by P163 Lefschetz Theorem on (1, 1) Classes.
$\Rightarrow$ $\operatorname{im} C_1 = \ker \iota_*$.

Claim: $\operatorname{im} C_1 = H^{1,1}(M) \cap H^2(M, \mathbb{Z})$.

Clearly $H^{1,1}(M) \cap H^2(M, \mathbb{Z}) \subset \operatorname{im} C_1 = \ker \iota_*$ by the proof the Lefschetz (1, 1) theorem. Given $\gamma \in \operatorname{im} C_1 = C_1(L)$, by the result of the last paragraph on P75 and the proposition on P141, $\gamma \in H^{1,1}(M, \mathbb{C})$. $\Rightarrow \gamma \in H^{1,1}(M, \mathbb{C}) \cap H^2(M, \mathbb{Z})$. ///

We have the following isomorphism

$$\frac{H^1(M, \mathcal{O}^*)}{\ker C_1} \cong \operatorname{im} C_1$$

$$\begin{array}{ccccc} \{\text{divisors on } M\} & \longrightarrow & H^1(M, \mathcal{O}^*) & \Rightarrow & [D_1 + D_2] \\ \Psi & & \Psi & & \| \quad \text{by P134.} \\ D & \longmapsto & [D]. & & [D_1] \otimes [D_2] \end{array}$$

$\Rightarrow$ if $c_1([D]) = 0$, then, by the Poincare duality the fundamental class of D is homologous to zero, see P61. Note.

$$\Rightarrow \quad \frac{\text{Divisors on } M}{\text{homological equivalence}} \cong \frac{H^1(M, \mathcal{O}^*)}{\ker C_1}$$

We shall prove that $\begin{cases} \rho_1 = b_1, \\ \rho_2 = b_2 - \rho. \end{cases}$

Proof. Recall that for a divisor D on M, $\Omega^p(*D)$ denotes the subsheaf of the sheaf $\mathcal{M}^p$ of all