$\Rightarrow f^*H = Q.$ ┘

Letting $\omega$ denote the class of a hyperplane in $Z_k \cong \mathbb{P}^k$, we have then

$$c(Q) = 1+\omega,$$

and since as $C^\infty$ bundles

$$V_{k+1} \times Z_k = S|_{Z_k} \oplus Q$$

$$1 = c(S|_{Z_k}) \cdot (1+\omega),$$

hence

$$c(S|_{Z_k}) = 1-\omega+\omega^2-\omega^3+\cdots,$$

i.e.,

$$c_r(S|_{Z_k}) = (-1)^r\omega^r.$$

∵

$$\begin{tikzcd} f^*H = Q \arrow[r] \arrow[d] & H \arrow[d] \\ Z_k = G(k,k+1) \arrow[r, "f"] & G(1,k+1) \end{tikzcd}$$

$\Rightarrow \quad c(Q) = 1+f^*\omega, \qquad \underset{c_1(H)}{\omega} \in H^2_{DR}(G(1,k+1))$

$\Rightarrow \quad 1 = c(S|_{Z_k}) \cdot (1+f^*\omega) = c(S|_{Z_k}) \cdot c(Q)$

$\Rightarrow \quad c(S|_{Z_k}) = 1 - f^*\omega + (f^*\omega)^2 - (f^*\omega)^3 + \cdots$

$\Rightarrow \quad c_r(S|_{Z_k}) = (-1)^r (f^*\omega)^r.$ ┘

Thus
$$\begin{aligned} c_r(S)(Z_r) &= c_r(S|_{Z_r}) \cdot (Z_r) \\ &= (-1)^r\omega^r(\mathbb{P}^r) \\ &= (-1)^r \end{aligned}$$

and the theorem is proved. Q.E.D.