We do not know that $\{D'\}$, where D' consists of distinct points, is a set of generic divisors of $|D|$

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and the generic divisors are hyperplane sections of C, dim. of $H^0(C, \mathcal{O}(D)) \geq$ dim. of the generic divisors. And if $(\sigma_1 = 0) \cap C = (\sigma_2 = 0) \cap C$, $\sigma_1 = \sigma_2$. $\Rightarrow$ dim of such generic divisors $= \dim H^0(\mathbb{P}^n, \mathcal{O}(H)) = n+1$.

More precisely, D_1, D_2 generic divisors s.t. $C \cap H_1 = C \cap H_2 = D_1 = D_2 \Rightarrow H_1 = H_2$, because D_i span H_i, otherwise we can get points p's in $C - D_i$ to get a hyperplane, which implies $\deg D_i > d$. *

Note that C must be nondegenerate. See the example below: Let $C = \mathbb{P}^1 \subset \mathbb{P}^2 \Rightarrow \deg \mathbb{P}^1 = 1 < 2 \cdot 2 = k$. genus $\mathbb{P}^1 = 0$. $\Rightarrow h^0(\mathbb{P}^1, \mathcal{O}(p)) = 2+1 = 3$ by the claim $\dim H^0(\mathbb{P}^1, \mathcal{O}(p)) - 1 = 2$. But $\dim H^0(\mathbb{P}^n, \mathcal{O}(H)) = n+1$ $\Rightarrow H^0(\mathbb{P}^1, \mathcal{O}(H)) = 2 \Rightarrow$ Contradiction.

Thus $h^0(K-D) = 0$, and by Riemann-Roch
$$g = d - h^0(D) + 1$$
$$\leq d - n.$$
Q.E.D

Since D is not special, $h^0(K-D) = 0$ by P245.
$\Rightarrow$ By Riemann-Roch, $h^0(D) = d - g + 1 + h^0(K-D)$,
$h^0(D) = d - g + 1 \Rightarrow g = d - h^0(D) + 1 \leq d - (n+1) + 1 = d - n$

"Comment: $H^0(\mathbb{P}^n, \mathcal{O}(H)) \longrightarrow H^0(C, \mathcal{O}(H))$ is injective, for if $H_1 \cap C = H_2 \cap C$, then $H_1 = H_2$ since C is nondegenerate.
$\Rightarrow \dim H^0(C, \mathcal{O}(H)) \geq \dim H^0(\mathbb{P}^n, \mathcal{O}(H)) = n+1 > \frac{d}{2} + 1$

PAGE