up to adding the real part of a holomorphic function.

Proof. By the $\bar\partial$ - Poincare lemma, locally

$$T = -\sqrt{-1}\,\bar\partial\eta$$

for some current η of type (1.0).

「 $T = \frac{\sqrt{-1}}{2}\sum T_{i\bar j}\, dz_i \wedge d\bar z_j$ locally Wrong!

$\Rightarrow dT = 0 \iff \partial T = 0 \Rightarrow \partial T_{i\bar j} = 0 \Rightarrow \exists\, f_{i\bar j}$ s.t

$T_{i\bar j} = T_{f_{i\bar j}}$, $f_{i\bar j}$ anti holomorphic by P380 Regularity for $\bar\partial$-operator.

$\Rightarrow T = \frac{\sqrt{-1}}{2}\sum f_{i\bar j}\, dz_i \wedge d\bar z_j = \frac{\sqrt{-1}}{2}\sum_i dz_i \wedge \left(\sum_j f_{i\bar j}\, d\bar z_j\right)$

$\Rightarrow$ By P 25. $\bar\partial$ - Poincare lemma, since $H^{0.1}_{\bar\partial}(\Delta) = 0$

and, since $\sum_j f_{i\bar j}\, d\bar z_j \in A^{0.1}(loc)$,

$\exists\ \eta \in A^{0.0}(loc)$ s.t $\bar\partial\eta_i = \sum_j f_{i\bar j}\, d\bar z_j$

$\Rightarrow T = \frac{\sqrt{-1}}{2}\sum_i dz_i \wedge \bar\partial\eta_i = -\frac{\sqrt{-1}}{2}\sum \bar\partial\eta_i \wedge dz_i$

$= -\frac{\sqrt{-1}}{2}\bar\partial\left(\sum \eta_i \wedge dz_i\right)$

$= -\frac{\sqrt{-1}}{2}\bar\partial\left(\sum \eta_i\, dz_i\right)$

For example, $T = T\, dz_1 \wedge d\bar z_1$

$\Rightarrow \partial T = 0 \Rightarrow \partial T \wedge dz_1 \wedge d\bar z_1$, $\frac{\partial T}{\partial z_1} \neq 0$, $\frac{\partial T}{\partial z_i} = 0$

$\Rightarrow$ Still $\partial T = 0$.

$\Rightarrow T(\phi) = -\frac{\sqrt{-1}}{2}\int \bar\partial\left(\sum \eta_i\, dz_i\right) \wedge \phi$,

Let $\eta = \frac{1}{2}\sum \eta_i\, dz_i$, locally, then $T = \sqrt{-1}\,\bar\partial\eta$

where η is the distribution represented by η.

Direct and Correct proof: we already proved that the complex of sheaves $0 \to \Omega^1 \to \mathcal{D}^{1,0} \xrightarrow{\bar\partial} \mathcal{D}^{1,1} \xrightarrow{\bar\partial} \mathcal{D}^{1,2} \to$ is exact on P382 ~ P383. Since $T \in \mathcal{D}^{1,1}$, $\exists\ \eta$