

$$- \dim(B+C) - \dim(A+C) + \dim(A+B+C).$$

$$\geq \alpha + \beta + \gamma - \dim(B+C) - \dim(A+C) + \dim(A+B+C) - \dim(A+B)$$

Since $\dim(B+C), \dim(A+C) \leq k$ and $\dim(A+B) \leq \dim(A+B+C)$,

$$\geq \alpha + \beta + \gamma - 2k = 2k+1 - 2k = 1.$$

$$\Rightarrow \dim(A \cap B \cap C) \geq 1.$$

□

Thus $\#(\sigma_a \cdot \sigma_b \cdot \sigma_c) = 0$ in $G(k, n)$ if

$$(k - \alpha + a_\alpha) + (k - \beta + b_\beta) + (k - \gamma + c_\gamma) > n-1,$$

i.e. if

$$a_\alpha + b_\beta + c_\gamma > n - k.$$

□ Let $k - \alpha + a_\alpha = x_\alpha$, $k - \beta + b_\beta = y_\beta$, & $k - \gamma + c_\gamma = z_\gamma$.

We can choose V_{n-x_α} and V'_{n-y_β} so that

$$\dim(V_{n-x_\alpha} \cap V'_{n-y_\beta}) = n - x_\alpha + n - y_\beta - n = n - (x_\alpha + y_\beta).$$

Choose V''_{n-z_γ} so that

$$\dim(V''_{n-z_\gamma} \cap (V_{n-x_\alpha} \cap V'_{n-y_\beta})) = n - z_\gamma - n - (x_\alpha + y_\beta)$$

$$- n = n - (x_\alpha + y_\beta + z_\gamma)$$

Thus if $x_\alpha + y_\beta + z_\gamma \geq n$,

$$\dim(V''_{n-z_\gamma} \cap V_{n-x_\alpha} \cap V'_{n-y_\beta}) = 0 \text{ so that}$$

$$\exists \text{ no } \Lambda \in \sigma_a(V) \cap \sigma_b(V') \cap \sigma_c(V'').$$

□

Suppose on the other hand that $a_\alpha + b_\beta + c_\gamma = n - k$, i.e., that generically chosen subspaces $V_{n-k+\alpha-a_\alpha}$, $V'_{n-k+\beta-b_\beta}$ and $V''_{n-k+\gamma-c_\gamma}$ will intersect