since $\sum \alpha_k = 0$.

$\ulcorner$ By the Bott Residue Formula,

$$\sum_{v(p)=0} \frac{P(A_p)}{\det(A_p)} = \int_M P\left(\frac{\sqrt{-1}}{2\pi} \Theta\right).$$

$P^n = \text{trace}^n$. $\quad M = \mathbb{P}^n$.

$$\Rightarrow \int_{\mathbb{P}^n} \left(\text{trace}\left(\frac{\sqrt{-1}}{2\pi}\Theta\right)\right)^n = \int_{\mathbb{P}^n} (C_1(\mathbb{P}^n))^n = (C_1(\mathbb{P}^n))^n(\mathbb{P}^n) = C_1(\mathbb{P}^n)^n$$

$$= \sum_{p_i} \frac{(\text{trace } A_{p_i})^n}{\det(A_{p_i})} = \sum_{i=0}^{n} \frac{\left(\sum_{j\neq i}(\alpha_j - \alpha_i)\right)^n}{\prod_{j\neq i}(\alpha_j - \alpha_i)}$$

If $\sum \alpha_k = 0$, $\quad \sum_{j\neq i}(\alpha_j - \alpha_i) = \sum_{j=0}^{n}(\alpha_j - \alpha_i)$

$$= \sum_{j=0}^{n} \alpha_j - (n+1)\alpha_i = -(n+1)\alpha_i \quad \lrcorner$$

"On Nonsense".

It has meaning. $\quad v(X) = \pi_* \sum_{i=0}^{n} \alpha_i X_i \frac{\partial}{\partial X_i}$

Let $\beta = \dfrac{\sum \alpha_i}{n+1}$.

$$v(X) - \pi_* \sum_{i=0}^{n} \beta X_i \frac{\partial}{\partial X_i} = v(X) = \pi_* \sum_{i=0}^{n} (\alpha_i - \beta) X_i \frac{\partial}{\partial X_i}$$

$\Rightarrow$ Put $\alpha_i' = \alpha_i - \beta$. $\Rightarrow$ α_i''s are all distinct and

$\sum \alpha_i' = \sum \alpha_i - (1+n)\beta = 0 \qquad \alpha_i' = 0 \Rightarrow \alpha_i = \beta$

$\Rightarrow$ For example, $n = 2$, $\alpha_1 = 1$, $\alpha_2 = 3$. $\alpha_0 = 2$

$\Rightarrow \alpha_0 = \dfrac{1+2+3}{3} = 2 = \beta \Rightarrow \alpha_i$ can be 0 (zero). $\lrcorner$