for all i. $\Rightarrow$ Again by Poincare duality, $\#(\alpha\cdot\sigma_b)=0$ for all σ_b implies that α is a torsion.
$\cap$ $H_{2k(n-k)-2\Sigma b_i}(G(k,n),\mathbb{Z})$,

But as we know, $H_*(G(k,n),\mathbb{Z})$ has no torsion.
$\Rightarrow \alpha=0 \Rightarrow$ This is impossible. Thus $\#(\sigma_a\cdot\sigma_b)\neq 0$ if $a_i = n-k-b_{k-i+1}$, for all i.

Computation.

Let $x_i = n-k+i-a_i$, $y_i = n-i+1-b_{k-i+1}$ as before.

$\Rightarrow \quad x_i+y_i = n-k+i-a_i + n-i+1-b_{k-i+1} = 2n-k+1-(a_i+b_{k-i+1})$

$= 2n-k+1 - n+k = n+1$.

Let $V_{x_i} = \{(*,\cdots \overset{x_i}{*},0\cdots 0)\in\mathbb{C}^n\} \Rightarrow \dim V_{x_i} = x_i$

$V'_{y_i} = \{(0\cdots 0\overset{x_i}{*},*\cdots *)\in\mathbb{C}^n\} \Rightarrow \dim V_{y_i} = n-x_i+1=y_i$

Claim: $\quad \sigma_a\cap\sigma_b = \{(0,0,\cdots 0\overset{x_1}{*}0\cdots 0\overset{x_2}{*}0\cdots \overset{x_k}{*}0\cdots 0)\in\mathbb{C}^n\} = \Lambda_0$

proof)

$\dim(\Lambda_0\cap V_{x_i}) = \dim\{(0,0,\cdots\overset{x_1}{*}0\cdots\overset{x_2}{*}0\cdots\overset{x_i}{*},0\cdots 0)\} = i$

$\dim(\Lambda_0\cap V'_{y_i}) = \dim\{(0,0,\cdots 0\overset{x_i}{*}0\cdots\overset{x_{i+1}}{*}0\cdots\overset{x_k}{*}0\cdots 0)\} = k-i+1$

$\Rightarrow \Lambda_0 \in \sigma_a\cap\sigma_b$.

Suppose $\Lambda\in\sigma_a\cap\sigma_b$.

$\Rightarrow \quad \dim(\Lambda\cap V_{x_1})\geq 1$ and $\dim(\Lambda\cap V'_{y_1})\geq k$

$\Rightarrow \Lambda\subset V'_{y_1} = \{(0,\cdots 0\overset{x_1}{*},*\cdots *)\} \Rightarrow \Lambda\cap V_{x_1}\subset V_{y_1}\cap V_{x_1}$

$= \{(0,\cdots 0\overset{x_1}{*}0\cdots 0)\} \Rightarrow \dim(\Lambda\cap V_{x_1}) = 1 \Rightarrow \Lambda\cap V_{x_1} = \langle e_{x_1}\rangle$.