then a_{k-r+1} must necessarily be $n-k$, i.e., $\sigma_a(V) \subset \{\Lambda \in G(k,n) : \Lambda \supset V_{k-r+1}\}$.

$\ulcorner$ Suppose $a_{k-r+1} \neq n-k$. $\Rightarrow$ Since $a \neq (\underset{a_1}{n-k}, \cdots \underset{a_{k-r}}{n-k}, \underset{a_{k-r+1}}{n-k-1}, \cdots \underset{a_k}{n-k-1})$, $a_{k-r+1} \neq n-k-1$.

$\Rightarrow \quad a_{k-r+1} \leq n-k-2$.

$\Rightarrow \quad a_1 + \cdots + a_{k-r} + a_{k-r+1} + \cdots + a_k \leq (n-k)(k-r) + r(n-k-2)$

$= (n-k)(k-r) + r(n-k) - 2r = (n-k)k - 2r$. $\cdots$ ①

$\operatorname{codim} \sigma_a = 2(n-k)k - 2r = 2\sum a_i$

$\Rightarrow \quad \sum a_i = k(n-k) - r \quad \cdots$ ②

From ① & ②, $k(n-k) - r \leq (n-k)k - 2r$.

$\Rightarrow \quad r \leq 0 \quad \Rightarrow \quad r = 0$ Contradiction to the fact that we assume $r > 0$ implicitly. $\lrcorner$

Thus if we take $e_1, \cdots e_{k-r+1}$ any basis for $V_{k-r+1} \subset \mathbb{C}^n$, the corresponding sections e_i of the trivial bundle $\mathbb{C}^n \times G(k,n)$ all lie in S over $\sigma_a(V)$.

$\ulcorner$ If $a \neq (n-k, \cdots n-k, n-k-1, \cdots n-k-1)$, then $a_{k-r+1} = n-k$.

$\Rightarrow$ If $\Lambda \in \sigma_a(V)$, $\dim(\Lambda \cap V_{n-k+(k-r+1)-a_{k-r+1}})$

$= \dim(\Lambda \cap V_{k-r+1}) \geq k-r+1 \quad \Rightarrow \quad \Lambda \supset V_{k-r+1}$

$\Rightarrow \quad \Lambda \in \{\Lambda \in G(k,n) \mid \Lambda \supset V_{k-r+1}\} \Rightarrow \sigma_a(V) \subset \{\Lambda \in G(k,n) : \Lambda \supset V_{k-r+1}\}$.

$$\begin{array}{ccc} G(k,n) \times \mathbb{C}^n & \qquad & G(k,n) \times \mathbb{C}^n \ni (\Lambda, e_i) \\ \downarrow & & \uparrow \quad \uparrow e_i \\ G(k,n) \supset \{\Lambda \in G(k,n) : \Lambda \supset V_{k-r+1}\} & & G(k,n) \ni \Lambda \end{array}$$