

(a) $q' \notin R$.

\Rightarrow By the argument above P987~P988 note, we can show that $\overline{p_L q'} = \overline{p_{L'} q'} \Rightarrow q' \in \overline{p_L p_{L'}} \cap S$
 $\Rightarrow h$ is tangent to S at q' .

(b) $q' \in R$

~~It is tangent to S at q' since q' is a double point.~~

$\sigma(q') \cap X = \sigma(q', \tilde{h}) \Rightarrow \overline{p_L q'}, \overline{p_{L'} q'} \in \sigma(q', \tilde{h})$ since $\overline{p_L q'} \in \sigma(p_L, h) \subset X$, and $\overline{p_L q'} \in \sigma(q')$, similarly for $\overline{p_{L'} q'}$. $\Rightarrow \tilde{h}$ contains $p_L, p_{L'}$, and $q' \Rightarrow$ If we assume $p_L \neq p_{L'}$, and $\overline{p_L q'} \neq \overline{p_{L'} q'}$, then $\tilde{h} = h$ since $h \ni p_L, p_{L'}, q'$. $\Rightarrow \sigma(q') \cap X = \sigma(q', h)$.
 $\Rightarrow L$ or L' is equal to $\sigma(q', h) \Rightarrow p_L$ or $p_{L'}$ is equal to q' . \Rightarrow Contradiction. $\Rightarrow q'$ can be not an element of R .

"For L, L' nonspecial, $h_L \cap S$ is a quartic with one ordinary double point."

"Lemma: Suppose $p_1, p_2 \notin R$ and $T_{p_i}(S) = h_i \notin R^*$.
 Then $h_1 \neq h_2$."

Proof) If $h_1 = h_2$, by the argument on P766~P767,
 $p_1^* = T_{h_1}(S^*) = T_{h_2}(S^*) = p_2^* \Rightarrow p_1 = p_2 \Rightarrow$ Contradiction.
 Thus $p_1 \neq p_2 \Rightarrow h_1 \neq h_2$."