Γ If we defined $e_K, \bar{e}_K, \bar{\iota}_K, \iota_K$, as before,

$$L(\eta) = \eta \wedge \omega. \Rightarrow$$

$$L = \frac{i}{2} \sum e_K \bar{e}_K.$$

i.e. $e_K(\varphi_I \wedge \bar{\varphi}_J) = \varphi_K \wedge \varphi_I \wedge \bar{\varphi}_J$

$\bar{e}_K(\varphi_I \wedge \bar{\varphi}_J) = \bar{\varphi}_K \wedge \varphi_I \wedge \bar{\varphi}_J$

and $\iota_K, \bar{\iota}_K$ are adjoints of $e_K, \bar{e}_K$ respectively.

Thus $\Lambda = -\frac{i}{2} \sum \bar{\iota}_K \iota_K$. exactly same as the previous Λ.

What about $\bar{\partial}$, ∂^*?

$$\bar{\partial}(f \varphi_I \wedge \varphi_{\bar{J}}) = \bar{\partial} f \wedge \varphi_I \wedge \varphi_{\bar{J}} + f\, \partial(\varphi_I \wedge \varphi_{\bar{J}})$$

$= \bar{\partial} f \wedge \varphi_I \wedge \varphi_{\bar{J}}$ + terms involving $d\varphi_i$'s.

$= \bar{v}_K(f)\, \varphi_K \wedge \varphi_I \wedge \varphi_{\bar{J}}$. + terms involving $d\varphi_i$'s.

Define ∂_K by $\partial_K(f \varphi_I \wedge \varphi_{\bar{J}}) = v_K(f)\, \varphi_I \wedge \varphi_{\bar{J}}$.

" $\bar{\partial}_K$ by $\bar{\partial}_K(f \varphi_I \wedge \varphi_{\bar{J}}) = \bar{v}_K(f)\, \varphi_I \wedge \varphi_{\bar{J}}$.

$$\langle -\bar{\partial}_K \eta, \psi\, d\varphi_L \wedge d\bar{\varphi}_M \rangle = (-\bar{v}_K(\eta_{L\bar{M}})\, d\varphi_L \wedge d\bar{\varphi}_M, \psi\, d\varphi_L \wedge d\bar{\varphi}_M \rangle = \int_M -\bar{v}_K(\eta_{L\bar{M}})\, \bar{\psi} \overset{X}{=} \int_M \eta_{L\bar{M}}\, \bar{v}_K(\bar{\psi})$$

$\cancel{\partial^* f \varphi_I \wedge \bar{\varphi}_J =}$

$\cancel{- * \partial * f \varphi_I \wedge \bar{\varphi}_J = - * \partial(\bar{f} \varphi_{I^\circ} \wedge \bar{\varphi}_{J^\circ})\, \epsilon}$

$\cancel{= -\epsilon * v_K(\bar{f})\, \bar{\varphi}_K \wedge \varphi_{I^\circ} \wedge \bar{\varphi}_{J^\circ} - * \epsilon}$ terms involving $d\varphi_i$'s

$\cancel{= -\epsilon \tau v_K(\bar{f})}$

$$\partial^*(f \varphi_I \wedge \bar{\varphi}_J) = - * \partial * (f \varphi_I \wedge \bar{\varphi}_J) = - * \partial \bar{f}\, \varphi_{I^\circ} \wedge \bar{\varphi}_{J^\circ}\, \epsilon$$

$= - * \epsilon\, v_K(\bar{f})\, \varphi_K \wedge \varphi_{I^\circ} \wedge \bar{\varphi}_{J^\circ} + - * \epsilon$ terms involving $d\varphi_i$'s

$$\Rightarrow \partial^*(f \varphi_I \wedge \bar{\varphi}_J)_{z_0} = - * \epsilon\, v_K(\bar{f})\, \varphi_K \wedge \varphi_{I^\circ} \wedge \bar{\varphi}_{J^\circ}\big|_{z_0} + 0.$$