We know that for $p+q \equiv 0\,(2)$,

$$(i)^{p-q}\,(-1)^{(p+q)(p+q-1)/2}\,Q > 0 \quad \text{on the real space}$$

$$(P^{p,q} \oplus P^{q,p}) \cap H^{p+q}(M, \mathbb{R}); \quad \text{since } Q(L\eta, L\xi) = Q(\eta, \xi),$$

we have

$$I(M) = \sum_{\substack{p+q \equiv 0\,(2) \\ \leq 2n}} (i)^{p-q}\,(-1)^{(p+q)(p+q-1)/2} \dim P^{p,q}(M).$$

$$= \sum_{\substack{p+q \equiv 0\,(2) \\ \leq 2n}} (-1)^{p} \dim P^{p,q}(M).$$

$\ulcorner$ $p+q = 2l$. $\quad p-q = p-(2l-p) = 2p-2l$.

$$i^{p-q}\,(-1)^{(p+q)(p+q-1)/2} = i^{2p-2l}\,{}^{l(2l-1)} = (-1)^{p-l}\,(-1)^{l(2l-1)}$$

$$\Rightarrow \quad p-l+2l^2-l \equiv p-2l \equiv p.$$

$$\Rightarrow \quad (-1)^{p-l+l(2l-1)} = (-1)^{p}. \quad \lrcorner$$

$\ulcorner$ Since $(P^{p,q} \oplus P^{q,p}) \cap H^{p+q}(M, \mathbb{R}) = \{\xi + \bar{\xi} \mid \xi \in P^{p,q}(M)\}$

$$I(M) = \sum_{\substack{p+q \equiv 0\,(2) \\ \leq 2n \\ p \leq q}} (-1)^{p} \dim P^{p,q}(M).$$

$\pm Q$ positive defite or negative definite on $\{L^{n-(p+q)/2} P^{p,q}(M) \oplus L^{n-(p+q)/2} P^{q,p}(M)\} \cap H^{p+q}(M, \mathbb{R})$ when $p \neq q$.

In case $p = q\,(= n)$, on $L^{0} P^{n,n}(M)$, since $P^{n,n}(M) = \overline{P^{n,n}(M)}$, $\pm Q$ is p, or n, definite on $P^{n,n}(M)$.

This is the reason we have only to consider the half. $\lrcorner$