No.

with the decomposition into type.

φ is harmonic $\Leftrightarrow$ $\bar\partial\varphi = 0$ & $\partial\varphi = 0$ $\Leftrightarrow$ $\Delta_{\bar\partial}\varphi = 0$ $\Leftrightarrow$ $\bar\partial\alpha = 0$ & $\partial\beta = 0$. $\Leftrightarrow$ α holomorphic & β antiholomorphic. See P81 ⏎

This will be the case if and only if $\partial\varphi = \bar\partial\varphi = 0$, or equivalently, if and only if $d\varphi = d^c\varphi = 0$.

⌈ $d = \partial + \bar\partial$ $\quad d^c = \frac{i}{4\pi}(\bar\partial - \partial)$ $\Rightarrow$ $\partial\varphi + \bar\partial\varphi = 0$ & $\bar\partial\varphi = \partial\varphi$ $\Leftrightarrow$ $\partial\varphi = \bar\partial\varphi = 0$. ⏎

If $d\varphi = 0$, then locally $\varphi = df$ for some C^∞ function f; we have

$$d^c\varphi = d^c df = -\frac{1}{\pi}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right) dx \wedge dy,$$

i.e., φ is harmonic $\Leftrightarrow$ f is harmonic in the usual sense of one complex variable. In particular, we see that the harmonic space $\mathcal{H}^1(S)$ does not depend on the choice of metric.

⌈ $d^c\varphi = d^c df = \frac{i}{4\pi}(\bar\partial - \partial)\left(\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy\right)$

$$= \frac{i}{4\pi}\left\{\bar\partial\left(\frac{\partial f}{\partial x}\right)dx + \bar\partial\left(\frac{\partial f}{\partial y}\right)dy - \partial\left(\frac{\partial f}{\partial x}\right)dx - \partial\left(\frac{\partial f}{\partial y}\right)dy\right\}$$

$$= \frac{i}{4\pi}\left\{\left(\frac{1}{2}\frac{\partial^2 f}{\partial x^2} + \frac{i}{2}\cancel{\frac{\partial^2 f}{\partial x\partial y}}\right)(-i\,dy\wedge dx) + \left(\frac{1}{2}\cancel{\frac{\partial^2 f}{\partial x\partial y}} + i\frac{1}{2}\frac{\partial^2 f}{\partial y^2}\right)dx\wedge dy\right.$$

$$\left. - \left(\frac{1}{2}\frac{\partial^2 f}{\partial x^2} - \frac{i}{2}\cancel{\frac{\partial^2 f}{\partial x\partial y}}\right) i\,dy\wedge dx - \left(\frac{1}{2}\cancel{\frac{\partial^2 f}{\partial x\partial y}} - \frac{i}{2}\frac{\partial^2 f}{\partial y^2}\right)dx\wedge dy\right\}$$

$$= \frac{i}{4\pi}\left(i\frac{\partial^2 f}{\partial x^2}dx\wedge dy + i\frac{\partial^2 f}{\partial y^2}dx\wedge dy\right)$$

$$= -\frac{1}{4\pi}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)dx\wedge dy.$$

$\Rightarrow$ We did not need the metric when we talk of harmonic. ⏎