If $\varphi^*(d_0 H|_{V'}) = d_0 H|_V$, then $\varphi^*(H|_{V'}) = H|_V$ is true?

Let $H|_{V'} = L'$ & $H|_V = L$.

$$c_1(\varphi^*(d_0 L')) = \varphi^*(c_1(d_0 L')) = \varphi^*(c_1(L'^{d_0}))$$
$$= \varphi^*(c_1(\overbrace{L' \otimes \cdots \otimes L'}^{d_0})) = \varphi^*(c_1(L') + \cdots + c_1(L'))$$
$$= \varphi^*(d_0\, c_1(L')) = d_0\, \varphi^*(c_1(L'))$$ by the properties of φ^* and c_1. --- ①

From the assumption $\varphi^*(d_0 L') = d_0 L$,

$$c_1(\varphi^*(d_0 L')) = c_1(d_0 L) = d_0\, c_1(L) \quad \text{--- ②}$$

$\Rightarrow$ By ① & ②, $c_1(\varphi^*(d_0 L')) = d_0\, \varphi^*(c_1(L')) = d_0\, c_1(L)$.

Once we know that $H^2(V, \mathbb{Z})$ has no torsion, $c_1(\varphi^*(L')) = \varphi^*(c_1(L')) = c_1(L)$ $\Longrightarrow$ By the previous observation, $\varphi^*(L') = L$, $\Longleftrightarrow$ $\varphi^*(H|_{V'}) = H|_V$.

So it remains to show that $H^2(V, \mathbb{Z})$ has no torsion.

By the Universal Coefficient Theorem,

$$0 \longrightarrow \underbrace{\mathrm{Ext}(H_1(V, \mathbb{Z}), \mathbb{Z})}_{=0} \longrightarrow H^2(V, \mathbb{Z}) \longrightarrow \underbrace{\mathrm{Hom}(H_2(V, \mathbb{Z}), \mathbb{Z})}_{=0}$$

$\Rightarrow$ $H^2(V, \mathbb{Z}) \cong \mathrm{Hom}(H_2(V, \mathbb{Z}), \mathbb{Z})$ $\Rightarrow$ $H^2(V, \mathbb{Z})$ has no torsion. See p189. Greenberg