Applying this lemma gives the first part of the Proposition. Suppose that $f = (f_1 \cdots f_r)$ is a regular sequence generating an ideal $I = I(f)$. Then

$$(*) \quad \mathrm{Ext}^k_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) = 0, \quad k < r; \quad \mathrm{Ext}^r_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) \cong \mathcal{O}/I.$$

The second isomorphism has the following functoriality property: Suppose that $I' = I(f')$ is a regular ideal contained in I, so that

$$f_i' = \sum_j a_{ij} f_j.$$

Denote by $\Delta = \det(a_{ij})$ the determinant of the matrix (a_{ij}). Then the diagram

$$(**) \quad \begin{tikzcd} \mathrm{Ext}^r_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) \arrow[r, "\sim"] \arrow[d] & \mathcal{O}/I \arrow[d, "\Delta"] \\ \mathrm{Ext}^r_{\mathcal{O}}(\mathcal{O}/I', \mathcal{O}) \arrow[r, "\sim"] & \mathcal{O}/I' \end{tikzcd}$$

is commutative. Moreover, the vertical map is injective.

Proof. The computation of the $\mathrm{Ext}^k_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O})$ follows from the previous lemma.

From the lemma, we get $\mathrm{Ext}^k_{\mathcal{O}}(\mathcal{O}/I, \mathcal{O}) = H^k(\mathrm{Hom}(E.(f), \mathcal{O}))$