For example

$$\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \quad a \neq 0.$$

$$\Rightarrow \begin{pmatrix} 1-x & a \\ 0 & 1-x \end{pmatrix} \Rightarrow \begin{pmatrix} 1-x-a\cdot\frac{1-x}{a}, & a \\ 0-(1-x)\frac{1-x}{a}, & 1-x \end{pmatrix}$$

$$= \begin{pmatrix} 0 & a \\ -\frac{(1-x)^2}{a}, & 1-x \end{pmatrix} \Rightarrow \begin{pmatrix} 0, & a \\ -\frac{(1-x)^2}{a}, & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a, & 0 \\ 0, & -\frac{(1-x)^2}{a} \end{pmatrix}$$

$\Rightarrow$ The minimal polynomial of $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ is $(x-1)^2$.

$\Rightarrow$ As $a \longrightarrow 0$, $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

This implies that the set $\mathcal{K}$ of all diagonalizable 2×2 matrices is not open, since $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is in $\mathcal{K}$.

Let $\mathcal{A}$ = the set of all matrices with distinct eigenvalues. $\Rightarrow$ $\mathcal{A}$ is open. for given $A_0 \in \mathcal{A}$ consider $P(A,z) = z^n + a_1(A)z^{n-1} + \cdots + a_n(A)$, $A \in GL_n$. where $P(A,z) = \det(A - zI)\cdot(-1)^n$.

$\Rightarrow$ $a_i(A)$ is holomorphic in a_{ij}'s.

Since $P(A_0,z)=0$ has no double root, by the inverse function theorem. $\exists\ \epsilon>0$ s.t for $A \in B(A_0,\epsilon)$, $P(A,z)=0$ has no double root. ($\because$ The roots of $P(A,z)=0$ are holomorphic functions of (a_{ij}).)

Since $P(A,z)=0$ has a double root when the dis-