$$\Rightarrow \begin{tikzcd} S \arrow[d] & \cong & Q^* \arrow[dr] & \\ G(n-k, n) & \leftrightarrow & & G(k, n) \end{tikzcd}$$

Note in particular that the universal subbundle $S \longrightarrow G(1, n) \cong \mathbb{P}^{n-1}$ is just the universal line bundle mentioned earlier.

$$\begin{tikzcd} S \arrow[d] & \subset & G(1, n) \times \mathbb{C}^n \\ G(1, n) \cong \mathbb{P}^{n-1} & & \end{tikzcd} \qquad S = \{ (\Lambda, v) \mid v \in \Lambda \}.$$

Now let $E \longrightarrow M$ be any holomorphic vector bundle of rank k on a complex manifold M, $V \subset H^0(M, \mathcal{O}(E))$ an n-dimensional vector space of global holomorphic sections, and suppose that the values $\{\sigma(x)\}_{\sigma \in V}$ of the sections σ in V span E_x for all $x \in M$. Then for each $x \in M$, the subspace $\Lambda_x \subset V$ of sections $\sigma \in V$ vanishing at x is an $(n-k)$-dimensional subspace; accordingly, we obtain a map

$$i_V : M \longrightarrow G(n-k, V) = G(k, V^*)$$

with

$$E = i_V^* S^* \quad \text{and} \quad V = i_V^* (H^0(G(k,n), \mathcal{O}(S^*)))$$

just as for line bundles.

$V = \langle \sigma_1, \sigma_2, \sigma_n \rangle, \qquad \langle \sigma_1(x), \cdots \sigma_n(x) \rangle = E_x.$