$$= \cdots + (-1)^{p+q-1} \dim H^{p-r,\,q+r-1} + (-1)^{p+q} \dim H^{p,q} + (-1)^{p+q+1} \dim H^{p+r,\,q-r+1} + \cdots$$

$\Rightarrow$ We get

$$\sum_{p,q} (-1)^{p+q} \dim E_r^{p,q} = \sum_{p,q} (-1)^{p+q} \dim H^{p,q}(E_r)$$
$$= \sum_{p,q} (-1)^{p+q} \dim E_{r+1}^{p,q} = \cdots = \sum_{p,q} (-1)^{p+q} \dim E_\infty^{p,q}$$

$$\Rightarrow \quad \sum_{p,q} (-1)^{p+q} \dim E_1^{p,q} = \sum_{p,q} (-1)^{p+q} \dim H_{\bar\partial}^{p,q}(M)$$
$$= \sum_{p,q} (-1)^{p+q} h^{p,q} = \sum_{p,q} (-1)^{p+q} \dim E_\infty^{p,q} = \sum_r (-1)^r \dim H_{DR}^r(M)$$
$$= \sum_r (-1)^r b_r = \chi(M).$$

At the other extreme, we suppose M is a noncompact complex manifold and that the Dolbeault cohomology

$$(*) \qquad H_{\bar\partial}^{p,q}(M) = 0, \qquad q > 0.$$

This happens if M is what is called a Stein manifold — e.g., in Section 3 of Chapter 0 we proved $(*)$ when
$$M = \Delta^{*\,k} \times \Delta^{n-k}$$
is a punctured polycylinder defined by
$$\{ z \in \mathbb{C}^n : \ |z_i| < 1, \ z_1, \cdots\cdots z_k \neq 0 \}.$$
If $(*)$ is satisfied, then $'E_1^{p,q} = 0$ for $q > 0$ and the first spectral sequence is trivial from E_2 onward; i.e., $'E_2 \cong {'E_\infty}$.

Clearly, $q > 0 \quad 'E_1^{p,q} = 0 \Rightarrow {'E_2^{p,q}} = \cdots = {'E_\infty^{p,q}} = 0.$
$q = 0 \quad 'E_1^{p,0} = H_{\bar\partial}^{p,0}(M) \qquad \bar\partial : H_{\bar\partial}^{p,0}(M) \longrightarrow H_{\bar\partial}^{p+1,0}(M).$