$\operatorname{sgn}\det(J_f(p)-I) = \#(\Delta\cdot\Gamma_f) = \iota_f(p)$ by the definition of the index. $\Rightarrow \sum_{f(p)=p} \iota_f(p) = \#(\Delta\cdot\Gamma_f)_{M\times M}$ in case $f$ has only non-degenerate fixed points.

$$\#(\Delta\cdot\Gamma_f) = \int_{\Gamma_f}\varphi_\Delta = \int_{\Gamma_f}\sum_{p,\mu}(-1)^{n-p}\varphi_{\mu,\mu,p,n-p}$$

$$= \sum_p \int_{\Gamma_f}(-1)^{n-p}\sum_\mu \pi_1^*\psi_{\mu,p}\wedge\pi_2^*\psi^*_{\mu,n-p} = \sum_p(-1)^{n-p}\int_{\tilde{f}(M)}\sum_\mu \pi_1^*\psi_{\mu,p}\wedge\pi_2^*\psi^*_{\mu,n-p}$$

$$= \sum_p (-1)^{n-p}\int_M\sum_\mu \tilde{f}^*\pi_1^*\psi_{\mu,p}\wedge\tilde{f}^*\pi_2^*\psi^*_{\mu,n-p} \quad (\text{by } \langle\tilde{f}^*\omega,\sigma\rangle = \langle\omega,\tilde{f}_*\sigma\rangle)$$

$$= \sum_p(-1)^{n-p}\int_M\sum_\mu \psi_{\mu,p}\wedge f^*\psi^*_{\mu,n-p} \quad (\text{by } \pi_1\circ\tilde{f} = id \ \& \ \pi_2\circ\tilde{f} = f)$$

$$= \sum_p (-1)^{n-p}\operatorname{tr}(f^*|_{H^{n-p}_{DR}(M)}), \ (\text{for } f^*: H^{n-p}_{DR}(M)\longrightarrow H^{n-p}_{DR}(M)$$

$$f^*\psi^*_{\mu,n-p} = \sum C_{\nu,\mu}\psi^*_{\nu,n-p} \Rightarrow \int_M \psi_{\mu,p}\wedge f^*\psi^*_{\mu,n-p}$$

$$= C_{\mu\ \mu} \Rightarrow \sum_\mu C_{\mu,\ \mu} = \text{trace of } (f^*|_{H^{n-p}_{DR}(M)}).\ )$$

$$= \sum_p (-1)^p \operatorname{tr}(f^*|_{H^p_{DR}(M)}).$$

The number $\sum(-1)^p\,\text{trace}(f^*|_{H^p_{DR}(M)})$ is called the Lefschetz number of the map $f$, and is usually denoted $L(f)$; we have proved the

Lefschetz Fixed-Point Formula.