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$\Rightarrow u_1^{n-1} du_1 \wedge df_2 \wedge \cdots \wedge df_n \equiv 0 \Rightarrow df_2 \wedge \cdots \wedge df_n \equiv 0$ modulo $du_1 \Rightarrow df_2' \wedge \cdots \wedge df_n' \equiv 0$.

Since z_0 is a smooth point on D_1, we can let $(f_1 = u_1, u_2, \cdots u_n)$ be coordinates around z_0.

$\Rightarrow df_1 \wedge \cdots \wedge df_n = du_1 \wedge df_2 \wedge \cdots \wedge df_n \equiv 0 \Rightarrow$ For example, $n=2$, $du_1 \wedge df_2 = du_1 \wedge (\frac{\partial f_2}{\partial u_1} du_1 + \frac{\partial f_2}{\partial u_2} du_2) \equiv 0$

$\Rightarrow \frac{\partial f_2}{\partial u_2} \equiv 0 \Rightarrow f_2$ is independent of u_2

$\Rightarrow df_2' = d f_2'(0, u_2) \equiv 0$.

In case $n=3$, $df_1 \wedge df_2 \wedge df_3 = du_1 \wedge df_2 \wedge df_3$

$= du_1 \wedge (\frac{\partial f_2}{\partial u_2} du_2 + \frac{\partial f_2}{\partial u_3} du_3) \wedge (\frac{\partial f_3}{\partial u_2} du_2 + \frac{\partial f_3}{\partial u_3} du_3)$

$= du_1 \wedge (\frac{\partial f_2}{\partial u_2}\frac{\partial f_3}{\partial u_3} - \frac{\partial f_2}{\partial u_3}\frac{\partial f_3}{\partial u_2}) du_2 \wedge du_3 \equiv 0$

$\Rightarrow \frac{\partial f_2}{\partial u_2}\frac{\partial f_3}{\partial u_3} = \frac{\partial f_2}{\partial u_3}\frac{\partial f_3}{\partial u_2}$

$\Rightarrow df_2' \wedge df_3' = d f_2(0, u_2, u_3) \wedge d f_3(0, u_2, u_3)$

$= (\frac{\partial f_2}{\partial u_2}\frac{\partial f_3}{\partial u_3} - \frac{\partial f_2}{\partial u_3}\frac{\partial f_3}{\partial u_2}) du_2 \wedge du_3 \equiv 0$.

On the other hand, if we let $f_i'(z_0) = w_i'$, then the equations $f_i'(z') = w_i'$ have z_0 as an isolated solution on D_1 near z_0.

If z_0 is not isolated, then for each n, $\exists\ \eta_n \in D_1$ s.t. $\|\eta_n\| < \frac{1}{n}$ and $f_2(\eta_n) = 0$. $\Rightarrow$ This contradicts to the fact that $f_1^{-1}(0) \cap f_2^{-1}(0) = \{0\}$. $\Rightarrow z_0$ is isolated solution for $f_i'(z') = w_i'$. $z_0 = (0, z_0') \Rightarrow f_i'(z_0) = w_i'$

Don't be confused with notations.