$f|_{U_1} : U_1 \longrightarrow f(U_1)$ has degree of $\{ f(\overset{p}{p_0}) = f(p) \}$
$f|_{U_2} : U_2 \longrightarrow f(U_2)$ " $\{ f(p_{n+1}) = f(p) \}$.
$\Rightarrow$ Take $\tilde{W} = f(U_1) \cap f(U_2)$
$f|_{U_1}^{-1}(\tilde{W}) \overset{\mathcal{O}_1}{=} \subset U_1$ $f|_{U_2}^{-1}(\tilde{W}) \overset{\mathcal{O}_2}{=} \subset U_2$. $\Rightarrow$ $f| : f|_{U_1}^{-1}(\tilde{W}) \longrightarrow \tilde{W}$
$f| : f|_{U_2}^{-1}(\tilde{W}) \longrightarrow \tilde{W}$ $\Rightarrow$ $f(V \cap \mathcal{O}_1) \cup f(V \cap \mathcal{O}_2)$ is an algebraic variety, which is equal to $f(V \cap (\mathcal{O}_1 \cup \mathcal{O}_2))$.
Here we counted some points outside $V \cap f|^{-1}(W')$. 끝

Note in particular that the disriminant, or branch locus, $D \subset W$, defined as the image $f(R)$ of the ramification divisor

$$\left\{ \frac{\partial(f_1, \cdots f_n)}{\partial(z_1, \cdots z_n)}(z) = 0 \right\},$$

is an analytic hypersurface. For $w \in W - D$, $h^{-1}(w) = \sum_\nu z_\nu(w)$, where the $z_\nu(w)$ are distinct. ∵ Since the Jacobian det. is not zero, f is locally biholomorphic. i.e., f is a covering map. 끝

Choosing a path $W(t)$ with $W(0) = 0$ and $W(t) = W - D$ for $t \neq 0$, we find the explicit good perturbation $f_t(z) = f(z) - W(t)$ of $f(z)$.

As another application of the lemma, we suppose that $h(z) \in \mathcal{O}(U)$ and consider the expression

$$H(z) = \prod_{\nu=1}^{d} (h(z) - h(z_\nu(f(z)))).$$

On the one hand, $H(z) \equiv 0$, since $z = z_\nu(f(z))$ for some ν. ∵ $z \in f^{-1}(f(z)) = \{z_\nu\}_{\nu=1}^{d}$. 끝