es – or rather their Poincare duals in homology – is available, and the remainder of this section will be spent in deriving it. The computation will not be made directly; instead, we will first compute the Chern classes of the universal bundles on the Grassmannians, and then by the functoriality of the Chern classes draw conclusions for general vector bundles.

Recall from Section 6 of Chapter 1 that for any strictly increasing flag

$$0 = V_0 \subset V_1 \subset \cdots \subset V_{n-1} \subset V_n = \mathbb{C}^n$$

of linear subspaces of $\mathbb{C}^n$, and for any nonincreasing sequence of k integers a_i : $0 \le a_i \le n-k$, we define the Schubert cycle $\sigma_a(V) \subset G(k,n)$ in the Grassmannian of k-planes in $\mathbb{C}^n$ by

$$\sigma_a(V) = \{ \Lambda : \dim(\Lambda \cap V_{n-k+i-a_i}) \ge i \}.$$

$\sigma_a(V)$ is an analytic subvariety of $G(k,n)$ of codimension $\sum a_i$, with fundamental class σ_a independent of the flag V; as we saw, the integral homology of the Grassmanian is freely generated by the classes σ_a.

「 See p196, Proposition 」

Recall also that the universal subbundle $S \to G(k,n)$ is defined to be the subbundle of the trivial bundle $\mathbb{C}^n \times G(k,n)$ whose fiber over a point $\Lambda \in G(k,n)$ is just the k-plane $\Lambda \subset \mathbb{C}^n$. Letting σ_a^* denote the Poincare dual of the cycle σ_a, our fundamental result is the