$$\begin{array}{ccccccc} 0 \longrightarrow & \mathcal{O}_M(V'-V) & \longrightarrow & \mathcal{O}_M(V') & \longrightarrow & \mathcal{O}_V(V') & \longrightarrow 0 \\ & \| & & \| & & \| & \\ & \mathcal{O}_M(L) & & \mathcal{O}_M(L') & & \mathcal{O}_V(V') & \end{array}$$

needed smoothness & irreducibility

$$\Rightarrow \quad \chi(L) = \chi(L') - \chi([V']|_V)$$

$$\begin{aligned} \Rightarrow \quad \chi([V']|_V) &= -\pi(V) + \deg([V']|_V) + 1 \\ &= -\pi(V) + [V']\cdot V + 1 \\ &= -\pi(V) + L'\cdot D + 1 \\ &= -\pi(D) + L'\cdot D + 1 \end{aligned}$$

since the intersection # depends only on homology classes and D & L' are homologous to V and V' respectively, see P144.

$$\Rightarrow \quad \chi([V']|_V) = -\frac{D\cdot D + D\cdot K}{2} + L'\cdot D$$

$$\begin{aligned} \Rightarrow \quad \chi(L) &= \chi(L') + \pi(D) - L'\cdot D - 1 \\ &= \chi(\mathcal{O}_M) + \frac{L'\cdot L' - L'\cdot K}{2} + \frac{D\cdot D + D\cdot K}{2} - L'\cdot D \end{aligned}$$

since $L' = [L+D] = [V']$, needed smoothness & irreducibility

$$\begin{aligned} &= \chi(\mathcal{O}_M) + \frac{L'\cdot L' - L'\cdot K + D\cdot D + D\cdot K - 2L'\cdot D}{2} \\ &= \chi(\mathcal{O}_M) + \frac{1}{2}\{(L+D)\cdot(L+D) - (L+D)\cdot K + D\cdot D + D\cdot K - 2L'\cdot D\} \\ &= \chi(\mathcal{O}_M) + \frac{1}{2}(L\cdot L + \cancel{2L\cdot D} + \cancel{D\cdot D} - L\cdot K - \cancel{D\cdot K} + \cancel{D\cdot D} + \cancel{D\cdot K} - \cancel{2L\cdot D} - \cancel{2D\cdot D}) \\ &= \chi(\mathcal{O}_M) + \frac{1}{2}(L\cdot L - L\cdot K) \end{aligned}$$

Note: $V' = M \cap H$ smooth $\Rightarrow$ V' irreducible

See P174 & P159, by Lefschetz hyperplane theorem,

$$H_0(V', \mathbb{Z}) \xrightarrow{\cong} H_0(M, \mathbb{Z}) = \mathbb{Z} \quad \Rightarrow \quad V' \text{ is connected}$$