Π According to P138 ~ P139.
the sheaf of meromorphic sections of K_M with poles of order ≤ 1 on V is identified with $\mathcal{O}(\Lambda^n T^*M \otimes [V]) = \mathcal{O}(K_M \otimes [V])$.

Given $\omega \in \mathcal{O}(K_M \otimes [V]) = \Omega^n_M(V)$, ω can be considered as a meromorphic section (meromorphic n-form) with a single pole along V and holomorphic elsewhere.
(or no pole)

$\Rightarrow$ We can write $\omega = \dfrac{g(z)\, dz_1 \wedge \cdots \wedge dz_n}{f(z)}$,

where $z = (z_1 \cdots z_n)$ are local coordinates on M and V is given locally by $f(z)$.

Then by the isomorphism. $K_M|_V = K_V \otimes N_V^*$,

ω corresponds to the form ω' s.t

$$\omega = \omega' \wedge \frac{df}{f}.$$

$$\omega|_{U_\alpha} = \omega'|_{U_\alpha} \wedge \frac{df_\alpha}{f_\alpha} \quad \text{where} \quad f|_{U_\alpha} = f_\alpha$$
$$\omega'|_{U_\alpha} = \omega'_\alpha$$
$$\omega|_{U_\alpha} = \omega_\alpha.$$

$\Rightarrow \dfrac{df_\alpha}{f_\alpha} = \dfrac{g_{\alpha\beta}\, df_\beta}{g_{\alpha\beta}\, df_\beta} \Rightarrow \left(\dfrac{df_\alpha}{f_\alpha}\right)$ defines a global meromorphic section of N_V^*, where

and $\omega_\alpha = h_{\alpha\beta}\, \omega_\beta = h_{\alpha\beta}\, \omega'_\beta \wedge \dfrac{df_\beta}{f_\beta}$

$= h_{\alpha\beta}\, \omega'_\beta \wedge \dfrac{df_\alpha}{f_\alpha} = \omega'_\alpha \wedge \dfrac{df_\alpha}{f_\alpha}. \Rightarrow \omega'_\alpha = h_{\alpha\beta}\, \omega'_\beta$

where $g_{\alpha\beta}$, $h_{\alpha\beta}$ are transition functions for $[V]$ and K_V respectively.