This says that the intersection
$$T_x(X) \cap T_y(X) = \bigcap_{x \in L} T_x(X)$$

will be two-dimensional for any pair of distinct points $x, y \in L$ if and only if it is two-dimensional for all pairs $x, y \in L$; in other words, the line L will be a multiple component of the intersection $T_x(X) \cap X$ for some $x \in L$ if and only if it is for all $x \in L$, if and only if the locus
$$\bigcap_{x \in L} T_x(X)$$

contains a 2-plane.

$\ulcorner$ $T_x(X) \cap T_y(X) = \bigcap_{x \in L} T_x(X) = T_{x'}(X) \cap T_{y'}(X)$
in other words, L is multiple $\Leftrightarrow$ the intersection of $T_x(X)$ with X fails to be transverse along L
$\Leftrightarrow$ for $x \neq y \in L$, $T_x(X) \cap T_y(X)$ is two-dimensional, since $T_x(X) = T_y(X)$ can not happen. See P981 note.
$\Leftrightarrow$ For any $y \in L$, $T_y(X) \cap T_z(X)$ is two-dimensional
$\Leftrightarrow$ $T_y(X) \cap X$ has a multiple component L.
$\lrcorner$

In this case, all the lines $\{l_x\}_{x \in L}$ of the corresponding pencil will be tangent to S; thus we see that

A line l_x of the complex X, other than a singular