On the other hand, a point (p,p) of intersection of $\sigma \times \tau$ with Δ corresponds to a point p of intersection of σ with τ, and examining the orientations, we find that for such a point p,

$$i_{(p,p)}(\sigma \times \tau, \Delta) = (-1)^{n-k} i_p(\sigma \cdot \tau)$$

Thus $\#(\sigma \cdot \tau) = (-1)^{n-k} i_{(p,p)}(\sigma \times \tau, \Delta) = \int_M \varphi \wedge \psi$,

i.e intersection of cycles in cycles in homology is Poincare dual to wedge product in cohomology.

$$\#(\sigma \cap \tau \cdot M) = \int_M \varphi \wedge \psi$$

Pf locally, $\sigma \times \tau \in R^n \times R^n$, σ k-vector, τ $(n-k)$-vector in R^n.

Let $(v_{11}, \cdots v_{1n}, v_{21}, v_{22} \cdots v_{2n})$ be a basis for $R^n \times R^n$.

~~$v_{ii} =$~~ $v_{1i} = v_{2i}$.

$$\Rightarrow (\sigma \times \tau)_1 = \sigma_{11} v_{11} + \cdots + \sigma_{n1} v_{1n} + \cancel{\tau_{11} v_{21} + \cdots + \tau_{n2}} v_{2n}$$

$$(\sigma \times \tau)_2 = \sigma_{12} v_{11} + \cdots + \sigma_{n2} v_{1n}$$

$$\vdots$$

$$(\sigma \times \tau)_k = \sigma_{1k} v_{11} + \cdots + \sigma_{nk} v_{kn}$$

$$(\sigma \times \tau)_{k+1} = 0\, v_{11} + \cdots + 0\, v_{kn} + \tau_{11} v_{21} + \cdots + \tau_{n1} v_{2n}$$

$$(\sigma \times \tau)_{k+2} = 0 + \tau_{12} v_{21} + \cdots + \tau_{n2} v_{2n}$$

$$\vdots$$

$$(\sigma \times \tau)_n = 0 + \tau_{1n-k} v_{2k+1} + \cdots + \tau_{nn-k} v_{2n}$$

$$\Rightarrow n \left| \left(\overbrace{\sigma_{11} \cdots \sigma_{1k}}^{k} \right| \overbrace{\cdots}^{(n-k)} \left| v_{21} \right. \cancel{\tau_{11} \cdots \tau_{1n-k}} \right.$$

see p 66 for supplement