

$$= \Omega_{\tilde{M}}^n((\tilde{L}^{K_1} + \tilde{K}_M^*) \otimes (\tilde{L}^{K'} - nE)) \quad K_1 + K' = K \geq K_0 = K_2 + K_1$$

if $K' \geq K_2$, $\Rightarrow K_1 + K' = K \geq K_1 + K_2 = K_0$.

Here we used the relation $K_{\tilde{M}} = \pi^* K_M + (n-1)E$ and $\mathcal{O}_{\tilde{M}}(E \otimes K_{\tilde{M}}) = \Omega_{\tilde{M}}^n(E)$ for any holomorphic vector bundle. refer to P153. Kodaira-Serre duality.

Now by hypothesis, $\tilde{L}^{K'} - nE$ has a positive definite curvature form on \tilde{M} ; $L^{K_1} + K_M^*$ has a positive curvature form on M , and so $(\tilde{L}^{K_1} + \tilde{K}_M^*)$ has a positive semidefinite one on \tilde{M} .

Γ $\tilde{L}^{K_1} + \tilde{K}_M^* = \pi^*(L^{K_1} + K_M^*) \geq 0 \Rightarrow$ positive semidefinite.
One thing we omitted to discuss is the following:

$$K_{\tilde{M}} = \pi^* K_M + (n-1)E = \tilde{K}_M + (n-1)E.$$

$$\Rightarrow K_{\tilde{M}}^* = \text{Hom}(K_{\tilde{M}}, \mathbb{C}) = \text{Hom}(\tilde{K}_M + (n-1)E, \mathbb{C})$$

$$= \tilde{K}_M^* + (n-1)E^* \Rightarrow K_{\tilde{M}}^* = \tilde{K}_M^* - (n-1)E.$$

In general, if L_1 & L_2 are line bundles,

$$\Rightarrow (L_1 \otimes L_2)^* = L_1^* \otimes L_2^*.$$

pf) $\{g_{\alpha\beta}\}$, $\{h_{\alpha\beta}\}$ are given as transition functions of L_1 & L_2 respectively.

$\Rightarrow \{g_{\alpha\beta} \otimes h_{\alpha\beta}\}$ is a set of transition functions of $L_1 \otimes L_2$. $\Rightarrow (L_1 \otimes L_2)^*$ has transition functions $\{g_{\alpha\beta}^{-1} \otimes h_{\alpha\beta}^{-1}\}$ by P67.

On the other hand, $L_1^* \otimes L_2^*$ has transition functions $\{g_{\alpha\beta}^{-1} \otimes h_{\alpha\beta}^{-1}\}$ which are the same as above.

Thus the line bundle $(\tilde{L}^{K_1} + \tilde{K}_M^*) + \tilde{L}^{K'} - nE$ is positive