In particular, we conclude that for any $\psi \in C_c^\infty(\mathbb{R}^n)$

$$D\psi_\epsilon \longrightarrow D\psi$$

uniformly, and consequently

$$T_\epsilon(\psi) \longrightarrow T(\psi) \quad \text{as } \epsilon \to 0.$$

Note that $\psi_\epsilon \longrightarrow \psi$ uniformly. Let

$$(DT_\psi)_\epsilon(x) = (T_{D\psi})_\epsilon(x) = \int D\psi(y)\, \chi_\epsilon(x-y)\, dy = (D\psi_\epsilon)(x)$$

$$= D((T_\psi)_\epsilon)(x) = D(T_{\psi_\epsilon})(x)$$

First, we will show that $\psi_\epsilon \longrightarrow \psi$ uniformly

$$\psi_\epsilon(x) = \int \psi(y)\, \chi_\epsilon(x-y)\, dy. \qquad \psi(x) = \int \psi(x)\, \chi_\epsilon(x-y)\, dy$$

$$\Rightarrow |\psi_\epsilon(x) - \psi(x)| = \left| \int (\psi(y) - \psi(x))\, \chi_\epsilon(x-y)\, dy \right| \le$$

$$\int_{\mathbb{R}^n} |\psi(y) - \psi(x)|\, \chi_\epsilon(x-y)\, dy.$$

Recall that $\operatorname{supp} \chi_\epsilon = \epsilon K$. $\Rightarrow$ $x - y \in \epsilon K$, $K = \operatorname{supp} \chi$.
$\Rightarrow$ For each $x \in \mathbb{R}^n$, $y \in x + \epsilon K$. contributes to the integral.

$$\Rightarrow \int_{\mathbb{R}^n} |\psi(y) - \psi(x)|\, \chi_\epsilon(x-y)\, dy = \int_{x+\epsilon K} |\psi(y) - \psi(x)|\, \chi_\epsilon(x-y)\, dy$$

$\Rightarrow$ If we choose ϵ sufficiently small, since K is bounded, given $\delta > 0$, $|\psi(y) - \psi(x)| < \delta$ for all $y \in x + \epsilon K$. Since ψ is uniformly continuous, δ is dependent