But the projection map
$$\pi : V^* \longrightarrow \mathbb{C}^k$$
$$: (z_1, \cdots z_n) \longmapsto (z_1, \cdots z_k)$$

expresses V^* as a d-sheeted branched cover of $\Delta' = \pi(\Delta)$ and consequently

$$c \int_{V^* \cap \Delta} dz_I \wedge d\bar{z}_I \leq d \cdot c \int_{\Delta'} dz_I \wedge d\bar{z}_I < \infty.$$

Q.E.D.

⌈ By P14, the assertion 2, the definition of the multiplicity (on P22) and the statement above (V meets each of the coordinate $(n-k)$-planes ($z_{i_1} = z_{i_2} = \cdots = z_{i_k} = 0$) only in discrete points), $\pi : V^* \longrightarrow (z_{i_1} = z_{i_2} = \cdots = z_{i_k} = 0)$ expresses V^* as a d-sheeted branched cover.

$$c \int_{V^* \cap \Delta} 1 \; dz_I \wedge d\bar{z}_I \leq d \cdot c \int_{\Delta'} \pi^*(1 \; dz_I \wedge d\bar{z}_I)$$
$$\parallel$$
$$d \cdot c \int_{\Delta'} dz_I \wedge d\bar{z}_I < \infty \quad ⌋$$

Note again the contrast to the C^∞ case, where the set of manifold points of the zero locus of a smooth function — e.g., $f(y) = (e^{-y^{-2}} - 1) \sin(1/y)$ — need not have locally finite area.

⌈ $f(y) = (e^{-y^{-2}} - 1) \sin(1/y) = 0 \Rightarrow e^{-y^{-2}} = 1 = \frac{1}{e^{y^{-2}}}$
$\Rightarrow e^{y^{-2}} = 1 \Rightarrow y^{-2} = 0 \quad y = \infty \Rightarrow \exists$ no such y.
$\sin(1/y) = 0, \; \frac{1}{y} = n\pi \Rightarrow y = 1/n\pi$.