readily solved in closed form; more generally, any integral

$$\int R(x, \sqrt{x^2+ax+b})\, dx,$$

for R a rational function, has a closed-form solution involving only elementary function. The solutions to integrals of this type have been known since the early days of calculus. For a long time however, mathematicians were unable to do much with the integrals

$$(**) \qquad \int \frac{dx}{\sqrt{x^3+ax^2+bx+c}}$$

or, more generally, the Abelian integrals

$$\int R(x, y)\, dx$$

where R is a rational function, and x and y are related by a polynomial equation $f(x, y) = 0$ of degree > 2.

In view of the genus formula of the last section, one reason for the difficulty is easy to spot: the first integral $(*)$ can be thought of as the line integral

$$\int \frac{dx}{y}$$

of the meromorphic form dx/y on the curve C given in terms of Euclidean coordinates x, y in $\mathbb{P}^2$ by $y^2 = x^2 + ax + b$. Now C is a conic curve, hence isomorphic to $\mathbb{P}^1$ via a polynomial map; if $t = t(x, y)$ is a Euclidean coordinate on $\mathbb{P}^1$, the meromorphic form dx/y on C