ed holomorphic covering of order ν, then the induced homomorphism π^* exhibits ${}_V\mathcal{O}$ as a finitely generated ${}_n\mathcal{O}$-module – indeed, as an integral algebraic extension of ${}_n\mathcal{O}$ of degree ν. Moreover, if V is irreducible, then the field ${}_V\mathcal{M}$ is an algebraic extension of ${}_n\mathcal{M}$ of degree ν and is generated by any element $f \in {}_V\mathcal{O}$ that seperates the sheets.

Proof. Since the germ $\pi : V \to \mathbb{C}^n$ can be represented by a finite branched holomorphic covering $\pi : V \longrightarrow W$ of pure order ν over an open subset $W \subseteq \mathbb{C}^n$, and since there obviously exists a function $f \in {}_V\mathcal{O}_V$ that seperates the sheets if the representative variety V is sufficiently small, the first statement of this corollary follows from Corollary 8 immediately.

" I think that the existence is not obvious.
For a sufficiently small representative variety V, V can be considered as a subset of $\mathbb{C}^N$. $\Rightarrow$ $\pi : V \longrightarrow W$ is a branched covering. $\Rightarrow$ $\exists$ regular part V_0 & W_0 s.t
$W_0 = \coprod_{i=1}^{m} W_i$, each W_i connected open subset of W_0.
For each $w_i \in W_i$, $\exists$ $v_{i1}, \cdots v_{i\nu}$ s.t $\pi(v_{i1}) = \cdots = w_i$.

$\Rightarrow$ We can find a holomorphic function f s.t
$f(v_{ij}) \neq f(v_{il})$ $j \neq l$ as follows:
Choose $\{b_{ij}\} \subset \mathbb{C}$ s.t b_{ij}'s are distinct each other.
Let $f(z_1, \cdots z_N) \in \mathbb{C}[z_1, \cdots z_N]$ with sufficiently many coefficient so that $\exists$ a solution for the equations