

② When we choose P_{11}, P_{12} , we take them outside $\overline{P_1 P_{11}}$. Remember $IP_{11}^{n-1} \cap \overline{P_1 P_{11}} = \emptyset$.

(ii) By P822 note, in other words, by applying the arguments on P822 note, we can conclude that

$T_{P_1}(F \cap \overline{V_8}) \cap T_{P_{11}}(F \cap \overline{V_8}) \cap T_{P_{12}}(F \cap \overline{V_8}) \cap (F \cap \overline{V_8})$ is the cone through $\overline{P_1 P_{11} P_{12}}$ over a smooth quadric of dimension 0. More precisely, choose an $(n-5)$ -plane IP_{11}^{n-5} not intersecting with $\overline{P_1 P_{11} P_{12}}$ in $T_{P_1} \cap T_{P_{11}} \cap T_{P_{12}}$.

$$\Rightarrow IP_{11}^{n-5} \cap \overline{V_8} \subset T_{P_1}(F \cap \overline{V_8}) \cap T_{P_{11}}(F \cap \overline{V_8}) \cap T_{P_{12}}(F \cap \overline{V_8}) \cap \overline{V_8}$$

Without loss of generality we may assume $\Rightarrow \{X_7=0\} \cap \{X_6=0\} \cap \{X_5=0\} \cap \overline{V_8} = IP^1$

$\Rightarrow IP_{11}^{n-5} \cap \overline{V_8}$ is a line since $IP_{11}^{n-5} \cap \overline{V_8} \subset IP^4$
 $\dim IP_{11}^{n-5} + \dim \overline{V_8} = n-5+1 > n+1$, and $IP_{11}^{n-5} \cap \overline{V_8}$ does not intersect with $IP^2 = \overline{P_1 P_{11} P_{12}}$.

$\Rightarrow IP_{11}^{n-5} \cap \overline{V_8} = IP^1 \Rightarrow$ By P134, $IP_{11}^{n-5} \cap \overline{V_8} \cap F$ is a smooth quadric of dim 0. $\Rightarrow IP_{11}^{n-5} \cap \overline{V_8} \cap F = \{P_{111}, P_{112}\}$.

We can see easily this by the following example.

We may assume $F = (X_0^2 + X_1^2 + \dots + X_{n+1}^2 = 0)$ & $\overline{V_8} = \{[* * \dots * 0 \dots 0]\}$, and find P_i 's. P_{ij} 's. ... //

$\Lambda \ni P_i, P_{ij}$. $q \in \Lambda \subset T_{P_i}(F) \cap T_{P_{ij}}(F) \cap F =$ The cone through $\overline{P_i P_{ij}}$ over $F \cap IP_{ij}^{n-3}$.

(i) $q \in \overline{P_i P_{ij}} \Rightarrow$ done

(ii) $q \notin \overline{P_i P_{ij}}$

$$\Rightarrow q \in \overline{px}, p \in \overline{P_i P_{ij}}, x \in F \cap IP_{ij}^{n-3}$$

$$\Rightarrow \overline{pq} \cap F \cap IP_{ij}^{n-3} \subset \Lambda \cap IP_{ij}^{n-3} \cap \overline{pq} \Rightarrow \bigcup_x$$