$\dim H^0(S, \mathcal{O}(D)) - \dim H^0(S, \mathcal{O}(D+P)) + 1 - \dim H^1(S, \mathcal{O}(D)) + \dim H^1(S, \mathcal{O}(D+P)) = 0 = h^0(S, \mathcal{O}(D)) - h^0(D+P) + 1 - h^1(D) + h^1(D+P)$

$\Rightarrow \quad h^0(D) - h^1(D) = h^0(D+P) - h^1(D+P) - 1$

$\Rightarrow \quad \chi(D) = \chi(D+P) - 1 \quad \Rightarrow \quad \chi(D+P) = \chi(D) + 1.$

$\chi(D+P) = \chi(\mathcal{O}_S) + c_1(D) + 1$

$\chi(D) = \chi(\mathcal{O}_S) + c_1(D)$

$\Rightarrow$ "From the classical R-R, we deduce the sheaf-theoretical formula, and we are going to prove this independently."

We want to prove $\chi(D) = \chi([D]) = \chi(\mathcal{O}_S) + c_1([D]) = \chi(\mathcal{O}_S) + c_1(L)$. for all line bundles.

We are going to use some sort of induction.

(1) We show it for the trivial bundle

(2) If it holds for a line bundle $L = [D]$, we will show that it holds for $[D+P]$ & $[D-P]$.

(1) It's done.

(2) Assume that it is true for $[D]$.

By the argument above, $\chi(D+P) = \chi(D) + 1$.

$\Rightarrow$ Since $\chi(D) = \chi(\mathcal{O}_S) + c_1(D)$, $\chi(D+P) = \chi(D) + 1 = \chi(\mathcal{O}_S) + c_1(D) + 1 = \chi(\mathcal{O}_S) + c_1(D+P)$. Similarly, $\chi(D-P) = \chi(D) - 1 = \chi(\mathcal{O}_S) + c_1(D) - 1 = \chi(\mathcal{O}_S) + c_1(D-P)$.

Given any divisor $D = \sum P_\lambda = P_1 + \cdots + P_\lambda$, we get D by adding $P_1, \cdots, P_\lambda$ to the trivial bundle. ⅃

This version of the Riemann-Roch formula, while not as explicit as the first, points the way toward gener-