Date

f is called the affine, or inhomogeneous form of F.

We now make the

Def. An algebraic variety $V \subset \mathbb{P}^n$ is the locus in $\mathbb{P}^n$ of a collection of homogeneous polynomials $\{F_\alpha(X_0, X_1, \cdots X_n)\}$.

An algebraic variety is clearly an analytic subvariety of $\mathbb{P}^n$ and will be considered primarily as such (i.e., an algebraic variety $V \subset \mathbb{P}^n$ is called smooth, irreducible, connected, etc. if it has these properties as an analytic subvariety of $\mathbb{P}^n$). Conversely, we will show that any analytic subvariety of projective space is expressible as the locus of homogeneous polynomials.

We have already done this in essen for hypersurfaces: if $V \subset \mathbb{P}^n$ is any divisor, the line bundle $[V]$ is of the form H^d for some d, and V is the zero locus of some section σ of $[V]$.

⟦ See P145. every divisor on $\mathbb{P}^n$ is linearly equivalent to a multiple of the hyperplane divisor H. See P135. the bottom line. ⟧