(c), (e), and (f) are also sufficient. For example, if $E \subset \mathcal{D}_K$ and $\|\phi\|_N \le M_N < \infty$ for every $\phi \in E$, then E is a bounded subset of $\mathcal{D}_K$ (Section 1.46), and now (b) implies that E is also bounded in $\mathcal{D}(\Omega)$.

「 To show that E is bounded in $\mathcal{D}(\Omega)$, given a nbd V of 0 in $\mathcal{D}(\Omega)$, we have to show that $\exists$ $s \in \mathbb{R}$ s.t if $\forall t > s$, $tV \supset E$, see P8.

$V \cap \mathcal{D}_K$ is open in $\mathcal{D}_K$ and $V \cap \mathcal{D}_K \ni 0$. $\Rightarrow$ Since E is bounded in $\mathcal{D}_K$, $\exists$ $s \in \mathbb{R}$ s.t if $\forall t > s$, then $t(V \cap \mathcal{D}_K) \supset E$. $\Rightarrow$ For all $t > s$, $tV \supset E$. $\Rightarrow$ E is bounded in $\mathcal{D}(\Omega)$.

Suppose $\{\phi_i\} \subset \mathcal{D}_K$ for some compact $K \subset \Omega$, and

$$\lim_{i,j\to\infty} \|\phi_i - \phi_j\|_N = 0 \qquad (N = 0, 1, 2, \cdots).$$

Then $\{\phi_i\}$ is Cauchy sequence in $\mathcal{D}(\Omega)$.

pf). Given an open set $V \ni 0$ in $\mathcal{D}(\Omega)$, $V \cap \mathcal{D}_K$ is open in $\mathcal{D}_K$. $\Rightarrow$ $\exists$ N s.t if $n, m > N$, then $\phi_n - \phi_m \in V \cap \mathcal{D}_K$. $\Rightarrow$ $\phi_n - \phi_m \in V$ $\Rightarrow$ $\{\phi_i\}$ is C.S in $\mathcal{D}(\Omega)$. 」

proof. Suppose first that $V \in \tau$. Pick $\phi \in \mathcal{D}_K \cap V$. By Theorem 6.4, $\phi + W \subset V$ for some $W \in \beta$. Hence

$$\phi + (\mathcal{D}_K \cap W) \subset \mathcal{D}_K \cap V.$$

「 $\phi + (\mathcal{D}_K \cap W) \subset \mathcal{D}_K \cap V$ clearly 」

Since $\mathcal{D}_K \cap W$ is open in $\mathcal{D}_K$, we have proved that