We may take homogeneous coordinates $X = [X_0, X_1, \cdots X_n]$ on $\mathbb{P}^n$ such that

$$\mathbb{P}^{n-k-1} = (X_0 = \cdots = X_k = 0), \qquad \mathbb{P}^k = (X_{k+1} = \cdots = X_n = 0),$$

$$\mathbb{P}^{n-k-2} = (X_0 = \cdots = X_{k+1} = 0), \qquad \mathbb{P}^{k+1} = (X_{k+2} = \cdots = X_n = 0);$$

in terms of these coordinates, π is given as before and

$$\pi'([X_0, X_1, \cdots X_n]) = [X_0, X_1, \cdots X_{k+1}]$$

so that π is just the composition of π' with projection from the point $(X_0 = \cdots = X_k = X_{k+2} = \cdots = X_n = 0)$ in $\mathbb{P}^{k+1}$ onto $\mathbb{P}^k$.

$$(X_{k+2} = \cdots = X_n = 0)$$

$$\begin{tikzcd}
 & & \mathbb{P}^{k+1} \ni [X_0, \cdots X_{k+1}] \arrow[dd] \\
{[X_0, X_1, \cdots X_n]} \in V \arrow[urr, "\pi'"] \arrow[drr, "\pi"'] & & \\
 & & \mathbb{P}^k \ni [X_0, X_1, \cdots X_k]
\end{tikzcd}$$

Note that, $\mathbb{P}^{n-k-2}$ having been chosen generically, the map π' will be one-to-one over an open set in its image: this will be the case as long as for some point p in V the $(n-k-1)$-plane $\overline{\mathbb{P}^{n-k-2}, p}$ meets V only at p — but for any p in V, the generic $(n-k-1)$-plane through p meets V only at p.