

On the other hand, given $D = \sum_{V_i} a_i V_i$,

we can find an open cover $\{U_\alpha\}$ of M s.t. in each U_α , every V_i appearing in D has a local defining function. $g_{i,\alpha} \in \mathcal{O}(U_\alpha)$.

For $\text{ord}_V(f_\alpha) = \text{ord}_V(f_\beta)$,

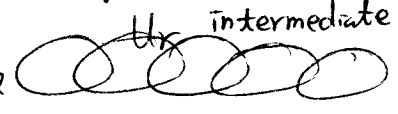
suppose $U_\alpha \cap V \neq \emptyset \neq U_\beta \cap V$.

Since V is connected, $V \subset \bigcup U_r$ and, $U_r \cap V \neq \emptyset$.

For any r s.t. $U_r \cap V \neq \emptyset$, $\exists \beta$ s.t. $U_r \cap U_\beta \neq \emptyset$ and $U_\beta \cap V \neq \emptyset$.

If $U_\alpha \cap U_\beta \neq \emptyset$, clearly $\text{ord}_V(f_\alpha) = \text{ord}_V(f_\beta)$ since

$$\frac{f_\alpha}{f_\beta} \in \mathcal{O}^*(U_\alpha \cap U_\beta).$$

If $U_\alpha \cap U_\beta = \emptyset$, \exists U_r 's s.t. $U_\alpha \cap U_r \neq \emptyset$ and $U_\beta \cap U_r \neq \emptyset$, and $U_\alpha \cap U_r \cap U_\beta \neq \emptyset$ (intermediate U_r 's). imagine.  \square

Since each V_i is closed in M , \exists an open cover $\{U_{i,\alpha}\}$ s.t. V_i has a local defining function $g_{i,\alpha} \in \mathcal{O}(U_{i,\alpha})$.

Since D is locally finite formal combination, \exists $\{O_\beta\}$ open covering of M s.t. each O_β meets only finite number of V_i 's.

Consider $\{U_{i,\alpha} \cap O_\beta\}$ which ^{is} open covering of M , since each O_β meets