$$v_3 = (\alpha_{31}-\alpha_{30})x_1\frac{\partial}{\partial x_1} + (\alpha_{32}-\alpha_{30})x_2\frac{\partial}{\partial x_2} + (\alpha_{33}-\alpha_{30})x_3\frac{\partial}{\partial x_3}$$

For example,

$$A = \begin{pmatrix} 10, & 100, & 1000 \\ 1, & 2, & 4 \\ -2, & -3, & -5 \end{pmatrix}$$

$\Rightarrow$ All 1×1 minor are distinct and nonzero.

$$\begin{vmatrix} 10, & 100 \\ 1. & 2 \end{vmatrix} = 20-100 = -80. \qquad \begin{vmatrix} 10, & 1000 \\ 1, & 4 \end{vmatrix} = 40-1000 = -960.$$

$$\begin{vmatrix} 100, & 1000 \\ 2. & 4 \end{vmatrix} = 400-2000 = -1600 \qquad \begin{vmatrix} 10, & 100 \\ -2, & -3 \end{vmatrix} = -30+200 = 170$$

$$\begin{vmatrix} 10, & 1000 \\ -2, & -5 \end{vmatrix} = -50+2000 = 1950 \qquad \begin{vmatrix} 100, & 1000 \\ -3, & -5 \end{vmatrix} = -50\overset{0}{\wedge}+3000 = 2500$$

$$\begin{vmatrix} 1. & 2 \\ -2, & -3 \end{vmatrix} = -3+4 = 1. \qquad \begin{vmatrix} 1. & 4 \\ -2, & -5 \end{vmatrix} = -5+8 = 3 \qquad \begin{vmatrix} 2. & 4 \\ -3. & -5 \end{vmatrix} = -10+12 = 2.$$

$$10\begin{vmatrix} 2, & 4 \\ -3, & -5 \end{vmatrix} - 100\begin{vmatrix} 1, & 4 \\ -2, & -5 \end{vmatrix} + 1000\begin{vmatrix} 1. & 2 \\ -2, & -3 \end{vmatrix} = 10\cdot 2 - 100\cdot 3$$

$$+ 1000\cdot 1 = 20-300+1000 = 720.$$

$\Rightarrow$ All 2×2 minor determinants are distinct and nonzero.

$\Rightarrow$ $v_1 = (2-1)x_1\frac{\partial}{\partial x_1} + (4-1)x_2\frac{\partial}{\partial x_2} = x_1\frac{\partial}{\partial x_1} + 3x_2\frac{\partial}{\partial x_2}$

$v_2 = (-3+2)x_1\frac{\partial}{\partial x_1} + (-5+2)x_2\frac{\partial}{\partial x_2}$

$= -x_1\frac{\partial}{\partial x_1} - 3x_2\frac{\partial}{\partial x_2}$

$\Rightarrow$ v_1 & v_2 are linearly dependent on $\mathbb{P}^2$.

I think we may change A as follows: