$V \cap H$ is non-degenerate in H. Obviously, if H' is near H sufficiently, $V \cap H'$ has n-linearly independent points. $\Rightarrow V \cap H'$ is again non-degenerate in $H' \Rightarrow$ The condition that $V \cap H$ be degenerate is a closed one on $H \in \mathbb{P}^{n*}$. If we have n linearly dependent points on V, we can choose n linearly independent points which are very near to those n-linearly dependent pts, by the following fact:

$$V \times V \times \cdots \times V \xrightarrow{\det} \mathbb{C} \quad \text{holomorphic locally.}$$

$\Rightarrow$ The set of n-linearly dependent ordered points is a variety. $\Rightarrow$ The set of n-linearly independent ordered points is dense.))

The former half of our assertion — that the generic hyperplane section of an irreducible variety is irreducible — is somewhat harder. We note first that in case V is smooth, this follows easily from the Bertini theorem and the Lefschetz theorem on hyperplane sections: by Bertini, the generic hyperplane section $H \cap V$ is smooth, and so by Lefschetz,

$$H_0(H \cap V, \mathbb{C}) \cong H_0(V, \mathbb{C}) \cong \mathbb{C}; \quad \text{i.e.,}$$

$H \cap V$ is connected.

Γ $H \cap V$ is connected $\Leftrightarrow (H \cap V)^* = H \cap V$ is irreducible, since $H \cap V$ is smooth. See P21, Prop.
For $H_0(H \cap V, \mathbb{C}) \cong H_0(V, \mathbb{C})$, see P157 ~ P158))
Thus, if $H \cap V$ were reducible, the components of $H \cap V$ would have to meet each other; but their