Next choose $V_{x_1} \subset V_{x_2} \subset \cdots \subset V_{x_i} \subset V_{x_{i+1}} \cdots \subset V_{x_k}$
and $V_{y_1} \supset V_{y_2} \supset \cdots \supset V_{y_i} \supset \cdots \supset V_{y_k}$.
Thus we find flags V, V' s.t, for some i,
$V_{x_i} \cap V'_{y_i} = (0)$. $\Rightarrow$ $\Lambda \cap V_{x_i} \cap V'_{y_i} \neq (0)$ if $\Lambda \in$
$\sigma_a(V) \cap \sigma_b(V')$. $\Rightarrow$ $\sigma_a(V) \cap \sigma_b(V') = \phi$.

So we can conclude that $\#(\sigma_a \cdot \sigma_b) = 0$ unless $a_i + b_{k-i+1} \leq n-k$ for all i. ⌟

Now suppose σ_a and σ_b are cycles of complementary dimension, so that

$$\sum a_i + \sum b_i = k(n-k);$$

then $a_i + b_{k-i+1} \leq n-k$ for all $i \Rightarrow b_{k-i+1} = n-k-a_i$, i.e., the cycle σ_a has intersection number zero with all Schubert cycles in complementary dimension except $\sigma_{n-k-a_n, \ldots, n-k-a_1}$.

Pf $\dim \sigma_a = k(n-k) - \sum a_i$ $\dim \sigma_b = k(n-k) - \sum b_i$
$\Rightarrow \dim \sigma_a + \dim \sigma_b = 2k(n-k) - \sum a_i - \sum b_i = k(n-k)$
$\Rightarrow \sum a_i + \sum b_i = k(n-k)$
$\sum a_i + \sum b_i = \sum_{i=1}^{k} (a_i + b_{k-i+1}) = k(k-k+n-k)$

If $a_i + b_{k-i+1} \leq n-k$ for all i, $\sum a_i + \sum b_i \leq k(n-k)$ $\Rightarrow$ $a_i + b_{k-i+1} = n-k$ should be valid. $\Rightarrow$ $b_{k-i+1} = n-k-a_i$.
$b_1 = n-k-a_k$ $b_2 = n-k-a_{k-1}$.
If $b_i \neq n-k-a_i$ for some i, then $a_i + b_{k-i+1} > n-k$.
$\Rightarrow$ By the conclusion above, $\#(\sigma_a \cdot \sigma_b) = 0$
$\Rightarrow$ $b_i = n-k-a_i$ for all i. $\#(\sigma_a \cdot \sigma_{n-k-a_k, n-k-a_{k-1}, \ldots, n-k-a_1}) \neq 0$.
since $\sigma_a \cap \sigma_{n-k-a_k \cdots} \neq \phi$ and analytic subvarieties intersect positively. ⌟