$$D_V = A \cap \sigma_1(V_3)$$
$$= \{ L \subset X : L \cap V_3 \neq \phi \} \subset A.$$

The self-intersection of D_V on A is given by

$$(D_V \cdot D_V)_A = (A \cdot \sigma_1 \cdot \sigma_1)_{G(2,6)}$$
$$= (A \cdot (\sigma_{1,1} + \sigma_2))_{G(2,6)} \quad \text{by P746}$$
$$= (A \cdot \sigma_{1,1})_{G(2,6)} + (A \cdot \sigma_2)_{G(2,6)}$$
$$= 16 + 16 = 32 \quad \text{by P778.}$$

We claim now that for a generic 3-plane V, the curve $D_V \subset A$ is smooth. Note that this does not follow immediately from Bertini's theorem: the divisors $\{D_V\}_{V \subset \mathbb{P}^5}$ are all linearly equivalent, but they do not form a linear system.

$\ulcorner$ $D_V = A \cap \sigma_1(V_3)$ and $\sigma_1(V_3)$ is a hyperplane section of G $\Rightarrow$ By the result on P145, since all hyperplanes are linearly equivalent, $\{D_V\}$ is linearly equivalent. $\lrcorner$

Indeed, the complete linear system $|\sigma_1|$ of hyperplane sections of the Grassmannian $G(2,6) \subset \mathbb{P}(\Lambda^2 \mathbb{C}^6)$ corresponds naturally to the projective space

$$\mathbb{P}(\Lambda^2 \mathbb{C}^6)^* = \mathbb{P}(\Lambda^4 \mathbb{C}^6);$$

and the map

$$G(4,6) \longrightarrow \mathbb{P}(\Lambda^4 \mathbb{C}^6)$$

given by

$$V_3 \longmapsto \sigma_1(V_3) \in |\sigma_1|$$