$\Rightarrow$ By the commutativity of the Global Duality Theorem II,

$$\underset{(\langle \psi, ? \rangle)}{\langle ?, \psi \rangle} = \sum_{p \in Z} \underset{(\langle \psi, \tau_p \rangle)}{\langle \tau_p, \psi \rangle}$$

$0 = ? \iff \underset{(\langle \psi, ? \rangle)}{\langle ?, \psi \rangle} = 0$ for all $\psi \in \mathrm{Ext}^2(S; \mathcal{O}, \mathcal{O})^* = H^2(S, \Omega^2)$,

by (**) on p.127

$\iff \exists\ e \in \mathrm{Ext}^1(S; I, \mathcal{O})$ s.t $\alpha(e)_p = \tau_p$ for each point $p \in Z$, $\iff \exists$ a holomophic rank-two vector bundle $E \longrightarrow S$ with $\Lambda^2 \mathcal{E} \cong \mathcal{O}$ and section $s \in H^0(S, \mathcal{O}(\mathcal{E}))$ that defines Z.

$H^0(S, \Omega^2) = \langle \psi_1, \psi_2, \cdots \psi_\ell \rangle$

$Z = \{ p_1, p_2, \cdots p_m \}$. $\quad m > 1$.

Choose $\tau_i \in \Lambda^2 T'_{p_i}(S)$, which is not zero.

$$\sum_{p_j \in Z} \langle \psi_i, a_j \tau_j \rangle = 0 = \sum_{p_j \in Z} a_j \langle \psi_i, \tau_j \rangle = 0$$

$$= \sum_{p_j \in Z} x_{ij} a_j, \quad \text{where } x_{ij} = \langle \psi_i, \tau_j \rangle$$

Question: Can we find a_j's which are not zero?
For example, $H^0(S, \Omega^2) = \langle \psi \rangle$ and $Z = \{p_1, p_2\}$.

$\Rightarrow \langle \psi, a_1 \tau_1 \rangle + \langle \psi, a_2 \tau_2 \rangle$

$= a_1 \psi(\tau_1) + a_2 \psi(\tau_2) = 0$

Suppose $\psi(\tau_1) = 0$ and $\psi(\tau_2) \neq 0 \Rightarrow a_2 = 0$ //

Suppose $H^0(S, \Omega^2) = \langle \psi_1, \psi_2 \rangle$.