Now $\mathcal{U}$ is covered by the open nbd W_p of all $p \in \mathcal{U}$, so by the sheaf property (i). $s = 0$ on $\mathcal{U}$.

Thus $\varphi(\mathcal{U})$ is injective.

② $\varphi(\mathcal{U}) = \varphi_{\mathcal{U}}$ is surjective.

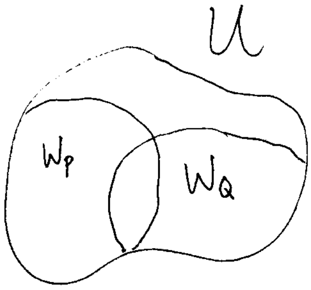
Given $t \in \mathcal{G}(\mathcal{U})$, for each $p \in \mathcal{U}$ $t_p \in \mathcal{G}_p$ is its germ at p.
Since φ_p is surjective, we have $s_p \in \mathcal{F}_p$ s.t $\varphi_p(s_p) = t_p$.
Let s_p be represented by a section $s(p) \in \mathcal{F}(V_p)$ where V_p is nbd of p.

$$\begin{tikzcd}
s(p) \in \mathcal{F}(V_p) \arrow[rr] \arrow[rr, bend left, dashed] \arrow[r, hook] \arrow[d, "\varphi_{V_p}"'] & \mathcal{F}(W_p) \arrow[r] \arrow[d, "\varphi_{W_p}"] & s_p \in \mathcal{F}_p \arrow[d, "\varphi_p"] \arrow[d, bend left] \\
\mathcal{G}(V_p) \arrow[rr, "\cancel{\mathcal{G}(W_p)}"] & & \mathcal{G}_p \ni t_p
\end{tikzcd}$$

Consider $\varphi(V_p)(s(p))$.
$\Rightarrow (\varphi(V_p)(s(p)))_p = t_p$

$\Rightarrow \exists$ open nbd $W_p \ni p$ s.t $\varphi(V_p)(s(p))|W_p = t|W_p$ and $W_p \subset V_p \cap \mathcal{U}$.

Now. $\mathcal{U}$ is covered by the open sets W_p for all $p \in \mathcal{U}$.



Consider $S(P)\big|_{W_P \cap W_Q}$ & $S(Q)\big|_{W_P \cap W_Q}$

$$\Rightarrow \varphi_{W_P \cap W_Q}(S(P)|_{W_P \cap W_Q}) = \varphi_{W_P \cap W_Q}(S(Q)|_{W_P \cap W_Q}) = t|_{W_P \cap W_Q}$$

$\Rightarrow$ By the injectiveness of φ, $S(P)|_{W_P \cap W_Q} = S(Q)|_{W_P \cap W_Q}$.