

The q th Chern polynomial is then

$$\begin{aligned}
 c_q(\mathbb{H}) &= \left(\frac{\sqrt{-1}}{2\pi}\right)^q \left\{ \sum_{\alpha_1 < \dots < \alpha_q} \left(\frac{1}{q!} \sum_{\pi} \operatorname{sgn} \pi \, \mathbb{H}^{\alpha_1}_{\alpha_{\pi(1)}} \wedge \dots \wedge \mathbb{H}^{\alpha_q}_{\alpha_{\pi(q)}} \right) \right\} \\
 &= \left(\frac{\sqrt{-1}}{2\pi}\right)^q (-1)^{q(q-1)/2} \sum_{\substack{\alpha_1 < \dots < \alpha_q \\ \mu_1 < \dots < \mu_q}} \frac{\operatorname{sgn} \pi}{q!} A_{\mu_1}^{\alpha_1} \wedge \dots \wedge \bar{A}_{\mu_1}^{\alpha_{\pi(1)}} \wedge \dots \wedge \bar{A}_{\mu_q}^{\alpha_{\pi(q)}} \\
 &= (\sqrt{-1})^{q^2} \sum_{\mu=(\mu_1, \dots, \mu_q)} \eta_{\mu} \wedge \bar{\eta}_{\mu},
 \end{aligned}$$

where $\eta_{\mu} = \left(\frac{1}{2\pi}\right)^q \frac{1}{q!} \sum_{\pi} \operatorname{sgn} \pi \cdot A_{\mu_1}^{\alpha_{\pi(1)}} \wedge \dots \wedge A_{\mu_q}^{\alpha_{\pi(q)}}$

is a form of type $(q, 0)$. It follows that

$$\int_Z c_q(\mathbb{H}) \geq 0$$

for any q -dimensional analytic subvariety Z in M .

According to P402,

$$\begin{aligned}
 p^q(A) &= \sum_{\#I=q} \det(A_{I,I}) \\
 &= \sum_{\alpha_1 < \dots < \alpha_q} \sum_{\pi}^{\operatorname{sgn} \pi} a_{\alpha_1, \alpha_{\pi(1)}} \dots a_{\alpha_q, \alpha_{\pi(q)}}
 \end{aligned}$$

$$\Rightarrow c_q(\mathbb{H}) = p^q\left(\frac{\sqrt{-1}}{2\pi} \mathbb{H}\right) = \sum_{\#I=q} \det\left(\frac{\sqrt{-1}}{2\pi} \mathbb{H}_{I,I}\right)$$

$$= \left(\frac{\sqrt{-1}}{2\pi}\right)^q \sum_{\#I=q} \det(\mathbb{H}_{I,I}) = \left(\frac{\sqrt{-1}}{2\pi}\right)^q \sum_{\alpha_1 < \dots < \alpha_q} \sum_{\pi} \operatorname{sgn} \pi \, \mathbb{H}^{\alpha_1}_{\alpha_{\pi(1)}} \wedge \dots \wedge \mathbb{H}^{\alpha_q}_{\alpha_{\pi(q)}}$$

$$= \left(\frac{\sqrt{-1}}{2\pi}\right)^q \sum_{\alpha_1 < \dots < \alpha_q} \sum_{\pi} \operatorname{sgn} \pi \sum_{\mu_1, \dots, \mu_q} A_{\mu_1}^{\alpha_1} \bar{A}_{\mu_1}^{\alpha_{\pi(1)}} \wedge A_{\mu_2}^{\alpha_2} \bar{A}_{\mu_2}^{\alpha_{\pi(2)}} \wedge \dots \wedge A_{\mu_q}^{\alpha_q} \bar{A}_{\mu_q}^{\alpha_{\pi(q)}}$$