$$= (-1)^{n-k} \times \det \left(\begin{array}{c|c} \sigma & \\ \tau & 0 \\ \hline I & I \end{array} \right) = (-1)^{n-k} \cdot \det \begin{pmatrix} \sigma \\ \tau \end{pmatrix} \cdot \det I$$

$$= (-1)^{n-k} \det \begin{pmatrix} \sigma \\ \tau \end{pmatrix} \Rightarrow i_{(p,p)}(\sigma \times \tau, \Delta) = (-1)^{n-k} i_p(\sigma \cdot \tau) \ \rfloor\rfloor$$

From P65, thus wedge product of forms corresponds, via the deRham isomorphism, to cup product of cocycles.

As an example, let us compute the homology algebra of $\mathbb{P}^n$. To do this, denote by $X = (X_0, X_1 \cdots X_n)$ Euclidean coordinate on $\mathbb{C}^{n+1}$, and $0 = V_0 \subset V_1 \subset \cdots \subset V_n \subset \mathbb{C}^n$ the flag in $\mathbb{C}^{n+1}$ given by

$$V_i = (X_n = \cdots = X_{i+1} = 0)$$

let $\mathbb{P}^k \subset \mathbb{P}^n$ be the image of V_{k-1}. As we have seen, the complement $\mathbb{P}^n - \mathbb{P}^{n-1}$ of the hyperplane $\mathbb{P}^{n-1}$ in $\mathbb{P}^n$ is $\mathbb{C}^n$ with Euclidean coordinates $X_0/X_n, \cdots X_{n-1}/X_n$; similarly, the complement of $\mathbb{P}^{k-1}$ in $\mathbb{P}^k$ is $\mathbb{C}^k$ with coordinates $X_0/X_k, X_1/X_k, \cdots X_{k-1}/X_k$.

We have therefore a cell-decomposition of $\mathbb{P}^n$,

$$\mathbb{P}^n = (\mathbb{P}^n - \mathbb{P}^{n-1}) \cup (\mathbb{P}^{n-1} \cup \mathbb{P}^{n-2}) \cup \cdots \cup (\mathbb{P}^1 - \mathbb{P}^0) \cup \mathbb{P}^0.$$

as a union of $2k$-cells $\mathbb{P}^k - \mathbb{P}^{k-1} \cong \mathbb{C}^k$, one for each $k = 0, \cdots n$. generalizing the familiar picture of the Riemann sphere. Since there are cells only