

in $\sigma(q', h) \Rightarrow h$ contains q' , P_L and $P_{L'} \Rightarrow h = h_L$, since we assumed implicitly $\overline{P_L, P_{L'}} \not\ni q'$.
 $(\because q' \in \overline{P_L, P_{L'}} \Rightarrow \overline{P_L, q'} = \overline{P_{L'}, q'})$

$\Rightarrow \sigma(q', h_L)$ special, and $\sigma(q', h)$ must be L or L' since L & L' are the possible lines in h_L . Again, contradiction.

② $\overline{P_L, q'}$ singular

$\Rightarrow h_1 \cap h_2$ is the only singular line in $\sigma(q') \Rightarrow h_1 \cap h_2 = \overline{P_L, q'} = \overline{P_{L'}, q'}$.

Thus, by ① & ②, $\overline{P_L, q'} = \overline{P_{L'}, q'}$.

If $q' \in R$, $\sigma(q') \cap X = \sigma(q', h)$ is a multiple component of $T_x(X) \cap X$ for all $x \in \sigma(q', h)$. Again L or L' must be special \Rightarrow Contradiction.

in case $x = \overline{P_L, q'}$ or $\overline{P_{L'}, q'}$ \Rightarrow

Thus, in general, if $h \in S^*$ is any hyperplane and L, L' the two pencils of X in h , h is tangent to S at the focus of the two confocal pencils of X containing the singular line $\overline{P_L, P_{L'}}$.

① $h \in R^* \Rightarrow \sigma(h) \cap X = \sigma(P_L, h)$, $P_L = P_{L'}$

By P174, h is tangent to S at every point of $h \cap S$.

$\overline{P_L, P_{L'}} \subset h$ and $P_L, P_{L'} \in \sigma(q, h_1) \cap \sigma(q, h_2)$ where $\sigma(q) \cap X = \sigma(q, h_1) \cup \sigma(q, h_2)$ by P168, $q \in h \cap S$

$\Rightarrow h$ is tangent to S at q . By P168, for any $x \in L$, x lies in the two confocal pencils. Say, $x \in \sigma(q, h_1) \cap \sigma(q, h_2)$.
 $\Rightarrow q \in h_1 \cap h_2 = x \Rightarrow q \in L \cap \sigma(P_L, h) \Rightarrow q \in h \Rightarrow q \in h \cap S$