$$\begin{pmatrix} \alpha a_0 + \beta b_0 \\ \alpha a_1 + \beta b_1 \\ 0 \end{pmatrix} = \begin{pmatrix} c_0 \\ c_1 \\ 0 \end{pmatrix}$$ by choosing proper $\alpha, \beta \in \mathbb{C}$.

Eith-er α or β can not be zero. otherwise $[(a_0, a_1, 0)] = [(c_0, c_1, 0)]$ or $[(b_0, b_1, 0)] = [(c_0, c_1, 0)]$. Contradiction.

$\Rightarrow$

$$\begin{pmatrix} \alpha a_0, & \beta b_0 & 0 \\ \alpha a_1, & \beta b_1 & 0 \\ 0, & 0 & 1 \end{pmatrix} : \mathbb{P}^2 \longrightarrow \mathbb{P}^2$$

$$\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] \longmapsto \left[\begin{pmatrix} a_0 \\ a_1 \\ 0 \end{pmatrix}\right]$$

$$\left[\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right] \longmapsto \left[\begin{pmatrix} b_0 \\ b_1 \\ 0 \end{pmatrix}\right]$$

$$\left[\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right] \longmapsto \left[\begin{pmatrix} c_0 \\ c_1 \\ 0 \end{pmatrix}\right].$$

$\Rightarrow$ Since $\begin{pmatrix} d_0 \\ d_1 \\ 0 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} a_0 \\ a_1 \\ 0 \end{pmatrix}$ & $\begin{pmatrix} b_0 \\ b_1 \\ 0 \end{pmatrix}$,

for proper γ, $$\begin{pmatrix} \alpha a_0, & \beta b_0 & 0 \\ \alpha a_1, & \beta b_1 & 0 \\ 0, & 0 & 1 \end{pmatrix} \left[\begin{pmatrix} 1 \\ \gamma \\ 0 \end{pmatrix}\right] = \left[\begin{pmatrix} d_0 \\ d_1 \\ 0 \end{pmatrix}\right].$$

If $\gamma \neq 0$, $\left[\begin{pmatrix} d_0 \\ d_1 \\ 0 \end{pmatrix}\right] = \left[\begin{pmatrix} a_0 \\ a_1 \\ 0 \end{pmatrix}\right]$ $\divideontimes$

$$\begin{pmatrix} \alpha a_0 & \beta b_0 & 0 \\ \alpha a_1 & \beta b_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left[\frac{x}{\alpha}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{y}{\beta}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] = \begin{pmatrix} \alpha a_0 & \beta b_0 & 0 \\ \alpha a_1 & \beta b_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left[\begin{pmatrix} 1 \\ \gamma \\ 0 \end{pmatrix}\right]$$

$$= \left[x\begin{pmatrix} a_0 \\ a_1 \\ 0 \end{pmatrix} + y\begin{pmatrix} b_0 \\ b_1 \\ 0 \end{pmatrix} \right] = \left[\begin{pmatrix} d_0 \\ d_1 \\ 0 \end{pmatrix}\right],$$ where $\begin{pmatrix} d_0 \\ d_1 \\ 0 \end{pmatrix} = x\begin{pmatrix} a_0 \\ a_1 \\ 0 \end{pmatrix} + y\begin{pmatrix} b_0 \\ b_1 \\ 0 \end{pmatrix}$ $x \neq 0 \neq y$.