on $\tilde{M}$, and by the Kodaira vanishing theorem,

$$H^1(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{L}^k - E)) = H^1(\tilde{M}, \Omega^n_{\tilde{M}}((\tilde{L}^{k_1} + \tilde{K}^*_M) + (\tilde{L}^{k'} - nE))$$
$$= 0 \quad \text{for } k \geq k_0.$$

$\ulcorner$ $k \geq k_0 = k_1 + k_2$ $\quad k = k_1 + k'$ $\quad k' \geq k_2$, where k_1 & k_2 are chosen so that they satisfy the desired conditions. $n = \dim \tilde{M} < 1 + n$ $\Rightarrow$ By the Kodaira vanishing theorem, $H^1(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{L}^k - E)) = 0$. $\lrcorner$

Hence the map

$$r_E : H^0(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{L}^k)) \longrightarrow H^0(E, \mathcal{O}_E(\pi^* L^k))$$

is surjective for $k \geq k_0$, and so assertion 1 is proved for x and y.

$\ulcorner$ From the exact sheaf sequence. $0 \to \mathcal{O}_{\tilde{M}}(\tilde{L}^k - E) \to \mathcal{O}_{\tilde{M}}(\tilde{L}^k) \xrightarrow{r_E} \mathcal{O}_E(\tilde{L}^k) \to 0$.

$$H^0(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{L}^k)) \xrightarrow{r_E} \underset{H^0(E,\ \mathcal{O}_E(\tilde{L}^k)) = H^0(E, \mathcal{O}_E(\pi^* L^k))}{\underset{\parallel}{H^0(\tilde{M}, \mathcal{O}_E(\tilde{L}^k))}} \longrightarrow \underset{0}{\underset{\parallel}{H^1(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{L}^k - E))}} \rightarrow$$

$\Rightarrow$ r_E is surjective. $\lrcorner$

Assertion 2 is proved similarly. Let $\tilde{M} \xrightarrow{\pi} M$ now denote the blow-up of M at x, $E = \pi^{-1}(x)$ the exceptional divisor. Again, the pullback map

$$\pi^* : H^0(M, \mathcal{O}_M(L^k)) \longrightarrow H^0(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{L}^k))$$

is an isomorphism. Further, if $\sigma \in H^0(M, \mathcal{O}_M(L^k))$, then $\sigma(x) = 0 \iff \tilde{\sigma} = \pi^* \sigma$ vanishes on E; thus π^* restricts to give an isomorphism