then the associated single complex is the de Rham complex $(A^*(M), d)$. In general not much seems to be known about the resulting Fröhlicher spectral sequences $\{'E_r\}$ and $\{''E_r\}$, both of which abut to $H^*_{DR}(M)$.

If, however M is compact Kähler, then every class $[a] \in {}'E_1^{p,q} \cong H_{\bar\partial}^{p,q}(M)$ has a harmonic representative for the $\bar\partial$-Laplacian $\Delta_{\bar\partial}$. By the Kähler assumption, $2\Delta_{\bar\partial} = \Delta_d$, and consequently $da=0$.

⌈ Here we are assuming that a is the harmonic representative. $\Rightarrow$ $\Delta_{\bar\partial} a = \Delta_d a = 0 \Leftrightarrow da=0$ & $d^* a = 0$. ⌋

Thus

$$'E_1 \cong {}'E_2 \cong \cdots \cong {}'E_\infty,$$

and the filtration on $H^*_{DR}(M)$ is the Hodge filtration defined by

$$F^p H^n_{DR}(M) \cong H^{n,0}(M) \oplus \cdots \oplus H^{p,n-p}(M).$$

⌈

$$\begin{array}{ccc} [a] \in {}'E_1^{p,q} & \xrightarrow{\partial} & 'E_1^{p+1,q} \\ \updownarrow \quad \wr\| & & \wr\| \\ a \in \mathcal{H}^{p,q}(M) & \xrightarrow{\partial} & \mathcal{H}^{p+1,q}(M) \end{array}$$

$da=0 \Leftrightarrow \partial a = 0$ & $\bar\partial a = 0$

$\Rightarrow$ $\operatorname{Ker}\partial = \mathcal{H}^{p,q}(M) \cong {}'E_1^{p,q}$ & $\operatorname{Im}\partial = 0$

$\Rightarrow$ $'E_2^{p,q} = {}'E_1^{p,q}$

Since the map from $'E_2^{p,q} \xrightarrow{\text{to}} {}'E_2^{p+1,q}$ is induced by $d = \partial + \bar\partial$, the exactly same argument can be applied,