4. One of the most important line bundles in general is the highest exterior power of the holomorphic cotangent bundle

$$K_M = \Lambda^n T^{*\prime} M ,$$

called the canonical bundle of the n-dimensional complex manifold M. Holomorphic sections of K_M are holomorphic n-forms, i.e $\mathcal{O}(K_M) = \Omega^n_M$.

We will compute the canonical bundle $K_{\mathbb{P}^n}$ of projective space: Let $Z_0, \cdots Z_n$ be homogeneous coordinates on $\mathbb{P}^n$, $w_i = z_i/z_0$ Euclidean coordinates on $U_0 = (z_0 \neq 0)$, and consider the meromorphic n-form

$$\omega = \frac{dw_1}{w_1} \wedge \frac{dw_2}{w_2} \wedge \cdots \wedge \frac{dw_n}{w_n} .$$

ω is clearly nonzero in U_0 with a single pole along each hyperplane $(Z_i = 0)$, $i = 1, 2, \dots n$.

Now if $w_i' = z_i/z_j$, $i = 0, \dots \hat{j} \dots n$ are Euclidean coordinates on $U_j = (z_j \neq 0)$, then

$$w_i = \frac{w_i'}{w_0'}, \quad i \neq j ; \quad w_j = \frac{1}{w_0'}$$

and so in terms of $\{w_i'\}$,

$$\omega = (-1)^j \cdot \frac{dw_0'}{w_0'} \wedge \cdots \wedge \widehat{\frac{dw_j'}{w_j'}} \wedge \cdots \wedge \frac{dw_n'}{w_n'} .$$

We omitted something, given as follows:

which gives $\dfrac{dw_i}{w_i} = \dfrac{dw_i'}{w_i'} - \dfrac{dw_0'}{w_0'}$, $i \neq j$;

$$\frac{dw_j}{w_j} = - \frac{dw_0'}{w_0'}$$