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$$(D_1, \cdots, D_n)_{\{0\}} = (D_2', \cdots, D_n')_{\{0\}}.$$

Proof. We may choose coordinates so that $f_1(z) = z_1$. Set $z = (z_1, z')$ and $f_i'(z') = f_i(0, z') = f_i|_{D_1}$. Then if $P = \{ |f_1(z)| = \cdots = |f_n(z)| = \varepsilon \}$ and $P' = \{ |f_2'(z')| = \cdots = |f_n'(z')| = \varepsilon \}$, we may iterate the Cauchy integral formula to obtain

$$\begin{aligned}(D_1, \cdots D_n)_{\{0\}} &= \left(\frac{1}{2\pi\sqrt{-1}}\right)^n \int_P \frac{dz_1}{z_1} \wedge \frac{df_2}{f_2} \wedge \cdots \wedge \frac{df_n}{f_n} \\ &= \left(\frac{1}{2\pi\sqrt{-1}}\right)^{n-1} \int_{P'} \frac{df_2'}{f_2'} \wedge \cdots \wedge \frac{df_n'}{f_n'} \\ &= (D_2', \cdots D_n')_{\{0\}}.\end{aligned}$$

Γ For example, $n = 2$. $f_1(z) = z_1$, $z = (z_1, z_2)$ and $f_2'(z^2) = f_2(0, z_2) = f_2|_{D_1} = f_2|_{\{z_1 = 0\}}$.

$\Rightarrow P = \{ |z_1| = |f_2(z)| = \epsilon \}$ and $P' = \{ |f_2'(z_2) = \epsilon \}$.

$$\begin{aligned}(D_1, D_2)_{\{0\}} &= \left(\frac{1}{2\pi\sqrt{-1}}\right)^2 \int_P \frac{dz_1}{z_1} \wedge \frac{df_2}{f_2} \\ &= \left(\frac{1}{2\pi\sqrt{-1}}\right)^2 \int_{|f_2|=\epsilon} \int_{|z_1|=\epsilon} \frac{dz_1}{z_1} \wedge \frac{df_2}{f_2} \\ &= \frac{1}{2\pi\sqrt{-1}} \int_{|f_2|=\epsilon} \frac{1}{2\pi\sqrt{-1}} \int_{|z_1|=\epsilon} \frac{dz_1}{z_1} \wedge \frac{\frac{\partial f_2}{\partial z_1} dz_1 + \frac{\partial f_2}{\partial z_2} dz_2}{f_2} \\ &= \frac{1}{2\pi\sqrt{-1}} \int_{|f_2|=\epsilon} \frac{1}{2\pi\sqrt{-1}} \int_{|z_1|=\epsilon} \frac{dz_1}{z_1} \wedge \frac{\frac{\partial f_2}{\partial z_2}}{f_2} dz_2 \\ &= \frac{1}{2\pi\sqrt{-1}} \int_{|f_2|=\epsilon}^{(0,z)} \frac{\frac{\partial f_2}{\partial z_2}(0, z_2)}{f_2(0, z_2)} dz_2\end{aligned}$$