Date

Now we have seen that if $E, E'$ are two hermitian vector bundles and if we give $E \otimes E'$ the induced metric, then

$$D_{E\otimes E'} = D_E \otimes 1 + 1 \otimes D_{E'}$$

and so $$\Theta_{E\otimes E'} = \Theta_E \otimes 1 + 1 \otimes \Theta_{E'}$$

where $D$, $\Theta$ always refer to the metric connection and curvature.

$\ulcorner$ See p74 lemma.

$$\bar{D} = D \otimes 1 + 1 \otimes D'.$$

$$\bar{D}(e_i \otimes e_j') = De_i \otimes e_j' + e_i \otimes D'e_j'$$

$$= \theta_{ki}\, e_k \otimes e_j' + e_i \otimes \theta'_{\ell j}\, e_\ell'$$

$$\bar{D}(\theta_{ki}\, e_k \otimes e_j') = d\theta_{ki} \otimes e_k \otimes e_j' - \theta_{ki} \wedge \bar{D}(e_k \otimes e_j')$$

$$\bar{D}(e_i \otimes \theta'_{\ell j} e_\ell') = De_i \wedge \theta'_{\ell j}\, e_\ell' + e_i \otimes D'(\theta'_{\ell j} e_\ell')$$

$$= \theta_{ki}\, e_k \wedge \theta'_{\ell j} e_\ell' + e_i \otimes D'^2 e_j'$$

$$\bar{D}(\theta_{ki} e_k \otimes e_j') = d\theta_{ki} \otimes e_k \otimes e_j' - \theta_{ki} \wedge (De_k \otimes e_j')$$
$$- \theta_{ki} \wedge (e_k \otimes De_j')$$
$$= d\theta_{ki} \;\text{——}\; - \theta_{ki} \wedge (\quad)$$
$$- \theta_{ki} \wedge (e_k \otimes \theta'_{\ell j} e_\ell')$$
$$= D^2 e_i \otimes e_j' - \theta_{ki} \wedge \theta'_{\ell j} \otimes e_k \otimes e_\ell'$$

$$\Rightarrow \bar{D}(e_i \otimes \theta'_{\ell j} e_\ell') + \bar{D}(\theta_{ki} e_k \otimes e_j') = D^2 e_i \otimes e_j'$$