for all $\psi \in A_c^{n,n-q}(\mathbb{C}^n)$.

$$\Rightarrow \int_{\mathbb{C}^n} K(\bar\partial(\rho\varphi))\wedge\psi + (-1)^q\int_{\mathbb{C}^n} K(\rho\varphi)\wedge\bar\partial\psi = \int_{\mathbb{C}^n}\rho\varphi\wedge\psi$$

$$\Rightarrow 0 = \int_{\mathbb{C}^n}\bar\partial(\underset{(0,q-1)}{K(\rho\varphi)}\wedge\underset{(n,n-q)}{\psi}) = \int_{\mathbb{C}^n} d(K(\rho\varphi)\wedge\psi)$$

$$= \int_{\mathbb{C}^n}\bar\partial(K(\rho\varphi))\wedge\psi + (-1)^{q-1}\int_{\mathbb{C}^n} K(\rho\varphi)\wedge\bar\partial\psi$$

$$\Rightarrow (-1)^q\int_{\mathbb{C}^n} K(\rho\varphi)\wedge\bar\partial\psi = \int_{\mathbb{C}^n}\bar\partial(K(\rho\varphi))\wedge\psi$$

$$\Rightarrow \int_{\mathbb{C}^n} K(\bar\partial(\rho\varphi))\wedge\psi + \int_{\mathbb{C}^n}\bar\partial(K(\rho\varphi))\wedge\psi = \int_{\mathbb{C}^n}\rho\varphi\wedge\psi$$

$\Rightarrow$ Since the equation is valid for all $\psi \in A_c^{n,n-q}(\mathbb{C}^n)$, we get

$$K(\bar\partial(\rho\varphi)) + \bar\partial(K(\rho\varphi)) = \rho\varphi. \quad \lrcorner$$

Restricting to V,

$$\varphi(z) = \bar\partial(K\rho\varphi)(z), \quad (z\in V).$$

$\ulcorner$ I don't know how to get $\varphi(z) = \bar\partial(K\rho\varphi)(z)$, $z\in V$ from

$$(\rho\varphi)(z) = \bar\partial(K\rho\varphi)(z) + K(\bar\partial(\rho\varphi))(z)$$

because for $z\in V$, $K(\bar\partial(\rho\varphi))(z) = 0$ is not clear.
The reason is that $\bar\partial(\rho\varphi)(w) = 0$ for all $w\in\mathbb{C}^n$.
might not be true