$$|\phi_\epsilon(x) - \phi(x)| = \left|\int (\phi(y) - \phi(x))\, \chi_\epsilon(x-y)\, dy\right|$$

$$= \left|\int_G (\phi(x-u) - \phi(x))\, \chi_\epsilon(u)\, dy\right| \qquad (\text{let } x-y=u)$$

$$= \left|\int_G (\phi(x-u) - \phi(x))\, \chi\left(\frac{u}{\epsilon}\right)\frac{1}{\epsilon^n}\, dy\right| \qquad \left(\text{let } \frac{u}{\epsilon} = y\right)$$

$$\le \int_G |\phi(x-\epsilon y) - \phi(x)|\, \chi(y)\, dy$$

$$\le \delta$$

$\Rightarrow$ Define $\Lambda(\phi) = \lim_{\epsilon \to 0} \Lambda(\phi_\epsilon)$.

We have to check if this is well-defined.
Suppose $\exists$ a sequence $\psi_n \in C_c^\infty(G)$ s.t $\psi_n \to \phi$ with respect to $\| \ \|_0$. $\Rightarrow$ Q?; $\underbrace{\lim_{n\to\infty} \Lambda(\psi_n)}_{a} = \underbrace{\lim_{\epsilon\to 0} \Lambda(\phi_\epsilon)}_{b}$.

$$|a-b| \le |\Lambda(\psi_n) - a| + |\Lambda(\psi_n) - \Lambda(\phi_\epsilon)| + |\Lambda(\phi_\epsilon) - b|$$

If we choose n large enough and ϵ sufficiently small, we can take care of the first and third terms.
To take care of $|\Lambda(\psi_n) - \Lambda(\phi_\epsilon)|$,

$|\Lambda(\psi_n) - \Lambda(\phi_\epsilon)| = |\Lambda(\psi_n - \phi_\epsilon)|$ $\Rightarrow$ We have only to prove that supp ψ_n is in some compact subset K in G.
$\Rightarrow$ We can bound $|\Lambda(\psi_n - \phi_\epsilon)| \le C\|\psi_n - \phi_\epsilon\|_0 \le C\|\psi_n - \phi\|_0 + C\|\phi - \phi_\epsilon\|_0$.

Note that $\psi_n \to \phi$ w.r.t $\| \ \|_0$.
Consider a set $E \subset C_c^0(G)$ whose support lie no $K \subset G$. Then there are functions $\phi_m \in E$ and there are