

is thus holomorphic in a nbd of p and vanishes exactly on $\overline{V_i}$. Q.E.D.

By P8, $\sigma_1(z'), \dots, \sigma_k(z')$ are holomorphic and since all z_n 's are in $\overline{\Delta}$, they are bounded. \Rightarrow By Riemann extension theorem (P9), $\sigma_1(z'), \dots, \sigma_k(z')$ are extended to Δ' . \Rightarrow The function

$$f_i(z) = z_n^k + \sigma_1(z') z_n^{k-1} + \dots + \sigma_k(z')$$

is holomorphic in a nbd of p' .

$$\Rightarrow \{f_i(z)=0\} \supset V_i \cap (\Delta - (g=0)) \subset \Delta$$

$$\Rightarrow \{f_i(z)=0\} \text{ is closed in } \Delta \Rightarrow \{f_i(z)=0\} \supset \overline{V_i}$$

$$\Rightarrow \{f_i(z)=0\} = \overline{V_i} \cap \Delta, \text{ since } \{f_i(z)=0\} \cap (\Delta - (g=0)) = V_i \cap (\Delta - (g=0))$$

Redundant! $\overline{V_i} \cap \Delta \subset \{f_i(z)=0\} \cap \Delta$, $\{f_i(z)=0\} \cap (\Delta - (g=0)) \subset \overline{V_i} \cap \Delta$.
 $\Rightarrow \{f_i(z)=0\} \cap \Delta \subset \overline{V_i} \cap \Delta$ since $\Delta - (g=0)$ is open dense, i.e., $\{g=0\}$ is a set of measure 0. \square

We take the dimension of an irreducible analytic variety V to be the dimension of the complex manifold V^* ; we say that a general analytic variety is of dimension k if all of its irreducible components are.

A note: if $V \subset M$ is an analytic subvariety of a complex manifold M , then we may define the tangent cone $T_p(V) \subset T_p(M)$ to V at any point $p \in V$ as follows: if $V = (f=0)$ is an an-