F_{12}, F_{34}, F_{56} span a hyperplane H_{123456} ⌟

3. Recall from our discussion of Grassmannians in Section 6 of Chapter 1 that if L_1, L_2, L_3, L_4 are disjoint lines in $\mathbb{P}^3$, then there are exactly two lines $L, L' \subset \mathbb{P}^3$ meeting all four.

⌜ See p206 ⌟

Now if the lines L_i all lie on S, then the lines L and L' meet S in four points, hence must also lie on S. Thus for any four disjoint lines on a cubic surface S there will be exactly two other lines on S meeting all four.

⌜ Since all L_i's lie on S, $\#(L \cap S) \geq 4$ and $\#(L' \cap S) \geq 4$. $\Rightarrow$ Since S is a cubic, $L \subset S$, and $L' \subset S$. ⌟

We now want to show that every smooth cubic surface S in $\mathbb{P}^3$ can be obtained by blowing up $\mathbb{P}^2$ in six points. We first locate six exceptional divisors on S to blow down; to find these we look for the cohomology classes they represent. Let $S_0 = \tilde{\mathbb{P}}^2_{p_1 \cdots p_6}$ be the cubic surface constructed above, and let