Moreover,

$$\omega = \frac{\sum \bar{v}^i dz_i}{\sum v^i \bar{v}^i} = \frac{\langle dz, v\rangle}{\langle v, v\rangle},$$

so

$$\bar{\partial}\omega = -\frac{\langle dz, dv\rangle}{\langle v, v\rangle} + \frac{\langle dz, v\rangle \wedge \langle v, dv\rangle}{\langle v, v\rangle^2}$$

Pf

$$\omega\left(v^j \frac{\partial}{\partial z_j}\right) = \frac{1}{\sum v^i \bar{v}^i} \sum \bar{v}^i v^i = 1.$$

$$\langle dz_i, dz_j\rangle = \delta_{ij}$$

$$\bar{\partial}\omega = -\frac{\underbrace{\langle dz, dv\rangle}_{= -\bar{\partial}\langle dz, v\rangle}}{\langle v, v\rangle} + \frac{\langle dz, v\rangle \wedge \underbrace{\bar{\partial}\langle v, v\rangle}_{= \langle v, \partial v\rangle,\ \partial v = dv}}{\langle v, v\rangle^2}. \quad \text{since } \bar{\partial} v = 0.$$

$\lrcorner$

Thus, since $\langle dz, v\rangle \wedge \langle dz, v\rangle = 0$,

$$\left(\frac{\sqrt{-1}}{2\pi}\right)^n \omega \wedge (\bar{\partial}\omega)^{n-1} = (-1)^{n-1}\left(\frac{\sqrt{-1}}{2\pi}\right)^n \frac{\langle dz, v\rangle \wedge \langle dz, dv\rangle^{n-1}}{\langle v, v\rangle^n}$$

$$= -C_n \frac{\sum_i (-1)^{i-1} \bar{v}^i d\bar{v}^1 \wedge \cdots \wedge \widehat{d\bar{v}^i} \wedge \cdots \wedge d\bar{v}^n \wedge dz_1 \wedge \cdots \wedge dz_n}{\langle v, v\rangle^n},$$

where C_n is the constant appearing in the Bochner–Martinelli formula from Section 1 of this chapter,

$$= -\frac{1}{\det A_p} \beta(v, \bar{v}),$$

where β is the form appearing in that formula.