$$\begin{array}{ccc} \tilde{\mathcal{U}}_2 & \longrightarrow & \mathbb{C}^2 \\ \Downarrow & & \Downarrow \\ p & \longmapsto & (z_1(p), z_2(p)) \\ \downarrow & \nearrow \psi & \\ (z_1(p), \frac{z_2}{z_1}(p)) \in \mathbb{C}^2 & & \end{array}$$

$\operatorname{Im}\psi$ contains an open nbd of $(0,0)$ except $\{(0,*)\}$ and $(0,0)$ — ②

From ① & ②, $\mathcal{U} = \tilde{\mathcal{U}}_1 \cup \tilde{\mathcal{U}}_2 \xrightarrow{\Phi} \mathbb{C}^2$

$$p \longmapsto (z_1(p), z_2(p))$$

map a nbd of C in M onto a nbd of $(0,0)$ in $\mathbb{C}^2$. This proves that $\iota_{L'}(C) \in \iota_{L'}(M)$ is a point corresponding to $(0,0)$ in $\mathbb{C}^2$ while some nbd of $\iota_{L'}(M)$ corresponds to an open nbd of $\mathbb{C}^2$. This implies that $\iota_{L'}(C)$ is a smooth point of $\iota_{L'}(M)$. $\Rightarrow$ Since the normal bundle of $\mathbb{P}^1 \cong C$ in M is just the universal bundle over C, by P145 & P146 (refer to P475), M is the blow up of $\iota_{L'}(M)$ at $\iota_{L'}(C)$.

5. Residues

Thus far in this book most of the methods we have developed for studing algebraic varieties have centered around the divisors - especially the linear systems - that lie on the variety. Not only does this technique generally suffice for obtaining a deep understanding of curves and sufaces, but it also entails a minimal amount of analytic and algebraic machinary. On the other hand, many of the outsta-