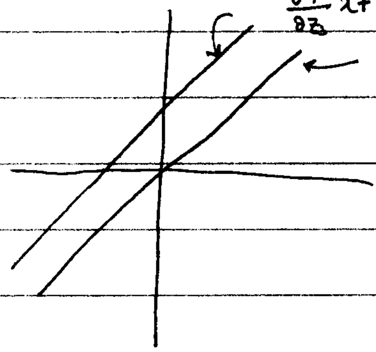
and similarly, we have

$$\alpha(z_1^\circ, \cdots)\frac{\partial f}{\partial z_3} + \beta(z_1^\circ, \cdots)\frac{\partial g}{\partial z_3} = \frac{\partial \gamma}{\partial z_3} \neq 0$$

since γ does not have a double root at $(z_1, z_2) \notin (D(\gamma) = 0)$.

$$\Rightarrow \begin{pmatrix} \frac{\partial f}{\partial z_4}, & \frac{\partial g}{\partial z_4} \\ \frac{\partial f}{\partial z_3}, & \frac{\partial g}{\partial z_3} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\partial \gamma}{\partial z_3} \end{pmatrix}$$

$$\Rightarrow \det \begin{pmatrix} \frac{\partial f}{\partial z_4}, & \frac{\partial g}{\partial z_4} \\ \frac{\partial f}{\partial z_3}, & \frac{\partial g}{\partial z_3} \end{pmatrix} \neq 0, \quad \text{otherwise} \quad \left(\frac{\partial f}{\partial z_4}, \frac{\partial g}{\partial z_4}\right) = 0$$



In case $\det \begin{pmatrix} \frac{\partial f}{\partial z_4}, & \frac{\partial g}{\partial z_4} \\ \frac{\partial f}{\partial z_3}, & \frac{\partial g}{\partial z_3} \end{pmatrix} \neq 0$,

consider the following map F:

$$(z_1, z_2, z_3, z_4) \longmapsto (z_1, z_2, f, g).$$

$$\Rightarrow \det \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\partial f}{\partial z_1}, & \frac{\partial f}{\partial z_2}, & \frac{\partial f}{\partial z_3}, & \frac{\partial f}{\partial z_4} \\ \frac{\partial g}{\partial z_1}, & \frac{\partial g}{\partial z_2}, & \frac{\partial g}{\partial z_3}, & \frac{\partial g}{\partial z_4} \end{pmatrix} = \det \begin{pmatrix} \frac{\partial f}{\partial z_3} & \frac{\partial f}{\partial z_4} \\ \frac{\partial g}{\partial z_3}, & \frac{\partial g}{\partial z_4} \end{pmatrix} \neq 0$$