and we can evaluate this intersection number by

$$\#(\Delta\cdot\Gamma_f) = \int_{\Gamma_f}\varphi_\Delta = \sum_p (-1)^{n-p}\int_{\Gamma_f}\sum_\mu \pi_1^*\psi_{\mu,p}\wedge\pi_2^*\psi^*_{\mu,n-p};$$

since $\tilde{f}^*\pi_2^* = f^*$, this is

$$= \sum_p (-1)^{n-p}\int_M \sum_\mu \psi_{\mu,p}\wedge f^*\psi^*_{\mu,n-p}$$

$$= \sum_p (-1)^{n-p}\cdot \operatorname{trace}(f^*|_{H^{n-p}_{DR}(M)})$$

$$= \sum_p (-1)^p \operatorname{trace}(f^*|_{H^p_{DR}(M)}).$$

$\Gamma_{\tilde{f}}(M) = \Gamma_f\cap\Delta(M)\ni(p,p)$ To be transverse at $(p,p)$, $\{\Delta_*(\frac{\partial}{\partial x_1}), \cdots \Delta_*(\frac{\partial}{\partial x_n}), \tilde{f}_*(\frac{\partial}{\partial x_1}), \cdots \tilde{f}_*(\frac{\partial}{\partial x_n})\}$ must generate $T_{(p,p)}(M\times M)$. As we computed above, the set of vectors is expressed as the matrix

$\begin{pmatrix} I_n, & I_n \\ I_n, & J_f(p)\end{pmatrix}$ in terms of the basis $(\frac{\partial}{\partial y_1}, \cdots \frac{\partial}{\partial z_1}\cdots)$.

$\Rightarrow$

$\det\begin{pmatrix} I_n, & I_n \\ I_n, & J_f(p)\end{pmatrix}\neq 0 \Leftrightarrow \Gamma_f$ and $\Delta$ intersect transversely at $(p,p)$.

$$\det\begin{pmatrix} I, & I \\ I, & J_f(p)\end{pmatrix} = \det\begin{pmatrix} I, & I-I \\ I, & J_f(p)-I\end{pmatrix} = \det\begin{pmatrix} I, & 0 \\ I, & J_f(p)-I\end{pmatrix}$$

$$= \det\begin{pmatrix} I, & 0 \\ I, & J_f(p)-I\end{pmatrix} = \det(J_f(p)-I)$$