「 All hypersurface V_C contain $W_2 = \{2L\}_{L \subset \mathbb{P}^2}$, since $C \cap L \neq \phi$ and $C \cap 2L = (C \cap L) \cup (C \cap L)$, i.e $2L$ is tangent to C. The proofs of 1, 2&2' are given later. 」

We can overcome this difficulty by blowing up. Precisely, let

$$\pi : \widetilde{W} \longrightarrow W$$

be the blow-up of W along the variety W_2 of double lines; for C any smooth conic, denote by $\widetilde{V}_C$ the proper transform of the subvariety $V_C \subset \mathbb{P}^5$. Then, once we verify assertions 1 and 2' above and the additional assertion that for $C_1 \cdots C_5$ generically chosen,

3. the proper transforms $\widetilde{V}_{C_1}, \cdots \widetilde{V}_{C_5}$, have no common points in the exceptional divisor of $\widetilde{W}$,

the answer to our original question will be simply the fivefold self-intersection of the divisor $\widetilde{V}_C$ on $\widetilde{W}$, and readily calculable. We will proceed with the computation, leaving the proof of assertions 1, 2' and 3 until later.

「 See P602 ~ P604 for blowing up submanifolds.
See P605 for the proper transform. 」