NO.
DATE / /

for the Dolbeault representative of $[\omega_t] \in H^{n-1}(U^*, \Omega^n)$, we see that the boundary integral

$$\int_{\partial U} \eta_{\omega_t}$$

is continuous in t. Going to the residue theorem, we find the principle of continuity:

$$(*) \qquad \lim_{t\to 0} \sum_{P_t \in f_t^{-1}(0)} \mathrm{Res}_{P_t}\, \omega_t = \sum_{P\in f^{-1}(0)} \mathrm{Res}_P\, \omega .$$

Γ

$$\lim_{t\to 0} \int_{\partial U} \eta_{\omega_t} = \lim_{t\to 0} \sum_{P_t\in f_t^{-1}(0)} \mathrm{Res}_{P_t} \omega_t = \int_{\partial U} \eta_\omega = \sum_{P\in f^{-1}(0)} \mathrm{Res}_P\, \omega$$

To apply this, we need to discuss pertubations of a given map $f: U \to \mathbb{C}^n$ having $f^{-1}(0) = \{0\}$. A family of maps $f_t : U \to \mathbb{C}^n$ defined and holomorphic in a nbd of $\bar{U}$, varying continuously with t and such that $f_0 = f$, is said to be a good perturbation of f in case f_t has only nondegenerate zeros for $t \neq 0$. We will be able to easily see the existence of good perturbations when we discuss finite holomorphic mappings below. For the moment they may be deduced from Sard's theorem as follows: Since the critical values of $f: U \to \mathbb{C}^n$ have measure zero in $\mathbb{C}^n$, we can find an arc $\gamma(t)$, $0 \leq t \leq \epsilon$, with $\gamma(0) = \{0\}$ and $\gamma(t)$ not a critical value for $t \neq 0$. Then

$$f_t(z) = f(z) - \gamma(t)$$

is a good perturbation of f.