

L.

By P479,  $r$  is given as follows: Given a generic hyperplane  $\bar{h}$ , for any  $p \in S$ , consider the line  $\overline{p.p_0}$ .  
 $\Rightarrow \exists$  an intersection point  $p' \in \bar{h} \cap \overline{p.p_0} \Rightarrow r(p) = p'$ .  
 $\Rightarrow$  For each point  $p \in h \cap S = C_h$ ,  $r(p) = p'$  for some  $p' \in \bar{h}$ , and conversely, for any <sup>generic</sup> point  $q \in \bar{h} \cap h$ ,  $\overline{p_0 q} \cap S \neq \emptyset \Rightarrow r(\tilde{C}_h) = h \cap \bar{h} = \mathbb{P}^1$ .  $\#(\overline{p_0 q} \cap S - \{p_0\}) = 2$ , since  $p_0$  is a double point and  $\deg S = 4$ .  $\Rightarrow r: \tilde{C}_h \rightarrow \mathbb{P}^1$  is a 2-sheeted cover.  $\Rightarrow$  By the formula on P255, the number of branch points  $b = 2 \cdot 2 + 2 \cdot 2 - 2 = 6$ .  
 See P940 note.  $\square$

The branch locus  $F \subset \mathbb{P}^2$  of  $r$  is thus a sextic plane curve without multiple components.

For generic hyperplane  $H \subset \mathbb{P}^3$ ,  $H \cap F$  is the branch locus of  $\tilde{C}_H \rightarrow H \cap \mathbb{P}^2$ .  $\Rightarrow$  By the argument above,  $\#(H \cdot F) = 6$ . Since  $r$  is a 2-sheeted cover, there is no multiple branch point.  $\Rightarrow F$  is a variety of degree 6 in  $\mathbb{P}^2 = \text{im } r$ , see the definition of branch locus on P217.  $\square$