

We may choose p_1 so that $\omega_1(p_1) \neq 0$ and then, subtracting a multiple of ω_1 from $\omega_2, \dots, \omega_g$, we may arrange that $\omega_2(p_1) = \dots = \omega_g(p_1) = 0$. Next, we may choose p_2 so that $\omega_2(p_2) \neq 0$, and then arrange as before that $\omega_3(p_2) = \dots = \omega_g(p_2) = 0$. Continuing in this way, the Jacobian matrix at D will be triangular with zeros below the diagonal and nonzero on the diagonal, and so has maximal rank at D .

$$\begin{aligned} \Gamma \quad \omega_1(p_2) = 0 &\Rightarrow \omega_1 = h(z) dz \Rightarrow \omega_1(p_2) = h(p_2) dz \\ &= 0 \Rightarrow h(p_2) = 0. \quad \text{If } \omega_1(p_1) \neq 0, h(p_1) \neq 0. \end{aligned}$$

$$\omega_1/dz_1 \neq 0 \text{ at } p_1 \quad \omega_2/dz_1 = 0 \text{ at } p_1 \quad \omega_1/dz_2 = 0(?) \text{ at } p_2 \quad \Rightarrow$$

Thus the map $\mu^{(g)}$ is not everywhere singular, and the Jacobian inversion theorem follows from the fact that any holomorphic map $f: M \rightarrow N$ between compact connected equidimensional complex manifolds is surjective if $|J(f)| \neq 0$. This follows immediately from the proper mapping theorem: $f(M) \subset N$ is an analytic subvariety and contains an open set, hence $f(M) = N$.

Γ Since $|J(f)| \neq 0$, $f(M)$ contains an open set by the inverse function theorem. See P34 for the proper mapping theorem. \Rightarrow

For a more elementary argument, let ω_N be a volume form on N . Since f is orientation preserving and $|J(f)| \neq 0$,