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S be an arbitrary smooth cubic in $\mathbb{P}^3$. Let $W = |3H| \cong \mathbb{P}^{19}$ be the linear system of all cubic surfaces in $\mathbb{P}^3$, and consider the incidence correspondence

$$X = \{ (S, p) : p \in S \} \subset W \times \mathbb{P}^3.$$

Γ $\dim H^0(\mathbb{P}^3, \mathcal{O}(3H)) = {}_6C_3 = \frac{6 \cdot 5 \cdot 4}{6} = 20$

$\Rightarrow \underset{\dim}{\wedge} |3H| = 20 - 1 = 19$

$\lrcorner$

The subset $V \subset W$ of singular cubics is a proper analytic subvariety of W, and so $W - V$ is connected; take $\gamma : I \longrightarrow W - V$ a C^∞ embedding of the unit interval $I \subset \mathbb{R}$ in $W - V$ with $\gamma(0) = S_0$, $\gamma(1) = S$.

Γ Let U be an open set in $\mathbb{P}^3$ holomorphic to $\Delta \subset \mathbb{C}^3$.

$$\Delta \times \mathbb{P}^{19} \supset \mathcal{S} = \{ \{ (x, y, z), [(a_1, a_2, \cdots a_{20})] \} \mid \begin{array}{l} (a_1 \sigma_1 + \cdots + a_{20}\sigma_{20})(x, y, z) = 0 \\ (a_1 \nabla \sigma_1 + \cdots + a_{20} \nabla \sigma_{20})(x, y, z) = 0 \} . \end{array}$$

where $H^0(\mathbb{P}^3, \mathcal{O}(3)) = \langle \sigma_1, \sigma_2, \cdots \sigma_{20} \rangle$.

$\Rightarrow$ V is locally expressed as the image of π_2, i.e., $\pi_2(\mathcal{S})$,
$\pi_2 : \Delta \times \mathbb{P}^{19} \longrightarrow \mathbb{P}^{19}$. $\Rightarrow$ By the proper mapping theorem, $\pi_2(\mathcal{S})$ is an analytic subvariety of $\mathbb{P}^{19}$, since $\mathbb{P}^3 \times \mathbb{P}^{19} \xrightarrow{\pi_2} \mathbb{P}^{19}$ is proper and $\mathcal{S}$ can be defined globally, ($\mathcal{S}$ is expressed locally as above) and clearly an algebraic subvariety of $\mathbb{P}^3 \times \mathbb{P}^{19}$.