Thus we proved that

$$\det\begin{pmatrix} h_{11}(x), & h_{12}(y), & h_{13}(z) \\ h_{21}(x), & h_{22}(y), & h_{23}(z) \\ h_{31}(x), & h_{32}(y), & h_{33}(z) \end{pmatrix} = 0 \text{ everywhere}$$

$\Rightarrow \exists$ a, b, c, s.t a, b, c constants.

$$a\begin{pmatrix} h_{11}(x) \\ h_{12}(x) \\ h_{13} \end{pmatrix} + b\begin{pmatrix} h_{21} \\ h_{22} \\ h_{23} \end{pmatrix} + c\begin{pmatrix} h_{31} \\ h_{32} \\ h_{33} \end{pmatrix} = 0.$$

For the general case, we can use the induction.

From the result above, $\begin{bmatrix} (\omega_1/dz_1)(P_1) & \cdots & (\omega_1/dz_d)(P_d) \\ \vdots & & \\ (\omega_g/dz_1)(P_1) & & (\omega_g/dz_d)(P_d) \end{bmatrix}$ has the

maximal rank for the generic points, since we can see the matrix as the following matrix

$$\begin{bmatrix} h_{11}(z_1), & h_{12}(z_2), & \cdots, & h_{1d}(z_d) \\ \vdots & \vdots & & \vdots \\ h_{g1}(z_1), & h_{g2}(z_2), & \cdots & h_{gd}(z_d) \end{bmatrix}.$$

Note that the set $\{(P_1, P_2 \cdots P_d) \mid \operatorname{rank}(\omega_i/dz_j(P_j)) \neq$ maximal rank $\}$ is analytic variety in S^d, since the det of every minor $d\times d$ matrix is zero, when $d \leq g$.

An effective divisor D such that $h^0(K-D) \neq 0$ is call