$\mathcal{O}_x$-module $\mathcal{F}(k)_x$ for $k \geq k_0$ and $x \in M$; i.e.,

$$H^0(M, \mathcal{F}(k)) \longrightarrow \mathcal{F}(k)_x / m_x \mathcal{F}(k) \longrightarrow 0,$$

where $m_x \subset \mathcal{O}_x$ is the maximal ideal.

(The equivalence of the two statements in this theorem is the Nakayama lemma. In general, $\mathcal{F}_x / m_x \mathcal{F}_x$ may be called the fiber of the coherent sheaf $\mathcal{F}$ at $x \in M$.)

Theorem B. $H^q(M, \mathcal{F}(k)) = 0$ for $q > 0$, $k \geq k_0$.

⌈ Obvious map: $H^0(M, \mathcal{F}(k)) \longrightarrow \mathcal{F}(k)_x / m_x \mathcal{F}(k)$. is $\sigma \longmapsto \sigma_x + m_x \mathcal{F}(k)$, σ_x is the germ of σ at $x \in M$. $m_x \mathcal{F}(k) = m_x \mathcal{F}(k)_x$.

In Theorem A, ($\Rightarrow$) clear and obvious.

($\Leftarrow$) $\exists\ \sigma^1 \cdots \sigma^m \in H^0(M, \mathcal{F}(k))$ s.t $\sigma^1_x + m_x \mathcal{F}(k), \cdots \sigma^m_x + m_x \mathcal{F}(k)$ generate $\mathcal{F}(k)_x / m_x \mathcal{F}(k)$. $\Rightarrow$ By the Nakayama lemma, P 680 ~ P681, $\sigma^1_x \cdots \sigma^m_x$ generates $\mathcal{F}(k)_x$. When $\mathcal{F} = \mathcal{O}(E)$,

$$\mathcal{F}_x / m_x \mathcal{F}_x = \frac{\mathcal{O}_x(\mathbb{C}^n)}{m_x \mathcal{O}_x} \cong \mathbb{C}^n = \text{fiber of } E \text{ at } x, \text{ where}$$

$$\mathcal{O}_x C(^{E|u}) = \mathcal{O} \oplus \cdots \oplus \mathcal{O}$$

$$m_x \mathcal{O}_x(^{E|u}) = m \oplus \cdots \oplus m.$$

For example, $n = 2$

$$\mathbb{C}^2 \xrightarrow{\ \phi\ } \frac{\mathcal{O} \oplus \mathcal{O}}{m(\mathcal{O} \oplus \mathcal{O})}$$

$$(\alpha, \beta) \longmapsto (\alpha, \beta) + m(\mathcal{O} \oplus \mathcal{O}).$$

Clearly ϕ is one to one, and given $(f, g) \in \mathcal{O}^{(2)}$, consider $(f(0), g(0)) \in \mathbb{C}^2$. $\Rightarrow$ $f - f(0) \in m\mathcal{O} = m$, $g - g(0) \in m\mathcal{O} = m$. $\Rightarrow$