$$\leq C \sum_{\zeta} \Big(\sum_{[\alpha]=s} |\zeta^{\alpha}|^2 \Big) |\varphi_{\zeta}|^2 + |\varphi_0|^2$$

$$= C \sum_{[\alpha]=s} \Big(\sum_{\zeta} |\zeta^{\alpha}|^2 |\varphi_{\zeta}|^2 \Big) + |\varphi_0|^2$$

$$= C \sum_{[\alpha]=s} \| D^{\alpha}\varphi \|_0^2 + |\varphi_0|^2 < \infty$$ since # of α satisfying $[\alpha]=s$ is finite.

For example, $n=2$, $(1+\|\zeta\|^2)^s = (1+\zeta_1^2+\zeta_2^2)^s \leq (2\zeta_1^2+2\zeta_2^2)^s$
$= 2^s(\zeta_1^2+\zeta_2^2)^s = 2^s \sum_{\alpha_1+\alpha_2=s} (\zeta_1^2)^{\alpha_1}(\zeta_2^2)^{\alpha_2} \cdot {}_sC_{\alpha_1} \leq 2^s M \sum (\zeta_1^2)^{\alpha_1}(\zeta_2^2)^{\alpha_2}$

for some constant $M>0$, $\Rightarrow$ $2^s M \sum_{\alpha_1+\alpha_2=s} (\zeta_1^2)^{\alpha_1}(\zeta_2^2)^{\alpha_2} \leq C_s \sum_{[\alpha]=s} |\zeta^{\alpha}|^2$ ⫽
We assume $\zeta \neq 0$.

From $$\sum_{[\alpha]\leq s} |\zeta^{2\alpha}| \leq (1+\|\zeta\|^2)^s \leq C_s \sum_{[\alpha]\leq s} |\zeta^{2\alpha}|$$

we see that on $C^s(T) \subset H_s$, the ~~Sob~~ Sobolev norm $\| \ \|_s$ is equivalent to $$\sum_{[\alpha]\leq s} \|D^{\alpha}\varphi\|_0^2$$ which we may describe as the L^2-norm of the function φ together with its derivative up to order s.

Γ $$\|\varphi\|_s^2 = \sum_{\zeta} (1+\|\zeta\|^2)^s |\varphi_{\zeta}|^2 \leq \sum_{\zeta} \Big(C_s \sum_{[\alpha]\leq s} |\zeta^{2\alpha}| \Big) |\varphi_{\zeta}|^2$$

$$= C_s \sum_{[\alpha]\leq s} \Big(\sum_{\zeta} |\zeta^{2\alpha}| |\varphi_{\zeta}|^2 \Big) = C_s \sum_{[\alpha]\leq s} \|D^{\alpha}\varphi\|_0^2$$

$$\sum_{[\alpha]\leq s} \|D^{\alpha}\varphi\|_0^2 = \sum_{[\alpha]\leq s} \Big(\sum_{\zeta} |\zeta^{2\alpha}| |\varphi_{\zeta}|^2 \Big) = \sum_{\zeta} \Big(\sum_{[\alpha]\leq s} |\zeta^{2\alpha}| \Big) |\varphi_{\zeta}|^2$$

$$\leq \sum_{\zeta} (1+\|\zeta\|^2)^s |\varphi_{\zeta}|^2 = \|\varphi\|_s^2$$