g, $p_0 \in S$ and $\omega_1, \cdots \omega_g$ a basis for $H^0(S, \Omega^1)$, for any $\lambda \in J(S)$ we can find g points $p_1, \cdots p_g \in S$ such that

$$(*) \qquad \mu\left(\sum_i (p_i - p_0)\right) = \lambda,$$

i.e., for any vector $\lambda \in \mathbb{C}^g$, we can find $p_1 \cdots p_g \in S$ and paths α_i from p_0 to p_i such that

$$\sum_i \int_{\alpha_i} \omega_j = \lambda_j \quad \text{for all } j.$$

Moreover, for generic $\lambda \in \mathbb{C}^g$, the divisor $\sum p_i$ is unique.

Proof. For now, we will just prove the result; in Section 7 of this chapter, after introducing Riemann's theta function, we will see how to solve the equation $(*)$ explicitly.

First let $S^{(d)}$ denote the set of effective divisors of degree d in S, i.e. the set of unordered d-tuples of points $\{p_1, p_2, \cdots p_d\}$ on S, not necessarily distinct. $S^{(d)}$ is the quotient of the d-fold product $S^d = S \times S \times \cdots \times S$ of S with itself d times by the action of the symmetric group Σ_d on d letters; as such it inherits from S^d the structure of a topological space. In fact, the the projection map $\pi : S^d \longrightarrow S^{(d)}$ gives $S^{(d)}$ the structure of a complex manifold: for a point $D = \sum p_i \in S^{(d)}$, let z_i be a local coordinate in a neighborhood U_i of p_i in S, where we take $U_i \cap U_j = \phi$ for $p_i \neq p_j$ and $z_i = z_j$ in $U_i = U_j$ for $p_i = p_j$. Then if we let $\sigma_1, \sigma_2, \cdots \sigma_d$ denote the elementary symmetric functions, by the fundamen-