

$$\Rightarrow T_x(X) = \{ X \in \mathbb{P}^5 \mid \sum X_i x_i = 0 \text{ and } \sum X_i \lambda_i x_i = 0 \}$$

$$\text{and } T_y(X) = \{ X \in \mathbb{P}^5 \mid \sum X_i y_i = 0 \text{ and } \sum X_i \lambda_i y_i = 0 \}.$$

$$\Rightarrow \langle x, (\lambda_0 x_0, \dots, \lambda_5 x_5) \rangle = \langle y, (\lambda_0 y_0, \dots, \lambda_5 y_5) \rangle$$

$$\text{since } T_x(X) = T_y(X).$$

$$\Rightarrow y = \alpha x + \beta (\lambda_0 x_0, \dots, \lambda_5 x_5)$$

$$(\lambda_0 y_0, \dots, \lambda_5 y_5) = \alpha' x + \beta' (\lambda_0 x_0, \dots, \lambda_5 x_5).$$

$$\Rightarrow y_i = \alpha x_i + \lambda_i \beta x_i \text{ \& } \lambda_i y_i = (\alpha' + \lambda_i \beta') x_i$$

$$\Rightarrow \alpha + \lambda_i \beta = (\alpha' + \lambda_i \beta') \lambda_i$$

$$\Rightarrow \alpha \lambda_i + \lambda_i^2 \beta = \alpha' + \lambda_i \beta' \Rightarrow \beta \lambda_i^2 + (\alpha - \beta') \lambda_i - \alpha' = 0, \quad (*)$$

$$\text{for, if } \lambda_i = \lambda_0, \text{ then } (X_0^2 + X_1^2 + \dots + X_5^2 = 0) \cap (\lambda_0 X_0^2 + \lambda_1 X_1^2 + \dots + \lambda_5 X_5^2 = 0) \ni (1, i, 0, 0, 0, 0) = z$$

$$\Rightarrow T_z(X) = T_z G = T_z H = (X_0 + i X_1 = 0) \Rightarrow \text{All } \lambda_i \text{'s are distinct.}$$

\Rightarrow We have a contradiction since $(*)$ must be valid for six λ_i 's ($\because (*)$ is quadratic).

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Let $\tilde{D} \subset A$ be the set of special lines of X , and

$$\Delta = \bigcup_{L \in \tilde{D}} L \subset X$$

the locus of all special lines. We can also write $\Delta = \{ x \in X : T_x(X) \cap X \text{ contains fewer than four lines} \}$

and

$$\{ x \in X : l_x \text{ is tangent to } S \} = \Sigma \cup \Delta.$$