$\Rightarrow$ If there are infinitely many x_i's, then $\exists$ a limit point x_0 in $f^{-1}(K)$. $\Rightarrow$ For any open set $U \ni x_0$, $U \cap M_i \neq \phi$, for i infinitely many. $\Rightarrow$ Contradiction, since M is expressed as the union of infinitely many irreducible components. Here $M = \bigcup M_i$ where M_i's are irreducible. $\Rightarrow$ If we prove $f(M_i)$ is analytic, then $f(M) = \bigcup f(M_i)$ is analytic, for. choose U_i's s.t $f(M_i) \cap U_i = (g_{i1} = \cdots = g_{ij} = 0)$

$\Rightarrow$ let $U = \bigcap U_i$ and, for exaple, if we have $f(M_1) \cap U_1 = (g_{11} = g_{12} = 0)$, $f(M_2) \cap U_2 = (g_{21} = g_{22} = g_{23} = 0)$, then.

$$(V_1 \cap V_2) \cup (U_1 \cap U_2 \cap U_3)$$
$$= \{V_1 \cup (U_1 \cap U_2 \cap U_3)\} \cap \{V_2 \cup (U_1 \cap U_2 \cap U_3)\}$$
$$= \{(V_1 \cup U_1) \cap (V_1 \cup U_2) \cap (V_1 \cup U_3)\} \cap \{(V_2 \cup U_1) \cap (V_2 \cup U_2) \cap (V_2 \cup U_3)\} \Rightarrow \{g_{11} g_{21} = 0 = g_{11} g_{22} = 0 = g_{11} g_{23} = 0 = g_{12} g_{21} = g_{12} g_{22} = g_{12} g_{23} = 0\}$$

2. The proof is by induction on $n = \dim M$. Let $M^* = M - M_s$ be the complex manifold of smooth points of M, and choose a point $p_0 \in M^*$ where the Jacobian matrix of $f: M^* \longrightarrow \mathbb{C}^N$ has maximum rank $k \leq n$. If $k < n$, by our assumption we may choose a k-dimensional analytic subvariety S in M passing through p_0 and such that $f|_S$ has maximum rank k.

「 See p 395, choose $\Lambda_k \subset T_pM$ s.t $\Lambda_k \cap \ker f_* = (0)$. 」