

By iterating the integral in the definition of the residue,

$$\begin{aligned} \operatorname{Res}_{z=0} \left(\frac{h(z, w) dz dw}{f(z, w) g(z, w)} \right) &= \left(\frac{1}{2\pi\sqrt{-1}} \right)^2 \int_{|g(z, w)|=\epsilon} \left(\int_{|z|=\epsilon} \frac{h(z, w) dz}{g(z, w) z} \right) dw \\ &= \frac{1}{2\pi\sqrt{-1}} \int_{|w|=\epsilon} \frac{h(0, w)}{g(0, w)} dw = \frac{1}{2\pi\sqrt{-1}} \int_{|w|=\epsilon} (\alpha + \dots) \frac{dw}{w} = \alpha \end{aligned}$$

Q.E.D.

$$\begin{aligned} \text{If } \frac{1}{2\pi\sqrt{-1}} \int_{|w|=\epsilon} \frac{h(0, w)}{g(0, w)} dw &= \frac{1}{2\pi\sqrt{-1}} \int_{|w|=\epsilon} \frac{\alpha w^{m_{w_0}-1} + \beta w^{m_{w_0}} + \dots}{w^{m_{w_0}} + \dots} dw \\ &= \frac{1}{2\pi\sqrt{-1}} \int_{|w|=\epsilon} (\alpha + \dots) \frac{dw}{w} = \alpha. \end{aligned}$$

□

The first nontrivial case is when $m=n=3$; then we obtain the classical statement:

Suppose that C and D are cubic curves meeting in nine points that are not necessarily distinct but that are simple points of C . Then any cubic E passing through eight of these points must contain the remaining one also.

$$\begin{aligned} \text{If } C = (f=0), \quad D = (g=0), \quad \deg f = \deg g = 3, \quad \{P_i\}_{i=1}^9, \quad E = (h=0) \\ \deg h = 3+3-3=3, \quad \deg(C \cdot E) = 9 \Rightarrow \sum_{i=1}^9 m_i P_i = C \cdot D \\ = m_1 P_1 + m_2 P_2 + \dots + m_9 P_9, \quad m_1 = m_2 = \dots = m_9 = 1. \end{aligned}$$