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$\Rightarrow \frac{\partial r}{\partial w}(z^*, w_1(z^*)) = 0$ r has a double zero at z^*
$\Rightarrow$ This proves that r has roots more than deg r.
$\Rightarrow$ This is possible only if $r=0$. $\Rightarrow$ $g = hf_1$

The general statement is:

For U a sufficiently small nbd of the origin and $f = (f_1, \cdots f_n) : U \longrightarrow \mathbb{C}^n$ a holomorphic mapping, the conditions

1. $f^{-1}(0) = \{0\}$,
2. codim $\{f_{i_1}(z) = \cdots = f_{i_k}(z) = 0\} = k$,
3. $f_1 \cdots f_n$ is a regular sequence

are all equivalent.

Since we shall be discussing regular sequences in detail in the section on Kozul complexes and shall give another proof of local duality there, we shall let our discussion in the case $n=2$ suffice for the moment.

The second result in local analytic geometry is the nullstellensatz for the ideal $\{f_1, \cdots f_n\}$:

There exists $k_i > 0$ such that
$$z_i^{k_i} \in \{f_1, \cdots f_n\}.$$

We will prove this in the next section on finite holomorphic mappings.