An extension of M by N is given by a short exact sequence

$$(E) \qquad 0 \longrightarrow N \longrightarrow E \longrightarrow M \longrightarrow 0.$$

The trivial or split extension is $M \oplus N$, and two extensions are equivalent in case there is a commutative diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & N & \longrightarrow & E & \longrightarrow & M & \longrightarrow & 0 \\ & & \| & & \downarrow & & \| & & \\ 0 & \longrightarrow & N & \longrightarrow & E' & \longrightarrow & M & \longrightarrow & 0 \end{array}$$

The name "Ext" is derived from the following

Lemma. There is a bijective correspondence between equivalence classes of extensions and $Ext^1_{\mathcal{O}}(M, N)$, with zero corresponding to the trivial extension.

Proof. Given an extension (E) as above, we have

$$Hom_{\mathcal{O}}(M, E) \longrightarrow Hom_{\mathcal{O}}(M, M) \xrightarrow{\partial} Ext^1_{\mathcal{O}}(M, N).$$

┌ See P686. $n=0$ second exact sequence ┘

The obstruction to splitting the sequence (E) is $\partial(1_M) \in Ext^1_{\mathcal{O}}(M, N)$, where 1_M is the identity