(Note that these two lines are necessarily distinct: if $T_p(S)$ met S in only one line L, $T_p(S)$ would have to be tangent to S everywhere along L, and so no other line on S could meet L. But the intersection of S with a general tangent plane $T_Q(S)$ not containing L will consist of a union of lines, and must meet L, so this can not be the case.)

Γ $T_p(S) \cap S = L$. $\Rightarrow$ Since $T_p(S) \cap S$ is a curve of degree 2 in $T_p(S)$, L has multiplicity 2. $\Rightarrow$ $T_p(S)$ must be tangent to S everywhere along L.

$\Rightarrow$ No other line on S could meet L, for, if not so, $r \in L \cap L'$ $\Rightarrow$ $T_r(S) \supset L \cup L'$ and $T_r(S) \supset T_p(S)$ $\Rightarrow$ $T_r(S) \supset 2L \cup L'$.
$\Rightarrow$ Contradiction to the fact $\dim T_r(S) = 2$.
By the previous argument, $S \cap T_Q(S) = \{\text{two lines}\}$
$T_Q(S) \cap L \neq \phi$ since $T_Q(S) \cong \mathbb{P}^2$ in $\mathbb{P}^3$.
Let $x \in T_Q(S) \cap L$. $\Rightarrow$ $\exists$ L'' line s.t. $L'' \ni x$.
and $L'' \subset T_Q(S) \cap S$. $\Rightarrow$ Contradiction to the fact that no other line on S does meet L. $\lrcorner$

Now, pick one line $L_0 \subset S$ and call any line on S an A-line if it is equal to or disjoint from L_0, a B-line if it meets L_0 in one point. If two lines $L, L' \neq L_0$ on S meet in a point, the plane they span