

$$f_*(T_p(B)) \subset T_q(f(B)) \quad \text{and} \quad f_*(T_p(f^{-1}(A))) \subset T_q A,$$

$$f_*(T_p M) \subset T_q(f(B)) + T_q A$$

$$\Rightarrow f_*(T_p M) + T_q A \subset T_q(f(B)) + T_q A$$

$\quad \quad \quad \text{"} T_q N$

$$\Rightarrow T_q(f(B)) + T_q A = T_q N.$$

In this context, the weaker form of duality may be restated once again as the assertion that the pairing

$$H_{DR}^k(M) \otimes H_{DR}^{n-k}(M) \longrightarrow \mathbb{R} \quad \text{given by}$$

$$([\varphi], [\psi]) \longmapsto \int_M \varphi \wedge \psi$$

is non-degenerate, or that for any closed k -form φ on M
 \exists an $(n-k)$ -cycle A , unique up to homology