

Thus we see that

$$\begin{array}{ccc}
 H^1(M, \mathbb{C}) & \longrightarrow & H^1(M, \mathcal{O}) \\
 \parallel \downarrow \cong & \searrow & \downarrow \cong \\
 H^1_{DR}(M, \mathbb{C}) & \xrightarrow{df} & \bar{\partial}f
 \end{array}$$

$\Rightarrow df \longmapsto \bar{\partial}f$. projection.

It follows that any cocycle $\gamma \in H^1(X, \mathcal{O}^*)$ in the kernel of c_1 is in the image of ι_2^* , i.e., is cohomologous to a cocycle with constant coefficients; thus any line bundle on X with Chern class 0 can be given by constant transition functions.

$$\begin{array}{ccccc}
 H^1(X, \mathcal{O}) & \longrightarrow & H^1(X, \mathcal{O}^*) & \xrightarrow{c_1} & H^2(X, \mathbb{Z}) \\
 \uparrow \iota_1^* & \nearrow \alpha & \uparrow \iota_2^* & \searrow \psi & \downarrow \cong \\
 H^1(X, \mathbb{C}) & \longrightarrow & H^1(X, \mathbb{C}^*) & \longrightarrow & H^2(X, \mathbb{Z}) \\
 \downarrow \alpha & \searrow \beta & & &
 \end{array}$$

$\Rightarrow \exists \beta \in H^1(X, \mathbb{C}^*)$ s.t. $\iota_2^*(\beta) = \gamma$.

Since β has constant coefficients, γ is cohomologous to $\iota_2^*\beta$ with constant coefficients.

See P133 for $H^1(X, \mathcal{O}^*)$.

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