of $\mathbb{P}^2$ at p_2. In this case, we say that a curve C in the plane contains p_1 and p_2 if C passes through p_2 and the curve $\pi^{-1}(C) - E$ contains p_1, i.e., if either C is smooth at p_2 with tangent line corresponding to p_1, or C is singular at p_2. Thus, for example, we say that the points p_1, p_2 and p_3 are collinear if the proper transform in $\widetilde{\mathbb{P}^2}$ of the line $\overline{p_2 p_3}$ contains p_1. Of course, the linear condition imposed by p_1 on the system of cubics is defined only on the subsystem of cubics passing through p_2, but the independence of the conditions imposed by $p_1 \cdots p_8$ is still well-defined.

"I think: $\pi^{-1}(C) - E$ should be corrected to $\pi^{*}C - E$."

"Fixing p_1 is determining the slope of a curve at p_2."

" At p_2, C has an ordinary double point or a cusp.

The argument for the lemma in case p_1 is infinitely near p_2 runs as follows: as before, we first want to show that any seven points $p_1, \cdots p_7$, with p_1 infinitely near p_2, impose independent conditions on cubics unless five are collinear.

"It means " If five are not collinear, $p_1 \cdots p_7$ impose independent conditions on cubics." $\Leftrightarrow$ $p_1 \cdots p_7$ fail to impose independent conditions on cubics only if five are collinear."

Assuming that no five are collinear, we know from