a line $l_0 \subset \mathbb{P}^n$ and contained in a 3-plane $S \subset \mathbb{P}^n$ containing l_0.

「 $\sigma_{n-2,n-3} = \{ \Lambda \in G(2, n+1) : \dim(\Lambda \cap V_2) \geq 1, \dim(\Lambda \cap V_4) \geq 2 \}$
$V_2 = l_0, \quad V_4 = S$ in $\mathbb{P}^n$.
$\Rightarrow$ $\Lambda = l$ meets with l_0 and is contained in S which is a 3-plane in $\mathbb{P}^n$. 」

Generically, $W' = W \cap S$ will be a smooth quadric surface in $S \cong \mathbb{P}^3$, with l_0 meeting it at two points p_1 and p_2; clearly any line $l \subset \tau(W) \cap \sigma_{n-2,n-3}$ will pass through either p_1 or p_2.

「 $l_0 \subset S$ (by def of $\sigma_{n-2,n-3}$) Given any line $l \in \tau(W) \cap \sigma_{n-2,n-3}$,
$l \subset W$ and $l_0 \cap l \neq \phi$ and $l \subset S$
$\Rightarrow$ $l \subset W' \Rightarrow l \cap l_0 \subset W' \cap l_0 = \{p_1, p_2\}$.
$\Rightarrow$ $l \cap l_0$ is $\{p_1\}$ or $\{p_2\}$
See P167. 」
" Comment: For a generic S, $W \cap S$ will be a quadric surface in S, but I don't see how it is smooth. unless W is smooth. "

But any line on W' through p_i must lie in the tangent plane $T_{p_i}(W')$; and $T_{p_i}(W') \cap W$ is a singular curve of degree 2, hence consists of two lines.

「 According to P176. $T_{p_i}(W') = \{$ The limiting position of chords $\lim_{\lambda \to 0} \overline{p, q(\lambda)}$, where $q(\lambda)$ is an arc in W' with $q(0) = p$. $\}$. Given any line l on W' through p_i,