$$= \left(\frac{\sqrt{-1}}{2\pi}\right)^q (-1)^{q(q-1)/2} \sum_{\alpha_1<\cdots<\alpha_q} \sum_{\pi} \sum_{\mu_1\cdots\mu_q} \mathrm{sgn}\,\pi\; A^{\alpha_1}_{\mu_1}\wedge\cdots\wedge A^{\alpha_q}_{\mu_q}\wedge \bar{A}^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge \bar{A}^{\alpha_{\pi(q)}}_{\mu_q}.$$

In computing the term above, the point is the following simple observation:

$$A^{\alpha_{\sigma(1)}}_{\mu_1}\wedge A^{\alpha_{\sigma(2)}}_{\mu_2}\wedge\cdots\wedge A^{\alpha_{\sigma(q)}}_{\mu_q}\wedge \bar{A}^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge \bar{A}^{\alpha_{\pi(q)}}_{\mu_q} \qquad \left\{\begin{matrix} \sigma(1) & \sigma(2) & & 1 & 2 \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ 1 & 2 & & \sigma^{-1}(1) & \sigma^{-1}(2)\end{matrix}\right\}$$

$$= \epsilon(\sigma)\, A^{\alpha_1}_{\mu_{\sigma^{-1}(1)}}\wedge A^{\alpha_2}_{\mu_{\sigma^{-1}(2)}}\wedge\cdots\wedge A^{\alpha_q}_{\mu_{\sigma^{-1}(q)}}\wedge \bar{A}^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge \bar{A}^{\alpha_{\pi(q)}}_{\mu_q}$$

$$= \epsilon(\sigma)\, A^{\alpha_1}_{\mu_{\sigma^{-1}(1)}}\wedge\cdots\wedge A^{\alpha_q}_{\mu_{\sigma^{-1}(q)}}\wedge \bar{A}^{\alpha_{\pi(\sigma^{-1}(1))}}_{\mu_{\sigma^{-1}(1)}}\wedge\cdots\wedge \bar{A}^{\alpha_{\pi(\sigma^{-1}(q))}}_{\mu_{\sigma^{-1}(q)}}\,\epsilon(\sigma^{-1})$$

$$\left\{\begin{matrix} \pi(1) & \pi(2) & & \pi(\sigma^{-1}(1)) & \\ \uparrow & \uparrow & \cdots & \uparrow & \cdots \\ 1 & 2 & & \sigma^{-1}(1) & \end{matrix}\right\} = A^{\alpha_1}_{\mu_{\sigma^{-1}(1)}}\wedge\cdots\wedge A^{\alpha_q}_{\mu_{\sigma^{-1}(q)}}\wedge \bar{A}^{\alpha_{\pi(\sigma^{-1}(1))}}_{\mu_{\sigma^{-1}(1)}}\wedge\cdots$$

Thus in case $\alpha_1<\cdots<\alpha_q$,
As $\mathrm{sgn}(\pi)\, A^{\alpha_1}_{\mu_1}\wedge\cdots\wedge A^{\alpha_q}_{\mu_q}\wedge \bar{A}^{\alpha_{\pi(1)}}_{\mu_1}\wedge\cdots\wedge \bar{A}^{\alpha_{\pi(q)}}_{\mu_q}$ term, if we allow unordered $\{\alpha_1\cdots\alpha_q\}$, then,

$$\mathrm{sgn}(\pi)\, A^{\alpha_{\sigma(1)}}_{\mu_1}\wedge\cdots\wedge A^{\alpha_{\sigma(q)}}_{\mu_q}\wedge \bar{A}^{\alpha_{\pi(\sigma(1))}}_{\mu_1}\wedge\cdots\wedge \bar{A}^{\alpha_{\pi(\sigma(q))}}_{\mu_q}$$
$$= \mathrm{sgn}(\pi)\, A^{\alpha_1}_{\mu_{\sigma^{-1}(1)}}\wedge\cdots\wedge A^{\alpha_q}_{\mu_{\sigma^{-1}(q)}}\wedge \bar{A}^{\alpha_{\pi(1)}}_{\mu_{\sigma^{-1}(1)}}\wedge\cdots\wedge \bar{A}^{\alpha_{\pi(q)}}_{\mu_{\sigma^{-1}(q)}}$$

For example, $q=2$, $K=3$. Consider $A^1_{\mu_1}\wedge A^2_{\mu_2}\wedge \bar{A}^1_{\mu_1}\wedge \bar{A}^2_{\mu_2}$.
For $(\mu_1,\mu_2)=(1,2)$,

$A^1_1\wedge A^2_2\wedge \bar{A}^1_1\wedge \bar{A}^2_2$, exchange 1 & 2 of α.

$$A^2_1\wedge A^1_2\wedge \bar{A}^2_1\wedge \bar{A}^1_2$$
$$= A^1_2\wedge A^2_1\wedge \bar{A}^1_2\wedge \bar{A}^2_1 \Rightarrow \text{for } (\mu_1,\mu_2)=(2,1),$$

one more $A^1_2\wedge A^2_1\wedge \bar{A}^1_2\wedge \bar{A}^2_{\mu_2}$ term is added.

Thus in general

$$C_q(\Theta) = \left(\frac{1}{2\pi}\right)^q (\sqrt{-1})^{q^2} \sum_{\substack{\alpha_1\cdots\alpha_q\\ \mu_1\cdots\mu_q}} \frac{1}{q!}\sum_{\pi} A^{\alpha_1}_{\mu_1}\wedge\cdots\wedge A^{\alpha_q}_{\mu_q}\wedge\cdots \bar{A}^{\alpha_{\pi(q)}}_{\mu_{(q)}}$$