

irreducible, g divides f . ($\because \deg g > \deg(\frac{\partial f}{\partial z_n})$).

$$\Rightarrow f = g_1^2 f.$$

$$\Rightarrow \exists g, s, t \quad g = \alpha f + \beta \frac{\partial f}{\partial z_n}, \quad g \neq 0 \in \mathcal{O}_n.$$

If g, α & β are not relatively prime, divide g, α & β by the greatest common divisor so that we may assume that g, α & β are relatively prime. By the assertion 2 (on P13).

$$\pi(\{f = \frac{\partial f}{\partial z_n} = 0\}) = \{g = 0\}.$$

Since $\{f = \frac{\partial f}{\partial z_n} = 0\}$ is the set of $\overset{\text{all}}{V(z', z_n)}$ s.t. \exists at least one (z', z'_n) s.t. $z'_n \neq z_n$.

$$f(z', z_n) = f(z', z'_n) = 0, \text{ i.e., } \{g = 0\} \supset$$

the set of all z' s.t. $f(z', z_n) = 0$ has a double root, for Δ some polydisc around p and Δ' a polydisc in \mathbb{C}^{n-1} , $\pi: V_i \cap (\Delta - (g=0)) \rightarrow \Delta' - (g=0)$ is a covering map. \square

Let $\{w_i(z')\}$ denote the z_n -coordinates of the points in $\pi^{-1}(z')$ for $z' = (z_1, \dots, z_{n-1}) \in \Delta' - (g=0)$ and let $\sigma_1(z'), \dots, \sigma_k(z')$ denote the elementary symmetric functions of the w_i . The functions σ_i are well-defined and bounded on $\Delta' - (g=0)$, and so extend to Δ' ; the function

$$f_i(z) = z_n^k + \sigma_1(z') z_n^{k-1} + \dots + \sigma_k(z')$$