$\sum x_i^2 + 2\sum x_i x_j$."

4. Fixed-Point and Residue Formulas

The Lefschetz Fixed-Point Formula

We now derive Lefschetz's formula for the number of fixed points, properly counted, of an endomorphism $f: M \to M$ on a compact oriented manifold M of dimension n in terms of the action of f^* on the cohomology of M. That such a formula should exist is not hard to see: a fixed point of f corresponds to a point of intersection of the graph $\Gamma_f \subset M \times M$ of f with the diagonal $\Delta \subset M \times M$, and as we have seen the intersection number $\#(\Gamma_f \cdot \Delta)_{M \times M}$ depends only on the homology classes of Γ_f and Δ in $M \times M$. ☞ See P51
Nor is the calculation itself difficult; it will come out readily once we have obtained an expression for the cohomology class $\eta_\Delta \in H^n(M \times M)$ of the diagonal $\Delta \subset M \times M$.
We do this as follows: First, for each q, let $\{\psi_{\mu,q}\}$ be a collection of closed q-forms on M representing a basis for $H^q_{DR}(M)$, and let $\{\psi^*_{\mu,n-q}\}$ be $(n-q)$-forms representing the dual basis for $H^{n-q}_{DR}(M)$, i.e., such that

$$\int_M \psi_{\mu,q} \wedge \psi^*_{\nu,n-q} = \delta_{\mu,\nu}.$$

Let π_1 and π_2 denote the two projection maps $M \times M \to M$. By the Künneth formula, the forms

$$\{\varphi_{\mu,\nu,p,q} = \pi_1^*\psi_{\mu,p} \wedge \pi_2^*\psi^*_{\nu,q}\}_{p+q=k}$$