Date

Suppose that $p_1 = p_2 \neq p_3 = q$. We will do the derivation on p . Now if $f \in M(S)$ with $(f) + D \geq 0$ then df is a meromorphic 1-form on S holomorphic on $S - \{p, q\}$, with no periods, no residues, and a pole of order $\leq 2, 3$ at p & q. Conversely, given any such differential η, the meromorphic function

$$f(p) = \int_{p_0}^{p} \eta$$

is well-defined and satisfies $(f) + D \geq 0$. Since $df = df' \iff f = f' + \lambda$, $\lambda \in \mathbb{C}$, we see that the dimension of $H^0(S, \mathcal{O}(2p+q))$ is one more than the dimension of the vector space V of differentials of the second kind holomorphic on $S - \{p, q\}$ with no periods and poles of order $\leq 2, 3$ at p, & q.

"By the Kodaira vanishing theorem, for any $p \in S$
$$H^1(S, \Omega^1(p)) = 0,$$
and so from the exact sequence
$$0 \to \Omega^1(p) \to \Omega^1(2p) \to \mathbb{C}_p \to 0.$$
we see that $\exists$ a meromorphic form on S, holomorphic on $S - \{p\}$ and having a double pole at p; clearly this form can not have any residue.

It follows that if we let z_λ be a local coordinate around the point p_λ, for any sequence a_1, a_2, a_3 of complex numbers there exists a meromorphic 1-form η_a on S, holomorphic on $S - \{p, q\}$, and having principal parts

$$\eta_a(z) = (a_1 z_1^{-2} + [0])\, dz_1 + (a_2 z_2^{-2} + [0])\, dz_2 + (a_3 z_3^{-2} + [0])\, dz_3 \text{''}$$

PAGE