Here we assume that $\det\left(\frac{\partial \pi_i(x)}{\partial x_j}\right) \neq 0$, $1 \le i, j \le k$.

$\Rightarrow$ By the inverse function theorem, $\exists\ G : \mathbb{C}^k \times \mathbb{C}^{n-k} \longrightarrow \mathbb{C}^n$
s.t $G(\pi(x), * \cdots *) = (x_1, \cdots x_n)$.
$\Rightarrow$ $G(\pi(x_1 \cdots x_k, 0 \cdots 0), 0, \cdots 0) = (x_1, \cdots x_k, 0 \cdots 0)$.
$\Rightarrow \exists\ g : V \longrightarrow U$ s.t $\pi(g(v)) = v$.
Not enough! Maybe I understand better sometime later. $\pi^{-1}(b_0)$ is a complex submanifold of E by the inverse function theorem, and is Kähler by P109.

Before giving the proof, we wish to suggest two interpretations of this result. One is as another reflection of the extraordinary topological properties, such as those encountered in Section 1 and 3 of Chapter 1, possessed by an algebraic variety. The other interpretation is as focusing attention on the extremely important role played by the monodromy group, which is by definition the image of $\pi_1(B, x_0)$ in $\mathrm{Aut}(H^*(G, \mathbb{Q}))$ under the representation obtained by displacing cycles around closed paths.

Proof. We first remark that a closed k-form φ given on the total space E defines classes in $E_r^{P,K}$ for all r, and moreover that multiplication by φ induces

$$\varphi : E_r^{P,q} \longrightarrow E_r^{P,q+K}$$

commuting with