This will provide a model for the formalism underlying the basic estimates; also by rendering transparent the behavior of the Euclidean Laplacian on the torus, we will gain some idea of what to expect in general.

Let T be the real torus $(\mathbb{R}/(2\pi\mathbb{Z}))^n$ with coordinates $x = (x_1, x_2 \cdots x_n)$. ~~Dono~~ Denote by $\mathcal{F}$ the space of formal Fourier series

$$u = \sum_{\xi \in \mathbb{Z}^n} u_\xi e^{i\langle \xi, x\rangle}$$

The Sobolev s-norm is given by

$$\|u\|_s^2 = \sum_\xi (1 + \|\xi\|^2)^s |u_\xi|^2,$$ and we define

the Sobolev spaces H_s by $H_s = \{ u \in \mathcal{F} ; \|u\|_s < \infty \}$

These are Hilbert spaces; we have clearly a sequence of inclusions

$$\supset H_{-n} \supset H_{-n+1} \supset \cdots \supset H_{-1} \supset H_0 \supset H_1 \supset H_2 \supset \cdots \supset H_n \supset \cdots$$

and we let $H_\infty = \bigcap_s H_s$, $H_{-\infty} = \bigcup H_s$.

Define an inner product on H_s by

$$\langle u, v \rangle = \sum_\xi (1 + \|\xi\|^2)^s u_\xi \overline{v_\xi}.$$

Given a Cauchy sequence $\{u_\ell\}$ in H_s, we have to find a limit point u_0 in H_s. To do this, first, find a convergent subsequence.

For each $\xi \in \mathbb{Z}^n$, we have a sequence $\{(1+\|\xi\|^2)^{\frac{s}{2}} (u_\ell)_\xi\}_{\ell=1}^\infty$.