Date /

② $d=n$. by P179 every irreducible nondegenerate curve of degree $d^{(=n)}$ in $\mathbb{P}^n$ is projectively isomorphic to the rational normal curve.

③ $n<d<2n$. by 251 the corollary of Clifford's theorem, $g \le d-n$, with equality if C is normal.

④ $d=2n$ by R-R. $h^0(D)=2n-g+1+h^0(K-D) \ge n+1$ ⊛

$\Rightarrow n+h^0(K-D) \ge g$.

$\deg(K-D) = \deg K - 2n = 2g-2-2n$

(i) $2g-2-2n \ge 0 \Rightarrow g \ge n+1$.

$\Rightarrow n+h^0(K-D) \ge n+1 \Rightarrow h^0(K-D) \ge 1 \Rightarrow D$ speial

$\Rightarrow$ By Clifford's theorem. $h^0(D)-1 \le n \Rightarrow h^0(D) \le n+1$

$\Rightarrow$ By ⊛, $h^0(D)=n+1$, $\dim|D| = n = \frac{2n}{2} = \frac{d}{2} \Rightarrow$ By again, Clifford's theorem, $D=0$, $D=K$ or C is hyperelliptic.

ⓐ $D=0 \Rightarrow h^0(0)=n+1$ ✗

ⓑ $D=K \Rightarrow h^0(K)=n+1=\dim H^0(C,\Omega^1)$

$=g$. $\Rightarrow C$ is a canonical curve

ⓒ C is hyperelliptic. by P247

✗, since we already have an embedding.

(ii) $2g-2-2n<0 \Rightarrow h^0(K-D)=0$. D is nonspecial

$h^0(D)=2n-g+1 \ge n+1 \Rightarrow g \le n$

⑤ $d \ge 2n$, $\Rightarrow g \le \frac{m(m-1)}{2}(n-1)+me$

where $m = \left[\frac{d-1}{n-1}\right]$, $d-1 = m(n-1)+\epsilon$

$d>2n \Rightarrow \left[\frac{d-1}{n-1}\right] \ge \left[\frac{2n+1-1}{n-1}\right] = \left[\frac{2(n-1)+2}{n-1}\right] \ge 2$

$\Rightarrow m \ge 2$ ~~$\Rightarrow \deg((l+m)D) \ge 2^{\#}(D) = 2\cdot 2n = 4n$~~

if $d<2n$, ~~$\Rightarrow$ if $d=2n-2$~~ $m=2$.

~~if $d<n$, $\Rightarrow m=0 \Rightarrow g\le 0 \Rightarrow \mathbb{P}^1 \cong C$ in $\mathbb{P}^n$~~