$\Rightarrow$ By the Tietze extension theorem, we can extend to $\overline{V}_2$.
Continue this process, we have an extension τ on $\bigcup V_i$.
$\Rightarrow$ $\tau|_{\bigcup V_i}$ is an extension we need.

Let's go back to our problem. We have $(k-r+1)$ everywhere linearly independent sections e_i on $\sigma_a(V)$. For each i, by the method above, we have an extension $\tilde{e}_i$ on an open set $V_i \supset \sigma_a(V)$. $\Rightarrow$ $V = \bigcap V_i$ is open. And since $\tilde{e}_i$'s are continuous, if necessary, by choosing smaller open set $V' \subset V$, we have $(k-r+1)$ everywhere linearly independent sections $\tilde{e}_i$ on V'.

Refer to P8. Th 3.2. Proof. (K-theory by M. Karoubi) ⌟

Let $S' \longrightarrow U$ be the trivial subbundle of $S|_U$ spanned by $\tilde{e}_1, \cdots \tilde{e}_{k-r+1}$, and let $S'' \longrightarrow U$ be the quotient of $S|_U$ by S'. Since S' is trivial, we have $C(S') = 1$; by the Whitney formula,

$$c(S|_U) = c(S''),$$

and hence

$$C_r(S)(\sigma_a(V)) = C_r(S'')((\sigma_a(V)) = 0,$$

since S'' has rank $r-1$.

Γ $S|_U \cong S'' \oplus S'$ $\Rightarrow$ $c(S|_U) = c(S'' \oplus S')$
$= c(S'')\, c(S') = c(S'')$.
$\Rightarrow$ $C_r(S)(\sigma_a(V)) = C_r(S'')\,(\sigma_a(V))$
$\operatorname{rank} S'' + \operatorname{rank} S' = \operatorname{rank} S = k = \operatorname{rank} S'' + k-r+1$
$\Rightarrow$ $\operatorname{rank} S'' = r-1$. $\Rightarrow$ $C_r(S'') = 0$ $\Rightarrow$ $C_r(S'')(\sigma_a(V)) = 0$ ⌟