the plane spanned by $L'$ and $L_0$ must meet $L$ in a point, which by definition can not be a point of $L_0$; so $L$ and $L'$ intersect.

┌ $\langle L', L_0 \rangle \cap L \neq \phi$, obviously. Again $T_p(S) \cap S = L' \cup L_0$, where $p \in L' \cap L_0$ ($\because$ $L'$ is B-line), by ① P735. note
$\Rightarrow L \cap L' \neq \phi$. ┘

Thus two lines on $S$ meet if and only if they are of different type; since there will be a unique B-line passing through every point of $L_0$ and likewise a unique A-line passing through each point of a fixed B-line, we see that the families of A-lines and B-lines are each parametrized by $\mathbb{P}^1$. In sum, then, the set of lines on $S$ consists of two disjoint families, each parametrized by $\mathbb{P}^1$, with two lines meeting if and only if they are from different families. It follows that $S \cong \mathbb{P}^1 \times \mathbb{P}^1$.

┌ Let $L_1, L_2$ be B-lines passing through $p \in L_0$.
$\Rightarrow L_1 \cap L_2 \ni p \Rightarrow L_1$ must be of different type from $L_2 \Rightarrow$ Contradiction. Similarly, we can show that a unique A-line passing through each point of a fixed B-line. Given a B-line $L$, for each $p \in L$, $\exists$ an A-line $L_p$. $L \cong \mathbb{P}^1 \Rightarrow \{L_p\}_{p \in L}$ is the family parametrized by $\mathbb{P}^1$.