where W_i's are irreducible submodules of W.

$\Rightarrow \quad Hv = mv$

$Hv = Hv_1 + Hv_2 + \cdots + Hv_n \quad \in W_1 \oplus \cdots \oplus W_n$

$\Rightarrow \quad Hv_1 = mv_1, \; Hv_2 = mv_2 \; \cdots \; Hv_n = mv_m.$

$\Rightarrow \quad V_m \cap (W_1 \oplus \cdots \oplus W_n) = (V_m \cap W_1) \oplus \cdots \oplus (V_m \cap W_n)$

$\ker X \subset W_1 \oplus W_2$

$v \in \ker X \quad \Rightarrow \quad v = w_1 + w_2$

$Xv = Xw_1 + Xw_2 = 0$ and $Xw_i \in W_i$

$\Rightarrow \quad Xw_i = 0. \quad \Rightarrow \quad \ker X = (\ker X \cap W_1) \oplus (\ker X \cap W_2)$

$Y \ker X = \oplus\, Y \ker X \cap W_i$ since $YW_i \subset W_i$

$\Downarrow v$

$\Rightarrow \quad v = \oplus v_i \qquad Yv = \oplus\, Yv_i$

$v_i \in \ker X \cap W_i. \qquad Yv_i \in Y(\ker X \cap W_i)$

$= Y \ker X \cap W_i$

$Y^2 \ker X = Y(\oplus\, Y \ker X \cap W_i)$

$= \oplus\, Y(Y \ker X \cap W_i) = \oplus\, Y^2(\ker X \cap W_i)$

$\oplus\, Y^2 \ker X \cap W_i$

"$\sqcap$ $Y^n \ker X \cap W_i = Y(\ker X \cap W_i)$

$v \in \ker X \qquad Y^n v \in W_i$

$Y^n v \in W_i \;\Rightarrow\; Y^n v = w_i \in W_i$

$\Rightarrow$ Let $v = \oplus v_i \qquad \Rightarrow 0 = Xv = \oplus\, Xv_i = 0$

$\Rightarrow \quad Xv_i = 0$ since $Xv_i \in W_i$

Choose $v_i \in \ker X$. al $v_i \in W_i$

$\Rightarrow \oplus\, Y^n v_i = w_i \quad \Rightarrow \quad Y^n v_j = 0$ for all $j \neq i$

$\Rightarrow \quad Y^n v_i = w_i$

$\Rightarrow \quad Y^n(\ker X \cap W_i) \ni Y^n v_i = w_i, \quad v_i \in W_i \cap \ker X$

$\Rightarrow Y^n \ker X \cap W_i \subset Y^n(\ker X \cap W_i)$

$\lrcorner$ "