Also, in case the quadric has odd dimension $n = 2m+1$ and $k = m$, the codimension $(m+1)(m+2)/2$ of the cycle $\Sigma_{m,2m+1}$ is exactly half the dimension of the Grassmannian $G(m+1, 2m+3)$, and so we may expect there to be a finite number of m-planes in the generic intersection of two quadrics in $\mathbb{P}^{2m+2}$; indeed, by our calculation this number is

$$\#(\Sigma_{m,2m+1} \cdot \Sigma_{m,2m+1}) = 2^{m+2} \cdot (\sigma_{m+1,m,\cdots 1} \cdot \sigma_{m+1,m,\cdots 1})$$
$$= 2^{m+2}.$$

We have already verified this in case $n=3$.

Γ $(m+1)(2m+3-m-1) = (m+1)(m+2)$ is the dimension of $G(m+1, 2m+3)$ over $\mathbb{C}$.

By P739, $a_i + b_{k+2-i} = n-k+1$ $\quad k=m, \ n = 2m+1$

$\Rightarrow \quad a_i + b_{m+2-i} = 2m+1-m+1 = m+2$

$\Rightarrow \quad b_{m+2-i} = m+2-a_i$

$\Rightarrow \quad b_{m+1} = m+2-(m+1) = 1 \qquad b_m = 2 \quad \cdots \qquad b_1 = m+1.$

$\Rightarrow \quad (\sigma_{m+1,m,\cdots 1} \cdot \sigma_{m+1,m,\cdots 1}) = 1.$

$\Sigma_{m,2m+1} = \{\Lambda_m \mid \Lambda_m \subset F_{2m+1} \subset \mathbb{P}^{2m+2}\}$ = Set of all m-planes in a smooth quadric of dim $(2m+1)$ in $\mathbb{P}^{2m+2}$.

Choose F^1_{2m+1}, F^2_{2m+1} smooth quadrics of dim $(2m+1)$ in $\mathbb{P}^{2m+2}$. $\Rightarrow$ $\#(\Sigma_{m,2m+1} \cdot \Sigma_{m,2m+1})$ = # of m-planes in the generic intersection of F^1_{2m+1} with F^2_{2m+1}. We know that $\#(\Sigma_{m,2m+1} \cdot \Sigma_{m,2m+1}) < \infty$.

For $n=3$ case, see P199, P550. $\quad \mathbb{P}^{4(=n+1)} \supset F_3$, $m=1$. ⅃