Γ See p39.

$$C^p(\underline{U}, K^q) = C^{p,q}$$

$$\bigoplus_{p+q=n} C^p(\underline{U}, K^q) = C^n(\underline{U})$$

$$D = d+\delta \ : \ C^n \longrightarrow C^{n+1}$$

A refinement $\underline{U}' < \underline{U}$ of coverings induces mappings

$$C^p(\underline{U}, K^q) \xrightarrow{\varphi} C^p(\underline{U}', K^q)$$ by the argument on P39

and so we have a map

$$\begin{array}{ccc} C^n(\underline{U}) & & C^n(\underline{U}') \\ \| & & \| \\ \bigoplus_{p+q=n} C^p(\underline{U}, K^q) & \xrightarrow{\varphi} & \bigoplus_{p+q} C^p(\underline{U}', K^q) \end{array}.$$

By the remark (on P39), this map commutes with $D = d+\delta$, since

$$\begin{aligned} (d(\varphi\sigma))_{\beta_0\cdots\beta_p} &= d(\varphi\sigma)_{\beta_0\cdots\beta_p} \\ &= d\,\sigma_{\varphi\beta_0\cdots\varphi\beta_p} \\ &= (d\sigma)_{\varphi\beta_0\cdots\varphi\beta_p} \\ &= (\varphi(d\sigma))_{\beta_0\cdots\beta_p}. \end{aligned}$$

$\Rightarrow$ φ induces a homomorphism

$$\varphi : H^n(C^*(\underline{U}), D) \longrightarrow H^n(C^*(\underline{U}), D).$$

Here we put $\rho_\varphi = \varphi$, where ρ_φ is the chain map on P39. ⌋

Now the spectral sequences $'E$, $''E$ associated to the double complex $(C^p(\underline{U}, K^q), \delta, d)$ behave well with respect to refinements of the covering, and passing to the