only if
$$Q(e_i, e_j) = b_{ij} + b_{ji} = 0$$
for all i and j, i.e., if and only if B is skew-symmetric.

Pf $Q(X) = \sum_{i=0}^{n} X_i X_{n+i+1} = \frac{1}{2}\left(\sum_{i=0}^{n} X_i X_{n+i+1} + \sum_{i=0}^{n} X_{n+i+1} X_i\right)$

$\Rightarrow Q = \begin{array}{cc} -n+1 & -n+1 \\ \left(\begin{array}{c|c} 0 & I \\ \hline I & 0 \end{array}\right) & \end{array}$ $Q(e_i) = 0$ for all i, since all e_i's lie on F.

$\Rightarrow \tilde{Q}(X, Y) = \frac{1}{2}\{\tilde{Q}(X+Y, X+Y) - \tilde{Q}(X, X) - \tilde{Q}(Y, Y)\}$
$\tilde{Q}(X, X) = Q(X)$.
$\Rightarrow \tilde{Q}(X, Y) = 0$ for all $X, Y \Leftrightarrow Q(Z) = 0$ for all Z.
$\Rightarrow \tilde{Q}(e_i, e_j) = 0$ for all i, j
$\Rightarrow$
$$(0 \cdots 0\, 1 \cdots 0, b_{i0} \cdots b_{in}) \left(\begin{array}{c|c} 0 & I \\ \hline I & 0 \end{array}\right) \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \\ b_{j0} \\ \vdots \\ b_{jn} \end{pmatrix} \begin{array}{c} \\ \\ j+1 \\ \\ \\ \\ \\ \\ \end{array}$$
$$= (b_{i0}, \cdots b_{in}, \underbrace{0 \cdots 1 \cdots 0}_{i+1}) \begin{pmatrix} 0 \\ \vdots \\ j \\ b_{j0} \\ \vdots \\ b_{jn} \end{pmatrix} = b_{ij} + b_{ji} = 0$$

⌟

The planes $\{\Lambda_B\}$ form an open set Γ_0 in one of the two families of n-planes on F. (Note that $\dim(\Lambda_B \cap \Lambda_{B'}) = n - \operatorname{rank}(B - B') \equiv n\ (2)$