coordinates $(z_1, z_2, \cdots z_n)$. We may assume that $f(z_1, z_2, \cdots z_n)$ is a Weierstrass polynomial in z_n and that

$$\varphi = \alpha(z)\, dz_1 \wedge \cdots \wedge dz_{n-1} \wedge d\bar{z}_1 \wedge \cdots \wedge d\bar{z}_{n-1},$$

since these forms generate all forms under coordinate stretchings,

$$z_1' = z_1, \cdots \quad z_{n-1}' = z_{n-1}, \quad z_n' = \beta_1 z_1 + \cdots + \beta_{n-1} z_n + z_n .$$

By p8. Weierstrass Preparation Theorem, $f = g \cdot h$, where g is a Weierstrass polynomial and $h(0) \neq 0$, in case f is not identically zero on the z_n-axis. (We can find a line on which f is not identically zero. $\Rightarrow$ By changing coordinates linearly, we can assume that the line is the z_n-axis. By using the partition of unity, we can make the problem local. $\partial\bar{\partial} \log |gh|^2 = \partial\bar{\partial} \log g h \bar{g} \bar{h}$ $= \partial\bar{\partial} \log|g|^2 + \underbrace{\partial\bar{\partial} \cancel{\log|h|^2}}_{=0} = \partial\bar{\partial} \log|g|^2$, which means that we may assume f is a Weierstrass polynomial in z_n.

I don't understand how these φ forms generate all forms under coordinate stretchings. I think I can get the correct proof. What we have to show is that $\log|f|$ is locally integrable and

$$\frac{\sqrt{-1}}{\pi} \int_M \log|f|\, \bar{\partial}\partial\varphi = \int_{Z^*} \varphi .$$

Since $|\log|f|| \leq |f| + \frac{1}{|f|}$, and $\left| \int_K \frac{1}{r}\, dz \wedge d\bar{z} \right| < \infty$,

$\log|f|$ is locally integrable.