on the quadric line complex, we obtain an embedding $E_L \subset \mathbb{P}^3$ of the curve B_L.

$\ulcorner$ $f_L : X \longrightarrow \mathbb{P}^3$ is the embedding of B_L. See P 796
B_L is isomorphic to the abstract curve B, see P 788 $\lrcorner$

Accordingly, we may define a map

$$\rho : A = \mathcal{J}(B) \longrightarrow \mathcal{J}(B)$$

by sending each line $L \in A$ to the class of the hyperplane bundle on $B \cong E_L \subset \mathbb{P}^3$; we ask now for a description of the map ρ.

$\ulcorner$ According to P313 & P782 & P784, $\mathcal{J}(B_L) \cong \mathrm{Pic}^0(B_L)$ = The group of holomorphic line bundles on B_L with Chern class zero i.e. degree 0. But hyperplane bundles on E_L are of form $\widehat{H \cap E_L}$, $H \subset \mathbb{P}^3$. $\Rightarrow \deg(H \cap E_L)$ is not zero. Strange. Let's see more. $\lrcorner$

To answer this question, we argue as follows. First, we note that the linear system associated to any divisor D of degree 5 on the curve B gives an embedding of B in $\mathbb{P}^3$ as a quintic curve E_D (Section 1, Chapter 2).