Now $m_{x_0}\mathcal{F}(k) = (m_{x_0}\mathcal{F})(k) = \mathcal{G}(k)$, where $\mathcal{G} = m_{x_0}\mathcal{F}$ is a coherent sheaf.

Γ By P697 & P612 note, m is coherent $\Rightarrow$ m_{x_0} is coherent by the argumet above. Clearly $m \times \mathcal{F}$ is a coherent sheaf. $\Rightarrow$

$$m \times \mathcal{F} \xrightarrow{\text{onto}} (m_{x_0}, \mathcal{F}_{x_0}) \xrightarrow{\text{onto}} m_{x_0}\mathcal{F}_{x_0} \to 0$$

$\Rightarrow$ $m_{x_0}\mathcal{F}_{x_0} = m_{x_0}\mathcal{F}$ is a coherent sheaf. ⅃

Using $H^1(M, \mathcal{G}(k)) = 0$ for $k \geq k_0$, we deduce that the gl obal sections $H^0(M, \mathcal{F}(k))$ generate the fiber of $\mathcal{F}(k)$ at x_0 for $k \geq k_0$.

Γ $H^0(M, \mathcal{G}(k)) \to H^0(M, \mathcal{F}(k)) \to H^0(M, \mathcal{F}(k)_{x_0}/m_{x_0}\mathcal{F}(k))$
$\to H^1(M, \mathcal{G}(k))$
$\quad = 0$ for $q > 0$ & for $k \geq k_0$.

$\Rightarrow$ $H^0(M, \mathcal{F}(k)) \longrightarrow H^0(M, \mathcal{F}(k)_{x_0}/m_{x_0}\mathcal{F}(k))$ onto

$\Rightarrow$ $H^0(M, \mathcal{F}(k))$ generate the fiber of $\mathcal{F}(k)$ at x_0, since the fiber of $\mathcal{F}(k)$ at x_0 = $\mathcal{F}(k)_{x_0}/m_{x_0}\mathcal{F}(k)$
$= H^0(M, \mathcal{F}(k)_{x_0}/m_{x_0}\mathcal{F}(k))$. ⅃

By the Nakayama lemma, these global sections gen erate the $\mathcal{O}_{x_0}$-module $\mathcal{F}(k)_{x_0}$ for $k \geq k_0$. By Oka's lemma. they generate the $\mathcal{O}_x$-modules $\mathcal{F}(k)_x$ for x near to x_0. The result now follows by the compact ness of M.