Notice the following:

$$0 \longrightarrow R_0 \longrightarrow E_0 \xrightarrow{\psi_0} I/I' \longrightarrow 0.$$

$\Rightarrow$ Clearly, $\psi_0(g e_0) = 0 = g\,\psi(e_0)$, $e_0 \in E_0$, $g \in I'$.
What about $g r_0$? $\Rightarrow$ $g r_0 \in \ker \psi_0$.

$$0 \longrightarrow R_1 \longrightarrow E_1 \xrightarrow{\psi_1} R_0 \longrightarrow 0$$

Question: $g r_1 \overset{?}{\in} \ker \psi_1$.

Wrong direction!

I don't know how to prove

$$\mathrm{Ext}^{r-k}_{\mathcal{O}}(I/I', \mathcal{O}/I'_{k+1}) \xrightarrow{f_k'} \mathrm{Ext}^{r-k}_{\mathcal{O}}(I/I', \mathcal{O}/I'_{k+1}) \text{ is zero.}$$

We know that given any $\varphi \in \mathrm{Ext}^{r-k}_{\mathcal{O}}(I/I', \mathcal{O}/I'_{k+1})$, then $\varphi = 0$ on $\partial E_{r-k+1}$ and $I'_{k+1} E_{r-k}$. But we should know that $\varphi = 0$ on $I'_k E_{r-k}$. Might be wrong. The authors may make a mistake again.

Putting this all together, we obtain a surjective map

$$\mathrm{Hom}_{\mathcal{O}}(I/I', \mathcal{O}/I'_{r-1}) \longrightarrow \mathrm{Ext}^{r-1}_{\mathcal{O}}(I/I', \mathcal{O}) \longrightarrow 0.$$

Since $f_k' = 0$, $\mathrm{Ext}^{0}_{\mathcal{O}}(I/I', \mathcal{O}/I'_{r-1}) \xrightarrow{\text{onto}} \mathrm{Ext}^{1}_{\mathcal{O}}(I/I', \mathcal{O}/I'_{r-2}) \longrightarrow \mathrm{Ext}^{2}_{\mathcal{O}}(I/I', \mathcal{O}/I'_{r-3}) \to \cdots \xrightarrow{\text{onto}} \mathrm{Ext}^{r-1}_{\mathcal{O}}(I/I', \mathcal{O})$ is onto

If $\varphi \in \mathrm{Hom}_{\mathcal{O}}(I/I', \mathcal{O}/I'_{r-1})$, then for any $g \in I/I'$

$$f_r'\varphi(g) = \varphi(f_r' g) = 0 \Rightarrow \varphi(g) = 0$$