

$\Rightarrow$  Let  $x_1, \dots, x_{2n}$  be the dual real coordinates (on  $V$ ) to  $\lambda_1, \dots, \lambda_{2n}$ .

$$\Rightarrow \omega = \frac{1}{2} \sum g_{i\bar{j}} dx_i \wedge d\bar{x}_j = \sum g_{i\bar{i}} dx_i \wedge d\bar{x}_i,$$

since, first put  $\omega = \frac{1}{2} \sum g_{i\bar{j}} dx_i \wedge d\bar{x}_j$  and calculate  $\omega(\lambda_i, \lambda_j) \equiv \omega(\frac{\partial}{\partial x_i}, \frac{\partial}{\partial \bar{x}_j})$ .

$\Rightarrow$

Now if  $\omega$  is nondegenerate - that is, if  $\omega^n \neq 0$ , as will be the case if  $\omega$  is positive - then  $g_{\alpha\bar{\alpha}} \neq 0$  for all  $\alpha$ , and we can take as our basis for the complex vector space  $V$  the vectors

$$e_\alpha = g_{\alpha\bar{\alpha}}^{-1} \lambda_\alpha, \quad \alpha = 1, \dots, n.$$

$\square$  If  $\omega$  is positive, by considering the metric induced by  $\omega$ , see P.29, we may express  $\omega$  as

$$\omega = \frac{\sqrt{-1}}{2} \sum \psi_i \wedge \bar{\psi}_i.$$

$\Rightarrow \omega^n = n! \Phi$ ,  $\Phi$  volume form on a manifold.

$\Rightarrow \omega^n \neq 0. \Rightarrow$  If  $\omega^n \neq 0$ , then  $g_{\alpha\bar{\alpha}} \neq 0$  for all  $\alpha$  clearly.

$\Rightarrow$

The period matrix of  $\Lambda \subset V$  will then be of the form

$$\Omega = (\Delta s, Z);$$