the argument above, $\sigma_1 - \alpha\sigma_2$ vanishes at q, in fact, vanishes along C. $\Rightarrow$ $\sigma_1 = \alpha\sigma_2$. along C. Similarly for any two σ_i, σ_j. $\Rightarrow$ $\iota_{L'}$ maps C to a point. ◻

To conclude the argument, we must show that $\iota_{L'}(C)$ is a smooth point of the image $\iota_{L'}(M)$; to see this, note that by the sequence $(*_{m-1})$ the restriction map

$$H^0(M, \mathcal{O}(L'-C)) \longrightarrow H^0(C, \mathcal{O}(H))$$

is surjective. Let $p_1 \neq p_2 \in C$, and let ζ_1 be a section of L' vanishing on C that restricts, via the map above, to a section of H over C vanishing at p_1; let ζ_2 be a global section of L' restricting to a section of H vanishing at p_2.

Γ Consider a section $\tau \in H^0(C, \mathcal{O}(H))$ s.t $\tau(p_1) = 0$.
$\Rightarrow$ Since $H^0(M, \mathcal{O}(L'-C)) \longrightarrow H^0(C, \mathcal{O}(H))$ is onto. $\exists\ \eta \in H^0(M, \mathcal{O}(L'-C))$ s.t $\eta|_C = \tau$. If we let $\sigma \in H^0(M, \mathcal{O}(C))$ s.t $(\sigma = 0) = C$, $\eta \otimes \sigma \in H^0(M, \mathcal{O}(L'))$. $\Rightarrow$
$\eta \otimes \sigma = 0$ on C and $\eta(p_1) \otimes \sigma(p_1) = 0$.

More clearly, let $\zeta_1 = \eta \otimes \sigma \in H^0(M, \mathcal{O}(L'))$
$\Rightarrow$ $\zeta_1 \otimes \sigma^{-1} = \eta \in H^0(M, \mathcal{O}(L'-C))$ $\Rightarrow$ $\eta|_C \in H^0(C, \mathcal{O}(H))$ becomes τ which vanishes at p_1. ◻

Let ζ_0 be any section of L' not vanishing identically on