

Let $x_i = n - k + i - a_i$. $y_i = n - i + 1 - b_{k-i+1}$

Since $x_i + y_i \leq n$. $x_i < x_{i+1}$, $y_i > y_{i+1}$.

We can find vector spaces. $V_{x_i}, W_{y_i} \subset \mathbb{C}^n$ s.t.

$$V_{x_i} \cap W_{y_i} = (0).$$

Choose any subspace $W_{y_2} \subset W_{y_1}$ since $y_2 < y_1$.

$\Rightarrow V_{x_1} \cap W_{y_2} = (0)$. Always we can find V_{x_2} s.t.

$V_{x_2} \supset V_{x_1}$ and $V_{x_2} \cap W_{y_2} = (0)$. since $x_2 + y_2 \leq n$.

Continue this procedure, then we get flags V, W s.t. V_{x_i} and W_{y_i} intersect only at the origin.

" $V_{x_i} \cap W_{y_2} = (0)$ $W_{y_2} = \langle w_1, \dots, w_{y_2} \rangle$, w_1, \dots, w_{y_2} lin. inde.

$\Rightarrow \exists v_1, \dots, v_{x_1}, \dots, v_{n-y_2}$ s.t. they are linearly indep.

and $\langle v_1, \dots, v_{x_1} \rangle = V_{x_1}$. $\{v_1, \dots, v_{x_1}, \dots, v_{n-y_2}, w_1, \dots, w_{y_2}\}$ is

a basis for \mathbb{C}^n . \Rightarrow Choose $v_1, v_2, \dots, v_{x_1}, \dots, v_{x_2}$.

so that $\langle v_1, \dots, v_{x_2} \rangle = V_{x_2}$ since $x_2 \leq n - y_2$. " \square

Consequently the cycles $\sigma_a(V)$ and $\sigma_b(V')$ can be made disjoint, i.e.,

$$\#(\sigma_a \cdot \sigma_b) = 0 \text{ unless } a_i + b_{k-i+1} \leq n - k, \text{ for all } i.$$

Υ If $a_i + b_{k-i+1} > n - k$ for some i ,

by the computation on p197, since $x_i + y_i$

$$= n - k + i - a_i + n - i + 1 - b_{k-i+1} \leq n, \text{ we can find}$$

$$V_{x_i}, V'_{y_i} \text{ s.t. } V_{x_i} \cap V'_{y_i} = (0).$$