Date

$k_1[\gamma_1] + k_2[\gamma_2] = 0 \in H_1(C, \mathbb{R})$,
which is impossible.

⌈ $\exists\ k_1, k_2 \in \mathbb{R}$ s.t $k_1 a_1 + k_2 a_2 = 0 = k_1\int_{\gamma_1}\omega + k_2\int_{\gamma_2}\omega$

$\Rightarrow$ Take the conjugate.

$\Rightarrow k_1\int_{\gamma_1}\overline{\omega} + k_2\int_{\gamma_2}\overline{\omega} = 0.$

$\omega \in H^0(C, \Omega^1) = H^{1,0}(C)$ $\Rightarrow$ Since $\dim H^0(C,\Omega^1) = 1$ and $H^0(C,\Omega^1) = H^{1,0}(C) = \overline{H^{0,1}(C)}$, $\overline{\omega} \in H^{0,1}(C)$.

$\Rightarrow k_1\int_{\gamma_1}\eta + k_2\int_{\gamma_2}\eta = 0$ for all $\eta \in H^1_{DR}(\mathbb{C}) = H^1_{DR}(C,\mathbb{C}) \overset{\shortparallel}{} H^1_{DR}(C,\mathbb{R})\otimes\mathbb{C}$

$\Rightarrow k_1\gamma_1 + k_2\gamma_2 = 0 \in H_1(C, \mathbb{R})$

$\Rightarrow$ Since γ_1 & γ_2 are linearly independent, it is impossible. ⌋

Thus a_1 and a_2 are independent over $\mathbb{R}$, and so the periods $\Lambda = \{n\cdot a_1 + m\cdot a_2\}_{n,m\in\mathbb{Z}} \subset \mathbb{C}$ of ω in C form a lattice in $\mathbb{C}$. Correspondingly, the value of the integral

$$\int_{p_0}^{p}\omega,$$

while not a well-defined number, is well-defined as a point of the complex torus $\mathbb{C}/\Lambda$.

⌈ Suppose α, β are paths from p_0 to p.

$\Rightarrow \alpha = \beta + n\cdot\gamma_1 + m\gamma_2$

$\Rightarrow \int_\alpha\omega = \int_\beta\omega + na_1 + ma_2$ ⌋

The first major step toward understanding integ