

unit for  $p \in Z$ . Thus, solving  $(**)$  and finding  $(*)$  are entirely equivalent.

$$\begin{aligned} \text{If } \underline{\text{Ext}}^1(I, L)_p &= \underline{\text{Ext}}^1(I_p, L_p) \cong \underline{\text{Ext}}^1_{\mathcal{O}_p}(\mathcal{O}_p/I_p, \mathcal{O}_p) \cong (\mathcal{O}_p/I_p)_p \\ &\quad \underline{\text{Ext}}^1(I_p, \mathcal{O}_p) \end{aligned}$$

$$= \mathcal{O}_{Z, p} \quad \text{by P 7.23}$$

(i)  $p \notin Z$ ,

$\Rightarrow$  From  $0 \rightarrow L \rightarrow \mathcal{E}^* \rightarrow I \rightarrow 0$ , we have

$$0 \rightarrow L_p \rightarrow \mathcal{E}_p^* \rightarrow I_p \rightarrow 0.$$

Since  $L_p \cong \mathcal{O}$  and  $I_p \cong \mathcal{O}_p/I_p \cong \mathcal{O}$ ,  $\mathcal{E}_p^* \cong \mathcal{O} \oplus \mathcal{O}$ .

(ii)  $p \in Z$

Again, from  $0 \rightarrow L \rightarrow \mathcal{E}^* \rightarrow I \rightarrow 0$ , we have

$$0 \rightarrow L_p \rightarrow \mathcal{E}_p^* \rightarrow I_p \rightarrow 0.$$

$\Rightarrow$  By the lemma on P 7.24, since  $e_p \in \underline{\text{Ext}}^1(I, L)_p = \underline{\text{Ext}}^1(I_p, L_p)$  is a unit,  $\mathcal{E}_p^*$  is free (= projective).

Thus  $\mathcal{E}_p^* \cong \mathcal{O}^{(k)}$ , for some  $k$ .

We have to show that  $\mathcal{E}^* \cong \mathcal{O}^{(k)}$  locally.

For example  $k=2$ ,  $\mathcal{E}_p^* \cong \mathcal{O} \oplus \mathcal{O}$ .

We will show that  $\exists$  open set  $U \ni p$  s.t.  $\mathcal{E}|_U \cong \mathcal{O}|_U \oplus \mathcal{O}|_U$ .

Suppose that  $\sigma_1, \sigma_2$  are basis for  $\mathcal{E}_p^*$ .