

$$\varphi \left(h^{-1} \frac{\partial}{\partial \bar{z}} \right) = h^{-1} \bar{a} h^2 = 1 \quad \bar{a} = h^{-1}.$$

$$\text{since } \left\langle h^{-1} \frac{\partial}{\partial \bar{z}}, h^{-1} \frac{\partial}{\partial \bar{z}} \right\rangle = 1.$$

$$\varphi \left(\frac{\partial}{\partial \bar{z}} \right) = h \Rightarrow \varphi = h d\bar{z}.$$

$$\Rightarrow d\varphi = \bar{\partial} h \wedge d\bar{z} = \frac{\bar{\partial} h}{h} \wedge \varphi. \Rightarrow \psi'' = \bar{\partial} \log h.$$

$$\begin{aligned} \Rightarrow \psi &= (\bar{\partial} - \partial) \log h. \Rightarrow \theta = -{}^t\psi = -(\bar{\partial} - \partial) \log h \\ &= (\partial - \bar{\partial}) \log h. \\ &= \frac{\partial \log h}{\partial z} dz - \frac{\partial \log h}{\partial \bar{z}} d\bar{z}. \end{aligned}$$

Since, $\theta \wedge \theta = 0$, by the Cartan structure equation

$$\textcircled{H} = d\theta = -2 \left(\frac{\partial^2}{\partial z \partial \bar{z}} \log h \right) dz \wedge d\bar{z}.$$

$$\begin{aligned} \left(\bar{\partial} \frac{\partial \log h}{\partial \bar{z}} \right) dz &= -\frac{1}{2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \log h \, dz \wedge d\bar{z} \\ - \left(\bar{\partial} \left(\frac{\partial \log h}{\partial \bar{z}} \right) \right) d\bar{z} &= -\frac{1}{2} \Delta \log h \cdot dz \wedge d\bar{z} \end{aligned}$$

\Rightarrow By comparing the curvature "matrix" \textcircled{H} with the associated (1,1)-form $\Phi = \frac{\bar{c}}{2} \varphi \wedge \bar{\varphi} = \frac{\bar{c}}{2} h^2 dz \wedge d\bar{z}$,

$$\text{we get } \bar{c} \textcircled{H} = K \cdot \Phi \text{ where } K = \frac{-\Delta \log h}{h^2} \text{ is}$$

the usual Gaussian curvature.

② $E \rightarrow M$ hermitian bundle, $S \subset E$ holo subbundle.
 $Q = E/S$ quotient bundle.

S^\perp has a hermitian structure and S^\perp is not a holo subbundle of E .

According to p71 (See Milnor p28), Q is isomorphic, as a C^∞ v.b., to the orthogonal complement S^\perp of S in E .

\Rightarrow Both S and Q inherit hermitian structures from E .

Let D_E, D_S & D_Q corresponding metric connections.