$$\begin{array}{ccccc} \mathbb{C}^{k\times n} & \longleftarrow & U_I \cap U_{I'} & \longrightarrow & \mathbb{C}^{k\times n} \\ \Psi & & \Psi & & \Psi \\ \Lambda^I & \longleftarrow & \Lambda & \longmapsto & \Lambda^{I'} \end{array}$$

Since Λ has unique representations Λ^I, $\Lambda^{I'}$ whose I th $k\times k$ minor & I'th $k\times k$ minor are the identities, $\Lambda^{I'} = (\Lambda^I_{I'})^{-1} \cdot \Lambda^I$, where $\Lambda^I_{I'}$ is the I'th $k\times k$ minor matrix of Λ^I.

$$\begin{array}{ccc} S|_{U_I} & \longrightarrow & U_I \times \mathbb{C}^k \\ \Psi & & \\ (\Lambda, v) & \longmapsto & (\Lambda, (x_1, \cdots x_k)), \text{ where} \end{array}$$

$$v = x_1 v_1 + \cdots + x_k v_k . \quad \begin{pmatrix} v_1 \\ \vdots \\ v_k \end{pmatrix} = \Lambda^I .$$

$$\begin{array}{ccc} S|_{U_{I'}} & \longrightarrow & U_{I'} \times \mathbb{C}^k \\ \Psi & & \\ (\Lambda, v) & \longmapsto & (\Lambda, (y_1 \cdots y_k)) \end{array}$$

$$v = y_1 w_1 + \cdots + y_k w_k , \quad \begin{pmatrix} w_1 \\ \vdots \\ w_k \end{pmatrix} = \Lambda^{I'} .$$

Let $g = (\Lambda^I_{I'})^{-1}$.

$$\Rightarrow \begin{pmatrix} w_1 \\ \vdots \\ w_k \end{pmatrix} = g \begin{pmatrix} v_1 \\ \vdots \\ v_k \end{pmatrix} = \begin{pmatrix} g_{11}, & \cdots & g_{1k} \\ \vdots & & \\ g_{k1}, & & g_{kk} \end{pmatrix} \begin{pmatrix} v_{11} & \cdots & v_{1n} \\ \vdots & & \\ v_{k1} . & \cdots & v_{kn} \end{pmatrix}$$

$$= \begin{pmatrix} g_{11}v_{11} + \cdots + g_{1k}v_{k1}, & g_{11}v_{12} + \cdots + g_{1k}v_{k2}, & \cdots \\ g_{21}v_{11} + \cdots + g_{2k}v_{k1}, & & \end{pmatrix}$$