Given $\omega \in \Omega^n_M(V)(U) = \mathcal{O}(K_M \otimes [V])(U)$, $\quad U \cap V = (f=0)$.

then $\omega = \eta \otimes \sigma$, $\quad \eta$ n-form on M. (over U)

$\Rightarrow$

let $\eta = g(z)\, dz_1 \wedge \cdots \wedge dz_n$.

$$\begin{tikzcd} {[V]|_U} \arrow[r] \arrow[d] & U \times \mathbb{C} \arrow[dl] \\ U \arrow[u, dashed, bend left, "\sigma"] & \end{tikzcd}$$

σ nonvanishing local section over U

σ exists since df is nonvanishing section of $[-V]|_U$ and we may take $\sigma \otimes df = 1$, in some sense $\sigma = \frac{1}{df}$.

As in the page P138 ~ P139, ω can be identified with

$$\frac{\omega}{f\sigma} = \frac{\eta \otimes \sigma}{f\sigma} = \frac{g(z)\, dz_1 \wedge \cdots \wedge dz_n}{f(z)}$$

, since $(f\sigma = 0) = V \cap U$ and $f\sigma$ is a section of $[V]$ over U.

Note: There exists a global meromph section τ of $[V]$ s.t $\tau|_U = f\sigma$. see back

Now, $\eta \otimes \sigma|_V$ becomes an element ω' in $\mathcal{O}(K_V)(U \cap V)$ by the algebraic identification $(K_M \otimes [V])|_V = K_V$.

If we apply tensor product of ω' with df, which corresponds to wedge product, we get

$$df \wedge \omega' \overset{\text{identification}}{=} \eta \otimes \sigma \otimes df = \eta = g(z)\, dz_1 \wedge \cdots \wedge dz_n$$