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is injective.

$\mathrm{Sym}^d(\mathbb{C}^{n+1*})$ let $V = \mathbb{C}^{n+1*} = \langle e_0^*, \cdots e_n^* \rangle$

$$\Rightarrow \begin{tikzcd} V \arrow[r, "f"] \arrow[dr] & S(V) \arrow[d] \\ & G \end{tikzcd} \quad S(V) = \frac{T(V)}{C}$$

where C is the ideal generated by $\{ v \otimes w - w \otimes v \mid v, w \in V \}$

$$T(V) = \bigoplus_{k=0}^{\infty} T^k(V)$$

$$T^k(V) = \overbrace{V \otimes \cdots \otimes V}^{k}$$

See p185 ~ p187 Hu. & p127. 5.16. Hartshorne

Let $P[x_0, x_1, \cdots x_n]$. be the polynomial algebra over $\mathbb{C}$.

$$\Rightarrow \begin{tikzcd} V \arrow[r, "f"] \arrow[dr, "g"'] & S(V) \arrow[d] \\ & P[x_0, x_1 \cdots x_n] \end{tikzcd}$$

$g: V \longrightarrow P[x_0, x_1 \cdots x_n]$ defined by

$e_i^* \longmapsto x_i$.

is a map from $V \longrightarrow P[x_0 \cdots x_n]$

$\Rightarrow$ By the universal property, $\exists$ a unique map $h: S(V) \longrightarrow P[x_0, x_1 \cdots x_n]$ whose inverse