$C \cap \ell = F_\lambda \cap F_{\lambda'} \cap \ell$, since C is the base locus of $\{F_\lambda\}$, $\lambda \neq \lambda'$, for some λ'.

$F_\lambda \cap F_{\lambda'} \cap \ell = F_{\lambda'} \cap \ell$ consists of two points, otherwise, $F_{\lambda'} \supset \ell \Rightarrow \mathbb{P}^1 = \ell \subset C \Rightarrow$ Contradiction.

If $\#(\ell \cdot C) = 2$, i.e., $\ell \cap C = \{p, q\}$, ${}^\forall r \in \ell$ s.t $r \neq p, q$, then $\exists\ F_\lambda$ s.t $F_\lambda \ni r$, for

$$F_\lambda = F + \lambda G \qquad F_\lambda(r) = F(r) + \lambda G(r) = 0$$

$$\Rightarrow \lambda = -\frac{F(r)}{G(r)}.$$

$\Rightarrow$ We can choose F_λ s.t $F_\lambda \ni r$.

Since $F_\lambda \supset C$, $F_\lambda \ni p, q, r$. $\Rightarrow \#(F_\lambda \cap \ell) \geq 3$

$\Rightarrow F_\lambda \supset \ell$, since F_λ is a quadric. $\lrcorner$

This being established, it is easy to compute the class of $V_0(L) \subset G(2,4)$: First, since a generic hyperplane $H \subset \mathbb{P}^3$ meets C in four points, $\binom{4}{2} = 6$ chords of C will lie in H,

so

$$\#(V_0(L) \cdot \sigma_{1,1}) = 6.$$

$\ulcorner$ By the lemma on P249, any three points in $H \cap C$ are not collinear. $\Rightarrow \binom{4}{2} = 6$ chords of C lie in H. $\#(V_0(L) \cdot \sigma_{1,1})$ = the number of chords of C lying in a generic hyperplane $= 6$.

$\lrcorner$