

on only  $\epsilon$ .

$$\Rightarrow |\psi_\epsilon(x) - \psi(x)| \leq \int_{\mathbb{R}^n} |\psi(y) - \psi(x)| \chi_\epsilon(x-y) dy = \int_{x+\epsilon K} |\psi(y) - \psi(x)| \chi_\epsilon(x-y) dy < \int_{x+\epsilon K} \delta \chi_\epsilon(x-y) dy < \int_{\mathbb{R}^n} \delta \chi_\epsilon(x-y) dy = \delta.$$

$\Rightarrow \psi_\epsilon \rightarrow \psi$  uniformly.

$$\begin{aligned} \Rightarrow (D\psi_\epsilon)(x) &= D\psi_\epsilon(x) = \int \psi(y) D\chi_\epsilon(x-y) dy \\ &= \int D\psi(y) \chi_\epsilon(x-y) dy \quad (\text{by integration parts}) \\ (D\psi)(x) &= \int D\psi(x) \chi_\epsilon(x-y) dy. \end{aligned}$$

$\Rightarrow$  Since  $D\psi \in C_c^\infty(\mathbb{R}^n)$ , by the argument above.

$D\psi_\epsilon \rightarrow D\psi$  uniformly.

$T_\epsilon(\psi) = T(\psi_\epsilon)$  by the assertion 2, since  $D\psi_\epsilon \rightarrow D\psi$  uniformly, which implies that  $\psi_\epsilon \rightarrow \psi$  in the  $C^\infty$  topology on  $C_c^\infty(\mathbb{R}^n)$ , by the continuity of  $T$ .

$$T(\psi_\epsilon) \rightarrow T(\psi) \quad \text{as } \epsilon \rightarrow 0, \Rightarrow T_\epsilon(\psi) \rightarrow T(\psi) \quad \text{as } \epsilon \rightarrow 0. \quad \square$$

There are, of course, many subtle questions about the convergence of the smoothing process in particular norms, but we need not get into these here.

A current  $T \in \mathcal{D}'(\mathbb{R}^n)$  may be considered as a differential form

$$T = \sum_{\#I=q} T_I dx_I$$