$\Rightarrow$ Q has eigenvales $\pm \delta_j i$, $j = 1, 2 \cdots n$.
$\Rightarrow$ Clearly, $\{\pm \delta_j i\}$ is invariant. $\Rightarrow$ $\{\delta_j\}$ is invariant, for we have a different and right argument below.

First of all, we will prove the following lemma.

Lemma: M module over a ring R, and L_1, L_2 module homomorphisms from M to M. ψ, ϕ are isomorphisms of M. And if $\phi \circ L_1 = L_2$. or $L_2 = L_1 \circ \psi$.
then $$M/\mathrm{im}\,L_1 \cong M/\mathrm{im}\,L_2.$$

pf). Define $\Phi : M/\mathrm{im}\,L_1 \longrightarrow M/\mathrm{im}\,L_2$ by

$$m + \mathrm{im}\,L_1 \longmapsto \phi(m) + \mathrm{im}\,L_2.$$

$$\phi(m + L_1(x)) = \phi(m) + \phi \circ L_1(x) = \phi(m) + L_2(x)$$

$\Rightarrow$ Φ is well-defined, and clearly Φ is onto.
$\phi(m) \in \mathrm{im}\,L_2$ $\Rightarrow$ $\phi(m) = L_2(y)$ $\Rightarrow$ $L_2(y) = \phi \circ L_1(y)$
$\Rightarrow$ Since ϕ is one to one, $m = L_1(y)$ $\Rightarrow$ Φ is one to one $\Rightarrow$ Φ is isomorphic.
Similarly, in case $L_2 = L_1 \circ \psi$,

$$\Psi : V/\mathrm{im}\,L_1 \longrightarrow V/\mathrm{im}\,L_2$$
$$v + \mathrm{im}\,L_1 \longmapsto v + \mathrm{im}\,L_2$$