Pf. Given a filtration on K^*, i.e.,

$$\begin{array}{ccccccc} K^0 & \longrightarrow & K^1 & \longrightarrow & K^2 & \longrightarrow & \cdots \\ \cup & & \cup & & \cup & & \\ F^1K^0 & \longrightarrow & F^1K^1 & \longrightarrow & F^1K^2 & \longrightarrow & \cdots \end{array}$$

$\Rightarrow$ then we have a filtration on the cohomology, i.e.

$$\begin{array}{ccc} H^1(K^*) & H^2(K^*) & \cdots \\ \cup & \cup & \\ F^1H^1(K^*) & F^1H^2(K^*) & \\ \cup & \cup & \\ F^2H^1(K^*) & F^2H^2(K^*) & \\ \vdots & \vdots & \end{array}$$

In our case, $K^n = \bigoplus_{p+q=n} K^{p,q}$, $K^{p,q} = A^{p,q}(M)$

$$F^pK^n = \bigoplus_{\substack{p'+q=n \\ p' \geq p}} A^{p',q}(M)$$

$$\Rightarrow \quad \begin{array}{ccc} H^r(K^*) & = & H^r_{DR}(M) \\ & & \cup \\ & & F^1H^r_{DR}(M) \\ & & \cup \\ & & \vdots \\ & & \cup \\ & & F^pH^r_{DR}(M) \\ & & \cup \\ & & F^{p+1}H^r_{DR}(M). \end{array}$$

$\Rightarrow$ By the result of Proposition on P440,

$$E_\infty^{p,q} \cong \frac{F^pH^{p+q}_{DR}(M)}{F^{p+1}H^{p+q}_{DR}(M)} \Rightarrow 0 \to F^{p+1}H^r_{DR}(M) \longrightarrow F^pH^r_{DR}(M) \to E_\infty^{p,r-p} \to 0$$

$$\Rightarrow \quad F^pH^r_{DR}(M) \cong F^{p+1}H^r_{DR}(M) \oplus E_\infty^{p,r-p}$$