

This was proved by Picard, and Severi showed that a multiple  $\lambda D$  ( $\lambda \in \mathbb{Z}$ ) is algebraically equivalent to zero (to be explained momentarily) if and only if there is a closed meromorphic 1-form whose residue is  $D$ . Combining these, it follows that the Neron-Severi group

$$NS(M) = \frac{\{\text{divisors on } M\}}{\{\text{divisors algebraically equivalent to zero}\}}$$

is finitely generated (theorem of the base).

⌈ If  $D$  is algebraically equivalent to zero,  $\exists$  a closed meromorphic 1-form  $\psi$  with  $R(\psi) = D$ .

$$\Rightarrow i(D) = 0 \Rightarrow i(NS(M)) \subset H^2(M, \mathbb{C})$$

$\Rightarrow$  Since  $H^2(M, \mathbb{C})$  is finite dimensional,  $i(NS(M))$  is finitely generated, ( $\because i$  is injective on  $NS(M)$ )  $\Rightarrow$

⌈ We will see this more clearly  $\Rightarrow$

The structure of the group of divisors on  $M$  may be pictured by the diagram

$$\begin{array}{ccc} & & \nearrow \text{see p. 44} \\ H^{1,1}(M) \cap H^2(M, \mathbb{Z}) & & \\ \underbrace{\text{Div} \supset \text{Div}_h}_{NS} & \supset & \underbrace{\text{Div}_a \supset \text{Div}_e}_{Pic^0} \\ & \underbrace{\hspace{10em}}_{Pic} & \end{array}$$

where  $\text{Div}_h$ ,  $\text{Div}_a$ ,  $\text{Div}_e$  are the divisors homologous, algebraically equivalent, and linearly equivalent to zero.