$\Rightarrow$ By P40.~P41 Cole (Theory of Ordinary Differential Equation), $f(\exp(tv))$ is differentiable in t. & v.

$\Rightarrow$ $e_1' = f_1(\exp v)\, e_1 + f_2(\exp v)\, e_2 \circ \exp$ is a section over $\exp^{-1}(\mathcal{U})$, which is an extension of e_1 at z_0.

Similarly, $e_2' = g_1 \circ \exp v\; e_1 + g_2(\exp v)\, e_2 \circ \exp$ is an extension of e_2 at z_0

Obviously $\nabla e_1'(z_0) = \nabla e_2'(z_0) = 0.$ $\lrcorner$

Γ_{11} Comment on notations.

$$e(\exp tv) = f_1(\exp tv)\, e_1(\exp tv) + f_2(\exp tv)\, e_2(\exp tv)$$

$$e(\exp v) = f_1(\exp v)\, e_1(\exp v) + f_2(\exp v)\, e_2(\exp v)$$

$$T_z M \xrightarrow{\exp} \mathcal{U} \subset M$$

$$\underset{\substack{\alpha(t) \\ \shortparallel \\ \exp tw}}{\cup} \qquad \alpha'(0) = w.$$

$$\nabla_w e = df_1(w) \otimes e_1 + f_1 \nabla_w e_1 + df_2(w) \otimes e_2 + f_2 \nabla_w e_2$$

$$\left(df_1(w) = \frac{d}{dt}\Big|_{t_0} f_1(\exp tw) \right) + \Theta_{11}(\exp tw)\Big|_{t_0} f_1(\exp 0w) + \Theta_{12}(w) f_1(z_0) = 0$$

$$\& \quad df_2 + \Theta_{21} f_1 + \Theta_{22} f_2 = 0 \text{ at } z_0. \text{ ''} \quad))$$

We will use the representation of Cech cohomology by harmonic forms to prove our first main result on the cohomology of vector bundles, the