Under the identifications

$$Q_2 \cong \mathcal{O}(e_r \otimes \Lambda^1 \mathbb{C}^{r-1}), \quad \partial(e_r \otimes e_j) = f_j e_r,$$
$$Q_1 \cong \mathcal{O}(e_r \otimes_{\mathbb{C}} \mathbb{C}) \cong \mathcal{O}\cdot e_r, \quad \alpha(g e_r) = g f_r,$$

the diagram is commutative.

Γ
$$\begin{array}{ccc} \alpha : Q_1 & \longrightarrow & I_r / I_{r-1} \\ \psi & & \psi \\ g e_r & \longmapsto & g f_r \end{array}$$
by the case $r = 1$.

$$\begin{tikzcd} & E_1 \arrow[r, "\partial"] \arrow[d] & I_r \arrow[r] \arrow[d] & 0 \\ Q_2 \arrow[r] & Q_1 \arrow[r] & I_r/I_{r-1} \arrow[r] & 0 \end{tikzcd}$$

$E_1 = \mathcal{O} \otimes_{\mathbb{C}} \Lambda^1 \mathbb{C}^r = \mathcal{O} \otimes_{\mathbb{C}} \mathbb{C}^r$

$g \otimes e_j \longmapsto \partial g e_j = g f_j$

$g \otimes e_j + F_1 = F_1$ if $j \neq r \mapsto 0$

$g \otimes e_r + F_1 = g e_r \in \mathcal{O}\cdot e_r$ if $j = r \mapsto g + I_{r-1}$

If $\alpha(g e_r) = 0$, then $g f_r \in \{f_1, \cdots f_{r-1}\}$, and so $g \in \{f_1 \cdots f_{r-1}\}$ by the regular sequence assumption. Thus $g e_r \in \partial Q_2$, and so the big diagram is commutative and exact. Since $H_*(F.) = 0 = H_*(Q)$, we deduce that $H_*(E.) = 0$ as desired.

Γ $\alpha(g e_r) = 0 \Rightarrow g f_r = \alpha(g e_r) \in I_r / I_{r-1} \Rightarrow g f_r \in \{f_1 \cdots f_{r-1}\} = I_{r-1}$. $\Rightarrow$ By the regular sequence assumption, $g \in I_{r-1}$. $\Rightarrow$ Since $g = h_1 f_1 + \cdots + h_{r-1} f_{r-1}$, $\partial(\sum_{i=1}^{r-1} h_i e_r \otimes e_i) = \sum_i h_i f_i e_r = g e_r \in \partial Q_2$.