$$\begin{tikzcd} E \arrow[r, "\tilde{\iota}_V"] \arrow[d] & Q \arrow[d] \\ M \arrow[u, bend left, dashed, "\sigma"] \arrow[r] & G(n-k.V) \arrow[u, bend right, dashed, "\tau"'] \\[-20pt] x \arrow[r, mapsto] & \Lambda_x \end{tikzcd}$$

$\tilde{\iota}_V(\sigma(x)) = \sigma' + \Lambda_x$, $\quad \phi(\sigma') = \sigma(x) = \sigma'(x)$.

$\Rightarrow \sigma' + \Lambda_x = \sigma + \Lambda_x = \tilde{\iota}_V(\sigma(x)). = \tau(\Lambda_x)$.

$\Rightarrow \sigma = \iota_V^* \tau$.

Given a section $\tau : G(n-k.V) \longrightarrow Q$, define $\sigma : M \longrightarrow E$ by $\sigma(x)$ as follows:

$$\tilde{\iota}_V(\sigma(x)) = \tau(\Lambda_x)$$

We have to check if σ is well-defined.

If $\tilde{\iota}_V(v) = \Lambda_x$, $\Rightarrow \sigma'(x) = v$.

$\Rightarrow \sigma' \in \Lambda_x \Rightarrow \sigma'(x) = v = 0 \Rightarrow \tilde{\iota}_V$ is injective.

$\Rightarrow \sigma$ is well-defined.

$\Rightarrow \iota_V^* \tau = \sigma \overset{?}{\in} V$.

For $\sigma(x_0) \in E_{x_0} \Rightarrow$ Since E_{x_0} is spanned by $\{\sigma(x_0)\}_{\sigma \in V}$, $\quad \sigma(x_0) = a_1 \sigma_1(x_0) + \cdots + a_n \sigma_n(x_0)$.

Let $\sigma' = a_1 \sigma_1 + \cdots + a_n \sigma_n$.

Consider $\sigma' - \sigma$. $\Rightarrow \tilde{\iota}_V(\sigma'(x_0) - \sigma(x_0)) = \Lambda_{x_0}$

$\Rightarrow \tilde{\iota}_V(\sigma'(x_0)) = \sigma' + \Lambda_{x_0} = \tilde{\iota}_V(\sigma(x_0)) = \tau(\Lambda_{x_0})$

We can go nowhere, so we have to find some other way.

Let V be spanned by $\{\sigma_1, \sigma_2, \cdots \sigma_n\}$.

Let $e_1, e_2 \cdots e_k$ be a frame over U.

$\Rightarrow$ Over U, $\quad \sigma_1 = a_{11} e_1 + a_{21} e_2 + \cdots + a_{k1} e_k$