4b. q infinitely near p infinitely near p_i.

┌ 3a. $p \neq q \in \mathbb{P}^2 - \{p_1 \cdots p_6\} \Leftrightarrow p \neq q \in \widetilde{\mathbb{P}}^2 - \cup E_i$

3b. $p \in E_i$, $q \in \widetilde{\mathbb{P}}^2 - \cup E_i \Leftrightarrow p$ infinitely near p_i, $q \in \mathbb{P}^2 - \{p_1 \cdots p_6\}$.

3c. $p \in E_i$, $q \in E_j \; {\scriptstyle i \neq j} \Leftrightarrow p$ infinitely near p_i, q infinitely near p_j

3d. $p \neq q \in E_i \Leftrightarrow p \neq q$ infinitely near p_i.

4a. $\iota_{\tilde{c}}$ has nonzero differential at p for $p \in \widetilde{\mathbb{P}}^2 - \cup E_i$

$\Leftrightarrow \iota_{\tilde{c}}$ has nonzero differential at $p \in \mathbb{P}^2 - \{p_1, p_2, \cdots p_6\}$

$\Leftrightarrow \iota_{\tilde{c}*}$ is injective at p

q is a tangent line at p, i.e. $q \in \mathbb{P}(T_p'(\widetilde{\mathbb{P}}^2))$. $\Rightarrow$ We have to show that $\iota_{\tilde{c}*}(q)$ is nonzero.

4b. $q \in \mathbb{P}(T_p'(\widetilde{\mathbb{P}}^2)) \Rightarrow$ Again we have to show that $\iota_{\tilde{c}*}(q) \neq 0$. ┘

In each of these cases, we see that since no three of the points p_i are collinear, no five of the points $p_1, \cdots p_6$, p, q are; and since the points p_i do not all lie on a conic, certainly $p_1, p_2, \cdots p_6$, p and q do not. By the lemma, then the points $p_1, p_2, \cdots p_6$, q and p impose independent conditions on cubics, and the map $\iota_{\tilde{c}}$ embeds $\widetilde{\mathbb{P}}^2$ as a cubic $S \subset \mathbb{P}^3$.

┌ Once we prove that $\iota_{\tilde{c}} : \widetilde{\mathbb{P}}^2 \longrightarrow \mathbb{P}^3$ is an