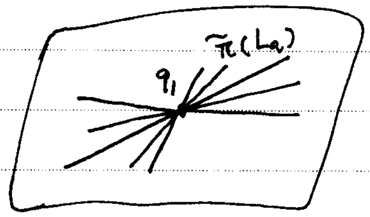
$\Rightarrow \tilde{\pi}(L)$ is a line.

If L' is an A-line, $L' \neq L$, suppose $\tilde{\pi}(L) \cap \tilde{\pi}(L') \neq \phi$. $\Rightarrow \exists\ a \in L, a' \in L'$ s.t p, a, a' are collinear.

$\Rightarrow \#(\overline{pa} \cap \tilde{S}) \geq 3 \Rightarrow \overline{pa} \subset \tilde{S} \Rightarrow \overline{pa} = L_1 \Rightarrow$

Contradiction to the fact that L' is an A-line.

Recall that two lines meet $\Leftrightarrow$ they are from different families. $\Rightarrow \{\tilde{\pi}(L_a)\}_{a \in L_1}$, where L_a is an A-line passing through $a \in L_1$, and each $\tilde{\pi}(L_a)$ contains q_1, since $L_a \cap L_1 \neq \phi$ and $\overline{pa} \cap H = L_1 \cap H = \{q_1\}$.



H

All lines (in H) containing q_1 form a pencil.

Similarly, we can show for B-lines that $\tilde{\pi}$(B-line) is in the pencil of lines through q_2.

$T_p'(\tilde{S}) \cap H$ is a line joining q_1 and q_2, since $p \notin H$, and $T_p'(\tilde{S}) \neq H$. $\Rightarrow$ For any $q \in T_p'(\tilde{S}) \cap H$, consider $\overline{pq}$. $\Rightarrow \overline{pq} \subset T_p'(\tilde{S}) \Rightarrow \exists\ \overline{pq} \in E$.

$\Rightarrow \tilde{\pi}(E) = \overline{q_1 q_2}$ ⌟

We see from this that $\tilde{\pi}$ is one-to-one on $\tilde{S} - \tilde{L}_1 - \tilde{L}_2$ and maps $\tilde{L}_1$ and $\tilde{L}_2$ onto q_1 and q_2 — i.e., $\tilde{\pi} : \tilde{S} \longrightarrow \mathbb{P}^2$ is just the blow-up of $\mathbb{P}^2$ at q_1 and q_2.

⌜ By the argument above, $\exists\ x \in \tilde{S}$ s.t $\tilde{\pi}(x) = q$ for a given $q \in H$, $q \neq q_1, q_2$.