

Two  $n$ -fold extensions are equivalent if there is a map of  $n$ -fold extensions

$$\begin{array}{ccccccc} 0 & \rightarrow & M' & \rightarrow & M_{n-1} & \rightarrow & \dots \rightarrow M_0 \rightarrow M \rightarrow 0 \\ & & \parallel & & \downarrow & & \downarrow \parallel \\ 0 & \rightarrow & M' & \rightarrow & M'_{n-1} & \rightarrow & \dots \rightarrow M'_0 \rightarrow M \rightarrow 0. \end{array}$$

To make this relation an equivalent relation, we need to do more work. For details, see Homology by S. MacLane P82 ~ P90. In Theorem 6.4 he showed

$\text{Ext}^n(C, A) \cong$  Set of all "congruence" classes  $\alpha =$  class of  $n$ -fold exact sequence  $S$  starting  $A$  and ending at  $C$ . Let's stop here on the Yoneda pairing.

$$\begin{array}{c} 0 \rightarrow M' \rightarrow M_{n-1} \rightarrow \dots \rightarrow M_0 \rightarrow M \rightarrow 0 \\ \quad \quad \quad \downarrow \\ 0 \rightarrow N \rightarrow N_{n-1} \rightarrow \dots \rightarrow N_0 \rightarrow M \rightarrow 0 \end{array} \quad \begin{array}{c} 0 \rightarrow M' \rightarrow M_{n-1} \rightarrow \dots \rightarrow M_0 \rightarrow M \rightarrow 0 \\ \quad \quad \quad \downarrow \\ 0 \rightarrow N \rightarrow N_{n-1} \rightarrow \dots \rightarrow N_0 \rightarrow M \rightarrow 0 \end{array}$$

Yoneda pairing

2. Applying the propagation principle; if  $\mathcal{F}, \mathcal{G}, \mathcal{H}$  are coherent sheaves on  $M$ , then there is

$$\underline{\text{Ext}}^p_{\mathcal{O}}(\mathcal{F}, \mathcal{G}) \otimes_{\mathcal{O}} \underline{\text{Ext}}^q_{\mathcal{O}}(\mathcal{G}, \mathcal{H}) \rightarrow \underline{\text{Ext}}^{p+q}_{\mathcal{O}}(\mathcal{F}, \mathcal{H})$$

inducing the previous pairing in each stalk.

Why do we need the propagation principle here?

Suppose we have  $\mathcal{F}, \mathcal{G}$ .

Consider  $\bigcup_{x \in M} \text{Ext}^p(\mathcal{F}_x, \mathcal{G}_x) = K$ .

$\Rightarrow$  To  $K$  be a sheaf, we need the following condition, according to Hartshorne's Algebraic Geometry P60