According to the general mechanism, once we have such a filtered complex $\{F^p A^*\}$, there is an associated spectral sequence $\{E_r\}$ with

$$E_\infty \Rightarrow H^*(A^*) = H^*_{DR}(E).$$

⌈ $H^*(A^*) = H^*_{DR}(E)$ by Proposition on P440 ⌋

We will calculate the terms E_1 and E_2.

Recall that

$$E_0^{p,q} = \frac{F^p \ A^{p+q}}{F^{p+1} \ A^{p+q}},$$

and d_0 is obtained from d by passing to the quotient. Taking a local product isomorphism

$$\pi^{-1}(U) \cong U \times F,$$

we may represent $E_0^{p,q}$ by forms

$$\varphi = \sum_{\# I = p} \eta_I(x, y, dy) \wedge dx_I,$$

where the η_I are q-forms on F.

⌈ $$E_0^{p,q} = \frac{F^p A^{p+q}}{F^{p+1} A^{p+q}} = \frac{\{\varphi = \sum_{\# I \geq p} \varphi_{IJ}(x, y) \, dx_I \wedge dy_J\}}{\{\varphi = \sum_{\# I \geq p+1} \varphi_{IJ}(x, y) \, dx_I \wedge dy_J\}}$$

$$= \{\varphi = \sum_{\substack{\# I = p \\ \# J = q}} \varphi_{IJ}(x, y) \, dx_I \wedge dy_J\}$$

⌋

Computing modulo $F^{p+1} A^*$,