By definition, $\partial(1) = -e$, where 1 means the constant function "one" under the identification $\mathrm{Hom}_{\mathcal{O}}(\mathcal{O}, \mathcal{O}) \cong \mathcal{O}$.

$\ulcorner$ $1 \in \mathcal{O}$ constant function.

$$\Rightarrow \quad \begin{array}{ccc} \mathrm{Hom}_{\mathcal{O}}(\mathcal{O}, \mathcal{O}) & \cong & \mathcal{O} \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ 1_{\mathcal{O}} & \longleftrightarrow & 1 \end{array}$$

where $1_{\mathcal{O}}$ is the identity map from $\mathcal{O}$ to $\mathcal{O}$.

For, given $\phi_f : \mathcal{O} \longrightarrow \mathcal{O}$ defined by $1 \longmapsto f$, $f \in \mathcal{O}$.

$$\begin{array}{ccc} \mathrm{Hom}_{\mathcal{O}}(\mathcal{O}, \mathcal{O}) & \longrightarrow & \mathcal{O} \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ \phi_f & \longmapsto & f \end{array}$$

defines an isomorphism.

$\Rightarrow$ $1_{\mathcal{O}}$ corresponds to $1 \in \mathcal{O}$.

By P722, given the extension $0 \to \mathcal{O} \xrightarrow{\alpha} E \xrightarrow{\beta} I \to 0$, we have

$$\begin{array}{ccccc} \mathrm{Hom}_{\mathcal{O}}(I, E) & \longrightarrow & \mathrm{Hom}_{\mathcal{O}}(I, I) & \longrightarrow & \mathrm{Ext}^1_{\mathcal{O}}(I, \mathcal{O}) \\ & & \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ & & \mathrm{id} & \longmapsto & \partial(\mathrm{id}) \end{array}$$

$$\begin{tikzcd}
0 \arrow[r] & \mathrm{Hom}(I, \mathcal{O}) \arrow[r] \arrow[d] & \mathrm{Hom}(I, E) \arrow[r] \arrow[d] & \mathrm{Hom}(I, I) \ni \mathrm{id} \arrow[d] & \\
0 \arrow[r] & \mathrm{Hom}(\mathcal{O}\oplus\mathcal{O}, \mathcal{O}) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}\oplus\mathcal{O}, E) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}\oplus\mathcal{O}, I) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathrm{Hom}(\mathcal{O}, \mathcal{O}) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}, E) \arrow[r] \arrow[d] & \mathrm{Hom}(\mathcal{O}, I) \arrow[r] \arrow[d] & 0 \\
& 0 & 0 & 0 &
\end{tikzcd}$$

$(g_1, g_2) \mapsto g_1 e_1 + g_2 e_2$

$\psi : \mathcal{O}\oplus\mathcal{O} \to I$, $(g_1, g_2) \mapsto g_1 f_1 + g_2 f_2$

$h \mapsto -h f_2 e_1 + h f_1 e_2$

where $0 \longrightarrow \mathcal{O} \longrightarrow \mathcal{O}\oplus\mathcal{O} \longrightarrow I \longrightarrow 0$ is a project-

$$\begin{array}{ccc} h & \longmapsto & (-hf_2, hf_1) \\ (g_1, g_2) & \longmapsto & g_1 f_1 + g_2 f_2 \end{array}$$

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