For example, let $n=3$, $V = \mathbb{P}^2$ and

$$Q = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 2 & 0 \\ 1 & 2 & 0 & 3 \end{pmatrix}. \Rightarrow \text{rank of } Q = 4.$$

$\Rightarrow F = \sum_{i,j=0}^{3} q_{ij} X_i X_j$ is smooth.

$F' = F \cap V = X_1^2 + 2X_1X_2 + 2X_2^2$

$$\frac{\partial F'}{\partial X_0} = 0 \qquad \frac{\partial F'}{\partial X_1} = 2X_1 + 2X_2 \qquad \frac{\partial F'}{\partial X_2} = 4X_2 + 2X_1$$

$\Rightarrow F'$ is singular at $[(1,0,0)]$, despite V does not intersect with the singular set of F. $\Rightarrow$ "Any plane in $\mathbb{P}^n$ disjoint from Λ intersects F smoothly" does not mean that any plane disjoint from Λ intersects F transversely i.e. $V \cap F$ is smooth, where V is a plane disjoint from Λ.

We can see most of this in terms of the Gauss map

$$\mathcal{G} : F \longrightarrow \mathbb{P}^{n*}$$

defined by sending a point $p \in F$ to its tangent plane $T_p(F) \in \mathbb{P}^{n*}$. Since the tangent plane to the quadric given by Q above at a point $p = [a_0, \cdots a_n]$ is

$$T_p(F) = \left(\sum_{i,j} q_{ij}\, a_j X_i = 0\right),$$