∵ $L = \sigma(p_L, h_L) \Rightarrow$ For generic L, $h_L \notin R^* \Rightarrow$ $h_L \in S^* - R^* \Rightarrow h_L$ is the tangent space of S at some point of S.

On S^*, a generic element $h \in S^*$ is a smooth point. $\Rightarrow$ By P762, h is a tangent space of S. Consider all tangent space of S, i.e. S^*.
$\Rightarrow$ Since S^* is smooth away from R^*, $T_x S \neq T_{x'} S$ if for generic $x \neq x'$, otherwise $T_x S$ is a double point of S^*, i.e., $T_x S \in R^*$. $\Rightarrow$ For generic L, h_L is a tangent space of S, and h_L can not be tangent at two distinct points. $\Rightarrow$ We can conclude that $h_L \cap S$ is smooth away from x, where $h_L = T_x S$.

To show that $C_L = h_L \cap S$ is a plane quartic with one ordinary double point, we have to show that, for generic $x \in S$, $T_x(S) \cap S$ has an ordinary double point at x.

Let $S = (F = 0)$ in $\mathbb{P}^3$. Suppose S is given as $(x_1, x_2, f(x_1, x_2))$ locally. $\Rightarrow$ Then, at (a,b), the tangent space at (a,b) is given as

$$\frac{\partial f}{\partial x_1}(a,b)\, x_1 + \frac{\partial f}{\partial x_2}(a,b)\, x_2 - x_3 = 0$$

$\Rightarrow S \cap T_{(a,b)} S = \{ x_3 = f(x_1, x_2) \} \cap \{ \frac{\partial f}{\partial x_1}(a,b)\, x_1 + \frac{\partial f}{\partial x_2}(a,b)\, x_2 = x_3 \} \Rightarrow f(x_1, x_2) - \frac{\partial f}{\partial x_1}(a,b)\, x_1 - \frac{\partial f}{\partial x_2}(a,b)\, x_2 = 0$.
$\Rightarrow$ It is a curve which has singular point at (a,b). Let $G(a,b,x_1,x_2) = f(x_1,x_2) - \frac{\partial f}{\partial x_1}(a,b)\, x_1 -$