$$\left\{\begin{matrix}\text{equivalent classes}\\ \text{of extensions}\end{matrix}\right\} \underset{\psi}{\overset{\Phi}{\rightleftarrows}} \operatorname{Ext}^1_{\Theta}(M,N)$$

$$[\,0 \to N \xrightarrow{\alpha} E \xrightarrow{\beta} M \to 0\,] \longmapsto \partial(1_M)$$

What is $\psi(\partial(1_M))$?

Let $\to E_2 \xrightarrow{\partial} E_1 \xrightarrow{\partial} E_0 \xrightarrow{\eta} M \to 0$ be a projective resolution for M.

$\Rightarrow$

```latex
\begin{tikzcd}
0 \arrow[r] & \mathrm{Hom}(M,N) \arrow[r] \arrow[d] & \mathrm{Hom}(M,E) \arrow[r] \arrow[d] & \mathrm{Hom}(M,M) \arrow[r] \arrow[d] & {} \\
0 \arrow[r] & \mathrm{Hom}(E_0,N) \arrow[r] \arrow[d] & \mathrm{Hom}(E_0,E) \arrow[r] \arrow[d, "\delta"'] & \mathrm{Hom}(E_0,M) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathrm{Hom}(E_1,N) \arrow[r] \arrow[d] & \mathrm{Hom}(E_1,E) \arrow[r] \arrow[d] & \mathrm{Hom}(E_1,M) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathrm{Hom}(E_2,N) \arrow[r] & \mathrm{Hom}(E_2,E) \arrow[r] & \mathrm{Hom}(E_2,M) \arrow[r] & 0
\end{tikzcd}
```

$1_M \mapsto \eta$, $\gamma \mapsto \eta$, $\gamma \mapsto \delta\gamma$, $\eta \mapsto 0$, $\omega \mapsto \delta\gamma$, $\omega = \partial(1_M)$

```latex
\begin{tikzcd}
 & & E_0 \arrow[d, dashed, "\gamma"'] \arrow[dr, "\eta"] & & \\
0 \arrow[r] & N \arrow[r, "\alpha"] & E \arrow[r, "\beta"] & M \arrow[r] & 0
\end{tikzcd}
```

$\Rightarrow \eta = \beta \circ \gamma$

```latex
\begin{tikzcd}
E_1 \arrow[r, "\delta(\gamma)"] \arrow[d, "\omega"'] & E \\
N \arrow[ur, "\alpha"'] &
\end{tikzcd}
```

$\Rightarrow \alpha \circ \omega = \delta(\gamma)$