

$$\Rightarrow \left| Q^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \lambda I \right|$$

$$= \left| \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right|$$

$$= \left| \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \end{pmatrix} + \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = \left| \begin{pmatrix} \lambda + a_{11} & 0 & 0 \\ a_{21} & \lambda & 0 \\ a_{31} & 0 & \lambda \end{pmatrix} \right|$$

$$= \lambda^2 (\lambda + a_{11}) = 0 \Rightarrow \lambda = 0 \text{ is a double root \& } \lambda = -a_{11}$$

At $\lambda = 0$ & $-a_{11}$, L has singular points, i.e. singular conic. \Rightarrow At $\lambda = 0$, i.e. F , $m_F(L, W_1) = 2$ since $\lambda = 0$ is a double root. At $\lambda = -a_{11}$, L has a singular cubic. By the argument above, L is the set of all conics passing p & p' and tangent to G at those two points, and if we let l & l' be the tangent lines of G at p & p' respectively, then $l + l'$ is a conic passing p & p' , and tangent to G at those two points. $\Rightarrow l + l' \in L \Rightarrow l + l'$ is the only singular conic in L other than F . \square

The reader may check that the tangent space