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We can describe the set of lines on a general smooth quadric S directly as follows: first, note that if Q is any point in the intersection of S with its tangent plane $T_p(S) \subset \mathbb{P}^3$ at P, the line $\overline{PQ}$ meets S three times – once at Q and twice at P – and so must lie in S.

Γ $\#(\overline{PQ} \cap S) \geq 3$. $\Rightarrow$ Since $\deg S = 2$, by P64, $S \supset \overline{PQ}$ ⅃

The locus $S \cap T_p(S)$ must therefore consist of a union of lines; since $S \cap T_p(S)$ has degree 2, it must consist of two lines.

Γ $S \cap T_p(S) \ni p$, and $S \cap T_p(S)$ has $\dim \geq 1$.
As we argued above, since $T_p(S)$ is a plane, $S \cap T_p(S)$ is a curve of degree 2. $\Rightarrow$ $S \cap T_p(S)$ is a set of two lines. ⅃

Conversely any line on S through P must lie in the locus $S \cap T_p(S)$, and so we see that through every point $P \in S$ there pass exactly two lines on S, comprising the locus $S \cap T_p(S)$.

Γ $p \in L \subset S$, $\Rightarrow$ $T_p(S) \supset L$, since L is a line.
$\Rightarrow$ $S \cap T_p(S) \supset L$. ⅃

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