— and hence x' — lies in the locus $T_x(G) \cap G$.

⟦ $\sigma_{2,1}(p,h)$ is a line by the result on P256 and $\sigma_{2,1}(p,h) \ni x$ $\Rightarrow$ Since $T_x(G) \cap G$ is the locus of lines in G through x, $\sigma_{2,1}(p,h) \subset T_x(G) \cap G$, and hence $x' \in \sigma_{2,1}(p,h) \subset T_x(G) \cap G$, since $l_{x'}$ passes through p and $l_{x'} \subset h = \overline{l_x, l_{x'}}$. ⟧

Note that if $x' \neq x$, then $\overline{x, x'} = \tilde{\Phi}(\sigma_{2,1}(p,h))$, $\tilde{\phi}: G(2,4) \to \mathbb{P}^5$.

The hyperplane section $T_x(G) \cap G$ thus contains the Schubert cycle $\sigma_1(l_x)$ of lines meeting l_x — but $\sigma_1(l_x)$ is itself a hyperplane section of G, and so we have:

For any $x \in G$
$$T_x(G) \cap G = \sigma_1(l_x).$$

⟦ $\sigma_1(l_x) \subset T_x(G) \cap G$

$\;\;\; \| \; H \cap G$ by the result on P256.

If $H \cap G \neq T_x(G) \cap G$, then

$H \cap T_x(G) \cap G = H \cap G$, since

$H \cap G \cap H \subset T_x(G) \cap G \cap H$.

$\Rightarrow$ If $T_x(G) \neq H$, then $H \cap G \subset \mathbb{P}^3 \cap G$

$\Rightarrow$ Since $\dim H \cap G = 3$, $H \cap G = \mathbb{P}^3$, and $\mathbb{P}^3 \subset G$.

$\Rightarrow$ $\sigma_1(l_x) = \mathbb{P}^3_{\ni x} \subset T_x(G) \cap G$ (the locus of lines through x and $T_x(G) \cap G$ is singular at x)

$\Rightarrow$ Contradiction. $\Rightarrow$ $T_x(G) \cap G = \sigma_1(l_x)$ ⟧