unless five are collinear.

Assume that $P_1, P_2, P_3, \cdots P_8$ fail to impose linearly independent conditions on cubics. Then, once we prove the first step, choose non collinear three points, call P_6, P_7, P_8. We may choose a conic τ s.t τ contains P_1, P_2, P_3, P_4, P_5. It is possible, since , let $\tau = a_1\tau_1 + \cdots + a_6\tau_6$,

$$\left.\begin{array}{l} \left(\frac{\partial\tau}{\partial x}, \frac{\partial\tau}{\partial y}\right)\cdot P_1 = 0 \quad \left(\frac{\partial\tau}{\partial x}, \frac{\partial\tau}{\partial y}\right)\cdot P_2 = 0 \\ \tau(P_3) = \tau(P_4) = \tau(P_5) = 0 \end{array}\right\} \begin{array}{l} 5 \text{ equction.} \\ \text{at most} \end{array}$$

$$\text{or} \quad \frac{\partial\tau}{\partial x} = \frac{\partial\tau}{\partial y} = 0 = \tau(P_3) = \tau(P_4) = \tau(P_5) = 0 \Big\} \begin{array}{l} 5 \text{ equctions} \\ \text{at most.} \end{array}$$

$\Rightarrow \exists$ a nontrivial solution of a_i's, where $H^0(\mathbb{P}^2, \mathcal{O}(2H)) = \langle \tau_i \rangle$.

Consider $\tau + L_{67}$, $\tau + L_{78}$, $\tau + L_{68}$.

$\Rightarrow$ By the first step and the assumption,

$\tau + L_{67} \ni P_8 \Rightarrow \tau \ni P_8$ by the non collinearity of P_6, P_7, P_8. Similary, $\tau \ni P_6, P_7$. $\Rightarrow \tau \ni P_i$, $i = 1, \cdots 8$.

Thus it remains to prove the first step.

The step of (i) is proved by the argument above P482.

(ii). Proof.

Choose $P_2' \in \mathbb{P}^2$ s.t $P_1, P_2', P_3, P_4, \cdots P_7$ with P_1 infinitely near P_3 have no five collinear points. $\Rightarrow$ By the argument on P482 *(or (i)) $P_1, P_2', P_3, P_4 \cdots P_7$ impose independent conditions on cubics. $\Rightarrow P_1, P_3, P_4 \cdots P_7$ impose independent