

Moreover, since $D_i \cdot D_j = 0$, the lines D_i on S are disjoint, so that the image $\pi_i(D_j)$ of D_j under the blowing-down π_i of D_i is again a smooth rational curve of self-intersection -1 .

Thus we can blow down all six divisors D_i in turn; let \tilde{S} be the surface obtained by blowing them down.

We observe first that the Betti numbers of \tilde{S} are

$$b^0(\tilde{S}) = b^4(\tilde{S}) = 1,$$
$$b^1(\tilde{S}) = b^3(\tilde{S}) = 0,$$
$$b^2(\tilde{S}) = b^2(S) - 6 = 7 - 6 = 1.$$
$$\begin{aligned} \mathbb{F} \quad \mathbb{C} &= H^0(\tilde{S}) = H^4(\tilde{S}) \quad \text{by Poincaré duality.} \\ H^1(\tilde{S}) &= H^3(\tilde{S}) \quad \text{"} \quad \text{and } H^1(\tilde{S}) = 0 \text{ since} \end{aligned}$$