

To construct $\text{Ext}_R^1(M, N)$, we start with part of a projective resolution

$$E_2 \rightarrow E_1 \rightarrow E_0 \rightarrow M \rightarrow 0$$

and take kernel/image in

$$\text{Hom}_R(E_0, N) \rightarrow \text{Hom}_R(E_1, N) \rightarrow \text{Hom}_R(E_2, N).$$

See P 684 for the definition. \Rightarrow

Thus a cycle gives a map $E_1/E_2 \rightarrow N$, and so a class in $\text{Ext}_R^1(M, N)$ gives the data

$$\begin{cases} 0 \rightarrow E_1/E_2 \rightarrow E_0 \rightarrow M \rightarrow 0, \\ E_1/E_2 \rightarrow N \end{cases}$$

$$\begin{array}{ccccc} \text{Hom}_R(E_0, N) & \xrightarrow{\delta} & \text{Hom}_R(E_1, N) & \xrightarrow{\delta} & \text{Hom}_R(E_2, N) \\ & & \downarrow & & \\ & & f & \longmapsto & \delta(f) = f \circ \partial \end{array}$$

If $\delta(f) = 0$, then $f = 0$ on $\partial(E_2) \subset E_1$.

E_1/E_2 means $E_1/\partial E_2$.

f defines a map from $E_1/\partial E_2$ to N as follows:

$$e_1 + \partial E_2 \longmapsto f(e_1).$$

Since $E_2 \xrightarrow{\partial} E_1 \xrightarrow{\partial} E_0 \xrightarrow{\partial} M \rightarrow 0$ is exact,

$$0 \rightarrow E_1/\partial E_2 \rightarrow E_0 \rightarrow M \rightarrow 0 \text{ is exact. } \Rightarrow$$