cteristic of the trivial line bundle, i.e

$$\chi(\mathcal{O}_M) = \sum (-1)^p h^p(M, \mathcal{O}(M\times\mathbb{C})).$$

「 $\dim H^p(M, \mathcal{O}(M\times\mathbb{C})) = \dim H^p(M, \mathcal{O}) = h^p(M, \mathcal{O}) = \dim H^{0,p}(M) = h^{0,p}(M)$ see P151.

Now for a line bundle $L$ on a Riemann surface $S$, by Kodaira – Serre duality we have

$$\begin{aligned}\chi(L) &= h^0(S, \mathcal{O}(L)) - h^1(S, \mathcal{O}(L)) \\ &= h^0(L) - h^0(K-L) \\ \chi(\mathcal{O}_S) &= h^{0,0}(S) - h^{0,1}(S) \\ &= 1-g,\end{aligned}$$

and so the Riemann – Roch formula reads simply

$$\chi(L) = \chi(\mathcal{O}_S) + c_1(L).$$

「 $\chi(L) = h^0(S, \mathcal{O}(L)) - h^1(S, \mathcal{O}(L))$

$= h^0(L) - \dim H^1(S, \mathcal{O}(L)) = h^0(L) - \dim H^{0,1}(S, L)$

$= h^0(L) - \dim H^{0,1}(L) = h^0(L) - h^{0,1}(L)$

Kodaira - Serre duality $\Rightarrow$ $= h^0(L) - \dim H^1(S, \Omega^1(K^*\otimes L)) = h^0(L) - \dim H^0(S, \Omega^1(L^*)) = h^0(L) - \dim H^0(S, \mathcal{O}(K\otimes L^*))$

$= h^0(L) - h^0(K-L)$

$\chi(\mathcal{O}_S) = h^{0,0}(S) - h^{0,1}(S) = \dim H^{0,0}(S) - \dim H^{0,1}(S) = \dim H^1(S) - \dim \overline{H^0(S, \Omega^1)} = 1 - g$

$\Rightarrow \chi(L) = h^0(L) - h^0(K-L) = d - g + 1 = d + \chi(\mathcal{O}_S)$

$= \chi(\mathcal{O}_S) + c_1(L).$