⌈⌈ $\dim(\mathbb{P}^{n-1} \cap T_pV) \geq (n-1) + \dim V - n = \dim V - 1.$

If π_p can be extended over the point p, then, for any point $q \in \mathbb{P}^{n-1} \cap T_pV$, there is a sequence $\{q_i\} \longrightarrow p$, and $\pi_p(q_i) \longrightarrow q$. This is nonsense, because $\exists$ at least two different limit points of one convergent sequence. ⌋⌋

Despite the fact that it is not everywhere defined, however, π_p is a natural geometric operation — as we have already had occasion to see in the previous section — and it is recognized as such in algebraic geometry. π_p is an example of a large class of transformations called rational maps, which we will now discuss.

We begin with a definition.

Definition. A rational (or meromorphic) map of a complex manifold M to projective space $\mathbb{P}^n$ is a map

$$f: z \longrightarrow [1, f_1(z), \cdots f_n(z)]$$

given by n global meromorphic functions on M. A rational map $f: M \longrightarrow N$ to the algebraic variety $N \subset \mathbb{P}^n$ is a rational map $f: M \longrightarrow \mathbb{P}^n$ whose image lies in N.

One difficulty in understanding rational maps $f: M \to \mathbb{P}^n$ is the fact that they are not, strictly speaking, maps: they need not be defined on all of M.

Let us first see how this occurs.

As we saw in several contexts in the chapter on curves, any collection $f_1, \cdots f_n$ of meromorphic functions on