$$2 \cdot \mathrm{Im} \int_{\delta_{g+i}} \omega_j$$

is positive definite, the imaginary part $\mathrm{Im}(Z)$ of Z is positive definite: this is the second Riemann bilinear relation.

Pf

$$(\omega_i, \omega_j) = \overline{(\omega_j, \omega_i)}$$

by the symmetry of Z

$$(\omega_i, \omega_j) = \sqrt{-1} \overline{\int_{\delta_{g+i}} \omega_j} - \sqrt{-1} \int_{\delta_{g+j}} \omega_i = \sqrt{-1} \overline{\int_{\delta_{g+i}} \omega_j} - \sqrt{-1} \int_{\delta_{g+i}} \omega_j = \sqrt{-1} \left(\overline{\int_{\delta_{g+i}} \omega_j} - \int_{\delta_{g+i}} \omega_j \right) = 2 \,\mathrm{Im} \int_{\delta_{g+i}} \omega_j$$

$((\omega_i, \omega_j))$ is positive definite $\Rightarrow$ $\left(\mathrm{Im} \int_{\delta_{g+i}} \omega_j\right)$ is positive definite $\Rightarrow$ $\mathrm{Im}({}^tZ)$ is positive defite $\Rightarrow$ $\mathrm{Im}\, Z$ is positive definite. ⫽

In sum, the two Riemann bilinear relations imply that for a normalized basis of $H^0(S, \Omega^1)$, the period matrix Ω of S has the form

$$\Omega = (I, Z), \text{ with } Z = {}^tZ, \ \mathrm{Im}\, Z > 0.$$

Abel's Theorem – Second Version

Let S as before be a Riemann surface of genus g, $D = \sum (p_\lambda - q_\lambda)$ a divisor of degree 0 on S, and consider the Abelian sum

$$\mu(D) = \left(\sum \int_{q_\lambda}^{p_\lambda} \omega_1, \cdots \sum \int_{q_\lambda}^{p_\lambda} \omega_g \right) \in J(S).$$