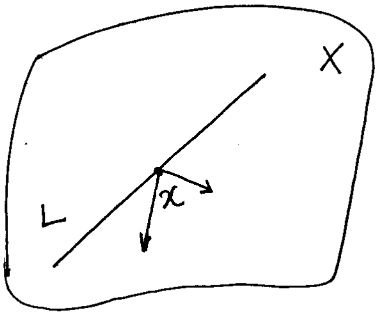
We can identify one of the rulings on the quadric $Q = \tilde{f}_L(F)$: first, for every $x \in L$, the image under $\tilde{f}_L$ of $\pi^{-1}(x) \subset \tilde{X}_L$ is just the line $V_3 \cap T_x(X)$ lying on Q.

$\ulcorner$



$\tilde{f}_L(\pi^{-1}(x)) \subset \tilde{f}_L(\pi^{-1}(L)) = \tilde{f}_L(F) = Q$

$\Rightarrow \tilde{f}_L(\pi^{-1}(x)) = T_x(X) \cap V_3 \subset Q$

$\Rightarrow T_x(X) \cap V_3$ is a line in Q

$\lrcorner$

Note that since L is nonspecial, for $x \neq x' \in L$, $T_x(X)$ meets $T_{x'}(X)$ only in L, so that the corresponding lines $\tilde{f}_L(\pi^{-1}(x))$ and $\tilde{f}_L(\pi^{-1}(x'))$ are disjoint; thus Q is a smooth quadric.

$\ulcorner$ Suppose $T_{x'}(X) \cap T_x(X) \supsetneqq L$, i.e., $T_x(X) \cap T_{x'}(X)$ is two-dimensional. $\Rightarrow$ By the observations on P792, $T_x(X) \cap T_{x'}(X)$ is 2-dim for all $x \neq x' \in L$ $\Rightarrow$ L is special. $\Rightarrow$ Since L is not special, $T_x(X) \cap T_{x'}(X) = L$. $\tilde{f}_L(\pi^{-1}(x)) \cap \tilde{f}_L(\pi^{-1}(x')) = (V_3 \cap T_x(X)) \cap (V_3 \cap T_{x'}(X))$
$= V_3 \cap (T_x(X) \cap T_{x'}(X)) = V_3 \cap L = \phi$

If Q is not smooth, then $Q = (X_0^2 + X_1^2 + X_2^2 = 0)$ or $Q = (X_0^2 + X_1^2 = 0)$ = Union of two hyperplanes? or $Q = (X_0^2 = 0)$)
$(X_0^2 + X_1^2 + X_2^2 = 0)$ = Union of lines passing the singular