$\psi \otimes \eta$ actually $= \psi \wedge \eta$ (we used previously)

$$- *_{M\times N}\, \overline{\partial_M}\, *_{M\times N} (\psi \otimes \eta) = - *_{M\times N}\, \overline{\partial_M}\, ((*_M \psi) \otimes (*_N \eta))\, (-1)^{(p+q)(p'+q')}$$
$$= - *_{M\times N} (\overline{\partial_M} *_M \psi) \otimes (*_N \eta)\, (-1)^{(p+q)(p'+q')}$$
$$= (-1)\; *_M (\overline{\partial_M} *_M \psi) \otimes *_N^2 \eta\, (-1)^{(p+q)(p'+q')} (-1)^{(m-p+m-q+1)(n-p'+n-q')}$$
$$= (-1)^{p'+q'}\, \overline{\partial_M}^* \psi \otimes \eta\, (-1)^{(p+q)(p'+q') + (p+q-1)(p'+q')}$$
$$= \overline{\partial_M}^* \psi \otimes \eta. \quad \text{adjoint of } \overline{\partial_M} \text{ only on } M.$$

$$- *_{M\times N}\, \overline{\partial_N}\, *_{M\times N} (\psi \otimes \eta) = - *_{M\times N}\, \overline{\partial_N}\, ((*_M \psi) \otimes (*_N \eta))\, (-1)^{(p+q)(p'+q')}$$
$$= - *_{M\times N} (*_M \psi) \otimes (\overline{\partial_N} *_N \eta)\, (-1)^{(p+q)(p'+q')} (-1)^{(m-p+m-q)}$$
$$= - \; *_M^2 \psi \otimes *_N \overline{\partial_N} *_N \eta\, (-1)^{(p+q)(p'+q') + p+q} (-1)^{(2m-p-q)(n-p'+n-q'+1)}$$

adjoint of $\overline{\partial_N}$ only on N

$$= *_M^2 \psi \otimes \overline{\partial_N}^{*\prime} \eta \quad (-1)^{(p+q)(p'+q') + p+q + (p+q)(p'+q'-1)}$$
$$= *_M^2 \psi \otimes \overline{\partial_N}^* \eta = (-1)^{p+q} \psi \otimes \overline{\partial_N}^* \eta.$$

$$\Rightarrow \quad \overline{\partial_M}^* \overline{\partial_M} + \overline{\partial_M}\, \overline{\partial_M}^* (\psi \otimes \eta)$$
$$= \overline{\partial_M}^* (\overline{\partial_M} \psi \otimes \eta) + \overline{\partial_M} (\overline{\partial_M}^{*\prime} \psi \otimes \eta)$$
$$= \overline{\partial_M}^* \overline{\partial_M} \psi \otimes \eta + \overline{\partial_M}\, \overline{\partial_M}^* \psi \otimes \eta = \Delta_M \psi \otimes \eta$$

$$\overline{\partial_N}^* \overline{\partial_N} + \overline{\partial_N}\, \overline{\partial_N}^* (\psi \otimes \eta)$$
$$= \overline{\partial_N}^* \overline{\partial_N} (\psi \otimes \eta) + \overline{\partial_N}\, \overline{\partial_N}^* (\psi \otimes \eta)$$
$$= (-1)^{p+q} \overline{\partial_N}^* (\psi \otimes \overline{\partial_N} \eta) + \overline{\partial_N} (\psi \otimes \overline{\partial_N}^* \eta)\, (-1)^{p+q}$$
$$= (-1)^{2(p+q)} (\psi \otimes \overline{\partial_N}^* \overline{\partial_N} \eta) + (\psi \otimes \overline{\partial_N}\, \overline{\partial_N}^* \eta)\, (-1)^{2(p+q)}$$
$$= \psi \otimes \overline{\partial_N}^* \overline{\partial_N} \eta + \psi \otimes \overline{\partial_N}\, \overline{\partial_N}^* \eta = \psi \otimes \Delta_N \eta. \;\lrcorner$$

Δ_M. Δ_N are Laplacians on M & N only
$\otimes$ can be considered as $\wedge$.

Now we come to the main point. If $\psi_1, \psi_2 \cdots$ are a complete set of eigenforms for Δ_M and η_1, η_2