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$$\omega = f(z,\bar z)\,dz = f'(z',\bar z')\,dz' = f'\frac{\partial z'}{\partial z}dz \quad\Rightarrow\quad f = f'\frac{\partial z'}{\partial z}$$

$\bar\partial\omega = \bar\partial f\wedge dz = \frac{\partial f}{\partial \bar z}d\bar z\wedge dz$ is well-defined, for

$$\bar\partial\omega = \bar\partial f'\wedge dz' = \frac{\partial f'}{\partial\bar z'}d\bar z'\wedge dz' = \frac{\partial f'}{\partial \bar z'}\frac{\partial\bar z'}{\partial\bar z}d\bar z\wedge\frac{\partial z'}{\partial z}dz$$
$$= \frac{\partial f'}{\partial\bar z'}\frac{\partial\bar z'}{\partial\bar z}\frac{\partial z'}{\partial z}\,d\bar z\wedge dz.$$

$\Rightarrow$ Since $\frac{\partial f'}{\partial \bar z'} = \left(\frac{\partial f'}{\partial z},\frac{\partial f'}{\partial\bar z}\right)\cdot\left(\frac{\partial z}{\partial\bar z'},\frac{\partial\bar z}{\partial\bar z'}\right) = \frac{\partial f'}{\partial\bar z}\frac{\partial\bar z}{\partial\bar z'}$

and $\frac{\partial f'}{\partial\bar z} = \left(\frac{\partial f'}{\partial z'},\frac{\partial f'}{\partial\bar z'}\right)\cdot\left(\frac{\partial z'}{\partial\bar z},\frac{\partial\bar z'}{\partial\bar z}\right) = \frac{\partial f'}{\partial\bar z'}\frac{\partial\bar z'}{\partial\bar z}$,

$$\frac{\partial f}{\partial\bar z} = \frac{\partial f'}{\partial\bar z}\frac{\partial z'}{\partial z} = \frac{\partial f'}{\partial\bar z'}\frac{\partial\bar z'}{\partial\bar z}\frac{\partial z'}{\partial z}. \qquad \text{done //}$$

$$U_\beta\cap U_\alpha\times\mathbb{C} \xrightarrow{\varphi_\alpha\circ\varphi_\beta^{-1}} U_\alpha\cap U_\beta\times\mathbb{C}$$
$$\underset{x+iy}{z} \longmapsto \underset{u+iv}{w(z)}$$

$J_{\mathbb{C}}(\varphi_\alpha\circ\varphi_\beta^{-1}) = ?$

$$(\varphi_\alpha\circ\varphi_\beta^{-1})_*\left(\frac{\partial}{\partial z}\right) = a_{11}\frac{\partial}{\partial w} + a_{21}\frac{\partial}{\partial\bar w}$$
$$(\varphi_\alpha\circ\varphi_\beta^{-1})_*\left(\frac{\partial}{\partial\bar z}\right) = a_{12}\frac{\partial}{\partial w} + a_{22}\frac{\partial}{\partial\bar w}$$

$\Rightarrow (\varphi_\alpha\circ\varphi_\beta^{-1})_*\left(\frac{\partial}{\partial z}\right)(w)$

$$= \frac{\partial w(\varphi_\alpha\circ\varphi_\beta^{-1}(z))}{\partial z} = \frac{\partial w}{\partial z} = a_{11} \qquad a_{21} = \frac{\partial\bar w}{\partial z} = 0 = a_{12} = 0$$
$$a_{22} = \frac{\partial\bar w}{\partial\bar z}$$

$J_{\mathbb{C}}(\varphi_\alpha\circ\varphi_\beta^{-1}) = \begin{pmatrix}\frac{\partial w}{\partial z} & 0\\ 0 & \frac{\partial\bar w}{\partial\bar z}\end{pmatrix}$ $\Rightarrow$ w.r.t $\{\frac{\partial}{\partial z},\frac{\partial}{\partial\bar z}\}$ & $\{\frac{\partial}{\partial w},\frac{\partial}{\partial\bar w}\}$, the transition funtion is

$$J_{\mathbb{C}}(\varphi_\alpha\circ\varphi_\beta^{-1}) = \begin{pmatrix}\frac{\partial w}{\partial z} & 0\\ 0 & \frac{\partial\bar w}{\partial\bar z}\end{pmatrix}$$

$\Rightarrow$ Transition funtn for $T'M$ is $\left(\frac{\partial w}{\partial z}\right)$ w.r.t $\{\frac{\partial}{\partial z}\}$ & $\{\frac{\partial}{\partial w}\}$

" $T''M$ is $\left(\frac{\partial\bar w}{\partial\bar z}\right)$ " $\{\frac{\partial}{\partial\bar z}\}$ & $\{\frac{\partial}{\partial\bar w}\}$.

$$J_{\mathbb{R}}(\varphi_\alpha\circ\varphi_\beta^{-1}) = \begin{pmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{pmatrix} \qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$