$$({}^tX\,,\ {}^tY)\begin{pmatrix}A & D\\ C. & B\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}$$

$$= ({}^tXA + {}^tYC,\ \ {}^tXD + {}^tYB)\begin{pmatrix}X\\ Y\end{pmatrix}$$

$$= {}^tXAX + {}^tYCX + {}^tXDY + {}^tYBY$$
$$= {}^tXAX + {}^tYBY + {}^tYCX + {}^tXDY$$

Let $X = X_+$. $Y = -X_+$.

$$\Rightarrow ({}^tX_+, -{}^tX_+)\begin{pmatrix}A & D\\ C & B\end{pmatrix}\begin{pmatrix}X_+\\ -X_+\end{pmatrix}$$

$$= {}^tX_+AX_+ + {}^tX_+B\,{}^tX_+ - {}^tX_+CX_+ + {}^tX_+D(-X_+)$$
$$= {}^tX_+(A+B)X_+ - {}^tX_+(C+D)X_+ \quad \cdots ①$$

$\Rightarrow$ Since X_+ is positive eigenvect for $C+D$ and $A+B$ is negative defite,

$${}^tX_+(A+B)X_+ - {}^tX_+(C+D)X_+ \leq 0. \quad (\text{strictly inequal})$$

Similary,

$${}^tX_0(A+B)X_0 - {}^tX_0(C+D)X_0 < 0 \quad \cdots ② \quad \text{since } {}^tX_0(C+D)X_0 = 0$$

In case $X = X_-$, take $Y = X_-$

$$\Rightarrow {}^tX_-(A+B)X_- + {}^tX_-(C+D)X_- < 0 \quad \cdots ③.$$

From ① & ② & ③, $\begin{pmatrix}A & D\\ C & B\end{pmatrix}$ is negative defte on the vector space V spanned by $\left\{\begin{pmatrix}X_+\\ -X_+\end{pmatrix}\ \begin{pmatrix}X_0\\ -X_0\end{pmatrix}\ \begin{pmatrix}X_-\\ X_-\end{pmatrix}\right\}$