of T_Δ of bitype $(0,*),(n,n-*)$. //

Recall from the subsection " Definitions; Residue Formulas" in Section 1 of this chapter the Bochner-Martinelli kernel on $\mathbb{C}^n \times \mathbb{C}^n$ is given by

$$k(z,\zeta) = C_n \frac{\overline{\sum \Phi_i(z-\zeta) \wedge \Phi(\zeta)}}{\|z-\zeta\|^{2n}},$$

where
$$\begin{cases} \Phi_i(x) = (-1)^{i-1} x_i\, dx_1 \wedge \cdots \wedge \widehat{dx_i} \wedge \cdots \wedge dx_n, \\ \Phi(x) = dx_1 \wedge \cdots \wedge dx_n. \end{cases}$$

This form has bitype $(0,*-1),(n,n-*)$ in the variable $(dz, d\bar{z})(d\zeta, d\bar{\zeta})$. Also, from $\partial \overline{\Phi_i(z-\zeta)} = \overline{\Phi(z-\zeta)}$ it follows that

$$\bar\partial k(z,\zeta) = 0 \quad \text{on } \mathbb{C}^n \times \mathbb{C}^n - \Delta,$$

so that the current defined by $k(z,\zeta)$ has distributional derivative $\bar\partial k$ supported on the diagonal.

$\ulcorner$ $\partial \Phi_i(z-\zeta) = \partial (-1)^{i-1}(z_i-\zeta_i)\, d(z_1-\zeta_1) \wedge \cdots \wedge \widehat{d(z_i-\zeta_i)} \wedge \cdots \wedge d(z_n-\zeta_n)$

$= (-1)^{i-1}(dz_i - d\zeta_i) \wedge d(z_1-\zeta_1) \wedge \cdots \wedge \widehat{d(z_i-\zeta_i)} \wedge \cdots \wedge d(z_n-\zeta_n)$

since $dz_i - d\zeta_i = \partial(z_i-\zeta_i)$

$\Rightarrow \overline{\partial \Phi_i(z-\zeta)} = \overline{\Phi(z-\zeta)}$. //

In fact, the homotopy formula proved in the subsection " Cohomology of Currents" is equivalent to the distributional equation

$$\bar\partial k = T_\Delta^0,$$

giving the desired " smoothing" of T_Δ^0 in $\mathbb{C}^n \times \mathbb{C}^n$.