

In intrinsic terms, the Jacobian variety $J(S) = V(S) / \Lambda(S)$, where $V(S) = H^0(S, \Omega^1)^*$ and the lattice $\Lambda(S) \cong H_1(S, \mathbb{Z})$ is embedded in $V(S)$ by integration.

¶ 'More on Riemann Conditions'

It is just to prove that an integral invariant 2-form is (1,1) type and positive. \Rightarrow Given $\omega = \frac{1}{2} \sum g_{ij} dx_i \wedge dx_j$, find conditions of Q and Ω on the normalized period matrix Ω .

$$\begin{array}{ccc} H_1(S, \mathbb{Z}) & \longrightarrow & V(S) = H^0(S, \Omega^1)^* \\ \downarrow \delta & \longmapsto & \downarrow \int_\delta \end{array}$$

Let $L_i \in H^0(S, \Omega^1)^*$ s.t. $L_i(\omega_j) = \delta_{ij}$, where $\{\omega_1, \dots, \omega_g\}$ is a basis for $H^0(S, \Omega^1)$.

$$\Rightarrow \int_\delta (\quad) = a_i L_i \Leftrightarrow (a_1, \dots, a_n) \Leftrightarrow \delta = a_i \delta_i.$$

where $\int_{\delta_i} \omega_j = \delta_{ij}$.

\Rightarrow

The polarizing form $\omega \in H^2(J(S), \mathbb{Z}) \cong \text{Hom}_{\mathbb{Z}}(\Lambda^2 H_1(S, \mathbb{Z}), \mathbb{Z})$ is the skew-symmetric bilinear form

$$Q: H_1(S, \mathbb{Z}) \otimes_{\mathbb{Z}} H_1(S, \mathbb{Z}) \longrightarrow \mathbb{Z}$$