$$= \frac{1}{2\pi\sqrt{-1}} \int_{|f_2(0,z_2)|=\epsilon} \frac{\frac{\partial f_2'(z_2)}{\partial z_2}}{f_2'(z_2)}\, dz_2$$

$$= \frac{1}{2\pi\sqrt{-1}} \int_{|f_2'(z_2)|=\epsilon} \frac{df_2'}{f_2'} = (D_2')_{\{0\}}.$$

Using this, we shall prove

The Jacobian $\partial(f_1,\cdots f_n)/\partial(z_1,\cdots z_n) \not\equiv 0$ and the local intersection number $(D_1 \cdots D_n)_{\{0\}} > 0$.

Proof. The proof is by induction on n, with the case $n=1$ being clear.

Pf $n=1$. $\frac{df}{dz} \not\equiv 0$ since $f^{-1}(0) = \{0\}$. i.e. f is nonconstant.

$1 = \frac{1}{2\pi\sqrt{-1}} \int_{|f|=\epsilon} \frac{df}{f} \not\equiv 0$. Wait a minute.

If $f(z) = z^2$, $\frac{df}{f} = \frac{2z\,dz}{z^2} \not\equiv 0$. f is not constant.

$\frac{1}{2\pi\sqrt{-1}} \int_{|z^2|=\epsilon} \frac{2dz}{z} = 1 \cdot 2 > 0$. D_1 is nonsingular at 0, but in this case, not.

Choose a point z_0 that is a smooth point on D_1 and is very close to the origin. If we assume that $df_1 \wedge \cdots \wedge df_n \equiv 0$ and set $f_i' = f_i|_{D_1}$, then $df_2' \wedge \cdots \wedge df_n' \equiv 0$ near z_0. Indeed, we may choose local coordinates $(u_1, u_2, \cdots u_n)$ around z_0 such that $f_1 = u_1^m$, where $m > 0$. Then $df_1 \wedge df_2 \wedge \cdots df \equiv 0$