$$\varphi - a_{-1}\frac{du}{u} = d\eta \iff \varphi - d\eta = a_{-1}\frac{du}{u}.$$

$\Rightarrow$ Since $[\varphi]=0$, $a_{-1}=0$ ($\because \frac{du}{u}$ is a generator of $H^1_{DR}(\Delta^*)$.). $\Rightarrow$ $\varphi = d\eta$, η has only a pole." ⌟

We now draw some conclusions from the lemma. The sheaves $\mathcal{A}^*(*D)$ admit partitions of unity, and therefore $H^q(M, \mathcal{A}^*(*D))=0$ for $q>0$ and, by the spectral sequence for hypercohomology,

$$\begin{aligned}\mathbb{H}^*(M, \mathcal{A}^*(*D)) &\cong H^*_d(H^0(M, \mathcal{A}^*(*D))) \\ &= H^*_{DR}(M-D) \\ &\cong H^*(M-D, \mathbb{C}).\end{aligned}$$

$\mathbb{F}$

$$''E_0^{p,q}(\underline{U}) = \frac{''F^p K^{p+q}}{''F^{p+1}K^{p+q}}, \text{ where } K^{p,q} = C^p(\underline{U}, \mathcal{A}^q(*D))$$

$$= \frac{C^q(\underline{U}, \mathcal{A}^p(*D)) \oplus C^{q-1}(\underline{U}, \mathcal{A}^{p+1}(*D)) \oplus \cdots\cdots \oplus \cdots\cdots \oplus C^0(\underline{U}, \mathcal{A}^{p+q}(*D))}{C^{q-1}(\underline{U}, \mathcal{A}^{p+1}(*D)) \oplus \cdots\cdots \oplus \cdots\cdots \oplus C^0(\underline{U}, \mathcal{A}^{p+q}(*D))}$$

$\cong C^q(\underline{U}, \mathcal{A}^p(*D)).$ $\qquad ''E_0^{p,q-1}(\underline{U}) \cong C^{q-1}(\underline{U}, \mathcal{A}^p(*D))$

$''E_0^{p,q+1}(\underline{U}) \cong C^{q+1}(\underline{U}, \mathcal{A}^p(*D))$

$$\begin{array}{ccccc} ''E_0^{p,q-1}(\underline{U}) & \xrightarrow{d_0} & ''E_0^{p,q}(\underline{U}) & \xrightarrow{d_0} & ''E_0^{p,q+1}(\underline{U}) \\ \| & & \| & & \| \\ C^{q-1}(\underline{U}, \mathcal{A}^p(*D)) & \xrightarrow{\delta} & C^q(\underline{U}, \mathcal{A}^p(*D)) & \xrightarrow{\delta} & C^{q+1}(\underline{U}, \mathcal{A}^p(*D)) \end{array}$$

$$\Rightarrow \quad ''E_1^{p,q}(\underline{U}) = H^q(\underline{U}, \mathcal{A}^p(*D))$$