ω and i^* is the restriction to X, $i^*\omega$ is the generator of $H^2(X, \mathbb{Z})$.

$\ulcorner$ $S \hookrightarrow X \hookrightarrow G \hookrightarrow \mathbb{P}^5$

By the definition on P171, $[S] \in H_4(\mathbb{P}^5) \Rightarrow [S] = d\,[H^3]$.

$\Rightarrow$ Since $H^2(X) \cong H^2(\mathbb{P}^5)$, and the generator of $H^2(X)$ is $i^*\omega$, where ω is the hyperplane class,

$[S] = n\,[X \cap H]$ for some $n \in \mathbb{Z}_+$. by Poincare duality. The degree of $X \cap H$ in $\mathbb{P}^5$ is 4 by the result on P550, i.e., $[X \cap H] = 4\,[H^3]$ in $H_4(\mathbb{P}^5)$.

$C_1^2 = H \cdot H = \deg S = H|_S \cdot H|_S = \#((H_1 \cap S) \cap (H_2 \cap S))$
$= \#((H_1 \cap H_2) \cap Q \cap Q') = \#((Q \cap H_1) \cap (Q \cap H_2)) = 4$, since $Q \cap H_1$ & $Q \cap H_2$ are quadric surface in $\mathbb{P}^3 (\in H_1, H_2)$, $Q \cap H_1 = (f = 0)$ $Q \cap H_2 = (g = 0)$. f, g quadric polynomials. and $Q \cap H_1 \sim 2H$, $Q' \cap H_2 \sim 2H$. where H is the hyperplane in $(H_1, H_2 =)\ \mathbb{P}^3 \subset \mathbb{P}^4$ $\lrcorner$

Note that this argument may be used in general to show that a smooth nondegenerate complete intersection of dimension in $\mathbb{P}^N$ can not contain a linear subspace of dimension $> n/2$.

$\ulcorner$ Def: A r-dimensional variety Y in $\mathbb{P}^n$ is a complete intersection if Y can be written as the intersection of $n-r$ hyper surfaces.