NO.

DATE . . ( )

On the other hand, if h is a generic hyperplane passing through p_0 and another double point p_i of S, then the curve C_h, having two double points is elliptic, and the map $r: \tilde{C}_h \longrightarrow \mathbb{P}^1$, expressing $\tilde{C}_h$ as a 2-sheeted cover of $\mathbb{P}^1$, can be branched at at most four points other than $r(p_i)$.

⌈⌈

$C_h' \longrightarrow C_h \subset \mathbb{P}^2$ is the desingularization

$\Rightarrow$ $\chi(C_h) = -2 + 2 = 0$ by the result on p508

since $\exists$ a smooth quartic in S which is holomologous to C_h. $\Rightarrow$ $0 = 2 - b_1$ $\Rightarrow$ $b_1 = 2$ $\Rightarrow$ $g(C_h) = \frac{b_1}{2} = 1$

$\Rightarrow$ C_h is elliptic.

Let $\tilde{\tilde{C}}_h$ be the desingularization of $\tilde{C}_h$.

$\Rightarrow$

$$\begin{tikzcd} \tilde{\tilde{C}}_h \arrow[r, "f"] & \tilde{C}_h \arrow[r, "r"] \arrow[d] & \mathbb{P}^1 \\ & C_h \arrow[ur] & \end{tikzcd}$$

$C_h \longrightarrow$ plane quartic with two double points.

$\Rightarrow$ $\tilde{\tilde{C}}_h$ is the desingularization of C_h, too.

$\Rightarrow$ $g(C_h) = g(\tilde{\tilde{C}}_h) = \dfrac{(4-1)(4-2)}{2} - 2 = 1$ by the