of points interior to U.

For $p_t \neq 0$, $f(p_t) \neq 0$.

∵ By assumption, $f^{-1}(0) = \{0\}$. $\Rightarrow$ If $p_t \neq 0$, then $f(p_t) \neq 0$.
Since $g_t(p_t) = 0 = A_t(p_t) \cdot f(p_t)$, $\det A_t(p_t) = 0$ ⏎

Suppose $f_1(p_t) \neq 0$ and denote by $A_{i,j}$ the i,jth minor of A. Then by Laplace's expansion of the determinant, for z near p_t.

$$\det A_t(z) = \sum_j (-1)^j A_{tj,1}(z)\, a_{tj,1}(z)$$

$$= \frac{1}{f_1(z)} \Big(\sum_{i,j} (-1)^j A_{tj,1}\, a_{tj,i}\, f_i(z) \Big),$$

since $\sum_j A_{tj,1} a_{tj,i} = 0$ for $i \neq 1$,

$$= \frac{1}{f_1(z)} \Big(\sum_j (-1)^j A_{tj,1}\, g_{t,j}(z) \Big).$$

∵ For z near p_t, $f_1(z) = 0$ by continuity of f_1

$$\det A_t(z) = \det \begin{pmatrix} (a_t)_{11}(z) & \cdots & (a_t)_{1n}(z) \\ (a_t)_{21} & \cdots & (a_t)_{2n} \\ \vdots & & \\ (a_t)_{n1} & \cdots & (a_t)_{nn} \end{pmatrix} = \sum_j (-1)^j (A_t)_{j,1}(z)\, (a_t)_{j,1}(z)$$

$$= \frac{1}{f_1(z)} \sum_j (-1)^j (A_t)_{j,1}(z)\, (a_t)_{j,1}(z)\, f_1(z) + \frac{1}{f_1(z)} \sum_{j,\, i\neq 1} (-1)^j (A_t)_{j,1}(z)$$

$(a_t)_{j,i}(z)\, f_i(z)$, since $\underset{\text{for fixed } i \neq 1}{\sum_j} (-1)^j (A_t)_{j,1}(z)\, (a_t)_{j,i}(z)\, f_i(z) =$