Since $^{\#}(D \cap z_4\text{-axis}) < \infty$, $\exists$ a circle $|z_4| = \epsilon$ with $z_1 = z_2 = z_3 = 0$. $\Rightarrow$ By the tube lemma again, $\exists\ \delta$ s.t $\{ |(z_1, z_2, z_3)| \leq \delta,\ |z_4| = \epsilon \} \cap D = \phi$. For each fixed z_4, $0 \neq |z_4| \leq \delta$, the integral

$$\frac{1}{2\pi\sqrt{-1}} \int_{|z_4| = \epsilon} \frac{dh(z_1, z_2, z_3, z_4)}{h(z_1, z_2, z_3, z_4)}.$$

is well-defined, continuous, and integer-valued. It follows that D meets each vertical disc $\{ z_4 = c,\ |(z_1, z_2, z_3)| \leqslant \epsilon,\ 0 < c \leqslant \delta \}$ the same number d of times. Thus, projecting $\overline{D}$ on the z_4-axis gives a proper mapping $\pi : \overline{D}| \xrightarrow{\text{restrict}} \{ (z_1, z_2, z_3) : |(z_1, z_2, z_3)| < \delta \}$ that restricts to a d-sheeted covering $\pi : D^{*} \longrightarrow \{ (z_1, z_2, z_3) : 0 < |(z_1, z_2, z_3)| < \delta \}$ over the punctured plane. Wrong, since h is not holomorphic and $(h = 0) \neq D \cap \Delta^{*} \times \Delta^{3}$.

Note: The general principle is this: Let $W \subset \Delta^n$ be a closed subset such that (1) the projection $W \xrightarrow{\pi} \Delta^k$ is proper, and (2) outside an analytic subvariety $Z \subset \Delta^k$ this projection $W^{*} \xrightarrow{\pi} \Delta^k - Z$ ($W^{*} = W - \pi^{-1}(Z)$) is an analytic branched covering. Then W is a k-dimensional analytic subvariety of Δ^n.

So far we saw that "proper" is needed for "boundedness".