

I need more explanations. If P_1, P_2, P_3 are collinear, \exists a line $L \Rightarrow A(P(z), P'(z)) = aP(z) + bP'(z) + C$ is a meromorphic function on C , where $L = \{ax + by + C = 0\}$.
with divisor $P_1 + P_2 + P_3 - 3P_0$.

"Comment" They showed the existence of a meromorphic function explicitly. They explained intricately, what I mean is that they used the intersection points of C & L .

Setting $P_i = [1, P(z_i), P'(z_i)]$, we may rewrite (*) in the form

$$(**) \begin{vmatrix} 1 & P(z_1) & P'(z_1) \\ 1 & P(z_2) & P'(z_2) \\ 1 & P(z_3) & P'(z_3) \end{vmatrix} = 0 \Leftrightarrow z_1 + z_2 + z_3 \equiv 0 \pmod{\Lambda}.$$

Clear. And $z_1 + z_2 \equiv -z_3$ gives a group structure. $z_1 = 0$ corresponds to P_0 plays a role as the identity.

This beautiful relation may be interpreted in several eventually equivalent ways. One is as the famous addition theorem for elliptic functions expressing

$P(-z_1 - z_2) = P(z_1 + z_2)$ and $P'(-z_1 - z_2) = -P'(z_1 + z_2)$ retroactively in terms of $P(z_1), P(z_2), P'(z_1)$ and $P'(z_2)$. Alternately,

we may give the group structure on the cubic curve C geometrically by making the construction with lines dictated by (**). In any case, the inversion of the elliptic integral via Abel's theorem and corresponding theory of cubic curves in the plane occupies a singular position of harmony and depth in the subject of algebraic geometry.