$\Rightarrow\ 0 \longrightarrow \mathcal{O}_M(E\otimes L^{m-1}) \xrightarrow{\otimes s_0} \mathcal{J}_{x,M}(E\otimes L^m) \longrightarrow \mathcal{J}_{x,V_\alpha}(E\otimes L^m) \to 0$

is exact.

To show the exactness, we have only to show that for each $p\in M$,

$$0 \longrightarrow \mathcal{O}_M(E\otimes L^{m-1})_p \longrightarrow \mathcal{J}_{x,M}(E\otimes L^m)_p \longrightarrow \mathcal{J}_{x,V_\alpha}(E\otimes L^m)_p \to 0$$

is exact. (See Hartshorne P 65 Caution 1.2.1).

It remains to show the following:

Given an exact sequence $0 \longrightarrow A \xrightarrow{f} B \xrightarrow{g} C \to 0$. is $0 \to A \xrightarrow{f} B' \xrightarrow{g} C' \longrightarrow 0$ exact?

Here $f(A) \subset B'$, and $g(B') = C'$.

Obviously, f is one to one, & $g\circ f = 0$.

$\ker g \ni b \Rightarrow \exists\ a\in A$ s.t $f(a) = b$

Since $f(A)\subset B'$, $b \in B'$, by the assumption,

Thus for $m > m_0' = \max(m_1, m_\alpha)$, we have

$$\begin{CD}
H^0(M, \mathcal{J}_x(E\otimes L^m)) @>{d_x}>> T_x^{*\prime}(M)\otimes (E\otimes L^m)_x \\
@V{r}VV @VV{p\otimes id}V \\
H^0(V_\alpha, \mathcal{J}_x(E\otimes L^m)) @>{d_x}>> T_x^{*\prime}(V_\alpha)\otimes (E\otimes L^m)_x .
\end{CD}$$

for all α, i.e., the map $(**)$ is surjective. Q.E.D.

「 By the arguments above, r is surjective, and $\omega_\alpha \otimes v \notin \ker(p\otimes id)$ for all $v \neq 0$.

Actually, we need more careful considerations.

Point is the following:

Given a vector $v \in T_x'(M)$, by the result