

$$\sigma_a(V) = \{ \Lambda : \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i \} \quad \text{and}$$

$$\sigma_b(V') = \{ \Lambda : \dim(\Lambda \cap V'_{n-k+i-b_{k-i+1}}) \geq i \}$$

be general Schubert cycles. Then for each  $i$  and any  $\Lambda \in \sigma_a(V) \cap \sigma_b(V')$ ,

$$\dim(\Lambda \cap V_{n-k+i-a_i}) \geq i,$$

$$\dim(\Lambda \cap V'_{n-k+(k-i+1)-b_{k-i+1}}) \geq k-i+1$$

$$\Rightarrow \Lambda \cap V_{n-k+i-a_i} \cap V'_{n-i+1-b_{k-i+1}} \neq (0).$$

Recall that.  $\dim(V+W) = \dim V + \dim W - \dim(V \cap W)$ .

$$\Rightarrow \dim(V \cap W) = \dim V + \dim W - \dim(V+W).$$

$$\begin{aligned} \dim(\Lambda \cap V_{n-k+i-a_i} \cap V'_{n-i+1-b_{k-i+1}}) &= \dim\{(\Lambda \cap V_{n-k+i-a_i}) \cap (\Lambda \cap V'_{n-i+1-b_{k-i+1}})\} \\ &= \dim(\Lambda \cap V_{n-k+i-a_i}) + \dim(\Lambda \cap V'_{n-i+1-b_{k-i+1}}) - \dim\{(\Lambda \cap V_{n-k+i-a_i}) + (\Lambda \cap V'_{n-i+1-b_{k-i+1}})\} \\ &\geq \dim(\Lambda \cap V_{n-k+i-a_i}) + k-i+1 - \dim \Lambda \geq i + k-i+1 - \dim \Lambda \\ &= k+1 - k = 1 \end{aligned}$$

$$\Rightarrow \Lambda \cap V_{n-k+i-a_i} \cap V'_{n-i+1-b_{k-i+1}} \neq (0).$$

But now if  $a_i + b_{k-i+1} > n-k$ , we have

$$\begin{aligned} (n-k+i-a_i) + (n-i+1-b_{k-i+1}) &= 2n-k+1-(a_i+b_{k-i+1}) \\ &\leq n, \end{aligned}$$

and so we can choose our flags  $V$  and  $V'$  such that  $V_{n-k+i-a_i}$  and  $V'_{n-i+1-b_{k-i+1}}$  intersect only at the origin.

$$\begin{aligned} n-k+i-a_i + n-i+1-b_{k-i+1} &= 2n-k-1+2-(a_i+b_{k-i+1}) \\ &\leq 2n-k-1+2-(n-k+1) = n \end{aligned}$$