pf) Let $int_{V^*}(V^* \cap H)$ = Set of interior points of $V^* \cap H$ in V^*. Assume that $int_{V^*}(V^* \cap H) \neq V^*$.

$\Rightarrow \exists$ a limit point p (in V^*) of $int_{V^*}(V^* \cap H)$ s.t

$p \notin H = (X_0 = 0)$. (We may assume $H = (X_0 = 0)$)

since $int_{V^*}(V^* \cap H)$ is open in V^*.

If not, every point in $V^* - int_{V^*}(V^* \cap H)$ is an element of $int_{V^*}(V^* \cap H) \cap H$. $\Rightarrow$ $V^* = V^* \cap H$ $\Rightarrow$ $V^* \subset V^* \cap H \subset V \cap H \subset V$ $\Rightarrow$ Since $V \cap H$ is closed & V^* is dense in V, $V = V \cap H$.

$$V^* = \{V^* - int_{V^*}(V^* \cap H)\} \cup int_{V^*}(V^* \cap H) \subset V^* \cap H \cup (V^* \cap H) = V^* \cap H$$

$\Rightarrow \exists$ U open in $\mathbb{P}^n$ s.t $U \ni p$, U connected.

$U \subset U_1 = (X_1 \neq 0) \cap U_0 = (X_0 \neq 0)$, without loss of generality.

Consider a holomorphic map $f: U \longrightarrow \mathbb{C}$

$$[X_0, X_1, \cdots X_n] \mapsto \frac{X_0}{X_1}.$$

Since p is a limit point of $int_{V^*}(V^* \cap H)$,

$U \cap int_{V^*}(V^* \cap H) \neq \phi$.

Since V^* is a submanifold of $\mathbb{P}^n$, we may assume that

$$\begin{array}{ccc} U \cap V^* & \cong & \mathbb{C}^k \\ \downarrow & & \downarrow \\ U & \cong & \mathbb{C}^n. \end{array}$$

$\Rightarrow$ If we consider f as a holomorphic function on $V^* \cap U$, then $f = 0$ on a nonempty open set of $V^* \cap U$. $\Rightarrow$ Since p is a limit of the nonempty open set of $V^* \cap U$, $f(p) = 0$. $\Rightarrow$ $H \ni p$ $\Rightarrow$