There results a big commutative diagram:

$$\begin{tikzcd}
 & & & 0 \arrow[d] & 0 \arrow[d] & & 0 \arrow[d] & 0 \arrow[d] & \\
F_\cdot : & & 0 \arrow[r] \arrow[d] & F_{r-1} \arrow[r] \arrow[d] & F_{r-2} \arrow[r] & \cdots \arrow[r] & F_1 \arrow[r] \arrow[d] & I_{r-1} \arrow[r] \arrow[d] & 0 \\
E_\cdot : & 0 \arrow[r] & E_r \arrow[r] \arrow[d] & E_{r-1} \arrow[r] \arrow[d] & E_{r-2} \arrow[r] \arrow[d] & \cdots \arrow[r] & E_1 \arrow[r] \arrow[d] & I_r \arrow[r] \arrow[d] & 0 \\
Q_\cdot : & 0 \arrow[r] & Q_r \arrow[r] \arrow[d] & Q_{r-1} \arrow[r] \arrow[d] & Q_{r-2} \arrow[r] \arrow[d] & \cdots \arrow[r] & Q_1 \arrow[r, "\alpha"] \arrow[d] & I_r/I_{r-1} \arrow[r] \arrow[d] & 0 \\
 & & 0 & 0 & 0 & & 0 & 0 &
\end{tikzcd}$$

We make the identification

$$Q_k \cong \mathcal{O}(e_r \otimes \Lambda^{k-1}\mathbb{C}^{r-1}).$$

This being done, for $J = (j_1, \cdots j_{k-1}) \subset (1, 2, \cdots r-1)$,

$$\partial(e_r \otimes e_J) = f_r e_J \pm e_r \otimes \partial e_J$$
$$\equiv e_r \otimes \partial e_J \quad \text{modulo } F_k.$$

Fr $\quad Q_k = \dfrac{E_k}{F_k} \longrightarrow \mathcal{O}(e_r \otimes \Lambda^{k-1}\mathbb{C}^{r-1})$

$\qquad\qquad\qquad\qquad\qquad\qquad$ $\|$ free $\mathcal{O}$ module generated by $e_r \otimes \Lambda^{k-1}\mathbb{C}^{r-1}$.

$$1 \otimes e_J + F_k \xmapsto{\ \phi\ } e_r \otimes e_{J-\{r\}}$$

$$1 \otimes e_{J-\{r\}} \wedge e_r + F_k \xleftarrow{\ \psi\ } e_r \otimes e_{J-\{r\}}$$

$$(e_r, e_{J-\{r\}})$$

If $r \notin J$, $\phi(\quad) = 0$.