represent a basis for $H^k_{DR}(M\times M)$.

$\ulcorner$ See P57 ~ P59. See the top line on P59. $\lrcorner$

The dual basis for $H^{2n-k}_{DR}(M\times M)$ is then represented by

$$\{ \varphi^*_{\mu,\nu,n-p,n-q} = (-1)^{q(p+q)} \pi_1^* \psi^*_{\mu,n-p} \wedge \pi_2^* \psi_{\nu,n-q} \}_{p+q=k}$$

since by a direct computation using iteration of the integral

$$\int_{M\times M} \varphi_{\mu,\nu,p,q} \wedge \varphi^*_{\mu',\nu',n-p',n-q'} = \delta_{\mu,\mu'} \cdot \delta_{\nu\nu'} \cdot \delta_{p,p'} \cdot \delta_{q,q'} .$$

$\ulcorner$
$$\int_{M\times M} \varphi_{\mu,\nu,p,q} \wedge \varphi^*_{\mu',\nu',n-p',n-q'} = \int_{M\times M} \pi_1^*\psi_{\mu,p} \wedge \pi_2^*\psi^*_{\nu,q} \wedge (-1)^{q'(p'+q')} \pi_1^*\psi^*_{\mu',n-p'} \wedge \pi_2^*\psi_{\nu',n-q'}$$

$$= (-1)^{q'(p'+q')} (-1)^{q(n-p')} \int_{M\times M} \pi_1^*(\psi_{\mu,p} \wedge \psi^*_{\mu',n-p'}) \wedge \pi_2^*(\psi^*_{\nu,q} \wedge \psi_{\nu',n-q'})$$

$$= (-1)^{q'(p'+q') + q(n-p') + q(n-q')} \int_M \psi_{\mu,p} \wedge \psi^*_{\mu',n-p'} \int_M \psi_{\nu',n-q'} \wedge \psi^*_{\nu,q} \quad (\text{see P58})$$

$$= (-1)^{q'(p'+q') + q(p'+q')} \delta_{\mu,\mu'} \delta_{p,p'} \delta_{\nu\nu'} \delta_{q,q'}$$

$$= (-1)^{(q+q')(p'+q')} \delta_{\mu,\mu'} \delta_{p,p'} \delta_{\nu,\nu'} \delta_{q,q'}$$

$$= \delta_{\mu,\mu'} \delta_{p,p'} \delta_{\nu\nu'} \delta_{q,q'} \quad \text{since if } q\neq q' \text{ then } 0. \lrcorner$$

The Poincaré dual η_Δ of the homology class of the diagonal $\Delta \subset M\times M$ is thus represented by the form

$$\varphi_\Delta = \sum_{p,\mu,\nu} c_{p,\mu,\nu} \varphi_{\mu,\nu,p,n-p},$$