Intersection Numbers of Analytic Varieties

Suppose that M is a compact, oriented manifold of real dimension n. Two closed currents T, S of complementary degrees have an intersection number defined by

$$T \cdot S = \int_M T_\epsilon \wedge S_\delta ,$$

where T_ϵ, S_δ are smooth forms in the cohomology classes defined by T and S using the isomorphism

$$H^*(\mathcal{D}^*(M), d) \cong H^*_{DR}(M).$$

⌈ T_ϵ, S_δ are not meant to be smoothings of T & S.

$$\begin{array}{ccc} H^q_{DR}(M) & \longrightarrow & H^q(\mathcal{D}^*(M), d) \\ \Psi & & \Psi \\ \eta & \longmapsto & T_\eta \end{array} \quad . \quad T_\eta(\varphi) = \int_M \eta \wedge \varphi .$$

$$\varphi \in A_c^{n-q}(M).$$ ⌋

This intersection number coincides with the usual topological one on piecewise smooth singular cycles, with the cup product on the smooth forms considered as currents, and with the usual pairing

$$\int_P \psi$$

of forms on cycles when $T = T_\psi$ for a smooth form ψ and