since $\Delta\eta$ is $C^\infty$, and so

$$\int_{\|x\|\leq\epsilon} \Delta\eta = O(\epsilon^n);$$

and where

$$B_\epsilon = \text{constant} \int_{\|x\|=\epsilon} \eta\sigma$$

for a suitable choice of the constant $C_n$. This proves the Poisson formula, and hence the lemma..

$\Gamma$

$$\lim_{\epsilon\to 0} \int_{\mathbb{R}^n - \{\|x\|\leq\epsilon\}} \frac{\Delta\eta(x)\, dx}{\|x\|^{n-2}} = \lim_{\epsilon\to 0} \int_{\mathbb{R}^n - \{\|x\|\leq\epsilon\}} - d\left(\frac{*d\eta}{r^{n-2}}\right) - \left(\frac{C_n}{n-2}\right)^{-1}$$

$$d(\eta\sigma) = \lim_{\epsilon\to 0} \int_{\|x\|=\epsilon} \frac{*d\eta}{\epsilon^{n-2}} + \frac{n-2}{C_n} \lim_{\epsilon\to 0} \int_{\|x\|=\epsilon} \eta\sigma$$

$$= \lim_{\epsilon\to 0} A_\epsilon + \lim_{\epsilon\to 0} B_\epsilon .$$

$$A_\epsilon = \frac{1}{\epsilon^{n-2}} \int_{\|x\|=\epsilon} * d\eta = \frac{1}{\epsilon^{n-2}} \int_{\|x\|\leq\epsilon} d(*d\eta)$$

$$= \frac{1}{\epsilon^{n-2}} \int_{\|x\|\leq\epsilon} -\Delta\eta \; dx$$

$$\Rightarrow \quad |A_\epsilon| \leq \frac{1}{\epsilon^{n-2}} \int_{\|x\|\leq\epsilon} |\Delta\eta|\, dx \leq \frac{1}{\epsilon^{n-2}} \int_{\|x\|\leq\epsilon} M \; dx$$

$$\leq \frac{1}{\epsilon^{n-2}} M \, C_n \, \epsilon^n = C_n M \epsilon^2 \quad \Rightarrow \quad \text{As } \epsilon\to 0,\ A_\epsilon \to 0.$$

where $|\Delta\eta| \leq M$..