

$S = T_P$ for a piecewise smooth chain P .

$$\overline{\Gamma} \quad S(\varphi) = \int_P \varphi = \int_M S_\delta \wedge \varphi \quad \varphi \in A_c^{n-p}(M), \quad S_\delta \in A_c^p(M).$$

$\Rightarrow S_\delta$ is the Poincaré-dual of P .

$$\int_P \varphi = \int_M S_\delta \wedge \varphi = (\pm 1) \int_M \varphi \wedge S_\delta = \pm (T \cdot S). \quad \square$$

In case M is a complex manifold of complex dimension n , the pairing

$$H_{\bar{\partial}}^{p,q}(M) \otimes H_{\bar{\partial}}^{n-p,n-q}(M) \longrightarrow \mathbb{C}$$

induces an intersection number on $\bar{\partial}$ -closed currents of complementary type (p, q) and $(n-p, n-q)$.

$$\overline{\Gamma} \quad \text{By } H_{\bar{\partial}}^{p,q}(M) \cong H^q(\mathcal{D}^{p,*}(M), \bar{\partial}), \quad p \geq 2. \quad \square$$

In case $p=q$ and T is a real (p, p) current and S a real $(n-p, n-p)$ current, then

$$dT=0 \Leftrightarrow \bar{\partial}T=0,$$

$$dS=0 \Leftrightarrow \bar{\partial}S=0,$$

and the intersection number of closed currents is the same in either the d or $\bar{\partial}$ sense.

$$\overline{\Gamma} \quad \text{By 3\&6, for real } T \in \mathcal{D}^{p,p}(M), \quad dT=0 \Leftrightarrow \bar{\partial}T=0$$

Let T_ϵ be a smooth form in the cohomology class def'd by