

locus of lines in G through x . Since V_2 is a α -plane through x , $V_2 \subset T_x(G) \cap G$.

\Rightarrow For any x_1, x_2, x_3 , $V_2 = \langle x_1, x_2, x_3 \rangle$,

$G \cap T_{x_i}(G) \supset V_2$, $i=1, 2, 3$.

$\Rightarrow G \cap T_{x_1}(G) \cap T_{x_2}(G) \cap T_{x_3}(G) = (G \cap T_{x_1}(G)) \cap (G \cap T_{x_2}(G))$

$\cap (G \cap T_{x_3}(G)) \supset V_2 \Rightarrow V_2 \subset \tilde{\phi}(\sigma_1(l_{x_1})) \cap \tilde{\phi}(\sigma_1(l_{x_2})) \cap \tilde{\phi}(\sigma_1(l_{x_3}))$

$= \{x \in G \mid l_{x_1} \cap l_{x_i} \neq \emptyset, i=1, 2, 3\}$.

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But the line $\overline{x_i x_j}$ lies in $V_2 \subset G$, and so the corresponding lines l_{x_i} and l_{x_j} must have a point p_{ij} in common.

$\square \quad \overline{x_i x_j} \subset G \iff l_{x_i} \cap l_{x_j} \neq \emptyset \Rightarrow p_{ij} = l_{x_i} \cap l_{x_j}$

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Since by hypothesis x_1, x_2 , and x_3 do not all lie on a Schubert cycle $\sigma_{2,1}(p, h)$, we must have either

1. p_{12}, p_{23} , and p_{13} are distinct, in which case a line $l \subset \mathbb{P}^3$ will meet l_{x_1}, l_{x_2} , and l_{x_3} , if and only if l lies in the hyperplane $h = \overline{p_{12}, p_{23}, p_{31}} = \overline{l_{x_1}, l_{x_2}, l_{x_3}}$; or

2. $p_{12} = p_{23} = p_{13}$, in which case, since l_{x_1}, l_{x_2} , and l_{x_3} can not be coplanar, a line $l \subset \mathbb{P}^3$ will meet l_{x_1} ,