$$* \bar\partial * \bar\partial \psi \equiv (-1)^q \sum_{k>q} f_{\bar k, k} \; \bar\varphi_1 \wedge \cdots \wedge \bar\varphi_q \; (-1)^{q^2}$$

$$+ \sum_{\substack{k>q \\ \ell \le q}} \underbrace{(-1)^{k-1} f_{\bar k, \ell} \; \bar\varphi_1 \wedge \cdots \wedge \hat{\bar\varphi}_\ell \wedge \cdots \wedge \bar\varphi_q \wedge \bar\varphi_k \; (-1)^{q+\ell+k}}_{(-1)^{q+\ell-1}}$$

This gives $* \bar\partial * \bar\partial \psi$, and the other term $\bar\partial * \bar\partial * \psi$ is similar but shorter:

$$* \psi = \bar f \; \bar\varphi_{q+1} \wedge \cdots \wedge \bar\varphi_n \wedge \Phi'$$

$$\bar\partial * \psi = \bar\partial \bar f \wedge \bar\varphi_{q+1} \wedge \cdots \wedge \bar\varphi_n \wedge \Phi' = \sum_{\ell \le q} \bar v_\ell(\bar f) \; \bar\varphi_\ell \wedge \bar\varphi_{q+1} \wedge \cdots \wedge \bar\varphi_n \wedge \Phi'$$

$$\equiv \sum_{\ell \le q} \bar f_{\bar \ell} \; \bar\varphi_\ell \wedge \bar\varphi_{q+1} \wedge \cdots \bar\varphi_n \wedge \Phi'.$$

$$* \bar\partial * \psi \equiv \sum_{\ell \le q} f_\ell \; \bar\varphi_1 \wedge \cdots \wedge \hat{\bar\varphi}_\ell \wedge \cdots \bar\varphi \wedge \cdots \bar\varphi_q \; (-1)^{\ell-1}$$

$$= \sum_{\ell \le q} (-1)^{\ell-1} f_\ell \; \bar\varphi_1 \wedge \cdots \wedge \hat{\bar\varphi}_\ell \wedge \cdots \wedge \bar\varphi_q$$

$$\bar\partial * \bar\partial * \psi \equiv \sum_{\substack{\ell \le q \\ k \le q}} (-1)^{\ell-1} \; \bar\partial f_\ell(\bar v_k) \; \bar\varphi_k \wedge \bar\varphi_1 \wedge \cdots \hat{\bar\varphi}_\ell \wedge \cdots \wedge \bar\varphi_q$$

$$+ \sum_{\substack{\ell \le q \\ k > q}} (-1)^{\ell-1} \; \bar\partial f_\ell(\bar v_k) \; \bar\varphi_1 \wedge \cdots \wedge \hat{\bar\varphi}_\ell \wedge \wedge \bar\varphi_q \wedge \bar\varphi_k \; (-1)^{q-1}$$

$$= \sum_{\ell \le q} f_{\ell . \bar \ell} \; \bar\varphi_1 \wedge \cdots \wedge \bar\varphi_q + \sum_{\substack{\ell \le q \\ k > q}} (-1)^{q+\ell} f_{\ell . \bar k} \; \bar\varphi_1 \wedge \cdots \wedge \hat{\bar\varphi}_\ell \cdots \wedge \bar\varphi_q \wedge \bar\varphi_k$$

$$\frac{\partial^2}{\partial \bar z_i \partial z_j} = \frac{\partial^2}{\partial z_j \partial \bar z_i}$$

Now $v_i(\bar v_j f) - \bar v_j(v_i f) \equiv A'(f)$, so that modulo first-order terms.