

$\sigma(p, h) \subset \sigma(p) \cap F$ . Since  $\sigma(p)$  is tangent to  $F$  at  $\text{for every } l$ ,  $\sigma(p) \cap F$  must be  $\sigma(p, h)$ .

Note that if, for two distinct  $l, l' \in \sigma(p, h)$ ,  $\pi(l) = \pi(l') = q$ , then  $\sigma(q) \cap F = \sigma(q, h_1) \cup \sigma(q, h_2)$  and  $\sigma(q)$  is tangent to  $F$  at  $l$  and  $l' \Rightarrow \sigma(q) \cap F = \sigma(q, h')$  and  $\sigma(q, h') \cap \sigma(p, h) \ni l, l' \Rightarrow \sigma(q, h') = \sigma(p, h) \Rightarrow p = q$  and  $h = h'$ .

Thus we have to divide into two cases.

(i)  $\pi(l) = p$  for all  $l \in \sigma(p, h)$

(ii)  $\{\pi(l)\}$  is a set of distinct elements in  $S$ .

We proved that in the case (i),  $\sigma(p, h) = X_p$ .

Case (ii).

$$\Rightarrow \pi(L) \cap R = \emptyset.$$

$\Rightarrow$  Note the following:

①  $\pi: L \rightarrow \pi(L)$  is biholomorphic

②  $\pi(L) \subset h \cap S = C_h$ , for  $\forall x \in L = \sigma(p, h)$

$$\Rightarrow \pi(x) \in l_x \subset h$$

③  $C_h = C_1 \cup L_1 \cup L_2$ , or  $C_1 \cup C_2$ ,

for  $\pi(L)$  is smooth, and  $\pi(L) = C_1$  or  $\pi(L) = L_i$ ,  $L_i$  line  $C_i$  a conic curve in  $S$ .

(Since  $\deg C_h = 4$ , and  $\pi$  is one to one, if  $\pi(L) = C_i$ , then  $C_i$  is smooth.)

Suppose  $X_h \neq \sigma(p, h)$  (Note  $\sigma(p, h) \subset X_h$  since  $\sigma(p, h) \subset \sigma(h)$ , and  $\sigma(p, h) \subset \Sigma \subset X$ .)

$$\Rightarrow X_h = \sigma(p, h) \cup \sigma(q, h), \quad p \neq q. \quad (i) \quad \pi(L) = C_1$$

For generic  $r \in \pi(L)$ , we have two distinct lines  $\overline{pr}$  and  $\overline{qr}$  contained in  $\sigma(r, h)$ .