$v_1 = (*, *, \cdots *, 1, 0, \cdots 0)$.

$\ulcorner$ This is possible since. $W_{a_1 \cdots a_k} \in U_I$ where $I = \{b_1 \cdots b_k\}$, $b_i = n-k+i-a_i$.

$$\Lambda \cap V_{I^\circ} = (0)$$

If $W \notin U_I$, this is impossible.

Example. $W_{2,1} = \{\Lambda \in G(2,4) ;\ \dim(\Lambda \cap \langle e_1 \rangle) = 1$
$\dim(\Lambda \cap \langle e_1, e_2, e_3 \rangle) = 2 \}$.

$a_1 = 2,\ a_2 = 1$

$b_1 = 4-2+1-2 = 1 \qquad b_2 = 4-2+2-1 = 3$

$\Rightarrow$ Consider $\Lambda_0 = \langle e_1, e_2 \rangle$. $\Rightarrow$ Clearly $\Lambda_0 \in W_{2,1}$, but $\Lambda_0 \cap V_1 = \Lambda_0 \cap \langle e_1 \rangle = \langle e_1 \rangle$ and $\Lambda_0 \cap \langle e_1, e_2, e_3 \rangle = \langle e_1, e_2 \rangle$. so that we can not choose v_2 s.t $\langle v_2, e_3 \rangle = 1$. $\lrcorner$

Now take v_2 so that v_1 and v_2 together span $\Lambda \cap V_{n-k+2-a_2}$, normalized so that

$$\langle v_2, e_{n-k+1-a_1} \rangle = 0, \qquad \langle v_2, e_{n-k+2-a_2} \rangle = 1.$$

$\ulcorner$ See the explanation above. $\lrcorner$

Continue in this way, choosing v_i so that $v_1, v_2 \cdots v_i$ span $\Lambda \cap V_{n-k+i-a_i}$ and such that

$$\langle v_i, e_{n-k+j-a_j} \rangle = \begin{cases} 0, & j < i, \\ 1, & j = i. \end{cases}$$

$\ulcorner$ $\langle v_1 \rangle = \Lambda \cap V_{n-k+1-a_1} \qquad \langle w, v_1 \rangle = \Lambda \cap V_{n-k+2-a_2}$

$\Rightarrow v_2 = \alpha v_1 + \beta w$. $0 = \langle v_2, e_{n-k+1-a_1} \rangle = \alpha \langle v_1, e_{n-k+1-a_1} \rangle +$