Date

as generator, we have an isomorphism

$$H_{2k}(\mathbb{P}^n, \mathbb{Z}) \cong \mathbb{Z}.$$

The degree of a k-dimensional variety $V \subset \mathbb{P}^n$ is its fundamental class in $H_{2k}(\mathbb{P}^n, \mathbb{Z})$ via this identification.

「 According to Note (P 61),
the fundamental class of a variety $V \subset M$ is the element of homology given by the linear functional

$$\begin{array}{ccl} H^{2k}_{DR}(M) & \longrightarrow & \mathbb{C} \qquad \text{by P56 \& P59} \\ \psi & & \\ [\varphi] & \longmapsto & \int_V \varphi = \#(\bar{V} \cdot B) \end{array}$$

where B is a $(2n-2k)$ cycle representing φ,

$\bar{V}$ is a $2k$ cycle in M.

$$\Rightarrow \quad [\bar{V}] \in H_{2k}(\mathbb{P}^n, \mathbb{Z}) \cong \mathbb{Z}.$$
$$\deg V.$$ 」

Alternative definitions abound. First, by Bertini applied to the smooth locus of V the generic $(n-k)$-plane $\mathbb{P}^{n-k} \subset \mathbb{P}^n$ will intersect V transversely, and so will meet V in exactly
$\#(\mathbb{P}^{n-k} \cdot V) = \text{degree}(V)$

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