Of course, this bound can be realized by any Riemann surface of genus $g = d-n$ and any linear system of degree d.

It remains now to find the maximal genus of a curve of degree d in $\mathbb{P}^n$ for $d > 2n$, or equivalently to find a sharper bound on the dimension of a linear system than that provided by Clifford's theorem, when $d \ll g$. We offer here an argument originally given by Castelnuovo in 1889.

Let $C \subset \mathbb{P}^n$ be a curve of degree d and genus g, with hyperplane section D. Consider the linear system $|kD|$ for $k = 1, 2, \cdots$. By our basic lemma, we can take the points of D to be in general position in a hyperplane in $\mathbb{P}^n$.

Let $m = [(d-1)/(n-1)]$ be the greatest integer less than or equal to $(d-1)/(n-1)$, and for each integer $k \leq m$ choose a set Γ of $k(n-1)+1$ points of D. We claim that the hyperplanes in $H^0(C, \mathcal{O}(kD))$ corresponding to the points of Γ are all independent; to prove it we will exhibit, for any point $q \in \Gamma$, a hypersurface of degree k in $\mathbb{P}^n$ containing $\Gamma - \{q\}$ but not q. This is easy: if we partition the remaining points of Γ into k sets

$$\{p_1^1, p_2^1, \cdots p_{n-1}^1\}, \{p_1^2 \cdots p_{n-1}^2\}, \cdots \{p_1^k, \cdots p_{n-1}^k\}$$

of $(n-1)$ points each, then each set $\{p_\alpha^i\}_\alpha$ will be linearly independent, and its linear span will not contain q.

Γ $\#\Gamma = k(n-1)+1 \Rightarrow \#(\Gamma - \{q\}) = k(n-1)$