From P404, we have

$$\sum_i P(A_1, \cdots, g'A_i - A_i g', \cdots A_k) = 0$$

if $I + g' \in GL_n$.

$\Rightarrow$ For $\|g - I\| < \epsilon$, ϵ small enough, $g \in GL_n$,

$$\sum_i P(A_1, \cdots, gA_i - A_i g, \cdots A_k) = 0.$$

Suppose, for some $g_0 \in GL_n$,

$$\sum_i P(A_1, \cdots, g_0 A_i - A_i g_0, \cdots A_k) = 0. \cdots (*)$$

Choose small enough $\delta > 0$ so that

and $I + h - g_0 \in GL_n$ for all h with $\|h - g_0\| < \delta$.

$\Rightarrow$ By the remark above,

$$\begin{aligned} 0 &= \sum_i P(A_1, \cdots, (h-g_0)A_i - A_i(h-g_0), \cdots A_k) \\ &= \sum_i P(A_1, \cdots, hA_i - g_0 A_i - A_i h + A_i g_0, \cdots A_k) \\ &= \sum_i P(A_1, \cdots, hA_i - A_i h, \cdots A_k) \\ &+ \sum_i P(A_1, \cdots, A_i g_0 - g_0 A_i, \cdots A_k) \\ &= \sum_i P(A_1, \cdots, hA_i - A_i h, \cdots A_k) \quad \text{by } (*). \end{aligned}$$

Consider $K = \{ g \in GL_n : \sum_i P(A_1, \ldots, gA_i - A_i g, \cdots A_k) = 0.$

$\Rightarrow$ K is open (by the argument above), and closed since if $g' \notin K$, then by continuity, $\exists\ \epsilon > 0$ s.t for h with $\|h - g'\| < \epsilon$