Put $A = \{ a \in F^p K^{p+q} ; \ da \in F^{p+2} K^{p+q+1} \}$

$B = F^{p+1} K^{p+q}$. $\quad C = (d(F^{p-1} K^{p+q-1}) + F^{p+1} K^{p+q}) \cap F^p K^{p+q}$

$\Rightarrow \quad B \subset C$.

$$\frac{\ker d_1}{\operatorname{Im} d_1} = \frac{A+B}{C} \cong \frac{A}{A \cap C}$$

$$= \frac{\{ a \in F^p K^{p+q} : da \in F^{p+2} K^{p+q+1} \}}{\{ a ; da \in F^{p+2} K^{p+q+1} \} \cap (d(F^{p-1} K^{p+q-1}) + F^{p+1} K^{p+q})}$$

since $A = \{ a \in F^p K^{p+q} : da \in F^{p+2} K^{p+q+1} \} \subset F^p K^{p+q}$.

Thus $$\frac{\ker d_1}{\operatorname{Im} d_1} = E_2^{p,q} = \frac{\{ a \in F^p K^{p+q} ; \ da \in F^{p+2} K^{p+q+1} \}}{(d(F^{p-1} K^{p+q-1}) + F^{p+1} K^{p+q}) \cap A}.$$

Here, the denominator is not a subgroup of the numerator; the meaning is that we take {denominator as written} $\cap$ {numerator}. A similar remark applies during the remainder of this proof.

Continuing in this way, we define in general

$$E_r^{p,q} = \frac{\{ a \in F^p K^{p+q} ; \ da \in F^{p+r} K^{p+q+1} \}}{d(F^{p-r+1} K^{p+q-1}) + F^{p+1} K^{p+q}},$$

and for $[a] \in E_r^{p,q}$ we define

$$d_r a = [da] \in \frac{\{ b \in F^{p+r} K^{p+q+1} ; \ db \in F^{p+r+2} K^{p+q+2} \}}{d(F^{p+1} K^{p+q}) + F^{p+r+1} K^{p+q+1}}$$

$$= E_r^{p+r,\, q-r+1}.$$

$\therefore \quad d_r : E_r^{p,q} \longrightarrow E_r^{p+r,\, q-r+1}$