

above as follows:

$P_0$ : a set of distinct points,  $\deg P_0 = d < \frac{n(n+3)}{2}$

As on P 712. Consider the blow-up  $S$  of  $\mathbb{P}^2$  along  $P_0$ . Let

$L = \pi^*H^n - E$ , etc, as above.  $\Rightarrow r = \dim |L| = \frac{n(n+3)}{2} - d + w > 0$

①  $|L|$  has no base curves

Clearly, since  $r = \dim |f_{P_0}(n)| = \frac{n(n+3)}{2} - d + w \geq 1$ ,  $|f_{P_0}(n)|$  contains at least a pencil even if  $|L|$  has a base curve. Since  $|L|$  has no base curves,

for general curves  $C, C' \in |f_{P_0}(n)|$   
 $C \cdot C'$  is a set of points.  $\Rightarrow$

$\Rightarrow$  For general curves  $C, C' \in |f_{P_0}(n)|$ ,  $\pi^*C - E$  &  $\pi^*C' - E$  are general curves in  $|L| = \pi^*H^n - E$ .

$\Rightarrow P = (\pi^*C - E) \cdot (\pi^*C' - E) = C \cdot C' - d = n^2 - d$

$\Rightarrow d + P = C \cdot C' = P + P_0$ . Here we didn't use the assertion anywhere.

②  $|L|$  has a base curve

$\Rightarrow$  As we observed above,  $|f_{P_0}(n)|$  contains at least a pencil.  $|L|$  can not have  $E_i$  as a base curve since

$\pi^*C_0 - E$  does not have any  $E_i$ ,  $E = E_1 + \dots + E_d$ .  $\Rightarrow$

Only possibility is that  $\{C\}_{C \in |f_{P_0}(n)|}$  has a base curve.

$\Rightarrow$

Since every curve in  $|f_{P_0}(n)|$  may be expressed as a polynomial of degree  $n$ , that means that those polynomials have a common factor. If a common factor is a polynomial of degree  $n$ . Since every curve is a polynomial of degree  $n$ , it is some constant multiple of the common