$$T : \mathcal{H}_0^{p,q}(M) \longrightarrow \mathcal{H}_0^{p,q}(M) \quad \text{compact, self-adjoint.}$$

$\mathcal{H}_0^{p,q}(M) = \bigoplus_m E(\rho_m)$, where ρ_m's eigenvalues & $\rho_m \neq 0$
$\dim E(\rho_m) < \infty$

$$T\varphi = \rho_m \varphi \iff \langle \varphi, \eta \rangle = \langle \rho_m \varphi, (I+\Delta)\eta \rangle$$

$$\varphi = (I+\Delta)\rho_m \varphi = \rho_m (I+\Delta)\varphi$$

$$\Rightarrow \frac{\varphi}{\rho_m} = \varphi + \Delta\varphi \quad \Rightarrow \Delta\varphi = \frac{\varphi}{\rho_m} - \varphi = \varphi\left(\frac{1-\rho_m}{\rho_m}\right)$$

Let $\lambda_m = \dfrac{1-\rho_m}{\rho_m}$, $0 = \lambda_0 < \lambda_1 < \cdots$ $\lambda_m \to \infty$ as $m \to \infty$.

Define the Green's operator by

$$G = 0 \quad \text{on } \mathcal{H}^{p,q}(M)$$

$$G(\varphi) = \frac{1}{\lambda_m}\varphi, \quad \varphi \in E\left(\frac{1}{1+\lambda_m}\right).$$

i.e G is the inverse of Δ on $\mathcal{H}^{p,q}(M)^{\perp}$.

(iii) $A^{p,q}(E)$ = E-valued (p,q)-type forms
$= \Gamma(\Lambda^p T^{*\prime}(M) \wedge \Lambda^q T^{*\prime\prime}(M) \otimes E)$.
$= \Gamma(\Lambda^p T^{*\prime}(M) \otimes \Lambda^q T^{*\prime\prime}(M) \otimes E)$

Then $F = \Lambda^p T^{*\prime}(M) \otimes \Lambda^q T^{*\prime\prime}(M) \otimes E \longrightarrow M$ is a bundle over M

By P93, the global Sobolev s-norm of sections $f \in C^\infty(M, F)$ can be defined.

Regularity Lemma I. Suppose that $\varphi \in \mathcal{H}_s^{p,q}(E)$, and