Applying $\otimes \mathcal{L}^{-k}$ to this exact sequence and setting $\mathcal{G} = \mathcal{G}'(-k)$, we obtain

$$0 \to \mathcal{G} \to \mathcal{E}_0 \to \mathcal{F} \to 0.$$

Γ $0 \to \mathcal{G}' \otimes \mathcal{L}^{-k} \to \mathcal{O}^{(p)} \otimes \mathcal{L}^{-k} \to \mathcal{F}(k) \otimes \mathcal{L}^{-k} \to 0.$

since $\mathcal{L}^{-k}$ is locally free $\mathcal{O}$-sheaf of rank 1. See P697.
(Consider them at stalk.)

$\mathcal{G}' \otimes \mathcal{L}^{-k} = \mathcal{G}'(-kH) = \mathcal{G}'(-k)$. $\mathcal{F}(k) \otimes \mathcal{L}^{-k} = \mathcal{F} \otimes \mathcal{L}^{k} \otimes \mathcal{L}^{-k} = \mathcal{F}$. $\mathcal{E}_0 = \mathcal{O}^{(p)} \otimes \mathcal{L}^{-k}$. locally free. ┘

Now apply the same procedure to $\mathcal{G}$, and keep on going. After at most $n = \dim_{\mathbb{C}} M$ steps we arrive at a local syzygy

$$(*) \qquad 0 \to \mathcal{E}_n \to \mathcal{E}_{n-1} \to \cdots \to \mathcal{E}_0 \to \mathcal{F} \to 0,$$

where each $\mathcal{E}_l$ is locally free

Γ Since $\mathcal{G}$ is coherent, $\mathcal{O}^{(q)} \to \mathcal{G} \to 0$ locally.
Point is , globally, not locally.

$\Rightarrow$ By Theorem A again, for large l,
$H^0(M, \mathcal{G}(l))$ generates each $\mathcal{O}_x$-module $\mathcal{G}(l)_x$.

$\Rightarrow \quad 0 \to \mathcal{G}'' \to \mathcal{O}^{(q)} \to \mathcal{G}(l) \to 0.$

Apply $\otimes \mathcal{L}^{-l}$. and we get an exact sequence

$$0 \to \mathcal{G}''' \to \mathcal{E}_1 \to \mathcal{G} \to 0, \quad \mathcal{E}_1 \text{ locally free}$$

Thus we have

$$0 \to \mathcal{G}''' \to \mathcal{E}_1 \to \mathcal{E}_0 \to \mathcal{F} \to 0.$$

Continue this, then

$$0 \to \mathcal{G}_0 \to \mathcal{E}_n \to \mathcal{E}_{n-1} \to \cdots \to \mathcal{E}_0 \to \mathcal{F} \to 0.$$