$\ulcorner$ $K : A_c^{n,\, n-q+1}(\mathbb{C}^n) \longrightarrow A^{n,\, n-q}(\mathbb{C}^n).$

$$K(z,w) = C_n \frac{\sum \overline{\Phi_i(z-w)} \wedge \Phi(w)}{\|z-w\|^{2n}}$$

$$\varphi \in A_c^{n,\, n-q+1}(\mathbb{C}^n) \Rightarrow \varphi = \sum_{\substack{\#I=n \\ \#J=n-q+1}} \varphi_{I\bar{J}}\, dz_I \wedge d\bar{z}_J$$

$$= \sum_{\#J=n-q+1} \varphi_{\bar{J}}\, dz \wedge d\bar{z}_J \quad , \; dz = dz_1 \wedge \cdots \wedge dz_n$$

$$\Rightarrow (K\varphi)(z) = \pm \Phi(z) \wedge \int_{w \in \mathbb{C}^n} K(z,w) \wedge \sum_{\#J=n-q+1} \varphi_{\bar{J}} \wedge d\bar{z}_J$$

$\Rightarrow$ Everything fits well each other. $\Rightarrow$ $K\varphi$ makes sense and we can write out the analogues of $K\varphi$ easily. Do it! $\lrcorner$

Then $KT \in \mathcal{D}^{0,\, q-1}(\mathbb{C}^n)$, and for a test form $\psi \in A_c^{n,\, n-q}(\mathbb{C}^n)$,

$$(\bar\partial(KT))(\psi) + (K(\bar\partial T))(\psi) = (KT)(\bar\partial\psi) + (\bar\partial T)(K\psi)$$

$$= T(K\bar\partial\psi + \bar\partial K\psi)$$

$$= T\psi,$$

so that, with this interpretation, the homotopy formula makes sense for compactly supported currents.