by setting, for $\sigma = (\sigma_0, \cdots \sigma_n)$ a section of $H^{\oplus(n+1)}$,

$$\mathcal{E}(\sigma) = \pi_* \left(\sum \sigma_i(X) \frac{\partial}{\partial X_i} \right).$$

┌ $H = J^* = \{ (Z, X) \in \mathbb{P}^n \times \mathbb{C}^{n+1} : X \in Z \}^*$

$\Rightarrow$ The fiber of the hyperplane line bundle $H \to \mathbb{P}^n$ over a point $x = \pi(X) \in \mathbb{P}^n$ corresponds to linear functionals on the line $\mathbb{C}\{X\} \subset \mathbb{C}^{n+1}$.

$\sigma = (\sigma_0, \cdots \sigma_n) \in H^{\oplus(n+1)}$. ┘

By the second observation the map $\mathcal{E}$ is surjective, with kernel the trivial line bundle spanned by the section $\tau = (X_0, X_1 \cdots X_n)$.

┌ By the observation 2, since $\{ \pi_* \frac{\partial}{\partial X_i} \}_{i=0,\cdots n}$ span $T'_x(\mathbb{P}^n)$, $\mathcal{E}$ is surjective.

Suppose $\mathcal{E}(\sigma) = 0$.

$\Rightarrow \mathcal{E}(\sigma) = \pi_* \left(\sum \sigma_i(X) \frac{\partial}{\partial X_i} \right) = 0 = \sum \sigma_i(X)\, \pi_* \frac{\partial}{\partial X_i}$

For example,

$$\Rightarrow \sum \sigma_i(X)\, \pi_* \frac{\partial}{\partial X_i} = -\sigma_0(X) \frac{X_1}{X_0^2} \frac{\partial}{\partial x_1} - \sigma_0(X) \frac{X_2}{X_0^2} \frac{\partial}{\partial x_2}$$

$$+ \frac{\sigma_1(X)}{X_0} \frac{\partial}{\partial x_1} + \sigma_2(X) \frac{1}{X_0} \frac{\partial}{\partial x_2} = 0$$

$$\Rightarrow \quad \sigma_0(X) \frac{X_1}{X_0^2} = \frac{\sigma_1(X)}{X_0} \qquad \sigma_0(X) \frac{X_2}{X_0^2} = \sigma_2(X) \frac{1}{X_0}$$

$$\Rightarrow \quad \sigma_0(X)\, X_1 = \sigma_1(X)\, X_0 \quad \Rightarrow \quad \sigma_0(X) : \sigma_1(X) = X_0 : X_1$$

$$\Rightarrow \quad \sigma_0(X) = \lambda X_0 \qquad \sigma_1(X) = \lambda X_1$$

$$\sigma_{[X_0, \cdots X_n], i}(\alpha X_0, \cdots \alpha X_n) = \alpha\, \sigma_{[X_0, \cdots X_n], i}(X_0, \cdots X_n)$$

e.x. $\sigma_{[X_0, X_1], 0}(\alpha X_0, \alpha X_1) = \alpha\, \sigma_{[X_0, X_1], 0}(X_0, X_1)$. $\sigma_{[X_0, X_1], 1}(\alpha X_0, \alpha X_1) = \alpha\, \sigma_{[X_0, X_1], 1}(X_0, X_1)$