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me as that of $\tilde{f}\,(=m_0)$.

For example, $f(z_1, z_2) = z_1 \cdot z_2$ vanishes to order 1
$f(z) = z_1^2 z_2$ vanishes to order 2. $\left.\frac{\partial f}{\partial z_i}\right|_{z=0} = 0$

$\Rightarrow \quad \left.\frac{\partial^2 f}{\partial z_i \partial z_j}\right|_{z=0} = 0$

The vanishing order depends on the degree of the polynomial.

By the definition of $mult_p(V)$ on $p \ge 0$, $mult_p(V) =$ the vanishing order of f at p. Since $\overline{\pi^{-1}(V) - E}$ is an algebraic variety obtained from $\pi^* V$ by getting rid of a divisor of type mE, we have

$$\tilde{V} = \pi^* V - mult_p(V) \cdot E$$
$$= \pi^* V - ord_E(\pi^* V) \cdot E, \quad ord_E(\pi^* f) = \text{the vanish-}$$

ing order of $\pi^* f$ along E. = the vanishing order of f at p

We had better use the definition for $\tilde{V}$ as follows

$$\tilde{V} = \overline{\pi^*(V - \{p\})}$$

Moreover, we see that

$$\tilde{V} \cap E = (z_i^{-m_0} \cdot f_m)$$
$$= \Big(\sum_{|a| = m_0} c_a \, l_1^{a_1} \cdots l_n^{a_n} \Big),$$

i.e., under the identification $E \cong \mathbb{P}(T_p'(M))$, $\tilde{V} \cap E$ is just the projective tangent cone to V at p. Figure 1 illustrates the case of a surface M and a curve V in M with an arbitrary double point at p.

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