$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\
0 \arrow[r] & \mathrm{Hom}(M, N') \arrow[r, "f_{-1}"] \arrow[d, "\delta"] & \mathrm{Hom}(M, N) \arrow[r, "g_{-1}"] \arrow[d, "\delta"] & \mathrm{Hom}(M, N'') \ni x \arrow[d, "\delta"] & \\
0 \arrow[r] & \mathrm{Hom}(E_0, N') \arrow[r, "f_0"] \arrow[d, "\delta"] & \mathrm{Hom}(E_0, N) \ni y \arrow[r, "g_0"] \arrow[d, "\delta"] & \mathrm{Hom}(E_0, N'') \ni \delta x \arrow[r] \arrow[d, "\delta"] & 0 \\
0 \arrow[r] & \mathrm{Hom}(E_1, N') \ni z \arrow[r, "f_1"] & \mathrm{Hom}(E_1, N) \ni \delta y \arrow[r, "g_1"] & \mathrm{Hom}(E_1, N'') \arrow[r] & 0
\end{tikzcd}$$

$$y \longmapsto \delta x \mapsto 0, \qquad z \longmapsto \delta y$$

$\Rightarrow$ Note that $\mathrm{Hom}(M, N') = \mathrm{Ext}^0_{\mathcal{O}}(M, N') \hookrightarrow \mathrm{Hom}(E_0, N')$.

$\Rightarrow \exists$ a long exact sequence.

$$0 \to \mathrm{Ext}^0_{\mathcal{O}}(M, N') \longrightarrow \mathrm{Ext}^0_{\mathcal{O}}(M, N) \to \mathrm{Ext}^0_{\mathcal{O}}(M, N'') \to \mathrm{Ext}^1_{\mathcal{O}}(M, N') \to$$

$$\underset{\mathrm{Ext}^1_{\mathcal{O}}(M, N)}{\underset{\parallel}{H^1(\mathrm{Hom}(E(M), N))}} \longrightarrow \underset{\mathrm{Ext}^1_{\mathcal{O}}(M, N'')}{\underset{\parallel}{H^1(\mathrm{Hom}(E(M), N''))}} \to \mathrm{Ext}^2_{\mathcal{O}}(M, N') \to \mathrm{Ext}^2_{\mathcal{O}}(M, N) \to$$

For example, given $0 \to N' \to N \to N'' \to 0$, we obtain

$$(**) \quad 0 \to \mathrm{Hom}_{\mathcal{O}}(M, N') \to \mathrm{Hom}_{\mathcal{O}}(M, N) \to \mathrm{Hom}_{\mathcal{O}}(M, N'') \to \mathrm{Ext}^1_{\mathcal{O}}(M, N'),$$

so that $\mathrm{Ext}^1_{\mathcal{O}}(M, \cdot)$ measures the extent to which $\mathrm{Hom}_{\mathcal{O}}(M, \cdot)$ fails to be right-exact.

We next shall prove:

$\mathrm{Ext}^q_{\mathcal{O}}(M, N) = 0$ for $q > 0$ and every $\mathcal{O}$-module $N \Leftrightarrow M$ is projective.

Proof. Clearly if M is projective, the higher Ext's are zero.