$$= \varphi(x)\ C_n^{-1} \int_0^\infty \chi_\epsilon(r) \left(\frac{1}{r^{n-1}}\right)^{-1} dr$$

$$= \varphi(x) \int_0^\infty \int_{\|x\|=r} \chi_\epsilon(r) \left(\frac{C_n}{r^{n-1}}\right)^{-1} \sigma(x)\, dr$$

$$= \varphi(x) \int_{\mathbb{R}^n} \chi_\epsilon(x)\, dx = \varphi(x).$$

Recall that $\left(\frac{C_n}{r^{n-1}}\right)^{-1} \sigma \wedge dr = \Phi = dx_1 \wedge \cdots \wedge dx_n$,

Since

$$\sigma \wedge dr = C_n \frac{* r dr}{r^n} \wedge dr = \frac{C_n}{r^n} * (r dr) \wedge dr$$

$$= \pm \frac{C_n}{r^n} dr \wedge *(r dr) = \pm \frac{C_n}{r^n} \langle dr, r dr \rangle \Phi$$

$$= \pm \frac{C_n}{r^{n-1}} \langle dr, dr \rangle \Phi = \pm \frac{C_n}{r^{n-1}} \Phi.$$

Note: "$\sigma = x\,dy - y\,dx$"

Suppose we have $\sigma = a\,dy + bx$ at (x_0, y_0).

When we have $g = \begin{pmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \end{pmatrix}$ which a proper orthogonal matrix, let's look at $R_{g^{-1}}^* \sigma$, closely.

Let $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ be a vector at (x_0, y_0).

$$(R_{g^{-1}}^* \sigma)\left(g\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right) = R_{g^{-1}}^* \sigma \left((a_{11} v_1 + a_{12} v_2)\frac{\partial}{\partial x} + (a_{21} v_1 + a_{22} v_2)\frac{\partial}{\partial y}\right)$$

Before we go further, let's assume

$R_{g^{-1}}^* \sigma = a'\,dy + b'x$. We want to find a relation beteween (a, b) & (a', b').