

$$C^0(U, \text{Hom}_\mathcal{O}(\mathcal{E}_2, \mathcal{F})) \oplus C^1(U, \text{Hom}_\mathcal{O}(\mathcal{E}_1, \mathcal{F})) \oplus C^2(U, \text{Hom}_\mathcal{O}(\mathcal{E}_0, \mathcal{F})).$$

$$D\varphi = 0 \Leftrightarrow \partial^* \varphi = 0, \quad \delta \varphi + \partial^* \eta = 0 \text{ and } \delta \eta = 0.$$

$$\partial^* \varphi = 0 \Rightarrow \pi \text{Hom}_\mathcal{O}(\mathcal{E}_1, \mathcal{F})(U_\alpha) \xrightarrow{\partial^*} \pi \text{Hom}_\mathcal{O}(\mathcal{E}_2, \mathcal{F})(U_\alpha)$$

$$\downarrow$$

$$\{\varphi_\alpha\} \longmapsto \{\varphi_\alpha \circ \partial\} = 0$$

$\Rightarrow \varphi_\alpha = 0$ on $\partial(\mathcal{E}_2(U_\alpha))$, i.e. $\varphi_\alpha: \partial(\mathcal{E}_2)(U_\alpha) \rightarrow \mathcal{F}(U_\alpha)$ is a zero map.

Note: φ_α may be considered as a sheaf map over U_α , from \mathcal{E}_1 to \mathcal{F} .

Thus, over U_α , we have

$$0 \rightarrow \frac{\mathcal{E}_1}{\partial \mathcal{E}_2} \xrightarrow{\partial} \mathcal{E}_0 \xrightarrow{\psi} \mathcal{F} \rightarrow 0 \text{ an exact sequence of sheaves}$$

and

$$\frac{\mathcal{E}_1}{\partial \mathcal{E}_2} \xrightarrow{\varphi_\alpha} \mathcal{F} \text{ a sheaf map.}$$

Consider the following short complexes of sheaves over U_α .

$$0 \rightarrow \mathcal{F} \xrightarrow{i} \frac{\mathcal{F} \oplus \mathcal{E}_0}{\mu(\frac{\mathcal{E}_1}{\partial \mathcal{E}_2})} \xrightarrow{\psi} \mathcal{F} \rightarrow 0$$

$$\xrightarrow{(f, e_0) + i \circ \mu} \psi(e_0)$$

$$\mu: \frac{\mathcal{E}_1}{\partial \mathcal{E}_2} \longrightarrow \mathcal{F} \oplus \mathcal{E}_0$$

Then, at each stalk, by the arguments on p 123