Note that since the tangent plane $T_y(Q)$ to Q at any point $y \in Q$ meets Q in the sum of two lines, one from each family, the 3-plane $\overline{x, T_y(Q)}$ meets G in the sum of a $\sigma(p)$ and a $\sigma(h)$, showing directly that

$$\sigma_1^2 = \sigma_{1,1} + \sigma_2 .$$

$\ulcorner$ $T_y(Q) \cap Q$ = Union of two lines by P479.
$= L_{\lambda_1} \cup L'_{\lambda_2}$

$\Rightarrow$ Since $\overline{x, T_y(Q)} = \bigcup_{z \in T_y(Q)} \overline{x, z}$,

$$\overline{x, T_y(Q)} = \overline{x, L_{\lambda_1} \cup L'_{\lambda_2}} = \overline{x, L_{\lambda_1}} \cup \overline{x, L'_{\lambda_2}}$$
$$= \sigma(p) \cup \sigma(h)$$

$\#(\overline{x, T_y(Q)} \cap G) = \#(\mathbb{P}^3 \cap G) = \#((\mathbb{P}^4 \cap G) \cap (\mathbb{P}^4 \cap G))$
$= \#(\sigma_1 \cap \sigma_1) = \sigma_1 \cdot \sigma_1 = \sigma_1^2$ by the result on P756
$= \sigma(p) + \sigma(h)$.

$\lrcorner$

Line Complexes

We have given, above and in Section 1 of this chapter, accounts of various cycles in the Grassmannian $G(2,4)$ arising from the geometry of $\mathbb{P}^3$. Of interest classically was the converse problem: to describe the geometry of the family of lines in