Thus, we consider $\omega(f_1', f_2, \cdots f_n)$ as defining a class in $H^{n-1}(\underline{U}', \Omega^n)$, where $\{U_1', U_2 \cdots U_n\} = \underline{U}'$ is the corresponding covering of $U^* = U - \{0\}$. Since $U_1 \subset U_1'$, there is a restriction mapping ρ' leading to a commutative diagram

$$\begin{tikzcd} H^{n-1}(\underline{U}', \Omega^n) \arrow[rr, "\rho'"] \arrow[dr, "\eta"'] & & H^{n-1}(\underline{U}, \Omega^n) \arrow[dl, "\eta'"] \\ & H_{\bar\partial}^{n,n-1}(U^*) & \end{tikzcd}$$

where η and η' are Dolbeault maps.

$\ulcorner$ $C^{n-1}(\underline{U}', \Omega^n) \ni \omega(f_1', f_2, \cdots f_n) = \dfrac{df_1'}{f_1'} \wedge \dfrac{df_2}{f_2} \wedge \cdots \wedge \dfrac{df_n}{f_n}$ on $U_1' \cap U_2 \cap \cdots \cap U_n$

$C^{n-1}(\underline{U}, \Omega^n) \ni \rho'\left(\dfrac{df_1'}{f_1'} \wedge \cdots \wedge \dfrac{df_n}{f_n}\right)$

on $U_1 \cap U_2 \cdots \cap U_n$.

$U_1 = U_1' \cap U_1'' = (U - D_1') \cap (U - D_1'') = U - (D_1' + D_1'')$

$D_1' \cap D_2 \cdots \cap D_n = D_1 \cap \cdots \cap D_n = \{0\}$.

Any restriction map commutes with any sheaf map, & coboundary map.

$$\Rightarrow \begin{tikzcd} H^{n-1}(U', \Omega^n) \arrow[r, "\rho'"] \arrow[d, "\eta"'] & H^{n-1}(\underline{U}, \Omega^n) \arrow[d, "\eta'"] \\ H_{\bar\partial}^{n,n-1}(U^*) \arrow[r, equal] & H_{\bar\partial}^{n,n-1}(U^*) \end{tikzcd}$$

see p39

$\lrcorner$

Setting $\eta(f_1, \cdots f_n) = \eta(\omega(f_1, \cdots f_n))$ and so forth, it follows from $(*)$ that

$$\eta(f_1, f_2, \cdots f_n) = \eta(f_1', f_2, \cdots f_n) + \eta(f_1'', f_2, \cdots f_n)$$

in $H_{\bar\partial}^{n,n-1}(U^*)$. (It is not the case that $\eta = \eta' + \eta''$ as