If we define the Hodge numbers.

$$h^{p,q}(M) = \dim H^q(M, \Omega^p),$$

then we have proved that

$$\begin{cases} h^{p,q}(M) < \infty \\ h^{n,n}(M) = 1 \quad \text{and} \quad h^{p,q}(M) = h^{n-p,n-q}(M) \\ h^{u,v}(M \times N) = \sum\limits_{\substack{p+r=u \\ q+s=v}} h^{p,q}(M)\, h^{r,s}(N). \end{cases}$$

In case M is Kähler, there will be additional deeper-lying relations among the Hodge numbers, such as

$$h^{p,q}(M) = h^{q,p}(M), \quad b_r(M) = \sum_{p+q=r} h^{p,q}(M)$$
$$h^{p,p}(M) \geq 1$$

where $b_r(M) = \dim H^r(M, \mathbb{C})$, is the r-th Betti number. These, and much more, will be derived in the next section.

One final comment. In general, the exterior product of harmonic forms is not harmonic. Similarly, the restriction of a harmonic form to a submanifold is generally not harmonic for the induced metric. Otherwise, the cohomology ring would have only those relations imposed by exterior algebra. Moreover, the two Laplacians on a hermitian manifold,

$$\Delta_{\bar\partial} = \bar\partial\bar\partial^* + \bar\partial^*\bar\partial$$
$$\Delta_d = dd^* + d^*d.$$

are generally unrelated. It is a miraculous fact that, when the metric is Kähler, both these general principles are violated and the theory of harmonic functions has an extraordinary amount of symmetry. More on this in the next section.