We can take l (in $\mathcal{O}_{n-1,z'}$) irreducible.
$\Rightarrow$ Since Γ and $\beta+hf$ are zeros on $\pi^{-1}(K)$, and l is irreducible in $\mathcal{O}_{n,(z',\, z_n)}$
l must divide Γ and $\beta+hf$ by Weak Nullstellensatz. Γ and $\beta+hf$ have a common factor $l \in \mathcal{O}_{n-1}$

"Correction"

I think, we have to correct the claim as follows:
$f, g \in \mathcal{O}_{n-1}[z_n]$. f, g Weierstrass polynomial in z_n.
$\Rightarrow$ Since f & g are relatively prime in $\mathcal{O}_n$, and in $\mathcal{O}_{n-1}[z_n]$, $\exists$ $\alpha(z_n)$, $\beta(z_n)$ s.t

$$\alpha(z_n) f + \beta(z_n) g = \overrightarrow{\gamma \in \mathcal{O}_{n-1}}^{\text{resultant}},$$

$\alpha(z_n), \beta(z_n) \in \mathcal{O}_{n-1}[z_n]$, by P343, Whitney.
Since $\mathcal{O}_{n-1}$ is UFD, $\gamma = \gamma_1 \cdots \gamma_e$. $\alpha(z_n) = \alpha_1 \cdots \alpha_p\, \alpha'(z_n)$ where $\alpha_1 \cdots \alpha_p \in \mathcal{O}_{n-1}$, similarly for $\beta(z_n)$.
$\Rightarrow$ We may assume that $\alpha(z_n)$, $\beta(z_n)$ & γ are relatively prime in $\mathcal{O}_n$. $\Rightarrow$ γ may not be the resultant of f and g. $\Rightarrow$ This can happen. For example,

$$f(x) = x^3 + x^2 + 4x + 4, \quad g(x) = x^2 + x + 2 \in \mathbb{Z}[x].$$

$\Rightarrow$

$R(f, g)$ = resultant of f & g

$$= \begin{vmatrix} 1 & 1 & 4 & 4 & 0 \\ 0 & 1 & 1 & 4 & 4 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 2 & 4 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \end{vmatrix} =$$