To prove the Riemann-Roch in this form, we note that it is clear for the trivial bundle, and show that it holds for any $L = [D]$, if and only if it holds as well for $L' = [D+p]$ and $L'' = [D-p]$, $p \in S$ any point.

⌈ $S \times \mathbb{C} = L$. $\Rightarrow c_1(L) = 0$. $H^p(S, \mathcal{O}(L)) = H^p(S, \mathcal{O})$
$\Rightarrow \chi(L) = \sum (-1)^p h^p(S) = \sum (-1)^p h^{0,p}(S) = \chi(\mathcal{O}_S)$ ⌋

This is easy: the exact sheaf sequence

$$0 \longrightarrow \mathcal{O}(D) \longrightarrow \mathcal{O}(D+p) \longrightarrow \mathbb{C}_p \longrightarrow 0$$

gives us the exact cohomology sequence

$$0 \longrightarrow H^0(S, \mathcal{O}(D)) \longrightarrow H^0(S, \mathcal{O}(D+p)) \longrightarrow \mathbb{C}_p \longrightarrow H^1(S, \mathcal{O}(D))$$

$$\longrightarrow H^1(S, \mathcal{O}(D+p)) \longrightarrow \underset{H^1(S, \mathbb{C}_p)}{\overset{0}{\shortparallel}},$$

and since the alternating sum of the dimensions of the vector spaces in an exact sequence is zero, this implies that

$$\chi([D+p]) = \chi([D]) + 1. \qquad \text{Q.E.D.}$$

⌈ $0 \longrightarrow \mathcal{O}(E \otimes [-D]) \longrightarrow \mathcal{O}(E) \longrightarrow \mathcal{O}_D(E|_D) \to 0$
$E = D \otimes [p]$, $D = p$

$\Rightarrow$ We get $0 \longrightarrow \mathcal{O}(D) \longrightarrow \mathcal{O}(D+p) \longrightarrow \mathbb{C}_p \longrightarrow 0$.

$H^1(S, \mathbb{C}_p) = H^1(p, \mathcal{O}) = H^{0,1}(p) = 0$ since $H^1(p) = 0$
$= H^{1,0}(p) \oplus H^{0,1}(p)$ by Hodge decomposition theorem, P116.