cases:

1. F meets $\sigma(p)$ transversely, i.e., X_p is a smooth conic curve.

$\ulcorner$ $F \cap \sigma(p) = F \cap G \cap \sigma(p) = X \cap \sigma(p)$ is a conic curve. $\lrcorner$

The locus of lines in X through p will then be a cone through p over a smooth conic curve (Figure 8). As we shall see, this is the generic case.

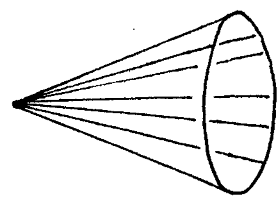

Figure 8

$\ulcorner$ $G(2,4) \xrightarrow{\tilde{\phi}} \mathbb{P}^5$ $\Rightarrow$ Since X_p in $\mathbb{P}^5$ is a smooth conic curve, $\tilde{\phi}^{-1}(X_p)$ is a smooth curve in $G(2,4)$. $\Rightarrow$ Assume that $p = [1,0,0,0]$.

$\Rightarrow$ We may express $\sigma(p) = \left\langle \begin{pmatrix} 1 & 0 & 0 & 0 \\ x_1, & x_2, & x_3, & x_4 \end{pmatrix} \right\rangle \subset G(2,4)$

On $(x_2 \neq 0)$, $\begin{pmatrix} 1, & 0 & 0 & 0 \\ 0, & 1 & \frac{x_3}{x_2}, & \frac{x_4}{x_2} \end{pmatrix} \Rightarrow \tilde{\phi}(\sigma(p)) = \langle e_1 \wedge (e_2 + \frac{x_3}{x_2} e_3 + \frac{x_4}{x_2} e_4) \rangle$

$= \{ [1, \frac{x_3}{x_2}, \frac{x_4}{x_2}, 0, 0, 0] \} = \{ [x_2, x_3, x_4, 0, 0, 0] \} = \tilde{\phi}(\sigma(p))$

$\Rightarrow$ By the result above, $\exists$ a quadratic homogeneous polynomial f in x_2, x_3, x_4. $(f=0)$ is a smooth curve

Thus, if we let $x_1 = 0$, $\tilde{\phi}^{-1}((f=0))$ in $\mathbb{P}^3$ is a cone through p over $(f=0)$. $\lrcorner$