$$\Rightarrow \int_{\mu\times\nu} \pi_1^*\varphi \wedge \pi_2^*\psi = \sum a_i b_j \int_{\Delta^{n-k}\times\Delta^k} p_1^* f_i^*\varphi \wedge p_2^*(g_j^*\psi)$$

$$= \sum a_i b_j \int_{\Delta^{n-k}} f_i^*\varphi \int_{\Delta^k} g_j^*\psi = \left(\sum a_i \int_{\Delta^{n-k}} f_i^*\varphi\right)\cdot\left(\sum b_j \int_{\Delta^k} g_j^*\psi\right)$$

$$= \int_\mu \varphi \cdot \int_\nu \psi \quad \lrcorner$$

On the other hand, if (p_1, p_2) is a point of intersection of $\sigma\times\tau$ with $\mu\times\nu$, then writing out the orientations, we see that

$$i_{(p_1,p_2)}(\sigma\times\tau, \mu\times\nu) = (-1)^{n-k}\, i_{p_1}(\sigma\cdot\mu)\cdot i_{p_2}(\tau\cdot\nu).$$

$\ulcorner$

$[\sigma_0^\circ, \sigma_1^\circ, \cdots \sigma_k^\circ]\,[\tau_0^\circ, \tau_1^\circ, \cdots \tau_{n-k}^\circ], [\mu_0^\circ \cdots \mu_{n-k}^\circ]\,[\nu_0^\circ \cdots \nu_k^\circ]$

$\Rightarrow$ $[\sigma_1^\circ-\sigma_0^\circ, \cdots \sigma_k^\circ-\sigma_0^\circ]$ can be considered as an orientation for σ.
$[\tau_1^\circ-\tau_0^\circ, \cdots \tau_{n-k}^\circ-\tau_0^\circ]$ " " " τ
$[\mu_1^\circ-\mu_0^\circ, \cdots \mu_{n-k}^\circ-\mu_0^\circ]$ " " " μ
$[\nu_1^\circ-\nu_0^\circ, \cdots \nu_k^\circ-\nu_0^\circ]$ " " " ν.

where, $\sigma_i^\circ-\sigma_0^\circ$ $\tau_j^\circ-\sigma_0^\circ$ represent vectors.

$$\Rightarrow [\sigma_1^\circ-\sigma_0^\circ \cdots \sigma_k^\circ-\sigma_0^\circ,\ \tau_1^\circ-\tau_0^\circ, \cdots \tau_{n-k}^\circ-\tau_0^\circ,\ \mu_1^\circ-\mu_0^\circ \cdots \nu_1^\circ-\nu_0^\circ \cdots \nu_k^\circ-\nu_0^\circ]$$

$$= [\sigma_1^\circ-\sigma_0^\circ \cdots \sigma_k^\circ-\sigma_0^\circ,\ \mu_1^\circ-\mu_0^\circ \cdots \mu_{n-k}^\circ-\mu_0^\circ,\ \tau_1^\circ-\tau_0^\circ, \cdots \tau_{n-k}^\circ-\tau_0^\circ,\ \nu_1^\circ-\nu_0^\circ, \nu_2^\circ-\nu_0^\circ, \cdots \nu_k^\circ-\nu_0^\circ]$$
$$\times\ (-1)^{(n-k)^2} = i_{p_1}(\sigma\cdot\mu)\ i_{p_2}(\tau\cdot\nu)$$

$$\Rightarrow\ i_{(p_1,p_2)}(\sigma\times\tau, \mu\times\nu) = (-1)^{(n-k)^2}\, i_{p_1}(\sigma\cdot\mu)\ i_{p_2}(\tau\cdot\nu) = (-1)^{n-k}\, i_{p_1}(\sigma\cdot\mu)\cdot i_{p_2}(\tau\cdot\nu).$$

Thus, $\#(\sigma\times\tau, \mu\times\nu) = (-1)^{n-k}\, \#(\sigma\cdot\mu)\cdot\#(\tau\cdot\nu)$

$$= (-1)^{n-k} \int_{\mu\times\nu} \pi_1^*\varphi \wedge \pi_2^*\psi;$$