⌈ $h^\circ(D) = 1 = \dim H^\circ(M, [D]) \Rightarrow H^\circ(M,[D]) = \langle s \rangle$, where s is a holomorphic section of $[D]$. $\Rightarrow H^\circ(M,[D]) \cong \mathbb{C}$. ⌋

Again by Riemann-Roch applied to the line bundle $K_C + 2D$,

$$\begin{aligned} h^\circ(K_C+2D) &= \deg(K_C+2D) - g + 1 + i(K_C+2D) \\ &= 2g-2+2g-g+1 \\ &= 3g-1, \end{aligned}$$

so that the space of meromorphic differentials having polar divisor $2D$ has dimension $3g-1$.

⌈ By Riemann-Roch theorem on P245,

$$\begin{aligned} h^\circ(K_C+2D) &= \deg(K_C+2D) - g+1 + h^\circ(K_C-(K_C+2D)) \\ &= 2g-2+2g-g+1+h^\circ(-2D). \end{aligned}$$

$\Rightarrow$ Since $\deg(-2D) < 0$, $h^\circ(-2D) = 0$, see P214.

$\Rightarrow h^\circ(K_C+2D) = 2g-2+2g-g+1 = 3g-1.$

$h^\circ(K_C+2D) = 3g-1 = \dim H^\circ(M, [K_C+2D])$ $= \dim H^\circ(M, \Omega^1(2D)) \Rightarrow$ The space of meromorphic differentials having polar divisor $2D$ has dimension $3g-1$. For K_C, see P471. ⌋

The equations

$$\sum \mathrm{Res}_{p_i}(\varphi) = 0$$

impose exactly $g-1$ independent conditions on this space, due to the residue theorem

$$\sum_i \mathrm{Res}_{p_i}(\varphi) = 0,$$