Since $\mathcal{E}^*$ is coherent, $\exists$ open set U' s.t
$\exists$ $\tau_1, \tau_2, \cdots \tau_\ell$ s.t τ_i's generate $\mathcal{E}^*(U')$ and $\#\tau_i$ is minimal. $\Rightarrow$ $(\tau_i)_p = a_{i1}\sigma_1 + a_{i2}\sigma_2$ for all i.
$\Rightarrow$ $\exists$ open set $U \ni x$ s.t

$$\tau_i = a_{i1}\sigma_1 + a_{i2}\sigma_2 \text{ over } U.$$

$\Rightarrow$ $\ell = 2$. $\Rightarrow$ $\mathcal{E}^*|_U \cong \mathcal{O}|_U \oplus \mathcal{O}|_U$.
Thus the coherent sheaf $\mathcal{E}^*$ is locally free. $\Rightarrow$
$\mathcal{E}^* = \mathcal{O}(E^*)$, E is a vector bundle of rank 2, since $\mathcal{E}_p^* \cong \mathcal{O}^{(2)}$ for $p \notin Z$.

(**) $\Rightarrow$ (*).
pf) By the explanation, we can find $\mathcal{E}^*$ s.t

$$0 \longrightarrow \mathcal{L} \longrightarrow \mathcal{E}^* \longrightarrow I \rightarrow 0 \quad \text{and}$$

$\mathcal{E}^*$ is locally free of rank 2.

(*) $\Rightarrow$ (**).
pf) Given $0 \longrightarrow \mathcal{L} \longrightarrow \mathcal{E}^* \longrightarrow I \rightarrow 0$, where $\mathcal{E}^*$ is locally free of rank 2,
by the lemma on p725, we get $e \in \mathrm{Ext}^1(S; I, \mathcal{L})$. For each point $p \in Z$, by the explanation on p727, $e_p \in \underline{\mathrm{Ext}}^1(I, \mathcal{L})_p = \mathrm{Ext}^1(I_p, \mathcal{L}_p)$ is a unit, since $\mathcal{E}_p^*$ is projective.
, by the lemma on p724,
$\Rightarrow$ We can find e satisfying the conditions of (**).
Thus, solving (**) $\Longleftrightarrow$ finding (*). ⌟