⌈ For each $\omega \in W$, $f^{-1}(\omega) = \{ z_1(\omega), \cdots z_d(\omega) \}$.

$$\sum_{\nu=1}^{d} h(z_\nu(\omega)) \cdot \varphi(\omega) = h(z_1(\omega))\varphi(\omega) + \cdots + h(z_d(\omega))\varphi(\omega) \quad \text{on } W$$

$$h(z_1(\omega))(f^*\varphi)(z_1) + h(z_2(\omega))(f^*\varphi)(z_2) + \cdots + h(z_d(\omega))(f^*\varphi)(z_d)$$

$$= h(z_1)\varphi(f(z_1)) + h(z_2)\varphi(f(z_2)) + \cdots + h(z_d)\varphi(f(z_d))$$

$$= h(z_1)\varphi(\omega) + \cdots + h(z_d)\varphi(\omega) \quad \text{on } U$$

$\Rightarrow$ Since the integral is the limit of sums above,

$$\int_W \sigma_h(\omega)\varphi(\omega) = \int_U h(z)(f^*\varphi)(z).$$ ⌋

"Here implicitly, we used the Sard's theorem, f is locally one to one almost everywhere."

By the regularity theorem from Section 1 of Chapter 3 it will suffice to show that $\bar\partial \sigma_h = 0$ in the sense of currents.

⌈ By the regularity for the $\bar\partial$-operator on p380, we have only to prove $\bar\partial \sigma_h = 0$ in the sense of currents. ⌋

⌈ Comment on σ_h more.

By the Sard's theorem. $f: U \longrightarrow W$ is a covering map on open subset of U which is dense. By the note above, clearly.

$$\int_W \sigma_h(\omega)\varphi(\omega) = \int_U h(z)(f^*\varphi)(z)$$

$\Rightarrow$ Once we prove $\bar\partial\sigma_h = 0$ in the sense of currents,

$\sigma_h = T_f$, $f \in \mathcal{O}(U)$. $\Rightarrow$ Since $f = \sigma_h$ on some open set, $f \equiv \sigma_h$ on U. $\Rightarrow$ σ_h is holomorphic on W. ⌋