factor. $\Rightarrow$ $\dim H^0(\mathbb{P}^2, \mathcal{O}([C_0])) = 1$ $\Rightarrow$ $\dim |\mathcal{J}_{P_0}(n)| = 0$ which contradicts to the assumption $\dim|L| = \dim|\mathcal{J}_{P_0}(n)| > 0$.

Looks better !!!

One more thing.

By comparing with P36 (def of $\mathcal{J}_V(U)$) and P703. I think $\mathcal{J}_P(K_S+L)$ is not correct. $\mathcal{J}_P(K_S+L)$ should be replaced by $I \otimes_{\mathcal{O}} \mathcal{O}(K_S)$, where I is the sheaf of ideals generated by s & $s' \in H^0(S, \mathcal{O}(L)) = H^0(\mathcal{O}_S(L))$.

Maybe, the authors tried to apply the assertion to the Reciprocity Formula I.

When $|L|$ has no base curves, for general curves $C, C' \in |\mathcal{J}_{P_0}(n)|$, $C \cap C'$ must be a set of points. I want to make it clearer, by considering an example.

First, note that since $|L| = \{\pi^*C - E\}$, $|\mathcal{J}_{P_0}(n)|$ has no base curves $\Rightarrow$ $\{f \mid (f=0) \in |\mathcal{J}_{P_0}(n)|\}$ has no common factor. For example, $\{f_1, f_2, f_3\}$ has no common factor. Let $f_3 = g_1 g_2$. g_1, g_2 irreducible. Consider $f_1 + \alpha f_2$, $\alpha \in \mathbb{C}$. Suppose g_1 divides $f_1 + \alpha_0 f_2$, $\alpha_0 \in \mathbb{C}$, and g_1 divides $f_1 + \alpha_0' f_2$, $\alpha_0' \neq \alpha_0$. $\Rightarrow$ g_1 must divide f_2, and so does for f_1. $\Rightarrow$ f_1, f_2 and f_3 have a common factor g_1 $\Rightarrow$ Contradiction. $\Rightarrow$ $\exists$ no other constant α s.t $f_1 + \alpha f_2$ has g_1 as a factor. Similarly, for g_2. $\Rightarrow$ This implies that for generic $\alpha \in \mathbb{C}$, $f_1 + \alpha f_2$ has neither g_1