$$\Rightarrow \quad {}'E_2^{p,0} = \frac{\overset{\nearrow \ker\partial \cap \ker\bar\partial}{\ker\partial} + \operatorname{im}\bar\partial}{\operatorname{im}\bar\partial} \Big/ \frac{\operatorname{im}\bar\partial + \operatorname{im}\partial}{\operatorname{im}\bar\partial} = \frac{\ker\partial + \operatorname{im}\bar\partial}{\operatorname{im}\bar\partial + \operatorname{im}\partial}$$

$$H_{\bar\partial}^{p-1,0}(M) \xrightarrow{\partial} \underset{\substack{\| \\ \frac{\ker\bar\partial}{\operatorname{Im}\bar\partial}}}{H_{\bar\partial}^{p,0}(M)} \xrightarrow{\partial} H_{\bar\partial}^{p+1,0}(M)$$

$$\Rightarrow \quad d(\ker\partial + \operatorname{im}\bar\partial) = d(\ker\partial \cap \ker\bar\partial + \operatorname{im}\bar\partial)$$
$$= \partial(\ker\partial \cap \ker\bar\partial + \operatorname{im}\bar\partial)$$
$$= \partial(\operatorname{im}\bar\partial) \quad \subset \quad \operatorname{im}\bar\partial \quad \text{by} \quad \partial\bar\partial + \bar\partial\partial = 0.$$

$\Rightarrow {}'E_2^{p-2,\overset{0}{1}} \to E_2^{p,0} \xrightarrow{d} {}'E_2^{p+2,\,0-2+1}$ and d is zero-map.

$\Rightarrow \quad \ker d = E_2^{p,0}$

$\Rightarrow \quad \operatorname{im} d = 0 \qquad \Rightarrow \quad {}'E_3^{p,0} = {}'E_2^{p,0}$

$\Rightarrow \quad {}'E_2^{p,0} \cong {}'E_3^{p,0} \cong \cdots \cong {}'E_\infty^{p,0}$

Thus $\quad {}'E_2 \cong {}'E_\infty$.

Note that $\quad {}'E_2^{p+2,-1} = 0$, for to get this, we need ${}'E_1^{p+2,-1}$ which is equal to $H_{\bar\partial}^{p+2,-1}(M) = 0$. $\lrcorner$

What this implies is

$$H_{DR}^*(M) \cong H_{DR}^*(M, \text{hol}),$$

where the right-hand side is the de Rham cohomology computed from the complex of holomorphic forms.

$\ulcorner$ Again by $E_r^{p,q} \cong \dfrac{F^p H_{DR}^{p+q}(M)}{F^{p+1} H_{DR}^{p+q}(M)}$, for sufficiently large r,

$$H_{DR}^q(M) \cong {}'E_\infty^{0,q} \oplus \cdots \oplus {}'E_\infty^{q,0} \cong {}'E_2^{0,q} \oplus \cdots \oplus {}'E_2^{q,0}$$
$$= H_0^{\circ}(H_{\bar\partial}^{0,q}(M)) \oplus \cdots \oplus H_\partial^q(H_{\bar\partial}^{q,0}(M)) \cong H_{DR}^*(M, \text{hol}) \quad \lrcorner$$