argue as follows: if the quadrics $F$ and $G$ contained a 2-plane $V_2 \subset \mathbb{P}^5$ in common, then the Gauss maps

$$\mathcal{G}_F : F \longrightarrow \mathbb{P}^{5*} \quad \text{and} \quad \mathcal{G}_G : G \longrightarrow \mathbb{P}^{5*}$$

would each map $V_2$ isomorphically (( see p135 )) onto the set $V_2^*$ of hyperplanes containing $V_2$. But then the isomorphism

$$\mathcal{G}_F^{-1} \circ \mathcal{G}_G : V_2 \longrightarrow V_2$$

would have a fixed point — i.e., for some $x \in V_2$ we would have $T_x(F) = T_x(G)$, contradicting the assumption that $F$ and $G$ meet transversely.

⌈ Since $F$ & $G$ are smooth, $\mathcal{G}_F$ & $\mathcal{G}_G$ are isomorphic by p135.

$$\mathcal{G}_F^{-1} \circ \mathcal{G}_G : \underset{\mathbb{P}^2}{\overset{}{V_2}} \longrightarrow \underset{\mathbb{P}^2}{V_2} \quad \text{is isomorphic.}$$

$$\Rightarrow \quad \mathcal{G}_F^{-1} \circ \mathcal{G}_F \in GL(3)/\mathbb{C}^* = PGL_3 \quad \text{by } p64 \sim p65.$$

Since $PGL_3$ is path-connected ($\because$ $GL(3)$ is path-connected), $(\mathcal{G}_F^{-1} \circ \mathcal{G}_G)^* = id^*$.

$$\Rightarrow L(\mathcal{G}_F^{-1} \circ \mathcal{G}_G) = \sum_p (-1)^p \operatorname{trace} (\mathcal{G}_F^{-1} \circ \mathcal{G}_G)^* |_{H^p_{DR}(\mathbb{P}^2)}$$

by the Lefschetz Fixed-Point Formula on p421.