Set $D_j = \frac{1}{i}\frac{\partial}{\partial x_j}$ and use the standard multiindex notations.

$$D^\alpha = D_1^{\alpha_1} D_2^{\alpha_2} \cdots D_n^{\alpha_n}, \quad \alpha = (\alpha_1, \alpha_2, \cdots \alpha_n)$$
$$[\alpha] = \alpha_1 + \alpha_2 + \cdots + \alpha_n$$
$$\xi^\alpha = \xi_1^{\alpha_1} \xi_2^{\alpha_2} \cdots \xi_n^{\alpha_n}.$$

By integration by parts,

$$\int_T D^\alpha \varphi \cdot \overline{\psi} = \int_T \varphi \, \overline{D^\alpha \psi}. \qquad \varphi, \psi \in C^\infty(T).$$

「For example $\int_T D_1 \varphi \cdot \overline{\psi} = \underbrace{\int_T D_1(\varphi \cdot \overline{\psi})}_{\substack{\| \\ 0 \text{ by Stoke's theorem}}} - \int_T \varphi \cdot \overline{D_1 \psi}$,

since $D_1(\varphi \overline{\psi}) = (D_1\varphi)\overline{\psi} + \varphi D_1\overline{\psi}$. and $D_1 \overline{\psi} = -i\frac{\partial}{\partial x_1}\overline{\psi}$
$= - \overline{-i\frac{\partial}{\partial x_1}\psi} = - \overline{D_1 \psi}$. 」

$\Rightarrow$ For $\varphi \in C^s(T)$, and $[\alpha] \leqslant s$,

$$(D^\alpha \varphi)_\xi = \int_T D^\alpha \varphi \, e^{-\langle \xi, x\rangle} dx = \int_T \varphi \, \overline{D^\alpha e^{+\langle \xi, x \rangle}} dx$$
$$= \int_T \xi^\alpha \varphi \, e^{-\langle \xi, x \rangle} dx = \xi^\alpha \varphi_\xi. \quad \text{i.e. } \|D^\alpha \varphi\|_0^2 = \sum_\xi |\xi^\alpha|^2 |\varphi_\xi|^2.$$

Thus there is an inclusion

$$C^s(T) \subset H_s ,$$

「 $\varphi \in C^s(T) \Rightarrow \|\varphi\|_s^2 = \sum_\xi (1 + \|\xi\|^2)^s |\varphi_\xi|^2$

But since $D^\alpha \varphi \in C^0(T) \subset H_0$, $\|D^\alpha\varphi\|_0^2 = \sum_\xi |\xi^\alpha|^2 |\varphi_\xi|^2 < \infty$.
~~$[\alpha] = s \Rightarrow (1 + \|\xi\|^2)^s \leq 2|\xi^\alpha|^2$ for~~ $|\xi|$ sufficiently large.
$[\alpha] = s \Rightarrow \quad (1 + \|\xi\|^2)^s \leq C \sum_{[\alpha]=s} |\xi^\alpha|^2$ for $\xi \neq 0$, for some constant $C > 0$. $\Rightarrow \quad \|\varphi\|_s^2 = \sum_{\xi \in \mathbb{Z}^n} (1 + \|\xi\|^2)^s |\varphi_\xi|^2$