We claim ψ_i extends to a meromorphic function on all of $\mathbb{P}^k$. If $p \in \pi(D)$ is any point and $f(X)$ a local defining function for $\pi(D)$ in a neighborhood Δ of p, then for m sufficiently large, the function

$$\varphi' = \varphi \cdot \pi^* f^m$$

will be holomorphic in $\pi^{-1}(\Delta)$.

⌈ Since $\pi(D)$ is an analytic hypersurface of $\mathbb{P}^k$, there is a homogeneous polynomial $f(X)$ defining $\pi(D)$ locally.

See P131 & P11 (Weak Nullstellensatz). $\Rightarrow$ $\exists$ m s.t. $\varphi' = \varphi \cdot \pi^* f^m$ is holomorphic in $\pi^{-1}(\Delta)$. ⌋

For $q \in \Delta - B$, then, let

$$\psi_i'(q) = \sum_{\alpha_1, \cdots \alpha_i} \left(\prod \varphi'(p_{\alpha_1}) \cdots \varphi'(p_{\alpha_i}) \right)$$

be the ith symmetric function of the values of φ' at the points of $\pi^{-1}(q)$; being bounded in any compact subset of Δ, ψ_i' likewise extends to a holomorphic function on Δ.

⌈ As in the case $\mathbb{P}^k - B - \pi(D)$, by the Riemann extension theorem, ψ_i' extends to a holomorphic function on Δ. ⌋

Writing $\psi_i = \dfrac{\psi_i'}{f^{i \cdot m}}$,