From ⊛ we have $\frac{X_5}{X_0} = \alpha t^2 + (1-\alpha)t'^2$

$$= \frac{X_1^2}{X_0X_3}\left(\frac{X_4}{X_1}\right)^2 + \left(1-\frac{X_1^2}{X_0X_3}\right)\frac{(X_2X_3-X_1X_4)^2}{(X_0X_3-X_1^2)^2}$$

$$= \frac{X_4^2}{X_0X_3} + \frac{(X_2X_3-X_1X_4)^2}{(X_0X_3-X_1^2)X_0X_3} = \frac{X_4^2(X_0X_3-X_1^2)+(X_2X_3-X_1X_4)^2}{(X_0X_3-X_1^2)X_0X_3}$$

$$\Rightarrow \frac{X_5}{X_0} = \frac{X_4^2X_0X_3 - \cancel{X_1^2X_4^2} + X_2^2X_3^2 + \cancel{X_1^2X_4^2} - 2X_1X_2X_3X_4}{(X_0X_3-X_1^2)X_0X_3}$$

$$\Rightarrow X_5(X_0X_3-X_1^2)\cancel{X_0}\cancel{X_3} = X_4^2\cancel{X_0}\cancel{X_3} + X_2^2\cancel{X_0}X_3^{\cancel{2}} - 2X_1X_2\cancel{X_3}X_4\cancel{X_0}$$

$$\Rightarrow X_5(X_0X_3-X_1^2) = X_4^2X_0 + X_3X_2^2 - 2X_1X_2X_4$$

§3. 1+1, 2.

$f : \mathbb{P}^2 \longrightarrow \mathbb{P}^5$ is an embedding, & $f(\mathbb{P}^2)$ is nondegenerate. irreducible of deg 4. ($S = f(\mathbb{P}^2)$)

" Given a nondegenerate, irreducible smooth surface $S \subset \mathbb{P}^5$ of deg 4. ?? Let p_1, p_2, p_3 be any 3 independent points of $f(\mathbb{P}^2)$, $V = \overline{p_1, p_2, p_3} \cong \mathbb{P}^2$ their linear span, and $\{H_\lambda\}_{\lambda\in\mathbb{P}^2}$ the net of hyperplanes in $\mathbb{P}^5$ containing V. Each hyperplane H_λ will then intersect S in 4 points: p_1, p_2, p_3 and an additional point we will call $q(\lambda)$. (In case H_λ is the hyperplane containing V and tangent to S at p_i, the point $q(\lambda) = p_i$.)

$\Rightarrow$ Every point of S will lie on a unique hyperplane H_λ, and so the map $q : \mathbb{P}^2 \longrightarrow S$ is an isomorphism. Since moreover nH is the unique line bundle of deg n on $\mathbb{P}^2$, it follows that every irreducible nondegenerate smooth surface of deg 4 in $\mathbb{P}^5$