

those n -points. $\Rightarrow a_0 Z_0 + \dots + a_n Z_n = 0$ defines a hyperplane H in \mathbb{P}^n . \Rightarrow By P64, $V \cap H$ is a curve, for, $V \cap H$ is an analytic subvariety of \mathbb{P}^n , $\Rightarrow \dim(V \cap H) = 1, 0$

① $\dim(V \cap H) = 1$, O.K. discrete

② $\dim(V \cap H) = 0 \Rightarrow V \cap H = \text{Set of } \text{points}$

$\Rightarrow \deg V \geq n$ Contradiction to the assumption that $\deg V \leq n-1$.

$\Rightarrow \dim(V \cap H) = \dim V$. \Rightarrow By §77. Th. 1.K (Whitney, Complex Analytic Variety), $V \subset H$. \Rightarrow

93. 1. 2.

Turning to the general case, we have to show that the generic hyperplane section $V \cap H$ of an irreducible non-degenerate variety V of dimension ≥ 2 is again irreducible and non-degenerate in H . The latter part is clear: the condition that $H \cap V$ be degenerate is a closed one on $H \in \mathbb{P}^{n*}$, and since V itself is non-degenerate, we can find n points of V spanning a hyperplane, so not every hyperplane section of V can be degenerate.

Γ Choose a point $p_0 \in V$. $\Rightarrow \exists$ a hyperplane H_1 containing p_0 .

\Rightarrow By non-degeneracy of V in \mathbb{P}^n , $\exists p_1 \in V$ s.t. $p_1 \notin H_1$. \Rightarrow

p_1 & p_0 are linearly independent. Continue this process, then we get $p_0, p_1, \dots, p_{n-1} \in V$ which are linearly independent.

\Rightarrow The set $\{p_0, p_1, \dots, p_{n-1}\}$ span a hyperplane H .

Consider $V \cap H$. $\Rightarrow V \cap H$ contains a set of n -linearly independent points $\{p_0, p_1, \dots, p_{n-1}\}$ which can not be contained in any $(n-2)$ -plane (which is a hyperplane of H). \Rightarrow