$$0 = \int_{\mathbb{C}^n \times \mathbb{C}^n} d(\psi(z) \wedge k(z,w) \wedge \varphi(w))$$

$$= \int_{\mathbb{C}^n \times \mathbb{C}^n} \bar\partial(\psi(z) \wedge k(z,w) \wedge \varphi(w))$$

$$= \int_{\mathbb{C}^n \times \mathbb{C}^n} \bar\partial \psi(z) \wedge k(z,w) \wedge \varphi(w) \pm \int_{\mathbb{C}^n \times \mathbb{C}^n} \psi(z) \wedge k(z,w) \wedge \bar\partial \varphi(w).$$

Γ $\psi(z) \wedge k(z,w) \wedge \varphi(w) \in A_c^{(n,\, n-q+1)+(0,\, q)+(n,\, n-q+q-1)}$

$$= A_c^{(2n,\, n+1+n-1)} = A_c^{(2n,\, 2n)}.$$

$\Rightarrow$ I think the test form ψ should be in $A_c^{n,\, n-q}(\mathbb{C}^n)$.

This equation says that, considering $K_\varphi = K\varphi \in A^{0,\, q-1}(\mathbb{C}^n)$ as a current operating on $A_c^{n,\, n-q+1}(\mathbb{C}^n)$,

(*) $\quad \bar\partial K_\varphi + K_\varphi \bar\partial = 0.$

Γ $(\bar\partial K_\varphi)(\phi) = (-1)^q K_\varphi(\bar\partial \phi)$, $\quad \phi \in A_c^{n,\, n-q}(\mathbb{C}^n)$

since $K_\varphi \in \mathcal{D}^{0,\, q-1}(\mathbb{C}^n)$

$$\bar\partial : \mathcal{D}^{0,\, q-1}(\mathbb{C}^n) \longrightarrow \mathcal{D}^{0,\, q}(\mathbb{C}^n).$$

$$\Rightarrow (\bar\partial K_\varphi)(\phi) = (-1)^q K_\varphi(\bar\partial\phi) = (-1)^q \int_{\mathbb{C}^n} K\varphi(z) \wedge \bar\partial \phi(z)$$

$$= (-1)^q \int_{\mathbb{C}^n} \left(\int_{\mathbb{C}^n} k(z,w) \wedge \varphi(w) \right) \wedge \bar\partial\phi(z)$$

$$= (-1)^q \int_{\mathbb{C}^n \times \mathbb{C}^n} k(z,w) \wedge \varphi(w) \wedge \bar\partial \phi(z) \quad = (-1)^q (-1)^{q-1}$$