

$H^0(M, \mathcal{O}_M(L)) \longrightarrow H^0(V, \mathcal{O}_V(L))$
 was surjective, i.e., that $H^1(M, \mathcal{O}_M(L-V)) = 0$.

□

$$\begin{array}{ccc}
 H^0(M, \mathcal{O}_M(L)) & \longrightarrow & H^0(M, L_x \oplus L_y) \\
 \text{onto} \downarrow & \curvearrowright & \nearrow \text{"onto" should be proved} \\
 H^0(V, \mathcal{O}_V(L)) = H^0(M, \mathcal{O}_V(L)) & & \\
 \downarrow & & \\
 H^1(M, \mathcal{O}_M(L-V)) = 0 & \text{(should be proved)} & \quad \square
 \end{array}$$

But this is very nearly presupposing the result to be proved: a priori, M need not have any divisors on it at all.

□ I think that they are talking about choosing a smooth hypersurface $V \ni x, y$, and $H^1(M, \mathcal{O}_M(L-V)) = 0$ i.e., M must have divisors satisfying some conditions. □

It is clear by now that our difficulty lies in the simple fact that, unless M is a Riemann surface, a point on M is not a divisor. We can overcome this problem by means of a beautiful classical construction called blowing up, which transforms points on a complex manifold into divisors.

Blowing Up.

We will first describe the blow-up of the origin in a disc Δ in \mathbb{C}^n . Let $Z = (Z_1, \dots, Z_n)$ be Euclidean