gent spaces to F at $p_{i_1}, p_{i_1 i_2}, \cdots p_{i_1 \cdots i_{m-1}}$, we find that the k-planes $\Lambda \subset F$ lying in $\sigma_{n-k, n-k-1, \cdots}$ are exactly the planes

$$\overline{p_{i_1}, p_{i_1 i_2}, \cdots p_{i_1 i_2 \cdots i_{k+1}}},$$

and there are 2^{k+1} of these.

Γ $\mathbb{P}^{n-2m+3}_{i_1 i_2 \cdots i_{m-1}} \subset T_{p_{i_1}}(F) \cap T_{p_{i_1 i_2}}(F) \cap \cdots \cap T_{p_{i_1 \cdots i_{m-1}}}(F) = \mathbb{P}^{n-m+2}$ is chosen so that it does not intersect with $\overline{p_{i_1} \cdots p_{i_1 \cdots i_{m-1}}} = \mathbb{P}^{m-2}$. (Note that $(n-2m+3) + (m-2)$ $= n-m+1 < n-m+2$)

$\overline{p_{i_1}, \cdots p_{i_1 i_2 \cdots i_{k+1}}} = \mathbb{P}^k \Rightarrow$ Since $\Lambda \ni p_{i_1}, p_{i_1 i_2}, \cdots p_{i_1 i_2 \cdots i_{k+1}}$, $\Lambda = \overline{p_{i_1} \cdots i_{k+1}} \in \sigma_{n-k, n-k-1, \cdots}$. There are two p_i's, two $p_{i_1 i_2}$'s, $\cdots$ $\Rightarrow \# \{\Lambda\} = \overbrace{2 \cdot 2 \cdots 2}^{k+1} = 2^{k+1}$.

$\Rightarrow \# \sigma_{n-k, n-k-1, \cdots} = 2^{k+1}$, since any $\Lambda \in \sigma_{n-k, n-k-1, \cdots}$ must contain $p_{i_1}, p_{i_1 i_2}, \cdots p_{i_1 i_2 \cdots i_{k+1}}$ points. ◻

Consequently $\# (\Sigma_{k,n} \cdot \sigma_{n-k, n-k-1, \cdots}) = 2^{k+1}$,

and we have

$$\Sigma_{k,n} \sim 2^{k+1} \cdot \sigma_{k+1, k, k-1, \cdots 1}$$

in the cohomology of $G(k+1, n+2)$.

Γ If $\sigma_{n-k, n-k-1, \cdots} \cap \Sigma_{k,n}$ has the multiplicity $r \geq 2$ at Λ, $\Lambda \in \sigma_{n-k, \cdots} \cap \Sigma_{k,n}$, then since Λ appears two times in F when V_{2m}'s search, $p_{i_1}, p_{i_1 i_2}, \cdots p_{i_1 i_2 \cdots i_{k+1}}$