for any particular choices of x and y, the above two maps are surjective for m sufficiently large.

⸢ For each pair $(x, y) \in M \times M$, $\exists$ $m_{(x,y)} \in \mathbb{Z}_+$ s.t

$$H^0(M, \mathcal{O}(E \otimes L^m)) \longrightarrow (E \otimes L^{m_{(x,y)}})_x \oplus (E \otimes L^{m_{(x,y)}})_y \text{ onto}$$

and

$$H^0(M, \mathcal{J}_x(E \otimes L^m)) \longrightarrow T_x^{*'} \otimes (E \otimes L^{m_{(x,y)}})_x .$$

$\Rightarrow$ By the same argument as p191, $\exists$ open set $U_x \times V_y$ satisfying the conditions above.

$\Rightarrow$ Since $M \times M$ is compact, take $m = \max\{ m_{(x_i, y_i)} \mid \bigcup U_{x_i} \times V_{y_i} = M \times M \}$. ⸥

We proceed by induction on the dimension of M. For any $x, y \in M$, consider the linear system of hyperplane sections of $M \subset \mathbb{P}^N$ containing x and y: by Bertini's theorem, the generic element of this system is smooth outside the base locus $\{x, y\}$ of the system, and it is easy to see that, unless M is a curve with $T_x(M) = T_y(M) \subset \mathbb{P}^N$ (which circumstance we can always avoid by embedding M differently), the generic element of the system will be smooth at x and y as well. Thus we can find a smooth hyperplane section $V = H \cap M$ of M containing x and y.

⸢① Let $\dim M \geq 2$ and H be the generic hyperplane in $\mathbb{P}^N$. $\Rightarrow$ $\dim(M \cap H) \geq 1$. $(M \cap H)_s \subset \langle x, y \rangle \cap M$,