

$$[a_3-3, a_2-3]$$

$$[a_d-d, a_{d+1}-d]$$

$$[-(d+1), a_d-(d+1)]$$

$$\dots a_4-4 < a_4-3 \leq a_3-3 < a_3-2 \leq a_2-2 \leq \overset{a_1-2 <}{\bigvee} a_1-1 \leq n-k$$

As we saw above, $d-1$ number of $c_i - i$'s are in those intervals, which are disjoint each other. "More closer look."

$$\text{Consider } [a_j - j, a_{j+1} - j] = A$$

$$[a_{j+1} - (j+1), a_j - (j+1)] = B.$$

$$\Rightarrow c_j - j \in [a_{j+1} - (j+1), a_{j+1} - j]$$

which contains A & B.

Except $c_j - j$, the rest are in one of those intervals. \Rightarrow We can conclude that at most one of $c_i - i$'s lies in each of the $(d+1)$ - closed disjoint intervals.

\Rightarrow We see that ^{exactly} one of those intervals will fail to contain an integer $c_i - i$. For, if $c_j - j \notin A \cup B$, since $c_j - j \in [a_{j+1} - (j+1), a_{j+1} - j]$, $c_j - j$ must be in $(a_j - (j+1), a_j - j)$,

$$\Rightarrow a_j - (j+1) < c_j - j < a_j - j$$

$\Rightarrow a_j - 1 < c_j < a_j \Rightarrow$ impossible because a_j & c_j are integers.

)