$$\mathbb{C}^{n-1} \xrightarrow{f} \mathbb{C}^n$$

$$(w_2, w_3 \cdots w_n) \longmapsto (0, w_2, w_3, \cdots w_n) \Rightarrow f^* dw' = dw'$$

$$f^*(dw_2) = dw_2 \qquad \Rightarrow f^* d\bar{w}' = d\bar{w}'$$

$$\psi(w_1 \cdot w_2 \cdots w_n) \Rightarrow (f^*\psi)(w_2 \cdots w_n)$$
$$= \psi(0, w_2 \cdots w_n)$$

④ $$\lim_{\epsilon \to 0} \int_{\partial D(\epsilon) \cap \Delta} \partial \log f_\alpha \wedge \psi = 2\pi i \int_{V \cap \Delta} \psi$$

for any polydisc Δ around any smooth point. $z_0 \in V$.

Here, polydisc Δ should be in some nbd of $z_0 \in V$.

$\Rightarrow$ Globally, $\lim_{\epsilon \to 0} \int_{\partial D(\epsilon)} \partial \log f_\alpha \wedge \psi = 2\pi i \int_V \psi$.

For, since V is compact, and V^* is dense, $\exists$ a finite # of smooth points $z_1 \cdots z_m$ & $\Delta_1, \Delta_2 \cdots \Delta_m$ s.t
$z_i \in \Delta_i$. $\qquad \cup \Delta_i \supset D = V$.

$\Rightarrow \exists$ a partition of unity, φ_i. s.t $\sum \varphi_i = 1$ on $D(\epsilon)$ of V

$\operatorname{supp} \varphi_i \subset \Delta_i$.

$$\Rightarrow \lim_{\epsilon \to 0} \int_{\partial D(\epsilon) \cap} \partial \log f_\alpha \wedge \psi = \lim_{\epsilon \to 0} \int_{\partial D(\epsilon)} \sum \varphi_i (\partial \log f_\alpha \wedge \psi)$$

$$= \lim_{\epsilon \to 0} \sum_i \int_{\partial D(\epsilon)} \varphi_i (\partial \log f_\alpha \wedge \psi) = \sum_i \lim_{\epsilon \to 0} \int_{\partial D(\epsilon)} \partial \log f_\alpha \wedge \varphi_i \psi$$

This is obvious since the integration is just the limit of sum.
"Sum of small region integration."
Note: Singular points set is measure zero. $\Rightarrow$ We can neglect the integration over the set.