

$\Rightarrow 0 = 2 - h^{0,1}(A) = 2 - g(A)$ by P494.

$$K_A \equiv 0 \Rightarrow m K_A \equiv 0 \Rightarrow H^0(A, \mathcal{O}(mK_A)) = \mathbb{C}$$

$\Rightarrow p_g(A) = 1 \Rightarrow$ By the definitions on P572 ~ P573,

The Kodaira number of A is 0, i.e., $\kappa(A) = 0$

\Rightarrow By the result on P583 ~ P584, A is an Abelian variety since $g(A) = 2$ and $\kappa(A) = 0$.

See P301 for the definition of Abelian variety. \square

The involutions ι and ι' are readily identified: Let z_1, z_2 be Euclidean coordinates on \mathbb{C}^2 and consider the holomorphic 1-forms dz_1, dz_2 on A : the forms

$$\omega_i = dz_i + \iota^* dz_i$$

are invariant under ι^* , and so we can write

$$\omega_i = j^* \tilde{\omega}_i$$

for $\tilde{\omega}_1, \tilde{\omega}_2$ holomorphic 1-forms on $S^1 \times \mathbb{R}$.

\square $\iota^* \omega_i = \iota^* dz_i + \iota^* \iota^* dz_i = \iota^* dz_i + dz_i = \omega_i$, since $\iota \circ \iota = \text{id}$. Define $j_* \omega_i$ by

$$j_* \omega_i(v) = \omega_i(\tilde{v}), \quad j_* \tilde{v} = v.$$

$\Rightarrow j_*$ is isomorphic, i.e., $j_*: T_x(A - j^{-1}R) \rightarrow T_{j(x)}(S^1 \times \mathbb{R})$ is isomorphic, since j is a covering map.

If $j_*(\tilde{v}') = v$, then $\iota_* \tilde{v} = \tilde{v}'$.

$$\Rightarrow j_* \omega_i(v) = \omega_i(\tilde{v}') = \omega_i(\iota_* \tilde{v}) = (\iota^* \omega_i)(\tilde{v}) = \omega_i(\tilde{v}) \Rightarrow j_* \omega_i \text{ is a well-defined form on } S^1 \times \mathbb{R}.$$

\Rightarrow Clearly, if we let $j_* \omega_i = \tilde{\omega}_i$, then