Date

$\Rightarrow$ Since $p' \sim p_3$, $\exists$ a meromorphic function f on C s.t $p' - p_3 = (f)$. $\Rightarrow$ f has a simple pole at p_3, elsewhere holomorphic. $\Rightarrow$ Since $g=1$, it is impossible unless $p' = p_3$. $\lrcorner$

In summary:

(*) $\quad z_1 + z_2 + z_3 \equiv 0\ (\Lambda) \Leftrightarrow p_1, p_2, p_3$ are collinear.

Γ $\mu^{(1)}(p_1) + \mu^{(1)}(p_2) + \mu^{(1)}(p_3) = \mu(p_1 - p_0) + \mu(p_2 - p_0) + \mu(p_3 - p_0)$

$$= \int_{p_0}^{p_1} \omega + \int_{p_0}^{p_2} \omega + \int_{p_0}^{p_3} \omega = \mu(p_1 + p_2 + p_3 - 3p_0) \equiv z_1 + z_2 + z_3 \equiv 0\ (\underset{\uparrow\ \text{periods}}{\Lambda})$$

$\Rightarrow$ $p_1 + p_2 + p_3 - 3p_0 \sim 0 \Leftrightarrow p_1 + p_2 + p_3 - 3p_0 = (f)$, f meromorphic function on C, by Abel's theorem.

If $p_1 + p_2 + p_3 - 3p_0 \sim 0$, then consider $A(x,y) \overset{L}{=} ax + by + c$ be the equation of the line L joining p_1 and p_2 in $\mathbb{P}^2$. $\Rightarrow$ By the argument above, p_3 will be the third point of intersection L with C.

$\Rightarrow$ p_1, p_2, p_3 are collinear and $\mu(p_1 + p_2 + p_3 - 3p_0) = 0$

$$= z_1 + z_2 + z_3 = \int_{p_0}^{p_1} \omega + \int_{p_0}^{p_2} \omega + \int_{p_0}^{p_3} \omega \equiv 0\ (\Lambda)$$

Suppose p_1, p_2, p_3 are collinear. $\overset{\text{since } \exists \text{ a meromorphic function } f}{\Rightarrow}$ $p_1 + p_2 + p_3 - 3p_0 \sim 0$

$\Rightarrow$ $\mu(p_1 + p_2 + p_3 - 3p_0) = 0 \Rightarrow \mu^{(1)}(p_1) = z_1,\ \mu^{(1)}(p_2) = z_2,\ \mu^{(1)}(p_3) = z_3$

$\Rightarrow$ $z_1 + z_2 + z_3 \equiv 0\ (\Lambda)$.

"All the way, we can assume that p_1, p_2 are not p_0. If so, by changing the infinity, we may avoid the situation."