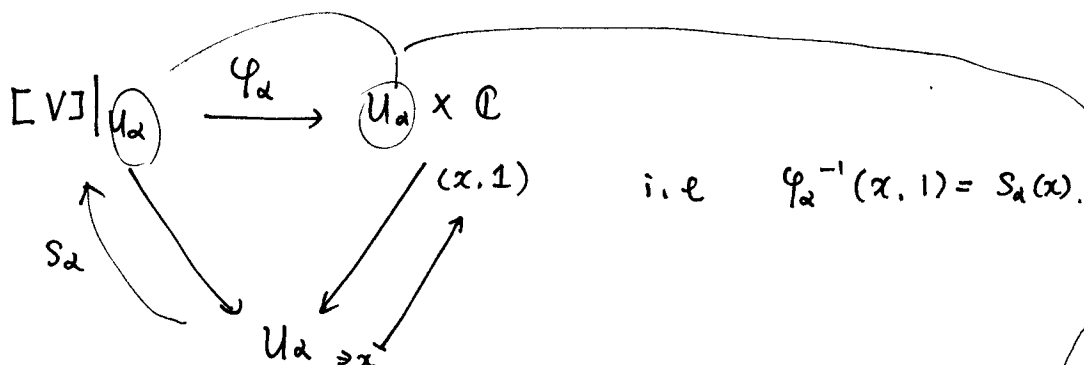


Consider  $\{(df_\alpha \otimes s_\alpha)\}$ , where



On  $U_\alpha \cap U_\beta \cap V$ ,  $df_\alpha \otimes s_\alpha \in N_V^*|_{U_\alpha} \otimes [V]|_{U_\alpha}$

$$df_\alpha \otimes s_\alpha = g_{\alpha\beta} df_\beta \otimes s_\alpha = df_\beta \otimes g_{\alpha\beta} s_\alpha. \text{ by } \otimes \downarrow$$

actually  
 $U_\alpha \cap V$ .

$$\varphi_\alpha^{-1}(x, 1) = s_\alpha(x)$$

$$\varphi_\beta^{-1}(x, 1) = s_\beta(x)$$

$$\varphi_\beta(\varphi_\alpha^{-1}(x, 1)) = \varphi_\beta(s_\alpha(x))$$

$$(x, g_{\beta\alpha}(x) \cdot 1) = \varphi_\beta(s_\alpha(x))$$

$$\begin{aligned} \varphi_\beta^{-1}(x g_{\beta\alpha}(x) \cdot 1) &= \varphi_\beta^{-1}(x, g_{\beta\alpha}(x) \cdot 1) = g_{\beta\alpha}(x) \varphi_\beta^{-1}(x, 1) \\ &= g_{\beta\alpha}(x) s_\beta(x) = s_\alpha(x) \Rightarrow g_{\beta\alpha} s_\beta = s_\alpha \Rightarrow s_\beta = g_{\alpha\beta} s_\alpha \end{aligned}$$

Thus  $df_\alpha \otimes s_\alpha = df_\beta \otimes g_{\alpha\beta} s_\alpha = df_\beta \otimes s_\beta$  on  $V \cap U_\alpha \cap U_\beta$ .  
This means that  $\{(df_\alpha \otimes s_\alpha)\}$  defines a global section of  $N_V^* \otimes [V]|_V$ .

Furthermore, since  $df_\alpha$  is nonzero everywhere &  $s_\alpha$  is nonzero everywhere, the global section is nonzero.

Since  $N_V^* \otimes [V]$  is a line bundle, it is the trivial bundle.

$$N_V^* \otimes [V]|_V = M|_V \times \mathbb{C}$$

$$\Rightarrow N_V^* \otimes [V]|_V \otimes [V]|_V = (M|_V \times \mathbb{C}) \otimes [V]|_V = [V]|_V$$

$$N_V^* = [V]|_V$$

□