$$= C\left(\frac{i}{2}\right)^{n-p}(-1)^{\frac{(n-p-1)(n-p)}{2}}(-1)^{(n-p)p}\sum_I |f_I|^2\, \varphi_1\wedge\cdots\wedge\varphi_n\wedge\bar\varphi_1\wedge\cdots\wedge\bar\varphi_n$$

since $\varphi_I\wedge\varphi_K = \epsilon\,\Phi$ $\quad \bar\varphi_I\wedge\bar\varphi_K = \epsilon\,\Phi'$, $\quad \Phi\wedge\Phi' = \varphi_1\wedge\cdots\wedge\varphi_n\wedge\bar\varphi_1\wedge\cdots\wedge\bar\varphi_n$

$$\Rightarrow\ Q(\xi,\bar\xi) = \int \xi\wedge\bar\xi\wedge\omega^{n-p}$$

$$= \left(\frac{i}{2}\right)^{n-p}(-1)^{\frac{(n-p-1)(n-p)}{2}+(n-p)p}\int \sum_I |f_I|^2\, \Phi\wedge\Phi'$$

Since $(-1)^{\frac{n(n-1)}{2}}(i)^n \int \sum_I |f_I|^2\, \Phi\wedge\Phi' > 0$. (See p 80),

$$i^p(-1)^{p(p-1)/2}Q(\xi,\bar\xi) = i^p(-1)^{p/2\,(p-1)}\left(\frac{i}{2}\right)^{n-p}(-1)^{\frac{(n-p-1)(n-p)}{2}+(n-p)p}$$
$$\times\int \sum_I |f_I|^2\, \Phi\wedge\Phi' = \left\{\frac{i^n}{2^{n-p}}(-1)^{\frac{n(n-1)}{2}}\int \sum |f_I|^2\, \Phi\wedge\Phi'\right\}\times(-1)^{\left\{\frac{p(p-1)}{2}+\frac{n(n-1)}{2}+(n-p)p+\frac{(n-p-1)(n-p)}{2}\right\}}.$$

$$\Rightarrow \text{Since } \frac{p(p-1)}{2}+\frac{n(n-1)}{2}+\frac{2(n-p)p+(n-p-1)(n-p)}{2} = \frac{p^2-p+n^2-n+2np-2p^2+(n-p)^2-n+p}{2} = \frac{p^2-p+n^2-n+2np-2p^2+n^2+p^2-2pn-n+p}{2}$$

$$= \frac{2n^2-2n}{2} = n^2-n = \text{even number.},\quad i^p(-1)^{\frac{p(p-1)}{2}}Q(\xi,\bar\xi) > 0.$$

In fact, it was in effect by deducing the Hodge–Riemann bilinear relations for holomorphic q-forms that we first proved that the holomorphic forms inject into the cohomology of a compact Kähler manifold.

Now let $\dim M = 2$; it remains only to prove the bilinear relations for $P^{1,1}$. Let ξ be a real, primitive harmonic $(1,1)$-form; in terms of a local unitary coframe φ_1, φ_2, we write

$$\xi = \sum \xi_{i\bar j}\,\varphi_i\wedge\bar\varphi_j.$$

Since ξ is real, $\bar\xi = \xi \Rightarrow \sum \xi_{i\bar j}\,\varphi_i\wedge\bar\varphi_j = \sum \bar\xi_{i\bar j}\,\bar\varphi_i\wedge\varphi_j$
$= -\sum \bar\xi_{j\bar i}\,\varphi_i\wedge\bar\varphi_j$. $\quad\Rightarrow\quad \xi_{i\bar j} = -\bar\xi_{j\bar i}$.