$$f^*\delta h = \delta f^* h.$$

$$\because \quad (f^*\delta h)_{\alpha\beta\gamma} = f^*((\delta h)_{\alpha\beta\gamma}) = (\delta h)_{\alpha\beta\gamma}\circ f$$
$$= (h_{\beta\gamma} - h_{\alpha\gamma} + h_{\alpha\beta})\circ f = h_{\beta\gamma}\circ f - h_{\alpha\gamma}\circ f + h_{\alpha\beta}\circ f$$
$$= f^*(h_{\beta\gamma}) - f^*(h_{\alpha\gamma}) + f^*(h_{\alpha\beta})$$
$$= (f^*h)_{\beta\gamma} - (f^*h)_{\alpha\gamma} + (f^*h)_{\alpha\beta} = (\delta(f^*h))_{\alpha\beta\gamma}.$$

$$\exp h = g \quad\Rightarrow\quad \exp(f^*h) = \exp(h\circ f) = g\circ f = f^*g.$$
$$= f^*\exp h$$

$$\begin{array}{ccc}
C^1(\underline{U}, \mathcal{O}^*) & \xrightarrow{\ f^*\ } & C^1(f^{-1}(\underline{U}), \mathcal{O}^*) \\
\quad \ni g & \longmapsto & \ni f^*g \\
\uparrow & & \uparrow \\
C^1(\underline{U}, \mathcal{O}) & \xrightarrow{\ f^*\ } & C^1(f^{-1}(\underline{U}), \mathcal{O}) \\
\ni h & \longmapsto & f^*h
\end{array}$$

We will be concerned in this subsection with giving two alternate interpretations of the Chern class; first, however, we want to make one observation:

Let $\mathcal{A}$ and $\mathcal{A}^*$ denote the sheaves of C^∞-functions and nonzero C^∞ functions, respectively. The transition functions of a C^∞ complex line bundle L, then, give a Čech cocycle

$$\{g_{\alpha\beta}\} \in C^1(M, \mathcal{A}^*),$$

and by the same argument as for holomorphic bundles, the bundle L is determined, up to C^∞ isomorphism by the cohomology class $[\{g_{\alpha\beta}\}] \in H^1(M, \mathcal{A}^*)$.