$= U' - W = U' \cap (N - W)$. For $x \in U$, ① $x \in W$
$\Rightarrow x \notin U - W \Rightarrow x \notin U' - W \Rightarrow x \in U'$
② $x \notin W \Rightarrow x \in U - W \Rightarrow x \in U' - W \Rightarrow x \in U'$
$\Rightarrow$ By ① & ②, $U \subset U'$. Similarly $U' \subset U \Rightarrow U = U'$.
By the note above, we have an open cover $\{g^{-1}(U_\alpha)\}$ over N
s.t.

$$\begin{tikzcd} g^*[D]|_{g^{-1}(U_\alpha)} \arrow[r, "\cong"] \arrow[d] & g^{-1}(U_\alpha) \times \mathbb{C} \arrow[dl] \\ g^{-1}(U_\alpha) & \end{tikzcd}$$

More precisely, for each $y \in W$, choose an open set U_y in N s.t. $U_y \cong \Delta$. $g(U_y - W)$ is open in M. $\Rightarrow \exists$ subset K in M s.t. $(\overline{g(U_y - W)})^\circ - g(U_y - W) = K$.
For each $z \in K$, consider an open set $U_z \subset \overline{g(U_y-W)}^\circ$ in M s.t. $U_z \cong \Delta$.
$\Rightarrow g^{-1}(U_z) \cap U_y = U' - W$, open set U' in N
Not satisfactory !!!

$$f : M - V \longrightarrow N \supset W$$

For each $y \in W$, $\exists$ open subset U_y in N s.t.
$$U_y \cap W = \{g_1 = g_2 = \cdots = g_e = 0\}.$$
Consider $f^{-1}(U_y)$. $\Rightarrow$ If $f^{-1}(U_y)$ contains an open set $O - V$, s.t. $O \overset{(\text{in } M)}{\cong} \Delta$, O nbd of a point of V, consider f^*g_1 on $O - V$. $\Rightarrow$ By Hartogs' theorem, f^*g_1 is extended to O. Similarly for $f^*g_2, \dots f^*g_e$.
$\Rightarrow \overline{f^{-1}(W) \cap O} = \{f^*g_1 = \cdots = f^*g_e = 0,\ f^*g_i$ the extension$\}$