It follows that for any $x, x' \in G$,

$$l_x \cap l_{x'} \neq \phi \Leftrightarrow x' \in T_x(G)$$
$$\Leftrightarrow \overline{x, x'} \subset G.$$

「 $\tilde{\phi}: G(2,4) \longrightarrow \mathbb{P}^5$ is the Plucker embedding.
$l_x \cap l_{x'} \neq \phi \Rightarrow x' \in T_x(G) \cap G$ by the argument above. $\Rightarrow x' \in T_x(G)$. Here $\tilde{\phi}(l_{x'}) = x'$ and the authors use $\tilde{\phi}(\sigma_1(l_x)) = \sigma_1(l_x)$ in terms of notations.
$x' \in T_x(G) \Rightarrow$

$x' \in T_x(G) \cap G$, by the assumption $x' \in G$
$\Rightarrow$ By the result above, $x' \in \sigma_1(l_x) = T_x(G) \cap G$
$\Rightarrow$ By the definition of $\sigma_1(l_x)$, $l_{x'} = \tilde{\phi}^{-1}(x')$ meets with l_x. $\Rightarrow l_{x'} \cap l_x \neq \phi$. Thus $l_x \cap l_{x'} \neq \phi \Leftrightarrow x' \in T_x(G)$.
If $\overline{x, x'} \subset G$, then obviously $\overline{x, x'} \subset T_x(G)$, and $x' \in T_x(G)$. Conversely, $x' \in T_x(G) \Rightarrow \overline{x, x'} \subset T_x(G)$.
$\Rightarrow x' \in T_x(G) \cap G = \sigma_1(l_x) \Rightarrow \sigma_{2,1}(p, h) \supset \overline{x, x'}$, and $\overline{x, x'} \subset \sigma_{2,1}(p, h) \subset T_x(G) \cap G \Rightarrow \overline{x, x'} \subset G$ //
」

We see from this that

Any line L lying on the Grassmannian is a Schubert cycle $\sigma_{2,1}(p, h)$.