Given a meromorphic form ω on V,

$$\omega = f_{01}\, d\left(\tfrac{X_1}{X_0}\right) + f_{02}\, d\left(\tfrac{X_2}{X_0}\right) + \cdots \qquad \text{on } U_0 = (X_0 \neq 0)$$

$$= f_{11}\, d\left(\tfrac{X_0}{X_1}\right) + f_{12}\, d\left(\tfrac{X_2}{X_1}\right) + \cdots \qquad \text{on } U_1 = (X_1 \neq 0)$$

$$\vdots$$

For simplicity, assume $V \subset \mathbb{P}^2$ and V is a curve.

$$\Rightarrow \omega = f_{01}\, d\left(\tfrac{X_1}{X_0}\right) + f_{02}\, d\left(\tfrac{X_2}{X_0}\right) \qquad \text{on } U_0 = (X_0 \neq 0)$$

$$= f_{11}\, d\left(\tfrac{X_0}{X_1}\right) + f_{12}\, d\left(\tfrac{X_2}{X_1}\right) \qquad \text{on } U_1 = (X_1 \neq 0)$$

$$= f_{21}\, d\left(\tfrac{X_0}{X_2}\right) + f_{22}\, d\left(\tfrac{X_1}{X_2}\right) \qquad \text{on } U_2 = (X_2 \neq 0)$$

(actually, $U_i \cap V$)

Let $\varphi = \dfrac{X_1}{X_0}$, $\psi = \dfrac{X_2}{X_0}$

$$\Rightarrow \omega = f_{01}\, d\varphi + f_{02}\, d\psi = f_{11}\, d\left(\tfrac{1}{\varphi}\right) + f_{11}\, d\left(\tfrac{\psi}{\varphi}\right)$$

$$= f_{11}\left(-\frac{1}{\varphi^2}\right) d\varphi + f_{11}\,(d\psi)\,\frac{1}{\varphi} + f_{11}\psi\left(-\frac{1}{\varphi^2}\right) d\varphi$$

$$= \left(-\frac{f_{11}}{\varphi^2} - \frac{f_{11}\psi}{\varphi^2}\right) d\varphi + \frac{f_{11}}{\varphi}\, d\psi.$$

$$\Rightarrow f_{01} = -\frac{f_{11}}{\varphi^2} - \frac{f_{11}\psi}{\varphi^2} \qquad f_{02} = \frac{f_{11}}{\varphi} \qquad \text{on } U_0 \cap U_1.$$

$\Rightarrow$ f_{01} & f_{02} can be extended to U_1, similarly for U_2.

$$\Rightarrow \omega = f_{01}\, d\left(\tfrac{X_1}{X_0}\right) + f_{02}\, d\left(\tfrac{X_2}{X_0}\right) \qquad \text{on } V$$

$\Rightarrow$ f_{01}, f_{02} are meromorphic on V $\Rightarrow$ f_{01}, f_{02} are rational.