$(n-1)$-planes on $\tilde{F}$ $\Leftrightarrow$ $\dim(\tilde{\Lambda}\cap\tilde{\Lambda}') \equiv (n-1)\ (2)$
$\Leftrightarrow$ $\dim(\Lambda\cap\tilde{\Lambda}) = \dim(\tilde{\Lambda}\cap\tilde{\Lambda}') + 1 \equiv n\ (2)$,
and we are done.

$\ulcorner$ $\dim(\tilde{\Lambda}\cap\tilde{\Lambda}') \equiv n-1\ (2)$ by the induction hypothesis
Since $\langle p, \tilde{\Lambda}\cap\tilde{\Lambda}'\rangle = \Lambda\cap\tilde{\Lambda}$, $\dim(\tilde{\Lambda}\cap\tilde{\Lambda}')+1 = \dim(\Lambda\cap\Lambda')$. $\lrcorner$

Suppose on the other hand that Λ and Λ' are disjoint. In this case, take any point $p\in\Lambda$ and set

$$\Lambda'' = \overline{\Lambda'\cap T_p(F), p}\ .$$

$\ulcorner$ Λ'' = Cone through p over $\Lambda'\cap T_p(F)$. See P172 $\lrcorner$

Now $T_p(F)$ can not contain Λ' — all n-planes in $T_p(F)\cap F$ contain p and hence meet Λ — so Λ'' is an n-plane and

$$\dim(\Lambda'\cap\Lambda'') = n-1,$$

and we deduce from our first argument that Λ' and Λ'' belong to opposite families.

$\ulcorner$ (Suppose $\Lambda'\subset T_p(F)$)
Suppose Λ' is an n-plane in $T_p(F)\cap F$ not containing p. $\Rightarrow$ $\overline{p,\Lambda'}$ is $(n+1)$-plane in F.
$\Rightarrow$ $\dim F = 2n^{=m}$ $\Rightarrow$ $n+1 > n$. $\Rightarrow$ Contradiction to the fact that F contains no linear space of dim