so the bundle $K_{\tilde{S}}$ is certainly not positive.

F By the lemma on P187, put $n=2$.
$\Rightarrow$ We get $\pi^* K_{\tilde{S}} + D_1 + \cdots + D_6 = K_S$.
By P476, $D \cdot K_{\tilde{S}} = \pi^* D \cdot \pi^* K_{\tilde{S}} = \pi^* D \cdot (K_S - D_1 - \cdots - D_6)$
$= \pi^* D \cdot K_S$

$\Rightarrow$ $K_S = -H \cap S$ in S.

$\pi^* D$ is an effective divisor. $\Rightarrow$ $\pi^* D \cdot K_S = -\pi^* D \cdot (H \cap S)$
< 0, since $\#(D_1 \cdot D_2) \geq 0$, for any two effective divisors D_1 & D_2, see P62, the cornerstone of algebraic geometry.
According to Castelnuovo-Enriques Criterion on P476, $\tilde{S}$ is a smooth algebraic surface.
If $K_{\tilde{S}}$ is positive, then, by the definition on P148, $\exists$ a metric on $K_{\tilde{S}}$ with curvature form Θ s.t. $(\sqrt{-1}/2\pi)\Theta$ is positive (1, 1)-form. For a smooth curve D in $\tilde{S}$,

$$\int_D (\sqrt{-1}/2\pi)\Theta = \int_D \eta_{K_{\tilde{S}}} = \eta_D \cdot \eta_{K_{\tilde{S}}} \geq 0$$

since $\sqrt{-1}/2\pi \, \Theta$ is a volume form on D, i.e.

$$\int_D \frac{\sqrt{-1}}{2\pi}\Theta = \mathrm{Vol}(D) \text{ w.r.t } \frac{\sqrt{-1}}{2\pi}\Theta .$$

$\Rightarrow$ $K_{\tilde{S}}$ can not be positive. ⌟

Consequently our argument that S is $\mathbb{P}^2$ blown up six times will be complete once we prove the