Since $\bar\partial \Phi$ and $P(\Theta)$ are both forms of top degree,

$$\iota(v)\,(\bar\partial\Phi + P(\Theta)) = 0$$

implies that

$$\bar\partial\Phi + P(\Theta) = 0$$

and we have constructed our explicit solution to $\bar\partial\Lambda = P(\Theta)$.

$\ulcorner$

$$\iota(v)\,\bar\partial\Phi = -\iota(v)\,P(\Theta) \quad\Rightarrow\quad \iota(v)\,(\bar\partial\Phi + P(\Theta)) = 0$$

$$\Phi_r \in A^{n,n-1} \quad\Rightarrow\quad \bar\partial\Phi = \sum_{i=0}^{n-1}\bar\partial\Phi_i \in A^{n,n}$$

$$P(\Theta) \in A^{n,n} \quad\Rightarrow\quad \bar\partial\Phi + P(\Theta) \in A^{n,n}(M).$$

$$\Rightarrow\quad \bar\partial\Phi + P(\Theta) = a\, dz_1\wedge\cdots\wedge dz_n \wedge d\bar z_1\wedge\cdots\wedge d\bar z_n \quad \text{locally}.$$

$$\Rightarrow\quad v = v^i\frac{\partial}{\partial z_i}$$

$$\iota(v)\,(a\, dz_1\wedge\cdots\wedge dz_n\wedge d\bar z_1\wedge\cdots\wedge d\bar z_n)$$

$$= a\, v^i (-1)^{i-1} dz_1\wedge\cdots\wedge \widehat{dz_i}\wedge\cdots\wedge dz_n\wedge d\bar z_1\wedge\cdots\wedge d\bar z_n = 0$$

$$\Rightarrow\quad a v^i = 0 \text{ for all } i \quad\Rightarrow\quad a = 0 \quad\Rightarrow\quad \bar\partial\Phi + P(\Theta) = 0. \quad \lrcorner$$

"Comment on sign."

$$\iota(v)\,\Theta = -\bar\partial E,$$

If we put

$$P_r(E,\Theta) = \binom{n}{r}\tilde P(\underbrace{-E,\cdots -E}_{n-r},\ \underbrace{\Theta,\cdots\Theta}_{r}),$$

then

$$\bar\partial P_r(E,\Theta) = \binom{n}{r}\sum_{i=1}^{n-r}\tilde P(-E,\cdots,\overset{i}{-\bar\partial E},\cdots -E,\Theta\cdots\Theta)$$

$$= \binom{n}{r}\sum_{i=1}^{n-r}\tilde P(-E,\cdots,\iota(v)\Theta,\cdots -E,\Theta,\cdots\Theta)$$

$$= (n-r)\binom{n}{r}\tilde P(\underbrace{-E,\cdots -E}_{n-r-1}\ \iota(v)\Theta,\cdots\Theta)$$