more than two points, or on any line meeting S in two points with intersecting tangent lines. The projection map $\pi_p|_S : S \longrightarrow \mathbb{P}^2$ will then have everywhere nonzero differential and will be at most 2-1 at isolated points; the image $\pi_p(S) \subset \mathbb{P}^2$ will be a plane algebraic curve whose only singularities are ordinary double points, or nodes — i.e., near a singular point, $\pi_p(S)$ will look like the union of two smooth analytic arcs meeting at a point with distinct tangents. (This discussion will be sharpened considerably in Section 4 of this chapter.)

Γ If a point $p \in \mathbb{P}^3$ does lie on any tangent line to S in $\mathbb{P}^3$, this implies that $p \in T(S) = \{ r \in \mathbb{P}^3 \mid r \in l,\ l \text{ is a tangent line to } S \}$. Note that $\dim T(S) \leqq 2$, since $T(S) \subset \{(q,r) \mid q \in S,\ r \in l,\ l \text{ tangent line to } S \text{ at } q \}$.
$\Rightarrow$ Generic points are not on a tangent line to S. Similarly, if a point $p \in \mathbb{P}^3$ is on a line meeting S in more than two points, p must satisfy an analytic function. $\Rightarrow$ Generic points do not lie on a line meeting S in more than two points. Similarly, generic points do not lie on a line meeting S in two points with intersecting tangent lines. Of course, we choose $p \notin S$.
Since p is not on a tangent line to S, π_p is an immersion, i.e., π_p has everywhere nonzero differential. By the picture below, we can easily see this.