curve C, let $V_C \subset W$ be the set of conic curves tangent to C (that is, having a point of intersection multiplicity ≥ 2 with C); V_C is a hypersurface of some degree d in $\mathbb{P}^5$.

「 $\forall p \in C \Rightarrow \exists$ an open set U_p of p in $\mathbb{P}^3$ s.t U_p is biholomorphic to an open set of $\mathbb{C}$.
We may assume that, on U_p, $X_0 \neq 0$.
$\Rightarrow$ By the inverse function theorem, on P18 ~ P19,
we may assume without loss of generality
$U_p \cap C = [X_0, X_1, X_0 g(X_1/X_0)]$, where g is a holomorphic map from an open subset of $\mathbb{C}$ to $\mathbb{C}$.

Given a conic $a_{00}X_0^2 + a_{11}X_1^2 + a_{22}X_2^2 + a_{01}X_0X_1 + a_{02}X_0X_2 + a_{12}X_1X_2 = 0$, say C', if C' is tangent to C on U_p,

$$a_{00} + a_{11}\left(\frac{X_1}{X_0}\right)^2 + a_{22}\, g\left(\frac{X_1}{X_0}\right)^2 + a_{01}\frac{X_1}{X_0} + a_{02}\, g\left(\frac{X_1}{X_0}\right) + a_{12}\frac{X_1}{X_0}\, g\left(\frac{X_1}{X_0}\right) = 0$$

has a multiple root on U_p.

Let $X_1/X_0 = \omega$.

$$\Rightarrow a_{00} + a_{11}\omega^2 + a_{22}\, g(\omega)^2 + a_{01}\omega + a_{02}\, g(\omega) + a_{12}\omega\, g(\omega)$$
$$= f(a_{00}, a_{11}, \cdots, \omega) = 0,$$

Suppose, for some $(a_{00}', a_{11}', a_{22}', \; a_{12}', \omega_0)$, $\omega_0 \in U_p$
$f(a_{00}', \cdots, \omega_0) = 0$, and $f(a_{00}', \cdots \omega) \not\equiv 0$ on U_p
i.e. $f(a_{00}', \cdots a_{12}', \omega)$ not identically zero on U_p.