Consequently, if we can show that

$$\mathrm{Res}_{\{0\}}\left(\frac{h(z,w)\, dz\wedge dw}{f(z,w)\ g(z,w)}\right) = 0 \iff \alpha = 0,$$

our assertion will follow from (*).

Pf

$$0 = \sum_{\nu} \mathrm{Res}_{p_\nu}\left\{\frac{h(z,w)\,dz\wedge dw}{f(z,w)\ g(z,w)}\right\} = \mathrm{Res}_{p_{\nu_0}}\left\{\frac{h(z,w)\,dz\wedge dw}{f(z,w)\ g(z,w)}\right\}$$

$$+ \sum_{\nu\neq\nu_0} \mathrm{Res}_{p_\nu}\left\{\frac{h(z,w)\,dz\wedge dw}{f(z,w)\,g(z,w)}\right\}$$ $\Rightarrow$ By choosing a holomorphic coordinates around p_ν, s.t $f(z,w)=z$,

$$\mathrm{Res}_{\{0\}}\left(\frac{h(z,w)\ dz\wedge dw}{f(z,w)\ g(z,w)}\right)$$

$$= \mathrm{Res}_{\{0\}}\left(\frac{h(z,w)\,dz\wedge dw}{z\ g(z,w)}\right) = \left(\frac{1}{2\pi\sqrt{-1}}\right)^2 \int_{|z|=\epsilon}\int_{|w|=\epsilon} \frac{h(z,w)}{z\ g(z,w)}\,dz\wedge dw$$

$$= \frac{1}{2\pi\sqrt{-1}} \int_{|w|=\epsilon} \frac{h(0,w)}{g(0,w)}\,dw.$$

But since $(E\cdot C)_{p_\nu} \geq (C\cdot D)_{p_\nu} = m_\nu$,

$$g(0,w) = a\,w^{m_\nu}$$
$$h(0,w) = a'\,w^{(E\cdot C)_{p_\nu}} \quad\Rightarrow\quad \frac{h(0,w)}{g(0,w)} = \frac{a'}{a}\,w^{(E\cdot C)_{p_\nu}-m_\nu}$$

$$\Rightarrow \frac{1}{2\pi\sqrt{-1}}\int_{|w|=\epsilon} \frac{a'}{a}\ w^{(E\cdot C)_{p_\nu}-m_\nu}\,dw = 0$$

$$\Rightarrow\quad \mathrm{Res}_{p_{\nu_0}}\left(\frac{h(z,w)\ dz\wedge dw}{f(z,w)\ g(z,w)}\right) = 0 \iff \alpha = 0$$