Contradiction to the assumption $p \notin H$.
$\Rightarrow$ We can conclude that $int_{V^*}(V^* \cap H) = V^*$.
$\Rightarrow$ $V^* \cap H \supset V^*$ $\Rightarrow$ $V^* = V^* \cap H$ $\Rightarrow$ $V = V \cap H$
$\Rightarrow$ $V \subset H$. Q.E.D.

In case V is not irreducible. $\Rightarrow$ Consider V^*.
$\Rightarrow$ $V^* = \cup V_i$, V_i connected component.
$\Rightarrow$ $V = \cup \overline{V_i}$, $\overline{V_i}$ irreducible component. by P21 Proposition. $\Rightarrow$ Once we prove that there exists a hyperplane H s.t $H \not\supset \overline{V_i}$. $\Rightarrow$ We can conclude that $H \cap V$ is an analytic subvariety of $k-1$ in $\mathbb{P}^n$.
For simplicity, we assume that $V = \overline{V_1} \cup \overline{V_2}$.
Choose points $[(a_0, \cdots a_n)] \in V_1$ & $[(b_0, \cdots b_n)] \in V_2$.
$\Rightarrow$ Take $(c_0, \cdots c_n)$ s.t

$$c_0 a_0 + \cdots + c_n a_n \neq 0 \neq c_0 b_0 + \cdots + c_n b_n$$

$\Rightarrow$ This implies that $H = (c_0 X_0 + \cdots + c_n X_n = 0)$ does not contain V_1 & V_2. (See P45 Whitney Lemma 2I for the idea).
$\Rightarrow$ By cutting V by hyperplanes, we have a linear subspace $\mathbb{P}^{n-k}$ s.t $V \cap \mathbb{P}^{n-k}$ is a set of discrete points.

" Comments on "First, by Bertini applied to the smooth locus of V the generic $(n-k)$-plane $\mathbb{P}^{n-k} \subset \mathbb{P}^n$ will intersect V transversely, and so will meet V in exactly $\#(\mathbb{P}^{n-k} \cdot V) =$ degree (V) points.