polar divisor of Θ. $\Rightarrow$ $f(M-W)-f(W)$ is in the polar divisor of Θ $\Rightarrow$ Since measure of $f(W)$ is zero, $f(M-W)$ is in the polar divisor of Θ. $\Rightarrow$ The divisor is closed in Δ^{n+1}, and $\overline{f(M-W)} = f(M-W) \cup f(W) = f(M)$ is contained in the divisor. ⌟

Equivalently, $f(M)$ is the divisor of the holomorphic function

$$e^{f\Theta}.$$

This completes the proof of the proper mapping theorem.

⌜ We are going to show by considering examples. First of all, $\int_1^z \frac{1}{z}\,dz = \int_1^z \partial \log z$ is well-defined on $\mathbb{C}-\{0\}$ (modulo 2π), since the residue at 0 is 1.

$\Rightarrow$ $e^{\int_1^z \frac{1}{z}dz}$ is well-defined on $\mathbb{C}-\{0\}$.

Now look how it behaves around $z=0$.

$$\int_1^z \frac{1}{z}\,dz = \int_0^1 \frac{1}{\alpha(t)}\,\alpha'(t)\,dt = \ln \alpha(t)\Big|_0^1 = \ln\alpha(1) - \ln 1$$

$$= \ln\alpha(1) \pmod{2\pi i}$$

$$= \ln z.$$

$\Rightarrow$ $\ln z = \ln|z| + i\theta$, θ, argument of z.

$\Rightarrow$ As $|z|\to 0$, $e^{\int_1^z \frac{1}{z}dz} = e^{i\theta}\cdot e^{\ln|z|} \longrightarrow 0$.