$$\Rightarrow \quad g'_{\alpha\beta} \frac{\sigma}{(s_0)_\beta} = \frac{\varphi_{\alpha\beta}}{g_{\alpha\beta}} \frac{\sigma_\beta}{(s_0)_\beta} = \frac{\sigma_\alpha}{(s_0)_\alpha}$$

$\Rightarrow \quad \dfrac{\sigma_\alpha}{(s_0)_\alpha}$ defines a global meromorphic section of E with poles of order $\leq a_i$ on V_i.

$\Rightarrow$ The tensoring with s_0 is onto.

If $s \in \mathcal{E}(D)$ with $s \otimes s_0 = 0$, $s\, s_\alpha = 0$ on M $\Rightarrow$ $s \equiv 0$ or $s_\alpha \equiv 0$; $\Rightarrow$ Since $s_\alpha \not\equiv 0$, s should be zero section. $s = 0$. ⌟

Thus in particular, if D is a smooth analytic hypersurface, the sequence of sheaves.

$$0 \longrightarrow \mathcal{O}_M(E \otimes [-D]) \xrightarrow{\otimes s_0} \mathcal{O}_M(E) \xrightarrow{r} \mathcal{O}_D(E|_D) \to 0$$

where r is the restriction map, is exact.
Henceforth, we shall make the identification $(*)$ implicitely and write $\mathcal{O}(D)$ for $\mathcal{O}([D])$.

⌜ Given $\sigma \in \mathcal{O}_D(E|_D)$, since D is smooth hypersurface, locally, $D \cong \mathbb{C}^{n-1} \times \{0\}$, cl. $\mathbb{C}^n \subset M$.
$\Rightarrow$ $\sigma : \mathbb{C}^{n-1} \longrightarrow E|_D$ can be extended to $\mathbb{C}^n$ by putting $\tilde{\sigma}(z_1 \cdots z_n) = \sigma(z_1 \cdots z_{n-1})$. simply. $\Rightarrow$ r is onto.

$\otimes s_0 \; (\mathcal{O}_M(E \otimes [-D])) = \mathcal{E}(-D) =$ The sheaf of sections of E vanishing to order $\geq a_i$ along V_i.
where $D = \sum a_i V_i$. $1 = a_i > 0$. $\Rightarrow$ $\mathcal{E}(-D)|_D = 0$.

$\Rightarrow$ $r \circ \otimes s_0 = 0$.
Given $\sigma \in \mathcal{O}_M(E)$ s.t $r(\sigma) = 0$, $\sigma|_D = 0$.