$$H^{0,p}_{\bar\partial}(M, L^{-\mu}\otimes E) \cong H^p(M, \mathcal{O}(L^{-\mu}\otimes E)) = 0$$

for $\mu \geq \mu_0$, $p<n$.

$\ulcorner$
$$H^q(M, \mathcal{O}(L^\mu\otimes E)) = H^q(M, \Omega^0(L^\mu\otimes E))$$
$$= H^{n-q}(M, \Omega^n((L^\mu\otimes E)^*)) = H^{n-q}(M, \Omega^n((L^\mu)^*\otimes E^*))$$
$$= H^{n-q}(M, \Omega^n(L^{-\mu}\otimes E^*)) = H^{n-q}(M, \Omega^0(K_M\otimes L^{-\mu}\otimes E^*))$$
$$= H^{n-q}(M, \mathcal{O}(L^{-\mu}\otimes E^*\otimes K_M))$$
$$= H^{0,n-q}_{\bar\partial}(M, L^{-\mu}\otimes E^*\otimes K_M)$$ by Dolbeault isomorphism.

$\Rightarrow$ If $q>0$, $\Leftrightarrow$ $n-q<n$, we have only to show that $\exists$ μ_0 s.t

$$H^{0,n-q}_{\bar\partial}(M, L^{-\mu}\otimes E^*\otimes K_M) = 0.$$

$\Rightarrow$ Let $n-q=p<n$. $E^*\otimes K_M = E$

$$\Rightarrow \quad H^{0,p}_{\bar\partial}(M, L^{-\mu}\otimes E) = H^p(M. \mathcal{O}(L^{-\mu}\otimes E)).$$ $\lrcorner$

Choose a metric in L such that $\omega = \frac{i}{2\pi}\Theta_L$ is positive, where Θ_L is the curvature form associated to the metric; let the metric on M be the one given by ω.

$\ulcorner$ $\omega = \frac{i}{2\pi}\Theta_L$ is positive $\Rightarrow$ $\omega = \frac{i}{2}\sum h_{ij}(z)\, dz_i\wedge d\bar z_j$.

$\Rightarrow$ Define a hermitian metric ds^2 by $\sum h_{ij}(z)\, dz_i \otimes d\bar z_j$