$$H^1((\mathbb{C}^*)^n, \mathcal{O}^*) \xrightarrow{c_1} H^2((\mathbb{C}^*)^n, \mathbb{Z}),$$

i.e., any line bundle on $(\mathbb{C}^*)^n$ is determined by its Chern class.

$\ulcorner$ See p27 & p46. $\Rightarrow$ $H^q((\mathbb{C}^*)^n, \mathcal{O}) = 0$ for $q > 0$.
From $0 \longrightarrow \mathbb{Z} \longrightarrow \mathcal{O} \longrightarrow \mathcal{O}^* \longrightarrow 0$, we have

$$H^1((\mathbb{C}^*)^n, \mathbb{Z}) \longrightarrow \underbrace{H^1((\mathbb{C}^*)^n, \mathcal{O})}_{=0} \longrightarrow H^1((\mathbb{C}^*)^n, \mathcal{O}^*) \xrightarrow[\cong]{c_1} H^2((\mathbb{C}^*)^n, \mathbb{Z})$$
$$\longrightarrow \underbrace{H^2((\mathbb{C}^*)^n, \mathcal{O})}_{=0} \longrightarrow$$

$\lrcorner$

For any L we can choose our basis $\lambda_1, \cdots \lambda_{2n}$ for Λ such that in terms of the dual coordinates $x_1, \cdots x_{2n}$ on V,

$$c_1(L) = \sum_{\alpha=1}^{n} \delta_\alpha \, dx_\alpha \wedge dx_{\alpha+n}.$$

$\ulcorner$ $c_1(L) \in H^2(V/\Lambda, \mathbb{Z})$ $\Rightarrow$ $c_1(L)$ is an integral, invariant 2-form on V/Λ. $\Rightarrow$ By $*$ on p305, $\exists\ \lambda_1, \cdots \lambda_{2n}$ for Λ s.t. in terms of the dual coordinates $x_1, x_2, \cdots, x_{2n}$ on V,

$$c_1(L) \underset{\uparrow}{=} \sum_{\alpha=1}^{n} \delta_\alpha \, dx_\alpha \wedge dx_{\alpha+n}, \quad \delta_\alpha \mid \delta_{\alpha+1}.$$

(in fact, represented by)

$\lrcorner$