

$\psi \circ \phi = id \Rightarrow \phi$ is one to one & ψ is onto.

(1) ψ is one to one, for, if $f^*H = g^*H$,
 $\Rightarrow f(p) = [s_0(p), \dots, s_N(p)] \Rightarrow$ Each $s_i(p)$ defines a holomorphic section of f^*H , as we noted above.
 $\Rightarrow \{s_0, \dots, s_N\}$ is linearly independent.

Assume $g(p) = [t_0(p), \dots, t_N(p)]$, \Rightarrow Each t_i is a linear combination of $\{s_0, \dots, s_N\}$, since $t_0, \dots, t_N, s_0, \dots, s_N$ are sections of $f^*H = g^*H$.

Not systematic, logical.

(i) Let $f: M \longrightarrow \mathbb{P}^{N_1}$ & $g: M \longrightarrow \mathbb{P}^{N_2}$
 $p \longmapsto [s_0(p), \dots, s_{N_1}(p)]$ $p \longmapsto [t_0(p), \dots, t_{N_2}(p)]$

where $\{s_0, \dots, s_{N_1}, t_0, \dots, t_{N_2}\}$ forms a basis for $E \subset H^1(M, \mathcal{O}(f^*H))$. $\Rightarrow \exists$ no relation between $\{s_0, \dots, s_{N_1}\}$ & $\{t_0, \dots, t_{N_2}\}$ in general.

(ii) Suppose not all $\{s_0, \dots, s_{N-1}\}$ vanish at any point of M .

$$\begin{array}{ccc}
 f_{N+1}^*(H) & \longrightarrow & H \\
 \downarrow & & \downarrow \\
 M & \xrightarrow{f_{N+1}} & \mathbb{P}^{N+1} \subset \mathbb{P}^N \\
 p \longmapsto & & [s_0(p), \dots, s_{N+1}(p)]
 \end{array}
 \quad f_{N+1}^*(H) = f_N^*(H)$$

Note that the degree of the image of M under i_E — that is, the intersection of M with n general hyperplanes in \mathbb{P}^N — is just the n -fold self-intersection of a representative divisor $D \in |E|$, that is,