$\Rightarrow \quad 0 \to \mathcal{O}^{(k_n)} \to \cdots \to \mathcal{O}^{(k_0)} \to \mathcal{F} \to 0$

$\Rightarrow \quad 0 \to \mathcal{O}^{(k_n)} \to \cdots \to \mathcal{O}^{(k_1)} \to \mathcal{R} \to 0$, where $0 \to \mathcal{R} \to \mathcal{O}^{(k_1)} \xrightarrow{F} \mathcal{O}^{(k_0)}$. $\Rightarrow$ By Oka's lemma & def of coherent, $\mathcal{R}$ is coherent sheaf in some nbd of z_0. $\Rightarrow$ By the induction assumption $\mathcal{H}_z^q(\mathcal{R})=0$ since $\mathcal{R}$ has a local syzygy of length $(n-2)$. Now consider

$$0 \to \mathcal{R} \to \mathcal{O}^{(k_1)} \to \mathcal{O}^{(k_0)} \to \mathcal{F} \to 0.$$

We can not go further. $\Rightarrow$ It is not good way.

$$0 \to \ker G \to \mathcal{O}^{(k_0)} \xrightarrow{G} \mathcal{F} \to 0$$

$\Rightarrow$ Since $0 \to \mathcal{O}^{(k_n)} \to \cdots \to \mathcal{O}^{(k_1)} \to \ker G \to 0$, $\ker G$ is coherent by the definition. $\Rightarrow$ The length is $(n-1)$. $\Rightarrow$ By the induction assumption, $\mathcal{H}_z^q(\ker G)=0$.

Consider the long exact sequence of the sequence

$$0 \to \ker G \to \mathcal{O}^{(k_0)} \to \mathcal{F} \to 0.$$

$\Rightarrow \quad H^q(U, \ker G) \to \underset{0}{\underset{\shortparallel}{H^q(U, \mathcal{O}^{(k_0)})}} \to H^q(U, \mathcal{F}) \to H^{q+1}(U, \underset{0}{\underset{\shortparallel}{\ker G)}}$ for $q \geq 0$.

$\Rightarrow \quad H^q(U, \mathcal{F}) = 0 \quad \Rightarrow \quad \mathcal{H}_z^q(\mathcal{F}) = \varinjlim_{z \in U} H^q(U, \mathcal{F}).$

A further property of coherent sheaves we shall repeatedly use is:

Given an exact sequence

$$0 \to \mathcal{F}' \to \mathcal{F} \to \mathcal{F}'' \to 0$$

of sheaves of $\mathcal{O}$-modules in which two of the three are coherent, then the remaining one is also.