$\Rightarrow u \dfrac{\partial u}{\partial x_1} = 0$ & $v \dfrac{\partial v}{\partial y_1} = 0$

If $u \neq 0$, $\dfrac{\partial u}{\partial x_1} = 0 = \dfrac{\partial v}{\partial y_1}$, and if $v \neq 0$, $\dfrac{\partial v}{\partial y_1} = \dfrac{\partial u}{\partial x_1} = 0$.

$\Rightarrow$ At $(z_1^\circ, z_2^\circ) \in \{|f| \neq 0\}$, with $Dg = 0$,

that $\dfrac{\partial u}{\partial x_1} = \dfrac{\partial u}{\partial x_2} = \dfrac{\partial v}{\partial y_1} = \dfrac{\partial v}{\partial y_2} = \dfrac{\partial v}{\partial x_1} = \dfrac{\partial v}{\partial x_2} = 0$

$\Leftrightarrow \dfrac{\partial f}{\partial z_1} = \dfrac{\partial f}{\partial z_2} = 0$.

$V = \{ \dfrac{\partial f}{\partial z_1} = 0 = \dfrac{\partial f}{\partial z_2} \}$ is an algebraic variety

(i) relatively prime in $\mathcal{O}_2$

$\dfrac{\partial f}{\partial z_1}, \dfrac{\partial f}{\partial z_2} \Rightarrow V$ is 0-dimensional $\Rightarrow V$ is a set of isolated points.

(ii) not relatively prime in $\mathcal{O}_2$.

$\dfrac{\partial f}{\partial z_1} = h k_1 \quad \dfrac{\partial f}{\partial z_2} = h k_2$. where k_1 & k_2 are relatively prime

$h(0,0) = 0$. $\Rightarrow$ On $\{h = 0\}$, f is constant, and $f = 0$ on $\{h = 0\}$. $\Rightarrow$ On $\{f \neq 0\}$, as we saw (i), the critical points are isolated.

In general we can do the process above, so we can conclude that $\epsilon > 0$ sufficiently small, f has a set of isolated critical points in $\overline{B}(0, \epsilon) - \{f = 0\}$.

$\Rightarrow$ Consider $g = |f|^2 : \overline{B}(0, \epsilon) \longrightarrow \mathbb{R}$

$\Rightarrow$ Since the critical points are isolated outside $\{f = 0\}$, we may choose $\delta > 0$ s.t g has no critical point in $g^{-1}(0, \delta]$. $\Rightarrow$ Then according to P12 Theorem 3.1 (Morse Theory by J. Milnor), $g^{-1}(\delta')$ is homotopy (diffeomorphic) equivalent to $g^{-1}(\delta)$,