$$\Rightarrow \quad \frac{dx}{2y} = -\frac{dy}{-3x^2-2ax-b} \quad \text{on } C$$

$$\Rightarrow \quad \frac{dx}{y} = \frac{2\,dy}{3x^2+2ax+b} \quad \text{on } C.$$

Since $x^3+ax^2+bx+c=0$ does not have a zero of order ≥ 2, $3x^2+2ax+b \neq 0$.

$\Rightarrow$ RHS is holomorphic on C since, on $(y=0 = x^3+ax^2+bx+c)$, $3x^2+2ax+b \neq 0$.

On $(y=0) \cap C$, $dx = 0$ must hold.

Let γ_1, γ_2 be closed loops on C generating $H_1(C,\mathbb{Z}) \cong \mathbb{Z} \oplus \mathbb{Z}$, and denote by

$$a_1 = \int_{\gamma_1} \omega, \quad a_2 = \int_{\gamma_2} \omega$$

the corresponding periods of ω. The general periods of ω on C will then be of the form $n \cdot a_1 + m \cdot a_2$, $n, m \in \mathbb{Z}$.

Pf α is a closed loop on C. $\Rightarrow$ $\alpha = n\gamma_1 + m\gamma_2$.

$$\Rightarrow \int_\alpha \omega = n\int_{\gamma_1} \omega + m\int_{\gamma_2} \omega = n \cdot a_1 + m\, a_2$$

If a_1 and a_2 were linearly dependent over $\mathbb{R}$, we could write

$$k_1 \int_{\gamma_1} \omega + k_2 \int_{\gamma_2} \omega = 0 \quad \text{for } k_1, k_2 \in \mathbb{R};$$

we would then have

$$k_1 \int_{\gamma_1} \overline{\omega} + k_2 \int_{\gamma_2} \overline{\omega} = 0,$$

and since ω and $\overline{\omega}$ generate $H^{1,0}(C) \oplus H^{0,1}(C) = H^1_{DR}(C)$, this would imply that