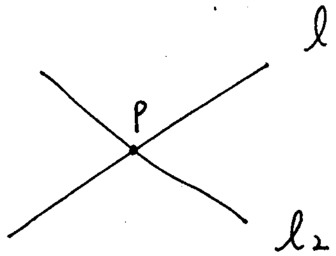
Without reference to blow-ups or cohomology, then, one could make the statement: " the condition that a conic be tangent to a conic C is equivalent to the condition that it contain either of two points, or be tangent to either of two lines"; this can be seen by noting that, as the conic C degenerates into a pair of lines $l_1 + l_2$, the variety V_C degenerates into the variety $I_{l_1} + I_{l_2} + I_{l_1 \cdot l_2}$, the latter component occurring with multiplicity 2.

$\ulcorner$ $C = l_1 + l_2$ Let $p = l_1 \cdot l_2 = l_1 \cap l_2$.



$\Rightarrow$ Clearly, $V_C \supset I_{l_1} \cup I_{l_2} \cup I_{l_1 \cdot l_2}$.

Note that the map $\phi : W \longrightarrow \{V_C\}_{C \in W}$
$C \longmapsto V_C$

is holomorphic, see P853 note.

$\Rightarrow$ Since $\deg V_C = 6$, for C smooth, V_C has degree 6 for C singular.

$\Rightarrow$ Since $I_l \sim 2\omega$ by P874 note, and I_{l_1} & I_{l_2} work in V_C in the same way,

$V_C \sim I_{l_1} + I_{l_2} + m I_p$, $m = 2$ ($\because$ $I_p \sim \omega$