

Writing out expansions around  $p$ , we have

$$F'(z)^2 = \frac{1}{z^6} + \frac{C}{z^2} + [-1]$$

and  $F(z)^3 = \frac{1}{z^6} + \frac{C'}{z^3} + \frac{C''}{z^2} + [-1],$

so that the meromorphic function

$$F'(z)^2 + C' F'(z) - F(z)^3 + (C'' - C) F(z)$$

is holomorphic away from  $p$ , with at most a single pole at  $p$ , hence equal to a constant.

$$\Gamma \quad F'(z) = \frac{1}{z^3} + [1] = \frac{1}{z^3} + a_1 z + a_2 z^2 + \dots$$

$$\begin{aligned} F'(z)^2 &= \frac{1}{z^6} + \frac{2a_1}{z^2} + \frac{2a_2}{z} + \dots \\ &= \frac{1}{z^6} + \frac{2a_1}{z^2} + \frac{1}{z}(2a_2 + \dots) \\ &= \frac{1}{z^6} + \frac{C}{z^2} + [-1] \end{aligned}$$

$$\begin{aligned} F(z) &= \frac{1}{z^2} + [1] = \frac{1}{z^2} + b_1 z + b_2 z^2 + \dots \\ &= \frac{1}{z^2} + z g(z) \end{aligned}$$

$$\begin{aligned} F(z)^3 &= \left( \frac{1}{z^2} + z g(z) \right)^3 = \frac{1}{z^6} + 3 \frac{g(z)}{z^3} + 3 g(z) + z^3 g^3(z) \\ z^3 g^3(z) &= \frac{1}{z^6} + \frac{C'}{z^3} + \frac{C''}{z^2} + \frac{1}{z}(c_0 + c_1 z + \dots) \\ &= \frac{1}{z^6} + \frac{C'}{z^3} + \frac{C''}{z^2} + [-1] \end{aligned}$$

$$\Rightarrow F'(z)^2 + C' F'(z) - F(z)^3 + (C'' - C) F(z) =$$