$$\begin{tikzcd} m_2 : A \arrow[r] & A \\ m_2^* B_{L_0} \arrow[r] \arrow[u, phantom, "\cup"] & B_{L_0} \arrow[u, phantom, "\cup"] \end{tikzcd} \quad \Rightarrow \quad m_2^* B_{L_0} \ni L \iff \underset{\substack{\| \\ 2L.}}{m_2(L)} \in B_{L_0}$$

In particular,

$$\#(\tilde{D}' \cdot B_{L_0}) = 4 \cdot (B_{L_0} \cdot B_{L_0}) = 8.$$

$\#(\tilde{D}' \cdot B_{L_0}) = \#(m_2^* B_{L_0} \cdot B_{L_0})$

Claim: $m_2^* B_{L_0} = 4 B_{L_0}$

To prove this, consider the following diagram, let $f = m_2$,

$$\begin{tikzcd} {[f^* B_{L_0}]} \arrow[r] \arrow[d, shift right] & {[B_{L_0}]} \arrow[d, shift right] \\ A \arrow[r, "f"] \arrow[u, shift right, "f^*\sigma"'] & A \arrow[u, shift right, "\sigma"'] \end{tikzcd} \qquad (\sigma = 0) = B_{L_0}$$

$\Rightarrow$ We have only to show that $c_1(f^* B_{L_0}) = 4\, c_1(B_{L_0})$, since $c_1(f^* B_{L_0})$ & $c_1(B_{L_0})$ are dual to $[f^* B_{L_0}]$ and $[B_{L_0}]$. $\Rightarrow$ By P307, since $J(B_{L_0}) = A$, we have $dx_1, \cdots dx_4$ for $H^1(J(B_{L_0}), \mathbb{Z}) = H^1(A, \mathbb{Z})$ s.t $\omega = dx_1 \wedge dx_3 + dx_2 \wedge dx_4$. $\Rightarrow$ By Lemma on P310, $c_1(B_{L_0}) = [\omega]$. $\Rightarrow$ $f^* c_1(B_{L_0}) = [f^* \omega]$
$= [f^* dx_1 \wedge f^* dx_3 + f^* dx_2 \wedge f^* dx_4]$.
$= [2\,dx_1 \wedge 2\,dx_3 + 2\,dx_2 \wedge 2\,dx_4] = 4[\omega] = 4\, c_1(B_{L_0})$
since $f^* dx_i = 2\,dx_i$. $\Rightarrow$ The claim is proved.