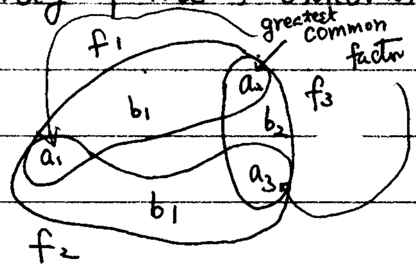
3. Last, let $V \subset U \subset \mathbb{C}^n$ be an analytic variety irreducible at $0 \in V$ such that for arbitrary small nbds Δ of 0 in $\mathbb{C}^n$, $\pi(V \cap \Delta)$ contains a nbd of 0 in $\mathbb{C}^{n-1}$. Write

$$V = \{ f_1(z) = \cdots = f_k(z) = 0 \}$$

near 0. Then the functions $f_i \in \mathcal{O}_n$ must all have a common factor in $\mathcal{O}_n$, since otherwise V would be contained in the common locus of two relatively prime functions, and by assertion 2, $\pi(V \cap \Delta)$ would be a proper analytic subvariety of $\mathbb{C}^{n-1}$.

For simplicity, let $k = 3$. see p268 note!
$V = \{ f_1(z) = f_2(z) = f_3(z) = 0 \}$. Suppose f_1, f_2, f_3 relat. prime.
If f_1 & f_2 are relatively prime, then $V \subset \{ f_1 = f_2 = 0 \}$ and it is done. If f_1 & f_2 are not relatively prime, then let h be a greatest common divisor of f_1 & f_2. $\Rightarrow$ h & f_3 must be relatively prime. $\Rightarrow$ $V \subset \{ h = f_3 = 0 \}$, for if $z \in V \Rightarrow f_1(z) = f_3(z) = 0$ Wrong!

For simplicity, let $k = 3$.
$V = \{ f_1 = f_2 = f_3 = 0 \}$. Suppose any of them are not relatively prime, otherwise done. Use a diagram.



$f_1 = a_1 b_1 a_2$

$f_2 = a_1 b_1 a_3$

$f_3 = a_2 b_2 a_3$