It leads to the following useful characterization of distributions.

6.8 Theorem If Λ is a linear functional on $\mathcal{D}(\Omega)$, the following two conditions are equivalent:

(a) $\Lambda \in \mathcal{D}'(\Omega)$.

(b) To every compact $K \subset \Omega$ corresponds a nonnegative integer N and a constant $C < \infty$ such that the inequality

$$|\Lambda \phi| \leq C \|\phi\|_N$$

holds for every $\phi \in \mathcal{D}_K$.

proof. This is precisely the equivalence of (a) and (b) in Theorem 6.6, combined with the description of the topology of $\mathcal{D}_K$ by means of the seminorms $\|\phi\|_N$ given in Section 6.2. ///

Consider $W = \{ \phi \in \mathcal{D}_K ; \ \|\phi\|_N \leq 1 \}$.

$\Rightarrow$ W is bounded. $\Rightarrow$ $\Lambda(W) \subset C \ B(0,1)$. $C > 0$.

$\Rightarrow$ $|\Lambda(W)| < C$ $\Rightarrow$ $\left| \Lambda\left(\frac{\phi}{\|\phi\|_N}\right) \right| < C$ for all $\phi \in \mathcal{D}_K$.

$\Rightarrow$ $|\Lambda(\phi)| < C \|\phi\|_N$.

Note: If Λ is such that one N will do for all K (but not necessarily with the same C), then the smallest such N is called the order of Λ. If no N will do for all K, then Λ is said to have infinite order.

$V_N = \{ \phi \in \mathcal{D}(\Omega) ; \ \|\phi\|_N < \frac{1}{N} \}$

Suppose that N is the order of Λ.

$\Rightarrow$ To every compact $K \subset \Omega$, $\exists$ C_K s.t $|\Lambda \phi| \lesssim C_K \|\phi\|_N$.