

According to P177, $\deg(f(P^2)) = \deg S = c_1(4H)^2$

since $f^*H = 4H$ and $\dim P^2 = 2$.

$\Rightarrow \deg S = 4 \cdot 4 = 16 \Rightarrow$ This is not true \Rightarrow See P690.

Furthermore, $\sigma_0 s_1 = f^* h_1$, $\sigma_0 s_2 = f^* h_2$, $\sigma_0 s_3 = f^* h_3$,
where h_1, h_2, h_3 are sections of $[H]$ over P^3 .

$\Rightarrow f(\{\sigma_0 s_i = 0\}) = H_i \cap f(P^2) = H_i \cap S$.

$f(x)$, $x \in \{\sigma_0 s_i = 0\} \Rightarrow \sigma_0 s_i(x) = h_i \circ f(x) = 0$

$\Rightarrow f(x) \in H_i$, H_i hyperplane represented by h_i .

$\Rightarrow f(x) \in f(P^2) \cap H_i$

Conversely, $f(x) \in H_i \Rightarrow h_i \circ f(x) = (\sigma_0 s_i)(x) = 0$

$\Rightarrow f(x) \in f(\{\sigma_0 s_i = 0\})$.

$\Rightarrow \{\sigma_0 s_i = 0\}$ is reducible.

$\Rightarrow f(\{\sigma_0 s_i = 0\})$ is reducible. for $i=1, 2, 3$.

Suppose $f(\{\tau=0\})$ is reducible.

$\Rightarrow f(\{\tau=0\}) = V_1 \cup V_2 \subset P^3$.

\Rightarrow Since $f^{-1}(V_1)$ & $f^{-1}(V_2)$ are varieties, and
 $f^{-1}(V_1) \cup f^{-1}(V_2) = \{\tau=0\}$, $\{\tau=0\}$ is reducible.

Contradiction.

\Rightarrow The ^{dimension of} reducible hyperplane sections is 2.

* V variety $f: M \rightarrow N$, $V \subset N$.

$\Rightarrow f^{-1}(V)$ is a subvariety of M .

Let $x \in f^{-1}(V)$. $\Rightarrow f(x) \in V \Rightarrow \exists$ open set U

s.t. $f(x) \in U$.

$$U \cap V = \{g_1 = g_2 = \dots = g_n = 0\}$$

$\Rightarrow f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V) = \{g_1 \circ f = \dots = g_n \circ f = 0\}$.