

$$\begin{aligned} \left\langle \sum \frac{\partial \bar{z}_k}{\partial \bar{w}_i} \frac{\partial}{\partial \bar{z}_k}, \sum \frac{\partial \bar{z}_l}{\partial \bar{w}_j} \frac{\partial}{\partial \bar{z}_l} \right\rangle &= \delta_{ij} \\ &= \sum \frac{\partial \bar{z}_k}{\partial \bar{w}_i} \overline{\frac{\partial \bar{z}_l}{\partial \bar{w}_j}} \left\langle \frac{\partial}{\partial \bar{z}_k}, \frac{\partial}{\partial \bar{z}_l} \right\rangle = \sum \frac{\partial \bar{z}_k}{\partial \bar{w}_i} \overline{\frac{\partial \bar{z}_l}{\partial \bar{w}_j}} h_{kl} \end{aligned}$$

$$\begin{aligned} \text{let } \left( \frac{\partial \bar{z}_k}{\partial \bar{w}_i} \right) &= b_{ki} \Rightarrow = \sum b_{ki} \overline{b_{lj}} h_{kl} \\ &= \sum (b^t)_{ik} h_{kl} \overline{b_{lj}} \\ &= (B^t h \bar{B})_{ij} = \delta_{ij} \end{aligned}$$

$$\Rightarrow B^t h \bar{B} = I. \quad \frac{\partial \bar{w}_i}{\partial \bar{z}_j} = a_{ji}$$

$$\begin{aligned} \text{and } \frac{\partial \bar{w}_i}{\partial \bar{w}_j} = \delta_{ij} &= \sum \frac{\partial \bar{w}_i}{\partial \bar{z}_k} \frac{\partial \bar{z}_k}{\partial \bar{w}_j} = \sum a_{ki} b_{kj} \\ &= ({}^t A B)_{ij} \end{aligned}$$

$$\Rightarrow {}^t A B = I. \quad \Rightarrow {}^t A = B^{-1}. \quad \Rightarrow B = ({}^t A)^{-1}.$$

$$A^{-1} h {}^t A^{-1} = I \Rightarrow h = A {}^t \bar{A}$$

To get  $B$ , apply the Gram-Schmidt process to  $\left\{ \frac{\partial}{\partial \bar{z}_k} \right\}$  and get  $A$ .  $\Rightarrow$

$$\boxed{h_{ij} = \sum_k \delta_{ij} + a_{ij,k} \bar{z}_k + \overline{a_{ij,k}} \bar{z}_k + [1]},$$

$$\begin{aligned} h_{ij} = \overline{h_{ji}} \Rightarrow \delta_{ij} + a_{ij,k} \bar{z}_k + \overline{a_{ij,k}} \bar{z}_k + [1] &= \overline{\delta_{ji} + a_{ji,k} \bar{z}_k + \overline{a_{ji,k}} \bar{z}_k + [1]} \\ &= \overline{a_{ji,k}} \bar{z}_k + [1] \end{aligned}$$

$$\Rightarrow \overline{a_{ji,k}} = a_{ij,k}$$

$$dw = 0 \Rightarrow \sum_{i,j,k} a_{ij,k} d\bar{z}_k + \overline{a_{ij,k}} d\bar{z}_k + d[2]$$

$$= \sum_{i,j,k} a_{ij,k} d\bar{z}_k + \overline{a_{ij,k}} d\bar{z}_k \quad \wedge \quad d\bar{z}_i \wedge d\bar{z}_j = 0$$

$$\begin{aligned} \Rightarrow \sum_{i,j,k} a_{ij,k} d\bar{z}_k \wedge d\bar{z}_i \wedge d\bar{z}_j &= - \sum_{i,j,k} \overline{a_{ij,k}} d\bar{z}_k \wedge d\bar{z}_i \wedge d\bar{z}_j = 0 \\ &\Downarrow \quad a_{ij,k} = a_{kji} \end{aligned}$$