Of course this formal computation is not correct, because in applying Stokes' theorem the singularities of the kernel along the diagonal come into the picture. Referring to the previously established Bochner–Martinelli formula,

$$\eta(0) = C_n \int_{\mathbb{C}^n} \bar\partial \eta(\zeta) \wedge \frac{\sum \overline{\Phi_i(\zeta)} \wedge \Phi(\zeta)}{\|\zeta\|^{2n}}$$

for $\eta \in C_c^\infty(\mathbb{C}^n)$, it seems pretty clear that the correction term that must be added to the right-hand side of $(*)$ is just the identity. This may be proved by writing everything out and using the Bochner–Martinelli formula, but since it is completely straightforward to carry out the computation, we will not do it here.

Γ

$$\lim_{\epsilon\to 0} \int_{\mathbb{C}^n\times\mathbb{C}^n - \{(z,w)\in\mathbb{C}^n\times\mathbb{C}^n :\ w\in B(z,\epsilon)\}} d\left(\psi(z) \wedge k(z,w) \wedge \varphi(w) \right)$$

$$= \lim_{\epsilon\to 0} \int_{\mathbb{C}^n\times \|w-z\|=\epsilon} \psi(z) \wedge k(z,w) \wedge \varphi(w) = \lim_{\epsilon\to 0} \int_{\mathbb{C}^n} \psi(z) \wedge \int_{\|w-z\|=\epsilon} k(z,w)\wedge \varphi(w)$$

$$= \int_{\mathbb{C}^n} \psi(z) \wedge \lim_{\epsilon\to 0} \int_{\|w-z\|=\epsilon} k(z,w)\wedge\varphi(w)$$

$$\lim_{\epsilon\to 0} \int_{\|w-z\|=\epsilon} k(z,w)\wedge\varphi(w) = \lim_{\epsilon\to 0} \int_{\|w-z\|=\epsilon} \frac{\sum \overline{\Phi_i(z-w)} \wedge \Phi(w)}{\|z-w\|^{2n}} \wedge \varphi(w) =$$