Date . .

To see this, let $z = (z_{\alpha\beta\gamma}) \in Z^2(M, \mathbb{Z})$; to find the image of z under the de Rham isomorphism, we take $f_{\alpha\beta} \in A^0(U_\alpha \cap U_\beta)$ s.t

$$z_{\alpha\beta\gamma} = f_{\alpha\beta} + f_{\beta\gamma} - f_{\alpha\gamma} \text{ in } U_\alpha \cap U_\beta \cap U_\gamma;$$

since $z_{\alpha\beta\gamma}$ is constant, $df_{\alpha\beta} + df_{\beta\gamma} - df_{\alpha\gamma} = 0$, so $(df_{\alpha\beta}) \in Z^1(M, \mathcal{Z}_k^1)$ and we can find $\omega_\alpha \in A^1(U_\alpha)$ such that

$$df_{\alpha\beta} = \omega_\beta - \omega_\alpha \text{ in } U_\alpha \cap U_\beta.$$

Pf If we let $z = (z_{\alpha\beta\gamma}) \in Z^2(M, \mathbb{Z})$, we can consider the element $z = (z_{\alpha\beta\gamma}) \in Z^2(M, \mathcal{A}^0)$ since $\mathbb{Z} \subset \mathcal{A}^0$, where $\mathcal{A}^0(U)$ is the set of C^∞-forms on $U \subset M$.

$\Rightarrow$ Then by p 42. $H^p(M, \mathcal{A}^{r,s}) = 0$ for $p > 0$

$\Rightarrow$ $H(M, \mathcal{A}^{0,0}) = 0 \iff H(M, \mathcal{A}^0) = 0$

$\iff$ $Z^2(M, \mathcal{A}^0) = \delta(C^1(\underline{U}, \mathcal{A}^0))$

$= \delta\left(\coprod_{\alpha \neq \beta} \mathcal{A}^0(U_\alpha \cap U_\beta)\right)$

$\Rightarrow$ Thus, $\exists\ f = (f_{\alpha\beta}) \in \coprod \mathcal{A}^0(U_\alpha \cap U_\beta)$ $A^0(U_\alpha \cap U_\beta)$ s.t $(z_{\alpha\beta\gamma}) = \delta(f_{\alpha\beta})$

$$\Rightarrow \quad z_{\alpha\beta\gamma} = f_{\alpha\beta} + f_{\beta\gamma} - f_{\alpha\gamma} \text{ in } U_\alpha \cap U_\beta \cap U_\gamma$$

$\Rightarrow$ Since $z_{\alpha\beta\gamma}$ is constant, $dz_{\alpha\beta\gamma} = 0 = df_{\alpha\beta} + df_{\beta\gamma} - df_{\alpha\gamma}$. $\Rightarrow$ If $(df)_{\alpha\beta} = df_{\alpha\beta}$,

$\Rightarrow$ $df_{\alpha\beta} \in A^1_d(U_\alpha \cap U_\beta)$. in partial. $df_{\alpha\beta} \in Z^1_d(U_\alpha \cap U_\beta)$