{p_1}W' \cap W$ consists of two lines.
$\dim T_{p_1}W' = 2 \Rightarrow T_{p_1}W' \cong \mathbb{P}^2$. $\dim W = n-1$.
$\Rightarrow T_{p_1}W' \cap W =$ curve or 2-plane.
Suppose $T_{p_1}W' \subset W$
Given any line $\ell \subset T_{p_1}W'$, since $T_{p_1}W' \subset W$, $\ell \subset W$ and since $S \cong \mathbb{P}^3$, $T_{p_1}S = S$.
$\Rightarrow \ell \subset S \Rightarrow \ell \subset W \cap S = W' \Rightarrow T_{p_1}W' \subset W'$
$\Rightarrow \mathbb{P}^2 \subset W' \Rightarrow$ subvariety of $W' \cong \mathbb{P}^2$ since $\mathbb{P}^2$ is irreducible.
and W' is quadric surface. $\Rightarrow$ Contradiction.

$\Rightarrow \dim(T_{p_1}W' \cap W) = 1 \Rightarrow T_{p_1}W' \cap W$ is a curve
Consider $T_{p_1}W' \cap W \cap \mathbb{P}^{n-1}$ for a generic $(n-1)$-plane $\mathbb{P}^{n-1}$.
$\Rightarrow \mathbb{P}^2 \cap W \cap \mathbb{P}^{n-1} = \mathbb{P}^1 \cap W \Rightarrow$ Since W is of deg 2, $\#((T_{p_1}W' \cap W)\cdot \mathbb{P}^{n-1}) = 2 = \#(\mathbb{P}^1 \cdot W)$
$\Rightarrow T_{p_1}W' \cap W$ is a curve of deg. 2.