$$(-1)\frac{1}{z^{n-r}}\frac{1}{z^2}\sum_{\#I=r}\prod_{k\in I}\left(\alpha_k-\frac{1}{z}\right)\frac{z^{n+1}}{\prod_{k=0}^{n}(\alpha_k z-1)}dz$$

$$=(-1)\frac{1}{z^{n-r+2}}\sum_{\#I=r}\prod_{k\in I}(z\alpha_k-1)\frac{z^{n+1}}{\prod_{k=0}^{n}(\alpha_k z-1)\,z^r}dz$$

$$=-\frac{1}{z}\sum_{\#I=r}\prod_{k\in I}(\alpha_k z-1)\frac{1}{\prod_{k=0}^{n}(\alpha_k z-1)}dz.$$

$$=-\frac{1}{z}\sum_{\#J=n-r+1}\frac{1}{\prod_{k\in J}(\alpha_k z-1)}dz$$

$$\Rightarrow\int_{\beta_e(0)}-\frac{1}{z}\sum_{\#J=n-r+1}\frac{1}{\prod_{k\in J}(\alpha_k z-1)}dz=Res_\infty(\varphi)$$

$$=(-1)\quad(-1)^{n-r+1}{}_{n+1}C_{n-r+1}=(-1)^{n-r}{}_{n+1}C_r=(-1)^{n-r}\binom{n+1}{r}.$$

$$\Rightarrow\quad Res_\infty(\varphi)=(-1)^{n-r}\binom{n+1}{r}=\sum_{i=0}^{n}\frac{\alpha_i^{n-r}\sum_{\#I=r,\,i\notin I}\prod_{k\in I}(\alpha_k-\alpha_i)}{\prod_{j\neq i}(\alpha_j-\alpha_i)}$$

by the residue theorem again.

$$\Rightarrow\quad c_1(\mathbb{P}^n)^{n-r}\cdot c_r(\mathbb{P}^n)=(-1)^{n-r}(n+1)^{n-r}(-1)^{n-r}\binom{n+1}{r}$$

$$={}_{n+1}C_r\,(n+1)^{n-r}={}_{n+1}C_r\,(n+1)^{n-r}$$

$$={}_{n+1}C_r\,(n+1)^{n-r}$$

Let $c_r(\mathbb{P}^n)=b\,\omega^r.\quad\Rightarrow c_1(\mathbb{P}^n)^{n-r}\cdot c_r(\mathbb{P}^n)=$
$(n+1)^{n-r}\omega^{n-r}\cdot b\,\omega^r=(n+1)^{n-r}b\,\omega^n.={}_{n+1}C_r\,(n+1)^{n-r}\omega^n$
$\Rightarrow\quad(n+1)^{n-r}b=\binom{n+1}{r}(n+1)^{n-r}\quad\Rightarrow b=\binom{n+1}{r}.$ ⤶