For $\{\alpha_1,\cdots\alpha_q\} = \alpha$ & $(\mu_1,\cdots\mu_q) = \mu$ given,

$$\sum_\pi \epsilon(\pi)\, A^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge A^{\alpha_{\pi(q)}}_{\mu_q} \wedge \sum_\sigma \epsilon(\sigma)\, \overline{A}^{\alpha_{\sigma(1)}}_{\mu_1}\wedge\cdots\wedge \overline{A}^{\alpha_{\sigma(q)}}_{\mu_q}$$

$$\left.\begin{aligned}
&= \sum_{\pi,\sigma} \epsilon(\pi)\,\epsilon(\sigma)\, A^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge A^{\alpha_{\pi(q)}}_{\mu_q}\wedge \overline{A}^{\alpha_{\sigma(1)}}_{\mu_1}\wedge\cdots\wedge\overline{A}^{\alpha_{\sigma(q)}}_{\mu_q}\\
&= \sum_{\pi,\sigma} \epsilon(\pi)\,\epsilon(\sigma)\, A^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge A^{\alpha_{\pi(q)}}_{\mu_q}\wedge \overline{A}^{\alpha_{\sigma(1)}}_{\mu_1}\wedge\cdots\wedge\overline{A}^{\alpha_{\sigma(q)}}_{\mu_q}\\
&= \sum_{\pi,\sigma} \epsilon(\pi)\,\epsilon(\sigma)\, A^{\alpha_1}_{\mu_{\pi^{-1}(1)}}\wedge\cdots\wedge A^{\alpha_q}_{\mu_{\pi^{-1}(q)}}\wedge \overline{A}^{\alpha_{\sigma(\pi^{-1}(1))}}_{\mu_{\pi^{-1}(1)}}\wedge\cdots\wedge\overline{A}^{\alpha_{\sigma(\pi^{-1}(q))}}_{\mu_{\pi^{-1}(q)}}\\
&= \sum_\pi\sum_\sigma \epsilon(\sigma\circ\pi^{-1})\, A^{\alpha_1}_{\mu_{\pi^{-1}(1)}}\wedge\cdots\wedge A^{\alpha_q}_{\mu_{\pi^{-1}(q)}}\wedge \overline{A}^{\alpha_{\sigma(\pi^{-1}(1))}}_{\mu_{\pi^{-1}(1)}}\wedge\cdots\wedge\overline{A}^{\alpha_{\sigma(\pi^{-1}(q))}}_{\mu_{\pi^{-1}(q)}}
\end{aligned}\right\}\text{not necessary}$$

$$= \sum_{\pi,\sigma} \epsilon(\sigma\circ\pi^{-1})\, A^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge A^{\alpha_{\pi(q)}}_{\mu_q}\wedge \overline{A}^{\alpha_{\sigma\circ\pi^{-1}(\pi(1))}}_{\mu_1}\wedge\cdots\wedge\overline{A}^{\alpha_{\sigma\circ\pi^{-1}(\pi(q))}}_{\mu_q}$$

$$= \sum_{\substack{\alpha'_1\cdots\alpha'_q\in\{\alpha_1\cdots\alpha_q\}\\ (\mu_1\cdots\mu_q)=\mu}} sgn(\pi)\, A^{\alpha'_1}_{\mu_1}\wedge\cdots\wedge A^{\alpha'_q}_{\mu_q}\wedge \overline{A}^{\alpha'_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge\overline{A}^{\alpha'_{\pi(q)}}_{\mu_q}$$

Let $\underset{(\mu_1\cdots\mu_q)}{\eta_{\mu}} = \sum_{\alpha_1<\cdots<\alpha_q} \frac{1}{q!}\,\epsilon(\pi)\, A^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge A^{\alpha_{\pi(q)}}_{\mu_q}\quad \left(\frac{1}{2\pi}\right)^{\frac{q}{2}}$

$$\Rightarrow (\sqrt{-1})^{q^2}\sum_\mu \eta_\mu\wedge\overline{\eta_\mu} = (\sqrt{-1})^{q^2}\left(\frac{1}{q!}\right)^2\left(\frac{1}{2\pi}\right)^q \sum_{\substack{\alpha_1<\cdots<\alpha_q\\ \mu,\pi}} q!\, sgn(\pi)\, A^{\alpha_1}_{\mu_1}\wedge\cdots\wedge A^{\alpha_q}_{\mu_q}\wedge\overline{A}^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge\overline{A}^{\alpha_{\pi(q)}}_{\mu_q} =$$

$$\left(\frac{\sqrt{-1}}{2\pi}\right)^q \frac{1}{q!}\sum_{\substack{\alpha_1<\cdots<\alpha_q\\ \mu,\pi}} sgn(\pi)\, A^{\alpha_1}_{\mu_1}\wedge\cdots\wedge A^{\alpha_q}_{\mu_q}\wedge\overline{A}^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge\overline{A}^{\alpha_{\pi(q)}}_{\mu_q}$$