$$= (-1)^n t^{-n} \left\{ 1 - \frac{\sum p^1(A)}{2} t + \left(\frac{\sum p^1(A)^2 + \sum p^2(A)}{12} \right) t^2 - \frac{\sum p^1(A)\, p^2(A)}{24} t^3 + \cdots \right\},$$

where the summations occur over increasing indices.

Γ

$$BAB^{-1} = \begin{pmatrix} \lambda_1 & * & * & \cdots & * \\ 0 & \lambda_2 & * & & * \\ \vdots & \vdots & \lambda_3 & & * \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & & \lambda_n \end{pmatrix}$$

$$\Rightarrow \det A = \Pi \lambda_i \qquad e^{tBAB^{-1}} = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} + t \begin{pmatrix} \lambda_1 & & * \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$

$$+ \frac{1}{2!} t^2 \begin{pmatrix} \lambda_1 & & * \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}^2 + \cdots + \frac{1}{n!} t^n \begin{pmatrix} \lambda_1 & & * \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}^n + \cdots$$

$$\Rightarrow e^{tBAB^{-1}} = \begin{pmatrix} e^{t\lambda_1} & * & & * \\ 0 & e^{t\lambda_2} & & \\ \vdots & & \ddots & \\ 0 & 0 & & e^{t\lambda_n} \end{pmatrix} \quad \text{since} \quad \begin{pmatrix} a_1 & & * \\ & \ddots & \\ 0 & & a_n \end{pmatrix} \begin{pmatrix} b_1 & & * \\ \vdots & \ddots & \\ 0 & & b_n \end{pmatrix}$$

$$= \begin{pmatrix} a_1 b_1, & * & \cdots & * \\ 0, & a_2 b_2, & \cdots & * \\ & & \ddots & \\ 0 & & & a_n b_n \end{pmatrix}.$$

For simplicity, $n = 2$.

$$\prod_{i=1}^{2} \frac{\lambda_i}{1 - e^{t\lambda_i}} = ? = (-1)^2 t^2 \prod_{i=1}^{2} \frac{\lambda_i t}{1 - e^{t\lambda_i}}$$

$$f(t) = \frac{-t}{1 - e^{t\lambda}} = f(0) + f'(0)\, t + \frac{f''(0)}{2!} t^2 + \frac{f'''(0)}{3!} t^3 + \cdots$$

as Taylor series.