

$P_f$  in  $M \times M$ , we find only that

$$\begin{aligned} L(f) &= \int_{P_f} \varphi_\Delta \\ &= \sum_{p,q} (-1)^{p+q} \int_{P_f} \sum_{\mu} \pi_1^* \psi_{p,q,\mu} \wedge \pi_2^* \psi_{n-p,n-q,\mu}^* \\ &= \sum_{p,q} (-1)^{p+q} \text{trace } f^*|_{H_{\frac{p,q}{2}}^{p,q}(M)}. \end{aligned}$$

According to P4.21,

$$\#(\Delta \cdot P_f) = \int_{P_f} \varphi_\Delta = L(f) = \sum_p (-1)^p \text{trace } (f^*|_{H_{pR}^p(M)})$$

$$= \sum_{p,q} (-1)^{p+q} \int_{P_f} \pi_1^* \psi_{p,q,\mu} \wedge \pi_2^* \psi_{n-p,n-q,\mu}^* = \sum_{p,q} (-1)^{p+q} \int_M \tilde{f}^* (\pi_1^* \psi_{p,q,\mu} \wedge \pi_2^* \psi_{n-p,n-q,\mu}^*)$$

$$\left\{ \begin{array}{l} \pi_2 \circ \tilde{f} = f : M \rightarrow P_f \rightarrow M \xrightarrow{f^*} H_{\frac{p,q}{2}}^{p,q}(M) \longrightarrow H_{\frac{p,q}{2}}^{p,q}(M) \\ \pi_1 \circ \tilde{f} = \text{id} \end{array} \right.$$

$$= \sum_{p,q} (-1)^{p+q} \int_M \psi_{p,q,\mu} \wedge f^* \psi_{n-p,n-q,\mu}^* = \sum_{p,q} (-1)^{p+q} \text{trace } (f^*|_{H_{\frac{p,q}{2}}^{p,q}(M)})$$

$$\text{Let } \varphi_\Delta = \sum c_{p,q,\mu,\nu} \varphi_{p,q,\mu,\nu} = \sum c_{p,q,\mu,\nu} \pi_1^* \psi_{p,q,\mu} \wedge \pi_2^* \psi_{n-p,n-q,\nu}^*.$$

$$\int_{M \times M} \varphi_\Delta \wedge \varphi_{n-p,n-q,\mu,\nu}^* = c_{p,q,\mu,\nu} \int_{M \times M} \varphi_{p,q,\mu,\nu} \wedge \varphi_{n-p,n-q,\mu,\nu}^*$$

$$\begin{aligned} \Rightarrow \int_{\Delta} \varphi_{n-p,n-q,\mu,\nu}^* &= c_{p,q,\mu,\nu} (-1)^{(n+n-p-q)(2n-p-q)} (-1)^{(p+q)(2n-p-q)} \\ &= c_{p,q,\mu,\nu} (-1)^{p+q} (-1)^{p+q} = c_{p,q,\mu,\nu} \end{aligned}$$

$$\begin{aligned} &= \int_{\Delta} \pi_1^* \psi_{n-p,n-q,\mu}^* \wedge \pi_2^* \psi_{p,q,\nu} = \int_M \psi_{n-p,n-q,\mu}^* \wedge \psi_{p,q,\nu} \\ &= (-1)^{p+q} \delta_{\mu,\nu} \end{aligned}$$