for any $\delta' > 0$, $\delta' < \delta$. $\Rightarrow$ $g^{-1}(\delta)$ is a smooth submanifold of $B(0,1) = D^2$, which is a 3-cycle, since $Dg \neq 0$ on all $g^{-1}(\delta)$.

$$\log f = \log|f| + i \arg f$$
$$\Rightarrow \quad i \arg f = \log f - \log|f| = \log \frac{f}{|f|}$$
$$\Rightarrow \quad \arg f = -i \log \frac{f}{|f|}$$

$$|f| = \epsilon \Rightarrow \arg f = -i \log f + i \log \epsilon$$
$$\Rightarrow d \arg f = -i \, d\log f = -i \frac{df}{f}$$

If we let $f = u + iv$, $df = du + i\,dv$.

$$-i\frac{df}{f} = -i \frac{du + i\,dv}{u + iv} = -i \frac{(du + i\,dv)(u - iv)}{u^2 + v^2}$$
$$= -i \frac{u\,du + v\,dv + i(dv)u - iv\,du}{u^2 + v^2} = -i \frac{0 + i(u\,dv - v\,du)}{\epsilon^2}$$
$$= \underbrace{\frac{u\,dv - v\,du}{\epsilon^2}}_{= d\theta} \geq 0$$

Remember that $df = 0$ on $T_p\{f = 0\}$ and $df \neq 0$ on $(T_p\{f=0\})^{\perp}$. But in our case not $f =$ constant, but $|f| =$ constant.

$$|f|^2 = C = f\bar{f} \Rightarrow d(f\bar{f}) = 0 = f\,d\bar{f} + \bar{f}\,df$$
$$\Rightarrow \frac{d\bar{f}}{\bar{f}} + \frac{df}{f} = 0 \Rightarrow \frac{df}{f} \text{ is pure imaginary}$$

That's why we multiply $\frac{df}{f}$ by $-i$.

For example for $\mathbb{C}^2$, $dz_1 = 0$ on $\{z_1 = 0\}$, $dz_1 \neq 0$ on $\{z_2 = 0\} = \{(z_1, 0) \mid z_1 \text{ varies}\}$ $\Rightarrow$ $df \neq 0$ on $|f| =$ constant since f varies along a circle. Some sort