$\exists$ exactly 12 sets of six disjoint lines on $S$.
This implies that $\eta_{ij} \cap \eta_{mn} \neq \phi$ if $\{i,j\} \cap \{m,n\} = \phi$.
Thus $\eta_{12} \oplus \eta_{34}$ is a line, and it can not be $\tau_i$ or $l_j$.
$\Rightarrow$ The only possibility is $\eta_{56}$. We proved that $\phi$ is homomorphism. Now it remains to show that $\phi$ is one to one. $\phi$ is one to on on $\{E_i\}$, clearly.

(i) On $\{G_i\}$,

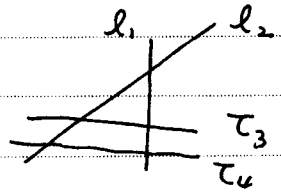
$\phi(G_1) = \phi(G_2) \Rightarrow l_1 = l_2$

$l_1$ is the unique line intersecting with $\tau_2, \cdots \tau_6$

$l_2$ " with $\tau_1, \tau_3, \cdots \tau_6$.

$\Rightarrow$ Contradiction. $\Rightarrow l_1 \neq l_2$

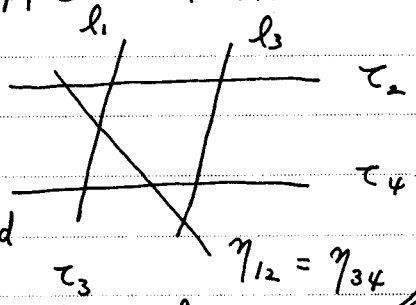
If $l_1 \cap l_2 \neq \phi$, then




$\Rightarrow$ By the second observation, $\tau_3 = \tau_4$.
Contradiction. $\Rightarrow l_1 \cap l_2 = \phi$.

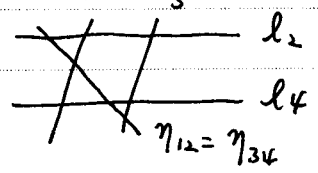
(ii) On $\{F_{ij}\}$.

By the argument on the third observation, we know that
$\phi(F_{ij}) = \eta_{ij} \neq \phi(F_{ik}) = \eta_{ik}$ if $j \neq k$.
Suppose $\phi(F_{12}) = \phi(F_{34})$. $\Rightarrow \eta_{12} = \eta_{34}$




$\Rightarrow \eta_{12} \oplus l_2 = \tau_1$

and




$\Rightarrow \eta_{12} \oplus l_2 = \tau_3$ since $\tau_3 \cap l_2 \neq \phi$
and $\tau_3 \cap \eta_{12} (= \eta_{34}) \neq \phi$.

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