ideal. From the exact sequence

$$0 \longrightarrow I \longrightarrow \mathcal{O} \longrightarrow \mathcal{O}/I \longrightarrow 0$$

and computation of Ext's in the section on Koszul complexes we have

$$\mathrm{Ext}^1_{\mathcal{O}}(I, \mathcal{O}) \cong \mathrm{Ext}^2_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) \cong \mathcal{O}/I.$$

⌜ By 686, we have

$$\mathrm{Ext}^1_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) \to \mathrm{Ext}^1_{\mathcal{O}}(\mathcal{O}, \mathcal{O}) \to \mathrm{Ext}^1_{\mathcal{O}}(I, \mathcal{O}) \to \mathrm{Ext}^2_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O})$$
$$\to \mathrm{Ext}^2_{\mathcal{O}}(\mathcal{O}, \mathcal{O}).$$

$\Rightarrow$ Since $\mathrm{Ext}^1_{\mathcal{O}}(\mathcal{O}, \mathcal{O}) = \mathrm{Ext}^2_{\mathcal{O}}(\mathcal{O}, \mathcal{O}) = 0$ by P686,

$\mathrm{Ext}^1_{\mathcal{O}}(I, \mathcal{O}) \cong \mathrm{Ext}^2_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) \cong \mathcal{O}/I$ by P690, proposition. ⌟

The second isomorphism depends on the choice of generators for I, but the assertion:

$$e \in \mathrm{Ext}^1_{\mathcal{O}}(I, \mathcal{O}) \cong \mathcal{O}/I \text{ is a unit} - \text{i.e., } e(0) \neq 0,$$

has intrinsic meaning, since if also $I = \{f_1', f_2'\}$, then

$$f_i' = \sum_j a_{ij} f_j \quad \text{and} \quad e = \Delta e',$$

where $\Delta = \det(a_{ij})$ is a unit.