$$\frac{\partial F}{\partial X_i}(p_{ij}) = q_{i,} = 0 \quad \text{for all } i = 0, \cdots \hat{n}, n+1$$

$$\Rightarrow \quad F\big|_{T_{p_i}(F) \cap T_{p_{ij}}(F)} = F\big|_{\mathbb{P}^{n-1}} \quad \text{has } Q$$

nonsingular.

$$= \left(\begin{array}{c|c|ccc|c|c}
0 & 0 & \cdot & \cdot & \cdot & 0 & q_{0,n+1} \\ \hline
0 & 0 & & & & q_{1n} & 0 \\ \hline
 & & * & \cdots & * & & \\
 & & \vdots & n \times n & & & \\
 & & * & & * & & \\ \hline
0 & \underset{q_{n1}}{q_{1n}} & 0 & \cdots & & & \\ \hline
\underset{q_{n+1,0}}{q_{0,n+1}} & 0 & & & & &
\end{array}\right)$$

$$\Rightarrow \quad F\big|_{\mathbb{P}^{n-3}_{ij}} = \sum_{i,j \geq 2}^{n-1} q_{ij} X_i X_j \quad \text{smooth in } \mathbb{P}^{n-3} \Rightarrow F \cap \mathbb{P}^{n-3}_{ij} = F_{ij}$$
is smooth.

$$F\big|_{\mathbb{P}^{n-1}} = \sum_{i,j=0}^{n-1} q_{ij} X_i X_j.$$ has a line as the singular set.

$\Rightarrow$ By p734 ⊛, $F \cap T_{p_i}(F) \cap T_{p_{ij}}(F)$ is the cone through $\overline{p_i p_{ij}}$ over F_{ij}.

$\lrcorner$

The third condition on $\sigma_{n-k,n-k-1,\cdots}$ says that any $\Lambda \in \Sigma_{k,n} \cap \sigma_{n-k,n-k-1,\cdots}$ meets the 5-plane $\overline{V_6}$ in a 2-plane.

$\ulcorner$ Since $\dim(\Lambda \cap V_6) \geq 3$, $\Lambda \cap \overline{V_6} \supseteq \mathbb{P}^2$.