The exterior derivative on smooth forms induces

$$d : \mathcal{D}^q(\mathbb{R}^n) \longrightarrow \mathcal{D}^{q+1}(\mathbb{R}^n).$$

defined by
$$(dT)(\varphi) = (-1)^{q+1}\, T(d\varphi), \qquad \varphi \in A_c^{n-q-1}(\mathbb{R}^n).$$
Then $d^2 = 0$.

「 $(d\,d\,T)(\varphi) = \pm\, dT(d\varphi) = \pm\, T(d^2\varphi) = 0. \Rightarrow d^2 T = 0$ 」

If $T = T_\psi$ for some smooth form $\psi \in A^q(\mathbb{R}^n)$, by Stokes' theorem

$$\begin{aligned} dT_\psi(\varphi) &= (-1)^{q+1} \int_{\mathbb{R}^n} \psi \wedge d\varphi \\ &= -\int_{\mathbb{R}^n} d(\psi\wedge\varphi) + \int_{\mathbb{R}^n} d\psi \wedge \varphi \\ &= T_{d\psi}(\varphi). \end{aligned}$$

「 $(dT_\psi)(\varphi) = (-1)^{q+1} T_\psi(d\varphi) = (-1)^{q+1} \int_{\mathbb{R}^n} \psi \wedge d\varphi$ (by Example 1)

$$\Rightarrow\quad d(\psi\wedge\varphi) = d\psi\wedge\varphi + (-1)^q\, \psi\wedge d\varphi$$
$$\Rightarrow\quad (-1)^{q+1}\, \psi \wedge d\varphi = d\psi\wedge\varphi - d(\psi\wedge\varphi)$$
$$\Rightarrow\quad (-1)^{q+1} \int_{\mathbb{R}^n} \psi\wedge d\varphi = \int_{\mathbb{R}^n} d\psi\wedge\varphi - \int_{\mathbb{R}^n} d(\psi\wedge\varphi).$$ by Stokes' theorem
$$= \int_{\mathbb{R}^n} d\psi \wedge \varphi = T_{d\psi}(\varphi).$$ 」

Similarly, for T_Γ as in the second example,

$$\begin{aligned} dT_\Gamma(\varphi) &= (-1)^{q+1} \int_\Gamma d\varphi \\ &= (-1)^{q+1} \int_{\partial\Gamma} \varphi \\ &= (-1)^{q+1}\, T_{\partial\Gamma}(\varphi). \end{aligned}$$