From this the formal rules

$$\left\{\begin{array}{l} \iota(v) : A^{p,q}(M) \longrightarrow A^{p-1,q}(M), \\ \iota(v)^2 = 0 \\ \qquad \iota(v)\bar\partial + \bar\partial\iota(v) = 0 \\ \iota(v)(\varphi\wedge\psi) = \iota(v)\varphi\wedge\psi + (-1)^{\deg\varphi}\varphi\wedge\iota(v)\psi, \end{array}\right.$$

are easily verified.

Pf

$$\langle \iota(v)\varphi, \eta\rangle = \langle\varphi, v\wedge\eta\rangle$$

$$\langle \iota(v)(\varphi\wedge\psi), \eta\rangle = \langle\varphi\wedge\psi, v\wedge\eta\rangle \quad \overset{?}{=}$$

$$\langle \iota(v)\varphi\wedge\psi, \eta\rangle + (-1)^{\deg\varphi}\langle\varphi\wedge\iota(v)\psi, \eta\rangle$$

Without loss of generality, assume. $\varphi = dz_I\wedge d\bar z_J$ $\psi = dz_{I'}\wedge d\bar z_{J'}$. $I\cap I = \phi$. $J\cap J' = \phi$. $v = \frac{\partial}{\partial z_1}$.

$$\Rightarrow \iota(v)(dz_I\wedge d\bar z_J \wedge dz_{I'}\wedge d\bar z_{J'}) = \left\{\begin{array}{l} 0 \quad \text{or} \\ dz_{I-\{1\}}\wedge d\bar z_J\wedge dz_{I'}\wedge d\bar z_{J'} \quad \text{or} \\ (-1)^{\deg\varphi} dz_I\wedge d\bar z_J\wedge dz_{I'-\{1\}}\wedge d\bar z_{J'} \end{array}\right.$$

$$i(v)\varphi\wedge\psi = \left\{\begin{array}{l} 0 \quad \text{or} \\ dz_{I-\{1\}}\wedge d\bar z_J\wedge dz_{I'}\wedge d\bar z_{J'} \quad \text{or} \end{array}\right.$$

$$(-1)^{\deg\varphi}\varphi\wedge\iota(v)\psi = \left\{\begin{array}{l} 0 \quad \text{or} \\ (-1)^{\deg\varphi} dz_I\wedge d\bar z_J\wedge dz_{I'-\{1\}}\wedge d\bar z_{J'} \end{array}\right.$$

$\Rightarrow$ It is valid. ⌟

In particular, contraction with v gives the Complex of