

$$m = \left\lfloor \frac{d-1}{n-1} \right\rfloor \Rightarrow m \leq \frac{d-1}{n-1} < m+1$$

$$m(n-1) \leq d-1 < m(n-1) + n-1$$

$$\Rightarrow 0 \leq d - m(n-1) - 1 < n-1$$

$$\Rightarrow m(d - m(n-1) - 1) < \frac{d-1}{n-1} \cdot (n-1) = d-1$$

$$\Rightarrow g \leq \frac{1}{2} \frac{d-1}{n-1} \left(\frac{d-1}{n-1} - 1 \right) (n-1) + d-1$$

$$= \frac{1}{2} (d-1) \left(\frac{d-1}{n-1} - 1 \right) + d-1 = \frac{1}{2} \frac{(d-1)^2}{n-1} + \frac{d-1}{2}$$

$$g - \frac{d-1}{2} \leq \frac{(d-1)^2}{2(n-1)} \quad \text{as } g > d-1$$

$$2g - d + 1 \leq \frac{(d-1)^2}{n-1} \quad \Leftrightarrow \quad n-1 \leq \frac{(d-1)^2}{2g - d + 1}$$

$$n \leq \frac{(d-1)^2}{2g - d + 1} + 1$$

$$m + \epsilon = \frac{d-1}{n-1}, \quad 0 < \epsilon < 1$$

$$\Rightarrow \frac{m(m-1)}{2} (n-1) + m(d - m(n-1) - 1) = \frac{1}{2} \left(\frac{d-1}{n-1} - \epsilon \right) \left(\frac{d-1}{n-1} - \epsilon - 1 \right) (n-1) +$$

$$\left(\frac{d-1}{n-1} - \epsilon \right) \left(d - \left(\frac{d-1}{n-1} - \epsilon \right) (n-1) - 1 \right) = \frac{n-1}{2} \left(\epsilon^2 - \epsilon \left(-1 + \frac{d-1}{n-1} \cdot 2 \right) + \right)$$

$$+ \left(\frac{d-1}{n-1} - \epsilon \right) \left(d - (d-1) + \epsilon(n-1) - 1 \right) = \frac{n-1}{2} \left(\epsilon^2 - \epsilon \left(1 + \frac{(d-1)2}{n-1} \right) + \right)$$

$$+ \epsilon(d-1) - \epsilon^2(n-1) =$$

$$- \frac{n-1}{2} \epsilon^2 - \frac{n-1}{2} \epsilon - \frac{(d-1)2}{2} \epsilon + \epsilon(d-1) +$$

$$= - \frac{n-1}{2} \epsilon^2 + \frac{n-1}{2} \epsilon + 0 = - \frac{n-1}{2} (\epsilon^2 - \epsilon) +$$

$$\Rightarrow \epsilon = \frac{1}{2} \Rightarrow \frac{1}{2} \left(\frac{d-1}{n-1} - \frac{1}{2} \right) \left(\frac{d-1}{n-1} - \frac{3}{2} \right) (n-1) + \frac{1}{2} (d-1) - \frac{n-1}{4}$$

$$= \frac{1}{2} \left\{ \left(\frac{d-1}{n-1} \right)^2 - 2 \left(\frac{d-1}{n-1} \right) + \frac{3}{4} \right\} (n-1) + \frac{d-1}{2} - \frac{n-1}{4}$$

$$= \frac{1}{2} \frac{(d-1)^2}{(n-1)} - (d-1) + \frac{3}{4} (n-1) + \frac{d-1}{2} - \frac{n-1}{4} = \frac{(d-1)^2}{2(n-1)} - \frac{d-1}{2} + \frac{n-1}{8} \geq g$$

Wrong
this is the reason why we have an upper bound on n.