Then $\sigma_1 = \tau l_1 \quad \sigma_2 = \tau l_2 \quad \sigma_3 = \tau l_3 \quad \sigma_4 = \tau l_4$.

$\Rightarrow l_1, l_2, \cdots l_4$ represent linearly independent lines in $\mathbb{P}^2$, which is impossible since

$\dim H^0(\mathbb{P}^2, \mathcal{O}(H)) = \binom{2+1}{2} = {}_3C_2 = 3$.

Thus C_0 must be a line ┘

If C_0 contains ≤ 4 points from Γ_0, then there will be a set Γ_0' of ≥ 3 points left over.

┌ $\#(\Gamma_0 \cap C_0) \leq 4 \Rightarrow \Gamma_0' = \Gamma_0 - (\Gamma_0 \cap C_0)$ and $\#\Gamma_0' \geq 3$ ┘

These will impose independent conditions on the linear system $|\mathcal{O}_{\mathbb{P}^2}(2)|$ of plane cubics, and consequently

$$\dim |\mathcal{J}_{\Gamma_0}(3)| = \dim |\mathcal{J}_{\Gamma_0'}(2)| \leq 5 - 3 = 2,$$

which is a contradiction. Q.E.D.

┌ The statement " These will impose independent conditions on the linear systems $|\mathcal{O}_{\mathbb{P}^2}(2)|$ of plane curves." is wrong, I think. For, if $\#(C_0 \cap \Gamma_0) \leq 2$, then $\#\Gamma_0' \geq 5$, so Γ_0' is not a set of $d < \frac{2(2+3)}{2} = 5$ points in $\mathbb{P}^2$, and we can apply this case to the Reciprocity Formula I.

But, I think, we can still derive a contradiction by modifying the above slightly.