and so we have

Chow's Theorem. Any analytic subvariety of projective space is algebraic.

⌈ If $q \notin V$, $\pi(q)$ may be in $\pi(V)$.

$\Rightarrow \tilde{F}(q) = F \circ \pi(q) = 0$.
$\Rightarrow \tilde{F}$ vanishes identically on $V$ but not at $p$.

Consider all homogeneous polynomials vanishing identically on $V$, which we denote by $I$.

$\Rightarrow$ $I$ is an ideal of $\mathbb{C}[X_0, X_1, \cdots X_n]$
$\Rightarrow$ Since $\mathbb{C}[X_0 \cdots X_n]$ is Noetherian, $I$ is finitely generated. (See p391. Th. 4.9 Hungerford p372. p375 Th 1.9)
$\Rightarrow I = \langle \tilde{F}_1, \tilde{F}_2, \cdots \tilde{F}_\ell \rangle$.

$$V = \bigcap_{f \in I} \{f = 0\} = \bigcap_{i=1}^{\ell} \{\tilde{F}_i = 0\}.$$

⌋

If $F(X_0, \cdots X_n)$ and $G(X_0, \cdots X_n) \neq 0$ are two homogeneous polynomials of the same degree $d$ in the homogeneous coordinates $X$ on $\mathbb{P}^n$, the quotient

$$\varphi([X]) = \frac{F(X)}{G(X)}$$

is a well-defined meromorphic function on $\mathbb{P}^n$; such a meromorphic function is called a rational function. Note that after dividing