┌ Consider a linear system $\{l_x \cdot S\}_{x \in L}$ on $S \cap h_L$.
$\Rightarrow$ By Bertini, for generic $x \in L$, $l_x \cap S - p_L$ is a set of distinct points. $\Rightarrow$ Since l_x must be tangent to S, l_x is tangent to S at p_L.
$\Rightarrow$ Since generic l_x is tangent to S at p_L, h_L is tangent to S at p_L.

┘

Definition. A line $L \subset X$ is called special if, equivalently,

1. $\dim(\bigcap_{x \in L} T_x(X)) = 2$; or

2. the locus $T_x(X) \cap X$ of lines in X through a generic point $x \in L$ consists of fewer than four lines;

3. $h_L = T_{p_L}(S)$, i.e. all the lines $\{l_x\}_{x \in L}$ are tangent to S at p_L.

┌ By P979 note, $1 \Leftrightarrow 2$. $2 \Leftrightarrow 3$ by the argument above on P791 & P792.

┘

┌ " $T_x(X) \neq T_y(X)$ for $x \neq y \in L$ "

If not, then we assume $G = (\sum X_i^2 = 0)$, $F = (\sum \lambda_i X_i^2 = 0)$ by P789, $x = (x_0, \cdots x_5)$ $y = (y_0, \cdots y_5)$