We want to show that $\iota_*(T_{x_0}(D_k - D_{k-1})) + T_{\iota(x_0)}\sigma_{1,1}(V) = T_{\iota(x_0)} G(k,n)$. by proving that

$$\iota_*(T_{x_0}(D_k - D_{k-1})) \not\subset T_{\iota(x_0)}\sigma_{1,1}(V)$$ since $\sigma_{1,1}(V)$ has the complex codimension 1 in $\sigma_1(V)$. In the same way, we can show that $\iota_*(T_x(M)) \not\subset T_{\iota(x)}\sigma_1(V)$, which implies that

$$\iota_*(T_x M) + T_{\iota(x)}\sigma_1(V) = T_{\iota(x)} G(k,n)$$

(Remember $\sigma_1(V)$ has the complex codimension 1 in $G(k,n)$).

For example, $n = 5$, $k = 4$, $r = 1$

$x' \in D_4 - D_3$ $\Rightarrow$ $\mathrm{cod}_{\mathbb{R}}(D_{3+1} - D_{\underset{+1}{2}}) = 2(4-3) = 2$

$$\Rightarrow \iota(x) = \left\langle \begin{array}{ccccc} & & k-1 & k-0 & k+1 \\ \multicolumn{5}{c}{\left(\begin{array}{ccccc} 1, & 0 & *, & *, & * \\ 0, & 1 & * & * & * \\ 0 & 0 & 0 & * & * \\ 0, & 0, & 0 & * & * \end{array}\right)} \end{array} \right\rangle \in \sigma_{1,1}(V), \quad x \in D_3 - D_2$$

$$\iota(x') = \left\langle \begin{array}{c} (1, 0, 0, *, *) \\ (0, 1, 0, *, *) \\ (0, 0, 1, *, *) \\ (0, 0, 0, 0, *) \end{array} \right\rangle \in \sigma_1(V)$$

Since

$$\begin{pmatrix} 1, & 0 & a_1, & a_2 \\ 0, & 1 & a_3, & a_4 \\ 0 & 0 & a_5, & a_6 \\ 0 & 0 & 0, & a_7 \end{pmatrix}^{-1} = \begin{pmatrix} 1, & 0, & -\frac{a_1}{a_5}, & \frac{a_1 a_6 - a_2 a_5}{a_5 a_7} \\ 0, & 1, & -\frac{a_3}{a_5}, & \frac{a_3 a_6 - a_5 a_4}{a_5 a_7} \\ 0, & 0, & \frac{1}{a_5}, & -\frac{a_6}{a_5 a_7} \\ 0, & 0, & 0, & \frac{1}{a_7} \end{pmatrix}$$ and