Substituting (**) in this exact sequence, we obtain

$$0 \longrightarrow H^1(M, \mathbb{C}) \longrightarrow \mathbb{H}^1(\Omega(*)) \xrightarrow{R} H^0(\bigoplus_{\substack{D \in \mathrm{Div} M \\ \text{irreducible}}} \mathbb{C}_D) \xrightarrow{i} H^2(M, \mathbb{C}) \rightarrow \mathbb{H}^2(\Omega(*)) \rightarrow G.$$

Γ $'E_2^{1,0} = H^1(M, \mathcal{H}^0(\Omega(*))) = H^1(M, \mathbb{C})$ since $\mathcal{H}^0(\Omega(*)) \cong \mathbb{C}$.

$'E_2^{0,1} = H^0(M, \mathcal{H}^1(\Omega(*))) = H^0(M, \bigoplus_{\substack{D \in \mathrm{Div} M \\ \text{irreducible}}} \mathbb{C}_D)$ since $\mathcal{H}^1(\Omega(*)) = \bigoplus_{\substack{D \\ \text{irreducible}}} \mathbb{C}_D$

by lemma on P457. $\quad \| \quad H^0(\bigoplus_{\substack{D \\ \text{irreducible}}} \mathbb{C}_D)$

$'E_2^{2,0} = H^2(M, \mathcal{H}^0(\Omega(*))) = H^2(M, \mathbb{C})$. ⅃

The interpretations of the maps in this sequence are (we omit the proofs that diagrams commute):

1. Using the previously established isomorphism
(***) $\qquad H^*(M, \mathbb{C}) \cong \mathbb{H}^*(\Omega^*)$,
the first map is the natural one

$$\mathbb{H}^1(\Omega^*) \longrightarrow \mathbb{H}^1(\Omega(*))$$

induced from the inclusions $\Omega^p \longrightarrow \Omega^p(*)$.

Γ
$$\begin{array}{ccc} 0 \longrightarrow & 'E_2^{1,0} & \longrightarrow \mathbb{H}^1(\Omega(*)) \\ & \cong & \nwarrow \\ & H^1(M, \mathcal{H}^0(\Omega(*))) & \mathbb{H}^1(\Omega^*) \\ & \| & \\ & H^1(M, \mathbb{C}) \cong \mathbb{H}^1(\Omega^*) & \end{array}$$

Everything should be natural. ⅃