We may therefore think of $\mathbb{H}^*(\Omega(*))$ as the de Rham cohomology of the function field of M.
For the other spectral sequence
(**) $\quad {}'E_2^{p,q} \cong H^p(M, \mathcal{H}^q(\Omega(*)))$.

$\ulcorner$ $\quad \Omega(*) : \Omega^0(*) \xrightarrow{d} \Omega^1(*) \to \cdots \to \Omega^n(*) \to$
$\mathcal{H}^p(\Omega(*))$ the pth cohomology sheaf
$\Rightarrow$ By p446,
$$ {}'E_2^{p,q} = H^p(M, \mathcal{H}^q(\Omega(*))) $$
$\lrcorner$

Now any spectral sequence gives an exact sequence in low degrees, which in this case is
$$0 \to {}'E_2^{1,0} \longrightarrow \mathbb{H}^1 \longrightarrow {}'E_2^{0,1} \xrightarrow{d_2} {}'E_2^{2,0} \longrightarrow \mathbb{H}^2 \to G \to 0,$$
where
$$G = \mathbb{H}^2/F^2\mathbb{H}^2 \quad \text{has the subgroup } G' = \mathbb{H}^2/F^1\mathbb{H}^2$$
with
$$G' \oplus G/G' \subset {}'E_2^{1,1} \oplus {}'E_2^{2,0} \; {}^{\to 0,2}$$
a subgroup of $\ker d_2$.

$\ulcorner$ Consider the following exact sequence
$$0 \to \ker d_2 \longrightarrow {}'E_2^{0,1} \xrightarrow{d_2} {}'E_2^{2,0} \longrightarrow \operatorname{coker} d_2 \to 0$$

① Note, from $\mathbb{H}^1/{}'F^1\mathbb{H}^1 \cong {}'E_\infty^{0,1}$
we have an exact sequence
$$0 \longrightarrow {}'F^1\mathbb{H}^1 \longrightarrow \mathbb{H}^1 \longrightarrow {}'E_\infty^{0,1} \longrightarrow 0$$

② Note, $\quad {}'E_2^{-2,0}=0 \xrightarrow{d_2} {}'E_2^{0,1} \xrightarrow{d_2} {}'E_2^{2,0} \xrightarrow{d_2} {}'E_2^{4,0-2+1}=0$