

We note that the function r^{-s} is locally integrable for $s < n$ but not for $s = n$.

By P136, Rudin's F.A., a complex function f is said to be locally integrable if f is measurable and $\int_K |f| < \infty$ for every compact $K \subset \mathbb{R}^n$.

Given a compact $K \subset \mathbb{R}^n$, \exists a ball $B(0, l)$ s.t. $B(0, l) \supset K$.

$$\int_{B(0, l)} r^{-s} dx_1 \wedge \dots \wedge dx_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n r_k^{-s} \Delta r \text{ (Volume of the sphere with radius } r_k) = \int_0^l r^{-s} C_n r^{n-1} dr$$

Since the volume of the sphere with radius r_k $= C_n r_k^{n-1}$, for example, $n=2$, $2\pi r_k$, $n=3$, $4\pi r_k^2$.

$$\Rightarrow C_n \int_0^l r^{-s} r^{n-1} dr = C_n \int_0^l r^{n-s-1} dr < \infty$$

if $n-s-1 \geq 0$, otherwise ∞ .

\Rightarrow If $s \leq n-1$, i.e. $s < n$, then r^{-s} is locally integrable and if $s = n$, r^{-s} is not locally integrable. \Downarrow

Define

$$\begin{aligned} \sigma &= C_n \frac{\sum \Phi_i(x)}{\|x\|^n} \\ &= C_n \frac{*r dr}{r^n} \end{aligned}$$

Recall $r dr \wedge *r dr = \langle r dr, r dr \rangle dx_1 \wedge \dots \wedge dx_n$
 $= \langle \sum x_i dx_i, \sum x_i dx_i \rangle dx_1 \wedge \dots \wedge dx_n$ by P82.
 $= \sum x_i^2 dx_1 \wedge \dots \wedge dx_n = r^2 dx_1 \wedge \dots \wedge dx_n$
 $r dr \wedge \sum \Phi_i(x) = \sum x_i dx_i \wedge \sum (-1)^{i-1} x_i dx_1 \wedge \dots \wedge \widehat{dx_i} \wedge \dots \wedge dx_n$