According to the spectral theorem for compact, self-adjoint operators, there is a Hilbert-space decomposition

$$\mathcal{H}_0^{p,q}(M) = \bigoplus_m E(\rho_m).$$

where ρ_m are the eigenvalues of T and $E(\rho_m)$ are the finite-dimensional eigenspaces. Since T is one to one, all $\rho_m \neq 0$; moreover, the equation

$$T\varphi = \rho_m \varphi$$

is the same as

$$(\varphi, \eta) = (\rho_m \varphi, (I+\Delta)\eta) \qquad (\eta \in A^{p,q}(M)).$$

which implies

$$\Delta\varphi = \left(\frac{1-\rho_m}{\rho_m}\right)\varphi. \qquad \text{in the weak sense.}$$

It follows that the eigenspaces for T and Δ are the same and are finite dimensional vector spaces consisting of C^∞-forms. The eigenvalues λ_m for Δ and ρ_m for T are related by

$$\lambda_m = \frac{1-\rho_m}{\rho_m} \qquad \rho_m = \frac{1}{1+\lambda_m}.$$

We may assume that

$$0 = \lambda_0 < \lambda_1 < \cdots$$

where $\lambda_m \uparrow \infty$, $\rho_m \downarrow 0$ as $m \to \infty$.
The harmonic space $\mathcal{H}^{p,q}(M)$ corresponds to $\lambda_0 = 0$.
For $\varphi \in \mathcal{H}^{p,q}(M)^\perp$,

$$\|\Delta\varphi\|_0 \geq \lambda_1 \|\varphi\|_0 \qquad (\lambda_1 > 0)$$

and if we define the Green's operator by