

From  $H(X_0(t), X_1(t), X_2(t))$ ,  $\frac{d^2}{dt^2} H(X_0(t), X_1(t), X_2(t)) \Big|_{t=0}$

$$= \frac{\partial^2 H}{\partial X_0^2}(p) X_0'(0)^2 + \frac{\partial^2 H}{\partial X_1^2}(p) X_1'(0)^2 + \frac{\partial^2 H}{\partial X_2^2}(p) X_2'(0)^2 + 2 \frac{\partial^2 H}{\partial X_0 \partial X_1}(p) X_0'(0) X_1'(0) \\ + 2 \frac{\partial^2 H}{\partial X_1 \partial X_2}(p) X_1'(0) X_2'(0) + 2 \frac{\partial^2 H}{\partial X_0 \partial X_2}(p) X_0'(0) X_2'(0) = 0 \quad \dots (1)$$

By p175,  $\frac{\partial H}{\partial X_0}(X(t)) X_0(t) + \frac{\partial H}{\partial X_1}(X(t)) X_1(t) + \frac{\partial H}{\partial X_2}(X(t)) X_2(t) = 0$ .

Differentiating w.r.t  $t$  and plugging in  $t=0$ , we get

$$\frac{\partial^2 H}{\partial X_0^2}(p) X_0'(0) X_0(0) + \frac{\partial^2 H}{\partial X_1^2}(p) X_1'(0) X_1(0) + \frac{\partial^2 H}{\partial X_2^2}(p) X_2'(0) X_2(0) \\ + \frac{\partial^2 H}{\partial X_0 \partial X_1}(p) (X_0'(0) X_1(0) + X_1'(0) X_0(0)) + \frac{\partial^2 H}{\partial X_1 \partial X_2}(p) (X_1'(0) X_2(0) + X_2'(0) X_1(0)) \\ + \frac{\partial^2 H}{\partial X_0 \partial X_2}(p) (X_0'(0) X_2(0) + X_2'(0) X_0(0)) = 0 \quad \dots (2)$$

Consider  $\frac{\partial H}{\partial X_0} X_0 + \frac{\partial H}{\partial X_1} X_1 + \frac{\partial H}{\partial X_2} X_2 = 0$ .

Apply the equation on p175 to  $\frac{\partial H}{\partial X_0} X_0 + \frac{\partial H}{\partial X_1} X_1 + \frac{\partial H}{\partial X_2} X_2 = 0$ , and we have

$$X_0 \frac{\partial}{\partial X_0} \left( \frac{\partial H}{\partial X_0} X_0 + \frac{\partial H}{\partial X_1} X_1 + \frac{\partial H}{\partial X_2} X_2 \right) + X_1 \frac{\partial}{\partial X_1} \left( \frac{\partial H}{\partial X_0} X_0 + \frac{\partial H}{\partial X_1} X_1 + \frac{\partial H}{\partial X_2} X_2 \right) + \\ X_2 \frac{\partial}{\partial X_2} \left( \frac{\partial H}{\partial X_0} X_0 + \frac{\partial H}{\partial X_1} X_1 + \frac{\partial H}{\partial X_2} X_2 \right) = \frac{\partial^2 H}{\partial X_1^2} X_1^2 + \frac{\partial^2 H}{\partial X_0^2} X_0^2 + \frac{\partial^2 H}{\partial X_2^2} X_2^2$$

$$+ 2 \frac{\partial^2 H}{\partial X_0 \partial X_1} X_0 X_1 + 2 \frac{\partial^2 H}{\partial X_1 \partial X_2} X_1 X_2 + 2 \frac{\partial^2 H}{\partial X_0 \partial X_2} X_0 X_2 = 0.$$