If we change the coordinate, $\varphi_i = \sum a_{ij}\, g_{jk}\, dz_k$
$= \sum (ag)_{ik}\, dz_k$

$\Rightarrow \psi'' = \bar\partial(ag)\,(ag)^{-1} = (\partial a)\, g\, g^{-1} a^{-1} = \bar\partial a\, a^{-1}$.
since $\bar\partial g = 0$. ($\because$ g is holomorphic).

$$\Lambda^k V \wedge \Lambda^l W = \Lambda^k(V) \otimes \Lambda^l(W). \quad \text{in case } V \cap W = \phi.$$

Let $v = v_1 \cdots v_n$ be the frame for $T'(M)$ dual to the frame $\varphi_1 \cdots \varphi_n$;
let θ be the connection matrix of D w.r.t the ~~fan~~ frame v and θ^* the matrix for D^* in the frame. $\varphi_1, \varphi_2 \cdots \varphi_n$.

$\Rightarrow D^{*\prime\prime} = \bar\partial \Rightarrow \theta^{*\prime\prime} = \psi''$. by lemma & observation.
$\Rightarrow \theta^* = \psi$. since $\theta^* + {}^t\bar\theta^*$ and $\psi + {}^t\bar\psi = 0$.

$$\Rightarrow \quad \theta = -{}^t\theta^* = -{}^t\psi$$

In summary, using the basic structure equation (*), we may determine the connection matrix $\theta = -{}^t\psi$ in the holomorphic tangent bundle $T'(M)$ by knowing the exterior derivatives of $d\varphi_i$ of a unitary coframe.

The vector $\tau = (\tau_1, \tau_2 \cdots \tau_n)$ is called the torsion;
a metric is called Kähler if its torsion vanishes.

Examples.

① M Riemann surface with local coordinate z;
A metric on M $ds^2 = h^2 dz \otimes d\bar z. = \varphi \otimes \bar\varphi$.
where $\varphi = h\,dz$. ($\Leftarrow$ if $\varphi = a\,dz \Rightarrow$. $a^2 dz \otimes d\bar z = h^2 dz \otimes d\bar z$)
Γ $h^2 = \langle \frac{\partial}{\partial z}, \frac{\partial}{\partial z} \rangle > 0$. $h > 0 \Rightarrow \varphi = \langle\ , a\frac{\partial}{\partial z} \rangle$ $\overset{\shortparallel}{a^2 = h^2}$