But all sections σ of H^d are of the form σ_F, and so

$$V = (\sigma_F) = (F(X_0, \cdots X_n) = 0)$$

is algebraic. In general, suppose $V \subset \mathbb{P}^n$ is a k-dimensional variety, $p \in \mathbb{P}^n$ any point not lying on V. We can find an $(n-k-1)$-plane $\mathbb{P}^{n-k-1}$ in $\mathbb{P}^n$ through p and missing V; let $\mathbb{P}^{n-k-2}$ be an $(n-k-2)$-plane in $\mathbb{P}^{n-k-1}$ disjoint from p.

$\ulcorner$ Let $V \subset \mathbb{P}^n$ be a k-dimensional variety. Then there exists a $(n-k)$-plane $\mathbb{P}^{n-k}$ in $\mathbb{P}^n$ meeting V in isolated points.

Since V is compact. $\exists$ a finite cover $\{U_i\}$ of $\mathbb{P}^n$ s.t $U_i \cap V = \{f_{1i} = 0, \cdots f_{ki} = 0\}$

and on each U_i, one of the homogeneous coordinates Z_j's of $\mathbb{P}^n$ is not zero.

$\mathbb{P}^{n-k}$ is determined by linearly indepedent $(n-k+1)$ points in $\mathbb{C}^{n+1}$, and

$\mathbb{P}^{n-k}$ is the intersection of k hyperplanes.

If we prove that $\exists$ a hyperplane $\mathbb{P}^{n-1}$ s.t $\mathbb{P}^{n-1}$ intersects with V transversally, by applying the same argument to $\mathbb{P}^{n-1} \cap V$,