at

$$\Delta \cdot I \subset I',$$

so that multiplication by Δ induces a map $\mathcal{O}/_I \xrightarrow{\Delta} \mathcal{O}/_{I'}$ going in the non obvious direction. By Cramer's rule

$$\Delta\, \delta_{ij} = \sum_k a_{kj} A_{ik},$$

where

A_{ik} is $\pm$ the (i,k)th cofactor of (a_{ij}).

Γ $BA = I = AB$.

$$\Rightarrow \quad b_{ij}\, A^1 + b_{i2} A^2 + \cdots + b_{ij} A^j + \cdots + b_{in} A^n = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} i$$

$$\left| A^1, \cdots A^{j-1}, \begin{array}{c} j \\ \begin{pmatrix} 0 \\ \vdots \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \end{array} A^{j+1} \cdots A^n \right| = \pm^r \text{ the } (i,j)\text{th cofactor of } (a_{ij})$$

$$= b_{ij}\,\Delta \quad \Rightarrow \quad b_{ij} = \frac{\pm\ (i,j)\text{th cofactor of } (a_{ij})}{\Delta}$$

$$= \frac{A_{ij}}{\Delta}$$

$$\Rightarrow \quad \left(\frac{A_{ij}}{\Delta} \right) \left(a_{ij} \right) = I = (a_{ij}) \left(\frac{A_{ij}}{\Delta} \right)$$

$$\Rightarrow \quad \delta_{ij} = \sum_k \frac{A_{ik}}{\Delta} a_{kj} \Rightarrow \Delta\, \delta_{ij} = \sum_k a_{kj} A_{ik}$$