

⌈ $\dim X = 3$ and $\deg X = 4 \Rightarrow$ For a generic 2-plane V_2 , $V_2 \cap X$ is a set of 4 distinct points.

⌋

By Bertini, the generic element of this pencil is smooth away from the base locus. But the base locus of this pencil consists of the 16 lines of X passing through the four points of $V_2 \cap X$, and the 16 lines lying in the hyperplane section $V_4 \cap X$ of X — 32 distinct lines in all.

⌈ $A \cap (\bigcap_{V_2 \subset V_3 \subset V_4} \sigma_i(V_3)) \ni L \Rightarrow L \subset X$ and $L \cap V_3 \neq \emptyset$
for all $V_3, V_2 \subset V_3 \subset V_4$.

$\Rightarrow L \cap V_2 \neq \emptyset \Rightarrow L \cap X \cap V_2 \neq \emptyset$

\Rightarrow If we let $X \cap V_2 = \{x_1, x_2, x_3, x_4\}$,

$L \subset \bigcup_{i=1}^4 T_{x_i}(X) \cap X \Rightarrow \bigcup_{i=1}^4 T_{x_i}(X) \cap X = 16$ distinct lines

OR, $L \subset X$ and $L \subset V_4 \Rightarrow L \cap V_3 \neq \emptyset$ for all $V_3 \subset V_4 \Rightarrow$ This L can not be one of those above, since V_4 does not contain any line on X meeting V_2 , i.e. V_4 does not contain any of lines in $T_{x_i}(X) \cap X$.

$V_4 \cap X = (V_4 \cap F) \cap (V_4 \cap G)$ is the smooth intersection of two smooth quadrics in $V_4 = \mathbb{P}^4$
 \Rightarrow The # of lines in $V_4 \cap X$ is $\#(4\sigma_{2,1} \cdot 4\sigma_{2,1}) = 16$ as we saw on P550. \Rightarrow The base locus consi-