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$= \det(I - \frac{\sqrt{-1}}{2\pi} t\ {}^t\overline{\Theta}) = \det(I + \frac{\sqrt{-1}}{2\pi} t\ {}^t\Theta) = \det(I + \frac{\sqrt{-1}}{2\pi} t\ \Theta)$

$= 1 + t\overline{C_1} + t^2 \overline{C_2} + \cdots + t^p \overline{C_p} + \cdots$

Suppose C_p is of type (r, s). $\Rightarrow$ $r + s = 2p$ & $\overline{C_p}$ is of type (s, r). $\Rightarrow$ $\overline{C_p} = C_p$ $\Rightarrow$ $r = s$ $\Rightarrow$ C_p is of type (p, p). ⌟

In case M is a compact Kähler manifold, these imply that

$$C_p(E) \in H^{p,p}(M) \cap H^{2p}(M, \mathbb{Z});$$

i.e., the Chern classes are integral and of Hodge type (p, p).

⌜ By Gauss-Bonnet Formula II, $C_p(E)$ is Poincare dual to the degeneracy cycle D_{k-p+1}. $\Rightarrow$ $C_p(E)(\alpha)$ is an integer whenever α is a cycle of real dimension $2p$ on M, see P412 ~ P413. $\Rightarrow$ $C_p(E) \in H^{2p}(M, \mathbb{Z})$. By Hodge Decomposition (P116) $H^{p,p}(M) \cong H_{\bar\partial}^{p,p}(M) \ni C_p(E)$. $\Rightarrow$ $C_p(E) \in H^{p,p}(M)$

$\Rightarrow$ $C_p(E) \in H^{p,p}(M) \cap H^{2p}(M, \mathbb{Z})$.

My argument above might be wrong, because we don't know whether the generic sections exist or not.

Probably they exist. To use the terminology Hodge type, maybe they need "Kähler condition".

$\mathcal{H}_d^{p,p}(M) \cong H^{p,p}(M)$. ⌟

In case M is a projective algebraic variety, which by the Kodiara embedding theorem is equivalent to the existence of a positive holomorphic line bundle $L \to M$, we can say more. We assume that L is the hyperpl-