$\Rightarrow$ We can conclude that every irreducible nondegenerate curve of degree n in $\mathbb{P}^n$ is projectively isomorphic to the rational normal curve. ⌟

2. In terms of Euclidean coordinates $s = Z_1/Z_0$, $t = Z_2/Z_0$ on $\mathbb{P}^2$, the Veronese map $f = \iota_{2H} : \mathbb{P}^2 \longrightarrow \mathbb{P}^5$ is given by

$$(s, t) \longmapsto [1, s, t, s^2, st, t^2].$$

⌜

$$[Z_0, Z_1, Z_2] \longmapsto [Z_0^2, Z_0Z_1, Z_0Z_2, Z_1^2, Z_1Z_2, Z_2^2]$$

$$\left[1, \frac{Z_1}{Z_0}, \frac{Z_2}{Z_0}\right] \longmapsto \left[1, \frac{Z_1}{Z_0}, \frac{Z_2}{Z_0}, \left(\frac{Z_1}{Z_0}\right)^2, \frac{Z_1}{Z_0}\frac{Z_2}{Z_0}, \left(\frac{Z_2}{Z_0}\right)^2\right]$$ ⌟

The image $S = f(\mathbb{P}^2)$ is a nondegenerate surface of degree $c_1(f^* H_{\mathbb{P}^5})^2 = c_1(2H_{\mathbb{P}^2})^2 = 4$; note that this degree is minimal in the sense of the last section.

⌜ Suppose S is degenerate. $\Rightarrow \exists$ a hyperplane $H \subset \mathbb{P}^5$ s.t $S \subset H$. $\Rightarrow \exists\ a_0, a_1 \cdots a_5$ s.t
$a_0 + s \cdot a_1 + a_2 t + a_3 s^2 + a_4 st + a_5 t^2 = 0$
for all s, t.
$\Rightarrow$ This is impossible, since $\{1, s, t, s^2, st, t^2\}$ is linearly independent.

$$\deg S = \#(S \cap \mathbb{P}^3) = \#(f(\mathbb{P}^2) \cap \mathbb{P}^4 \cap \mathbb{P}^4)$$
$$= \#(\mathbb{P}^2 \cap f^{-1}(\mathbb{P}^4) \cap f^{-1}(\mathbb{P}^4)) = \#(f^{-1}(\mathbb{P}^4) \cap f^{-1}(\mathbb{P}^4))$$ see p59

$\Rightarrow$ Since $f^{-1}(\mathbb{P}^4)$ corresponds to $c_1(f^* H_{\mathbb{P}^5})$,
$f^{-1}(\mathbb{P}^4) \cap f^{-1}(\mathbb{P}^4)$ corresponds to $c_1(f^* H_{\mathbb{P}^5})^2 = c_1(f^* H_5) \wedge c_1(f^* H_5)$.