

$$\Rightarrow \frac{\partial}{\partial \bar{z}} W = \sum \left(\delta_{ij} + \sum_k (a_{ij\bar{k}} w_k + a_{ij\bar{k}} \bar{w}_k + b_{j\bar{k}i} w_k + \bar{b}_{i\bar{k}j} \bar{w}_k) \right)$$

$$\wedge d\bar{w}_i \wedge d\bar{w}_j + [1].$$

$$\Rightarrow a_{ij\bar{k}} = -b_{j\bar{k}i} \Rightarrow b_{j\bar{k}i} = -a_{ij\bar{k}} = -a_{k\bar{j}i} = b_{j\bar{i}k}$$

\Downarrow This satisfies the normalization.

$$\text{and } \bar{b}_{i\bar{k}j} = -\overline{a_{j\bar{k}i}} = -a_{ij\bar{k}}$$

So that the coordinate change does in fact satisfy the condition (*). Q.E.D. \Downarrow

Remark: I think [2] should be [1] or [2] means that the term vanishes up to order 1 not order 2.

See Wells. (P. 80. Lemma 2.3 Differential Analysis on Complex Manifolds.)

$$\Rightarrow h(z) = I + O(|z|^2)$$

Here $O(|z|^2)$ means that $|O(|z|^2)| \leq C|z|^2$ for some $C > 0$.

$o(|z|^2)$ means that $\lim_{|z| \rightarrow 0} \frac{o(|z|^2)}{|z|^2} = 0$

See Hardy. (P. 164. 89. Symbols O, o A course of Pure Mathematics.)

Another way of expressing this condition that is useful in computation is to say that for each point $z \in M$, we can find a unitary coframe $\varphi_1, \dots, \varphi_n$ for the metric in some nbd of z_0 s.t. $d\varphi_i(z_0) = 0$.

Since we have a coordinate chart (z) s.t. $\langle \frac{\partial}{\partial z_i}, \frac{\partial}{\partial z_j} \rangle = \delta_{ij}$, applying Gram-Schmidt process to $\langle z \rangle$, we get $\varphi_i \Rightarrow \varphi_i(z_0) = 0$ since