Date

In other words, $\sigma = (\sigma_\alpha)$ $\quad S_0 = (S_{0\alpha})$

$$f = \frac{\sigma}{S_0} = \left(\frac{\sigma_\alpha}{S_{0\alpha}}\right) \Rightarrow \frac{\sigma_\alpha}{S_{0\alpha}} = \frac{g_{\alpha\beta}\,\sigma_\beta}{S_{0\beta}\,g_{\alpha\beta}} = \frac{\sigma_\beta}{S_{0\beta}} ; U_\alpha \cap U_\beta \to \mathbb{C}.$$

$\frac{\sigma}{S_0}$ has only a simple pole at p. See P136.

But such a function assumes the value ∞, and hence every value λ, exactly once, and so gives an isomorphism $f : S \longrightarrow \mathbb{P}^1$. Thus,

any compact Riemann surface of genus 0 is the Riemann sphere $\mathbb{P}^1$

Γ $\quad f : S \longrightarrow \mathbb{C} \cup \{\infty\} = \mathbb{P}^1$

$\quad q \longmapsto [(F(q), G(q))]$ since f is a rational function, see P P168.

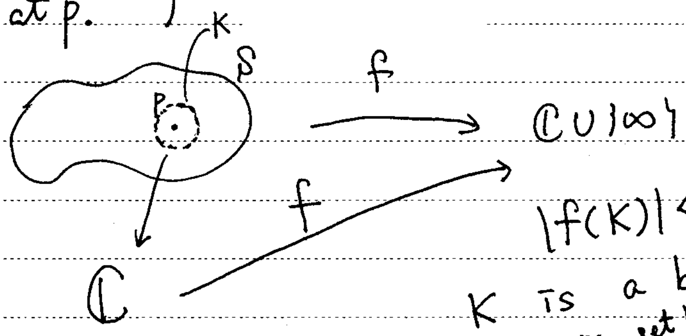
Around $p \in S$, $\quad \mathbb{C} \longrightarrow \mathbb{C}$, $\quad z \longmapsto f(z)$.

$\Rightarrow$ f has a simple pole at the origin (corresponds to p) $\Rightarrow$ f is of form $\frac{a_{-1}}{z} + a_0 + a_1 z + \cdots + a_n z^n$.

See P241 Ex 7 Silverman.

But since $f \not\to \infty$ as $z \to \infty$, $a_1 = \cdots = a_n = 0$

($\because$ Globally, f is a meromorphic function with a simple pole only at p.)



$|f(K)| < \infty$

K is a boundary of an open set U homeomorphic to $\mathbb{C}$.