irreducible 3-dimensional family of 2-planes in F_{λ_1}
$\Rightarrow$ By the technique on P988 ~ P989, $\exists$ 2-plane Λ_1 s.t. $\Lambda_1 \supset L_2$. In case F_{λ_1} singular, $\overline{L_2, p}$ is Λ_1, where p is the singular point of F_{λ_1}. In case F_{λ_1} smooth, if Λ_1 & Λ_2 contain L_2, $\dim(\Lambda_1 \cap \Lambda_2) = 1$, since $\Lambda_1 \neq \Lambda_2$ and $\Lambda_1 \cap \Lambda_2 = L_2$. $\Rightarrow 1 \not\equiv 2 \pmod 2$ $\Rightarrow$ Λ_1 & Λ_2 belong to opposite families by P935.
$\Rightarrow$ $\exists$ a unique Λ_1 containing L_2.

By P988, $\Lambda_i \cap X$ is a conic curve in Λ_i. $\Rightarrow$
$\Lambda_i \cap X = L_2 \cup M_i'$.

$$\begin{tikzcd}
B_{L_0} \arrow[r, "t_{L_0 - L_2}"'] \arrow[d, "\pi_0"'] & B_{L_2} \arrow[d, "\pi_2"] \\
\mathbb{P}^1 \arrow[r, equal] & \mathbb{P}^1
\end{tikzcd}$$

$M_1 \in B_{L_0} \Rightarrow t_{L_0 - L_2}(M_1) = M_1 + L_0 - L_2$
As we saw above, $\pi_0(\overline{L_0, M_1}) = \lambda_1$, and $\overline{L_0, M_1} \subset F_{\lambda_1}$.
$\pi_2(\overline{L_2, M_1'}) = \lambda_1$, and $\overline{L_2, M_1'} \subset F_{\lambda_1}$. Note here that $\overline{L_0, M_1}$ & $\overline{L_2, M_1'}$ belong to the same family.
$\Rightarrow t_{L_0 - L_2}(M_1) = M_1'$. $\Rightarrow M_1' = M_1 + L_0 - L_2$. But since $L_0 = 0$, $M_1' = M_1 - L_2$. Similarly, we get $M_2' = M_2 - L_2$.

Finally, the lines L_2, M_1', and M_2' span a 3-plane