$$= C_n \int_{\|z_\alpha\|=\epsilon} \frac{\sum \overline{\Phi_i(w_\alpha)} \wedge \det(J(f))\, dz_{\alpha_1}\wedge\cdots\wedge dz_{\alpha_n}}{\|w_\alpha\|^{2n}}$$

$$= C_n \int_{\|z_\alpha\|=\epsilon} \frac{\sum \overline{\Phi_i(w_\alpha)} \wedge dw_{\alpha_1}\wedge\cdots\wedge dw_{\alpha_n}}{\|w_\alpha\|^{2n}} \quad \frac{\det(J(f))}{\det(I-J(f))}$$

$$= C_n \int_{\|w_\alpha\|=\epsilon} \frac{\sum \overline{\Phi_i(w_\alpha)} \wedge dw_{\alpha_1}\wedge\cdots\wedge dw_{\alpha_n}}{\|w_\alpha\|^{2n}} \quad \frac{\det(J(f))}{\det(I-J(f))}.$$

(since $K(0, w_\alpha)$ is concentrated at the origin)

$$= \frac{\det(J(f x_\alpha))}{\det(I-J(f x_\alpha))} = \frac{\det(B_\alpha)}{\det(I-B_\alpha)}.$$

I don't know whether the above computation is correct or not.

Summary

To compute $\eta^\circ_\Delta(\Gamma_f)$, we considered the distribution T°_Δ given by

$$T^\circ_\Delta(\varphi) = \int_\Delta \sum_q \varphi^{(n,n-q),(0,q)}.$$

T°_Δ corresponds to the cohomology class η°_Δ.
We need to know a concrete differential form representing η°_Δ. To get the form, we get $\varphi = T^\circ_\Delta - \partial k$.
s.t.
① φ is $\bar\partial$-closed current representing η°_Δ
② φ is smooth in some open set containing Γ_f
③ $\varphi = -\partial k$ outside Δ.

Thus we computed $\eta^\circ_\Delta(\Gamma_f)$ by using $-\partial k$, where K is locally the Bochner-Martinelli kernel.