If F has degree d, then the fundamental class of $V = (\sigma_F)$ is $\eta_V = c_1(H^d)$ – that is, d times the class of a hyperplane – so V has degree d.

⌈ $V = (F=0) \Rightarrow \eta_V = c_1(H^d)$ since F defines a section of H^d, see p165 and p141.
$c_1(H^d) = d\, c_1(H) = d$ in $H^2(\mathbb{P}^n, \mathbb{Z}) \cong \mathbb{Z} \cong H_{2n-2}(\mathbb{P}^n; \mathbb{Z})$
see p145 & p144. ($c_1(H) = 1$, see p144 ~ p145) ⌋

Alternatively, if

$$[Y_0, Y_1] \xrightarrow{\mu} [a_0 Y_0 + b_0 Y_1, \cdots \quad a_n Y_0 + b_n Y_1]$$

is a generic line in $\mathbb{P}^n$, the pullback $\mu^* F$ of F to $\mathbb{P}^1$ will be homogeneous of degree d in Y_0 and Y_1, and so by the fundamental theorem of algebra will have exactly d roots. The degree of V is thus the degree d of the polynomial F.

⌈ $\mu : \mathbb{P}^1 \longrightarrow \mathbb{P}^n$

$$[Y_0, Y_1] \longmapsto [a_0 Y_0 + b_0 Y_1, \cdots \quad a_n Y_0 + b_n Y_1]$$

$$\begin{tikzcd} \mathbb{P}^1 \arrow[r, "\mu"] \arrow[dr, "F \circ \mu"'] & \mathbb{P}^n \arrow[d, "F"] \\ & \mathbb{C} \end{tikzcd}$$

$\Rightarrow F \circ \mu : \mathbb{P}^1 \longrightarrow \mathbb{C}$ will be a homogeneous polynomial of deg d.

$\Rightarrow$ Consider $f(x) = F \circ \mu(Y_0, Y_1) / Y_0^d$, where $x = Y_1 / Y_0$.

$\Rightarrow$ $f = 0$ has exactly d roots, since we choose a generic line in $\mathbb{P}^n$, i.e. for generic (a_i, b_i)'s, $\deg f = d$.