$$\Rightarrow \operatorname{Ext}_{\mathcal{O}}^{r-1}(I/I', \mathcal{O}) = 0. \qquad \text{Q.E.D.}$$

$\ulcorner$ $\varphi(f_r' g) = 0$ since $f_r' g \in I'$ and $\varphi : I/I' \longrightarrow \mathcal{O}/I'_{r-1}$.

$\Rightarrow$ $f_r' \, \varphi(g) = 0 \iff (f_r' + I'_{r-1})(\varphi(g) + I'_{r-1}) = I'_{r-1}$

$\Rightarrow$ $\varphi(g) \in I'_{r-1}$ since $f_r' + I'_{r-1}$ is not a zero divisor.

$\Rightarrow$ $\operatorname{Hom}_{\mathcal{O}}(I/I', \mathcal{O}/I'_{r-1}) = 0 \Rightarrow \operatorname{Ext}_{\mathcal{O}}^{r-1}(I/I', \mathcal{O}) = 0$

If the previous statement is correct, then

$$\operatorname{Ext}_{\mathcal{O}}^{r-k}(I/I', \mathcal{O}/I'_{k-1}) = 0 \text{ for all } 1 \le k \le r \qquad \lrcorner$$

We now return to the local duality theorem. Let $I = \{f_1, \cdots f_n\}$ and $I' = \{f_1', \cdots f_n'\}$ be regular ideals with $I' \subset I$, and denote by Ω^n the stalk at the origin of the sheaf of holomorphic n-forms. A choice of coordinates $z_1, \cdots z_n$ near the origin induces an isomorphism

$$\mathcal{O} \cong \Omega^n$$

given by

$$g(z) \longrightarrow g(z)\, dz_1 \wedge \cdots \wedge dz_n .$$

We recall that the pairing

$$\operatorname{res}_f : \mathcal{O}/I \otimes \mathcal{O}/I \longrightarrow \mathbb{C}$$

defined by

$$\operatorname{res}_f(h, g) = \operatorname{Res}_{\{0\}} \left(\frac{g(z)\, h(z)\, dz_1 \wedge \cdots \wedge dz_n}{f_1(z) \cdots f_n(z)} \right)$$

depends on the choice of generators f_i for I and local coordinates $z_1, \cdots z_n$. The behavior of res_f under changing generators for I or changing local coordinates is given by the transformation formula. This brings us to the