Now let M be a compact hermitian manifold with hermitian connection in the tangent bundle. Denote by $\mathcal{H}_s^{p,q}(M)$. the completion of $A^{p,q}(M)$ in the Sobolev s-norm, $\| \; \| = \| \; \|_0$, and define the Dirichlet inner product and Dirichlet norm, respectively, by

$$\begin{aligned}\mathcal{D}(\varphi,\psi) &= (\varphi,\psi) + (\bar\partial\varphi,\bar\partial\psi) + (\bar\partial^*\varphi,\bar\partial^*\psi)\\ &= (\varphi,\ (I+\Delta)\psi)\end{aligned}$$

$$\begin{aligned}\mathcal{D}(\varphi) = \mathcal{D}(\varphi,\varphi) &= \|\varphi\|^2 + \|\bar\partial\varphi\|^2 + \|\bar\partial^*\varphi\|^2.\\ &= \langle \varphi,\ (I+\Delta)\varphi \rangle\end{aligned}$$

The basic estimate in the theory is provided by

Garding's Inequality. For $\varphi \in A^{p,q}(M)$,

$$\|\varphi\|_1^2 \leqslant C\,\mathcal{D}(\varphi) \qquad C>0.$$

We remark that the operator $I+\Delta$, rather than just the Laplacian Δ, is being used, since $\Delta \geq 0$ implies that $I+\Delta$ has no kernel and therefore we have a better chance of inverting it.

Γ $(I+\Delta)\varphi = 0 \iff -\varphi = \Delta\varphi \Rightarrow \underset{\langle\bar\partial\varphi,\bar\partial\varphi\rangle + \langle\bar\partial^*\varphi,\bar\partial^*\varphi\rangle}{\underset{\shortparallel}{\langle\Delta\varphi,\varphi\rangle}} \geq 0$

$\Rightarrow$ But $\langle\Delta\varphi,\varphi\rangle = \langle-\varphi,\varphi\rangle = -\langle\varphi,\varphi\rangle < 0$. unless $\varphi = 0$.

One use of the Garding inequality will be to prove the

Regularity Lemma I. Suppose that $\varphi \in \mathcal{H}_s^{p,q}(M)$, and that $\psi \in \mathcal{H}_0^{p,q}(M)$ is a weak solution of the equation $\Delta\psi = \varphi$. in the sense $(\psi, \Delta\eta) = \langle\varphi, \eta\rangle$