$I = \{ (p, \Lambda_n) : p \in \Lambda \subset \widetilde{F} \} \subset \widetilde{F} \times G(n+1, 2n+2).$

As before, the fibers of the projection map $\pi_2 : I \longrightarrow \Sigma_n \subset G(n+1, 2n+2)$ are isomorphic to $\mathbb{P}^n$;

Σ_n = Set of all n-planes on F_{2n}
$\Rightarrow \pi_2^{-1}(\Lambda) = \{ p \mid p \in \Lambda \} \cong \mathbb{P}^n.$

and the fibers of $\pi_1 : I \longrightarrow \widetilde{F}$ is isomorphic to Σ_{n-1}.

$\forall p \in \widetilde{F}$, $\pi_1^{-1}(p)$ = Set of all $(n-1)$-planes on $\widetilde{F}_{2n-2}$
$= \Sigma_{n-1}$.

In this case, however, by induction Σ_{n-1} is disjoint union of two irreducible varieties of dimension $n(n-1)/2$. The connected components of the fibers of π_1 thus constitute an unbranched 2-sheeted cover of $\widetilde{F}$, which, since $\widetilde{F}$ is rational and hence simply connected, must be disconnected.

Let $\Sigma_{n-1} = A \cup B$. A, B are two irreducible families.
$\Rightarrow A \cap B = \phi$ since otherwise $A \& B$ are reducible.
$\dim A = \dim B = n(n-1)/2$.
According to P495, example 3, $\widetilde{F}$ is birational to $\mathbb{P}^{2n}$,
i.e., by the definition on P493, $\widetilde{F}$ is rational.
$\Rightarrow$ By the result 5 on P494, $\pi_1(\widetilde{F}) = \pi_1(\mathbb{P}^{2n}) = 0$
$\Rightarrow$ $\widetilde{F}$ is simply connected.