By Poincare duality this linear functional determines a cohomology class $\eta_V \in H_{DR}^{2n-2k}(M)$, called the fundamental class of V.

Γ
$$L : \begin{array}{ccc} H_{DR}^{2k}(M) & \longrightarrow & \mathbb{R} \\ \Downarrow & & \\ [\varphi] & \longmapsto & \int_V \varphi . \end{array} \quad \text{defined by}$$

Since
$$\begin{array}{ccccc} H_{DR}^{2k}(M) & \otimes & H_{DR}^{2n-2k}(M) & \longrightarrow & \mathbb{R} \\ [\varphi_1] & \otimes & [\varphi_2] & \longmapsto & \int_M \varphi_1 \wedge \varphi_2 \end{array}$$
is nondegenerate, $\exists \quad \eta_V \in H_{DR}^{2n-2k}(M)$ s.t
$$L([\varphi]) = \int_V \varphi = \int_M \varphi \wedge \eta_V$$
see p59 $\lrcorner$

We may also define the fundamental class of V by means of the intersection pairing. For any homology class $\alpha \in H_{2n-2k}(M, \mathbb{Z})$ we may find a cycle A representing α and intersecting V transversely in smooth points. In fact, the intersection number

$$\#(V \cdot A) = \sum_{p \in A \cap V} \iota_p(V, A)$$

– where V again is given the natural orientation – depends only on the homology class α: if $A' \sim A$, then since the singular locus of V has real codimension ≥ 2 we can find a $(2n-2k+1)$-chain C on M avoiding the singular set of V, meeting V tran