The phrases compact Riemann surface and smooth algebraic curve or just curve will be used pretty much interchangeably from now on. This is somewhat imprecise, as a smooth algebraic curve may be thought of as carrying the additional structure of an imbedding – i.e., as a Riemann surface S together with a line bundle $L \to S$ and subspace $E \subset H^0(S, \mathcal{O}(L))$ – but hopefully no confusion should arise. What is important is the ability to think alternately of the abstract analytic object – the compact Riemann surface – and the algebraic object – the zeros of polynomials in $\mathbb{P}^N$; this is implicit in the use of the two terminologies.

「 $S \subset \mathbb{P}^N \Rightarrow \exists$ a line bundle $L = H|_S$ and $E = H^0(S, \mathcal{O}(L)) = \iota^* H^0(\mathbb{P}^N, \mathcal{O}(H))$ see P 177, where $\iota : S \hookrightarrow \mathbb{P}^N$ inclusion embedding. 」

As we saw in Section 4 of Chapter 1, the variety $C(S)$ of chords of an algebraic curve $S \subset \mathbb{P}^N$ is a closed subvariety of $\dim \leq 3$ in $\mathbb{P}^N$.

「 In Section 3, Algebraic Varieties, of Chapter 1, p173 $\dim C(V) \leq 2 \dim V + 1. \Rightarrow \dim C(S) \leq 3$. 」

Projecting from a point $p \notin C(S)$ to any hyperplane $H \subset \mathbb{P}^N$ gives an embedding of S in $H \cong \mathbb{P}^{N-1}$; thus any curve can be smoothly embedded in 3-space $\mathbb{P}^3$. We can not, in general, embed a curve in $\mathbb{P}^2$. Given a smooth curve $S \subset \mathbb{P}^3$, however, we can find a point $p \in \mathbb{P}^3$ that does not lie on any tangent line to S in $\mathbb{P}^3$, or on any line meeting S in