

be extended to \mathbb{C} . $-dw/w^2$ can not be extended to \mathbb{C} .

$j: A \rightarrow S \subset \mathbb{P}^3$ is holomorphic \Rightarrow Locally,

$$j(z_1, z_2) = (f_1(z_1, z_2), f_2(z_1, z_2), f_3(z_1, z_2))$$

\Rightarrow We can express $\tilde{\omega}_i$ on $S-R$ as, locally,

$$\tilde{\omega}_i = g_{i1}(u_1, u_2, u_3) du_1 + g_{i2} du_2 + g_{i3} du_3$$

$$\Rightarrow j^* \tilde{\omega}_i = g_{i1} \circ j \cdot j^* du_1 + g_{i2} \circ j \cdot j^* du_2 + g_{i3} \circ j \cdot j^* du_3$$

$$\Rightarrow j^* du_1 = df_1 = \frac{\partial f_1}{\partial z_1} dz_1 + \frac{\partial f_1}{\partial z_2} dz_2$$

$$j^* du_3 = df_3 = \frac{\partial f_3}{\partial z_1} dz_1 + \frac{\partial f_3}{\partial z_2} dz_2$$

\Rightarrow Since $f_i, \frac{\partial f_i}{\partial z_j}$ are bounded, if $\tilde{\omega}_i$ is unbounded,

g_{ij} is unbounded for some j , and $g_{ij} \circ j = j^* g_{ij}$

is unbounded. \Rightarrow Contradiction to the fact

$\tilde{\omega}_i$ is bounded. Here $g_{ij} \circ j = g_{ij}(f_1(z_1, z_2), f_2(z_1, z_2), f_3(z_1, z_2))$. Thus we see that $\tilde{\omega}_i$ is bounded on $S-R$ since S is compact.

$\Rightarrow \pi^* \tilde{\omega}_i$ is bounded on $\Sigma - \pi^{-1}R$, since π^* is just a pull-back.

\Rightarrow By Riemann extension theorem on P^1 ,

$\pi^* \tilde{\omega}_i$ can be extended to Σ .

→ 'More precisely', for example, $f: \mathbb{C} \rightarrow \mathbb{C}$
 $z \mapsto z^2$

$$f^*(g(w) dw) = g \circ f(z) \cdot \frac{df}{dz} dz$$

$$= g \circ f(z) \cdot 2z dz \Rightarrow \text{If } g(w) = w^{-n} \tilde{g}(w),$$

$$\text{then } g \circ f(z) = z^{-2n} \tilde{g}(z^2) \Rightarrow f^*(g dw) \text{ can}$$