homomorphic function on S, which must be constant. $\Rightarrow H^0(S, \mathcal{O}(K-D)) = 1$.

If $K \neq D$, let $\sigma \in H^0(S, \mathcal{O}(K-D))$.

$\Rightarrow$ Since $\deg(K-D) = 0 = \deg K - \deg D = 2g-2-(2g-2)$, $(\sigma = 0) \sim K-D$. $\Rightarrow (\sigma = 0)$ is an effective divisor

$\Rightarrow$ Effective divisors have positive degrees unless they are nonvanishing. If σ is nonvanishing, σ is not a section of $[K-D]$ unless $[K-D]$ is trivial, which means $K = D$. $\Rightarrow H^0(S, \mathcal{O}(K-D)) = \phi$. ⌟

This implies that: if $S \subset \mathbb{P}^{g-1}$ is any nondegenerate curve of genus g and degree $2g-2$, then S is a canonical curve.

⌜ $S \subset \mathbb{P}^{g-1}$, H hyperplane of $\mathbb{P}^{g-1}$.

$\Rightarrow \#(S \cdot H) = \deg S = c_1([H]|_S) = 2g-2$

$\Rightarrow$ By the note above, $[H]|_S = K_S \Rightarrow h^0(K-H) = 1$ and $[H]|_S \neq K_S \Rightarrow h^0(K-H) = 0$.

We don't need this

I think that the statement is as follows; more precisely

If $S \subset \mathbb{P}^{g-1}$ is a nondegenerate curve of genus g and degree $2g-2$ embedded from S' by i_L, then S is a canonical curve, where L is a line bundle over S'!

$$\begin{tikzcd} L \arrow[d] & H \arrow[d] \\ S' \arrow[r, hook, "i_L"] & S \subset \mathbb{P}^{g-1} \end{tikzcd}$$

$\Rightarrow$ We are going to show that $L = K_{S'}$. If not, By the argument above, $h^0(L) = g-1$. This contradicts

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I have a corrected proof of this see last yellow page.