$\Rightarrow \Lambda_0 \in \overline{W}$.

Actually, it is possible to do the process because of the following observation.

Suppose $\begin{pmatrix} A_1, & B_1 \\ A_2, & B_2 \\ \vdots & \vdots \\ A_{10}, & B_{10} \\ \hline A_{11} & B_{11} \\ \vdots & \\ A_{100} & B_{100} \end{pmatrix}$ has rank 10.

and $\begin{pmatrix} B_1 \\ \vdots \\ B_{100} \end{pmatrix}$ has rank 5. Assume that (A_1, B_1) $\cdots$ (A_{10}, B_{10}) are linearly independent. $B_9, B_{10}, B_{11}, B_{12}, B_{13}$ are linearly independent.

Claim: We can find 5-linearly independent vectors in $\{B_1, B_2 \cdots B_{10}\}$.

To prove the claim, we have only to show that $B_{11}, \cdots B_{13}$ can be expressed as a linear combination of $B_1 \cdots B_{10}$.

Consider (A_{11}, B_{11}). $\Rightarrow$ $(A_{11}, B_{11}) = x_1(A_1, B_1) + x_2(A_2, B_2) + \cdots + x_{10}(A_{10}, B_{10})$ $\Rightarrow$ $B_{11} = x_1 B_1 + \cdots + x_{10} B_{10}$.

$\Rightarrow$ Similarly, we can show for B_{12}, B_{13}.

Thus if we have to make $\{(A_i, B_i)\}$ rank 21 and $\{B_i\}$ rank 20, first make $B_1', B_2' \cdots B_5', B_6 \cdots B_{10}$ to be linearly independent in case $B_6 \cdots B_{10}$ are linearly independent. $\Rightarrow$ $\{(A_1, B_1'), \cdots (A_5, B_5'), (A_6, B_6)$