L must be one of the lines F_{ij}, G_i.

⊩ Since $1 = 3H\cdot\pi(L) + \sum_{p_i\in\pi(L)} E_i\cdot E_i$, if $\pi(L)$ is a line,

$3H\cdot\pi(L) = 3$, and $\sum_{p_i\in\pi(L)} E_i\cdot E_i = -2$. $\Rightarrow$ $\pi(L)$ contains exactly two of the points p_i. If $\pi(L)$ is a conic, $3H\cdot\pi(L) = 6$, and $\sum_{p_i\in\pi(L)} E_i\cdot E_i = -5$. $\Rightarrow$ $\pi(L)$ contains exactly five of the points p_i's. $\Rightarrow$ $\pi(L) = \tilde{L}_{ij}$ or G_i. $\left(\overset{\shortparallel}{F_{ij}}\right)$ ⌟

Thus there are exactly 27 lines on the cubic surface we have constructed: six E_i's, 15 F_{ij}'s, and six G_i's.

⊩ Six E_i's, ${}_6C_2 = 15$ F_{ij}'s ${}_6C_5 = 6$ G_i's. These are all the lines on S, since we proved that a line $L \neq E_i$ for $i=1,\cdots 6$. is either F_{jk} or G_l. ⌟

The incidence relations among the lines are clearly seen from their description as curves in $\mathbb{P}^2$: the line E_i will meet all lines on S coming from plane curves passing through p_i, that is, F_{ij} for any j and G_j for any $j \neq i$.

⊩ Clearly $\tilde{L}_{ij} \cap E_i \neq \phi$ in $\tilde{\mathbb{P}}^2$ and $E_i \cap G_j \neq \phi$ in $\tilde{\mathbb{P}}^2$ for $j \neq i$. $\tilde{L}_{jk} \cap E_i = \phi$ if $j \overset{\text{or}}{\vee} k = i$. $G_i \cap E_i = \phi$. ⌟