$\Rightarrow$ By the argument above, we get $\phi\psi$, $\phi\varphi$ satisfying

Laplacian on M.

$$\int_{\mathbb{R}^n} \phi\psi\Delta\eta = \int_{\mathbb{R}^n} \phi\varphi\,\eta \quad \text{for all } \eta \in C_c^\infty(\mathbb{R}^n).$$

Note that $\forall \eta \in C_c^\infty(\mathbb{R}^n)$, η can be considered as an elmt. of $\overset{P.o}{\mathcal{H}}(M)$

Now it is clear that we have only to show the following:

constant

$$P = C\,\Delta + \sum a_k(x)\frac{\partial}{\partial x_k}$$

Euclidean Laplacian

$\Rightarrow$ If $\displaystyle\int_{\mathbb{R}^n} \psi(x)\, P(\eta)(x)\, dx = \int_{\mathbb{R}^n} \varphi(x)\,\eta(x)\, dx$ for all $\eta \in C_c^\infty(\mathbb{R}^n)$

, $\psi \in \mathcal{H}_0$ and $\varphi \in \mathcal{H}_s$, then $\psi \in \mathcal{H}_{s+2}$.

In complex case, it can be done, I think. I am sure 98%.

Proof. $\|Pu\|_0^2 + \|u\|_0^2 \geq C\|u\|_1^2$ $C > 0$. for u compactly supported. since we can use the Stokes' theorem

Let $\psi = u$.

We define the smoothing

$$u_\epsilon(x) = \int_{\mathbb{R}^n} u(y)\,\chi_\epsilon(x-y)\,dy$$

as above. The L^2-norm

$$\|u_\epsilon - u\|_0^2 \longrightarrow 0 \quad \text{as } \epsilon \to 0,$$

since the convergence $u_\epsilon \to u$ is uniform.