If $[a]$ is a class in $E_1^{p,q}$ as just above, then

$$da \in \frac{\{ b \in F^{p+1} K^{p+q+1} : db \in F^{p+2} K^{p+q+2} \}}{d(F^{p+1} K^{p+q}) + F^{p+2} K^{p+q+1}}$$

defines a class in $E_1^{p+1,q}$, and this gives the differential

$$d_1 : E_1^{p,q} \longrightarrow E_1^{p+1,q}.$$

$\ulcorner$ $[a] = a + d(F^p K^{p+q-1}) + F^{p+1} K^{p+q}$,

$da \in F^{p+1} K^{p+q+1}$

$$\Rightarrow [da] \in \frac{\{ b \in F^{p+1} K^{p+q+1} ; db \in F^{p+2} K^{p+q+2} \}}{d F^{p+1} K^{p+q} + F^{p+1+1} K^{p+q+1}}$$

since $da \in F^{p+1} K^{p+q+1}$ and $d\,d(F^p K^{p+q-1}) = 0$. $\lrcorner$

It follows that

$$\operatorname{Ker} d_1 = \frac{\{ a \in F^p K^{p+q} : da \in F^{p+2} K^{p+q+1} \}}{d(F^p K^{p+q-1}) + F^{p+1} K^{p+q}},$$

$$\operatorname{im} d_1 = \frac{d(F^{p-1} K^{p+q-1})}{d(F^p K^{p+q-1}) + F^{p+1} K^{p+q}},$$

so that

$$E_2^{p,q} = \frac{\{ a \in F^p K^{p+q} ; da \in F^{p+2} K^{p+q+1} \}}{d(F^{p-1} K^{p+q-1}) + F^{p+1} K^{p+q}}.$$