As a corollary we deduce the theorem of the base: $NS(M)$ is a finitely generated group of rank $\rho = b_2 - p_2$.

Pf By p.457, $\dfrac{\{\text{divisors on } M\}}{\text{homological equivalence}} = \dfrac{Div(M)}{\equiv}$

$\Rightarrow$ rank $NS(M) = \rho = b_2 - p_2$.

$$\begin{array}{ccc} Div M/\sim & \xrightarrow{\phi} & H^2(M, \mathbb{Z}) \\ \cup & & \cup \\ \langle D \rangle & \longmapsto & \eta_D \end{array}$$

ϕ is one to one. $\Rightarrow$ Since $H^2(M,\mathbb{Z})$ is finitely generated, $\operatorname{im}\phi$ is finitely generated. $\Rightarrow$ $Div M/\sim$ is finitely generated.

I think we can not prove $NS(M)$ is finitely generated by the results of Picard & Severi. For, if $D \in Div M$ s.t $\eta_D = 0$, then we only can conclude that $\langle D \rangle \in NS(M)$ is a torsion element.

$$\begin{array}{ccccc} NS(M) = & \text{free part} & \oplus & \text{torsion part} \\ & \downarrow & \psi & \downarrow \\ H^2(M, \mathbb{Z}) = & \text{free part} & \oplus & \text{torsion part.} \end{array}$$

$\Rightarrow$ $\psi|_{\text{torsion part}}$ need not be injective.

Note: A subgroup of a finitely generated abelian group is finitely generated.