

By Prop., $A_{nn}(F^{n-p+1}(\wedge^n T(E))) = \{ \ell \in (\wedge^n T(E))^* \mid \ell(F^{n-p+1}(\wedge^n T(E))) = 0 \}$.

Since $F^{n-p+1}(\wedge^n T(E)) \supset F^{n-(p+1)+1}(\wedge^n T(E))$,

$$F^p(\wedge^n T^*(E)) \supset F^{p+1}(\wedge^n T^*(E)). \quad \Rightarrow$$

This gives a filtration $F^p A^n(E)$ on the space of C^∞ differential forms of degree n on E , and setting $A^n = A^n(E)$, we have

$$\begin{cases} A^n = F^0 A^n \supset F^1 A^n \supset \dots \supset F^n A^n \supset F^{n+1} A^n = 0 \\ d: F^p A^n \longrightarrow F^p A^{n+1}. \end{cases}$$

For example, $\sigma \in A^n(E) = P(\wedge^n T(E))$.

$$F^1 \sigma \in F^1 A^n, \quad q \in E, \quad \sigma_q \in \wedge^n T_q(E)$$

$$F^1 \sigma_q \in F^1 \wedge^n T_q(E) = \wedge^1 T_q(F) \wedge \wedge^{n-1} T_q(E).$$

$$\text{Note that } \wedge^{n+1} T_q(E) = 0 \Rightarrow F^{n+1} A^n = 0 \quad \Rightarrow$$

To picture this filtration, we choose local product coordinates (x, y) in E with $\pi(x, y) = x$. Then $T_p(F)$ is spanned by the vectors $\partial/\partial y_i$, and

$$F^p A^n = \left\{ \varphi = \sum_{\substack{\#I + \#J = n \\ \#I \geq p}} \varphi_{IJ}(x, y) dx_I \wedge dy_J \right\},$$

from which the two above properties of the filtration are apparent.

Since $\#I \geq p$, $F^p A^n \supset F^{p+1} A^n$. J