$\mathbb{C}^n$ is

$$\omega = \frac{\sqrt{-1}}{2} \sum dz_i \wedge d\bar{z}_i ,$$

and so for $c = (\sqrt{-1}/2)^k (-1)^{k(k-1)/2} \cdot k!$

$$\omega^k = c \cdot \sum_{\# I = k} dz_I \wedge d\bar{z}_I .$$

$$\omega^k = \left(\frac{\sqrt{-1}}{2}\right)^k \left(\sum dz_i \wedge d\bar{z}_i\right)^k$$

$$= (\sqrt{-1}/2)^k (dz_1 \wedge d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_2 \wedge \cdots dz_k \wedge d\bar{z}_k) \cdot k! + \cdots$$

$$= (\sqrt{-1}/2)^k (-1)^{1+2+\cdots+(k-1)} \underset{I = \{1,2,\cdots k\}}{dz_I \wedge d\bar{z}_I} \cdot k! + \cdot$$

$$= (\sqrt{-1}/2)^k (-1)^{k(k-1)/2} \cdot k! \sum_{\# I = k} dz_I \wedge d\bar{z}_I .$$

Thus it will suffice to prove that

$$c \int_{V^* \cap \Delta} dz_I \wedge d\bar{z}_I < \infty$$

for $I = \{1, 2, \cdots k\}$, Δ a small polydisc around the origin.

Once we prove the finiteness for $I = \{1, 2, \cdots k\}$, we can prove it for any I in the same way, which completes the proof.