$dx_1' \wedge dx_2' = dx_1 \wedge dx_2$ is well-known.

(ii) $n = 4$

$\Rightarrow \quad x_1 \, dx_2 \wedge dx_3 \wedge dx_4 - x_2 \, dx_1 \wedge dx_3 \wedge dx_4 + x_3 \, dx_1 \wedge dx_2 \wedge dx_4 - x_4 \, dx_1 \wedge dx_2 \wedge dx_3$.

Consider

$$\begin{pmatrix} a_1, & -b_1 & 0 & 0 \\ b_1, & a_1 & 0 & 0 \\ 0, & 0, & a_2, & -b_2 \\ 0, & 0, & b_2, & a_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} \quad \cdots (*)$$

where the matrix above is orthogonal, i.e., $a_i^2 + b_i^2 = 1$, $i = 1, 2$.

$\Rightarrow \quad x_1' \, dx_2' \wedge dx_3' \wedge dx_4' - x_2' \, dx_1' \wedge dx_3' \wedge dx_4' + x_3' \, dx_1' \wedge dx_2' \wedge dx_4'$
$- \; x_4' \, dx_1' \wedge dx_2' \wedge dx_3' \quad \cdots \quad ⊛$

$\Rightarrow$ From the argument of (i), $dx_3' \wedge dx_4' = dx_3 \wedge dx_4$
$dx_1' \wedge dx_2' = dx_1 \wedge dx_2$.

$\Rightarrow \quad ⊛ = x_1' \, dx_2' \wedge dx_3 \wedge dx_4 - x_2' \, dx_1' \wedge dx_3 \wedge dx_4 + x_3' \, dx_1 \wedge dx_2 \wedge dx_4' - x_4' \, dx_1 \wedge dx_2 \wedge dx_3'$
$= (x_1' \, dx_2' - x_2' \, dx_1') \wedge dx_3 \wedge dx_4 + (x_3' \, dx_4' - x_4' \, dx_3') \wedge dx_1 \wedge dx_2 = (x_1 \, dx_2 - x_2 \, dx_1) \wedge dx_3 \wedge dx_4 + (x_3 \, dx_4 - x_4 \, dx_3) \wedge dx_1 \wedge dx_2$ (by the argument of (i), $x_1' dx_2' - x_2' dx_1' = x_1 \, dx_2 - x_2 \, dx_1$, & $x_3' dx_4' - x_4' dx_3' = x_3 \, dx_4 - x_4 \, dx_3$)
$= x_1 \, dx_2 \wedge dx_3 \wedge dx_4 - x_2 \, dx_1 \wedge dx_3 \wedge dx_4 + x_3 \, dx_1 \wedge dx_2 \wedge dx_4 - x_4 \, dx_1 \wedge dx_2 \wedge dx_4$.

If we let $z_1 = x_1 + i x_2$, $z_2 = x_3 + i x_4$, $\alpha = a_1 + i b_1$ & $\beta = a_2 + i b_2$, (*) may be expressed as follows:

$$\begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} z_1' \\ z_2' \end{pmatrix}.$$

In general, $\alpha = \alpha_1 + i\alpha_2$, $\beta = \beta_1 + i\beta_2$, $\gamma = \gamma_1 + i\gamma_2$, $\omega = \omega_1 + i\omega_2$
$z_1 = x_1 + i y_1$, $z_2 = x_2 + i y_2$, $z_1' = x_1' + i y_1'$ & $z_2' = x_2' + i y_2'$.