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$H^q(\mathcal{U}, \mathcal{E}) = 0, \quad q>0.$

Use the induction on the length of a local syzygy.

$$0 \longrightarrow \mathcal{R} \longrightarrow \mathcal{O}^{(k)} \longrightarrow \mathcal{E} \longrightarrow 0$$

where $H^q(\mathcal{U}', \mathcal{R}) = 0, \quad q>0$ small open $\mathcal{U}' \cong \mathbb{C}^n$

$\Rightarrow$ By the exact sequence of cohomology,

$$\begin{array}{ccccccc} H^q(\mathcal{U}', \mathcal{R}) & \longrightarrow & H^q(\mathcal{U}', \mathcal{O}^{(k)}) & \longrightarrow & H^q(\mathcal{U}', \mathcal{E}) & \longrightarrow & H^{q+1}(\mathcal{U}', \mathcal{R}) \\ \| & & & & & & \| \\ 0 & & & & & & 0 \end{array}$$

$\Rightarrow$ $H^q(\mathcal{U}', \mathcal{O}^{(k)}) = H^q(\mathcal{U}', \mathcal{E}) = 0$ by P46

$$\| \quad \sum_k H^q(\mathcal{U}', \mathcal{O}) = \sum_k H^q(\mathbb{C}^n, \mathcal{O})$$

For each r, we have small $\mathcal{U}_r \cong \mathbb{C}^n$. s.t.

$$H^q(\mathcal{U}_r, \underline{Ext}_{\mathcal{O}}^r(\mathcal{F}, \mathcal{G})) = 0$$

since $\underline{Ext}_{\mathcal{O}}^r(\mathcal{F}, \mathcal{G})$ is coherent and $\underline{Ext}_{\mathcal{O}}^{n+l}(\mathcal{F}, \mathcal{G}) = 0$ for $l \geq 2$.

Let $\mathcal{U}$ be the smallest open set among $\mathcal{U}_r$'s.

$\Rightarrow$ $H^q(\mathcal{U}, \underline{Ext}_{\mathcal{O}}^{*}(\mathcal{F}, \mathcal{G})) = 0, \quad q>0.$

According to P706, ${}'E_2^{p,q} = H^p(\mathcal{U}, \underline{Ext}_{\mathcal{O}}^q(\mathcal{F}, \mathcal{G}))$

For $p \frown 0$, ${}'E_3^{p,q} = {}'E_2^{p,q}$ since

$$\begin{array}{ccccc} {}'E_2^{p-2,q+1} & \longrightarrow & {}'E_2^{p,q} & \longrightarrow & {}'E_2^{p+2,q-1} = H^{p+2}(\mathcal{U}, \underline{Ext}_{\mathcal{O}}^q(\mathcal{F}, \mathcal{G})) = 0 \\ \| & & & & \\ 0 & & & & \end{array}$$

$\cdots \Rightarrow \quad {}'E_3^{0,q} = {}'E_4^{0,q} = \cdots = {}'E_\infty^{0,q}$ for all q