

$$F(N), 1 \otimes \partial_N) = \text{Tor}_{p+q+1}^{\sigma}(N, M). \text{ Refer to p142. Hu. Homological } \Rightarrow$$

$p+q \geq 0$

For Ext the situation is more complicated and necessitates discussing injective resolutions for the second factor. Since this will not be required for our discussion, we won't get into these matters.

Next, we observe that

$$(*) \quad \text{Ext}_0^{\sigma}(M, N) \cong \text{Hom}_0(M, N).$$

Proof. If $E_1 \rightarrow E_0 \rightarrow M \rightarrow 0$ is exact, then so is

$$0 \rightarrow \text{Hom}_0(M, N) \rightarrow \text{Hom}_0(E_0, N) \xrightarrow{\delta} \text{Hom}(E_1, N). \quad \text{Q.E.D.}$$

$$\Gamma \quad \text{Ext}_0^{\sigma}(M, N) = H^0(\text{Hom}_0(E(M), N)) = \ker \delta = \text{Hom}_0(M, N)$$

$$\text{Comment on } \text{Tor}_{p+q+1}^{\sigma}(M, N) \cong \text{Tor}_{p+q+1}^{\sigma}(N, M)$$

It is valid for $p+q \geq 0$. For $p+q+1=0$,

$$\text{Tor}_0^{\sigma}(M, N) = M \otimes N, \text{ for}$$

$$\rightarrow E_1 \otimes N \xrightarrow{\partial \otimes 1} E_0 \otimes N \rightarrow 0$$

$$\Rightarrow \text{Tor}_0^{\sigma}(M, N) = \frac{E_0 \otimes N}{\text{Im}(\partial \otimes 1)} = H_0(E(M) \otimes N)$$

$$\cong \frac{E_0}{\text{Im} \partial} \otimes N$$

$$\frac{E_0 \otimes N}{\text{Im}(\partial \otimes 1)} \xrightarrow{\phi} \frac{E_0}{\text{Im} \partial} \otimes N$$

$$(e_0 \otimes n) + \text{Im}(\partial \otimes 1) \xrightarrow{\phi} e_0 + \text{Im} \partial \otimes n$$

$$(e_0 \otimes n) + \text{Im}(\partial \otimes 1) \xleftarrow{\psi} e_0 + \text{Im} \partial \otimes n \quad \text{J}$$

Refer to p136. Hu.

Introduction to Homological Algebra.