sheaves

$$\Omega^n \xrightarrow{\iota(v)} \Omega^{n-1} \to \cdots \to \Omega^{(1)} \xrightarrow{\iota(v)} \mathcal{O}.$$

$\iota(v) : \Omega^p \longrightarrow \Omega^{p-1}$
$(\overset{\shortparallel}{A^{p,0}}) \quad (\overset{\shortparallel}{A^{p-1,0}})$.

We note that the image of $\Omega^1 \xrightarrow{\iota(v)} \mathcal{O}$ is the ideal sheaf I of Z: in fact, near a zero of v the above sequence is the Koszul complex associated to regular ideal $\{v_1(z), \cdots v_n(z)\}$.

$$\begin{array}{ccc} \Omega^1(U) & \xrightarrow{\iota(v)} & \mathcal{O}(U) \\ \psi & & \psi \\ dz_i & \longmapsto & v_i \end{array} \Rightarrow I_U = \{v_1, \cdots v_n\}.$$

$$\Omega^n(U) \xrightarrow{\iota(v)} \Omega^{n-1}(U) \longrightarrow \cdots \to \Omega^K(U) \xrightarrow{\iota(v)} \Omega^{K-1}(U) \to \cdots$$

$$dz_J \longmapsto \sum_{\alpha=1}^{K} (-1)^{\alpha-1} v_{j_\alpha} dz_{J-\{j_\alpha\}} = \sum (-1)^{\alpha-1} v_{j_\alpha} dz_{j_1} \wedge \cdots \wedge \widehat{dz_{j_\alpha}} \wedge \cdots \wedge dz_{j_K}$$

$v(0) = (v_1(0), \cdots) = 0$

Since 0 is an isolated point, $v^{-1}(0) = \{0\}$.

Consequently, we have a very natural global syzygy for $\mathcal{O}_Z = \mathcal{O}/I$, one which will be used to calculate global Ext.

For this we observe the commutative diagram

$$\begin{array}{ccc} \underline{\mathrm{Hom}}_{\mathcal{O}}(\Omega^p, \Omega^n) & \xrightarrow{\sim} & \Omega^{n-p} \\ \iota(v)^* \downarrow & & \downarrow \iota(v) \\ \underline{\mathrm{Hom}}_{\mathcal{O}}(\Omega^{p+1}, \Omega^n) & \xrightarrow{\sim} & \Omega^{n-p-1} \end{array}$$