$$\Rightarrow\quad b_{I^\circ \bar J^\circ}\ \varphi_I\wedge\overline{\varphi_J}\wedge\varphi_{I^\circ}\wedge\overline{\varphi_{J^\circ}} = \overline{\eta_{I\bar J}}\ \cancel{2^{p+q}}\ \Phi(z)$$
$$= \overline{\eta_{I\bar J}}\ \cancel{2^{p+q}}\ C_n\ \varphi_1\wedge\cdots\wedge\varphi_n\wedge\overline{\varphi_1}\wedge\cdots\wedge\overline{\varphi_n}$$

$I=\{i_1\cdots i_p\}\qquad i_1<\cdots<i_p$

$J=\{j_1\cdots j_q\}\qquad j_1<\cdots<j_q.$

$I^\circ=\{i^\circ_1\cdots i^\circ_{n-p}\}\qquad i^\circ_1<\cdots<i^\circ_{n-p}\qquad J^\circ=\{j^\circ_1,\cdots j^\circ_{n-q}\}\ \ j^\circ_1<\cdots<j^\circ_{n-q}.$

$$\{\underbrace{1,\cdots n}_{i},\ \underbrace{1',2',\cdots n'}_{j}\}\xrightarrow{\ \sigma\ }\{i_1\cdots i_p,\ j_1\cdots j_q,\ i^\circ_1\cdots i^\circ_{n-p},\ j^\circ_1\cdots j^\circ_{n-q}\}$$

Let $\epsilon(\sigma)$ = sign of σ.

$$\Rightarrow\quad b_{I^\circ\bar J^\circ}\ \epsilon(\sigma)\ \varphi_1\wedge\cdots\wedge\varphi_n\wedge\overline{\varphi_1}\wedge\cdots\overline{\varphi_n} = \overline{\eta_{I\bar J}}\ \cancel{2^{p+q}}\ C_n\ \varphi_1\wedge\cdots\varphi_n\wedge\overline{\varphi_1}\wedge\cdots\overline{\varphi_n}$$
$$\Rightarrow\quad b_{I^\circ\bar J^\circ}\ \epsilon(\sigma)=C_n\overline{\eta_{I\bar J}}\ \cancel{2^{p+q}}\qquad\Rightarrow\quad b_{I^\circ\bar J^\circ}=\epsilon(\sigma)\ \overline{\eta_{I\bar J}}\ \cancel{2^{p+q}}\ C_n.$$

$$C_n=(-1)^{\frac{n(n-1)}{2}}\frac{i^n}{\cancel{2^n}}\qquad\Rightarrow\quad C_n\ \cancel{2^{p+q}}=(-1)^{\frac{n(n-1)}{2}}\,i^n\ \cancel{2^{p+q-n}}$$

Let $K_n=(-1)^{\frac{n(n-1)}{2}}\,i^n.\qquad\Rightarrow\quad b_{I^\circ\bar J^\circ}=\epsilon(\sigma)\ K_n\ \cancel{2^{p+q-n}}\ \overline{\eta_{I\bar J}}.$

Here $I^\circ=\{1,2\cdots n\}-I$, $J^\circ=\{1,2\cdots n\}-J$. //

Thus $\displaystyle *\eta = K_n\ \cancel{2^{p+q-n}}\sum_{I,J}\epsilon_{IJ}\ \overline{\eta_{I\bar J}}\ \varphi_{I^\circ}\wedge\overline{\varphi_{J^\circ}}.$

The signs work out so that
$$**\eta=(-1)^{p+q}\eta.$$

Γ

$$*\eta = *\left(\eta_{I\bar J}\ \varphi_I\wedge\overline{\varphi_J}\right) = \epsilon_{IJ}\ \overline{\eta_{I\bar J}}\ \varphi_{I^\circ}\wedge\overline{\varphi_{J^\circ}}$$

$$**\eta = \epsilon_{IJ}\ *\left(\overline{\eta_{I\bar J}}\ \varphi_{I^\circ}\wedge\overline{\varphi_{J^\circ}}\right)=\epsilon_{IJ}\ \epsilon_{I^\circ J^\circ}\ \eta_{I\bar J}\ \varphi_I\wedge\overline{\varphi_J}$$

$\epsilon_{I^\circ J^\circ}\Rightarrow\ \tau:\{1,\cdots n,\ 1',\cdots n'\}\longrightarrow\{i^\circ_1\cdots i^\circ_{n-p},\ j^\circ_1\cdots j^\circ_{n-q},\ i_1\cdots i_p,\ j_1\cdots j_q\}$

$\{i^\circ_1\cdots i^\circ_{n-p},\ j^\circ_1\cdots j^\circ_{n-q},\ i_1\cdots i_p,\ j_1\cdots j_q\}\to\{i_1,\cdots i_p,\ j_1\cdots j_q,\ i^\circ_1\cdots i^\circ_{n-p},\ j^\circ_1\cdots j^\circ_{n-q}\}$

has the sign $(-1)^{p(n-p+n-q)+q(n-p+n-q)}$