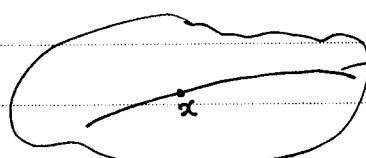


$$\begin{aligned}
\sigma_1 &= \sigma_1 + 0 + 0 + \dots + 0 + 0 + \dots + 0 \\
\sigma_2 &= 0 + \sigma_2 + 0 + 0 + \dots + 0 + 0 + \dots + 0 \\
\sigma_3 &= 0 + 0 + \sigma_3 + 0 + \dots + 0 + 0 + \dots + 0 \\
&\vdots \\
\sigma_{k-1} &= 0 + \dots + 0 + \sigma_{k-1} + 0 + \dots + 0 \\
\sigma_k &= * \sigma_1 + * \sigma_2 + * \sigma_3 + \dots + * \sigma_{k-1} + 0 + \dots + 0 \\
\sigma_{k+1} &= * \sigma_1 + \dots + * \sigma_{k-1} + * \sigma_k + 0 + \dots + 0 \\
&\vdots \\
\sigma_n &= * \sigma_1 + \dots + * \sigma_{k-1} + * \sigma_k + 0 + \dots + 0
\end{aligned}$$

$$\Rightarrow L(x) = \begin{pmatrix} (1, 0, 0, \dots, 0, *, *, \dots, *) \\ (0, 1, 0, \dots, 0, *, *, \dots, *) \\ \vdots \\ k-1 (0, 0, 0, \dots, 1, *, *, \dots, *) \\ k (0, 0, 0, \dots, 0, 0, *, \dots, *) \\ k+1 (0, 0, 0, \dots, 0, 0, 0, \dots, 0) \\ \vdots \\ n (0, 0, 0, \dots, 0, 0, 0, \dots, 0) \end{pmatrix}$$

Since $\text{codim}_{\mathbb{R}}(D_k - D_{k-1}) = 2$, and σ_k intersects the subspace $\langle \sigma_1, \dots, \sigma_{k-1} \rangle$, we can choose

 a point x' near x s.t. $x' \in M - (D_k - D_{k-1})$ and $\sigma_1, \dots, \sigma_{k-1}, \sigma_k$ are linearly independent at x' .

$$\begin{aligned}
\sigma_1 &= \sigma_1 + 0 + \dots + 0 + 0 + \dots + 0 \\
\sigma_2 &= 0 + \sigma_2 + 0 + \dots + 0 + 0 + \dots + 0 \\
&\vdots \\
\sigma_{k-1} &= 0 + 0 + 0 + \dots + 0 + \sigma_{k-1} + 0 + \dots + 0 \\
\sigma_k &= 0 + 0 + 0 + \dots + 0 + 0 + \sigma_k + 0 + \dots + 0 \\
\sigma_{k+1} &= * \sigma_1 + * \sigma_2 + * \sigma_3 + \dots + * \sigma_{k-1} + * \sigma_k + 0 + \dots + 0 \\
&\vdots \\
\sigma_n &= * \sigma_1 + * \sigma_2 + * \sigma_3 + \dots + * \sigma_{k-1} + * \sigma_k + 0 + \dots + 0
\end{aligned}$$