Thus F is a double point of W_1; indeed, since the polynomial $|\lambda Q + Q'|$ will have degree < 2 exactly when the determinant of the upper left-hand 2×2 minor of Q' is zero, we deduce that the tangent cone to W_1 at a point $F \in W_2$ is just the locus of quadrics tangent to the singular line of F.

$$\begin{vmatrix} q'_{00} & q'_{01} & q'_{02} & q'_{03} \\ q'_{10} & q'_{11} & q'_{12} & q'_{13} \\ q'_{20} & q'_{21} & q'_{22}+\lambda & q'_{23} \\ q'_{30} & q'_{31} & q'_{32} & q'_{33}+\lambda \end{vmatrix} = |\lambda Q + Q'|$$

$$= \begin{vmatrix} q'_{00} & q'_{01} \\ q'_{10} & q'_{11} \end{vmatrix} \begin{vmatrix} q'_{22}+\lambda & q'_{23} \\ q'_{32} & q'_{33}+\lambda \end{vmatrix} + \lambda (\qquad) + (\quad)$$

is a polynomial of degree < 2

$$\Leftrightarrow \begin{vmatrix} q'_{00} & q'_{01} \\ q'_{10} & q'_{11} \end{vmatrix} = 0$$

By the argument on P176, $L = \{\lambda Q + Q'\}$ is a tangent line to W_1 at $F \Leftrightarrow m_F(L \cdot W_1) \geq 3$ since F is a double point of $W_1 \Leftrightarrow$

$$\begin{vmatrix} q'_{00} & q'_{01} \\ q'_{10} & q'_{11} \end{vmatrix} = 0 \qquad \Leftrightarrow$$