Indeed, we can give an explicit recipe for the reconstruction of X from B: first embed B in $\mathbb{P}^3$ as a quintic curve E_D — by the above, it will not matter what divisor D we employ for the embedding. Then blow up $\mathbb{P}^3$ along the curve E_D, and blow down the family of proper transforms in $\widetilde{\mathbb{P}}^3_{E_D}$ of the tri-chords of E_D into a curve.

D. a divisor of degree 5 on B.

① $H^0(B, \mathcal{O}(D)) = \langle \sigma_0, \sigma_1, \sigma_2, \sigma_3 \rangle$

$$\begin{array}{ccc} B & \longrightarrow & \mathbb{P}^3 \\ \rotatebox[origin=c]{-90}{$\in$} & & \rotatebox[origin=c]{-90}{$\in$} \\ p & \longmapsto & [\sigma_0(p), \sigma_1(p), \sigma_2(p), \sigma_3(p)] \end{array}$$

see P1004 note.

② Consider the blow-up $\widetilde{\mathbb{P}}^3_{E_D}$ of $\mathbb{P}^3$ along E_D.

$$\widetilde{\mathbb{P}}^3_{E_D} \xrightarrow{\pi} \mathbb{P}^3$$

Question? Does there exist $L \in A$ s.t $E_L = E_D$ up to automorphisms of $\mathbb{P}^3$? $\Rightarrow$ Since $\rho : \mathcal{J}(B) \to \mathrm{Pic}^0(B)$ is surjective, $\exists$ $L \in A$ s.t

$\rho(L) = [E_L \cap H - 2K_B - p_0] = [D - 2K_B - p_0]$

$\Rightarrow$ More precisely, $E_L \cap H = \tilde{f}^*(E_L \cap H)$, $\tilde{f} : B \to E_L \subset \mathbb{P}^3$.

$\Rightarrow$ $[E_L \cap H] = [D]$. $\Rightarrow$ By P492, $E_L = E_D$ up to automorphisms of $\mathbb{P}^3$. Thus $\widetilde{\mathbb{P}}^3_{E_D} \cong \widetilde{X}_L$, by P604. As the note on P800, by blowing down the proper