holomorphic on $S - \{p_\lambda\}$, with no periods, no residues, and a pole of order ≤ 2 at each p_λ.

$\ulcorner$ Since $D = \sum p_\lambda$, and p_λ's are distinct, f must have simple poles so that $(f) + D \geq 0$.
Locally, $f = \frac{h}{g}$.

$$\Rightarrow df = \frac{\frac{\partial h}{\partial z} g - h \frac{\partial g}{\partial z}}{g^2} dz$$

If $g = 0$, $h \neq 0$ at p, $\mathrm{ord}_p(g) = 1$ or 0. $\Rightarrow \frac{\partial g}{\partial z} \neq 0$ at p
$\Rightarrow df$ has double pole at p. Thus f has only a pole of order 2 at each p_λ.

$$\int_{\delta_i} df = f(\delta_i(0)) - f(\delta_i(1)) = 0$$ by the Fundamental Theorem of Calculus. $\lrcorner$

Conversely, given any such differential η, the meromorphic function

$$f(p) = \int_{p_0}^{p} \eta$$

is well-defined and satisfies $(f) + D \geq 0$.

$\ulcorner$ Since η has no residues and no periods, f is well-defined i.e. if we do the integral by using any path from p_0 to p, we get the same value.
Suppose η has a pole at p_λ of order ≤ 2. f has a pole of order ≤ 1, since $df = \eta$. $\lrcorner$