in the normal bundle to B in A, then the hom-otopy type of

$$A_t = \{ x \in A ; \varphi(x) \leq t \},$$

remains the same as long as t does not cross a critical value (this is obvious; we just retract along the gradient vector field of φ), and changes by attaching a cell of dimension k when we cross a critical value whose Hessian has exactly k negative eigenvalues. (This requires a local analysis of the Morse function φ around the critical point x_ν, and is the main step.)

Now let M be a compact, complex manifold, $L \to M$ a positive holomorphic line bundle, and $s \in H^0(M, \mathcal{O}(L))$ a holomorphic section whose zero divisor $V = (s)$ is a smooth hypersurface. Choose a metric for $L \to M$ such that $\frac{i}{2\pi} \Theta = \frac{i}{2\pi} \partial\bar\partial \log |s|^{-2}$ is positive and set $\varphi(x) = \log |s|^2$.

φ – or a function near φ in the C^2-topology – may be used as a Morse function (the fact that $\varphi : M \to [-\infty, \infty)$ with $\varphi^{-1}(-\infty) = V$ causes no essential difficulty; what is important is that $d(|s|) \neq 0$ along V).

Pf $s : M \longrightarrow L$ holomorphic section

$\varphi(x) = \log |s|^2$.

Since $(s) = V$, $s = 0$ on V. $\Rightarrow$ On V, φ takes the value $-\infty$.

Since $\frac{i}{2\pi} \Theta = \frac{i}{2\pi} \partial\bar\partial \log |s|^{-2} = -\frac{i}{2\pi} \partial\bar\partial \log |s|^2$ is