$E_r \cong \mathcal{O}, \quad E_r' \cong \mathcal{O}.$

A_r is just the determinant Δ.

$\ulcorner$

$$\begin{tikzcd} e_I' \arrow[dd, mapsto] & \in E_k' \arrow[r, "\partial"] \arrow[dd] & E_{k-1}' \arrow[dd] \\ & & \\ & E_k \arrow[r, "\partial"] & E_{k-1} \end{tikzcd}$$

$$A_k(e_I') = A_1(e_{i_1}') \wedge \cdots A_1(e_{i_k}')$$
$$\parallel$$

$$\sum a_{i_1 j_1} e_{j_1} \wedge \cdots \wedge a_{i_k, j_k} e_{j_k} = a_{i_1 j_1} \cdots a_{i_k j_k} e_{j_1} \wedge \cdots \wedge e_{j_k}$$

$$\partial (a_{i_1 j_1} \cdots a_{i_k j_k} e_{j_1} \wedge \cdots \wedge e_{j_k}) = a_{i_1 j_1} \cdots a_{i_k j_k} (-1)^{\nu-1} f_{j_\nu} e_{j_1} \wedge \cdots \hat{e}_{j_\nu} \wedge \cdots e_{j_k}$$

On the other hand,

$$\partial e_I' = \sum (-1)^{\nu-1} f_{i_\nu}' e_{i_1}' \wedge \cdots \wedge \hat{e}_{i_\nu}' \cdots \wedge e_{i_k}'$$

$$\Rightarrow A_{k-1}(\partial e_I') = \sum (-1)^{\nu-1} f_{i_\nu}' A_1(e_{i_1}') \wedge \cdots \wedge \widehat{A_1(e_{i_\nu}')} \wedge \cdots \wedge A_1(e_{i_k}')$$

$$= (-1)^{\nu-1} f_{i_\nu}' a_{i_1 j_1} e_{j_1} \wedge \cdots \widehat{a_{i_\nu j_\nu} e_{j_\nu}} \wedge \cdots \wedge a_{i_k j_k} e_{j_k}$$

$$= (-1)^{\nu-1} f_{i_\nu}' a_{i_1 j_1} \cdots \widehat{a_{i_\nu j_\nu}} \cdots a_{i_k j_k} e_{j_1} \wedge \cdots \wedge \hat{e}_{j_\nu} \cdots \wedge e_{j_k}$$

$$= \partial (A_k(e_I')) \text{ by replacing by } f_{i_\nu}' = a_{i_\nu j_\nu} f_{j_\nu}$$

$$A_r : E_r' \longrightarrow E_r'$$
$$\begin{array}{l} \rotatebox{90}{$\in$} \\ e_I' \\ \parallel \end{array}$$
$$e_1' \wedge \cdots \wedge e_r' \longmapsto A_1(e_1') \wedge \cdots \wedge A_1(e_r')$$
$$= \Delta \, e_1 \wedge \cdots \wedge e_r .$$

$\lrcorner$

It remains to prove the injectivity. Before doing this, it might be instructive to verify directly th-