boundaries, $\{\sigma_\alpha^k, \tau_\alpha^k\}$ a basis for the cycles and $\partial \mu_\alpha^k = \sigma_\alpha^{k-1}$.

Now let $\{\sigma_\alpha'^k, \tau_\alpha'^k, \mu_\alpha'^k\}$ be a similarly constructed basis for the chains of N, and let A be a cycle in $M \times N$, expressed as a linear combination of the product of the basis elements in M and N.

Since the products $\sigma_\alpha^k \times \sigma_\beta'^l$, $\sigma_\alpha^k \times \tau_\beta'^l$ and $\tau_\alpha^k \times \sigma_\beta'^l$ are the boundaries of $\mu_\alpha^{k+1} \times \sigma_\beta'^l$, $\mu_\alpha^{k+1} \times \tau_\beta'^l$, and $(-1)^k \tau_\alpha^k \times \mu_\beta'^{l+1}$, respectively, we may, after replacing A with a homologous cycle, assume that no such terms appear in the expression for A. Also, if a term

$\sigma_\alpha^k \times \mu_\beta'^l$ appears in A, we may remove it by subtracting from A the boundary

$$\partial(\mu_\alpha^{k+1} \times \mu_\beta'^l) = \sigma_\alpha^k \times \mu_\beta'^l + (-1)^{k+1} \mu_\alpha^{k+1} \times \sigma_\beta'^{l-1}$$

Thus we can write

$$A = \sum a_{\alpha\beta kl} \tau_\alpha^k \times \tau_\beta'^l + \sum b_{\alpha\beta kl} \tau_\alpha^k \times \mu_\beta'^l + \sum c_{\alpha\beta kl} \mu_\alpha^k \times \tau_\beta'^l$$
$$+ \sum d_{\alpha\beta kl} \mu_\alpha^k \times \mu_\beta'^l + \sum e_{\alpha\beta kl} \mu_\alpha^k \times \sigma_\beta'^l.$$

Taking the boundary, we have

$$0 = \partial A = \sum b_{\alpha\beta kl} \tau_\alpha^k \times \sigma_\beta'^{l-1} (-1)^k + \sum c_{\alpha\beta kl} \sigma_\alpha^{k-1} \times \tau_\beta'^l$$
$$+ \sum d_{\alpha\beta kl} \sigma_\alpha^{k-1} \times \mu_\beta'^l + \sum d_{\alpha\beta kl} (-1)^k \mu_\alpha^k \times \sigma_\beta'^{l-1}$$
$$+ \sum e_{\alpha\beta kl} \sigma_\alpha^{k-1} \times \sigma_\beta'^l +$$

Now all the terms in the sum are linearly independent, and so each must be zero.: thus
$b_{\alpha\beta kl} = c_{\alpha\beta kl} = d_{\alpha\beta kl} = e_{\alpha\beta kl} = 0$ for each $\alpha\, \beta .\, k\, l$.