$$\varphi = \{\varphi_\alpha\} \longmapsto \{(\delta\varphi)_{\alpha\beta}\} = \{\varphi_\beta - \varphi_\alpha\} = \{-(\partial^*\eta)_{\alpha\beta}\}$$
$$= \{-\partial^*\eta_{\alpha\beta}\}$$

Furthermore, the sheaf map is isomorphic.

$\Rightarrow$ Define $\mathcal{E} = \dfrac{\coprod \mathcal{E}_\alpha}{\text{"}x_\alpha \sim \varphi_{\alpha\beta}(x_\alpha)\text{"},\ x_\alpha \in \mathcal{E}_\alpha}$

"some sort of patch"

which is well-defined, since

$$\begin{tikzcd} & \mathcal{E}_\alpha|_{U_\alpha\cap U_\beta\cap U_\gamma} \arrow[dl, "\varphi_{\alpha\beta}"'] \arrow[dr, "\varphi_{\alpha\gamma}"] & \\ \mathcal{E}_\beta|_{U_\alpha\cap U_\beta\cap U_\gamma} \arrow[rr, "\varphi_{\beta\gamma}"] & & \mathcal{E}_\gamma|_{U_\alpha\cap U_\beta\cap U_\gamma} \end{tikzcd} \quad \text{commutative}$$

$$\varphi_{\beta\gamma}\circ\varphi_{\alpha\beta}\,(f, e_\circ) = ((id - \eta_{\beta\gamma})\oplus id)\circ((id-\eta_{\alpha\beta})\oplus id)(f, e_\circ)$$
$$= ((id-\eta_{\beta\gamma})\oplus id)\,(f - \eta_{\alpha\beta}(e_\circ);\ e_\circ) = (f - \eta_{\alpha\beta}(e_\circ) - \eta_{\beta\gamma}(e_\circ),\ e_\circ)$$
$$= (f - \eta_{\alpha\gamma}(e_\circ),\ e_\circ) = ((id - \eta_{\alpha\gamma})\oplus id)(f, e_\circ) = \varphi_{\alpha\gamma}(f, e_\circ)$$

since $(\delta\eta)_{\alpha\beta\gamma} = \eta_{\beta\gamma} - \eta_{\alpha\gamma} + \eta_{\alpha\beta} = 0 \Leftrightarrow \delta\eta = 0$ ⅃

## Points on a Surface and Rank-Two Vector Bundles

As an application of the global duality theorem (I), we shall discuss the following question:* Given an algebraic surface $S$ and sheaf of regular ideals $\mathcal{I} \subset \mathcal{O}$ with $\mathrm{supp}(\mathcal{O}/\mathcal{I})$ a set of points $Z$, we define $\mathcal{O}_Z = \mathcal{O}/\mathcal{I}$ and ask whether there