⌈ If $\Lambda \cap V_2$ is a plane. $\Lambda = V_2 \Rightarrow \Lambda \cap F = V_2 \cap F$ can not be smooth, since $\Lambda \cap F$ is a double line.
$\#((\Lambda \cap V_2)\cdot(V_2 \cap F)) = \#(\ell \cdot (V_2 \cap F)) = \#(\ell \cdot C) = 2$
But since $\Lambda \cap F$ is a double line, $\Lambda \cap V_2 = \ell$ is tangent to $V_2 \cap F = C$. We may assume that
$C: X_0^2 + X_1^2 + X_2^2 = 0$; $p = [p_0, p_1, p_2]$. $p_0 \neq 0$
$\Rightarrow p_0^2 + p_1^2 + p_2^2 \neq 0$. If we let $[X_0, X_1, X_1]$ be the points whose tangent lines pass through p, then $(X_0 Z_0 + X_1 Z_1 + X_2 Z_2 = 0)$ tangent line passes $p \Rightarrow X_0 p_0 + X_1 p_1 + X_2 p_2 = 0$ and $X_0^2 + X_1^2 + X_2^2 = 0$

$$\Rightarrow \left(\frac{p_1}{p_0} X_1 + \frac{p_2}{p_0} X_2\right)^2 + X_1^2 + X_2^2 = 0$$

$$\Rightarrow (p_1^2 + p_0^2) X_1^2 + 2 p_1 p_2 X_1 X_2 + (p_2^2 + p_0^2) X_2^2 = 0 \quad \cdots \circledast$$

$\Rightarrow$ Since the discriminant is $p_0^2 (p_0^2 + p_1^2 + p_2^2)$, $\circledast$ has two distinct points as solutions. $\Rightarrow \exists$ two distinct tangent lines of C passing p.

⌋

Let x_1, x_2 denote the points of tangency of L_1, L_2 with C; the locus of lines on $Q = F \cap V_4$ through x_i is just $T_{x_i}(Q) \cap Q$.

⌈ Let ℓ be a line on Q through x_i. $\Rightarrow \ell \subset T_{x_i}(Q)$
$\Rightarrow \ell \subset T_{x_i}(Q) \cap Q \Rightarrow$ The locus of lines on Q through $x \subset T_{x_i}(Q) \cap Q$, and since $T_{x_i}(Q) \cap Q$ is