We now return to the reciprocity law and deduce some consequences. First, consider the case where $\eta = \omega'$ is a holomorphic 1-form; write Π'^i for the periods of ω'. Since ω' has no residues, the formula reads

$$\sum_{i=1}^{g} \left(\Pi^i \Pi'^{g+i} - \Pi^{g+i} \Pi'^i \right) = 0.$$

This is the first Riemann bilinear relation in the periods. In particular, if $\omega = \omega_i$, $\omega' = \omega_j$ are elements of our normalized basis, all but two terms in the expression on the right vanish, and we have

$$\int_{\delta_{g+i}} \omega_j - \int_{\delta_{g+j}} \omega_i = 0,$$

i.e., the right-hand block Z in the period matrix above is symmetric.

Γ $\quad \Pi^i = \int_{\delta_i} \omega \qquad \Pi'^j = \int_{\delta_j} \omega'$

$\Rightarrow$ If $\omega = \omega_i$, $\omega' = \omega_j$, then $\Pi'^{g+i} - \Pi^{g+j} = 0$ i.e.,

$$\int_{\delta_{g+i}} \omega_j - \int_{\delta_{g+j}} \omega_i = 0 \Rightarrow \text{This says that } Z \text{ is symmetric.}$$

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Note that since the quadratic form on $H^0(S, \Omega^1)$ given by

$$(\omega_i, \omega_j) = \sqrt{-1} \int_S \omega_i \wedge \overline{\omega_j} = \sqrt{-1} \int_{\delta_{g+i}} \overline{\omega_j} - \sqrt{-1} \int_{\delta_{g+j}} \omega_i =$$