The fact that the volume of a complex submanifold S of the complex manifold M is expressed as the integral over S of a globally defined differential form on M is quite different from the real case. For a C^∞ arc

$$t \longmapsto (x(t), y(t))$$

in $\mathbb{R}^2$, for example, the element of arc length is given by

$$(x'(t)^2 + y'(t)^2)^{\frac{1}{2}}\, dt,$$

which is not, in general, the pullback of any differential form in $\mathbb{R}^2$.

Γ Suppose that $(\alpha'(t)^2 + \beta'(t)^2)\, dt$ is the pullback of one form $f(x,y)\, dx + g(x,y)\, dy$ in $\mathbb{R}^2$.

$\Rightarrow \quad h^*(f(x,y)\, dx + g(x,y)\, dy)$

$\quad = f(\alpha(t), \beta(t))\, h^*dx + g(\alpha(t), \beta(t))\, h^*dy.$

where $h : \mathbb{R} \longrightarrow \mathbb{R}^2$

$\qquad t \longmapsto (\alpha(t), \beta(t)).$

$\Rightarrow \quad f(\alpha(t), \beta(t))\, dx \circ h + g(\alpha(t), \beta(t))\, dy \circ h$

$\quad = f(\alpha(t), \beta(t))\, d\alpha(t) + g(\alpha(t), \beta(t))\, d\beta(t)$

$\quad = f(\alpha(t), \beta(t))\, \alpha'(t)\, dt + g(\alpha(t), \beta(t))\, \beta'(t)\, dt \qquad f \,\&\, g.$

$\quad = (f\alpha' + g\beta')\, dt = (\alpha'(t)^2 + \beta'(t)^2)\, dt$

$\Rightarrow \quad f(\alpha(t), \beta(t)) = \alpha'(t), \quad g(\alpha(t), \beta(t)) = \beta'(t) \Rightarrow \exists$ no such

Thus the element of arc length given by

$$(x'(t)^2 + y'(t)^2)\, dt$$

is not, in general, the pullback of any differential form in $\mathbb{R}^2$. This implies that not every curve (in $\mathbb{R}^2$) has the volume element induced from