$$\Rightarrow \left(\frac{\partial h_{11}}{\partial z_2} dz_2 + \frac{\partial h_{11}}{\partial \bar{z}_2} d\bar{z}_2\right) \wedge dz_1 \wedge d\bar{z}_1 + \left(\frac{\partial h_{12}}{\partial z_2} dz_2 + \frac{\partial h_{12}}{\partial \bar{z}_1} d\bar{z}_1\right) \wedge dz_1 \wedge d\bar{z}_2$$

$$+ \left(\frac{\partial h_{21}}{\partial z_1} dz_1 + \frac{\partial h_{21}}{\partial \bar{z}_2} d\bar{z}_2\right) \wedge dz_2 \wedge d\bar{z}_1 + \left(\frac{\partial h_{22}}{\partial z_1} dz_1 + \frac{\partial h_{22}}{\partial \bar{z}_1} d\bar{z}_1\right) \wedge dz_2 \wedge d\bar{z}_2$$

$$\Rightarrow \frac{\partial h_{11}}{\partial z_2} = + \frac{\partial h_{21}}{\partial z_1} \qquad \Rightarrow g_1 \text{ s.t } \frac{\partial g_1}{\partial z_1} = h_{11} \text{ and } \frac{\partial g_1}{\partial z_2} = h_{21}$$

$$\Rightarrow \frac{\partial h_{12}}{\partial z_2} = \frac{\partial h_{22}}{\partial z_1} \qquad g_2 \text{ s.t } \frac{\partial g_2}{\partial z_1} = h_{12},\ \frac{\partial g_2}{\partial z_2} = h_{22}$$

Actudly. $h = T_\omega \in \mathcal{A}^{1,1} \Rightarrow$ By the exactness of the complex of sheaves

$$0 \to \mathbb{C} \to C^\infty \xrightarrow{d} \mathcal{A}^1 \xrightarrow{d} \mathcal{A}^2 \xrightarrow{d} \mathcal{A}^3 \to$$

$\Rightarrow \exists\ \eta \in \mathcal{A}^1$ s.t $d\eta = T_\omega$.

Again by the excctness of the complex of sheaves

$$0 \to \bar{\Omega}^1 \to \mathcal{A}^{0,1} \xrightarrow{\partial} \mathcal{A}^{1,1} \xrightarrow{\partial} \mathcal{A}^{2,1} \to$$

since $dT_\omega = \partial T_\omega = 0$, $\exists\ \phi \in \mathcal{A}^{0,1}$ s.t $\partial\phi = T_\omega$

Similarly, $\bar{\partial}\psi = T_\omega$. ψ is of type (1.0). $\Rightarrow \partial\psi$ is a form of type $(2,0)$. and $\bar{\partial}(\partial\psi) = 0$. $\Rightarrow \partial\psi$ is a closed holomorphic 2-form. $\Rightarrow$ By the d-Poincare lemma, $\partial\psi = d\zeta$. for a holomorphic 1-form ζ. $\Rightarrow T_\omega = \bar{\partial}\eta'$ where $\eta' = \psi - \zeta$ satisfies $\partial\eta' = 0$. $\Rightarrow$ By the ∂-Poincare lemma, $\eta' = \partial\gamma$. for some $\gamma \in C^\infty$. Choose $\varphi = \frac{1}{2}(\gamma + \bar{\gamma}) \Rightarrow \partial\bar{\partial}\varphi = T_\omega$.

Imitate the proof of Lemma ($\partial\bar{\partial}$-Poincare lemma) on P387.