/

3. Sheaves and Cohomology.

Def: X top. space.
A presheaf $\mathcal{F}$ of abelian groups on X consists of the data.
(a). for every open subset $U \subset X$, an abelian group $\mathcal{F}(U)$ &
(b). for every inclusion $V \subseteq U$ of open substs of X.
a morphism of abelian groups $\rho_{UV} : \mathcal{F}(U) \longrightarrow \mathcal{F}(V)$.

subject to the conditions.
(i) $\mathcal{F}(\phi) = \phi$.
(ii) ρ_{UU}. identity map $\mathcal{F}(U) \longrightarrow \mathcal{F}(U)$
(iii). if $W \subseteq V \subseteq U$, open. , $\Rightarrow$ $\rho_{UW} = \rho_{VW} \circ \rho_{UV}$

As a matter of terminology, if $\mathcal{F}$ is a presheaf on X, we refer to $\mathcal{F}(U)$ as the sections of the presheaf $\mathcal{F}$ over the open set U. We call ρ_{UV} restriction map. and we sometimes write $s|_V$ instead of $\rho_{UV}(s)$ if $s \in \mathcal{F}(U)$.

Def: A presheaf $\mathcal{F}$ on X (top) is a sheaf if it satisfies the following conditions.

(i). If $\sigma \in \mathcal{F}(U \cup V)$ and $\sigma|_U = \sigma|_V = 0$,
$\sigma = 0$.
(ii). For any pair of open sets $U. V \subset X$, and
$\sigma \in \mathcal{F}(U)$, $\tau \in \mathcal{F}(V)$. s.t $\sigma|_{U\cap V} = \tau|_{U \cap V}$,
then $\exists$ $s \in \mathcal{F}(U \cup V)$ s.t $s|_U = \sigma$ $s|_V = \tau$.

Examples.
① X top. A abelian group.
We define the constant sheaf $\mathcal{A}$ on X determined by A as follows: