$\Rightarrow T_P - T_\psi \in \mathcal{D}^q(M)$ s.t $(T_P - T_\psi)(\varphi) = 0$ for all $\varphi \in A^{n-q}(M)$.

See P 77 ~ P 98 de Rham. specially P 95. Theorem 17 & 17'.

Once we have Theorems 17 & 17', $T_P - T_\psi = dT$, where $T \in \mathcal{D}^{q-1}(M)$. $\Rightarrow$ Since $T$ may be considered as a $(q-1)$-form, with distribution coefficients

$T_P - T_\psi = \quad dT_\eta$

Then the equation above becomes

$$dT_\eta - T_{d\eta} = T_P,$$

which is a residue formula of the sort discussed above.

Pf $\varphi \in A_c^q(M-\Gamma)$

$\Rightarrow T_P(\varphi) = \quad 0 \quad \Rightarrow$

$(T_\psi + dT_\eta)(\varphi) = 0 \quad \Rightarrow (T_\psi + T_{d\eta})(\varphi) = 0$

$\Rightarrow \psi = -d\eta$ on $M - \Gamma$. ∎

Before doing this, we note that if $M$ is a complex manifold, then we also have the complex $(\mathcal{D}^{p,*}(M), \bar\partial)$ of currents of type $(p,q)$. We will also prove that the map

$$H_{\bar\partial}^{p,*}(M) \longrightarrow H^*(\mathcal{D}^{p,*}(M), \bar\partial)$$

is an isomorphism. Since both proofs are essentially the same, we will do the complex case.

Let $\mathcal{D}^{p,q}$ be the sheaf of currents of type $(p,q)$. Then there is a complex of sheaves