Def: E, E' hermitian v.b. we define a natural metric on $E \otimes E'$ given by

$$\langle \lambda \otimes \lambda, \delta \otimes \delta' \rangle = \langle \lambda, \delta \rangle \cdot \langle \lambda', \delta' \rangle. \quad \text{for } \lambda, \delta \in E_x.$$

$\lambda', \delta' \in E'_x$.

lemma: D_E, $D_{E'}$, $D_{E\otimes E'}$ metric connections on E, E' & $E \otimes E'$ respectively. let $D_E \otimes 1$ be the connection on $E \otimes E'$ given by $D_E \otimes 1(\xi \otimes \eta) = D\xi \otimes \eta$, define $1 \otimes D_{E'}$ analogously. Then we have $D_{E\otimes E'} = D_E \otimes 1 + 1 \otimes D_{E'}$.

pf) $(D_E \otimes 1 + 1 \otimes D_{E'})''(\xi \otimes \eta) = (D_E'' \otimes 1 + 1 \otimes D_{E'}'')(\xi \otimes \eta)$
$\Rightarrow \bar\partial \otimes 1 + 1 \otimes \bar\partial = \bar\partial$

$\xi, \eta \in \mathcal{A}^0(E)$, $\xi', \eta' \in \mathcal{A}^0(E')$.

$d(\xi \otimes \xi', \eta \otimes \eta') \overset{?}{=} \langle D_E\xi \otimes \xi' + \xi \otimes D_{E'}\xi', \eta \otimes \eta' \rangle + \langle \xi \otimes \xi', D_E\eta \otimes \eta' + \eta \otimes D_{E'}\eta' \rangle = \langle D_E\xi, \eta \rangle \langle \xi', \eta' \rangle + \langle \xi, \eta \rangle \langle D_{E'}\xi', \eta' \rangle + \langle \xi, D_E\eta \rangle \langle \xi', \eta' \rangle + \langle \xi, \eta \rangle \langle \xi', D_{E'}\eta' \rangle = \langle \xi', \eta' \rangle \, d\langle \xi, \eta \rangle + \langle \xi, \eta \rangle \, d\langle \xi', \eta' \rangle$
$\Rightarrow d(\langle \xi, \eta \rangle \langle \xi', \eta' \rangle) = \langle \xi', \eta' \rangle \, d\langle \xi, \eta \rangle + \langle \xi, \eta \rangle \, d\langle \xi', \eta' \rangle$.

$\Rightarrow d\langle \xi \otimes \xi', \eta \otimes \eta' \rangle = \langle D_E\xi \otimes \xi' + \xi \otimes D_{E'}\xi', \eta \otimes \eta' \rangle + \langle \xi \otimes \xi', D_E\eta \otimes \eta' + \eta \otimes D_{E'}\eta' \rangle \Rightarrow D_E \otimes 1 + 1 \otimes D_{E'} = D_{E\otimes E'}$

Finally, note that a hermitian metric on the holo. bundle E induces a metric on E^*, — if e unitary frame for E, e^* the dual frame for E^*, set $\langle e_i^*, e_j^* \rangle = \delta_{ij}$ — and the metric connection D^* on E^* can be defined by the requirement

$$d\langle \sigma, \tau \rangle = \langle D\sigma, \tau \rangle + \langle \sigma, D^*\tau \rangle$$

for $\sigma \in \mathcal{A}^0(E)(U)$, $\tau \in \mathcal{A}^0(E^*)(U)$.