Date

This means that $\# V \cap \mathbb{P}^1 = d$ for a generic $\mathbb{P}^1 \subset \mathbb{P}^n$.

$\#(F=0 \cap \mu(\mathbb{P}^1)) = d$.

$F=0 \cap [a_0 Y_0 + b_0 Y_1, \cdots a_n Y_0 + b_n Y_1]$

$F(a_0 Y_0 + b_0 Y_1, \cdots a_n Y_0 + b_n Y_1) = 0 = F \circ \mu(Y_0, Y_1)$

A basic fact about degree is that it is multiplicative with respect to intersections. Since a $\mathbb{P}^{n-k_1}$ and a $\mathbb{P}^{n-k_2}$ intersect transversely in a $\mathbb{P}^{n-k_1-k_2}$, the degree of the intersection of two varieties meeting transversely almost everywhere is the product of their degrees. More generally if V and W are varieties of degree d_1 and d_2 in $\mathbb{P}^n$ intersecting in a variety of the appropriate dimension, $\{Z_i\}$ the irreducible components of $V \cap W$, then

$$d_1 \cdot d_2 = \sum_i \mathrm{mult}_{Z_i} (V \cdot W) \ \mathrm{degree}(Z_i)$$

with $\mathrm{mult}_{Z_i}(V \cdot W)$ defined as in Section 4 of Chapter 0.

Pf

$[V] \in H_{2k_1}(\mathbb{P}^n, \mathbb{Z}) = H^{2n-2k_1}(\mathbb{P}^n, \mathbb{Z}) \ni \eta_V$

$[W] \in H_{2k_2}(\mathbb{P}^n, \mathbb{Z}) = H^{2n-2k_2}(\mathbb{P}^n, \mathbb{Z}) \ni \eta_W$.

$\Rightarrow [V \cap W] \in H_{2k_1+2k_2-2n}(\mathbb{P}^n, \mathbb{Z}) \cong H^{4n-2k_1-2k_2}(\mathbb{P}^n, \mathbb{Z}) \ni \eta_V \wedge \eta_W$

$$\Rightarrow \begin{array}{ccccc} H^{2n-2k_1}(\mathbb{P}^n, \mathbb{Z}) & \otimes & H^{2n-2k_2}(\mathbb{P}^n, \mathbb{Z}) & \longrightarrow & H^{4n-2k_1-2k_2}(\mathbb{P}^n, \mathbb{Z}) \\ \cup\!\!\!\!\!\!\;\;\text{\rotatebox{90}{$\in$}} & & \text{\rotatebox{90}{$\in$}} & & \text{\rotatebox{90}{$\in$}} \\ d_1 \omega_1 = \eta_V & & \eta_W = d_2 \omega_2 & \longmapsto & \eta_V \wedge \eta_W \\ & & & & \| \\ & & & & d_1 d_2 (\omega_1 \wedge \omega_2) \end{array}$$

$\omega_1 \wedge \omega_2 \in$ generator of $H^{4n-2k_1-2k_2}(\mathbb{P}^n, \mathbb{Z})$

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