Date

$[0, Z_1, Z_1] \Rightarrow Z_2 = Z_1 \Rightarrow \dfrac{Z_2 - Z_1}{Z_0}$ has no zero at $[0, 1, 1]$.

Thus we have a meromorphic function on C with a simple pole at $[0, 1, -1]$, which is holomorphic elsewhere.

$$\begin{array}{ccc}
\mathbb{C} & \longrightarrow & \mathbb{P}^1 \\
\rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\
[Z_0, Z_1, Z_2] & \longmapsto & [Z_0, Z_2 - Z_1] \\
\downarrow & & \downarrow \\
\mathbb{C}^2 \ni & & \mathbb{C} \ni \\
\left(\dfrac{Z_1}{Z_0}, \dfrac{Z_2}{Z_0}\right) & \longmapsto & \dfrac{Z_2 - Z_1}{Z_0} = y - x = t(x, y) \\
\| & & \\
(x, y) & &
\end{array}$$

$$\Rightarrow \int \frac{dx}{y} = \int \frac{dx}{\sqrt{x^2 + ax + b}} \Rightarrow$$

$$t + x = \sqrt{x^2 + ax + b} \Rightarrow \frac{dt}{dx} = \frac{x + \frac{a}{2}}{\sqrt{x^2 + ax + b}} - 1$$

$$= \frac{x + \frac{a}{2} - \sqrt{x^2 + ax + b}}{\sqrt{x^2 + ax + b}} = \frac{\frac{a}{2} - t}{\sqrt{x^2 + ax + b}} = \frac{\frac{a}{2} - t}{y}$$

$$\Rightarrow \frac{dt}{\frac{a}{2} - t} = \frac{dx}{y}$$

$$\Rightarrow \int \frac{dx}{y} = \int \frac{dt}{\frac{a}{2} - t}, \quad R(t) = \left(\frac{a}{2} - t\right)^{-1}$$

$$= -\ln\left|t - \frac{a}{2}\right| + C \leftarrow \text{constant.}$$

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This is the proof of the statement above that $\int \frac{dx}{y}$ can be thought of as the line integral i.e. the integral over line

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