for $H_1(S, \mathbb{Z})$ as on p. 227 such that no point p_λ, q_λ lies on one of the paths δ_j, and let $\omega_1, \cdots, \omega_g$ be normalized basis for $H^0(S, \Omega^1)$ with respect to $\{\delta_1, \cdots \delta_{2g}\}$. By the lemma, there exists a differential of the third kind with residues $a_\lambda / 2\pi\sqrt{-1}$ at p_λ, $b_\lambda / 2\pi\sqrt{-1}$ at q_λ: any two such forms differ by a holomorphic form on S, and hence there exists a unique such form η such that the A-periods

$$N^i = \int_{\delta_i} \eta = 0, \quad i = 1, 2, \cdots g.$$

⌈⌈ If $\eta_1 = \sigma_1/s_0$ and $\eta_2 = \sigma_2/s_0$ are such two forms, then $\sigma_1/s_0 - \sigma_2/s_0$ is holomorphic on S. Let η_0 be a such form and let $\int_{\delta_i} \eta = x_i$. Consider $\eta = \eta_0 - x_i \omega_i$.

$$\Rightarrow \int_{\delta_i} \eta = \int_{\delta_i} \eta_0 - x_i \omega_i = 0.$$

Suppose η_1, η_2 are such two forms.

$$\Rightarrow \quad \eta_1 - \eta_2 = a_1 \omega_1 + \cdots + a_g \omega_g$$

$$\int_{\delta_i} \eta_1 - \eta_2 = 0 = a_i \Rightarrow \eta_1 = \eta_2 \Rightarrow \text{Unique} \; \lrcorner\lrcorner$$

The problem now is to alter η so as to make all its B-periods integral; clearly we can do this without disturbing the singularities of η or the integrality of its A-periods only by adding on an integral linear combination of the forms ω_i. To see if this is possible, we read off the B-periods of η by the reciprocity law: since $N^i = 0$ for $i = 1, 2, \cdots g$, we have for each i,

$$N^{g+i} = \sum_\lambda a_\lambda \int_{p_0}^{p_\lambda} \omega_i + \sum_\lambda b_\lambda \int_{p_0}^{q_\lambda} \omega_i = \sum_\lambda \int_{q_\lambda}^{p_\lambda} \omega_i .$$