not be bounded. In general, the same arguments works, I think, even for g not meromorphic.
Let's accept !!!

$H^0(\Sigma, \Omega^1) = H^{1,0}(\Sigma) = 0$ since $H^1(\Sigma) = 0 = H^{1,0}(\Sigma) \oplus H^{0,1}(\Sigma)$ since Σ is simply connected and $\pi_1(\Sigma) = H_1(\Sigma) = 0$ by P770 and P494.

$\Rightarrow \omega_i \equiv 0$.

Suppose G is smooth quadric in $\mathbb{P}^4$.

$\Rightarrow$ By Lefschetz hyperplane section,

$$H^i(G) \cong H^i(G \cap H) \quad \text{for } i \leq \underbrace{\#}_{3 = \dim G} - 2 = 2$$

$\Rightarrow \quad H^1(G) = H^1(G \cap H) = 0$ since $G \cap H \cong \mathbb{P}^1 \times \mathbb{P}^1$.

$\Rightarrow$ We can show that $H^1(\Sigma) = 0$.

┘

Thus $\iota^* dz_i = -dz_i$, i.e., ι is the standard involution of the Abelian variety $A = \mathbb{C}^2/\Lambda$ induced by the map $(z_1, z_2) \mapsto (-z_1, -z_2)$ on $\mathbb{C}^2$; precisely the same argument shows that the involution ι' is likewise induced by the involution $z \mapsto -z$ on $\mathbb{C}^2$, but with a different choice of base point.

┌ $\iota : A = \mathbb{C}^2/\Lambda \longrightarrow \mathbb{C}^2/\Lambda$

$$[z_1, z_2] \longmapsto [f_1(z_1, z_2), f_2(z_1, z_2)].$$

$$\iota^* dz_1 = \frac{\partial f_1}{\partial z_1} dz_1 + \frac{\partial f_1}{\partial z_2} dz_2 = -dz_1 \Rightarrow \frac{\partial f_1}{\partial z_1} = -1$$