

$$\delta(a^*, b^*, c^*) = \delta(a, b; c) \stackrel{?}{=} 0 \text{ if either}$$

1. $c_i^* < a_i^*$ or $c_i^* < b_i^*$ or
2. $l(c^*) < l(a^*)$ or $l(c^*) < l(b^*)$

$$\Leftrightarrow \begin{array}{l} 1. \quad l(c) < l(a) \text{ or } l(c) < l(b) \text{ or } \\ 2. \quad c_i < a_i \text{ or } c_i < b_i. \end{array} \quad (\text{true}) \quad \text{))}$$

We turn now to the original problem of computing $\delta(a, b; c)$ for general a, b, c . We will first give a reduction that allows us to compute effectively in many cases.

Our basic technique is simply a linear algebra reduction to smaller Grassmannians. For example, consider a triple of indices $\alpha, \beta, \gamma, s, t$ $\alpha + \beta + \gamma = 2k + 1$. Then for any k -plane $\Lambda \in \sigma_a(V) \cap \sigma_b(V') \cap \sigma_c(V'')$

$$\dim(\Lambda \cap V_{n-k+\alpha-a\alpha}) \geq \alpha.$$

$$\dim(\Lambda \cap V'_{n-k+\beta-b\beta}) \geq \beta$$

$$\dim(\Lambda \cap V''_{n-k+\gamma-c\gamma}) \geq \gamma.$$

$$\Rightarrow \dim(\Lambda \cap V_{n-k+\alpha-a\alpha} \cap V'_{n-k+\beta-b\beta} \cap V''_{n-k+\gamma-c\gamma}) \geq 1.$$

$$\text{Let } \Lambda \cap V_{n-k+\alpha-a\alpha} = A, \quad \Lambda \cap V'_{n-k+\beta-b\beta} = B.$$

$$\Lambda \cap V''_{n-k+\gamma-c\gamma} = C.$$

$$\Rightarrow \dim(A \cap B \cap C) = \dim(A \cap B) + \dim(B \cap C) + \dim(A \cap C) \\ - \dim A - \dim B - \dim C + \dim(A+B+C).$$

$$= \dim A + \dim B - \dim(A+B) + \dim B + \dim C - \dim(B+C)$$

$$+ \dim A + \dim C - \dim(A+C) - \dim A - \dim B - \dim C$$

$$+ \dim(A+B+C) = \dim A + \dim B + \dim C - \dim(A+B)$$