$\mathcal{V}_\rho(U) = \{ \sigma : U \longrightarrow \tilde{U} \times_{\pi_1} V$ defined by $x^* \longmapsto [(x, v)] \}$

Third, the qth direct image sheaf $R^q_\pi(Q)$ is the sheaf constructed in this way from the representation of $\pi_1(B, x_0)$ on $H^q(F_{x_0}, Q)$.

$R^q_\pi(Q) = \mathcal{V}_\rho$

It is instructive to sketch the derivation of the Leray spectral sequence in de Rham cohomology. At any point $p \in E$ we let

$$T_p(F) = \ker \{ \pi_* : T_p(E) \longrightarrow T_{\pi(p)}(B) \}$$

be the tangent space to the fiber $F_{\pi(p)}$ passing through p.

$E|_U \cong U \times F \qquad T(E|_U) \cong TU \times TF \longrightarrow TU$

$\Rightarrow \quad T_p E \cong T_x F, \quad (\pi \varphi, x) \longleftrightarrow p.$

Setting

$$F^p(\Lambda^n T_p(E)) = (\Lambda^p T_p(F)) \wedge (\Lambda^{n-p} T_p(E))$$

defines a filtration $\{ F^p(\Lambda^n T(E)) \}$ on the exterior powers of the tangent bundle $T(E)$, and we let $\{ F^p(\Lambda^n T^*(E)) \}$ be the dual filtration of the exterior powers of $T^*(E)$ given by

$$F^p(\Lambda^n T^*(E)) = \mathrm{Ann}\,(F^{n-p+1}(\Lambda^n T(E))).$$