Here $\mathcal{U}$ is an open set containg more than one point in P_0. In other words, we may not define a map from $\mathcal{O}(L)$ to $\mathcal{J}_{P_0}(L)$.

$$\begin{tikzcd}
 & \mathcal{O}(L) \arrow[d, dashed, "?"'] \arrow[dr] & & \\
0 \arrow[r] & \mathcal{J}(L) \arrow[r] & \mathcal{J}_{P_0}(L) \arrow[r] & \mathcal{O}(L)/\mathcal{J}(L) \arrow[r] & 0
\end{tikzcd}$$

Anyway, as the authords claimed on P 705 ~ P706, they can remedy this subtle problem, by tensoring $\mathcal{L}^k$. So I am comfortable, and let's return to the original problem. Believe that I am right roughly on the maps defined on note P678. So, by taking Hom functor, we have

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\
0 \arrow[r] & \mathrm{Hom}(\mathcal{O}(L)/\mathcal{J}_P(L), \Omega^2) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{J}_{P_0}(L), \Omega^2) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{J}(L), \Omega^2) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathrm{Hom}(\mathcal{O}(L), \Omega^2) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}\oplus\mathcal{O}\oplus\mathcal{O}(L), \Omega^2) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}\oplus\mathcal{O}, \Omega^2) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathrm{Hom}(\mathcal{O}\oplus\mathcal{O}, \Omega^2) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}(L^*)\oplus\mathcal{O}\oplus\mathcal{O}, \Omega^2) \arrow[r] \arrow[d, "\phi"'] & \mathrm{Hom}(\mathcal{O}(L^*), \Omega^2) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathrm{Hom}(\mathcal{O}(L^*), \Omega^2) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}(L^*), \Omega^2) \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & 0 \\
 & 0 & 0 & 0 &
\end{tikzcd}$$

0-stage, 1-stage, 2-stage

$\psi' \mapsto \psi$, $\psi'(0, h_1, h_2) = 0$, $-\psi \mapsto -\psi$

JONG IE WAK YUN PILL