⋮

for v_k. $n-(n-k+k-a_k)+1+k-1$ entries are specified.

$$\Rightarrow \sum_{i=1}^{k} n-(n-k+i-a_i)+1+(i-1)$$

$$= \sum_{i=1}^{k} (k-i+a_i)+1+i-1 = \sum_{i=1}^{k} k + a_i$$

$$= k^2 + \sum_{i=1}^{k} a_i \text{ entries are determined.}$$

$$\Rightarrow (n-k)k + k^2 - k^2 - \sum_{i=1}^{k} a_i = (n-k)k - \sum_{i=1}^{k} a_i.$$

Thus

$$W_{a_1 \cdots a_k} \cong \mathbb{C}^{k(n-k)-\sum a_i} \underset{\substack{\uparrow \\ \text{homeomorphism}}}{\cong} D^{2\{k(n-k)-\sum a_i\}}$$

Since we have cells only in even dimensions, all boundary maps are zero, and we deduce the

Proposition. The integral homology of the Grassmannian $G(k,n)$ has no torsion and is freely generated by the cycles $\sigma_{a_1 \cdots a_k} = [\overline{W_{a_1 \cdots a_k}}]$ in real codimension $2\sum a_i$, where $\{(a_1 \cdots a_k)\}$ ranges over all nonincreasing sequences of integers between 0 and $n-k$. In particular, all cohomology in $G(k,n)$ is analytic.

Γ $W_{a_1 \cdots a_k} \underset{\substack{\uparrow \\ \text{homeo}}}{\cong} D^{2\{k(n-k)-\sum a_i\}}$

All cohomology in $G(k,n)$ is represented by an analytic subvariety $\overline{W}$ in $G(k,n)$. $\Rightarrow$ We say that all cohomology is analytic.