$\Rightarrow$ Consider $a_1 b_1 \quad b_2$ & $a_2 b \; a_3$. Let $g_1 = a_1 b_1 \quad b_2$ & $g_2 = f_3$ $\Rightarrow$ Clearly, g_1 & g_2 are relatively prime. $V \ni z \Rightarrow z \in \{g_1 = g_2 = 0\}$ for $z \in f_1^{-1}(0) \cap f_2^{-1}(0)$. Wrong Again.

$V = \{f_1 = f_2 = f_3 = 0\}$.

$\Rightarrow$ Without loss of generality, we may assume that $(f_1 = 0)$, $(f_2 = 0)$ & $(f_3 = 0)$ does not contain the z_n-axis. $\Rightarrow$ We may assume that f_1, f_2 & f_3 are Weierstrass polynomials in z_n. Suppose f_1 & f_2 are relatively prime. $\Rightarrow$ $f_1 = h g_1$, $f_2 = h g_2$, where g_1 & g_2 are relatively prime and $g_1, g_2 \in \mathcal{O}_{n-1}[z_n]$.

$\Rightarrow$ By P343, Whitney Th. 8B, $\exists \; \alpha(z_n) \; \beta(z_n)$ s.t $\alpha g_1 + \beta g_2 = \gamma \in \mathcal{O}_{n-1}$.

$\Rightarrow$ Consider $\alpha f_1 + \beta f_2$ & f_3.

$\Rightarrow$ If h & f_3 are relatively prime, then $\alpha f_1 + \beta f_2$ & f_3 are relatively prime since $\alpha f_1 + \beta f_2 = h(\alpha g_1 + \beta g_2) = h\gamma$ and γ & f_3 are relatively prime ($\because \; f_3(z_n) = z_n^{\ell} + \cdots$). Thus we can conclude that either $\alpha f_1 + \beta f_2$ & f_3 are relatively prime or that f_1, f_2 & f_3 have a common factor.

Note: Suppose that $(f=0)$ does not contain the z_n-axis. $\Rightarrow$ $\exists$ an open set $\mathcal{U} \ni [z_n]$ in $G(1,n)$ s.t any element $x \in \mathcal{U}$ does not lie in $(f=0)$ by Weierstrass Preparation Theorem.