Consequently the number of vertices in our triangulation of S is

$$n \cdot C_0 - \sum_{q \in S} (v(q)-1),$$

and the Euler characteristic

$$\chi(S) = n \cdot C_2 - n \cdot C_1 + n \cdot C_0 - \sum_{q \in S} (v(q)-1)$$

$$= n \cdot \chi(S') - \sum_{q \in S} (v(q)-1),$$

so $g(S) = n \cdot (g(S')-1) + 1 + \frac{1}{2} \sum_{q \in S} (v(q)-1).$

$\ulcorner$ Let $P_1, P_2, \cdots P_{C_0}$ be the points in B'.

$\Rightarrow$

$$\#(f^{-1}(P_1)) = n - \sum_{q \in f^{-1}(P_1)} (v(q)-1) \quad — ①$$

$$\#(f^{-1}(P_2)) = n - \sum_{q \in f^{-1}(P_2)} (v(q)-1) \quad — ②$$

$$\vdots \qquad\qquad \vdots$$

$$\#(f^{-1}(P_{C_0})) = n - \sum_{q \in f^{-1}(P_{C_0})} (v(q)-1). \quad — C_0$$

Note that if $P \notin B'$, $\# f^{-1}(p) = n$, because $v(q) = 1$, $q \in f^{-1}(p)$.

$\Rightarrow$ ① $+ \cdots +$ C_0 $\Rightarrow$ The number of vertices is $n \cdot C_0 - \sum_{\substack{q \in f^{-1}(P_i) \\ 1 \le i \le C_0}} (v(q)-1)$.

$\Rightarrow$ By the note above, $\sum_{q \in f^{-1}(P_i)} (v(q)-1) = \sum_{q \in S} (v(q)-1),$