

i.e. consider all hyperplanes in  $W = \mathbb{P}^5$  containing  $C'$ .  $\Rightarrow$  The set of hyperplane sections has a base point  $C'$ , and, for a generic hyperplane  $H$ ,  $H \cap V_C = V_1 \cup V_2 \cup \dots \cup V_e$ ,  $V_i$  smooth in  $H$  except  $C'$ , of course,  $\dim$  of  $V_i$  is different.

Again, for a generic hyperplane  $H'$  in  $W$ ,  $H \cap H' \cap V_C = V_1' \cup V_2' \cup \dots \cup V_e'$ ,  $V_i'$  smooth in  $H \cap H'$ , each  $\dim$  is different. except  $C'$ .

Continue this procedure, then we have  $H \cap H' \cap H'' \cap H''' = l$ , and  $l \cap V_C$  is smooth, which implies that  $l \cap V_C$  is a set of distinct points, except  $C'$ . Refer to p174, p14 & p137.

According to the argument above,  
 $\#(l \cap V_C - C') \leq 4$ . But since  $\#(l \cdot V_C) = 6$ ,  
 $\text{mult}_{C'}(l, V_C) \geq 2$ , i.e.  $l$  is tangent to  $V_C$  at  $C'$ .  
 Since  $l (= L)$  is a generic line in  $H_p$ ,  
 $H_p$  is a tangent plane of  $V_C$  at  $C'$ . "

If  $C'$  is simply tangent to  $C$  at only one point  $q$ ,

for a generic pencil  $L$  through  $C'$ , by the argument above, since  $L$  gives a 4-sheeted, branched cover of  $\mathbb{P}^1$  with a simple branch point at  $q$ .  $\Rightarrow \#(L \cap V_C - C') \leq 5$ , counting multiplicity.

Suppose  $L \cap V_C - C'$  is a set of distinct points less than 5, here we may choose  $L$  s.t