

that may be obtained from  $\mathbb{P}^2$  by a series of blow-ups and blow downs.

## 2. Rational Maps

### Rational and Birational Maps

One of the basic geometric operations on algebraic varieties  $V \subset \mathbb{P}^n$  is the projection

$$\pi_p: V \longrightarrow \mathbb{P}^{n-1}$$

of a variety  $V$  from a point  $p \in \mathbb{P}^n$  not lying on  $V$  to a hyperplane. In Chapter 2 we saw that, if  $V$  is a curve, the map  $\pi_p$  is well-defined even in case  $p$  lies on  $V$ :

$\pi_p$ , defined a priori only on  $V - \{p\}$ , may be extended by mapping  $p$  to the image in  $\mathbb{P}^{n-1}$  of the tangent line to  $V$  at  $p$ .

¶ See P216 (Chapter 2), and refer to P497 =)

In general, however, if  $V$  has dimension greater than one and  $p \in V$ , the map  $\pi_p$  is not well-defined at, nor can it be extended over, the point  $p$ . This is not hard to see: for any point  $q \in \mathbb{P}^{n-1} \cap T_p(V)$  in the image of the tangent plane to  $V$  at  $p$  there is a sequence  $\{q_i\}$  of points on  $V - \{p\}$  tending to  $p$ , such that  $\pi_p(q_i)$  tends to  $q$ .