$\{U_\alpha\}$ open cover of M and $\{g_{\alpha\beta}\}$ transition funcs
$\{U'_\beta\}$ refinement of $\{U_\alpha\}$ and $g'_{\alpha\beta} = g_{\rho(\alpha),\rho(\beta)}$.
where $\rho : I' \longrightarrow I$, see P39.

$$\Rightarrow \frac{\coprod U_\alpha \times \mathbb{C}}{(x,z)\sim(x, g_{\alpha\beta}(x)z)} = L \qquad \frac{\coprod U'_\alpha \times \mathbb{C}}{(x,z)\sim(x, g'_{\alpha\beta}(x)z)} = L'$$

Question: L is isomorphic to L' ?

Define $\phi : \dfrac{\coprod U'_\alpha \times \mathbb{C}}{(x,z)\sim(x, g'_{\alpha\beta}(x)z)} \longrightarrow \dfrac{\coprod U_\alpha \times \mathbb{C}}{(x,z)\sim(x, g_{\alpha\beta}(x)z)}$

by
$$\begin{array}{ccc} U'_\alpha \times \mathbb{C} & \longrightarrow & U_{\rho(\alpha)} \times \mathbb{C} \\ \cup & & \\ [(x,z)] & \longmapsto & [(x,z)] \end{array}$$
if $x \in U'_\alpha$ and $U'_\alpha \subset U_{\rho(\alpha)}$.

$$\begin{array}{c} (x, g'_{\alpha\beta}(x)z) \in U'_\beta \times \mathbb{C} \\ \downarrow \\ (x, g_{\rho(\alpha)\rho(\beta)}(x)z) \in U_{\rho(\beta)} \times \mathbb{C} \end{array} \Rightarrow \phi \text{ is well-defined}$$

$\Rightarrow$ Obviously, ϕ is one to one, onto.

$\Rightarrow$ ϕ is isomorphic.