$$\begin{tikzcd}
0 \arrow[r] \arrow[d] & H_i(E) \oplus H_i(\tilde{M}^*) \arrow[r] \arrow[d, "\pi_*"] & H_i(\tilde{M}) \arrow[r] \arrow[d, "\pi_*"] & \mathbb{Z} \arrow[r, "g_1"] \arrow[d, "\cong"] & * \arrow[d, "\cong"] \\
0 \arrow[r] & 0 \oplus H_i(M^*) \arrow[r] & H_i(M) \arrow[r] & \mathbb{Z} \arrow[r, "g_2"] & *
\end{tikzcd}$$

$$\Rightarrow \quad \begin{tikzcd}
0 \arrow[r] \arrow[d] & H_i(E) \oplus H_i(\tilde{M}^*) \arrow[r] \arrow[d, "\pi_*"] & H_i(\tilde{M}) \arrow[r] \arrow[d, "\pi_*"] & \ker g_1 \arrow[r] \arrow[d, "\cong"] & 0 \\
0 \arrow[r] & H_i(M^*) \arrow[r] & H_i(M) \arrow[r] & \ker g_2 \arrow[r] & 0
\end{tikzcd}$$

$\Rightarrow$ Since $\ker g_1$ & $\ker g_2$ are free,

$$H_i(\tilde{M}) = \ker g_1 \oplus H_i(E) \oplus H_i(\tilde{M}^*)$$
$$H_i(M) = \ker g_2 \oplus H_i(M^*) \cong \ker g_1 \oplus H_i(\tilde{M}^*).$$
$$\Rightarrow \quad H_i(\tilde{M}) = H_i(E) \oplus \ker g_1 \oplus H_i(\tilde{M}^*) = H_i(E) \oplus H_i(M) \quad \lrcorner$$

Since all the cohomology of $E \cong \mathbb{P}^{n-1}$ is represented by analytic cycles,

$$h^{i,i}(\tilde{M}) = h^{i,i}(M) + 1, \qquad i > 0,$$

with all other Hodge numbers of $\tilde{M}$ equal to those of M.

$\ulcorner$ From $H_i(\tilde{M}) = H_i(E) \oplus H_i(M)$, we have

$$H^n(\tilde{M}) = H^n(E) \oplus H^n(M)$$
$$\Rightarrow \quad \bigoplus_{p+q=n} H^{p,q}(\tilde{M}) = \bigoplus_{p+q=n} \left(H^{p,q}(E) \oplus H^{p,q}(M) \right) \cdots (*)$$

$$\Rightarrow \quad p \neq q \qquad H^{p,q}(\tilde{M}) = H^{p,q}(E) \oplus H^{p,q}(M) = H^{p,q}(M)$$

since $H^{p,q}(\mathbb{P}^{n-1}) = 0$ ($\because$ all the cohomology of $\mathbb{P}^{n-1}$ is represented by analytic cycles and any analytic co-