First of all, we have to notice the following fact. (See p 211. M. Greenberg. (25.11))

Given an open cover $\{V_\alpha\}$ of M, $\exists$ an open cover $\{U_\alpha\} \cup \{N - M\}$ of N.

Consider the collection of open covers $\underset{V}{\mathfrak{U}}$ of N choosen as above. Then by (25.11),

$$\varinjlim_{\mathfrak{U}} H^p(\mathfrak{U}, \mathcal{F}) = H^p(N, \mathcal{F}).$$

Clearly $H^p(\mathfrak{U}, \mathcal{F}) = H^p(\mathfrak{V}, \mathcal{F}) \Rightarrow \varinjlim_{\mathfrak{U}} H^p(\mathfrak{U}, \mathcal{F}) = \varinjlim_{\mathfrak{V}} H^p(\mathfrak{V}, \mathcal{F}) = H^p(M, \mathcal{F})$.

The definition of $H^*(M, \mathcal{F})$ as a direct limit is, in practice, more or less impossible to work with.
What is needed is a simple sufficient condition on a cover $\mathfrak{U}$ for

$$H^*(\mathfrak{U}, \mathcal{F}) = H^*(M, \mathcal{F})$$

and this is provided by the

Leray Theorem: If $\mathfrak{U}$ is acyclic for the sheaf $\mathcal{F}$ in the sense that

$$H^q(U_{i_1} \cap \cdots \cap U_{i_p}, \mathcal{F}) = 0 \quad q > 0 \text{ for any } i_1 \cdots i_p.$$

then $H^*(\mathfrak{U}, \mathcal{F}) \cong H^*(M, \mathcal{F})$.

We will prove the Leray theorem in those cases where it will be used.