Proof. We let φ be a closed meromorphic 1-form given in a sufficiently small polycylindrical nbd W of a point $x_0 \in M$. The polar divisor of φ is $D = D_1 + \cdots + D_k$, where the D_i are irreducible and are divisors of holomorphic functions $f_i \in \mathcal{O}(W)$.

By P131, $\varphi = \sum \frac{g_\ell}{f_\ell} dz_\ell$. g_ℓ & f_ℓ are relatively prime for each ℓ. $\Rightarrow$ For each ℓ, we have a polar divisor. i.e.,

$= \sum_V \mathrm{ord}_V(f_\ell) \cdot V$, where V is a irreducible hypersurface in W.

By the same argument as above, if $W^* = W - D$, then $H_1(W^*, \mathbb{Z})$ is generated by 1-cycles γ_i consisting of circles turning once around D_i.

$$\begin{array}{ccc} H_1(W^*, \mathbb{Z}) & \longrightarrow & H_1(W, \mathbb{Z}) = 0 \\ \Psi & & \Psi \\ \gamma & \longmapsto & 0 \end{array}$$

$\Rightarrow$ By the argument on P455, $\gamma = \sum m_i \gamma_{D_i}$.

If
$$\lambda_i = \frac{1}{2\pi\sqrt{-1}} \int_{\gamma_i} \varphi,$$
then
$$\varphi - \sum_i \lambda_i \frac{df_i}{f_i} = \psi$$
will have no periods, and consequently
$$g = \int \psi$$
will be a meromorphic function in W with