

A natural question to ask is whether we can characterize geometrically the classes in homology that are Poincare dual to classes in one of these factors. For example, we say a homology class $\gamma \in H_{2p}(M, \mathbb{Z})$ is analytic if it is a rational linear combination of fundamental classes of analytic subvarieties of M ; we say a cohomology class is analytic if ^{dually} its Poincare dual is. Now, we have seen for purely local reasons that if $V \subset M$ is an analytic subvariety of dimension $n-p$ and ψ any differential form on M ,

$$\int_V \psi = \int_V \psi^{n-p, n-p}.$$

(See P23 for notation and P32 for evidence, $V \subset M$ is an analytic subvariety of dimension $n-p$ and ψ any differential form on M .))

Thus if η is the harmonic form on M representing the cohomology class η_V and ψ any harmonic form,

$$\int_M \psi \wedge \eta = \int_V \psi = \int_V \psi^{n-p, n-p} = \int_M \psi \wedge \eta^{p,p}.$$

i.e. $\eta = \eta^{p,p}$, and so we see that any analytic cohomology class of degree $2p$ is of pure type (p, p) .

$$\Gamma \int_M \psi \wedge \eta = \int_V \psi = \int_V \psi^{n-p, n-p} \text{ see p.59.}$$