$\Gamma \quad 0 \longrightarrow \mathcal{O}_{\tilde{U}}(\tilde{U}\times\mathbb{C}\otimes[-E]) \longrightarrow \mathcal{O}_{\tilde{U}}(\tilde{U}\times\mathbb{C}) \longrightarrow \mathcal{O}_E(E\times\mathbb{C}) \to 0$

$\tilde{U} = \pi^{-1}(U) = \{(z, l) \in U \times \mathbb{P}^{n-1} \mid z \in l\}$.
$\tilde{U}_i = (l_i \neq 0) \subset \tilde{U}$

Let $\sigma_i : \tilde{U}_i \longrightarrow \mathbb{C}$ defined by
$(z, l) \longmapsto z_i$

$\Rightarrow \quad \sigma_i = \dfrac{z_i}{z_j}\sigma_j = g_{ij}\sigma_j$, where $g_{ij} = \dfrac{z_i}{z_j}$.

$\Rightarrow \{(\sigma_i, \tilde{U}_i)\}$ defines a section of $[E]$ over $\tilde{U}$,
since $z_i = 0 \Leftrightarrow \sigma_i = 0$ implies $(\overset{E}{\sigma} = 0)$ in $\tilde{U}_i$.

Consider $\pi^* f \otimes \sigma_i^{-1}(z, l) = \dfrac{f(z)}{z_i} : \tilde{U}_i \longrightarrow \mathbb{C}$.
$\Rightarrow$ We have to compute $\pi^* f \otimes \sigma_i^{-1}$ at $(0, l)$.
To do this, we need to compute.

$$\lim_{t\to 0} \pi^* f \otimes \sigma_i^{-1}(tl, l) = ?$$

$$\lim_{t\to 0} \frac{f(tl)}{tl_i} = \lim_{t\to 0} \frac{f(tl_1, tl_2, \cdots tl_n) - f(0, 0 \cdots 0)}{t\, l_i} = \lim_{t\to 0} \frac{1}{tl_i}\big(f(tl_1 \cdots tl_n)$$
$$- f(0, tl_2, \cdots tl_n) + f(0, tl_2, \cdots tl_n) - f(0, 0, tl_3, \cdots tl_n) + f(0, 0, tl_3 \cdots)$$
$$- \cdots$$
$$= \frac{1}{l_i}\frac{d}{dt} f(tl) = \frac{1}{l_i}\left(\frac{\partial f}{\partial z_1}, \frac{\partial f}{\partial z_2}, \cdots \frac{\partial f}{\partial z_n}\right)\cdot(l_1, l_2, \cdots l_n)$$
$$= \frac{l_1}{l_i}\frac{\partial f}{\partial z_1} + \frac{l_2}{l_i}\frac{\partial f}{\partial z_2} + \frac{l_3}{l_i}\frac{\partial f}{\partial z_3} + \cdots + \frac{\partial f}{\partial z_i} + \frac{l_{i+1}}{l_i}\frac{\partial f}{\partial z_{i+1}} + \cdots \frac{l_n}{l_i}\frac{\partial f}{\partial z_n}$$

$\tilde{U}_i|_E = \mathcal{O}_i = (l_i \neq 0) \subset \mathbb{P}^{n-1}$.