

$$\Rightarrow (E_{\Omega^{r,*}})^{p,q}_x = \begin{cases} H^p(M, \Omega^r) & \text{if } q=0 \\ 0 & \text{if } q>0 \end{cases} \quad \text{for all } x \geq 2.$$

$$\begin{aligned} \Rightarrow \text{by } H(E_r) &= E_{r+1} \\ H^q(M, \Omega^{r,*}) &= (E_{\Omega^{r,*}})^{0,q}_2 \oplus \dots \oplus (E_{\Omega^{r,*}})^{q,0}_2 \\ &= 0 \oplus \dots \oplus H^q(M, \Omega^r). \\ &= H^q(M, \Omega^r) \quad \text{--- ①} \end{aligned}$$

On the other hand, by the partition of unity argument $H^q(M, \mathcal{Q}^{*,*}) = 0$, see P42, and

$$("E_{\mathcal{Q}^{r,*}})^{p,q}_2 = H^q_d(H^p(M, \mathcal{Q}^{r,*})).$$

$$\begin{aligned} H^q(M, \mathcal{Q}^{r,p-1}) &\xrightarrow{\bar{\partial}} H^q(M, \mathcal{Q}^{r,p}) \xrightarrow{\bar{\partial}} H^q(M, \mathcal{Q}^{r,p+1}) \\ \text{if } q > 0, \text{ since } H^q(M, \mathcal{Q}^{r,*}) &= 0 \end{aligned}$$

$$("E_{\mathcal{Q}^{r,*}})^{p,q}_2 = 0$$

$$\text{if } q=0 \quad H^0(M, \mathcal{Q}^{r,p-1}) = A^{r,p-1}(M) \quad \dots$$

$$\Rightarrow ("E_{\mathcal{Q}^{r,*}})^{p,q}_2 = H^{r,p}_{\bar{\partial}}(M)$$

$$\Rightarrow ("E_{\mathcal{Q}^{r,*}})^{p,q}_2 = \begin{cases} H^{r,p}_{\bar{\partial}}(M) & q=0 \\ 0 & q>0 \end{cases}$$

$$\begin{aligned} \Rightarrow H^q(M, \mathcal{Q}^{r,*}) &= ("E_{\mathcal{Q}^{r,*}})^{0,q}_2 \oplus \dots \oplus ("E_{\mathcal{Q}^{r,*}})^{q,0}_2 \\ &= 0 \oplus \dots \oplus H^{r,q}_{\bar{\partial}}(M) \\ &= H^{r,q}_{\bar{\partial}}(M) \quad \text{--- ②} \end{aligned}$$

By ① ② & ③

$$H^{r,q}_{\bar{\partial}}(M) \cong H^q(M, \Omega^r) \quad \sqcup$$

3. The complex of holomorphic forms. We now show how