$$\dim \bar{E} \leq \deg \bar{E} - 1 - (g-d+s-1)$$
$$= (2g-2-d) - 1 - (g-d+s-1)$$
$$= g-s-2.$$

⌈ Let $E = H \cap \iota_K(\mathcal{S}) - D$, where, we have to count multiplicities of points in $H \cap \iota_K(\mathcal{S})$.

Apply (*) to E. $\Rightarrow$ $\dim \bar{E} \leq \deg E - 1 - \dim|E| \leq (2g-2) - d - 1 - \dim|E|$.

But since $\dim|E| \geq \dim(H \cap \iota_K(\mathcal{S}) - \bar{D}) = \ell = g-d+s-1$,

$$\dim \bar{E} \leq 2g-2-d-1-\dim|E| \leq 2g-2-d-1-\ell$$
$$= 2g-2-d-1-(g-d+s-1) = g-s-2. \quad ⌋$$

But now the hyperplanes in $\mathbb{P}^{g-1}$ containing E will cut out on $\mathcal{S}$ a linear subseries of $|D|$ having dimension at least

$$(g-1) - (g-s-2) - 1 = s,$$

that is,

$$\dim|D| \geq s = d-1-\dim \bar{D}$$

and so the Riemann-Roch formula is proved.

Q.E.D.

⌈ Let H be a hyperplane in $\mathbb{P}^{g-1}$ contaning E.

$\Rightarrow$ $(H \cap \iota_K(\mathcal{S}) - \bar{E})$ is a subseries of an effective divisor linearly equivalent to D. If we denote it by D', $\dim \bar{D}' = (g-1) - \dim \bar{E} - 1$.

$\Rightarrow$ $\dim \bar{D}' = g-2-\dim \bar{E} \geq g-2-(g-s-2) = s$

$\Rightarrow$ $\exists$ points $p_1, p_2, \cdots p_\ell \in \mathcal{S}$, s.t. $\iota_K(p_1), \iota_K(p_2) \cdots \iota_K(p_\ell)$ are linearly independent in $\mathbb{P}^{g-1}$, $\ell \geq s+1$.