

$$\frac{\partial f_\alpha}{\partial z_1} \Big|_V \Big/ \frac{\partial f_\beta}{\partial z_1} \Big|_V = \frac{\partial \varphi_\alpha \varphi_\beta}{\partial z_1} \Big|_V \Big/ \frac{\partial f_\beta}{\partial z_1} \Big|_V.$$

Warning:

$$\begin{array}{ccc} \mathbb{C}^n & \xleftarrow{\varphi_\alpha} U_\alpha & \xrightarrow{f_\alpha} \mathbb{C} \\ & \cup & \\ \mathbb{C}^{n-1} & \xleftarrow{} U_\alpha \cap V & \xrightarrow{f_\alpha} \mathbb{C}. \end{array}$$

$$\mathbb{C}^n \xleftarrow{\varphi_\beta} U_\beta \xrightarrow{f_\beta} \mathbb{C}$$

$$\Rightarrow \begin{array}{ccc} \mathbb{C}^n & \xleftarrow{\varphi_\beta} U_\alpha \cap U_\beta & \xrightarrow{\varphi_\alpha} \mathbb{C}^n \\ \mathbb{C}^{n-1} & \xleftarrow{\varphi_\beta} U_\alpha \cap U_\beta \cap V & \xrightarrow{\varphi_\alpha} \mathbb{C}^{n-1}. \end{array}$$

$$\begin{aligned} \varphi_\alpha \circ \varphi_\beta^{-1}(\underbrace{z_1 \dots z_n}_{z_{\beta,1} \dots z_{\beta,n}}) &= (\underbrace{\varphi_{\alpha,1} \dots \varphi_{\alpha,n}}_{z_{\alpha,1} \dots z_{\alpha,n}}) \\ \frac{\partial \varphi_{\alpha,1} \circ \varphi_\beta^{-1}}{\partial z_{\beta,2}} &= 0. \end{aligned}$$

$$\frac{\partial f_\alpha}{\partial z_{\alpha,1}} = \frac{\partial f_\alpha}{\partial z_{\beta,1}} \frac{\partial z_{\beta,1}}{\partial z_{\alpha,1}} + \frac{\partial f_\alpha}{\partial z_{\beta,2}} \frac{\partial z_{\beta,2}}{\partial z_{\alpha,1}} + \dots + \frac{\partial f_\alpha}{\partial z_{\beta,n}} \frac{\partial z_{\beta,n}}{\partial z_{\alpha,1}}$$

$$(z_{\alpha,1}, z_{\alpha,2}, z_{\alpha,3}) \xrightarrow{\varphi_\beta \circ \varphi_\alpha^{-1}} (z_{\beta,1}, z_{\beta,2}, z_{\beta,3})$$

$$(0, z_{\alpha,2}, z_{\alpha,3}) \longmapsto (0, z_{\beta,2}, z_{\beta,3})$$

$$\text{At } \{0, z_{\alpha,2}, z_{\alpha,3}\}, \quad \frac{\partial z_{\beta,2}}{\partial z_{\alpha,1}} =$$

So far Nonsense