$\sigma|_{U_\alpha} \in \Omega^p(*D)(U_\alpha) = \bigcup_{k\geq 0} \Omega^p(kD)(U_\alpha)$. $\quad \underline{U} = \{U_\alpha\}$

$\Rightarrow$ $\exists$ $k_1 \geq k_0$ s.t σ has poles of order less than k over D, where k_0 is the #, $H^q(M, \Omega^p(kD)) = 0$ for $q>0$, $k \geq k_0$.

$\Rightarrow$ We may consider σ as an element in $H^q(M, \Omega^p(k_1D))$ since $\Omega^p(k_1D)$ is identified with the sheaf of meromorphic p-forms with poles of order $\leq k_1$ on D.

$\Rightarrow$ Since $H^q(M, \Omega^p(kD)) = 0$ for $q>0$, $k \geq k_0$, $\exists$ an element $\tau \in C^{p-1}(\underline{U}, \Omega^p(k_1D))$ s.t $\delta\tau = \sigma$, if necessary, we may choose a refinement of $\underline{U}$. Now consider τ as an element of $C^{p-1}(\underline{U}, \Omega^p(*D))$, in other words,

$$\begin{array}{ccc} C^{p-1}(\underline{U}, \Omega^p(k_1D)) & \xrightarrow{\iota} & C^{p-1}(\underline{U}, \Omega^p(*D)) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ \tau & \longmapsto & \iota(\tau) \end{array}.$$

$\Rightarrow$ We can conclude that $H^q(M, \Omega^p(*D)) = 0$ for $q>0$.

Therefore by the same argument on Note P456,

$$\mathbb{H}^q(M, \Omega^*(*D)) = \mathbb{H}^q(M, \mathcal{A}^*(*D)) = {}''E_2^{q,0} = H_d^q(H^0(M, \Omega^q(*D)) = H^q_{DR}(M-D, alg).$$

$\Rightarrow$ $H^q_{DR}(M-D, alg) = H^q_{DR}(U, alg) = H^q(U, \mathbb{C})$.

As on P456 note,

$$\Omega^{q-1}(*D)(M) \xrightarrow{d} \Omega^q(*D)(M) \xrightarrow{d} \Omega^{q+1}(*D)(M)$$