

As far as I understand, the authors confuse me on this Bochner-Martinelli kernel.

$$\begin{aligned}
 \int_{\|z\|=\epsilon} \bar{H}^* k(z) &= \int_{\|z\|=\epsilon} k(z+f(z), z) = \int_{\|z\|=\epsilon} C_n \frac{\sum (-1)^{i-1} \bar{f}_i d\bar{f}_1 \wedge \dots \wedge d\bar{f}_n}{\|f(z)\|^{2n}} \\
 \dots d\bar{f}_n \wedge dz_1 \wedge \dots \wedge dz_n &= \int_{\|z\|=\epsilon} C_n \frac{\sum (-1)^{i-1} \bar{f}_i d\bar{f}_1 \wedge \dots \wedge d\bar{f}_n}{\|f(z)\|^{2n}} \frac{dz_1 \wedge \dots \wedge dz_n}{J_f(z)} \\
 &= \int_{\|z\|=\epsilon} C_n \frac{\sum (-1)^{i-1} \bar{f}_i d\bar{f}_1 \wedge \dots \wedge d\bar{f}_n}{\|f(z)\|^{2n}} \frac{J_f(z)}{J_f(z)} \\
 &= \int_{\|f(z)\|=\epsilon} C_n \frac{\sum (-1)^{i-1} \bar{f}_i d\bar{f}_1 \wedge \dots \wedge d\bar{f}_n}{\|f(z)\|^{2n}} \frac{1}{J_f(z)} = J_f(0)^{-1} = \frac{1}{J_f(0)}.
 \end{aligned}$$

where $J_f(0) \neq 0$. \square

We now know that for any type of isolated fixed point,

$$\int_{\|z\|=\epsilon} \bar{H}^* k = \text{Res}_{z=0} \left(\frac{dz_1 \wedge \dots \wedge dz_n}{f_1(z) \dots f_n(z)} \right).$$

This leads to a corresponding extension of the holomorphic Lefschetz theorem.

Not for any type of isolated fixed point, but for $J_f(0) \neq 0$.

$$\int_{\|z\|=\epsilon} \bar{H}^* k = \int_{\|z\|=\epsilon} \bar{g}^{-1} \eta_{\omega} = \frac{1}{J_f(0)} = \text{Res}_{z=0} \left(\frac{dz_1}{f_1(z)} \right).$$