⌈ In $\tilde{U}_i \cap \tilde{U}_j$, $l_i \neq 0 \neq l_j$

$$z(i)_k = \frac{z_k}{z_i} = \frac{z_k}{z_j}\frac{z_j}{z_i} = z(j)_k \cdot z(j)_i^{-1}$$

$$z(i)_j = \frac{z_j}{z_i} = \left(\frac{z_i}{z_j}\right)^{-1} = (z(j)_i)^{-1}$$

$$z_i = \frac{z_i}{z_j} z_j = z(j)_i \, z_j .$$ ⌋

Now, since $E = (z_i)$ in $\tilde{U}_i$, the line bundle $[E]$ is given in $\tilde{U}$ by transition functions

$$g_{ij} = z(j)_i = \frac{z_i}{z_j} = \frac{l_i}{l_j}, \text{ in } U_i \cap U_j$$

and so we can realize $[E]|_{\tilde{U}}$ by identifying the fiber

(*) $$[E]_{(z,l)} = \{ \lambda (l_1, l_2, \ldots l_n),\ \lambda \in \mathbb{C} \}.$$

⌈ By P133 ~ P134, on $U_\alpha \cap U_\beta$, the transition function $g_{\alpha\beta}$ is given by $\dfrac{f_\alpha}{f_\beta}$, where f_α, f_β are local defining functions on U_α & U_β respectively.

Since z_i, z_j are local defining functions for E on $\tilde{U}_i$, $\tilde{U}_j$ respectively, the transition function

$$g_{ij} = \frac{z_i}{z_j} = \frac{l_i}{l_j} \text{ in } \tilde{U}_i \cap \tilde{U}_j .$$

$z_i = z(i)_i \qquad z_j = z(j)_j \quad \Rightarrow \quad \dfrac{z(i)_i}{z(j)_j} = \dfrac{z_i}{z_j} = g_{ij}$

$\tilde{U}_i \times \mathbb{C} \qquad \tilde{U}_j \times \mathbb{C}$

$((z,l), \alpha) \qquad ((z,l), \beta) \Longrightarrow g_{ij}\, \beta = \alpha$ ⌋

↓

$\left(\frac{z_1}{z_i}, \frac{z_2}{z_i}, \ldots \frac{z_{i-1}}{z_i}, z_i, \frac{z_{i+1}}{z_i}, \ldots \frac{z_n}{z_i}\right)$