Since the subspace $\iota(x) = \Lambda \subset \mathbb{C}^n$ corresponds to linear functionals on the fiber E_x of E over x, moreover, we see as before that $\iota^*(S) = E^*$, i.e.,

$$\iota^*(S^*) = E.$$

⌈ See note P 533 93. 4. 17.

$$\begin{tikzcd} E \arrow[r] \arrow[d] & Q \arrow[r] \arrow[d] & S^* \arrow[d] \\ M \arrow[r, "\iota"] & G(n-k, V) \arrow[r, phantom, "\cong"] & G(k, V^*) \end{tikzcd}$$

$$G(n-k, V) = \langle \sigma_1, \cdots \sigma_n \rangle$$

$$\iota^* S^* = E$$

To each functional on the fiber E_x, some element in Λ corresponds. $\Rightarrow$ $\iota^* S^* = E$ ⌋

Now for each $r = 1, 2, \cdots k$ let $V_{n-k+r-1} = \{e_{k-r+2}, \cdots e_n\} \subset \mathbb{C}^n$. Then for any $x \in M$, the k-plane $\Lambda = \iota(x) \in G(k.n)$ will intersect $V_{n-k+r-1}$ in a space of dimension r or greater if and only if the sections $\sigma_1, \cdots, \sigma_{k-r+1}$ are linearly dependent at x – i.e., $\iota(M)$ meets the Schubert cycle $\sigma_{r,1}(V) \subset G(k.n)$ exactly in the degeneracy set D_{k-r+1} of the sections $\sigma_1, \cdots \sigma_k$.

⌈ First of all, we need to check if ι is well-defined. Let $\omega_1, \cdots \omega_k$ be a local frame for E, and $\omega_1' \cdots \omega_k'$ be another frame.

$$\Rightarrow \quad \sigma_1 = v_{11}\omega_1 + \cdots + v_{k1}\omega_k = v_{11}'\omega_1' + \cdots + v_{k1}'\omega_k'$$