the spectral sequences i.e.,

$$j_* : E_r^{p,q}(\underline{U}, \mathcal{L}^*) \longrightarrow E_r^{p,q}(\underline{U}, \mathcal{K}^*) .$$

Specially,

$$j_* : E_2^{p,q}(\underline{U}, \mathcal{L}^*) \longrightarrow E_2^{p,q}(\underline{U}, \mathcal{K}^*)$$

and

$$\begin{array}{ccc} j_* : E_2^{p,q}(\mathcal{L}^*) & \longrightarrow & E_2^{p,q}(\mathcal{K}^*) \\ \| & & \| \\ H^p(X, \mathcal{H}^q(\mathcal{L}^*)) & & H^p(X, \mathcal{H}^q(\mathcal{K}^*)) \end{array}$$

is isomorphic since $\mathcal{H}^q(\mathcal{L}^*) \xrightarrow{j_*} \mathcal{H}^q(K^*)$ is isomorphic. $\Rightarrow$ $j_* : E_r^{p,q}(\mathcal{L}^*) \to E_r^{p,q}(\mathcal{K}^*)$ is isomorphic for $r \geq 2$.

$$\Rightarrow \quad F^1\mathbb{H}^q(X, \mathcal{L}^*) \oplus E_r^{0,q}(\mathcal{L}^*) = \mathbb{H}^q(X, \mathcal{L}^*)$$

$$= F^2\mathbb{H}^q(X, \mathcal{L}^*) \oplus E_r^{1,q}(\mathcal{L}^*) \oplus E_r^{0,q}(\mathcal{L}^*)$$

$$\cdots = E_r^{0,q}(\mathcal{L}^*) \oplus \cdots \oplus E_r^{n,q-n}(\mathcal{L}^*) \oplus \cdots$$

$$= E_r^{0,q}(\mathcal{K}^*) \oplus \cdots \oplus E_r^{n,q-n}(\mathcal{K}^*) \oplus \cdots$$

$$= \mathbb{H}^q(X, \mathcal{K}^*) .$$

Here are some examples.

1. De Rham's theorem revisited. Suppose M is a manifold and $(\mathcal{Q}^*, d)$ the de Rham complex of sheaves of smooth forms

$$\mathcal{Q}^0 \xrightarrow{d} \mathcal{Q}^1 \xrightarrow{d} \mathcal{Q}^2 \to \cdots .$$

We denote by $\mathbb{R}^*$ the trivial complex

$$\mathbb{R} \longrightarrow 0 \longrightarrow 0 \longrightarrow \cdots$$

with $\mathbb{R}$ in degree zero and nothing elsewhere. By the d-Poincaré lemma,

$$\mathcal{H}^q(\mathcal{Q}^*) = 0 \quad \text{for } q > 0, \quad \mathcal{H}^0(\mathcal{Q}^*) \cong \mathbb{R} .$$