this decomposition is compatible with the decomposition of V into eigenspaces V_m for H. We also see that the maps

$$V_m \underset{X^m}{\overset{Y^m}{\rightleftarrows}} V_{-m} \quad \text{are isomorphisms}$$

Finally, in general,

$$(\ker X) \cap V_k = \ker (Y^{k+1} : V_k \longrightarrow V_{-k-2})$$

We return now to our compact complex manifold M with Kähler metric $ds^2 = \sum \varphi_i \otimes \overline{\varphi_i}$. First, we want to compute the commutator $[L, \Lambda]$ of the operators L and Λ; this may be done on $\mathbb{C}^n$ using the operators $e_k, \overline{e}_k, \overline{\iota}_k$, and ι_k defined earlier. Recall that

$$L = \frac{i}{2} \sum e_k \overline{e}_k \quad \text{and} \quad \Lambda = -\frac{i}{2} \sum \overline{\iota}_k \iota_k ;$$

we have then

$$[L, \Lambda] = \frac{1}{4}\Big(\sum_{k,l} e_k \overline{e}_k \overline{\iota}_l \iota_l - \sum_{k,l} \overline{\iota}_l \iota_l e_k \overline{e}_k \Big)$$

$$= \frac{1}{4} \sum_{k \neq l} (e_k \overline{e}_k \overline{\iota}_l \iota_l - \overline{\iota}_l \iota_l e_k \overline{e}_k)$$

$$+ \frac{1}{4} \sum_k (e_k \overline{e}_k \overline{\iota}_k \iota_k - \overline{\iota}_k \iota_k e_k \overline{e}_k).$$

Since $v, Yv, Y^2 v, \cdots Y^n v$ generate $V(n)$,
$$(I + Y)^n = \sum_{r=0}^{n} {}_nC_r Y^r.$$

$\Rightarrow$ For a fixed $v \in V(n)$, $Iv \; Yv \; Y^2 v \cdots Y^n v$ are basis elements for $V(n)$.
In expanding of $(I + Y)^n$, it is like choosing Y's independently of order. $\Rightarrow$ $V(n) \cong \mathrm{Sym}^n(\mathbb{C}^n)$

$v \in V_m \Rightarrow v = v_1 + \cdots + v_n \in W_1 \oplus \cdots \oplus W_n.$