i.e., $k \le m/2$.

⌜ Suppose $\mathbb{P}^{m+1} = \langle e_0, e_1, \cdots e_k, \cdots e_{m+1} \rangle$ and $\Lambda_k = \langle e_0, e_1, \cdots e_k \rangle$.
From $\{e_{k+1}, \cdots e_{m+1}\}$, form a $(m-k-1)$-dimensional plane $\mathbb{P}^{m-k-1}$.
$\Rightarrow \langle \mathbb{P}^{m-k-1}, \Lambda_k \rangle \cong \mathbb{P}^m$.
Thus the question is "How many $\mathbb{P}^{m-k-1}$ planes in $\mathbb{P}^{m-k}$?"
, more precisely, "what is the dimension of the set of all $\mathbb{P}^{m-k-1}$ planes in $\mathbb{P}^{m-k}$?"
$\Rightarrow$ The dimension is the dimension of the hyperplanes in $\mathbb{P}^{m-k}$, so the dimension is $m-k$.
Thus since $\mathcal{G}(p)$ contains Λ, $p \in \Lambda$, $\mathcal{G}(p)$ lies in the set of all hyperplanes containing Λ which is of dimension $(m-k)$. $\Rightarrow$ The image $\mathcal{G}(\Lambda)$ lies entirely in the $(m-k)$-dimensional subspace of $\mathbb{P}^{m+1*}$ of planes through Λ.
$\Rightarrow$ Since $\mathcal{G}(\Lambda)$ form a k-dimensional linear subspace of $\mathbb{P}^{m+1*}$, $k \le m-k$. $\Rightarrow$ $k \le m/2$. ⌟

To prove the remainder of the proposition we use an induction on n. Let $\Sigma_n' \subset G(n+1, 2n+3)$ be the set of n-planes Λ lying on a smooth quadric F of odd dimension $2n+1$ in $\mathbb{P}^{2n+2}$, and $\Sigma_n \subset G(n+1, 2n+2)$ the family of n-planes on a smooth $F_{2n} \subset \mathbb{P}^{2n+1}$.

⌜ The set of n-planes Λ on F of odd dim. $2n+1$ in $\mathbb{P}^{2n+1}$ is contained in $G(n+1, 2n+3)$. Similarly, $\Sigma_n \subset$