

$$\begin{aligned}
\Rightarrow \psi'' - \frac{\psi''(u')_{-1}}{u_k} &= \sum_{j=-N}^{-2} \psi''(u')_j u_k^j + \sum_{j=0}^{\infty} \psi''(u')_j u_k^j \\
&= \frac{\partial}{\partial u_k} \left(\sum_{j=-N}^{-2} \psi''(u')_j \frac{u_k^{j+1}}{j+1} + \sum_{j=0}^{\infty} \psi''(u')_j \frac{u_k^{j+1}}{j+1} \right) \\
\eta &= \sum_{j=-N}^{-2} \psi''(u')_j \frac{u_k^{j+1}}{j+1} + \sum_{j=0}^{\infty} \psi''(u')_j \frac{u_k^{j+1}}{j+1}
\end{aligned}$$

$\Rightarrow \eta$ has one less order pole than ψ'' in u_k and η has the same order pole in u' .

If $k=1$, $u' = (u_0)$. \Rightarrow Since $H^q(\Delta^n, \Omega^p) = 0$, $q > 0$,
 $H^*(M, \mathbb{C}) \cong H_{DR}^*(M, \text{hol})$.

\Rightarrow The theorem is valid, i.e. φ closed holomorphic p -form on Δ^n , $\Rightarrow \exists \eta$ holo. s.t. $\varphi = d\eta$.

\Rightarrow By induction, we prove the theorem for $u' = (u_1)$
 done. \square

"Comment on k where $\varphi = \psi' + \psi'' \wedge du_k$.

Don't be confused with k in $P^*(k, n)$. "

Clearly, $\tilde{\varphi} = \varphi - d\eta = \xi' + \xi'' \wedge \frac{du_k}{u_k}$,
 where $\xi'' \equiv 0(u', du')$ and $\xi' \equiv 0(du')$.

$$\begin{aligned}
\Gamma \quad \eta &= \eta_I d^u_I, \quad I \subset \{1, 2, \dots, k-1\}. \\
d\eta &= \frac{\partial \eta_I}{\partial u_k} du_k \wedge du_I + \frac{\partial \eta_I}{\partial u_j} du_j \wedge du_I, \quad j=1, \dots, k-1. \\
\varphi - d\eta &= \psi' + \psi'' \wedge du_k - d\eta
\end{aligned}$$