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$$\mu = \sum \frac{\sqrt{-1}}{2\pi} \log c_\alpha \cdot e_\alpha \in V,$$

we have

$$L' = \tau_\mu^* L.$$

$$\mu = \sum \frac{\sqrt{-1}}{2\pi} \log c_\alpha \cdot e_\alpha \quad \Rightarrow \quad \mu_\alpha = \frac{\sqrt{-1}}{2\pi} \log c_\alpha$$

$\Rightarrow$ $\tau_\mu^* L$ is given by multipliers

$$\begin{aligned} e'_{\lambda_\alpha}(z) &= 1 \\ e'_{\lambda_{n+\alpha}}(z) &= e^{-2\pi i (z_\alpha + \mu_\alpha)} = e^{-2\pi i z_\alpha} \cdot e^{-2\pi i \mu_\alpha} \\ &= e^{\log c_\alpha} e^{-2\pi i z_\alpha} = c_\alpha e^{-2\pi i z_\alpha} \\ &= c_\alpha e_{\lambda_{n+\alpha}}(z). \end{aligned}$$

$$\Rightarrow L' = \tau_\mu^* L$$

Thus, to prove that any line bundle having the same Chern class as L must be a translate of L, it will suffice to show that any line bundle with Chern class 0 can be realized by constant multipliers.

Point: $L_1 = \dfrac{V \times \mathbb{C}}{(z, \zeta) \sim (z+\lambda, e^1_\lambda(z) \cdot \zeta)}$ $\quad L_2 = \dfrac{V \times \mathbb{C}}{(z, \eta) \sim (z+\lambda, e^2_\lambda(z) \cdot \eta)}$

$$L_1 \otimes L_2 = \frac{V \times \mathbb{C}}{(z, \zeta) \sim (z+\lambda, e^1_\lambda(z)\, e^2_\lambda(z) \cdot \zeta)},$$ since $e_\lambda(z)$'s are