

$$\begin{aligned}
\Rightarrow \partial \bar{\partial} V &= \partial \left(\frac{\partial V}{\partial \bar{z}_j'} \right) \wedge d \bar{z}_j' \\
&= \frac{\partial^2 V}{\partial z_i' \partial \bar{z}_j'} d z_i' \wedge d \bar{z}_j' \\
&= \frac{\partial^2 V}{\partial z_i' \partial \bar{z}_j'} \frac{\partial z_i'}{\partial z_n} d z_n \wedge \frac{\partial \bar{z}_j'}{\partial \bar{z}_k} d \bar{z}_k \\
&= \frac{\partial}{\partial z_i'} \left(\frac{\partial V}{\partial \bar{z}_l} \frac{\partial \bar{z}_l}{\partial \bar{z}_j'} \right) \frac{\partial z_i'}{\partial z_n} d z_n \wedge \frac{\partial \bar{z}_j'}{\partial \bar{z}_k} d \bar{z}_k \\
&= \left(\frac{\partial}{\partial z_i'} \left(\frac{\partial V}{\partial \bar{z}_l} \right) \right) \frac{\partial \bar{z}_l}{\partial \bar{z}_j'} \frac{\partial z_i'}{\partial z_n} d z_n \wedge \frac{\partial \bar{z}_j'}{\partial \bar{z}_k} d \bar{z}_k \\
&\quad + \frac{\partial V}{\partial \bar{z}_l} \frac{\partial}{\partial z_i'} \left(\frac{\partial \bar{z}_l}{\partial \bar{z}_j'} \right) \boxed{} \\
&= \frac{\partial^2 V}{\partial z_p \partial \bar{z}_l} \frac{\partial z_p}{\partial z_i'} \frac{\partial \bar{z}_l}{\partial \bar{z}_j'} \frac{\partial z_i'}{\partial z_n} d z_n \wedge \frac{\partial \bar{z}_j'}{\partial \bar{z}_k} d \bar{z}_k \\
&= \frac{\partial^2 V}{\partial z_p \partial \bar{z}_l} \frac{\partial z_p}{\partial z_i'} \frac{\partial z_i'}{\partial z_n} \frac{\partial \bar{z}_l}{\partial \bar{z}_j'} \frac{\partial \bar{z}_j'}{\partial \bar{z}_k} d z_n \wedge d \bar{z}_k \\
&= \frac{\partial^2 V}{\partial z_p \partial \bar{z}_l} \frac{\partial z_p}{\partial z_n} \frac{\partial \bar{z}_l}{\partial \bar{z}_k} d z_n \wedge d \bar{z}_k = \frac{\partial^2 V}{\partial z_p \partial \bar{z}_l} d z_p \wedge d \bar{z}_k
\end{aligned}$$

Since $\frac{\partial z_p}{\partial z_n} = \delta_{p,n}$ $\frac{\partial \bar{z}_l}{\partial \bar{z}_k} = \delta_{l,k}$ $\frac{\partial z_p}{\partial z_i'} \frac{\partial z_i'}{\partial z_n} = \frac{\partial z_p}{\partial z_n}$ (\because

$\frac{\partial z_p}{\partial \bar{z}_l} = 0$) J

We have a class of functions defined on a complex analytic variety and no longer merely on a complex vector space.