$\cdot\Gamma$ Let $S = F \cap \sigma(h)$ be the smooth conic curve in $\sigma(h)$. We may assume that $\sigma(h) = \{[*,*,*,0,0,0]\}$, and $S = X_0^2 + X_1^2 + X_2^2 = 0$. Suppose $\tilde{\Phi}(a_0X_0 + a_1X_1 + a_2X_2 = 0) \in S$, where $\tilde{\Phi} : G(2,4) \longrightarrow G \subset \mathbb{P}^5$.

$\Rightarrow$ Assume $a_2 \neq 0$. $\Rightarrow$ $[1, 0, -\frac{a_0}{a_2}, 0 \cdots]$ & $[0, 1, -\frac{a_1}{a_2}, 0 \cdots]$ lie in $a_0X_0 + a_1X_1 + a_2X_2 = 0$

$\Rightarrow$ $\tilde{\Phi}(a_0X_0 + a_1X_1 + a_2X_2 = 0) = (e_1 - \frac{a_0}{a_2}e_3) \wedge (e_2 - \frac{a_1}{a_2}e_3)$

$= e_1 \wedge e_2 - \frac{a_0}{a_2} e_3 \wedge e_2 - \frac{a_1}{a_2} e_1 \wedge e_3$ by P256.

$\Rightarrow$ Since $[1, -\frac{a_1}{a_2}, +\frac{a_0}{a_2}]$ lies in S,

$$1 + \left(\frac{a_1}{a_2}\right)^2 + \left(\frac{a_0}{a_2}\right)^2 = 0. \Rightarrow a_0^2 + a_1^2 + a_2^2 = 0$$

Conversely, given a line $a_0X_0 + a_1X_1 + a_2X_2 = 0$ with $a_0^2 + a_1^2 + a_2^2 = 0$, we can show that the line is tangent to $S = X_0^2 + X_1^2 + X_2^2 = 0$. Assume $a_0 \neq 0$.

$X_0 = -\frac{a_1}{a_0}X_1 - \frac{a_2}{a_0}X_2$. Plug in S.

$$\left(\frac{a_1}{a_0}X_1 + \frac{a_2}{a_0}X_2\right)^2 + X_1^2 + X_2^2 = 0$$

$$\frac{a_1^2}{a_0^2}X_1^2 + \frac{a_2^2}{a_0^2}X_2^2 + \frac{2a_1a_2}{a_0^2}X_1X_2 + X_1^2 + X_2^2 =$$

$$\frac{a_1^2 + a_0^2}{a_0^2}X_1^2 + \frac{a_2^2 + a_0^2}{a_0^2}X_2^2 + \frac{2a_1a_2}{a_0}X_1X_2$$

$$= -\frac{a_2^2}{a_0^2}X_1^2 - \frac{a_1^2}{a_0^2}X_2^2 + \frac{2a_1a_2}{a_0}X_1X_2 = -\left(\frac{a_2}{a_0}X_1 - \frac{a_1}{a_0}X_2\right)^2$$

$= 0 \Rightarrow a_1X_2 = a_2X_1 \Rightarrow X_2 = a_2 \quad X_1 = a_1 \Rightarrow X_0 = a_0$

$\Rightarrow$ $a_0X_0 + a_1X_1 + a_2X_2 = 0$ is tangent to S at $[a_0, a_1, a_2]$. $\lrcorner$