are given topological spaces X, Y with a continuous mapping

$$f: X \longrightarrow Y$$

and sheaf $\mathcal{F}$ over X. The qth direct image sheaf is the sheaf $R_f^q(\mathcal{F})$ on Y associated to the presheaf

$$U \mapsto H^q(f^{-1}(U), \mathcal{F}).$$

The Leray spectral sequence, which exists under very mild restrictions (cf. the references at the end of this chapter) is a spectral sequence $\{E_r\}$ with

$$\begin{cases} E_\infty \Rightarrow H^*(X, \mathcal{F}), \\ E_2^{p,q} = H^p(Y, R_f^q(\mathcal{F})). \end{cases}$$

Suppose that $E \xrightarrow{\pi} B$ is a differentiable fiber bundle with compact fiber F. Then E, B, and F are manifolds, π is a C^∞ mapping, and

$$\pi^{-1}(U) \cong U \times F$$

for sufficiently small open sets $U \subset B$. For the constant sheaf Q on E, by the Künneth formula

$$H^q(\pi^{-1}(U), Q) \cong H^q(F, Q).$$

Pf $H^q(\pi^{-1}(U), Q) = H^q(U \times F, Q) = H^0(U, Q) \otimes H^q(F, Q)$ since $H^i(U, Q) = 0$ for $i > 0$ ($\because$ U contractible).

Q may be a misprint, and $\mathbb{C}$ might be correct for the case.

This suggests that as a first approximation

$$R_f^q(Q) \cong H^q(F, Q)$$

is a constant sheaf on B.

Pf $R_f^q(Q)_x = \lim\limits_{U \ni x} H^q(\pi^{-1}(U), Q) = H^q(F, Q).$