$$\mathrm{Res}_{\{0\}}\left(\frac{J_f(z)\,dz_1\wedge\cdots\wedge dz_n}{f_1\cdots f_n}\right) = \mathrm{Res}_{\{0\}}\left(\frac{df_1\wedge\cdots\wedge df_n}{f_1\cdots f_n}\right).$$

Thus

$$\mathrm{Res}_{\{0\}}\left(\frac{df_1'\wedge\cdots\wedge df_n'}{f_1'\cdots f_n'}\right) = \mathrm{Res}_{\{0\}}\left(\Delta\frac{df_1\wedge\cdots\wedge df_n}{f_1'\cdots f_n'} + g\,\frac{dz_1\wedge\cdots\wedge dz_n}{f_1'\cdots f_n'}\right)$$

$$= \mathrm{Res}_{\{0\}}\left(\Delta\frac{df_1\wedge\cdots\wedge df_n}{f_1'\cdots f_n'}\right) + \mathrm{Res}_{\{0\}}\left(g\,\frac{dz_1\wedge\cdots\wedge dz_n}{f_1'\cdots f_n'}\right)$$

$= \mathrm{Res}_{\{0\}}\left(\frac{df_1\wedge\cdots\wedge df_n}{f_1\cdots f_n}\right)$ which implies that the local intersection number is independent of the choice of generators f_i in $I(f)$.

(b) The intersection number is linear in each divisor D_i.

Proof. If $D_1 = D_1' + D_1''$ corresponds to the factorization $f_1 = f_1' f_1''$, then clearly

(*) $\quad \omega(f_1, f_2, \cdots f_n) = \omega(f_1', f_2, \cdots f_n) + \omega(f_1'', f_2, \cdots f_n)$.

Pf

$$\omega(f_1, f_2) = \frac{df_1}{f_1}\wedge\frac{df_2}{f_2} = \frac{df_1'f_1''}{f_1'f_1''}\wedge\frac{df_2}{f_2} = \frac{f_1'df_1''+f_1''df_1'}{f_1'f_1''}\wedge\frac{df_2}{f_2} = \left(\frac{df_1''}{f_1''}+\frac{df_1'}{f_1'}\right)\wedge\frac{df_2}{f_2} = \frac{df_1'}{f_1'}\wedge\frac{df_2}{f_2} + \frac{df_1''}{f_1''}\wedge\frac{df_2}{f_2}$$

$= \omega(f_1', f_2) + \omega(f_1'', f_2)$.

This is not yet enough to prove linearity, owing to the complicated nature of the path of integration $\Gamma = \{\,|f_i| = \varepsilon_i\,\}$ in the definition of the residue. What is suggested is that we use the Dolbeault isomorphism to convert Γ into the sphere $\|z\| = \varepsilon$.