

$$(3) \quad \mathcal{O}_K \cap (\phi + W_\phi) = \phi + (\mathcal{O}_K \cap W_\phi) \subset E.$$

Γ $W_\phi \cap \mathcal{O}_K = \{ \psi \in \mathcal{O}_K : \|\psi\|_N < \delta \}$ is open in \mathcal{O}_K ?

If we choose δ large enough, $V_\delta = \{ \phi \in \mathcal{O}_K : \|\phi\|_E < \frac{1}{\delta} \}$
 $\subset W_\phi \cap \mathcal{O}_K$, since $\|\phi\|_E \geq \|\phi\|_N$. Here $\frac{1}{\delta} < \delta$.

$\Rightarrow W_\phi \cap \mathcal{O}_K$ is open in \mathcal{O}_K for all $K \subset \Omega$. $\Rightarrow W_\phi \in \tau$.

Compare with P16, note.

$\mathcal{O}_K \cap (\phi + W_\phi) \ni \phi + \psi \in W_\phi \Rightarrow \phi + \psi \in \mathcal{O}_K$, but since $\phi \in E \in \tau_K \Rightarrow \phi \in \mathcal{O}_K \Rightarrow \psi \in \mathcal{O}_K \Rightarrow \phi + \psi \in \phi + (\mathcal{O}_K \cap W_\phi)$
 Converse is easier. Clearly $\phi + (\mathcal{O}_K \cap W_\phi) \subset E$, by (2). \sqcup

If V is the union of these sets $\phi + W_\phi$, one for each $\phi \in E$, then V has the desired property.

$$\Gamma \quad V = \bigcup_{\phi \in E} (\phi + W_\phi) \Rightarrow V \cap \mathcal{O}_K = \left(\bigcup_{\phi \in E} (\phi + W_\phi) \right) \cap \mathcal{O}_K$$

$$= \bigcup_{\phi \in E} ((\phi + W_\phi) \cap \mathcal{O}_K) = \bigcup_{\phi \in E} \phi + (\mathcal{O}_K \cap W_\phi) \subset E$$

$$\Rightarrow \text{Since } V \supset E, \quad E = V \cap \mathcal{O}_K. \quad \sqcup$$

For (c), consider a set $E \subset \mathcal{O}(\Omega)$ which lies in no \mathcal{O}_K .
 Then there are functions $\phi_m \in E$ and there are distinct points $x_m \in \Omega$, without limit point in Ω , such that $\phi_m(x_m) \neq 0$ ($m=1, 2, 3, \dots$).

Γ For each N , $E \not\subset \mathcal{O}_{K_N} \Rightarrow \exists \phi_N \in E$ s.t.
 $\text{supp } \phi_N \not\subset K_N \Rightarrow \exists$ a point $x_N \in \Omega$, s.t. $\phi_N(x_N) \neq 0$,
 $x_N \in K_N^c$. We may choose distinct points, and the points
 have no limit in Ω . If \exists a limit point in Ω ,
 the limit point is in \checkmark some compact set K_N . $\Rightarrow *$, since
 \exists infinitely many x_N 's. which is impossible. \sqcup