$\Rightarrow$ Obviously, for sufficiently small $\epsilon > 0$, if $\| A - A' \| < \epsilon$,

$$\det \begin{bmatrix} e_1 + A_1' \\ \vdots \\ e_k + A_k' \\ e_{j_1} \\ \vdots \\ e_{j_{n-k}} \end{bmatrix} \neq 0, \text{ i.e } V_{I'^0} \cap \langle e_i + A_i' \rangle = \{0\}.$$

since det is continuous. ⌟

Comment : $\Lambda \cap V_{I^0} = \{0\} \iff \Lambda + V_{I^0} = \mathbb{C}^n$. ⌟

But this is clear: if, for $\Lambda \in U_I \cap U_{I'}$, we let $\Lambda^I_{I'}$ be the I'th $k \times k$ minor of Λ^I, then

$$\Lambda^{I'} = (\Lambda^I_{I'})^{-1} \cdot \Lambda^I,$$

and since the entries of $(\Lambda^I_{I'})^{-1}$ vary holomorphically with the entries of Λ^I, $\varphi_I \circ \varphi_{I'}^{-1}$ is holomorphic.

$\ulcorner$ For $\Lambda \in U_I \cap U_{I'}$, without loss of generality, assume that

$$\varphi_I(\Lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 & a_{11} & \cdots & a_{1\,n-k} \\ 0 & 1 & \ddots & & \vdots & & \\ \vdots & & & & & & \\ 0 & 0 & & 1 & a_{k1} & \cdots & a_{k,n-k} \end{pmatrix} \in \mathbb{C}^{k(n-k)},$$

and

$$\varphi_{I'}(\Lambda) = \begin{array}{c} \overbrace{\qquad\qquad}^{I'} \\ \begin{pmatrix} * & \cdots & 1 & 0 & \cdots & 0 & * & * \\ * & \cdots & 0 & \ddots & & 0 & \vdots & \vdots \\ \vdots & & & & & \vdots & & \\ * & \cdots & 0 & 0 & & 0\ 1 & * & * \end{pmatrix} \end{array} \in \mathbb{C}^{k(n-k)}$$

$\Rightarrow$ By P 483, note, $\varphi_{I'}(\Lambda) = g\, \varphi_I(\Lambda)$, $g \in GL_k$.

$\Rightarrow$ g^{-1} must be the I'th $k \times k$ minor of $\varphi_I(\Lambda)$, i.e. g is the $\{1, 2, \cdots k\}$ minor of $\varphi_{I'}(\Lambda)$,

$$\varphi_{I'}(\Lambda) = \Big(\underbrace{\overbrace{\begin{matrix} * & & 1 \\ * & & \\ \vdots & & \\ * & & 0 \end{matrix}}^{k}}_{g^{-1}} \;\; \begin{matrix} 0 & * & * \\ \ddots & & \\ 1 & * & * \end{matrix} \Big)$$