

$\Rightarrow \overline{f^{-1}(W)}$ (the closure of $f^{-1}(W)$ in M) is an analytic subvariety of M . $\Rightarrow V \cup \overline{f^{-1}(W)} = V \cup f^{-1}(W)$ ($\because f^{-1}(W)$ has ^{every} limit point ^{except itself} in V) is an analytic subvariety of M .

Two things should be noted:
 $\overline{f^{-1}(U_y - W)}^\circ = f^{-1}(U_y - W)$ is of measure 0. & perhaps $f^{-1}(U_y)$ contains an $\mathcal{O} - V$, $\mathcal{O} \cong \Delta$ in M , \mathcal{O} a nbhd of a point of V .

The total transform $D \in \text{Div}(M)$
 $\Rightarrow D = \sum a_i V_i$, V_i an irreducible hypersurface in M
 $\Rightarrow P \subset M \times N$
 $\downarrow \pi_1$ $\Rightarrow \pi_1^{-1}(V_i)$ is an algebraic subvariety of P , since $M \times N \subset \mathbb{P}^n \times \mathbb{P}^m$.

\Rightarrow By the proper mapping theorem, $\pi_2(\pi_1^{-1}(V_i))$ is an analytic subvariety of N . $\dim \pi_1^{-1}(V_i) = \dim M - 1 = \dim N - 1$
 $\Rightarrow \dim \pi_2(\pi_1^{-1}(V_i)) = \dim M - 1 = \dim N - 1$, since P is some sort of diagonal in $M \times N$ ($\because M \stackrel{f}{\rightarrow} N$ are birational)
 $\Rightarrow f: M \rightarrow N$ is generically one to one. \square

The reader may verify that the total transform map preserves linear equivalence while the proper transform map does not.

\square Since $f: M \rightarrow N$ is generically one to one,
 $P \xrightarrow{\pi_1} M$ & $P \xrightarrow{\pi_2} N$ are generically one to one.