these out.

⌜ If $(K\varphi)(z)$ has compact support, $\exists\ r$ s.t for all $z \in B(0,r)^c$, $K(\varphi)(z)=0$. For exaple, $n=2$, $q=1$

$$K(\varphi)(z) = \int_{\mathbb{C}^2} \frac{-(z_1-\omega_1)+(z_2-\omega_2)}{\|z-\omega\|^4}\,\varphi(\omega) = 0 \quad \text{for all } z \in \mathbb{C}^2 - B(0,r).$$

I think that the statement " $K\varphi$ does not have compact support " is not quite right, because there might be some $\varphi \in A_c^{0,q}(\mathbb{C}^n)$ s.t $K(\varphi)=0$. Maybe the authors thought by looking at $k(z, z-u) \wedge \varphi(z-u)$, specially $\varphi(z-u)$, $(K\varphi)(z)$ does not have compact support in general. ⌟

What we need to know about K is the homotopy formula

$$\bar\partial K + K\bar\partial = \text{identity}.$$

Proof. Since

$$\bar\partial\left(\frac{\sum \overline{\Phi_i(\zeta)}}{\|\zeta\|^{2n}}\right) = \frac{n\,\overline{\Phi(\zeta)}}{\|\zeta\|^{2n}} - \frac{n\,\bar\partial(\zeta,\zeta)\wedge\overline{\sum \Phi_i(\zeta)}}{\|\zeta\|^{2n+2}}$$

$$= \frac{n}{\|\zeta\|^{2n}}\left[\overline{\Phi(\zeta)} - \frac{(\sum \zeta_i\, d\bar\zeta_i)\wedge(\overline{\sum \Phi_i(\zeta)})}{\|\zeta\|^2}\right] = 0,$$