

$\Rightarrow \phi$  is injective. Let  $(s_1=0)=p$ .  
 $\Rightarrow \frac{\omega}{s_1} \in H^0(S, \Omega'(-p))$ .  
 $\Rightarrow \exists$  no  $\sigma \in H^0(S, \Omega'(-p-q))$  s.t.  $\phi(\sigma) = \frac{\omega}{s_1}$ .  
 If so,  $\frac{\omega}{s_1} = \frac{\sigma}{s_0} \Rightarrow \frac{\omega(q)}{s_1(q)} \neq 0 = \frac{\sigma(q)}{s_0(q)} = 0$ .  
 $\Rightarrow$  Contradiction  $\Rightarrow \phi$  is not onto  $\Rightarrow$   
 $h^0(K-p) > h^0(K-p-q)$ .

②  $p=q$ .  $H^0(S, \Omega'(-2p)) \xrightarrow{\phi} H^0(S, \Omega'(-p))$   
 $\frac{\omega}{s_0} \longmapsto \frac{\sigma}{s_0}$

Let  $\omega \in H^0(S, \Omega')$  with a single zero at  $p$ .

Consider  $\frac{\omega}{s_1}$ . If  $\sigma \in H^0(S, \Omega'(-2p))$  s.t.

$$\phi(\sigma) = \frac{\omega}{s_1} = \frac{\sigma}{s_0} \Rightarrow \frac{\omega(p)}{s_1(p)} \neq 0 = \frac{\sigma(p)}{s_0(p)} = 0$$

$\Rightarrow$  Contradiction  $\Rightarrow h^0(K-p) > h^0(K-2p)$ .

$(\Leftarrow)$  ①  $p, q$  distinct,  $\exists \eta \in H^0(S, \Omega'(-p))$  s.t.  
 $\text{im } \phi \not\supset \eta$ .  $\Rightarrow \frac{\omega}{s_0} \neq \eta$  for all  $\eta$ .

Comment on  $H^0(S, \mathcal{O}(p))$ .  $\sigma \in H^0(S, \mathcal{O}(p))$  with  $\sigma(p) \neq 0$

$\Rightarrow \frac{\sigma}{s_0}$  is meromorphic on  $S$  with a single pole at  $p$  and holomorphic on  $S-3p$ .  $\Rightarrow S \cong \mathbb{P}^1$ , where  $(s_0=0)=p$ .

$\Rightarrow \sigma = 0$  at  $p \Rightarrow \frac{\sigma}{s_0}$  is holomorphic  $\Rightarrow \sigma = c\sigma'$

$\Rightarrow \dim H^0(S, \mathcal{O}(p)) = 1 = h^0(p) \Rightarrow$  By R-R,

$$h^0(K-p) = g-1.$$

If  $\exists$  no  $\eta \in H^0(S, \Omega'(-p))$  s.t.  $\eta(q) \neq 0$ ,

$\psi: H^0(S, \Omega'(-p)) \longrightarrow H^0(S, \Omega'(-p-q))$  given by