Let's continue. $\omega = h^0(S, \mathcal{J}_P(K_S+L)) = h^0(S, \mathcal{J}_P(\pi^*H^{n-3}))$
$= h^0(S, \mathcal{J}_P(n-3)) = \dim|\mathcal{J}_P(n-3)| + 1$. $\lrcorner$

As a first illustration we shall show how the reciprocity formula may be used to derive the properties of linear systems of cubics that arose in Section 1 of Chapter 4 in our study of the cubic surface. We begin with:

A set Γ_0 of seven points imposes independent conditions on $|\mathcal{O}_{\mathbb{P}^2}(3)|$, unless five of the seven are collinear.

Proof. If there are two cubics $C, C' \in |\mathcal{J}_{\Gamma_0}(3)|$ without a common component, then for the residual set Γ we have $h^0(\mathcal{J}_\Gamma) = 0$, and by (the most trivial case of) the reciprocity formula the points Γ_0 impose independent condition on $|\mathcal{O}_{\mathbb{P}^2}(3)|$.

$$\Gamma_9 = C \cdot C' = \underset{7}{\underset{\parallel}{\Gamma_0}} + \Gamma. \qquad \#\Gamma = 2$$

$\Rightarrow \omega = h^0(\mathcal{J}_\Gamma(3-3)) = \dim H^0(S, \mathcal{J}_\Gamma^{(0)}) = 0$
since $H^0(S, \mathcal{O}) \cong \mathbb{C}$, and so $(\sigma = 0) = S$ or ϕ, for all $\sigma \in H^0(S, \mathcal{J}_\Gamma^{(0)}) \subset H^0(S, \mathcal{O})$, but $(\sigma = 0) = \Gamma$. $\lrcorner$

So we may assume that $\dim|\mathcal{J}_{\Gamma_0}(3)| \geq 3$ and that any two cubics in this linear system have a common