five conics the hypersurfaces $V_{C_1}, \cdots V_{C_5} \subset W$

1. met transversely away from the subvariety of singular conics, and
2. Contained no singular conics in common,

the answer to our question would be easy: it would just be the fivefold self-intersection $(\deg V_C)^5$ of $V_C$ in $W \cong \mathbb{P}^5$.

⌈ Since every smooth conic is projectively isomorphic each other, $V_C$ is homologous to $V_{C'}$, $C, C'$ smooth conics. ⌋

Unfortunately, matters are not so simple: while assertion 1 above is the case, and half of assertion 2, namely,

2'. for $C_1, \cdots C_5$ generically chosen, no conic consisting of two distinct lines will be tangent to all five

holds, the problem is that all the hypersurface $V_C \subset W$ will contain the subvariety

$$W_2 = \{ 2L \}_{L \subset \mathbb{P}^2}$$

of double lines.