and, taking adjoints,

$$\bar\partial^* = -\sum \partial_k \bar\iota_k$$

$$\partial^* = -\sum \bar\partial_k \iota_k .$$

L is defined as exterior product with the standard Kähler form defined on $\mathbb{C}^n$, so

$$L = \frac{i}{2}\sum e_k \bar e_k$$

and, taking the adjoint,

$$\Lambda = -\frac{i}{2}\sum \bar\iota_k \iota_k .$$

So

$$\Lambda\partial = -\frac{i}{2}\sum_{k,l} \bar\iota_k \iota_k \partial_l e_l \overset{\text{since } \partial_l \text{ commutes with } \bar\iota_k, \iota_k,}{=} -\frac{i}{2}\sum_{k,l} \partial_l \bar\iota_k \iota_k e_l$$

$$= -\frac{i}{2}\Big(\sum_k \partial_k \bar\iota_k \iota_k e_k + \sum_{k\neq l} \partial_l \bar\iota_k \iota_k e_l\Big).$$

To evaluate the first term, write

$$-\frac{i}{2}\sum_k \partial_k \bar\iota_k \iota_k e_k = \frac{i}{2}\sum \partial_k \bar\iota_k(-2 + e_k \iota_k)$$

$$= \frac{i}{2}\sum \partial_k \bar\iota_k e_k \iota_k - i\sum \partial_k \bar\iota_k$$

$$= \frac{i}{2}\sum \partial_k(-e_k \bar\iota_k)\iota_k - i\sum \partial_k \bar\iota_k$$

$$= -\frac{i}{2}\sum \partial_k e_k \bar\iota_k \iota_k - i\sum \partial_k \bar\iota_k \quad \text{--- ①}$$

For the second term,

$$-\frac{i}{2}\sum_{k\neq l} \partial_l \bar\iota_k \iota_k e_l \overset{\bar\iota_k e_l + e_l \bar\iota_k = 0}{=} -\frac{i}{2}\sum_{k\neq l} \partial_l \bar\iota_k(-e_l \iota_k)$$

$$= \frac{i}{2}\sum_{k\neq l} \partial_l \bar\iota_k e_l \iota_k = -\frac{i}{2}\sum_{k\neq l} \partial_l (e_l \bar\iota_k)\iota_k$$

$$= -\frac{i}{2}\sum_{k\neq l} \partial_l e_l \bar\iota_k \iota_k \quad \text{--- ②}$$

Thus $\Lambda\partial = -\frac{i}{2}\sum_{k,l} \partial_l e_l \bar\iota_k \iota_k + i\sum \partial_k \bar\iota_k$ by adding ① & ②,