The local intersection number is the topological degree $\deg(f)$ of f.

Proof. The form β gives an integer generator of $H^{2n-1}_{DR}(S^{2n-1}(\varepsilon))$ for any sphere $\|w\| = \varepsilon$.

Γ Since $d\beta = 0$ & $\int_{\|x\|=\epsilon} \beta = 1$, β is an integer generator of $H^{2n-1}_{DR}(S^{2n-1}(\epsilon))$. ┘

By definition of the degree,

$$\deg(f) = \int_{\|z\|=\varepsilon} f^*\beta = \int_{\|z\|=\varepsilon} \eta(f_1, \cdots f_n) = (D_1, \cdots, D_n)_{\{0\}}$$

by our basic integral formula.

Finite Holomorphic Mappings

We now want to tie in the local intersection number with the properties of f viewed as a holomorphic mapping $f : U \to \mathbb{C}^n$. For this, the following standard terminology will be useful: On a complex manifold M a zero cycle is a formal finite sum

$$\Gamma = \sum_\nu m_\nu P_\nu$$

of points $P_\nu \in M$ with multiplicities $m_\nu \in \mathbb{Z}$. We set

$$\underbrace{P + P + \cdots + P}_{k} = k \cdot P .$$