The graph of this map is just the main irreducible component of incidence correspondence $I \subset V^k \times G(k, n+1)$ given by

$$I = \{ (p_1, \cdots p_k ; \Lambda) : p_i \in \Lambda \text{ for all } i \}.$$

$\ulcorner$ $\overbrace{\mathbb{C}^{n+1} \times \cdots \times \mathbb{C}^{n+1}}^{k} \longrightarrow \mathbb{C}$

$\{ (z_{11}, \cdots z_{1,n+1}), \quad \cdots \quad (z_{k1}, \cdots z_{k,n+1}) \} \longmapsto$ det of some $k \times k$ matrix of

$$\begin{pmatrix} z_{11}, & \cdots & z_{1,n+1} \\ \vdots & & \\ z_{k1}, & \cdots & z_{k,n+1} \end{pmatrix}$$

$\Rightarrow$ We have ${}_{n+1}C_k$ number of $k \times k$ minors, i.e., ${}_{n+1}C_k$ linearly independent holomorphic functions on $(\mathbb{C}^{n+1})^k$.

$\Rightarrow$ Let $f_i$'s be the ${}_{n+1}C_k$ linearly independent functions above.

$\Rightarrow$ $V^k - V^k \cap \{ f_1 = \cdots \} \longrightarrow G(k, n+1)$ is holomorphic.

Suppose rank $\begin{pmatrix} \frac{\partial \varphi_1}{\partial z_1} & \frac{\partial \varphi_1}{\partial z_2}, & \frac{\partial \varphi_1}{\partial z_3} \\ \frac{\partial \varphi_2}{\partial z_1}, & \frac{\partial \varphi_2}{\partial z_2}, & \frac{\partial \varphi_2}{\partial z_3} \end{pmatrix} = 2$ almost everywhere.

$\Rightarrow$ $\det \begin{pmatrix} \frac{\partial \varphi_1}{\partial z_1}, & \frac{\partial \varphi_1}{\partial z_2} \\ \frac{\partial \varphi_2}{\partial z_1}, & \frac{\partial \varphi_2}{\partial z_2} \end{pmatrix}$ & $\det \begin{pmatrix} \frac{\partial \varphi_1}{\partial z_1}, & \frac{\partial \varphi_1}{\partial z_3} \\ \frac{\partial \varphi_2}{\partial z_1}, & \frac{\partial \varphi_2}{\partial z_3} \end{pmatrix}$ are linearly

independent. So what !!!

Do not think complicatedly?

$$g : M^* \longrightarrow G(k+1, n+1) \longrightarrow \mathbb{P}(\Lambda^{k+1} \mathbb{C}^{n+1})$$