On the other hand, since projective modules are free, for M projective we have

$$0 \longrightarrow P \longrightarrow Q$$

$$\Downarrow$$

$$0 \longrightarrow M \otimes_{\mathcal{O}} P \longrightarrow M \otimes_{\mathcal{O}} Q .$$

┌ By P212. Th. 5.7, Algebra by Hungerford and by Th 5.11. on P215. Algebra by Hungerford. See Ex 8(b) on P217, Algebra by Hungerford. ┘

Consequently, both the functors $\mathrm{Hom}_{\mathcal{O}}(M, \cdot)$ and $M \otimes_{\mathcal{O}} \cdot$ are exact for M projective. We shall explore this systematically in the next discussion.

Homological Algebra

We begin by remembering a series of definitions, most of which are probably familiar from algebraic topology.

(a) A complex is given by either

$$(K_\cdot) \qquad \rightarrow K_n \xrightarrow{\partial} K_{n-1} \xrightarrow{\partial} \cdots \qquad \partial^2 = 0$$

or

$$(K^\cdot) \qquad \rightarrow K^n \xrightarrow{\delta} K^{n+1} \xrightarrow{\delta} \cdots \qquad \delta^2 = 0 .$$

Here the K's will always be finitely generated $\mathcal{O}$-modules and maps $\mathcal{O}$-module homomorphisms, although most of the present discussion goes over to modules