$\Rightarrow \ \|\bar{\nabla}\phi\|^2 \le C'\{\langle\Delta\phi.\phi\rangle + \|\phi\|^2\}.$

$\& \ \|\nabla\phi\|^2 \le C''\{\langle\Delta\phi.\phi\rangle + \|\phi\|^2\} \quad \Rightarrow \quad \|\phi\|_1 \le C\,\mathcal{D}(\phi).$ ⌋

" Comment : The key point is the Garding inequality & the existence of $T$ which is compact, self-adjoint, & the inverse of $I+\Delta$. "

Now if $M$ is Kähler with associated (1.1)-form $\omega$, we define the operator

$$L : A^{p.q}(E) \longrightarrow A^{p+1,\,q+1}(E) \quad \text{by setting.}$$

for $\eta \in A^{p.q}(M)$ and $s \in A^0(E)$,

$$L(\eta \otimes s) = \omega \wedge \eta \otimes s \ ;$$

let $\Lambda = L^*$ be the adjoint of $L$.

⌈ Locally, $L$ is defined by

$$L(\eta \otimes s) = \omega \wedge \eta \otimes s, \quad \text{where } \eta \in A^{p.q}(U) \text{ and } s \in A^0(E|_U)$$

$$E|_U \cong U \times \mathbb{C}^n, \qquad U \cong \mathbb{C}^k.$$

On $U \cap V$, $\quad L(\eta_u^i \otimes s_u^i) = L(\eta_v^i \otimes s_v^i)$ if

$\eta_u^i \otimes s_u^i = \eta_v^i \otimes s_v^i$ on $U \cap V$.

since $(\eta_u^i \otimes s_u^i) \wedge \omega = \eta_u^i \wedge \omega \otimes s_u^i$ is well-defined globally, as the def of the exterior derivative. ⌋

If $D = D' + D''$ ($D'' = \bar{\partial}$) is the metric connection on $E$, then we have the basic identity