Since $z_{\alpha\beta\gamma}$ is constant, $(\delta a^{1.0})_{\alpha\beta\gamma} = C_{\alpha\beta\gamma}\, z_{\alpha\beta\gamma}$ for some $C_{\alpha\beta\gamma}$ constant, in case $z_{\alpha\beta\gamma}$ is nonzero.

$$
\begin{aligned}
0 = (\delta\delta a^{1.0})_{\alpha\beta\gamma\delta} &= (\delta a^{1.0})_{\beta\gamma\delta} - (\delta a^{1.0})_{\alpha\gamma\delta} + (\delta a^{1.0})_{\alpha\beta\delta} \\
&\quad - (\delta a^{1.0})_{\alpha\beta\gamma} \\
&= C_{\beta\gamma\delta}\, z_{\beta\gamma\delta} - C_{\alpha\gamma\delta}\, z_{\alpha\gamma\delta} + C_{\alpha\beta\delta}\, z_{\alpha\beta\delta} - C_{\alpha\beta\gamma}\, z_{\alpha\beta\gamma} \\
&= C_{\beta\gamma\delta}\,(\log g_{\beta\gamma} + \log g_{\gamma\delta} - \log g_{\beta\delta}) \\
&\quad - C_{\alpha\gamma\delta}\,(\log g_{\alpha\gamma} + \log g_{\gamma\delta} - \log g_{\alpha\delta}) \\
&\quad + C_{\alpha\beta\delta}\,(\log g_{\alpha\beta} + \log g_{\beta\delta} - \log g_{\alpha\delta}) \\
&\quad - C_{\alpha\beta\gamma}\,(\log g_{\alpha\beta} + \log g_{\beta\gamma} - \log g_{\alpha\gamma}) \\
&= (C_{\alpha\beta\delta} - C_{\alpha\beta\gamma}) \log g_{\alpha\beta} + (C_{\beta\gamma\delta} - C_{\alpha\beta\gamma}) \log g_{\beta\gamma} \\
&\quad + (C_{\beta\gamma\delta} - C_{\alpha\gamma\delta}) \log g_{\gamma\delta} + (C_{\alpha\beta\delta} - C_{\beta\gamma\delta}) \log g_{\beta\delta} \\
&\quad + (C_{\alpha\beta\gamma} - C_{\alpha\gamma\delta}) \log g_{\alpha\gamma} + (C_{\alpha\gamma\delta} - C_{\alpha\beta\delta}) \log g_{\alpha\delta}
\end{aligned}
$$

$$\Rightarrow \quad C_{\alpha\beta\delta} = C_{\alpha\beta\gamma} = C_{\beta\gamma\delta} = C_{\alpha\gamma\delta}$$

$$\Rightarrow \quad \delta a^{1.0} = C\,(z_{\alpha\beta\gamma}), \text{ where } C = C_{\alpha\beta\gamma} = \cdots .$$

We don't know whether $\{\log g_{\alpha\beta}\}$ is a set of linearly independent.

⊛ We are going to explain why the following diagram is commutative, by considering an example.

$$
\begin{tikzcd}
{}'E_2^{0,1} \arrow[r, "d_2"] \arrow[d] & {}'E_2^{2,0} \arrow[d] \\
H^0(\underline{U}, \bigoplus_D \mathbb{C}_D) \arrow[d] & H^2(\underline{U}, \mathbb{C}) \\
H^1(\underline{U}, \mathcal{O}^*) \arrow[ur, "c_1"'] &
\end{tikzcd}
$$

c_1 Chern class