then we can hope to obtain the same sort of picture as on the torus. (Let's see this later.)

We conclude the Fourier series discussion with some remarks concerning distributions, defined as the linear functions.

$\lambda : C^\infty(T) \longrightarrow \mathbb{C}$, which are continuous

in the sense that $|\lambda(\varphi)| \leq C_\lambda \sup_{\substack{[\alpha] \leq k \\ x \in T}} |D^\alpha \varphi(x)|$ for some k.

Each distribution generates a formal Fourier series

$\sum \lambda_\xi e^{i\langle \xi, x\rangle}$ where $\lambda_\xi = \lambda(e^{-i\langle \xi, x\rangle})$.

It follows from the def of continuity of λ and the above estimate on $\sup_{x\in T} |D^\alpha \varphi(x)|$ that each distribution λ (See p 88)

is a continuous linear function on H_s for some s.

⊢ Suppose $\lambda : C^\infty(T) \longrightarrow \mathbb{C}$. and $|\lambda(\varphi)| \leq C_\lambda \sup_{\substack{[\alpha] \leq k \\ x \in T}} |D^\alpha \varphi(x)|$ for some k.

If $\varphi \in H_{[\frac{n}{2}]+1+k}$, let $s = [\frac{n}{2}] + 1 + k$.

$\sup_{\substack{\cancel{[\alpha]\leq k} \\ x \in T}} |D^\alpha \varphi(x)| \leq \|\varphi\|_{[\frac{n}{2}]+1+[\alpha]}$ by p 88.

$\Rightarrow \sup_{[\alpha]\leq k} |D^\alpha \varphi(x)| \leq \|\varphi\|_{[\frac{n}{2}]+1+[\alpha]} \leq \|\varphi\|_{[\frac{n}{2}]+1+k} = \|\varphi\|_s$

$\Rightarrow |\lambda(\varphi)| \leq C_\lambda \|\varphi\|_s$ which we will get by assuming λ can be defined on H_s.

$\Rightarrow \lambda$ is uniformly continuous on $C^\infty(T)$ with norm $\| \;\|_s$.