codimension,

$$\delta(a, b; c) = \begin{cases} 1, & \text{if } b_i \leq c_i \leq b_{i-1}, \\ 0, & \text{otherwise}. \end{cases}$$

We have, setting $k = \ell(c)$,

$$\delta(a, b; c) = \#(\sigma_a \cdot \sigma_b \cdot \sigma_{c_1-c_k, \dots c_1-c_2, 0}) \text{ in } G(k, k+c_1)$$

To start, suppose that $c_i < b_{i-1}$ for some i. Then we have

$c_1 + b_{i-1} + (c_1 - c_i) \geq 2c_1 + 1$, and applying the second reduction formula with $\alpha = 0$, $\beta = i-1$, and $\gamma = k-i+1$, we obtain

$$\delta(a, b; c) = \#(\sigma_a \cdot \sigma_{b_1-1, \dots b_{i-1}, b_i \dots} \cdot \sigma_{c_1-c_k-1, \dots c_1-c_i-1, c_1-c_{i+1}, \dots}) \text{ in } G(k, k+c_1-1)$$

$$= \delta(a, b'; c')$$

where $b' = b_1-1, \dots b_{i-1}-1, b_i, \dots$

and $c' = c_1-1, \dots c_{i-1}-1, c_i \dots$.

$\ulcorner$ $c_1 + b_{i-1} + c_1 - c_i \geq 2c_1 + 1$. since $b_{i-1} > c_i$.

$\alpha + \beta + \gamma = 0 + i-1 + k-i+1 = k$.

$a_0 + b_{i-1} + c''_{k-i+1} = k+c_1 - k + b_{i-1} + c''_{k-i+1}$

$= c_1 + b_{i-1} + c_1 - c_i \geq 2c_1 + 1 = 2(k+c_1-k)+1$

$\Rightarrow$ We can apply the second reduction formula.

$\Rightarrow \delta(a, b; c'') = \#(\sigma_a \cdot \sigma_{b_1-1, \dots b_{i-1}, b_i \dots} \cdot \sigma_{c_1-c_k-1, \dots c_1-c_i-1, c_1-c_{i+1} \dots})$

$= \delta(a, b'; c')$. $c' = c_1 - 1, \dots c_{i-1}-1, c_i \dots, \dots$ $\lrcorner$

Now $(b_i \leq c_i \leq b_{i-1} \text{ for all } i) \Leftrightarrow (b'_i \leq c'_i \leq b'_{i-1} \text{ for all } i)$ and of course $b'_{i-1} - c'_i = b_{i-1} - c_i - 1 \geq 0$.