Conversely, $\delta^* \beta^* = 0$, since, given $\sigma \in C^p(\underline{U}, \mathcal{F})$, with $\delta\sigma = 0$,

$\beta\sigma \in C^p(\underline{U}, \mathcal{G})$

$$
\begin{tikzcd}
0 \arrow[r] & C^p(\underline{U}, \mathcal{E}) \arrow[r, "\alpha"] & C^p(\underline{U}, \mathcal{F}) \arrow[r, "\beta"] & C^p(\underline{U}, \mathcal{G}) \arrow[r] & 0 \\
 & & \sigma \arrow[r, mapsto] \arrow[d, "\delta"] & \beta\sigma \arrow[d, "\delta"] \arrow[ddll, bend right] & \\
 & & 0 & 0 & \\
0 \arrow[r] & C^{p+1}(\underline{U}, \mathcal{E}) \arrow[r] & C^{p+1}(\underline{U}, \mathcal{F}) \arrow[r] & C^{p+1}(\underline{U}, \mathcal{G}) \arrow[r] & 0 \\
 & 0 \arrow[r, mapsto] & 0 \arrow[r, mapsto] & 0 &
\end{tikzcd}
$$

What is $\delta^* \beta\sigma$ ?

$\delta^*(\beta\sigma) = 0$.

The remaining stages are similar but easier.

The most common application of the exact cohomology sequence associated to a sheaf sequence

$$0 \longrightarrow \mathcal{E} \xrightarrow{\alpha} \mathcal{F} \xrightarrow{\beta} \mathcal{G} \longrightarrow 0$$

is to answer the question: given a global section $\sigma$ of $\mathcal{G}$, when is $\sigma$ the image under $\beta$ of a global section of $\mathcal{F}$? The answer, according to the exact sequence, is that this is the case exactly when $\delta^* \sigma = 0$ in $H^1(M, \mathcal{E})$.

$$
\begin{array}{ccccccc}
H^0(M, \mathcal{E}) & \longrightarrow & H^0(M, \mathcal{F}) & \longrightarrow & H^0(M, \mathcal{G}) & \xrightarrow{\delta^*} & H^1(M, \mathcal{E}) \longrightarrow \cdots \\
 & & ? & \longmapsto & \sigma & \longmapsto & 0
\end{array}
$$

For example, consider the exact sequence

$$0 \longrightarrow \mathcal{O} \xrightarrow{\alpha = i} \mathcal{M} \xrightarrow{\beta = \text{projection}} \mathcal{PP} \longrightarrow 0$$

on a Riemann surface $M$.