Lemma. $K_{\tilde M} = \pi^* K_M + (n-1)E$.

pf). This is easy in case M has a nontrivial meromorphic $(n,0)$ type "n-form" ω. In terms of local coordinates $z_1, z_2, \cdots z_n$ in a nbd U of x, write

$$\omega(z) = \frac{f(z)}{g(z)} \cdot dz_1 \wedge \cdots \wedge dz_n .$$

Now let $z(i)_j$ be local coordinates in as before. The map π is given in $\tilde U_i$ by

$$(z(i)_1, \cdots\cdots z(i)_n) \longmapsto (z(i)_1 z_i, \cdots z_i, \cdots z(i)_n z_i)$$

and so

$$\pi^*\omega = \pi^*(f/g) \cdot d(z(i)_1 z_i) \wedge \cdots \wedge dz_i \wedge \cdots \wedge d(z(i)_n z_i)$$
$$= \pi^*(f/g) \cdot z_i^{(n-1)} \, dz(i)_1 \wedge \cdots\cdots \wedge dz(i)_n .$$

「 Remember that π is given as follows:

$$\pi : U \times \mathbb{P}^{n-1} \longrightarrow U$$
$$(z, \ell) \longmapsto z.$$
$$\pi : \tilde U_i \longrightarrow U$$
$$(z(i)_1, \cdots z(i)_n) \longmapsto (z_1, z_2 \cdots z_n) = \left(\frac{z_1}{z_i} z_i, \frac{z_2}{z_i} \cdot z_i, \cdots z_i, \right)$$
$$= (z(i)_1 z_i, \; z(i)_2 z_i, \cdots, z_i, \cdots, z(i)_n z_i).$$

$\pi^* dz_1 = d\pi^* z_1 = d(z(i)_1 z_i) = (dz(i)_1) z_i + z(i)_1 dz_i$

since $\pi^* z_1(z,\ell) = z_1(\pi_*(z,\ell)) = z_1(z) = z_1 = \frac{z_1}{z_i} z_i$
$= z(i)_1 z_i$. 」

Thus we see that in a nbd of $E = \pi^{-1}(x_0)$, the divisor $(\pi^*\omega)$ is given by $\pi^*(\omega) + (n-1)E$.

「 From $\pi^*\omega = \pi^*(f/g) \cdot z_i^{(n-1)} \, dz(i)_1 \wedge \cdots \wedge dz(i)_n$,

$$(\pi^*\omega = 0) = (z_i^{(n-1)} = 0) + (\pi^*(f/g) = 0)$$
$$= (n-1)E + \pi^*(f=0) = (n-1)E + \pi^*(\omega = 0)$$ 」

Since clearly $(\pi^*\omega = 0) = \pi^*(\omega = 0)$ away from E,
$K_{\tilde M} = [(\pi^*\omega)] = \pi^* K_M + (n-1)E$ as desired.