where $\bar{\partial}\lambda$ means the derivative in the direction λ.

$\Rightarrow$ By the Cauchy integral formula

$$-\int_{|z|\leq r} \bar{\partial} f \wedge dz = -\int_{|z|=r} \frac{f(z)\, dz}{z} + 2\pi\sqrt{-1}\, f(0),$$

let $f(z) = \int_{\mathbb{P}^{n-1}} (\pi'^{*}\varphi)(z, l) \wedge \pi'^{*}((p^{*}\Omega)^{n-1})$.

$$\Rightarrow \quad f(0) = \int_{\mathbb{P}^{n-1}} (\pi'^{*}\varphi)(0, l) \wedge \pi'^{*}((p^{*}\Omega)^{n-1})$$

$$= \int_{\mathbb{P}^{n-1}} \varphi(0)\, \pi'^{*}((p^{*}\Omega)^{n-1}) = \varphi(0) \text{ volume of } \mathbb{P}^{n-1} = \varphi(0)$$

by P31. & P150

$\mathrm{Vol}(\mathbb{P}^{n-1}) = 1$. $\int_{\mathbb{P}^n} \omega^n = 1$.

P122

$$-\int_{|\lambda|=r} \frac{d\lambda}{\lambda} \int_{\mathbb{P}^{n-1}} (\pi'^{*}\varphi)(z, l) \wedge \pi'^{*}((p^{*}\Omega)^{n-1})$$

$$+ 2\pi\sqrt{-1}\, \varphi(0) = -\int_{B[r]} \bar{\partial}\varphi \wedge \beta$$

Since $-\int_{|\lambda|=r} \frac{d\lambda}{\lambda} \int_{\mathbb{P}^{n-1}} (\pi'^{*}\varphi)(z, l) \wedge \pi'^{*}((p^{*}\Omega)^{n-1})$

$$= -\int_{\partial B[r]} \varphi\, \beta,$$

$$-\int_{B[r]} \bar{\partial}\varphi \wedge \beta = -\int_{\partial B[r]} \varphi\, \beta + 2\pi\sqrt{-1}\, \varphi(0).$$

$$\Rightarrow \quad 2\pi\sqrt{-1}\, \varphi(0) = -\int_{B[r]} \bar{\partial}\varphi \wedge \beta + \int_{\partial B[r]} \varphi\, \beta.$$

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