map from M to itself. This gives the map from extensions to $\mathrm{Ext}^1_{\mathcal{O}}(M, N)$.

$\ulcorner$ If (E) splits, then $\mathrm{Hom}_{\mathcal{O}}(M, E) \to \mathrm{Hom}_{\mathcal{O}}(M, M)$ is onto. $\Rightarrow$ ∂ is zero map.
$\Rightarrow$ $\partial(1_M) = 0$. Conversely, if $\partial(1_M) = 0$, then $\exists$ f s.t $f : M \to E$ and

$$\begin{tikzcd} E \arrow[r] & M \arrow[r] & 0 \\ & M \arrow[ul, "f"] \arrow[u, "1_M"'] \arrow[ul, phantom, bend left, "\circlearrowright"] & \end{tikzcd}$$
commutative.

$\Rightarrow$ This implies that $0 \to N \to E \to M \to 0$ splits.

$$\begin{tikzcd} 0 \arrow[r] & N \arrow[r] \arrow[d, equal] & E \arrow[r] \arrow[d] & M \arrow[r] \arrow[d, equal] & 0 \\ 0 \arrow[r] & N \arrow[r] & E' \arrow[r] & M \arrow[r] & 0 \end{tikzcd}$$

$\Rightarrow$
$$\begin{tikzcd} \mathrm{Hom}_{\mathcal{O}}(M, E) \arrow[r] \arrow[d] & \mathrm{Hom}_{\mathcal{O}}(M, M) \arrow[r, "\partial"] \arrow[d, equal] & \mathrm{Ext}^1_{\mathcal{O}}(M, N) \arrow[d, equal] \\ \mathrm{Hom}_{\mathcal{O}}(M, E') \arrow[r] & \mathrm{Hom}_{\mathcal{O}}(M, M) \arrow[r, "\partial"] & \mathrm{Ext}^1_{\mathcal{O}}(M, N) \end{tikzcd}$$
$1_M \in \mathrm{Hom}_{\mathcal{O}}(M, M)$ (top and bottom)

$\Rightarrow$ $\partial(1_M) = \partial(1_M)$.
This gives the map from equivalence classes of extensions to $\mathrm{Ext}^1_{\mathcal{O}}(M, N)$. $\lrcorner$