

$r=0 \Rightarrow C_0 = n-k$. formally, \Rightarrow If $a_i + b_{k-i} \geq n-k+1$, for some i ,
then $a_i + b_{k-i} + C_0 \geq n-k+1 + n-k = 2(n-k)+1$.

$$\alpha = i \quad \beta = k-i \quad r=0 \Rightarrow \alpha + \beta + r = k.$$

Now we have to prove that

$$\#(\sigma_a \cdot \sigma_b \cdot \sigma_c) = \#(\sigma_{a_1-1, \dots, a_i-1, a_{i+1}, \dots, a_k} \cdot \sigma_{b_1-1, b_2-1, \dots, b_{k-i}-1, b_{k-i+1}, \dots, b_k} \cdot \sigma_c)_{G(k+1, n)}.$$

Start with $\#(\sigma_a \cdot \sigma_b \cdot \sigma_c)$.

$$\begin{aligned} \#(\sigma_a \cdot \sigma_b \cdot \sigma_c)_{G(k, n)} &= \#(\sigma_a^* \cdot \sigma_b^* \cdot \sigma_c^*)_{G(n-k, n)} \\ &= \#(\sigma_{a^*-a_{a_i}}^* \cdot \sigma_{b^*-b_{b_{k-i}}}^* \cdot \sigma_{c^*-c_{c_0}}^*)_{G(n-k+1, n-1)} \quad \text{if } a_{a_i}^* + b_{b_{k-i}}^* = i + k - i \\ &= \#(\sigma_{a_1-1, \dots, a_i-1, a_{i+1}, \dots, a_k} \cdot \sigma_{b_1-1, \dots, b_{k-i}-1, b_{k-i+1}, \dots, b_k} \cdot \sigma_{c_1, c_2, \dots, c_k})_{G(k+1, n)} \\ &= \#(\sigma_{a_1-1, \dots, a_i-1, a_{i+1}, \dots, a_k} \cdot \sigma_{b_1-1, \dots, b_{k-i}-1, b_{k-i+1}, \dots, b_k} \cdot \sigma_{c_1, c_2, \dots, c_k})_{G(k+1, n)} \end{aligned}$$

since $c_0^* = 0$.

I am going to prove the previous note, i.e.
Reduction Formula I is valid for $\beta=r=k$, $a_i=n-k$,
and $r=k$. $a_i + b_{k+1-i} = n-k$ for any i .

Obviously, they are valid, since the conditions above satisfy the requirement of Reduction Formula I.
See p521 note. \square

Note also that if the sequence $b_1-1, \dots, b_{\beta-1}, b_{\beta+1}, \dots$ appearing in the formula is no longer nonincreasing - i.e., if $b_\beta = b_{\beta+1}$ - then the intersection number is zero: just apply the formula to $\alpha, \beta+1, r$.