⊢ $\mathcal{O}_Z = \mathcal{O}/I$. By assumption, the support of Z is of dim zero. $\Rightarrow$ $\mathcal{O}_Z \neq 0$ at finite set of points. So does $\underline{Ext}^n_{\mathcal{O}}(\mathcal{O}_Z, \Omega^n)$.

$H^0(M, \mathcal{O}_Z) = \bigoplus_{p \in Z} \mathcal{O}_{Z,p}$ since every element in $H^0(M, \mathcal{O}_Z)$ is determined by the values at $p \in Z$. $\cdots$ ⊛

$H^0(M, \underline{Ext}^n_{\mathcal{O}}(\mathcal{O}_Z, \Omega^n)) \ni \sigma \Rightarrow \sigma : M \longrightarrow \underline{Ext}^n_{\mathcal{O}}(\mathcal{O}_Z, \Omega^n)(M)$.

$\Rightarrow \sigma(x) \in \underline{Ext}^n_{\mathcal{O}}(\mathcal{O}_Z, \Omega^n)_x = Ext^n_{\mathcal{O}}(\mathcal{O}_{Z,x}, \Omega^n_x)$

$\Rightarrow \sigma = (\sigma(p))_{p \in Z}$. $\Rightarrow$ $H^0(M, \underline{Ext}^n_{\mathcal{O}}(\mathcal{O}_Z, \Omega^n)) = \bigoplus_{p \in Z} \underline{Ext}^n_{\mathcal{O}_p}(\mathcal{O}_{Z,p}, \Omega^n_p)$, which is the same as ⊛ in principle.

$H^q(M, \mathcal{O}_Z) \underset{\substack{\nwarrow \\ \text{by } \wedge \\ p\,6p_0}}{=} H^q(M, \underline{Ext}^n_{\mathcal{O}}(\mathcal{O}_Z, \Omega^n)) = 0$, as follows.

by $Ext^n_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) = \mathcal{O}/I$

$$H^q(M, \mathcal{O}_Z) = \bigoplus_{p \in Z} H^q(M, \mathcal{O}_{Z,p})$$

$H^q(M, \mathcal{O}_{Z,p} \otimes \mathcal{L}^k) = 0$ for $q > 0$ and $k \geq k_0$. but $\mathcal{O}_{Z,p} \otimes \mathcal{L}^k \cong \mathcal{O}_{Z,p}$. $\Rightarrow$ $H^q(M, \mathcal{O}_{Z,p}) = 0$. $\Rightarrow$ $H^q(M, \mathcal{O}_Z) = 0$

$\cong \mathcal{O}_{Z,p} \otimes \mathcal{O}_p$ ⌟

Adding up the local duality theorems in each point $p \in Z$ gives the Global Duality Theorem I. Let $I \subset \mathcal{O}$ be a sheaf of regular ideals s.t $Z = \mathrm{supp}(\mathcal{O}/I)$ has zero. Then there is a nondegenerate pairing

$$H^q(M, \mathcal{O}_Z) \otimes Ext^{n-q}(M; \mathcal{O}_Z, \Omega^n) \longrightarrow \mathbb{C}$$

that is functorial in the sheaf of ideals I.

⊢ $q = 0$. $H^0(M, \mathcal{O}_Z) \cong \bigoplus_{p \in Z} \mathcal{O}_{Z,p}$. $Ext^n(M; \mathcal{O}_Z, \Omega^n) \underset{\Uparrow}{=} H^0(M, \underline{Ext}^n_{\mathcal{O}}(\mathcal{O}_Z, \Omega^n)) = \bigoplus_{p \in Z} \underline{Ext}^n_{\mathcal{O}}(\mathcal{O}_Z, \Omega^n)_p$

by the second property.