Since $[D_1] \otimes [D_2]^*$ has zero Chern class,

$$[D_1] \otimes [D_2]^* = P_{\xi_0}$$

for some ξ_0.

╔ $c_1([D_1] \otimes [D_2]^*) = c_1([D_1]) + c_1([D_2]^*) = c_1([D_1]) - c_1([D_2]) = 0$, see P139. ╛

By the last result proved in the subsection "Intrinsic Formulations" in Section 6 of Chapter 2, we may find a line bundle $L \longrightarrow M$ and sections $\theta_\xi \in H^0(M, \mathcal{O}(L \otimes P_\xi))$ such that $\theta_\xi \neq 0$ for generic ξ.

╔ See P309 (Proposition). $L \cong L_\xi \otimes P_\xi^*$ a positive line bundle. I don't read the section and I don't understand it. So I just accept the statement as a fact. ╛

In fact from the proof we may assume that $\theta_e, \theta_{\xi_0} \neq 0$. Setting $D_\xi = (\theta_\xi)$ from $[D_1 - D_2 + D_e] = [D_{\xi_0}]$, we deduce that the linear equivalence

$$D_1 + D_e \sim D_2 + D_{\xi_0}$$

holds.

╔ I guess. $P_{\xi_0} = [D_{\xi_0} - D_e]$ and so set $D_\xi = (\theta_\xi)$.

$\Rightarrow [D_1 - D_2 + D_e] = [D_{\xi_0}]$

$\Rightarrow D_1 - D_2 \sim D_{\xi_0} - D_e \quad \Rightarrow \quad D_1 + D_e \sim D_2 + D_{\xi_0}$

$\Rightarrow D_1 + D_e = D_2 + D_{\xi_0} + (f = 0)$.

Consider $\{(\lambda_0 f + \lambda_1 = 0)\}$, $[(\lambda_0, \lambda_1)] \in \mathbb{P}^1$. see P144