To begin with, if $p \in \mathbb{P}^3$ is a generic point, then the set of quadrics in W through p forms a generic net N_p, with p as a base point.

⌜ Clearly, the set of quadrics in W through p forms a generic net since p & W are generic. ⌟

By the argument given in the count for $V_1(N)$, the lines through p lying on a pencil of quadrics from W will just be the lines joining p to the other seven base points of N_p. Consequently,

$$\#(V_1(W) \cdot \sigma_2) = 7.$$

⌜ As we saw above, if we choose a generic net of quadrics, then the base points is a set of 8 distinct points. Choose one of them, let p the point. Let N_p be the net, and $\{p, p_2, \cdots p_8\}$ be the base points.
⇒ By the argument of $V_1(N)$, $\overline{Pp_i}$'s are the lines containing p and contained in a pencil of quadrics from W.
⇒ $\#(V_1(W) \cdot \sigma_2) = 7$
More precisely, if the line l contains p and l is contained in a pencil of quadrics from W. ⇒ $l \subset \{a\sigma_0 + b\sigma_1\}$, i.e $l \subset \sigma_0, \sigma_1$.