is a quadric.

⌟

The map $\tilde{f}_L$ is best understood by cases: for each point $r \in V_3$, let $V_2(r)$ be the 2-plane spanned by r and L, and write

$$G \cdot V_2(r) = L + L_1, \quad F \cdot V_2(r) = L + L_2.$$

⌜ $G \cap V_2(r) \supset L$, and since $G \cdot V_2(r)$ is a conic, $G \cdot V_2(r) = L + L_1$. Similarly, $F \cdot V_2(r) = L + L_2$.

⌟

There are then a number of possibilies:

1. Generically, L, L_1 and L_2 are all distinct, and L_1 meets L_2 at a point $p \in X$ not on L. (See Figure 26.) In this case $V_2(r)$ will not be tangent to X anywhere along L, so

$$\tilde{f}_L^{-1}(r) = \{p\}.$$

⌜ By Bertini's theorem, for generic hyperplane $H \supset L$, $H \cap X$ is smooth away from L. $\Rightarrow$ For generic r, $V_2(r) \cap X$ is smooth away from L $\Rightarrow$ $V_2(r) \cap X - L$ is a set of points. $\Rightarrow$ If $L_1 = L_2$, since $V_2(r) \cap X$ $= L + L_1$,