$\mathcal{F}$ is coherent if locally it has a representation.

$$\mathcal{O}^{(p)} \longrightarrow \mathcal{O}^{(q)} \longrightarrow \mathcal{F} \longrightarrow 0.$$

$\Leftrightarrow$ For given any point $z_0 \in M$, $\exists$ $U \ni z_0$ s.t

$$\mathcal{O}^{(p)}/U \xrightarrow{\phi} \mathcal{O}^{(q)}/U \xrightarrow{\psi} \mathcal{F}/U \longrightarrow 0$$ is exact,

for some p & q.

$\Rightarrow$ $\{\phi(e_i)\}$ is a set of generators for the relations and $\{\psi(e_i')\}$ is a set of generators for the $\mathcal{F}/U$. //

Here are some remarks and examples. The gist throughout is that Oka's lemma allows properties in the local ring $\mathcal{O}_{z_0}$ to propagate to the same properties in nearby local rings $\mathcal{O}_z$. We shall refer to this as the propagation principle: it gives rise to the name coherent.

We begin by noting that:

Coherent sheaves admit local syzygies

$$0 \to \mathcal{O}^{(k_n)} \longrightarrow \mathcal{O}^{(k_{n-1})} \longrightarrow \cdots \longrightarrow \mathcal{O}^{(k_0)} \longrightarrow \mathcal{F} \longrightarrow 0.$$

Proof. By definition we have

$$\mathcal{O}^{(p)} \longrightarrow \mathcal{O}^{(q)} \longrightarrow \mathcal{F} \longrightarrow 0$$

in a nbd U of z_0. Applying Oka's lemma, we find

$$\mathcal{O}^{(r)} \longrightarrow \mathcal{O}^{(p)} \longrightarrow \mathcal{O}^{(q)} \longrightarrow \mathcal{F} \longrightarrow 0$$

in a possibly smaller nbd U' of z_0.

$\ulcorner$ Consider the sheaf map F : $\mathcal{O}^{(p)}/U \longrightarrow \mathcal{O}^{(q)}/U$.