

Thus ^{locally} we can lift $-\frac{3}{2}\alpha$ to $U \cap Y'$ and U . By the partition of unity, we have a lifted vector field v and v' on X' and Y' respectively. Let φ_t be the flow of v . $\Rightarrow \varphi_t|_{Y'}$ is the flow of v' . \Rightarrow By the argument above, $\varphi_1: S \rightarrow S_0$ is diffeomorphism, and $\varphi_1|_H: S \cap H \rightarrow S_0 \cap H$ is diffeomorphism, too.

The application of the argument above to hypersurfaces of degree d in \mathbb{P}^n is clearly plausible. \square

Now by the adjunction formula applied to $S \subset \mathbb{P}^3$,

$$\begin{aligned} K_S &= (K_{\mathbb{P}^3} + S)|_S \\ &= -H|_S \end{aligned}$$

and similarly $K_{S_0} = -H|_{S_0}$.

$$\begin{aligned} \text{By P147, } K_S &= (K_{\mathbb{P}^3} \otimes [S])|_S \\ &= ([-4H] \otimes [3H])|_S \text{ by P146} \\ &= [-H]|_S. \end{aligned} \quad \square$$

Since our diffeomorphism $\varphi: S \rightarrow S_0$ carries $S \cap H$ to $S_0 \cap H$, we deduce that

$$c_1(K_S) = \varphi^* c_1(K_{S_0}).$$

$$\begin{array}{ccc} i^*[H] & \longrightarrow & [H] \\ \downarrow & & \downarrow \\ S & \xrightarrow{i} & \mathbb{P}^3 \end{array} \quad \Rightarrow \quad i^*[H] = [S \cap H] = [H]|_S = -K_S = K_S^*$$

\Rightarrow Similarly, $i^*[H] = -K_{S_0} = K_{S_0}^*$.