Suppose E passes $P_2, \cdots P_q$. $\Rightarrow$ $(C \cdot E)_{P_2} \geq m_2 = 1$
$(C \cdot E)_{P_3} \geq m_3 = 1$. $\cdots$ $(C \cdot E)_{P_q} \geq m_q = 1$. $(C \cdot E)_{P_1} \geq 0$.
$\Rightarrow$ By the example above, $(C \cdot E)_{P_1} \geq 1$ $\Rightarrow$ E passes P_1 too.

This fact was known in 1748 to Euler, who remarked that as a consequence polynomial functions in two or more variables would necessarily be much more complicated than in one variable, since then it is not generally the case that a set of mn points in plane is the common zero locus of a pair of polynomials.

「 Make-up. If E passes through eight points, then for some ν_0, $(C \cdot E)_{P_{\nu_0}} \geq m_{\nu_0} - 1$. & $(C \cdot E)_{P_\nu} \geq m_\nu$ for $\nu \neq \nu_0$. By the example, we get $(C \cdot E)_{P_{\nu_0}} \geq m_{\nu_0}$. $\Rightarrow$ E passes the remaining point with the same multiplicity $(C \cdot D)_{\nu_0}$.

「 On $\dim_{\mathbb{C}}(\mathcal{O}/I(f_1 \cdots f_n)) = d$

See P35. Corollary 8. Introduction to Holomorphic Functions of Several Variables by R. Gunning.

We had better wait until Section 3.