which tends to $\varphi(0)$ as $\epsilon \to 0$.

$\ulcorner$ $\underbrace{\min_{x \in \epsilon K} \varphi(x)}_{L_1(\epsilon)} \leq \varphi(x) \leq \underbrace{\max_{x \in \epsilon K} \varphi(x)}_{L_2(\epsilon)}$ for all $x \in \epsilon K$, and for a fixed ϵ.

$$\Rightarrow \underbrace{\int_{\mathbb{R}^n} L_1 \chi_\epsilon(x)\, dx}_{L_1(\epsilon)} \leq \int_{\mathbb{R}^n} \varphi(x) \chi_\epsilon(x)\, dx \leq \int_{\mathbb{R}^n} L_2 \varphi(x)\, dx$$

$$L_1(\epsilon) \leq \int_{\mathbb{R}^n} \varphi(x) \chi_\epsilon(x)\, dx \leq L_2(\epsilon)$$

As $\epsilon \to 0$, $\min_{x \in \epsilon K} \varphi(x) \longrightarrow \varphi(0)$. and $\max_{x \in \epsilon K} \varphi(x) \longrightarrow \varphi(0)$. $\lrcorner$

Having "smoothed" the δ-function, for a general distribution $T \in \mathcal{D}(\mathbb{R}^n)$, we consider the function.

$$T_\epsilon(x) = T_y(\chi_\epsilon(x-y)),$$

where we use the subscript y on T to indicate that we consider $\chi_\epsilon(x-y)$ as a function of y and apply T accordingly. $T_\epsilon(x)$ is a C^∞-function on $\mathbb{R}^n$ with derivatives

$$D^\alpha T_\epsilon(x) = \pm T_y(D^\alpha_x(\chi_\epsilon(x-y))).$$

$\ulcorner$ $D^\alpha_x T_\epsilon(x) = D^\alpha_x T_y(\chi_\epsilon(x-y))$.

$$D^1_x T_\epsilon(x) = \lim_{h \to 0} \frac{T_y(\chi_\epsilon(x_1+h-y_1, x_2-y_2 \cdots)) - T_y(\chi_\epsilon(x_1-y_1, \cdots))}{h}$$

$$= \lim_{h \to 0} T_y\left(\frac{\chi_\epsilon(x_1-y_1+h, \cdots) - \chi_\epsilon(x_1-y_1, \cdots)}{h} \right)$$

because of the continuity of T_y.

$$= T_y(D^1 \chi_\epsilon(x-y)) \Rightarrow D^\alpha T_\epsilon(x) = T_y(D^\alpha_x \chi_\epsilon(x-y)). \lrcorner$$