

$\Rightarrow$  By the result on P757,

$$\sigma_1(l_x) = \sigma_1(l_y) \quad \text{for some } x \neq y.$$

This is absurd, for  $\exists$  a line  $l$  s.t.  
 $l \cap l_x \neq \emptyset$ , but  $l \cap l_y = \emptyset$  since  $l, l_x, l_y \subset \mathbb{P}^3$   
and  $l_x \neq l_y$ .

Thus  $\dim \sigma(p) = 2, \Rightarrow \dim \{T_x(G)\}_{x \in \sigma(p)} = 2$  in  $\mathbb{P}^{5*}$ .

$\sqcup$

Thus  $X_p$  must be a line, that is,

For each  $p \in \mathbb{P}^3$ , the lines of  $X$  through  $p$  form  
a pencil  $\sigma(p, h)$ .

$\Gamma$  Suppose  $X_p = \sigma(p)$ .  $\Rightarrow \sigma(p) \subset H \Rightarrow$  By the  
result above,  $H = T_x(G)$  for some  $x \in G$ .

$\Rightarrow X = H \cap G = T_x(G) \cap G$  is singular which  
is impossible since we assume that  $X$  is  
smooth.  $\Rightarrow$  The lines of  $X$  through  $p = X_p =$   
 $\sigma(p) \cap H = X \cap \sigma(p) = G \cap H \cap \sigma(p)$

$\sqcup$

Likewise,  $H$  can not contain the  $\alpha$ -plane  $\sigma(h)$   
for any hyperplane  $h \subset \mathbb{P}^3$ , and so

For each hyperplane  $h \subset \mathbb{P}^3$ , the lines of  $X$   
lying in  $h$  form a pencil  $\sigma(p, h)$ .