Maybe we misunderstood the filtration $F^pH^*(K^*)$.

$$\begin{array}{ccccccc} K^{q-1} & \longrightarrow & K^q & \longrightarrow & K^{q+1} & \longrightarrow & \cdots \\ \cup & & \cup & & \cup & & \\ F^pK^{q-1} & \longrightarrow & F^pK^q & \longrightarrow & F^pK^{q+1} & & \end{array}$$

$$F^pH^q(K^*) \equiv \frac{\{a \in F^pK^q ;\ da = 0\}}{d(K^{q-1})}$$ is acceptable.

If we use this definition,

$$\frac{F^pH^{p+q}(K^*)}{F^{p+1}H^{p+q}(K^*)} = \left(\frac{\{a \in F^pK^{p+q};\ da=0\}}{d(K^{p+q-1})} \right) \Big/ \left(\frac{\{a \in F^{p+1}K^{p+q};\ da=0\}}{d(K^{p+q-1})} \right)$$

$$= \frac{\{a \in F^pK^{p+q};\ da=0\} + dK^{p+q-1}}{\{a \in F^{p+1}K^{p+q};\ da=0\} + dK^{p+q-1}}$$

$$\downarrow h$$

$$\frac{\{a \in F^p:\ da=0\} + dK^{p+q-1} + F^{p+1}K^{p+q}}{dK^{p+q-1} + F^{p+1}K^{p+q}}.$$

$\Rightarrow$ Clearly h is well-defined. and onto.

Suppose $h[a] = 0$.

$\Rightarrow\ a = dl + x,\quad l \in K^{p+q-1},\quad x \in F^{p+1}K^{p+q}$

$\Rightarrow\ [dl] = 0,\quad x = a - dl \in F^{p+1}K^{p+q}$ and

$dx = da - ddl = 0 \Rightarrow x \in \{a \in F^{p+1};\ da=0\}$.

$\Rightarrow [x] = 0 \Rightarrow [a] = 0 \Rightarrow h$ is one to one

$\Rightarrow h$ is isomorphic $\Rightarrow$ This implies that

$F^pH^{q+p}(K^*)/F^{p+1}H^{p+q}(K^*) \cong \{a \in F^pK^{p+q};\ da=0\}/$