Moreover, multiplication by L^k commutes with the differentials in the spectral sequence, as follows by the formal rules for calculating with $\iota(v)$. Therefore, to prove that $d_1'=0$, by using the primitive decomposition and hard Lefschetz it will suffice to prove that this is the case on primitive cohomology.

┌ $\iota(v)(L(\varphi)) = \iota(v)(\omega\wedge\varphi) = \iota(v)(\omega)\wedge\varphi + (-1)^{\deg\omega}\omega\wedge \iota(v)\varphi = 0\wedge\varphi + (-1)^2\omega\wedge\iota(v)\varphi = \omega\wedge\iota(v)\varphi = L(\iota(v)\varphi)$

by Lichnerowicz' lemma.

$\Rightarrow$ $\iota(v)$ commutes with L and so does with L^k.

$$\begin{array}{ccc} H^{n-p,q}(M) & \xrightarrow{\iota(v)} & H^{n-p-1,q}(M) \\ \parallel & & \parallel \\ \oplus L^k P^{n-p-k,q-k} & \longrightarrow & \oplus L^k P^{n-p-1-k,q-k}(M) \end{array}$$

$\Rightarrow$ We have only to prove that $\iota(v): P^{n-p-k,q-k} \to P^{n-p-1-k,q-k}(M)$ is zero. ┘

Let $\psi \in P^{n-q-k,q}(M) \subset {}'E^{q+k,q}$.

┌ $P^{n-q-k,q}(M) \subset H^{n-q-k,q}(M) \cong {}'E_1^{q+k,q}$ ┘

Then, essentially repeating the argument from the degeneration of the Leray spectral sequence in Section 6 of Chapter 3,

$$0 = \iota(v)(\omega^{k+1}\psi) = \omega^{k+1}\iota(v)\psi$$