Since $\det A = b_1$, the transformation law states exactly that the diagram

$$\begin{tikzcd}
\mathcal{O}/I(f) \otimes \mathcal{O}/I(f) \arrow[d, "\alpha"'] \arrow[r, "\mathrm{res}_f"] & \mathbb{C} \arrow[d, equal] \\
\mathcal{O}/I(f') \otimes \mathcal{O}/I(f') \arrow[u, shift left=6, "\pi"] \arrow[r, "\mathrm{res}_{f}"] & \mathbb{C}
\end{tikzcd}$$

is commutative; i.e.

$$\mathrm{res}_f(g, h) = \mathrm{res}_{f'}(b_1 g, h)$$

for all $g, h \in \mathcal{O}$.

$\ulcorner$

$$\mathrm{res}_f(g, h) = \left(\frac{1}{2\pi\sqrt{-1}}\right)^n \int_{|f_i| = \epsilon_i} \frac{g h \, dz_1 \wedge \cdots \wedge dz_n}{f_1 \cdots f_n}$$

$$= \left(\frac{1}{2\pi\sqrt{-1}}\right)^n \int_{|f_i'| = \epsilon_i'} \frac{g h \cdot \det A \, dz_1 \wedge \cdots \wedge dz_n}{f_1' \cdots f_n'}, \qquad f' = Af$$

$$= \left(\frac{1}{2\pi\sqrt{-1}}\right)^n \int_{|f_i'| = \epsilon_i} \frac{g h \, b_1 \, dz_1 \wedge \cdots \wedge dz_n}{f_1' \cdots f_n'} \qquad \text{since } \det A = b_1$$

$$= \mathrm{res}_{f'}(b_1 g, h)$$

$$\begin{tikzcd}
g \arrow[d, mapsto] & \mathcal{O}/I(f) \arrow[d, "\alpha"'] & \otimes & \mathcal{O}/I(f) \arrow[r, "\mathrm{res}_f"] \arrow[dr, phantom, "\circlearrowright"] & \mathbb{C} \arrow[d, equal] \\
b_1 g \in & \mathcal{O}/I(f') & \otimes & \mathcal{O}/I(f') \arrow[u, "\pi"'] \arrow[r, "\mathrm{res}_f"] & \mathbb{C}
\end{tikzcd}$$

$g \in \mathcal{O}/I(f)$, $h \in \mathcal{O}/I(f)$, $h \in \mathcal{O}/I(f')$, $h \mapsto h$

$\lrcorner$