Now $[S] = d \cdot [H]$ in $H_2(\mathbb{P}^2; \mathbb{Z})$, so the sheet number of the projection map π_p is d; by the Riemann-Hurwitz formula,

$$\chi(S) = d \cdot \chi(\mathbb{P}^1) - \sum_{q \in S} (v(q) - 1)$$

$$= 2d - d(d-1)$$

and so

$$g(S) = \frac{2 - \chi(S)}{2} = \frac{(d-1)(d-2)}{2}.$$

⌈ By P145, $[H]$ is a generator of $H_2(\mathbb{P}^2; \mathbb{Z})$.

$$\pi_p : S \longrightarrow H.$$

$$\Rightarrow \pi_{p*} : H_2(S, \mathbb{Z}) \longrightarrow H_2(H, \mathbb{Z})$$

$$[S] \longmapsto n[H].$$

(See P 576 for clear explanation)

Since S has degree d, $[S] = d[H]$.

$\pi_{p*}[S] = n[H] \Rightarrow \pi_p(S)$ is homologous $nH \Rightarrow$ π_p covers H generically n times. and H is the connected cover of H.

$$g(S) = \frac{2 - \chi(S)}{2} = \frac{2 - (2d - d(d-1))}{2} = \frac{2 - 2d + d^2 - d}{2}$$

$$= \frac{d^2 - 3d + 2}{2} = \frac{(d-1)(d-2)}{2}$$ ⌋

A second way to arrive at this formula is by the adjunction formula from Section 2 of Chapter 1. It gives

$$K_S = K_{\mathbb{P}^2}|_S \otimes N_S = (K_{\mathbb{P}^2} + S)|_S.$$

⌈ By P147, adjunction formula II, $K_V = (K_M \otimes [V])|_V$