$\Rightarrow C \cong \mathbb{P}^1$, and $\tilde{C} = \pi^* C - E_1 - E_2 - E_3 - E_4 - \cdots - E_6$.

$\Rightarrow \tilde{C}\cdot\tilde{C} = \pi^*C \cdot \pi^* C - 6 = C\cdot C - 6 = 4-6 = -2$.

$$\Rightarrow \pi(C) = 0 = \frac{\tilde{C}\cdot\tilde{C} + K_{\tilde{S}}\cdot\tilde{C}}{2} + 1 = \frac{-2 + K_{\tilde{S}}\cdot\tilde{C}}{2} + 1 \Rightarrow$$

$K_{\tilde{S}}\cdot\tilde{C} = 0 \Rightarrow$ The canonical bundle of $\tilde{S}$ is negative, a contradiction. ⌟

Thus if $\tilde{S} \cong \tilde{\mathbb{P}}^2_{p_1,\cdots p_6}$, the points p_i necessarily satisfy the conditions 1 and 2 of P. 480; thus we see that

Every smooth cubic surface $S \subset \mathbb{P}^3$ may be obtained by blowing up $\mathbb{P}^2$ at six points $p_1, \cdots p_6$, no three collinear and not all six on a conic, and embedding the blow-up in $\mathbb{P}^3$ by the proper transform of the linear system of cubics passing through the points p_i.

⌜ This is the rephrasement of the results on P481 and P489 above. ⌟

In particular, we see that our discussion of the lines on the surface constructed before applies to all smooth cubics.

As we will see in the following sections, the quadric and cubic surfaces are the only smooth hypersurfaces in $\mathbb{P}^3$