As another application, we can prove

Pascal's Theorem. The pairs of opposite sides of a hexagon inscribed in a smooth conic Q meet in three collinear points.

Proof. Suppose that $L_1 L_2 L_3 L_4 L_5 L_6$ is the inscribed hexagon. Take $C = L_1 + L_3 + L_5$, $D = L_2 + L_4 + L_6$, and $E = Q + \overline{P_{52} P_{14}}$ where $P_{ij} = L_i \cap L_j$. Then E passes through the remaining point P_{36} of $C \cap D$. Q.E.D.

┌ Cubic is a curve of degree 3.
Conic is a curve of degree 2.

$C = L_1 + L_3 + L_5 = \{ (a_{11} X_0 + a_{12} X_1 + a_{13} X_2)(a_{31} X_0 + a_{32} X_1 + a_{33} X_2)(a_{51} X_0 + a^5_2 X_1 + a^5_3 X_2) = 0 \}$

$D = L_2 + L_4 + L_6 = \{ (a_{21} X_0 + a_{22} X_1 + a_{23} X_2)(a_{41} X_0 + a_{42} X_1 + a_{43} X_2)(a_{61} X_0 + a_{62} X_1 + a_{63} X_2) = 0 \}$.

$Q = \{ f = 0 \}$, f irreducible of degree 2.

$\Rightarrow$ Q + L is going to be a cubic

$E = Q + \overline{P_{41} P_{25}}$ $P_{36} = L_3 \cap L_6$, $P_{25} = L_2 \cap L_5$, $P_{41} = L_1 \cap L_4$

$\Rightarrow$ E passes eight points $\Rightarrow$ By the classical theorem above, E passes the remaining point P_{36}. $\Rightarrow$ $\overline{P_{41} P_{25}}$ passes through P_{36}. $\Rightarrow$ P_{41}, P_{25} & P_{36} are collinear. ┘

There is also a

Converse to Pascal's Theorem. If $H = L_1 L_2 L_3 L_4 L_5 L_6$ is