Claim : $\varinjlim_{\underline{\mathcal{U}}} H_\delta ( H(C^p(\underline{\mathcal{U}}, K^q))) = H^p(X, \mathcal{H}^q)$
$= \varinjlim_{\underline{\mathcal{U}}} H^p(\underline{\mathcal{U}}, \mathcal{H}^q)$

pf) $$H_\delta(H(C^p(\underline{\mathcal{U}}, K^q))) = \frac{\ker \delta}{\operatorname{im} \delta}$$

where
$$\begin{array}{ccc} H(C^p(\underline{\mathcal{U}}, K^q)) & \xrightarrow{\delta} & H(C^{p+1}(\underline{\mathcal{U}}, K^q)) \\ \| & & \| \\ \dfrac{\ker d^{p,q}}{\operatorname{im} d^{p,q-1}} & & \dfrac{\ker d^{p+1,q}}{\operatorname{im} d^{p+1,q-1}} \end{array}$$

$$C^p(\underline{\mathcal{U}}, K^{q-1}) \xrightarrow{d^{p,q-1}} C^p(\underline{\mathcal{U}}, K^q) \xrightarrow{d^{p,q}} C^p(\underline{\mathcal{U}}, K^{q+1}) \xrightarrow{d^{p,q+1}} C^p(\underline{\mathcal{U}}, K^{q+2}) \to$$
$$C^{p+1}(\underline{\mathcal{U}}, K^{q-1}) \xrightarrow{d^{p+1,q-1}} C^{p+1}(\underline{\mathcal{U}}, K^q) \xrightarrow{d^{p+1,q}} C^{p+1}(\underline{\mathcal{U}}, K^{q+1})$$

$\Rightarrow$ Any element in $\varinjlim_{\underline{\mathcal{U}}} H_\delta( H(C^p(\underline{\mathcal{U}}, K^q))$ is expressed

as $$\sigma + \operatorname{im} d^{p,q-1} + \frac{\delta(\ker d^{p-1,q}) + \operatorname{im} d^{p,q-1}}{\operatorname{im} d^{p,q-1}},$$

$\sigma \in \ker d^{p,q}$. s.t $\delta\sigma \in \operatorname{im} d^{p+1,q-1}$.

$$\begin{array}{ccccc} H(C^{p-1}(\underline{\mathcal{U}}, K^q)) & \xrightarrow{\delta} & H(C^p(\underline{\mathcal{U}}, K^q)) & \xrightarrow{\delta} & H(C^{p+1}(\underline{\mathcal{U}}, K^q)) \\ \| & & \| & & \| \\ \dfrac{\ker d^{p-1,q}}{\operatorname{im} d^{p-1,q-1}} & & \dfrac{\ker d^{p,q}}{\operatorname{im} d^{p,q-1}} & & \dfrac{\ker d^{p+1,q}}{\operatorname{im} d^{p+1,q-1}} \end{array}$$