Note first that the planes h_i' are linearly independent: if all four contained a point q, then all four points p_i would have to lie in the plane swept out by the pencil X_q of lines of X through q; dually, the points $\{p_i'\}$ are independent.

$\ulcorner$ Suppose h_i''s are linearly dependent.

$h_1' \cap h_2' \cap h_3' \neq \phi$, and $h_1' \cap h_2' \cap h_3' \cap h_4' \neq \phi$

for otherwise $\cap h_i' = \phi$

$h_i' = a_{i0}X_0 + a_{i1}X_1 + a_{i2}X_2 + a_{i3}X_3 = 0$

$\Rightarrow$ h_i's linearly independent $\Leftrightarrow$ $\{(a_{i0}, \cdots a_{i3})\}$ linearly independent $\Rightarrow$ $\nexists$ no nontrivial X_i's s.t $a_{i0}X_0 + \cdots + a_{i3}X_3 = 0$ for all $i = 1, 2, 3, 4$.

First of all, $\overline{q, p_i} \in \sigma(q)$ for all i.

$q \in \cap h_i'$ $\Rightarrow$ $\overline{q, p_i} \in \sigma(p_i, h_i')$

$\Rightarrow$ $\tilde{\Phi}(\overline{q, p_i}) \in X_{p_i} = \tilde{\Phi}(\sigma(p_i)) \cap H = \tilde{\Phi}(\sigma(p_i, h_i'))$

$\Rightarrow$ $\tilde{\Phi}(\overline{q, p_i}) \in H$, and clearly $\overline{q, p_i} \in \sigma(q)$ for all i

$\Rightarrow$ $\tilde{\Phi}(\overline{q, p_i}) \in \tilde{\Phi}(\sigma(q)) \cap H = X_q$ for all i

$\Rightarrow$ $\overline{q, p_i} \subset h$ for all i, since $\tilde{\Phi}(\sigma(q)) \cap H = X_q = \tilde{\Phi}(\sigma(q, h))$ for some hyperplane $h \subset \mathbb{P}^3$.

$\Rightarrow$ Contradiction, since p_i's span $\mathbb{P}^3$.

Note: ① $\mathbb{C}^4 \longrightarrow \mathbb{C}^{4*}$

$\quad e_i \longrightarrow e_i^*$ $\quad e_i = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ $\quad e_i^*(e_j) = \delta_{ij}$

$\Rightarrow$ $\exists$ an induced isomorphism between $\mathbb{P}^3$ and $\mathbb{P}^{3*}$.

② $h_i \subset \mathbb{P}^3$ and h_i's linearly independent.