$$0 \longrightarrow \mathcal{O}_M(-1) \xrightarrow{\sigma} \mathcal{O}_M \longrightarrow \mathcal{O}_H \longrightarrow 0,$$

where $\sigma \in H^0(M, \mathcal{O}(1))$ defines H and $\mathcal{O}_H$ is the usual structure sheaf on the complex manifold H. Applying $\otimes \mathcal{F}$ to this sequence gives ($\mathcal{O} = \mathcal{O}_M$)

$$0 \longrightarrow \underline{\mathrm{Tor}}_1^{\mathcal{O}}(\mathcal{O}_H, \mathcal{F}) \longrightarrow \mathcal{F}(-1) \longrightarrow \mathcal{F} \longrightarrow \mathcal{F}_H \longrightarrow 0,$$

where $\mathcal{F}_H = \mathcal{F} \otimes_{\mathcal{O}_M} \mathcal{O}_H$ is a coherent sheaf of $\mathcal{O}_H$-modules, and where we used that $\underline{\mathrm{Tor}}_1^{\mathcal{O}}(\mathcal{O}_M, \mathcal{F}) = 0$.

Γ

$$0 \longrightarrow \mathcal{O}_M(-H) \longrightarrow \mathcal{O}_M \longrightarrow \mathcal{O}_M|_H \longrightarrow 0$$

$\Rightarrow$ By the sequence on p 100,

$$\underline{\mathrm{Tor}}_1^{\mathcal{O}}(\mathcal{O}_M(-H), \mathcal{F}) \longrightarrow \underset{(\overset{\shortparallel}{0})}{\underline{\mathrm{Tor}}_1^{\mathcal{O}}(\mathcal{O}_M, \mathcal{F})} \longrightarrow \underline{\mathrm{Tor}}_1^{\mathcal{O}}(\mathcal{O}_H, \mathcal{F}) \longrightarrow$$

$$\underline{\mathrm{Tor}}_0^{\mathcal{O}}(\mathcal{O}_M(-H), \mathcal{F}) \longrightarrow \underline{\mathrm{Tor}}_0^{\mathcal{O}}(\mathcal{O}_M, \mathcal{F}) \longrightarrow \underline{\mathrm{Tor}}_0^{\mathcal{O}}(\mathcal{O}_H, \mathcal{F}) \longrightarrow 0$$

$$\begin{array}{ccc} \shortparallel & \shortparallel & \shortparallel \\ \mathcal{O}_M(-H) \otimes \mathcal{F} & \mathcal{F} \otimes_{\mathcal{O}} \mathcal{O}_M & \mathcal{F} \otimes_{\mathcal{O}} \mathcal{O}_H \\ \shortparallel & \shortparallel & \shortparallel \\ \mathcal{F}_{-H} = \mathcal{F}(-1) & \mathcal{F} & \mathcal{F}_H \end{array}$$

Since $\mathcal{F}$ is coherent, $\exists$ a local syzygy

$$0 \rightarrow \mathcal{O}^{(k_n)} \longrightarrow \cdots \quad \mathcal{O}^{(k_0)} \longrightarrow \mathcal{F} \rightarrow 0$$

$\Rightarrow 0 = \underline{\mathrm{Tor}}_1^{\mathcal{O}}(\mathcal{F}, \mathcal{O}) = \underline{\mathrm{Tor}}_1^{\mathcal{O}}(\mathcal{O}, \mathcal{F}) = 0$

Easy way.

$$\begin{array}{ccccccc} 0 & \longrightarrow & \mathcal{O}_M & \longrightarrow & \mathcal{O}_M & \longrightarrow & 0 \\ \shortparallel & & \shortparallel & & & & \\ \mathcal{E}_1 & & \mathcal{E}_0 & & & & \end{array}$$

$\Rightarrow \quad \underline{\mathrm{Tor}}_1^{\mathcal{O}}(\mathcal{O}, \mathcal{F}) = 0.$ $\lrcorner$

The sheaf $\mathcal{G} = \underline{\mathrm{Tor}}_1^{\mathcal{O}}(\mathcal{O}_H, \mathcal{F})$ is a coherent sheaf of