$\bar\partial^2 = 0 \Rightarrow \bar\partial^{*2} = 0$.

We now digress for a moment to explain the origins of the terminology Laplacian and harmonic. Provided we work with compactly supported forms, the above definitions are valid for any complex manifold. It is reasonable to expect the case of $\mathbb{C}^n$ with the Euclidean metric to provide a good local approximation to what is going on. Suppose we take $q=p=0$ and write $dz = dz_1 \wedge \cdots \wedge dz_n$. Then, for $f \in C_c^\infty(\mathbb{C}^n)$,

$$\Delta(f) = \bar\partial^*\bar\partial f = \bar\partial^*\Big(\sum_j \frac{\partial f}{\partial \bar z_j} d\bar z_j\Big) = -*\bar\partial\Big(\sum \pm \overline{\Big(\frac{\partial f}{\partial \bar z_j}\Big)}\, dz \wedge d\bar z_j^\circ\Big)$$

$$= * \sum \frac{\partial}{\partial \bar z_j}\overline{\Big(\frac{\partial f}{\partial \bar z_j}\Big)}\, dz \wedge d\bar z = \quad = -*\bar\partial * \Big(\sum_j \frac{\partial f}{\partial \bar z_j} d\bar z_j\Big)$$

$$= -*\bar\partial\Big(\sum_j *\Big(\frac{\partial f}{\partial \bar z_j} d\bar z_j\Big)\Big) = -*\bar\partial \sum_j (-1)^{n+j-1}\overline{\frac{\partial f}{\partial \bar z_j}}\, dz \wedge d\bar z_j^\circ$$

$$= -*\sum_j (-1)^{n+j-1} \frac{\partial}{\partial \bar z_j}\overline{\Big(\frac{\partial f}{\partial \bar z_j}\Big)}\, d\bar z_j \wedge dz \wedge d\bar z_j^\circ$$

$$= -*\sum_j \frac{\partial}{\partial \bar z_j}\overline{\Big(\frac{\partial f}{\partial \bar z_j}\Big)}\, dz \wedge d\bar z = -\sum_j \frac{\partial^2 f}{\partial z_j \partial \bar z_j}$$

$$= -\sum_j \frac{\partial^2 f}{\partial z_j \partial \bar z_j} = -\frac{1}{4}\sum_j\Big(\frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial y_j^2}\Big) f$$

$$\frac{1}{4}\Big(\frac{\partial}{\partial x_j} - i\frac{\partial}{\partial y_j}\Big)\Big(\frac{\partial}{\partial x_j} + i\frac{\partial}{\partial y_j}\Big) = \frac{1}{4}\Big(\frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial y_j^2}\Big)$$

We find that, up to a constant, $\Delta(f)$ is the usual Laplacian on functions in $\mathbb{C}^n \cong \mathbb{R}^{2n}$. Later on, in the discussion of Kähler manifolds, this computation will be extended to show that

$$\Delta(f\, dz_I \wedge d\bar z_J) = -\sum_j \frac{\partial^2 f}{\partial z_j \partial \bar z_j}\, dz_I \wedge d\bar z_J.$$

which explains the terminology for compactly supported forms in $\mathbb{C}^n$.