seen that F has at least the 15 singular points $r(p_i)$, $i=1,\cdots 15$.

⌈ We know that for each double point $p \in S$, $r(p)$ is a double point of F, by the argument above. Since $\deg S = 4$, it is impossible to hold $r(p_i) = r(p_j)$, $i \neq j$.
⌋

From the chain of inequalities

$$15 \leq \sum_{i \neq j} d_i d_j + \sum \frac{(d_i-1)(d_i-2)}{2}$$
$$= \frac{1}{2}\left(\sum d_i\right)^2 - \sum \frac{d_i}{2} - \sum (d_i - 1)$$
$$= 18 - 3 - \sum (d_i - 1),$$

we conclude that $d_i = 1$ for all i, i.e., that F consists of the sum of six distinct lines L_i.

⌈ $15 \leq 15 - \sum (d_i - 1) \Rightarrow \sum (d_i - 1) = 0$
$\Rightarrow d_i = 1. \Rightarrow F_i$ is a line, say L_i.
⌋

F then has exactly the 15 double points $L_i \cdot L_j$; these must, of course, be the images $r(p_i)$.