수정 된 같았은
93. 7. 23

$$(\sqrt{-1})^{\overset{n-p}{\wedge}}_{(-1)} T_Z(\varphi \wedge \overline{\varphi}) = (\sqrt{-1})^{\overset{n-p}{\wedge}}_{(-1)} \int_{Z^*} \varphi \wedge \overline{\varphi} \quad , \quad \varphi \in A_c^{n-p,0}(M)$$

To $\overset{\text{for}}{\vee}$ T_Z be positive, we need adjustments for the definitions again.

$$T = \sum_{\substack{\#I=p \\ \#J=q}} T_{I\bar{J}}\, dz_I \wedge d\bar{z}_J$$

$$\Phi\, T(\varphi\, dz_{I^\circ} \wedge d\bar{z}_{J^\circ}) = T_{I\bar{J}}(\varphi)\, dz_I \wedge d\bar{z}_J \wedge dz_{I^\circ} \wedge d\bar{z}_{J^\circ}$$
$$= \epsilon(I, J, I^\circ, J^\circ)\, T_{I\bar{J}}(\varphi)\, dz \wedge d\bar{z}$$

I have a better idea! Change the map above as follows:

$$\varphi \longmapsto (-1)^{\frac{(n-p)(n-p-1)}{2}} \int_{Z^*} \varphi \quad , \quad \varphi \in A_c^{n-p,n-p}(M)$$

$$\Rightarrow \quad T(\varphi) = (-1)^{\frac{(n-p)(n-p-1)}{2}} \int_{Z^*} \varphi . \quad \Rightarrow \quad T \text{ is real}$$

To show T is positive,

$$(\sqrt{-1})^{n-p}\, T(\varphi \wedge \overline{\varphi}) \overset{?}{\geq} 0 \quad \text{for } \varphi \in A_c^{n-p,0}(M).$$

$$\Rightarrow \quad (\sqrt{-1})^{n-p} (-1)^{\frac{(n-p)(n-p-1)}{2}} \int_{Z^*} (\varphi \wedge \overline{\varphi}) = \text{volume}(Z^*) \times$$

some constant ≥ 0 by P32 Proposition.

By the note, for real $T \in \mathcal{D}^{p,p}(M)$,

$$dT = 0 \iff \bar{\partial} T = 0 .$$

$$(\bar{\partial} T)(\varphi) = (-1)\, T(\bar{\partial}\varphi) = (\pm) \int_{Z^*} \bar{\partial}\varphi \underset{\text{by } \int_{Z^*} \partial\varphi = 0}{=} \pm \int_{Z^*} d\varphi$$

$\varphi \in A_c^{n-p,n-p-1}$

$= 0$ by P33 Stokes' Theorem for Analytic Varieties.