$$\Rightarrow \quad c_1(E\otimes L) = [\,\text{trace } \Theta_{E\otimes L}]$$
$$= [\,tr\,\Theta_E + tr\,\Theta_L \cdot r\,] = c_1(E) + r\,c_1(L).$$

4. Finally for now, if Θ is the curvature matrix of a connection in a complex vector bundle E, then the dual connection in E^* has curvature matrix $-\Theta$; thus
$$c_r(E^*) = (-1)^r c_r(E).$$

「
$$d\,e_i^*(e_j) = (D^* e_i^*)(e_j) + e_i^*(D e_j) = 0$$
$$= \theta^*_{i\alpha} e^*_\alpha(e_j) + e_i^*(\theta_{jk} e_k)$$
$$= \theta^*_{ij} + \theta_{ji}$$
$$\Rightarrow \quad \theta_{ji} = -\theta^*_{ij} \quad \Rightarrow \quad \theta^*_{ij} = -({}^t\theta)_{ij}$$

$$\Rightarrow \quad \Theta^* = d\theta^* - \theta^*\wedge\theta^* = -d\,{}^t\theta - {}^t\theta\wedge{}^t\theta$$
$$= -{}^t(d\theta) + {}^t(\theta\wedge\theta) = -{}^t(d\theta - \theta\wedge\theta) = -{}^t\Theta.$$
$$\Rightarrow \quad c_r(E^*) = (-1)^r c_r(E)$$

We can use the Whitney product formula to evaluate the Chern classes $c_i(\mathbb{P}^n)$ of projective space, as follows. Let $\pi : \mathbb{C}^{n+1} - \{0\} \longrightarrow \mathbb{P}^n$ be the standard projection map; let $X_0, \cdots X_n$ be linear coordinates on $\mathbb{C}^{n+1}$ and
$$x_i = X_i / X_0, \quad i = 1, 2, \cdots n$$
corresponding affine coordinates on $\mathbb{P}^n$. Then we have
$$\pi^* dx_i = \frac{X_0 \cdot dX_i - X_i \cdot dX_0}{X_0^2},$$
and so, at a point $X \in \mathbb{C}^{n+1}$, the image under π of the