$$\Rightarrow \quad \begin{array}{l} 0 \to \Omega^p \to \mathcal{A}^{p,0} \xrightarrow{\bar\partial} Z_{\bar\partial}^{p,1} \to 0 \\ 0 \to Z_{\bar\partial}^{p,1} \to \mathcal{A}^{p,1} \xrightarrow{\bar\partial} Z_{\bar\partial}^{p,2} \to 0 \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ 0 \to Z_{\bar\partial}^{p,q} \to \mathcal{A}^{p,q} \xrightarrow{\bar\partial} Z_{\bar\partial}^{p,q+1} \to 0 \end{array}$$

are exact for all p,q. Since $H^r(M, \mathcal{A}^{p,q}) = 0$, for $r > 0$ and for all p,q (see p 42), the long exact cohomology sequences associated to these sheaf sequences give us

$$H^q(M, \Omega^p) \cong H^{q-1}(M, Z_{\bar\partial}^{p,1}) \quad \left(\overset{0}{\overset{\|}{H^{q-1}(M, \mathcal{A}^{p,0})}} \to H^{q-1}(M, Z_{\bar\partial}^{p,1}) \xrightarrow{\cong} H^q(M, \Omega) \to \underset{\overset{\|}{0}}{H^q(M, \mathcal{A}^{p,0})} \right)$$

$$\cong H^{q-2}(M, Z_{\bar\partial}^{p,2})$$

$$\vdots$$

$$\cong H^1(M, Z_{\bar\partial}^{p,q-1})$$

$$\cancel{H^0(M, \mathcal{A}^{p,q-1}) \to H^0(M, Z_{\bar\partial}^{p,q-1}) \to H^1(M, \mathcal{A} Z_{\bar\partial}^{p-})}$$

$$H^0(M, Z_{\bar\partial}^{p,q-1}) \to H^0(M, \mathcal{A}^{p,q-1}) \xrightarrow{\bar\partial^*} H^0(M, Z_{\bar\partial}^{p,q}) \to H^1(M, Z_{\bar\partial}^{p,q-1}) \to \underset{\overset{\|}{0}}{H^1(M, \mathcal{A}^{p,q-1})}$$

$$\Rightarrow \quad H^1(M, Z_{\bar\partial}^{p,q-1}) \cong \frac{H^0(M, Z_{\bar\partial}^{p,q})}{\bar\partial^* H^0(M, \mathcal{A}^{p,q-1})}$$

$$= \frac{Z_{\bar\partial}^{p,q}(M)}{\bar\partial \mathcal{A}^{p,q-1}(M)} = \frac{Z_{\bar\partial}^{p,q}(M)}{\bar\partial A^{p,q-1}(M)} = H_{\bar\partial}^{p,q}(M).$$

As an application, we will prove a special case of Leray's theorem: for a locally finite cover $\underline{\mathcal{U}} = \{U_\alpha\}$ of M that is acyclic for the structure sheaf $\mathcal{O}$, we have $H^*(\underline{\mathcal{U}}, \mathcal{O}) = H^*(M, \mathcal{O})$

Def: A locally finite cover $\underline{\mathcal{U}} = \{U_\alpha\}$ of M is acyclic for the structure sheaf $\mathcal{O}$ provided that $H^p(U_{\alpha_1} \cap \cdots \cap U_{\alpha_q}, \mathcal{O}) = 0$ for $p > 0$.