$$\Rightarrow \quad f(z_1) + f(z_2) = \sigma_1(z_1', z_2')$$
$$f(z_1)\, f(z_2) = \sigma_2(z_1', z_2').$$

$\Rightarrow$ Obviously, $\sigma_i(z_1', z_2')$ is a symmetric function in z_1, z_2, $i = 1, 2$ $\Rightarrow$ $\sigma_i(z_1', z_2')$ may be expressed in terms of $\sigma_1(z_1, z_2)$ & $\sigma_2(z_1, z_2)$.

Locally,
$$f(z_1) = a_0 + a_1 z_1 + a_2 z_1^2 + \cdots$$
$$f(z_2) = a_0 + a_1 z_2 + a_2 z_2^2 + \cdots$$
$$\Rightarrow f(z_1) + f(z_2) = 2a_0 + a_1\sigma_1 + a_2(\sigma_1^2 - 2\sigma_2) + \cdots$$

Similarly we may express $f(z_1) f(z_2)$ in terms of σ_1, σ_2.

$$f(z_1) f(z_2) = a_0^2 + a_0 a_1 (z_1 + z_2) + a_1^2 z_1 z_2 + a_2 a_0 (z_1^2 + z_2^2) + \cdots$$
$$= a_0^2 + a_0 a_1 \sigma_1 + a_1^2 \sigma_2 + a_2 a_0 (\sigma_1^2 - 2\sigma_2) + \cdots$$

$\Rightarrow$ $\psi \circ \phi^{-1}$ is holomorphic in σ_1 & σ_2 (z_1, z_2).

The other way is easier if we notice the following

$$V_1 \times V_2 \longrightarrow \mathbb{C} \times \mathbb{C} \xrightarrow{\ h\ } \mathbb{C} \times \mathbb{C}$$
$$(q_1, q_2) \longmapsto (z_1'(q_1), z_2'(q_2)) \longmapsto (z_1' + z_2' = \sigma_1', \; z_1' z_2' = \sigma_2')$$

$$\downarrow$$
$$\mathbb{C} \times \mathbb{C}$$
$$(z_1(q_1), z_2(q_2))$$
$$\downarrow$$
$$\mathbb{C} \times \mathbb{C}$$
$$(\underbrace{z_1(q_1) + z_2(q_2)}_{= \sigma_1}, \; \underbrace{z_1(q_1) z_2(q_2)}_{= \sigma_2})$$

Note that since $z_1' = z_2'$ as a function on V_1, V_2, and $q_1 \neq q_2$, $z_1'(q_1) \neq z_2'(q_2)$.

Once we prove h is invertible, then since σ_1, σ_2 are functions of z_1', z_2', σ_1, σ_2 are functions of $\sigma_1(z_1', z_2')$ & $\sigma_2(z_1' z_2')$.