$\Rightarrow a_1 \psi_1(P_d) + \cdots + a_{r+1}\psi_{r+1}(P_d) = 0$ if Z satisfies the Cayley - Bacharach property.

Let $V = \langle (\psi_1(P_1), \cdots \psi_{r+1}(P_1)), \cdots (\psi_1(P_{d-1}), \cdots \psi_{r+1}(P_{d-1}))\rangle$

$\Rightarrow \dim V \leq d-1. \Rightarrow \dim V^{\perp} \geq (r+1)-(d-1) = r-d+2.$

$\Rightarrow$ Given $\omega \in V^{\perp}$, $\omega \perp (V \oplus \langle(\psi_1(P_d), \cdots \psi_{r+1}(P_d))\rangle)$ by the Cayley - Bacharach property. $\Rightarrow \dim(V \oplus \langle(\psi_1(P_d), \cdots \psi_{r+1}(P_d))\rangle) \leq d-1$

$\Rightarrow [V] \subset \mathbb{P}^r \Rightarrow \dim [V] = \dim \overline{Z} = d-2+p, \; p \geq 0.$

See P309. Hartshorne for secant.

References

We give a few sources to assist the reader in amplifying the discussion in this chapter. These will also serve as a guide to the literature.

Section 3

L. Hormander, Introduction to Complex Analysis in Several Complex Variables

R. Gunning, Introduction to Holomorphic Functions of Several Variables Vol. II & III.