If $g \in I_2'$, then $g = a_1 f_1' + a_2 f_2'$. $\Rightarrow$ $g + I_1' = a_2 f_2' + I_1'$
$\Rightarrow$ $a_2 f_2' \in I_1'$ $\Rightarrow I_1' = a_2 f_2' + I_1' = (a_2 + I_1')(f_2' + I_1') \underset{\text{by regularity}}{\Rightarrow} a_2 \in I_1'$
$\Rightarrow$ $g \in I_1'$.

Applying the exact sequences of Ext in the second variable gives

$$\mathrm{Ext}_{\mathcal{O}}^{r-2}(I/I', \mathcal{O}/I_1') \longrightarrow \mathrm{Ext}_{\mathcal{O}}^{r-1}(I/I', \mathcal{O}) \xrightarrow{f_1'} \mathrm{Ext}_{\mathcal{O}}^{r-1}(I/I', \mathcal{O})$$

$$\mathrm{Ext}_{\mathcal{O}}^{r-3}(I/I', \mathcal{O}/I_2') \longrightarrow \mathrm{Ext}_{\mathcal{O}}^{r-2}(I/I', \mathcal{O}/I_1') \xrightarrow{f_2'} \mathrm{Ext}_{\mathcal{O}}^{r-2}(I/I', \mathcal{O}/I_1')$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\mathrm{Ext}_{\mathcal{O}}^{0}(I/I', \mathcal{O}/I_{r-1}') \longrightarrow \mathrm{Ext}_{\mathcal{O}}^{1}(I/I', \mathcal{O}/I_{r-2}') \xrightarrow{f_{r-1}'} \mathrm{Ext}^{1}(I/I', \mathcal{O}/I_{r-2}').$$

See the front page for the multiplication by f_i'.

Now, and this is the point, the maps

$$\mathrm{Ext}_{\mathcal{O}}^{r-k}(I/I', \mathcal{O}/I_{k-1}') \xrightarrow{f_k'} \mathrm{Ext}_{\mathcal{O}}^{r-k}(I/I', \mathcal{O}/I_{k-1}')$$

are all zero, since these maps are $\mathcal{O}$-linear, and therefore the multiplication by f_k' can be moved from the factor $\mathcal{O}/I_{k-1}'$ to I/I', where it is zero.

$$\mathrm{Hom}_{\mathcal{O}}(E_{r-k}, \mathcal{O}/I_{k-1}') \xrightarrow{f_k'} \mathrm{Hom}_{\mathcal{O}}(E_{r-k}, \mathcal{O}/I_{k-1}')$$
$$\begin{array}{c} \uplus \\ g \end{array}$$

$$g(e_I) = h + I_{k-1}', \quad h \in \mathcal{O}$$

$$\Rightarrow \quad (f_k' g)(e_I) = f_k' g(e_I) = f_k' h + I_{k-1}' \in I/I_{k-1}'.$$