$$g_{i0}\, g_{0j} = g_{ij} = \frac{z_i}{z_j} \quad \Rightarrow \sigma \text{ is the global section of } [E] \text{ on } \tilde{M}_x \text{ with } (\sigma = 0) = E.$$
$$\overset{\shortparallel}{z_i \cdot \frac{1}{z_j}}$$

$[E]\big|\tilde{\mathcal{U}} = \{((z, \ell), v) \in \tilde{\mathcal{U}} \times \mathbb{C}^n ;\ v \in \ell\}$
See P145, & on P466 in note.
$\Rightarrow$ We give a metric h_1 on $[E]$ induced by the natural metric on $\tilde{\mathcal{U}} \times \mathbb{C}^n$, i.e. $\|v = (v_1 \cdots v_n)\|$
$= \sqrt{v_1^2 + \cdots + v_n^2}$. ⌟

For $\epsilon > 0$, denote by $\mathcal{U}_\epsilon$ the ball $(\|z\| < \epsilon)$ around x in $\mathcal{U}$ and set $\tilde{\mathcal{U}}_\epsilon = \pi^{-1}(\mathcal{U}_\epsilon)$; let ρ_1, ρ_2 be a partition of unity for the cover $\{\tilde{\mathcal{U}}_{2\epsilon}, \tilde{M} - \tilde{\mathcal{U}}_\epsilon\}$ of M, and let h be the global metric given by
$$h = \rho_1 \cdot h_1 + \rho_2 \cdot h_2 .$$

⌜ $\operatorname{supp} \rho_1 \subset \tilde{\mathcal{U}}_{2\epsilon} \Rightarrow \rho_1 = 0$ on $M - \tilde{\mathcal{U}}_{2\epsilon}$. $\Rightarrow \rho_2 = 1$.
$\operatorname{supp} \rho_2 \subset \tilde{M} - \tilde{\mathcal{U}}_\epsilon \Rightarrow \rho_2 = 0$ on $\tilde{\mathcal{U}}_\epsilon \Rightarrow \rho_1 = 1$ ⌟

We will compute the curvature of $[E]$ with this metric. For notational convenience, let $\Omega_{[E]}$ denote $i/2\pi$ times the curvature $\Theta_{[E]}$ of $[E]$. It is necessary to consider three cases:

1. On $\tilde{M}_x - \tilde{\mathcal{U}}_{2\epsilon}$, $\rho_2 \equiv 1$ so $|\sigma|^2 \equiv 1$; consequently
$$\Omega_{[E]} = dd^c \log \frac{1}{|\sigma|^2} \equiv 0 .$$

⌜ From the above, on $\tilde{M} - \tilde{\mathcal{U}}_{2\epsilon}$, $\rho_1 = 0$ and $\rho_2 = 1$.
$\Rightarrow h = h_2 \Rightarrow |\sigma| \equiv 1$. Actually, we put $\tilde{\mathcal{U}}_0 = \tilde{\mathcal{U}}$ in the previous argument.