$j_K = i_0 + i_1 + \cdots + i_{K-1} + K.$ 」

Thus the number of monomials of degree d in $X_0, X_1, \ldots X_n$ is just the number $\binom{d+n}{n}$ of subsets of order n in a set of order $d+n$, and so

$$h^0(\mathbb{P}^n, \mathcal{O}(H^d)) = \binom{d+n}{n}.$$

「 $\binom{d+n}{n} = {}_{d+n}C_n.$ 」

Note that the locus of a homogeneous polynomial $F(X_0, X_1, \cdots X_n)$ of degree d in the homogeneous coordinates X_i may also be given in terms of Euclidean coordinates $x_i = X_i / X_0$, $i = 1, 2, \cdots n$ in $(X_0 \neq 0)$ by the inhomogeneous polynomial of degree $\leq d$.

$$f(x_1, x_2, \cdots x_n) = F(1, x_1, \cdots x_n) = \frac{1}{X_0^d} F(X_0, \cdots X_n),$$

and conversely any such polynomial

$$f(x_1, x_2, \cdots x_n) = \sum a_{i_1 \cdots i_n} x_1^{i_1} \cdots x_n^{i_n}$$

corresponds to a homogeneous polynomial

$$F(X_0, \cdots X_n) = \sum a_{i_1 \cdots i_n} X_0^{d - \Sigma i_k} X_1^{i_1} \cdots X_n^{i_n}.$$

「 $F(X_0 \cdots X_n) = X_0^d f(x_1 \cdots x_n) = X_0^d \sum a_{i_1 \cdots i_n} x_1^{i_1} \cdots x_n^{i_n}$

$= X_0^d \sum a_{i_1 \cdots i_n} \frac{X_1^{i_1}}{X_0^{i_1}} \cdots \frac{X_n^{i_n}}{X_0^{i_n}} = \sum a_{i_1 \cdots i_n} X_0^{d - \Sigma i_k} X_1^{i_1} \cdots X_n^{i_n}$ 」