points on S. We will do this first by an Euler characteristic argument, as follows: Let $\{H_\lambda\}$ be a generic pencil of hyperplanes in $\mathbb{P}^3$ — specially, one such that for each $p \in R$, p lies on a unique H_λ and H_λ is generic among hyperplanes containing p; and such that the pencil $\{H_\lambda \cap S\}$ is Lefschetz on $S - R$.

⌜ Choose generic hyperplanes H_1 & H_2 s.t. $H_i \not\ni p$, for all $p \in R$. $\Rightarrow$ $\{H_1 + \lambda H_2\}$ has a unique element H_λ containing p, for each $p \in R$.

I can not assure myself that for a generic pencil $\{H_\lambda\}$, $\{H_\lambda \cap S\}$ is Lefschetz on $S-R$. But I can get some idea as follows: For a plane curve C, the condition that C has one double point as the only singularity, is the weakest one. ...... Pathological ...
In some sense, Lefschetz pencil is generic. ⌟

Let
$$\{C_\lambda = \pi^{-1}(H_\lambda)\}$$
be the corresponding pencil of curves on Σ.

⌜ $\pi : \Sigma \longrightarrow S$, $[H] \downarrow \uparrow \sigma_1, \sigma_2$ on S $\Rightarrow$ $\pi^*\sigma_1$ & $\pi^*\sigma_2$ form a pencil on Σ. ⌟