$\Rightarrow$

$$
\begin{tikzcd}
0 \arrow[r] & \mathrm{Hom}(I, \mathcal{O}) \arrow[r] \arrow[d] & \mathrm{Hom}(E, \mathcal{O}) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}, \mathcal{O}) \ni id_{\mathcal{O}} \arrow[d] & \\
0 \arrow[r] & \mathrm{Hom}(\mathcal{O}\oplus\mathcal{O}, \mathcal{O}) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}\oplus(\mathcal{O}\oplus\mathcal{O}), \mathcal{O}) \ni p : (g_1, g_2, g_3) \mapsto g_1 \arrow[r] \arrow[d, "\delta"'] & \mathrm{Hom}(\mathcal{O}, \mathcal{O}) \ni id \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathrm{Hom}(\mathcal{O}, \mathcal{O}) \ni -\phi_e \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}, \mathcal{O}) \ni -\phi_e : h \mapsto -h\ell = \delta(p) \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & 0 \arrow[r] & 0 \arrow[r] & 0 \arrow[r] & 0
\end{tikzcd}
$$

$$
\begin{aligned}
\delta(p) : \mathcal{O} &\longrightarrow \mathcal{O} \\
h &\longmapsto \delta(p)(h) = p(\partial h) \\
&\qquad = p(-h\ell, -hf_2, hf_1) \\
&\qquad = -h\ell = -\phi_e(h). \quad \partial(1)
\end{aligned}
$$

$\Rightarrow$ We proved $\partial(id_{\mathcal{O}}) = -\phi_e$. $\Rightarrow$ $\partial(id_{\mathcal{O}}) = -e$. //

Identifying $\mathrm{Ext}^1_{\mathcal{O}}(I, \mathcal{O})$ with $\mathcal{O}/I$ and using that ∂ is $\mathcal{O}$-linear, if e is a unit,

$$\partial(e^{-1}) = -1 \in \mathcal{O}/I.$$

It follows that ∂ is surjective, and so $\mathrm{Ext}^1_{\mathcal{O}}(E, \mathcal{O}) = 0$. It is trivially the case that $\mathrm{Ext}^q_{\mathcal{O}}(E, \mathcal{O}) = 0$ for $q \geq 2$.

「 By the commutative diagram, we have a projective resolution for E, i.e.,

$$\cdots \longrightarrow 0 \longrightarrow \underset{= E_1}{\mathcal{O}} \longrightarrow \underset{= E_0}{\mathcal{O}\oplus(\mathcal{O}\oplus\mathcal{O})} \longrightarrow E \longrightarrow 0$$