On properties of $\mathbb{R}^{2n}$ — subharmonic functions.

Properties of functions of class (C^2)

(1) If we consider in D an analytic mapping

$$z_k = \psi_k(t_1, \cdots t_p)$$

in which the ψ_k are holomorphic, we deduce from the fact that

$$L(V) = d_z d_{\bar{z}} V = d_t d_{\bar{t}} V \geq 0$$

that the image of V in C^p is plurisubharmonic.

⌈ $$L(V) = d_z d_{\bar{z}} V = d_z \left(\frac{\partial V}{\partial \bar{z}_q} d\bar{z}_q \right) = \frac{\partial^2 V}{\partial z_p \partial \bar{z}_q} \cdot dz_p \wedge d\bar{z}_q$$

$$d_{\bar{z}} V = \bar{\partial}_z V = \frac{\partial V}{\partial \bar{z}_k} d\bar{z}_k = \frac{\partial V}{\partial \bar{t}_\ell} \frac{\partial \bar{t}_\ell}{\partial \bar{z}_k} \frac{\partial \bar{z}_k}{\partial \bar{t}_i} d\bar{t}_i$$

$$= \frac{\partial V}{\partial \bar{t}_\ell} \frac{\partial \bar{t}_\ell}{\partial \bar{t}_i} d\bar{t}_i = \frac{\partial V}{\partial \bar{t}_\ell} d\bar{t}_\ell = \bar{\partial}_t V$$

$\Rightarrow \bar{\partial}_z = \bar{\partial}_t$. Similarly, we get $\partial_z = \partial_t$.

$\Rightarrow \partial_z \partial_{\bar{z}} V = \partial_t \bar{\partial}_t V = L(V)$ ⌋

The trace of $V \circ \psi$ on an embedded analytic variety $(p<n)$ is plurisubharmonic, and in particular, on a complex straight line

$$z_k = z_k^0 + a_k t$$

is $\mathbb{R}^2$ — subharmonic.

(2) Suppose V is subharmonic and of class (C^2). Then Stokes' theorem yields

$$\int_B \Delta V \, d\tau = \int_{B^*} \frac{\partial V}{\partial \eta} d\sigma = \int \frac{\partial V}{\partial r}(x^0 + r\vec{\alpha}) r^{2n-1} d\omega_{2n-1}^{(\alpha)}$$