$\Rightarrow$ Given $\epsilon > 0$, $\exists$ N s.t $\|u_l - u_m\|_s < \epsilon$ if $l, m \geq N$.

$\Rightarrow$ Since $|(1+\|\xi\|^2)^{\frac{s}{2}}((u_l)_\xi - (u_m)_\xi)|^2 \leq \sum_{\xi \in \mathbb{Z}^n} (1+\|\xi\|^2)^s |(u_l - u_m)_\xi|^2 < \epsilon^2$,

$\{(1+\|\xi\|^2)^{\frac{s}{2}} (u_l)_\xi\}_{l=1}^{\infty}$ is a Cauchy sequence for each $\xi \in \mathbb{Z}^n$.

$\Rightarrow \exists$ a limit point $(1+\|\xi\|^2)^{\frac{s}{2}} (u_0)_\xi$.

Consider $u_0 = \sum_{\xi \in \mathbb{Z}^n} (u_0)_\xi \, e^{i\langle \xi, x \rangle}$.

We claim that u_0 is the limit point of $\{u_l\}$.

Note that $\|u_l - u_0\|_s \leq \epsilon$ if $l \geq N$, for,

$\|u_l - u_m\|_s^2 = \sum_{\xi \in \mathbb{Z}^n} (1+\|\xi\|^2)^s |(u_l)_\xi - (u_m)_\xi|^2 < \epsilon$

$\Rightarrow$ $\|u_l - u_0\|_s^2 = \sum_{\xi \in \mathbb{Z}^n} (1+\|\xi\|^2)^s |(u_l)_\xi - (u_0)_\xi|^2 \leq \epsilon$ as $m \to \infty$.

" For a fixed $\xi_0 \in \mathbb{Z}^n$, $(1+\|\xi_0\|^2)^s |(u_l)_{\xi_0} - (u_0)_{\xi_0}|^2 +$

$\sum_{\xi \neq \xi_0} (1+\|\xi\|^2)^s |(u_l)_\xi - (u_m)_\xi|^2 \leq \epsilon$ for each (every) m and a fixed l.

$\Rightarrow$ Keep doing this business for ξ, $\Rightarrow$ we get the desired."

Suppose not, i.e $\sum (1+\|\xi\|^2)^s |(u_l)_\xi - (u_0)_\xi|^2 > \epsilon$

Since $(1+\|\xi\|^2)^s |(u_l)_\xi - (u_0)_\xi|^2 \leq \epsilon$ for each ξ,
by considering this as a sequence indexed by ξ,
we have to have a finite number of ξ's s.t

$$\sum_{\substack{\xi \in \mathbb{Z}^n \\ \#\xi < \infty}} (1+\|\xi\|^2)^s |(u_l)_\xi - (u_0)_\xi|^2 > \epsilon.$$

, which contradicts to the above process. $\lrcorner\!\lrcorner$