$\ulcorner$

$$\begin{array}{ccc} C & \xrightarrow{\pi} & \mathbb{P}^1 \\ \rotatebox{90}{$\in$} & & \\ x & \longmapsto & [a, b], \end{array} \quad \text{where} \quad L = \langle F, G \rangle$$

$$aF(x) + bG(x) = 0.$$

Note, here, that $F(x) = G(x) = 0$, since L is a generic pencil. Given any $[a, b] \in \mathbb{P}^1$, $\exists$ four points x's s.t $aF(x) + bG(x) = 0$ on C.

π is holomorphic, for $[F(x), G(x)]$ is holomorphic, and $\mathbb{P}^1 \xrightarrow{\cong} \mathbb{P}^{1*}$ is biholomorphic.

$\Rightarrow$ π is a 4-sheeted cover of $\mathbb{P}^1$.

By the Riemann-Hurwitz formula on P218,

$$g(C) = 4 \cdot (g(\mathbb{P}^1) - 1) + 1 + \frac{1}{2} \sum_{q \in C} (v(q) - 1).$$

Since the definition of the branch locus of π is

$$\sum_{q \in C} (v(q) - 1) q,$$

the number of branch points of π, counting multiplicity, is $\sum_{q \in C} (v(q) - 1)$, see P 219.

$\Rightarrow$ Let $b = \sum_{q \in C} (v(q) - 1)$.

$$\Rightarrow \quad b = 2g(C) - 8(g(\mathbb{P}^1) - 1) - 2$$
$$= 0 - 0 + 8 - 2 = 6,$$
since $\deg C = 2$, $\deg \mathbb{P}^1 = 1$, and $g(C) = 0 = g(\mathbb{P}^1)$ by the genus formula.

$\lrcorner$