$\Rightarrow$ The degree of the intersection of two varieties meeting transversely is the product of their degrees.

By P65, the topological intersection of two analytic varieties V and W is given by

$$V \cdot W = \sum_{Z_i \subset V \cap W} \mathrm{mult}_{Z_i}(V \cdot W) \cdot Z_i .$$

$$\Rightarrow \#((V \cdot W) \cdot \mathbb{P}^{n-k_1-k_2}) = \#\Big(\sum_{Z_i \subset V \cap W} \mathrm{mult}_{Z_i}(V \cdot W) Z_i \cdot \mathbb{P}^{n-k_1-k_2}\Big)$$
$$= \sum_{Z_i \subset V \cap W} \mathrm{mult}_{Z_i}(V \cdot W) \#(Z_i \cdot \mathbb{P}^{n-k_1-k_2}) = \sum_{Z_i \subset V \cap W} \mathrm{mult}_{Z_i}(V \cdot W) \,\mathrm{degree}(Z_i)$$

92, 12.30.

This is of particular interest in the case of complementary dimension. For example, if C and D are two curves in $\mathbb{P}^n$ of degree d_1 and d_2 and having no component in common — that is, intersecting only in points — we see that they can have at most $d_1 d_2$ points of intersection.

┌ We count double points one.

This is a weak form of

Bezout's theorem. Two relatively prime polynomials $f(x,y)$, $g(x,y) \in \mathbb{C}[x,y]$ of degree d_1 & d_2 can have at most $d_1 d_2$ simultaneous solutions.

┌ C, D curves of $\mathbb{P}^2$ $\Rightarrow$ $C = \{f(X_0, X_1) = 0\}$ $D = \{g(X_0, X_1) = 0\}$ where f & g are homogeneous polynomials. Since f & g are relatively prime, $C \cap D$ is a set of discrete points.