We write $w = f(z)$, denote by $K = dw_1/w_1 \wedge \cdots dw_n/w_n$ the Cauchy kernel, and set

$$\omega(f_1 \cdots f_n) = f^*K = \frac{df_1}{f_1} \wedge \cdots \wedge \frac{df_n}{f_n} .$$

The local intersection number is defined by

$$(D_1 \cdots D_n)_{\{0\}} = \operatorname{Res}_{\{0\}} \omega(f_1 \cdots f_n).$$

We shall give a list of its properties:

(a) $(D_1 \cdots D_n)_{\{0\}}$ is an integer that depends only on the ideal $I(f)$ and not the choice of generators f_i. In particular, it depends only on the divisors D_i and not on their defining functions.

Proof. $(1/2\pi\sqrt{-1})^n \omega(f_1 \cdots f_n)$ represents an integral cohomology class in $H^n_{DR}(U-D)$, and so the intersection number is an integer.

⌈ Since $\frac{1}{2\pi\sqrt{-1}} \frac{dw_i}{w_i}$ represents an integral cohomology class in $H^1_{DR}(U-\{0\})$, where U is a ball around the origin, $f^*\left(\frac{1}{2\pi\sqrt{-1}} \frac{dw_i}{w_i}\right)$ is an integral cohomology in $H^1_{DR}(U-D)$ too. ⌋

If $f_i' = \sum a_{ij} f_j$

where $\Delta = \det(a_{ij}) \neq 0$, then

$$\frac{df_1' \wedge \cdots \wedge df_n'}{f_1' \cdots f_n'} = \Delta \frac{df_1 \wedge \cdots \wedge df_n}{f_1' \cdots f_n'} + g \frac{dz_1 \wedge \cdots \wedge dz_n}{f_1' \cdots f_n'} ,$$

where g is in the ideal.