

make a $(N-1)$ -dimensional "cone". \Rightarrow We can choose a coordinate system so that the coordinate projections $f(M_\lambda) \rightarrow \Delta^n$ are all proper mappings. "We may shrink Δ^n ". \Rightarrow

To complete the argument we make an observation: in $\mathbb{C} \times \mathbb{C}^p \times \mathbb{C}^q$ with coordinates $(u, v_1, \dots, v_p, w_1, \dots, w_q) = (u, v, w)$ we suppose given a closed subset S' of the polycylinder $\Delta \times \Delta^p \times \Delta^q$ defined by $|u| \leq \epsilon, |v_i| \leq \epsilon, |w_i| \leq \epsilon$. Suppose that we let $S_0 = S' \cap \{u=0\}$, and assume that the projection $S_0 \rightarrow \Delta^p$ induced by $(0, v, w) \rightarrow v$ is proper. Then, taking a smaller ϵ if necessary, the projection $S' \rightarrow \Delta \times \Delta^p$ induced by $(u, v, w) \rightarrow (u, v)$ will again be proper.

□ Once we proved the observation, then the argument is completed. For,

$$\Delta^N = \Delta \times \Delta^{N-1} = \Delta \times \Delta^n \times \Delta^{N-1-n}$$

$$f(M) = S \text{ (let)}$$

Let $n=p, N-1-n=q$. By the observation, the projection $f(M) = S \rightarrow \Delta \times \Delta^n = \Delta^{n+1}$ is proper, i.e., $g = \pi \circ f$ is proper. Thus it remains to show the observation.