respectively, i.e.,

$$\begin{tikzcd} {[\tilde{C}]} \supset {[\tilde{C}]}|_U \arrow[d] \arrow[r] & U \times \mathbb{C} \arrow[dl, bend left] \\ \tilde{\mathbb{P}}^2 \supset U \arrow[u, bend right, "\tilde{\sigma}_i"'] \arrow[ur, "(x',\ f_i(x'))"'] & \end{tikzcd}$$

$$\underset{x'}{\psi}$$

Let $b_1 f_1 + \cdots + b_4 f_4$.

$\Rightarrow$ At p, $\quad b_1 f_1 + \cdots + b_4 f_4 = 0 \quad [(f_1(p), f_2(p), f_3(p), f_4(p))] \in \mathbb{P}^3$

Choose a coordinate system (x, y) on U.

$$b_1 f_1(x, g(x)) + b_2 f_2(x, g(x)) + \cdots + b_4 f_4(x, g(x)) = 0$$

$$\Rightarrow b_1 \nabla f_1(p)\cdot(1, g'(x)) + b_2 \nabla f_2(p)\cdot(1, g'(x)) + \cdots = 0$$

$$\Rightarrow (b_1 \nabla f_1(p) + \cdots + b_4 \nabla f_4(p)) \cdot (1, g'(x)) = 0$$

$\Rightarrow$ In case q is infinitely near p,

$$(b_1 \nabla f_1(p) + \cdots + b_4 \nabla f_4(p)) \cdot q = 0$$

If q is represented by (q_1, q_2),

$$0 = b_1\left(\frac{\partial f_1}{\partial x} q_1 + \frac{\partial f_1}{\partial y} q_2\right) + b_2\left(\frac{\partial f_2}{\partial x} q_1 + \frac{\partial f_2}{\partial y} q_2\right) + \cdots + b_4\left(\frac{\partial f_4}{\partial x} q_1 + \frac{\partial f_4}{\partial y} q_2\right).$$

$\Rightarrow$ Since p and q impose linearly independent conditions,

$(f_1(p), \cdots f_4(p))$ & $\left(\frac{\partial f_1}{\partial x} q_1 + \frac{\partial f_1}{\partial y} q_2, \cdots, \frac{\partial f_4}{\partial x} q_1 + \frac{\partial f_4}{\partial y} q_2\right)$

are linearly independent. --- ⊛

On the other hand

$$\iota_{\tilde{C}}(x') = [(f_1(x'), f_2(x'), f_3(x'), f_4(x'))].$$

$$\Rightarrow \iota_{\tilde{C}}(x, y) = \left(\frac{f_2(x, y)}{f_1}, \frac{f_3}{f_1}, \frac{f_4}{f_1}\right) \quad \text{locally}$$