

the cone through x_i over a smooth quadric curve, $T_{x_i}(Q) \cap Q$ is the locus of lines on Q through x_i .

□

Let U_i be a 2-plane in $T_{x_i}(Q)$ containing p and not containing x_i ; then $F \cap U_i$ is a smooth conic curve C_i and $T_{x_i}(Q) \cap Q$ is just the locus of lines through x_i meeting C_i .

▮ Suppose that q is a singular point of $F \cap U_i$.

$$\Rightarrow T_q F \supset U_i \ni p. \Rightarrow p \neq q, \text{ since } p \notin F.$$

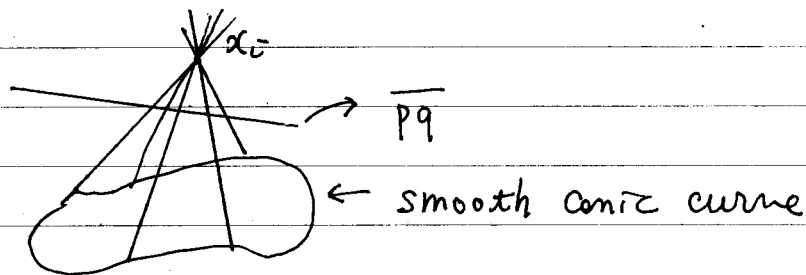
$$\Rightarrow \overline{pq} \subset U_i \subset T_q(F) \Rightarrow \overline{pq} \subset U_i \subset T_{x_i}(Q)$$

Since \overline{pq} is tangent to F at q , and $\deg F = 2$,

$$\overline{pq} \subset F \cap U_i. \Rightarrow T_{x_i}(Q) \subset V_u \Rightarrow \overline{pq} \subset V_u$$

$$\Rightarrow \overline{pq} \subset F \cap V_u = Q \Rightarrow \overline{pq} \subset T_{x_i}(Q) \cap Q$$

\Rightarrow Since $T_{x_i}(Q) \cap Q$ is the locus of lines through x_i , it is the cone through x_i over a smooth conic curve. So if \overline{pq} does not pass through x_i , then we have a contradiction i.e.,



the smooth conic curve contains a line.