$\Rightarrow$ By the Corollary 8, $1 + d_g I, \cdots \quad g^{d-1} + d_g I$ generate $\dfrac{d_g \mathcal{O}_z}{d_g I}$, and are linearly independent over $\dfrac{f^* \mathcal{O}_w}{f^* m_w}$ since d_g is isomorphic.

$\Rightarrow \quad \dfrac{d_g \mathcal{O}_z}{d_g I}$ is a complex vector space of dimension ν

Clearly $\quad \mathcal{O}_z / I \cong \dfrac{d_g \mathcal{O}_z}{d_g I}$ as a $\dfrac{f^*\mathcal{O}_w}{f^* m_w}$ module $\Rightarrow \dim_{\mathbb{C}} \mathcal{O}_z / I = \nu$.

I want to make clear on the linearly independence of $1 + d_g I, \cdots, g^{d-1} + d_g I$ in $\dfrac{d_g \mathcal{O}_z}{d_g I}$, one more time.

Suppose they are linearly dependent. As before, we have the following.

$$(a_0 + d_g I) + (a_1 g + d_g I) + \cdots + (a_{d-1} g^{d-1} + d_g I) = d_g I, \quad a_i \in f^* \mathcal{O}_w$$

$\Rightarrow \quad a_0 + a_1 g + \cdots + a_{d-1} g^{d-1} \in d_g I$

$\Rightarrow \quad a_0 + a_1 g + \cdots + a_{d-1} g^{d-1} = d_g \, (b_1 f_1 + \cdots + b_n f_n), \quad b_i \in \mathcal{O}_z$

$= (b_{10} + b_{11} g + \cdots + b_{1,d-1} g^{d-1}) \, d_g f_1, \qquad b_{ij} \in f^* \mathcal{O}_w.$

$+ (\qquad\qquad) \, d_g f_2$

$\vdots$

$\Rightarrow \quad a_0 = d_g \cdot (b_{10} f_1 + \cdots + b_{n0} f_n) \in d_g f^* m_w \subset f^* m_w$

$\vdots$

since $d_g = f^* d_g'$, and