Since the type of Θ is clearly invariant under change of frame, we see that the curvature matrix of the metric connection on a hermitian bundle is a hermitian matrix of $(1,1)$-forms.

Computations of the metric connection & curvature matrices of hermitian bundles in two special cases.

Recall that for E a hermitian bundle with metric D, the metric D^* on E^* satisfies

$$d(\tau(\sigma)) = \tau(D\sigma) + D^*\tau(\sigma) \quad \text{for all } \sigma \in \mathcal{A}^0(E)(U),\ \tau \in \mathcal{A}^0(E^*)(U).$$

In particular, if e unitary frame for E and e^* dual frame for E^*, θ and θ^* corresponding connection matrices, $\Rightarrow$

$$0 = d\langle e_i, e_j^*\rangle = \theta_{ij} + \theta_{ji}^* . \quad \Rightarrow \quad \theta = -{}^t\theta^*$$

In view of this, a special situation holds when we consider the metric connection on the holomorphic tangent bundle of a hermitian manifold: we can compare the dual connection D^* on the holomorphic cotangent bundle with the ordinary exterior derivative. Thus.

$$D^* : \underset{\parallel\ (T^{*\prime}M)}{A^{1,0}} \longrightarrow A^{1,0} \otimes A^1 = A^{1,0} \otimes (A^{1,0} \oplus A^{0,1}) = (A^{1,0} \otimes A^{1,0}) \oplus (A^{1,0} \otimes A^{0,1})$$

$$d : \underset{\parallel\ (T^{*\prime}M)}{A^{1,0}} \longrightarrow A^{2,0} \oplus (A^{1,0} \otimes A^{0,1})$$

$$\Lambda^2(T^*M) = \Lambda^2(T^{*\prime}M \oplus T^{*\prime\prime}M) = \sum_{k+l=2} \Lambda^k(T^{*\prime}M) \otimes \Lambda^l(T^{*\prime\prime}M)$$

$$= \Lambda^2(T^{*\prime}M) \oplus \underset{\Lambda^1(T^{*\prime}M) \wedge \Lambda^1(T^{*\prime\prime}M) = T^{*\prime}M \wedge T^{*\prime\prime}M}{\underset{\parallel\cong}{\Lambda^1(T^{*\prime}M) \otimes \Lambda^1(T^{*\prime\prime}M)}} \oplus \Lambda^2(T^{*\prime\prime}M)$$