

"Ext" suggests extensions, and it was in this context that Ext' was first defined. We shall discuss this in Section 4, where it will arise quite naturally in context.

Another possible interpretation of Ext is pertaining to some sort of duality, since it reflects the properties of passing from an \mathcal{O} -module M to the "dual" \mathcal{O} -module $\text{Hom}_{\mathcal{O}}(M, \cdot)$. This will be made precise in the next section, where, in particular, the local duality theorem will be put in intrinsic form.

Thus Ext has two quite different faces, each interesting in its own right.

The Koszul Complex and Applications

Koszul Complex. We continue using the notation \mathcal{O} for the local ring of germs of analytic functions defined in some nbd of the origin in \mathbb{C}^n . Suppose $f_1, \dots, f_r \in \mathcal{O}$; denoted by $I_k = \langle f_1, \dots, f_k \rangle$ the ideal generated by the first k functions, and set $I = I_r$.

Definition. (f_1, \dots, f_r) is a regular sequence if f_k is not a zero divisor in \mathcal{O}/I_{k-1} for $k=1, \dots, r$.

We recall that the geometric interpretation, mentioned above and proved in the case $n=2$, that is equivalent to $\text{codim } V_k = k$, where $V_k = \{f_1(z) = \dots = f_k(z) = 0\}$.