$\Rightarrow$ By syzygy theorem,
since $0 \to (\mathcal{G}_0)_x \to (\mathcal{E}_n)_x \to \cdots \to (\mathcal{E}_0)_x \to \mathcal{F}_x \to 0$ at each stalk at x, $(\mathcal{G}_0)_x = 0$. $\Rightarrow$ $\mathcal{G}_0 = 0$.

$\Rightarrow$ We have a global syzygy

$$0 \to \mathcal{E}_n \to \mathcal{E}_{n-1} \to \cdots \to \mathcal{E}_0 \to \mathcal{F} \to 0.$$

Note that for $0 \le l \le n-1$, $\mathcal{E}_K$ is a direct sum of copies of $\mathcal{L}^{m_K}$, but all we can say about the last term is that $\mathcal{E}_n = \mathcal{O}(E)$ for some holomorphic vector bundle $E \to M$. The existence of such global syzygies for coherent sheaves is of fundamental importance.

For example $n=2$.

$$0 \to \mathcal{G}' \to \mathcal{O}^{(P)} \to \mathcal{F}(k) \to 0$$

Apply $\otimes \mathcal{L}^{-k}$,

$$0 \to \mathcal{G}' \otimes \mathcal{L}^{-k} \to \mathcal{O}^{(P)} \otimes \mathcal{L}^{-k} \to \mathcal{F} \to 0$$

Let $\mathcal{E}_0 = \mathcal{O}^{(P)} \otimes \mathcal{L}^{-k}$ which is locally free and is a direct sum of copies of $\mathcal{L}^{-k}$, i.e., $\mathcal{E}_0 = \underbrace{\mathcal{L}^{-k} \oplus \cdots \oplus \mathcal{L}^{-k}}_{p}$.

$\Rightarrow$ By Theorem A, again,

$$0 \to \mathcal{G}_1 \to \mathcal{O}^{(q)} \to \mathcal{G}' \otimes \mathcal{L}^{-k} \otimes \mathcal{L}^{l} \to 0$$

if $\mathcal{G}' \otimes \mathcal{L}^{-k}$ is not locally free, for large l.

$$\Rightarrow \quad 0 \to \mathcal{G}_1 \otimes \mathcal{L}^{-l} \to \underset{\substack{\parallel \\ \mathcal{E}_1}}{\mathcal{O}^{(q)} \otimes \mathcal{L}^{-l}} \to \mathcal{G}' \otimes \mathcal{L}^{-k} \to 0$$

$\Rightarrow$ $\mathcal{E}_1$ is locally free and a direct sum of $\mathcal{L}^{-l}$, i.e.,

$$\mathcal{E}_1 = \underbrace{\mathcal{L}^{-l} \oplus \cdots \oplus \mathcal{L}^{-l}}_{q}.$$