

$$\begin{aligned}
&= \sum_{k \leq p} (-1)^k f(\alpha_{\bar{i}_0 \dots \bar{i}_k \dots \bar{i}_{p+1}} \otimes \beta_{\bar{i}_{p+1} \dots \bar{i}_{p+q+1}}) \\
&+ \sum_{k \geq p+1} (-1)^k f(\alpha_{\bar{i}_0 \dots \bar{i}_p} \otimes \beta_{\bar{i}_p \dots \bar{i}_k \dots \bar{i}_{p+q+1}}) \\
&= \sum_{k \leq p} (-1)^k f(\alpha_{\bar{i}_0 \dots \bar{i}_k \dots \bar{i}_{p+1}} \otimes \beta_{\bar{i}_{p+1} \dots \bar{i}_{p+q+1}}) + (-1)^{p+1} f(\alpha_{\bar{i}_0 \dots \bar{i}_p} \otimes \beta_{\bar{i}_{p+1} \dots \bar{i}_{p+q+1}}) \\
&+ \sum_{k \geq p+1} (-1)^k f(\alpha_{\bar{i}_0 \dots \bar{i}_p} \otimes \beta_{\bar{i}_p \dots \bar{i}_k \dots \bar{i}_{p+q+1}}) + (-1)^{p+2} f(\alpha_{\bar{i}_0 \dots \bar{i}_p} \otimes \beta_{\bar{i}_{p+1} \dots \bar{i}_{p+q+1}}) \\
&= f\left(\left(\sum_{k=0}^{p+1} \alpha_{\bar{i}_0 \dots \bar{i}_k \dots \bar{i}_{p+1}}\right) \otimes \beta_{\bar{i}_{p+1} \dots \bar{i}_{p+q+1}}\right) \\
&+ (-1)^p f\left(\alpha_{\bar{i}_0 \dots \bar{i}_p} \otimes \left(\sum_{k=0}^{q+1} (-1)^k \beta_{\bar{i}_p \dots \bar{i}_{p+k} \dots \bar{i}_{p+q+1}}\right)\right) \\
&= f\left((\delta\alpha)_{\bar{i}_0 \dots \bar{i}_{p+1}} \otimes \beta_{\bar{i}_{p+1} \dots \bar{i}_{p+q+1}}\right) + (-1)^p f\left(\alpha_{\bar{i}_0 \dots \bar{i}_p} \otimes (\delta\beta)_{\bar{i}_{p+1} \dots \bar{i}_{p+q+1}}\right) \\
&= f(\delta\alpha \otimes \beta)_{\bar{i}_0 \dots \bar{i}_{p+q+1}} + (-1)^p f(\alpha \otimes \delta\beta)_{\bar{i}_0 \dots \bar{i}_{p+q+1}} \\
&= \text{RHS}
\end{aligned}$$

Thus  $f$  induces a map from  $H^p(X, \mathcal{G}) \otimes H^q(X, \mathcal{G})$  to  $H^{p+q}(X, \mathcal{A})$ , since  $\delta\alpha = 0$ ,  $\delta\beta = 0$ .  
 $\Rightarrow \delta(\alpha \otimes \beta) = \delta\alpha \otimes \beta + (-1)^p \alpha \otimes \delta\beta = 0$   
 and  $\delta(f(\alpha \otimes \beta)) = f(\delta(\alpha \otimes \beta)) = 0$   $\square$

In particular,

$$\Omega^p \otimes \Omega^q \longrightarrow \Omega^{p+q}$$

induced by the exterior product of holomorphic differential forms induce

$$(*) \quad H^*(M, \Omega^p) \otimes H^*(M, \Omega^q) \longrightarrow H^*(M, \Omega^{p+q}).$$

On the other hand, the pairing