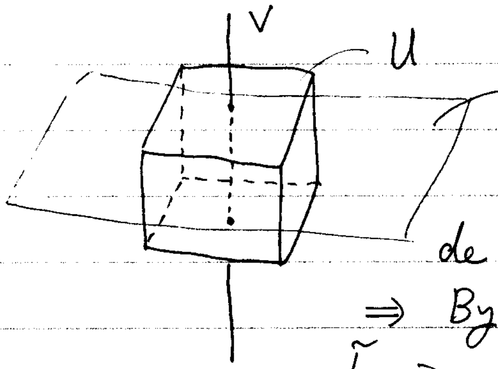
⌈ Suppose $(f)_\infty = 2V_1 + 3V_2$, where V_1 & V_2 are hypersurfaces in $M-V$. $\Rightarrow$ $\overline{(f)_\infty} = \overline{V_1} \cup \overline{V_2}$.
$\Rightarrow$ If $p \in V_1 \cap V_2$, $\exists$ $(g_1 = 0) = V_1$, $(g_2 = 0) = V_2$.
$\Rightarrow$ $g_1^2 g_2^2 = g$. $\Rightarrow \tilde{h} = f \cdot g$ is holomorphic in $U \cap (M-V)$.



For each section of U, by using Hartogs' Theorem, we can extend $\tilde{h}$ to U. $\Rightarrow$ We can conclude h is bounded on $U-V$.
$\Rightarrow$ By Riemann Extension Theorem, $\tilde{h}$ is holomorphic. Here we concerned about the holomorphy along V, which is explained by Riemann Extension Theorem. (We don't need R.E.T. See (Hartogs' Th. P7 $\tilde{h}(z_1, z_2, z_3) = \frac{1}{2\pi i}\int_{|w_2|=r} \frac{f(z_1, w_2, z_3)}{w_2 - z_2} dw_2$))

Since the question of whether $\overline{(f)_\infty}$ is an analytic variety is local around a point of M, the theorem is reduced to

Levi Extension Theorem (II). In the polycylinder Δ^n in $\mathbb{C}^n$ let V be a codimension ≥ 2 analytic subvariety, and D a subvariety of codimension 1 in $\Delta^n - V$. Then the closure $\overline{D}$ of D in Δ^n is analytic.

Proof. This is a geometric variant of Hartogs' theorem.