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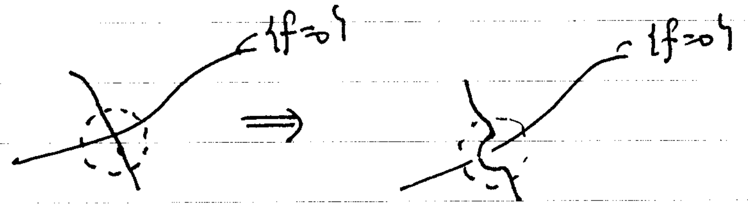
"Comment on the path-connectedness of the complement of an algebraic variety"

We have only to prove that $\{f=0\}^c$ is path-connected. By Weierstrass preparation theorem & the fact that $\dim_{\mathbb{R}} \{f=0\}^c$ is ≥ 2, $\{f=0\}^c$ is path-connected.

In a different way, consider $\{f=0\}$ and a curve joining any two points. Consider a general position of the curve with $\{f=0\}$. $\Rightarrow$ Since $\{f=0\}$ has codimension ≥ 2, we have a curve which does not intersect with $\{f=0\}$.

Return to Weierstrass preparation theorem. Given p, q distinct two points $\notin \{f=0\}$, first draw a curve joining p & q which does not intersect the branch locus. Second, for any point which is not on the branch locus, consider an small open set which looks like a $\mathbb{C}^{n-1}$ plane in $\mathbb{C}^n$. We can draw an arc as follows.



Case 2: $\det A(0) \neq 0$ but f possibly degenerate.

Since the result is local around the origin, we may shrink U and assume that $\det A(z) \neq 0$ in $\overline{U}$. If f_t is a good purturbation of $f=f_0$, then $g_t = A \cdot f_t$ is a good perturbation of $g=g_0$, and by continuity and