NO.

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Since by the theorem of P317,

$h^0(2B_{L_0}) = h^0(2B_{L_0} + \frac{1}{2}\mu) = 4$,

we see that both j and j' are given by complete linear systems, it follows that

The Kummer surfaces S and S^* are projectively isomorphic.

⌈ $h^0(2B_{L_0}) = h^0(2B_{L_0} + \frac{1}{2}\mu)$ since $[2B_{L_0} + \frac{1}{2}\mu] = \tau^*_{\frac{1}{2}\mu}[2B_{L_0}]$.

First of all, recall that $[B_{L_0}] \longrightarrow A$ is a principal polorized bundle with $C_1(B_{L_0})$.

$\Rightarrow$ $C_1(B_{L_0}) = dx_1 \wedge dx_3 + dx_2 \wedge dx_4$, and $C_1(2B_{L_0}) = 2\, dx_1 \wedge dx_3 + 2\, dx_2 \wedge dx_4$

$\Rightarrow$ By Theorem on P317, $\dim H^0(A, \mathcal{O}(2B_{L_0})) = 2 \cdot 2 = 4$

$\Rightarrow$ By P177.

$$j : A \longrightarrow \mathbb{P}^3, \quad x \longmapsto [\sigma_0(x), \sigma_1(x), \cdots \sigma_3(x)]$$

$$j' : A \longrightarrow \mathbb{P}^{3*} = \mathbb{P}^3, \quad x \longmapsto [\tau^*_{\frac{1}{2}\mu}\sigma_0(x), \cdots \tau^*_{\frac{1}{2}\mu}\sigma_3(x)]$$

where $H^0(A, \mathcal{O}(2B_{L_0})) = \langle \sigma_0, \sigma_1, \sigma_2, \sigma_3 \rangle$.

and j & $\tilde{j} = \tau^*_{\frac{1}{2}\mu} j'$ are projectively isomorphic

$\Rightarrow$ The images of those two maps are projectively isomorphic, i.e., S & S^* are projectively isomorphic. Actually, we have to trace the isomorphism of $\mathbb{P}^{3*} \cong \mathbb{P}^3$ in details. But I think the arguments above are enough. ⌋