By the guess above, i.e $\pi^{-1}(U) = M - \pi^{-1}(D) = M - D'$ (we may assume or guess $[D'] \longrightarrow M$ is positive)

$\Rightarrow$ By Grothendieck's Algebraic de Rham theorem,
$H^p_{DR}(\pi^{-1}(U), alg) \cong H^p(\pi^{-1}(U), \mathbb{C}) \cong H^p(U, \mathbb{C})$
since π is a diffeomorphism.
$\Rightarrow$ $H^p(U, \mathbb{C})$ may be computed from the complex $\Omega^*(U, alg)$.

Differentials of the Second Kind.

Let M be a smooth algebraic variety. A differential of the first kind is the classical terminology for a holomorphic p-form on M. By Hodge theory these inject to give the part $H^{p,0}(M)$ of the cohomology $H^p(M, \mathbb{C})$ of M.

According to P110, 2, the holomorphic p-forms $H^0(M, \Omega^p)$ inject into the cohomology $H^p_{DR}(M)$, i.e., every such η is closed, and is never exact.

By 116, Hodge decomposition, $H^{p,0}(M) \cong H^0(M, \Omega^p)$, and $H^p(M, \mathbb{C}) \cong \bigoplus_{i+j=p} H^{i,j}(M)$.

Definition. A differential of the second kind is given by a closed meromorphic p-form φ on M such that, for some divisor D with complement $U = M - D$, φ is holomorphic in U and is in the