$$
\begin{tikzcd}
\mathcal{G}(U) \arrow[r] \arrow[rr, bend left, "\phi_U"] & \mathcal{G}(V) \arrow[r, dotted] & \mathcal{G}_p \\
\mathcal{F}(U) \arrow[u, "\varphi_U"] \arrow[ur] \arrow[r, "\rho_{UV}"] \arrow[rr, bend right, "\psi_U"'] & \mathcal{F}(V) \arrow[u, "\varphi_V"'] \arrow[r, dashed] & \mathcal{F}_p \arrow[u, "\varphi_p\,?"']
\end{tikzcd}
$$

$\phi_U \circ \varphi_U$

$$\phi_V \circ \varphi_V \circ \rho_{UV} = \phi_V \circ \rho'_{UV} \circ \varphi_U = \phi_U \circ \varphi_U$$

$\Rightarrow \exists\ \varphi_p : \mathcal{F}_p \longrightarrow \mathcal{G}_p$. by the property of a direct system, see M. Greenberg. p209 ~ p210, for more details.

pf). ($\Rightarrow$) Trivial. since φ_U is isomorphic for each U.

($\Leftarrow$). Sufficient to show that $\varphi_U : \mathcal{F}(U) \longrightarrow \mathcal{G}(U)$ is an isomorphism. since we can define an inverse morphism ψ by $\psi(U) = \varphi(U)^{-1}$. for each U.

① $\varphi(U)$ is injective.

$s \in \mathcal{F}(U)$ s.t $\varphi(U)(s) = 0$ in $\mathcal{G}(U)$. $\Rightarrow$ for every point $p \in U$, the image $\varphi(s)_p$ of $\varphi(s)$ in the stalk $\mathcal{G}_p$ is 0.

$$
\begin{tikzcd}
s \in \mathcal{F}(U) \arrow[r] \arrow[r, bend left, "s_p"] \arrow[d, "\varphi(U) = \varphi_U"'] & \mathcal{F}_p \arrow[d, "\varphi_p"] \\
\varphi_U(s) = 0 \in \mathcal{G}(U) \arrow[r] \arrow[r, bend right, "\varphi(s)_p = 0"'] & \mathcal{G}_p
\end{tikzcd}
$$

Since φ_p is injective for each p, we deduce $s_p = 0$ in $\mathcal{F}_p$ for each $p \in U$.

* To say that $s_p = 0$ means that $s = 0$ on $W_p \subset U$ where W_p is an open nbd of p.