

$$K \times \mathbb{P}^1 \subset \Delta \times \mathbb{P}^1. \quad \text{See p 395}$$

⌋

But by the calculation above the ratio f/g is constant on every connected component of $V-B$ and so V can meet only finitely many divisors D_λ away from the base locus of $\{D_\lambda\}$. Q.E.D

⌈ $V-B = \bigcup C_\alpha$, $C_\alpha \cap C_\beta = \emptyset$ if $\alpha \neq \beta$.
and C_α is connected.

$\Rightarrow \frac{f}{g}$ is constant on each C_α . If $\frac{f}{g} = \lambda \in \mathbb{C}$,

$C_\alpha \subset V_\lambda =$ locus of singular points in D_λ .

Suppose we have infinitely many C_α 's.

\Rightarrow Choose $a_\alpha \in C_\alpha$ for each α .

$\Rightarrow \{a_\alpha\}$ is infinite in the compact manifold M

$\Rightarrow \exists$ a limit a_0 . \Rightarrow Contradiction, since $\{C_\alpha\}$ is discrete. (See Weierstrass Preparation Theorem. \Rightarrow It tells $\{a_\alpha\}$ is discrete.)

$\{C_\alpha\}$ is supposed to be discrete. Why?

$V-B = \bigcup C_\alpha \Rightarrow$ Each C_α is open in $V-B$.
 \uparrow compact. \leftarrow closure in M .

$\overline{V-B} = \overline{\bigcup C_\alpha} \supset \bigcup \overline{C_\alpha} \Rightarrow \overline{C_\alpha} = C_\alpha$ since
if $\frac{f}{g}$ is constant on C_α , $\frac{f}{g}$ is constant on $\overline{C_\alpha}$ too.

$\Rightarrow \{C_\alpha\}$ is discrete.

⌋

The essential point here is that a pencil $\{D_\lambda\}_{\lambda \in \mathbb{P}^1}$ with base locus B gives a holomorphic mapping

$$M-B \longrightarrow \mathbb{P}^1$$

since by linearity every $p \in M-B$ is on a unique D_λ .