Given a continuous linear functional λ on H_s,

let $\lambda(e^{i\langle\xi,x\rangle}) = \overline{\lambda_\xi}$ and $u = \sum \lambda_\xi e^{i\langle\xi,x\rangle}$.

$\Rightarrow$ We have to show that $u \in H_{-s}$.

$$\lambda(v) = \lambda\left(\sum v_\xi e^{i\langle\xi,x\rangle}\right) = \sum v_\xi \overline{\lambda_\xi}$$

$$= \sum v_\xi (1+\|\xi\|^2)^{\frac{s}{2}} (1+\|\xi\|^2)^{-\frac{s}{2}} \overline{\lambda_\xi} = \sum (1+\|\xi\|^2)^{\frac{s}{2}} v_\xi (1+\|\xi\|^2)^{-\frac{s}{2}} \overline{\lambda_\xi}.$$

$\Rightarrow$ Let $v_\xi' = (1+\|\xi\|^2)^{\frac{s}{2}} v_\xi$. $\overline{\lambda_\xi'} = (1+\|\xi\|^2)^{-\frac{s}{2}} \overline{\lambda_\xi}$.

$\Rightarrow$ $v_\xi' \in \mathbb{C}$. $\lambda_\xi' \in \mathbb{C}$.

$\Rightarrow$ $(v_\xi') = v'$ $(\lambda_\xi') = \mu'$

$\Rightarrow$ Obviously, v', μ' are elent of $H \subset \mathbb{C} \times \cdots \times \cdots \times$
$= \prod_{\#\text{ of } \mathbb{Z}^n} \mathbb{C}$ which is a Hilber space with inner product

$$\langle v', \mu' \rangle = \sum v_\xi' \overline{\mu_\xi'}.$$

$\Rightarrow$ We can consider λ as a continuous linear fcntl on H.

$\Rightarrow$ By a well-known theorem on Hilber spaces,

$\Rightarrow$ $\lambda(v') = \langle v', w \rangle$ $w \in H$.

$\Rightarrow$ $\langle w, w \rangle < \infty \iff \sum |w_\xi|^2 < \infty$.

$\Rightarrow$ $\langle v', w - \mu' \rangle = 0 \Rightarrow w = \mu'$

$\Rightarrow$ $\langle \mu', \mu' \rangle = \langle w, w \rangle < \infty$.

We can prove $w_\xi = \mu_\xi'$ by choosing $v' = \begin{cases} v_{\xi'}' = 1, & \text{if } \xi' = \xi \\ v_{\xi'}' = 0 & \text{if } \xi' \neq \xi. \end{cases}$ ⟫