let $\Lambda = \overline{L, L'}$ be the 2-plane spanned by L and L'. There is then a unique quadric $F_{\lambda(L')}$ in the pencil $\{F_\lambda\}$ containing the 2-plane Λ. To see this, let $q \in \Lambda$ be any point of Λ lying off L and L'. (See Figure 22.) q is then contained in some quadric $F_{\lambda(L')}$ — but $F_{\lambda(L')}$, containing L, L' and q, has three points of intersection with any line L'' in Λ through q, and so contains L''; thus $F_{\lambda(L')}$ contains Λ.

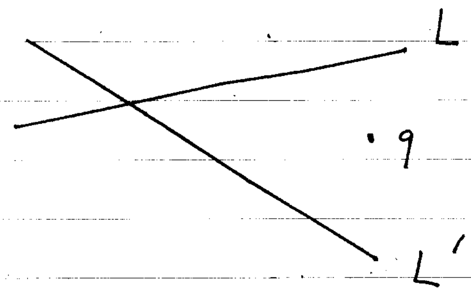

Figure 22.

$\ulcorner$ $L, L' \subset X = F \cap G \Rightarrow {}^\forall x \in L \Rightarrow \lambda F + G \ni x$
$\Rightarrow L, L' \subset \lambda F + G$ $\qquad (\because F(x) = G(x) = 0)$
$F(q) \neq 0 \neq G(q) \Rightarrow \exists\, \lambda^{\in \mathbb{P}^1}$ s.t $(\lambda F + G)(q) = 0$.
Let $F_{\lambda(L')} = \lambda F + G \supset L, L', q$. We do not prove yet that $F_{\lambda(L')} \supset \Lambda$. If $\Lambda \supset \underset{\text{generic}}{L''} \ni q$, then $L'' \cap L \neq \phi$ and $L'' \cap L' = \phi$. $\Rightarrow$
$F_{\lambda(L')} \supset \{q, L'' \cap L, L'' \cap L'\}$. $\Rightarrow \#(F_{\lambda(L')} \cap L'') \geq 3$
$\Rightarrow$ Since $\deg F_{\lambda(L')} = 2$, $F_{\lambda(L')}$ contains L''. Here L'' is generic line in Λ, then L'' does not pass $L \cap L'$. $\Rightarrow$ $\#(F_{\lambda(L')} \cap L'') \geq 3 \Rightarrow$ Since $F_{\lambda(L')}$ contains