quently $W_{a_1 \cdots a_k}$ will be empty unless $a_1 \cdots a_k$ is a nonincreasing sequence of integers $\le n-k$.

$\dim(\Lambda + V_{n-k+i-a_i}) = \dim \Lambda + \dim V_{n-k+i-a_i} - \dim(\Lambda \cap V_{n-k+i-a_i})$
$= k + n - k + i - a_i - i = n - a_i \le n$

Since $\dim \Lambda = k \le \dim(\Lambda + V_{n-k+i-a_i}) = n - a_i$, $a_i \le n-k$.

Suppose $a_1, a_2, \cdots a_k$ is an increasing sequence.

$\Rightarrow$ If we let $b_i = n-k+i-a_i$, $b_i - b_{i+1} = n-k+i-a_i - n + k - i - 1 + a_{i+1} = a_{i+1} - a_i - 1 \ge 0 \Leftrightarrow b_i \ge b_{i+1}$

$\Rightarrow \{b_i\}$ is nonincreasing. $\Rightarrow \{\dim(\Lambda \cap V_{b_i})\}$ must be nonincreasing. $\Rightarrow$ Contradiction to the fact that $\dim(\Lambda \cap V_{b_i}) = i$ is increasing.

Since $\dim(\Lambda \cap V_{n-k+i-a_i}) = i \Leftrightarrow$ the rank of the last $k \times (k+a_i-i)$ minor of a matrix representative for $\Lambda$ is exactly $k-i$, it follows that the closure

$$\overline{W_{a_1, \cdots a_k}} = \{\Lambda : \dim(\Lambda \cap V_{n-k+i-a_i}) \ge i\}$$

is an analytic subvariety of $G(k,n)$.

Let $\Lambda = \left\langle \begin{pmatrix} v_{11}, & v_{12}, & \cdots & v_{1n} \\ v_{21}, & v_{22}, & \cdots & v_{2n} \\ \vdots & & & \\ v_{k1}, & v_{k2}, & \cdots & v_{kn} \end{pmatrix} \right\rangle$. $\Rightarrow$ Since

$\dim \Lambda \cap V_{n-k+i-a_i} = i$. We can choose $v_1, \cdots v_i$ s.t. $v_1, \cdots, v_i \in V_{n-k+i-a_i}$. $\Rightarrow$ $v_{1,n-k+i-a_i+1} = \cdots = v_{1n} = 0$
$v_{2,n-k+i-a_i+1} = \cdots = v_{2n} = 0$ $\cdots$ $v_{i,n-k+i-a_i+1} = \cdots = v_{in} = 0$.
and $\dim \langle v_1, \cdots v_i \rangle = i$. --- ⊛

$\Rightarrow$ $\Lambda$ is represented by a matrix

$$\begin{array}{c} \qquad\qquad\qquad\qquad\qquad \overbrace{\qquad\qquad}^{k+a_i-i} \\ \begin{pmatrix} v_{11}, & \cdots & v_{1,n-k+i-a_i}, & 0 & \cdots & 0 & 0 \\ \vdots & & & \vdots & & \vdots & \vdots \\ v_{i1}, & \cdots & v_{i,n-k+i-a_i}, & 0 & \cdots & 0 & 0 \\ v_{i+1,1} & & & * & \cdots & * & * \\ \vdots & & & \vdots & & & \\ v_{k,1} & \cdots & & * & \cdots & * & * \end{pmatrix} \end{array}$$

$\Rightarrow$ Since rank $\Lambda = k$, by ⊛, the last $k \times (k+a_i-i)$ minor has rank of $k-i$.