$\phi_i \longrightarrow 0$ in some $\mathcal{D}_K$, and the restriction of Λ to this $\mathcal{D}_K$ is bounded.

⌈ If E is bounded in $\mathcal{D}_K$, for $V \ni 0$, $E \subset tV$. But $V = \mathcal{D}_K \cap W$. $W \in \mathcal{D}(\Omega)$. $\Rightarrow$ If we have $\mathcal{O} \in \mathcal{D}(\Omega)$ s.t $\mathcal{O} \ni 0$, $s(\mathcal{D}_K \cap \mathcal{O}) \supset E$ $\Rightarrow$ $s\mathcal{O} \supset E$ $\Rightarrow$ E is bounded in $\mathcal{D}(\Omega)$. The restriction of Λ to this $\mathcal{D}_K$ is bounded. ⌋

Th. 1.32, applied to $\Lambda : \mathcal{D}_K \longrightarrow Y$, shows that $\Lambda\phi_i \to 0$ in Y. Thus (b) implies (c).

⌈ Since $\mathcal{D}_K$ is metrizable, we can apply Th 1.32 to our case. ⌋

Assume (c) holds, $\{\phi_i\} \subset \mathcal{D}_K$, and $\phi_i \longrightarrow 0$ in $\mathcal{D}_K$. By (b) of Theorem 6.5, $\phi_i \longrightarrow 0$ in $\mathcal{D}(\Omega)$.

⌈ W open in $\mathcal{D}(\Omega)$. $W \ni 0$. $\Rightarrow$ $W \cap \mathcal{D}_K$ open in $\mathcal{D}_K$ $\Rightarrow \exists$ N s.t if $i > N$. $\phi_i \in W \cap \mathcal{D}_K$. $\Rightarrow$ $\phi_i \in W$. ⌋

Hence (c) implies that $\Lambda\phi_i \longrightarrow 0$ in Y, as $i \to \infty$. Since $\mathcal{D}_K$ is metrizable, (d) follows.

⌈ Here we used again Th 1.32. P23.
Consider $\Lambda : \mathcal{D}_K \longrightarrow Y$. $\Rightarrow$ By (c), if $\phi_i \longrightarrow 0$ in $\mathcal{D}_K$, then $\Lambda\phi_i \longrightarrow 0$ in Y, since $\phi_i \longrightarrow 0$ in $\mathcal{D}(\Omega)$.
$\Rightarrow$ By the equivalences of Th 1.32, Λ is continuous. ⌋

To prove that (d) implies (a), let U be a convex balanced nbd of 0 in Y, and put $V = \Lambda^{-1}(U)$. Then V is convex and balanced.