set of discrete points transversely. $\Rightarrow$ Not clear crystally.

Pf $\dim V^* \cap \mathbb{P}^{n-k+1} = 1$. and $V^* \cap \mathbb{P}^{n-k+1}$ is open in $V \cap \mathbb{P}^{n-k+1}$.

$\Rightarrow$ Clearly $(V \cap \mathbb{P}^{n-k+1})^*$ is a smooth curve, and $(V \cap \mathbb{P}^{n-k+1})_s$ is an analytic subvariety of $V \cap \mathbb{P}^{n-k+1}$ by $p \geq 1$.

$\Rightarrow$ $(V \cap \mathbb{P}^{n-k+1})_s$ is a set of discrete points by Whitney.

$\Rightarrow$ We can find a generic hyperplane H s.t

$V \cap \mathbb{P}^{n-k+1} \cap H$ = a set of discrete points not containg any point of $(V \cap \mathbb{P}^{n-k+1})_s$

$\Rightarrow$ A generic $(n-k)$-plane $\mathbb{P}^{n-k}$ intersects V transversely.

In case $V \subset \mathbb{P}^n$ is a hypersurface, we have seen that it may be given in terms of homogeneous coordinates $X_0, \cdots X_n$ on $\mathbb{P}^n$ as the locus

$V = (F(X_0, X_1, \cdots X_n) = 0)$ of a homogeneous polynomial F.

Pf V is an analytic subvariety of $\mathbb{P}^n$. $\Rightarrow$ By Chow's theorem, V is an algebraic variety of $\mathbb{P}^n$. $\Rightarrow$ $V = \{F_1 = 0\} \cap \cdots \cap \{F_m = 0\}$, $F_1 \cdots F_m$ homogeneous polys. But since V is a hypersurface, $V = (F = 0)$.