

angent vectors  $\{\omega_\alpha\}$  for  $T_x^{*'}(M)$ , we can find a smooth hyperplane section  $V_\alpha$  of  $M$  through  $x$ , such that  $\omega_\alpha$  is not in the kernel of the natural projection map  $T_x^{*'}(M) \rightarrow T_x^{*'}(V_\alpha)$ .

For a given cotangent vector  $\omega_\alpha \in T_x^{*'}(M)$ , we can find a smooth hyperplane section  $V_\alpha$  of  $M$  through  $x$ , s.t.  $\omega_\alpha$  is not in the kernel of the natural projection map  $T_x^{*'}(M) \rightarrow T_x^{*'}(V_\alpha)$ .

First of all, consider the linear system of hyperplane sections of  $M \subset \mathbb{P}^N$  containing  $x$ .

$\Rightarrow$  By the same argument as above, generic hyperplane sections  $H \cap M$  are smooth for all  $y \in H \cap M$ .

For generic hyperplane sections  $V = H \cap M$ ,  $\omega_\alpha$  is not in the kernel of the natural projection map  $T_x^{*'}(M) \rightarrow T_x^{*'}(V)$ .

$\Rightarrow \{H\} \cap \{H'\} = \{H''\}$ .

$\Rightarrow V_\alpha = V' = H'' \cap M$  is smooth s.t.  $\omega_\alpha$  is not in the kernel of the natural projection map  $T_x^{*'}(M) \rightarrow T_x^{*'}(V)$ .

Then by induction we can find  $m_\alpha$  such that for  $m > m_\alpha$ , the differential map

$H^0(V_\alpha, \mathcal{I}_x(E \otimes L^m)) \rightarrow T_x^{*'}(V_\alpha) \otimes (E \otimes L^m)_x$  is surjective.

Since  $\dim V_\alpha = \dim M - 1 < \dim M$ , by induction, the differential map is surjective.  $\square$