Kodaira-Serre duality

$H^0(C, \mathcal{O}(K_C - (m-k)H)) = H^0(C, \mathcal{O}(-2H - (m-k)H))$

$= H^0(C, \Omega^1(-(m-k)H)) = H^0(C, \mathcal{O}(k-m-2)H)$

$\Rightarrow$ Since $k-m-2 \leq m+1-m-2 = -1$, $H^0(C, \mathcal{O}((k-m-2)H) = 0$

$\Rightarrow$ $H^1(C, \mathcal{O}(L+kC)) = 0$ for $k \leq m+1$. P153 & P146. ⌟

It follows from the long exact cohomology sequence associated to $(*_k)$ that $H^1(M, \mathcal{O}(L+(k-1)C)$ surjects onto $H^1(M, \mathcal{O}(L+kC))$ for $k \leq m+1$; and since by hypothesis $H^1(M, \mathcal{O}(L)) = 0$,

$$H^1(M, \mathcal{O}(L+kC)) = 0 \quad \text{for } k \leq m+1,$$

so the restriction map $H^0(M, \mathcal{O}(L+kC)) \to H^0(C, \mathcal{O}((m-k)H))$ is surjective for $k \leq m+1$.

⌜ $H^1(M, \mathcal{O}(L+(k-1)C)) \to H^1(M, \mathcal{O}(L+kC)) \to \underset{\overset{\|}{0}}{H^1(C, \mathcal{O}(L+kC))}$

$\Rightarrow$ $H^1(M, \mathcal{O}(L+(k-1)C))$ surjects onto $H^1(M, \mathcal{O}(L+kC))$

$\Rightarrow$ $0 = H^1(M, \mathcal{O}(L)) \longrightarrow H^1(M, \mathcal{O}(L+C))$ onto $\Rightarrow \underset{\overset{\|}{0}}{H^1(M, \mathcal{O}(L+C))}$

$H^1(M, \mathcal{O}(L+C)) \to H^1(M, \mathcal{O}(L+2C))$ onto

$\vdots$

$H^1(M, \mathcal{O}(L+(k-1)C)) \to H^1(M, \mathcal{O}(L+kC))$ onto

$\Rightarrow$ $H^1(M, \mathcal{O}(L+kC)) = 0$ for $k \leq m+1$.

$H^0(M, \mathcal{O}(L+kC)) \to H^0(C, \mathcal{O}(L+kC)) \to \underset{\overset{\|}{0}}{H^1(M, \mathcal{O}(L+kC))} \to$ for $k \leq m+1$.

$\Rightarrow$ $H^0(M, \mathcal{O}(L+kC)) \longrightarrow H^0(C, \mathcal{O}(L+kC))$ is surjective ⌟

In particular, this tells us that the linear system $|L+kC|$ has no base points on C for $k \leq m$; since $|L|$

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