extension is precisely ν. Moreover, the discriminant $d_f \in {}_n\mathcal{O}_W$ of the polynomial $P_f(X)$ is not identically zero on any connected component; thus, multiplying by d_f describes an isomorphism ${}_V\mathcal{O}_V \longrightarrow d_f \cdot {}_V\mathcal{O}_V$ of ${}_n\mathcal{O}_W$-modules.

More precisely,

$$\begin{array}{cccc} d_f : & {}_V\mathcal{O}_V & \longrightarrow & d_f \cdot {}_V\mathcal{O}_V \subset \pi^*({}_n\mathcal{O}_W)[f] \\ & \rotatebox{90}{$\in$} & & \\ & g & \longmapsto & Q_{f,g}(f) = d_f \cdot g \end{array}$$

$\Rightarrow$ If $d_f \cdot g = 0$, since $d_f \cdot {}_V\mathcal{O}_V \subset {}_V\mathcal{O}_V$ which is integral domain and $d_f \neq 0$, g must be 0. $\Rightarrow$ d_f is injective. Clearly $(\pi^* h) g \longmapsto \pi^* h \, Q_{f,g}(f)$

$$\begin{array}{c} \pi^* h \, Q_{f,g}(f) \\ \| \\ Q_{f,(\pi^* h)\cdot g}(f) \\ \| \\ \pi^* h (d_f \cdot g) \in \pi^*({}_n\mathcal{O}_W)(f) \end{array}$$

Since $\pi^* h \, Q_{f,g}(X) \in \pi^*({}_n\mathcal{O}_W)[X]$, ($\because$ $\pi^*({}_n\mathcal{O}_W)[f]$ is naturally an $\pi^*({}_n\mathcal{O}_W)$-module).

Finally Theorem 6 also shows that whenever $g \in {}_V\mathcal{O}_V$, then the product $d_f \cdot g$ can be expressed as a linear combination of the elements $1, f, \cdots f^{\nu-1}$ with coefficients from ${}_n\mathcal{O}_W$ ($= \pi^* {}_n\mathcal{O}_W$). This latter module is actually a free module of rank ν, since any nontrivial linear relation among the generators $1, f, \cdots f^{\nu-1}$ would yield a nontrivial polynomial relation $P(f) = 0$ for $P(X) \in {}_n\mathcal{O}_W[X]$ of degree less than ν, and as already noted there can be no such polynomial relation. That suffices to conclude the proof.

9. Corollary. If $\pi : V \longrightarrow \mathbb{C}^n$ is the germ of a finite branch