$= g - \dim \langle (\omega_1(p_1), \omega_2(p_1)), (\omega_1(p_2), \omega_2(p_2)), \ldots (\omega_1(p_3), \omega_2(p_3)) \rangle$

$=$ dim { vectors in Z_2 perpendicular to $\omega_1(p_i), \omega_2(p_i)$ }

$=$ # of hyperplanes in $\mathbb{P}^1$.

$\Rightarrow$ $h^0(K-D) =$ # of hyperplanes in $\mathbb{P}^{g-1}$ in general.

$\Rightarrow$ $h^0(D) = d - g + 1 + h^0(K-D) = d - \{ g - h^0(K-D) - 1 \}$

$\Rightarrow$ $g - h^0(K-D) - 1 =$ dimension of the linear space spanned by $i_K(p_i)$'s. (projective space) 」

Here, of course, we take the "linear span" of a point p_i with multiplicity a_i in D to be the span of p_i together with the first $a_i - 1$ derivatives of the canonical map.

「 $g = 4$, $1 \le i \le 3$

$$\Rightarrow \quad p_1 \longmapsto \left[\frac{\omega_1}{dz_1}, \frac{\omega_2}{dz_1}, \frac{\omega_3}{dz_1}, \frac{\omega_4}{dz_1} \right] \in \mathbb{P}^3$$

$$p_2 \longmapsto \left[\frac{\omega_1}{dz_2}, \frac{\omega_2}{dz_2}, \frac{\omega_3}{dz_2}, \frac{\omega_4}{dz_2} \right] \in \mathbb{P}^3$$

$$p_3 \longmapsto \left[\frac{\omega_1}{dz_3}, \frac{\omega_2}{dz_3}, \frac{\omega_3}{dz_3}, \frac{\omega_4}{dz_3} \right] \in \mathbb{P}^3.$$

$$\Rightarrow h^0(K-D) = \dim \text{ of } \left\{ v \in \mathbb{C}_4 \;\middle|\; \overline{v} \perp \left(\frac{\omega_1}{dz_1} \cdots \frac{\omega_4}{dz_1}\right) \cdots \overline{v} \perp \left(\frac{\omega_1}{dz_3}, \cdots \frac{\omega_4}{dz_3}\right) \right\} = 4 - \operatorname{rank}\left\{ \left(\frac{\omega_1}{dz_1} \cdots\right) \cdots \left(\frac{\omega_1}{dz_3}, \frac{\omega_4}{dz_3}\right) \right\}$$

$$\overline{(a_1, a_2, a_3, a_4)} \perp \left(\frac{\omega_1}{dz_1} \cdots \frac{\omega_4}{dz_1} \right) \cdots$$

$$\Rightarrow \quad a_1 \frac{\omega_1}{dz_1} + a_2 \frac{\omega_2}{dz_1} + \cdots + a_4 \frac{\omega_4}{dz_1} = 0$$

$$\vdots \qquad\qquad \vdots$$

$$a_1 \frac{\omega_1}{dz_3} + a_2 \frac{\omega_2}{dz_3} + \cdots + a_4 \frac{\omega_4}{dz_3} = 0$$

$$\Rightarrow \quad a_1 \begin{pmatrix} \frac{\omega_1}{dz_1} \\ \vdots \\ \frac{\omega_1}{dz_3} \end{pmatrix} + a_2 \begin{pmatrix} \frac{\omega_2}{dz_1} \\ \vdots \end{pmatrix} + \cdots + a_4 \begin{pmatrix} \frac{\omega_4}{dz_1} \\ \vdots \\ \frac{\omega_4}{dz_3} \end{pmatrix} = 0.$$