The main property of the Ext functor is:

Short exact sequences of $\mathcal{O}$-modules

$$\begin{cases} 0 \to M' \to M \to M'' \to 0, \\ 0 \to N' \to N \to N'' \to 0, \end{cases}$$

induce long exact sequences

$$\begin{cases} \cdots \to \mathrm{Ext}^n_{\mathcal{O}}(M, N) \to \mathrm{Ext}^n_{\mathcal{O}}(M', N) \to \mathrm{Ext}^{n+1}_{\mathcal{O}}(M'', N) \to \cdots \\ \cdots \to \mathrm{Ext}^n_{\mathcal{O}}(M, N) \to \mathrm{Ext}^n_{\mathcal{O}}(M, N'') \to \mathrm{Ext}^{n+1}_{\mathcal{O}}(M, N') \to \cdots, \end{cases}$$

of Ext's.

Proof. First we note that a short exact sequence of free $\mathcal{O}$-modules splits, as indicated by the dotted arrow in the diagram

$$\begin{tikzcd} 0 \arrow[r] & E' \arrow[r] & E \arrow[r] & E'' \arrow[r] \arrow[l, bend right, dashed] & 0. \end{tikzcd}$$

$\ulcorner$ By the projectiveness of E'',

$$\begin{tikzcd} E'' \arrow[d, dashed] \arrow[dr, "id"] \arrow[dr, phantom, "\circlearrowleft", shift right=3] & & \\ E \arrow[r] & E'' \arrow[r] & 0 \end{tikzcd}$$

$\lrcorner$

Thus $E \cong E' \oplus E''$, and consequently

$$0 \to \mathrm{Hom}_{\mathcal{O}}(E'', N) \to \mathrm{Hom}_{\mathcal{O}}(E, N) \to \mathrm{Hom}_{\mathcal{O}}(E', N) \to 0$$

is exact for any $\mathcal{O}$ module N. Choose projective resolutions so that

$$0 \to E.(M') \to E.(M) \to E.(M'') \to 0$$

is exact, it follows that

$$0 \to \mathrm{Hom}_{\mathcal{O}}(E.(M''), N) \to \mathrm{Hom}_{\mathcal{O}}(E.(M), N) \to \mathrm{Hom}_{\mathcal{O}}(E.(M'), N) \to 0$$