Let $\eta \in \mathcal{H}^{p,q}(L)$ be a harmonic form. Then

$$\Theta = D^2 = \bar\partial D' + D'\bar\partial, \quad \text{so from } \bar\partial\eta = 0$$

$$\Theta\eta = \bar\partial D'\eta,$$

and

$$2i\langle \Lambda\Theta\eta, \eta\rangle = 2i\langle \Lambda\bar\partial D'\eta, \eta\rangle = 2i\langle (\bar\partial\Lambda - \tfrac{i}{2}D'^{*})D'\eta, \eta\rangle$$
$$= \langle D'^{*}D'\eta, \eta\rangle = \langle D'\eta, D'\eta\rangle \geq 0,$$

since $\langle \bar\partial\Lambda D'\eta, \eta\rangle = \langle \Lambda D'\eta, \bar\partial^{*}\eta\rangle = 0$.

$$\Theta = D^2 = (D' + \bar\partial)(D' + \bar\partial) = D'D' + \bar\partial D' + D'\bar\partial + \bar\partial^2$$
$$= D'^2 + \bar\partial D' + D'\bar\partial$$

But since Θ is $(1,1)$-form, $D'^2 = 0$.

$\Rightarrow \quad \Theta = \bar\partial D' + D'\bar\partial$.

$\Delta\eta = 0 \iff \bar\partial\eta = 0 \;\&\; \bar\partial^{*}\eta = 0$.

$\Rightarrow \quad \Theta\eta = \bar\partial D'\eta$.

$$i\langle \Lambda\Theta\eta, \eta\rangle = i\langle \Lambda\bar\partial D'\eta, \eta\rangle = i\langle (\bar\partial\Lambda - iD'^{*})D'\eta, \eta\rangle$$
$$= i\langle \bar\partial\Lambda D'\eta, \eta\rangle + \langle D'^{*}D'\eta, \eta\rangle = i\langle \Lambda D'\eta, \bar\partial^{*}\eta\rangle$$
$$+ \langle D'\eta, D'\eta\rangle = 0 + \langle D'\eta, D'\eta\rangle \geq 0.$$

Similarly, $2i\langle \Theta\Lambda\eta, \eta\rangle = 2i\langle D'\bar\partial\Lambda\eta, \eta\rangle =$
$2i\langle D'(\Lambda\bar\partial + \frac{i}{2}D'^{*})\eta, \eta\rangle = -\langle D'D'^{*}\eta, \eta\rangle = -\langle D'^{*}\eta, D'^{*}\eta\rangle$
≤ 0.

$$i\langle \Theta\Lambda\eta, \eta\rangle = i\langle (\bar\partial D' + D'\bar\partial)(\Lambda\eta), \eta\rangle$$
$$= i\langle \bar\partial D'\Lambda\eta, \eta\rangle + i\langle D'\bar\partial\Lambda\eta, \eta\rangle = i\langle D'\Lambda\eta, \bar\partial^{*}\eta\rangle$$
$$+ i\langle D'\bar\partial\Lambda\eta, \eta\rangle = 0 + i\langle D'\bar\partial\Lambda\eta, \eta\rangle$$
$$= i\langle D'(\Lambda\bar\partial + iD'^{*})\eta, \eta\rangle = i\langle D'\Lambda\cancel{\bar\partial\eta}, \eta\rangle - \langle D'D'^{*}\eta, \eta\rangle$$
$$= -\langle D'D'^{*}\eta, \eta\rangle = -\langle D'^{*}\eta, D'^{*}\eta\rangle \leq 0.$$

Combining, $2i\langle [\Lambda, \Theta]\eta, \eta\rangle \geq 0$.