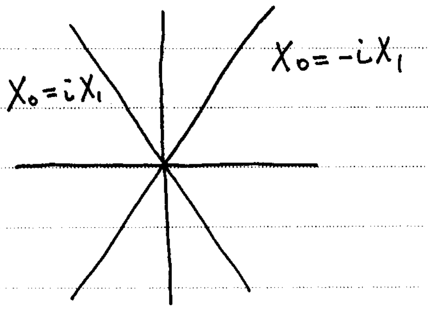
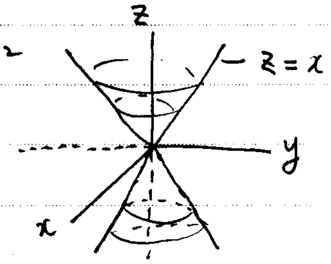


$\Rightarrow$ $[(0,0,0,1)]$ is the vertex.

$z^2 = x^2 + y^2$



For $X_0^2 + X_1^2 = 0$, $X_0 = \pm i X_1$. $\Rightarrow$ $[(X_0, X_1, X_2, X_3)] =$
$[(X_0, \pm i X_0, X_2, X_3)]$ $\Rightarrow$ $\{[(X_0, iX_0, X_2, X_3)]\}$ is a plane $\cong \mathbb{P}^2$. $\lrcorner$

The Cubic Surface

We now describe a smooth cubic surface in $\mathbb{P}^3$. We will first construct such a surface by blowing up six "general" points in $\mathbb{P}^2$ and embedding the blown-up surface in $\mathbb{P}^3$ as a cubic; we will then show that in fact every nonsingular cubic surface may be obtained in this way.

Choose six points $p_1, p_2, \cdots p_6 \in \mathbb{P}^2$ such that

1. $p_1, \cdots p_6$ do not all lie on a conic curve; and
2. no three of them lie on a line.

Let $\tilde{\mathbb{P}}^2 \xrightarrow{\pi} \mathbb{P}^2$ be the blow up of $\mathbb{P}^2$ at $p_1, \cdots p_6$, E_i the exceptional divisor over p_i, and consider the complete linear system $|\tilde{C}|$ where

$$\tilde{C} = \pi^* 3H - E_1 - E_2 - \cdots - E_6.$$

If C is any cubic curve in the plane passing through