for E in terms of which the connection metrix θ_β vanishes at x_0

$\ulcorner$ $P(\Theta_\alpha) = P(\Theta_\beta)$ See p401 & note p318 back. $\lrcorner$

Thus $d\,P(\Theta_\alpha) = d\,P(\Theta_\beta)$

$$= \sum \pm \tilde{P}(\Theta_\beta, \cdots d\theta_\beta \wedge \theta_\beta - \theta_\beta \wedge d\theta_\beta, \cdots \Theta_\beta)$$

$$\Rightarrow d\,P(\Theta_\beta)(x_0) = 0$$

$$\Rightarrow d\,P(\Theta) \equiv 0.$$

In order to prove part 2, we need to establish an identity for invariant forms. We consider the holomorphic function on GL_n given by

$$f(g) = P(gA_1g^{-1}, \cdots gA_kg^{-1})$$

for any choice of $A_1, \cdots A_k \in M_n$.

$\ulcorner$ $P : \overbrace{M_n \times \cdots \times M_n}^{k} \longrightarrow \mathbb{C}$ is a k-linear form. $\lrcorner$

Using as coordinates on GL_n the entries of $g' = g - I$, we compute the linear term f_1 of the power series expansion for f around I. First,

$$(I + g')^{-1} = I - g' + [2].$$

$\ulcorner$ $(1+x)(1 - x + x^2 - x^3 + x^4 - \cdots) = 1$ formally. $x \in \mathbb{R}$

$$\Rightarrow (I + g')(I - g' + g'^2 - g'^3 + \cdots) = I$$

$$\Rightarrow g'^2 - g'^3 + \cdots = (g'^2)(I - g' + \cdots) = [2]$$

$\lrcorner$

Thus