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Let $P = \sum_{I=\{i_1<\cdots<i_{k-1}\}} P_I e_I$. $\Rightarrow$ $i(e_1^*)P = 0$ $\Rightarrow$ $\langle i(e_1^*)P, \zeta\rangle = 0$ for all $\zeta \in \Lambda^{k-2}V^*$.

$\zeta = e_J^* = e_{j_1}^* \wedge \cdots e_{j_{k-2}}^*$ if $J \not\ni 1$.

$\Rightarrow \langle i(e_1^*)P, \zeta\rangle = \langle P, e_1^* \wedge e_J^*\rangle = P_{\{1\}\cup J} = 0$

$\Rightarrow$ $P = \sum_{I \not\ni 1} P_I e_I$

Let $\eta = \sum_{L=\{1\}\cup I} \eta_L e_L$ where $I \not\ni 1$. $\eta_L = \eta_{\{1\}\cup I} = P_I$.

$\Rightarrow \langle i(e_1^*)\eta, e_I^*\rangle = \langle \eta, e_1^* \wedge e_I^*\rangle = \eta_{\{1\}\cup I} = P_I$

$= \langle P, e_I^*\rangle$ for $I \not\ni 1$.

If $I \ni 1$, $P_I = 0$. $\eta_L = 0$. $L = \{1\}\cup I$.

$\Rightarrow$ $i(e_1^*)\eta = P$. $\Rightarrow$ $\ker i(e_1^*) \subset \operatorname{Im} i(e_1^*)$.

$\Rightarrow$

$$0 \to \Lambda^n V \to \Lambda^{n-1} V \to \cdots \to \Lambda^2 V \to V \to \mathbb{C} \to 0 \quad \text{exact.}$$

The basis for our intrinsic formulation of local duality is that, under the identifications

$$\operatorname{Hom}(\Lambda^k V, \mathbb{C}) \cong \Lambda^k V^* \cong \Lambda^n V^* \otimes \Lambda^{n-k} V,$$

the above sequence is self-dual in the sense that the diagram

$$\begin{array}{ccc}
\operatorname{Hom}(\Lambda^k V, \mathbb{C}) & \xrightarrow{\sim} & \Lambda^n V^* \otimes \Lambda^{n-k} V \\
\Big\downarrow {\scriptstyle i(v^*)^*} & & \Big\downarrow {\scriptstyle 1 \otimes i(v^*)} \\
\operatorname{Hom}(\Lambda^{k+1} V, \mathbb{C}) & \xrightarrow{\sim} & \Lambda^n V^* \otimes \Lambda^{n-(k+1)} V
\end{array}$$

is commutative.

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