$$M - V \xrightarrow{f} \mathbb{P}^n \supset U_0 = (X_0 \neq 0) \longrightarrow \mathbb{C}^n$$

$$x \longmapsto (f_1(x), f_2(x), \cdots f_n(x))$$

$$x \longmapsto [1, f_1(x), \cdots f_n(x)]$$

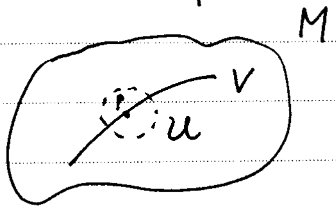
$$f^*\left(\frac{X_1}{X_0}\right) = \frac{X_1}{X_0}(f(x)) = f_1(x) \quad \cdots \quad f^*\left(\frac{X_n}{X_0}\right) = f_n(x).$$

Consider the holomorphic function f_i on $M-V$.

$\Rightarrow$ By the Levi extension theorem (I) on P396, f_i is extended to M as a meromorphic function, denote it by f_i.

$\Rightarrow$ $f = [1, f_1(x), \cdots f_n(x)]$ is rational.

Note that the extension is unique: locally an extension is expressed as $\frac{h_1}{k_1}$, h_1 k_1 relatively prime



If $\frac{h_1}{k_1} = \frac{h_2}{k_2}$ on $U-V$, then (h_2, k_2 relatively prime.)

$$h_1 k_2 - k_1 h_2 = 0 \text{ on } U-V.$$

$\Rightarrow$ By the identity theorem, $h_1 k_2 - k_1 h_2 = 0$ on U.

$\Rightarrow$ In $\mathcal{O}_p$, $h_1 = \ell h_2$ and $k_2 = \ell' k_1$

$\Rightarrow$ $h_1 k_2 = \ell h_2 \ell' k_1 = k_1 h_2 \Rightarrow \ell \ell' = 1$

$\Rightarrow$ $\dfrac{h_1}{k_1} = \dfrac{\ell h_2}{\frac{1}{\ell'} k_2} = \dfrac{h_2}{k_2}$

not necessary.

$\Rightarrow$ $\dfrac{h_1}{k_1} = \dfrac{h_2}{k_2}$ on U $\Rightarrow$ The extension is unique.

This affords a second point of view on rational maps, namely

A rational map

$$f : M \longrightarrow N$$