Γ

$$H_{-s} \otimes H_s \longrightarrow \mathbb{C}$$
$$(u\ ,\ v) \longmapsto \langle u, v\rangle = \sum u_\xi \overline{v_\xi}.$$

$$|\langle u, v\rangle| = \left|\sum u_\xi \overline{v_\xi}\right| = \left|\sum (1+\|\xi\|^2)^{-\frac{s}{2}} u_\xi\,(1+\|\xi\|^2)^{\frac{s}{2}} \overline{v_\xi}\right|$$
$$\leq \left(\sum (1+\|\xi\|^2)^{-s} |u_\xi|^2\right)^{\frac{1}{2}} \left(\sum (1+\|\xi\|^2)^{s} |v_\xi|^2\right)^{\frac{1}{2}} = \|u\|_{-s}\,\|v\|_s$$

$$H_{-s} \longrightarrow H_s^*$$
$$u \longmapsto \lambda_u$$

$\lambda_u(v) = \langle v, u\rangle. \Rightarrow \lambda_u$ is a continuous linear functional on H_s.

$\lambda_u(v) = 0 = \sum v_\xi \overline{u_\xi} = 0$ for all v.
$\Rightarrow \overline{u_\xi} = 0 \Rightarrow u = 0. \Rightarrow \lambda_u = 0 \Rightarrow$ one to one.

Given any linear functional λ on H_s,
let $\lambda(e^{i\langle \xi, x\rangle}) = \overline{\lambda_\xi}$, $\Rightarrow \sum \overline{\lambda_\xi}\, e^{i\langle \xi, x\rangle} = u$.
is in H_{-s}, for, since H_s is a Hilbert space,
$\exists\ \eta \in H_s$ s.t

~~$\lambda(\eta) = \langle$~~ $\lambda(v) = \langle v, \eta\rangle$