To compute curvatures, choose a unitary frame $e_1 \cdots e_r$ for E s.t $e_1, e_2, \cdots e_s$ is a frame for S.
Using this frame and our lemma, the connection matrix for E is

$$\theta_E = \left(\begin{array}{c|c} \theta_s , & A \\ \hline -{}^t\bar{A} , & \theta_Q \end{array} \right). \qquad {}^t\bar{\theta}_E + \theta_E = 0$$

$${}^t\bar{\theta}_E = \left(\begin{array}{cc} {}^t\bar{\theta}_s & -A \\ \hline {}^t\bar{A} & {}^t\bar{\theta}_Q \end{array} \right) \Rightarrow \quad {}^t\bar{\theta}_s + \theta_s = 0 \ \& \ {}^t\bar{\theta}_Q + \theta_Q = 0$$

$$\Rightarrow \quad {}^t\bar{\theta}_E + \theta_E = 0.$$

$\ulcorner$ Since $D_E = D_E|_{a^\circ(S)} + D_E|_{a^\circ(Q)} = D_S + D_E|_{a^\circ(S)} - D_S + D_Q$
$= D_S + A + D_Q$,

$1 \le i \le s$.

$$D_E e_i = D_S e_i + A e_i + \cancel{D_Q e_i}^{\,0} = \underline{\theta_{s_{ij}} e_j + A_{i\,j+s}\, e_{s+j}}.$$

$1 \le i \le r-s$

$$D_E e_{i+s} = D_Q e_{i+s} + (-{}^t\bar{A}) e_{i+s}.$$

$$D_E = D_E|_{a^\circ(S)} + D_E|_{a^\circ(Q)} = D_E|_{a^\circ(Q)} - D_Q + D_Q$$
$$= -{}^t\bar{A} + D_Q .. \quad \lrcorner$$

where θ_S, θ_Q are connection matrices for S and Q.

$$\Rightarrow \Theta_E = d\theta_E - \theta_E \wedge \theta_E = \begin{bmatrix} d\theta_S - \theta_S \wedge \theta_S + A \wedge {}^t\bar{A}, & * \\ * & d\theta_Q - \theta_Q \wedge \theta_Q + {}^t\bar{A} \wedge A \end{bmatrix}$$

$$\Rightarrow \quad \Theta_S = \Theta_E|_S - A \wedge {}^t\bar{A} \quad \& \quad \Theta_Q = \Theta_E|_Q - {}^t\bar{A} \wedge A.$$