

$$\Rightarrow (dz, z) \wedge (d\bar{z}, z) = 0.$$

$$\begin{aligned} (\partial \bar{\partial} \log \|z\|^2)^{n-1} &= \left( \frac{\langle dz, dz \rangle}{\langle z, z \rangle} - \frac{\langle dz, z \rangle \wedge \langle z, d\bar{z} \rangle}{\langle z, z \rangle^2} \right)^{n-1} \\ &= \left( \frac{\langle dz, dz \rangle}{\langle z, z \rangle} \right)^{n-1} + \bigcirc \langle dz, z \rangle + \dots \end{aligned}$$

$$\Rightarrow r = C_n (\partial \log \|z\|^2) \wedge (\partial \bar{\partial} \log \|z\|^2)^{n-1}$$

$$= C_n \frac{\langle dz, z \rangle}{\langle z, z \rangle} \wedge \left( \frac{\langle dz, dz \rangle}{\langle z, z \rangle} \right)^{n-1} + \bigcirc \langle dz, z \rangle + \dots$$

$$= C_n \frac{\langle dz, z \rangle}{\langle z, z \rangle} \wedge \frac{\langle dz, dz \rangle^{n-1}}{\langle z, z \rangle^{n-1}} = C_n \frac{\langle dz, z \rangle \wedge \langle dz, dz \rangle^{n-1}}{\langle z, z \rangle^n}$$

$$= C_n \frac{\langle dz, z \rangle \wedge \langle dz, dz \rangle^{n-1}}{\|z\|^{2n}}$$

The numerator is

$$C_n' \left( \sum_i \bar{z}_i dz_i \right) \wedge \left( \sum_j dz_j \wedge d\bar{z}_j \right)^{n-1}$$

$$= C_n'' \left( \sum (-1)^{i-1} \bar{z}_i d\bar{z}_1 \wedge \dots \wedge \widehat{d\bar{z}_i} \wedge \dots \wedge d\bar{z}_n \wedge dz_1 \wedge \dots \wedge dz_n \right),$$

which implies the result.

$$\begin{aligned} \mathbb{F} \quad C_n' & \left( \bar{z}_1 dz_1 + \bar{z}_2 dz_2 \right) \wedge \left( dz_1 \wedge d\bar{z}_1 + dz_2 \wedge d\bar{z}_2 \right) \\ &= C_n' \left( \bar{z}_2 dz_2 \wedge dz_1 \wedge d\bar{z}_1 + \bar{z}_1 dz_1 \wedge dz_2 \wedge d\bar{z}_2 \right) \\ &= C_n' \left( (-1) \bar{z}_2 dz_1 \wedge dz_2 \wedge d\bar{z}_1 + (-1) \bar{z}_1 dz_1 \wedge dz_2 \wedge d\bar{z}_2 \right) \end{aligned}$$

$$C_n' \left( \bar{z}_1 dz_1 + \bar{z}_2 dz_2 + \dots + \bar{z}_n dz_n \right) \wedge \left( \underbrace{dz_1 \wedge d\bar{z}_1 + dz_2 \wedge d\bar{z}_2 + \dots + dz_n \wedge d\bar{z}_n}_{(dz_i \wedge d\bar{z}_i)} \right)^{n-1}$$

$$= C_n' \bar{z}_i \wedge \widehat{dz_i} (dz_1 \wedge d\bar{z}_1) \wedge (dz_2 \wedge d\bar{z}_2) \wedge \dots \wedge (dz_n \wedge d\bar{z}_n) \cdot n!$$

$$= C_n'' \sum \bar{z}_i dz_i \wedge (dz_1 \wedge d\bar{z}_1) \wedge \dots \wedge \widehat{(dz_i \wedge d\bar{z}_i)} \wedge \dots \wedge (dz_n \wedge d\bar{z}_n)$$