

Since $\Lambda \in \underbrace{\sigma_{\bar{j}, \bar{j}}}_{k-\bar{i}+1}(V)$, $\dim(\Lambda \cap V_{n-k+k-\bar{i}+1-\bar{j}}) \geq k-\bar{i}+1. \dots \textcircled{0}$

$$V_{n-\bar{i}-\bar{j}+1} = \{e_{\bar{i}+\bar{j}}, e_{\bar{i}+\bar{j}+1}, \dots, e_n\}.$$

By $\textcircled{*}$, $\dim(\Lambda \cap V_{n-\bar{i}-\bar{j}+1}) \leq k-l < k-(\bar{i}-\bar{j}) = k-\bar{i}+\bar{j}. \dots \textcircled{**}$

Thus if $\bar{j}=1$, $\textcircled{**}$ contradicts to $\textcircled{0}$. $\Rightarrow x \in D_i^{(1)}$

In case $\bar{j}=2$, $l > \bar{i}-2$.

By $\textcircled{**}$, $\dim(\Lambda \cap V_{n-\bar{i}-\bar{j}+1}) = k-\bar{i}+1 \dots$

Since $V_{n-\bar{i}-1} = \{e_{\bar{i}-2}, e_{\bar{i}-1}, \dots, e_n\}$, and $l = \bar{i}-1$ or \bar{i} ,

by the argument on P361,

$\dim(\Lambda \cap V_{n-\bar{i}-1}) \geq k-l+1 \geq k-(\bar{i}-2)+1 = k-\bar{i}+3$. which contradicts to $\textcircled{**}$.

In the case $\bar{j}=3, \dots$, we can see what will happen as before (on P363). \Rightarrow We get a contradiction \Rightarrow

$\Rightarrow \iota^{-1}(\sigma_{\bar{j}, \bar{j}}) \subset D_i^{(\bar{j})} \Rightarrow$ Together with the previous result, we obtain $\iota^{-1}(\sigma_{\bar{j}, \bar{j}}) = D_i^{(\bar{j})}$. \Rightarrow

Composing ι with the isomorphism $*: G(k, n) \rightarrow G(n-k, n)$, we find that

$$D_i^{(\bar{j})} = (*\iota)^{-1}(\underbrace{\sigma_{k-\bar{i}+1}, \dots, k-\bar{i}+1}_{\bar{j}});$$

and since $c_r(E) = (*\iota)^* \sigma_r$, we may combine this with Giambelli's formula on P. 205 to obtain

Porteous' Formula. For $\sigma_1, \dots, \sigma_i$ suitably generic, the Poincare dual of the degeneracy cycle $D_i^{(\bar{j})}$ is

$$D_i^{(\bar{j})*} = \det \begin{bmatrix} c_{k-\bar{i}+1}(E), & \dots & c_{k-\bar{i}+\bar{j}}(E) \\ \vdots & & \vdots \\ c_{k-\bar{i}-\bar{j}+2}(E), & \dots & c_{k-\bar{i}+1}(E) \end{bmatrix}$$