$\Rightarrow$ For each $x \in M$, $\exists$ an open set $U_x \subset M$ s.t $U_x \ni x$, and

there exists a section $\bar{f}_x \in \dfrac{\mathcal{M}^*(U_x)}{\mathcal{O}^*(U_x)}$ s.t

$$(\bar{f}_x)_p = \varinjlim_{V} \bar{f}_x = f(p), \quad \text{where } V \ni p.$$

$\Rightarrow \{U_x\}_{x \in M}$ is an open covering of M. with

$$\bar{f}_x = f_x\,\mathcal{O}^*(U_x) \in \frac{\mathcal{M}^*(U_x)}{\mathcal{O}^*(U_x)}, \quad \text{where } f_x \in \mathcal{M}^*(U_x).$$

Suppose $U_x \cap U_y \neq \phi$. We want to show that

$$\frac{f_x}{f_y} \in \mathcal{O}^*(U_x \cap U_y).$$

For a fixed point $p \in U_x \cap U_y$, $(\bar{f}_x)_p = (\bar{f}_y)_p$

$$\Leftrightarrow \quad \bar{f}_x|V_p = \bar{f}_y|V_p \quad \Rightarrow \quad f_x|_{V_p}\,\mathcal{O}^*(V_p) = f_y|_{V_p}\,\mathcal{O}^*(V_p)$$

$$\Leftrightarrow \quad \frac{f_x|V_p}{f_y|V_p} \in \mathcal{O}^*(V_p).$$

Since $\{V_p\}_{p \in U_x \cap U_y}$ is an open covering of $U_x \cap U_y$, and

$$\frac{f_x}{f_y}\Big|_{V_p \cap V_q} = \frac{f_x|V_p \cap V_q}{f_y|V_p \cap V_q} = \frac{f_x|V_q \cap V_p}{f_y|V_q \cap V_p} = \frac{f_x}{f_y}\Big|_{V_q \cap V_p},$$

$\dfrac{f_x}{f_y} \in \mathcal{O}^*(U_x \cap U_y)$ by the property (2) (P 35) of the sheaf. $\lrcorner\!\lrcorner$ $\mathrm{Ord}_{V_p}(f)$ is independent of p.