More explanation on the exceptional case.

The exception case implies that some of $\frac{n(n-3)}{2}+1$-points are not linearly independent in $\mathbb{P}^N$. So, some curve of degree $n-3$ possesses more than $\frac{n(n-3)}{2}$ points.

First choose N-points $P_1, P_2 \cdots P_N$.
Choose a curve B of degree $n-3$ passing through $P_{N+1}, \cdots P_{n^2}$.
$\Rightarrow$ B might pass through P_ν, for $\nu \leq N$.
$\Rightarrow$ We relabel $p_1, \cdots p_{\frac{n(n+3)}{2}}$ so that the curve B of degree $n-3$ passes through exactly

$$P_K, P_{K+1}, \cdots P_{n^2}, \quad K \leq \frac{n(n+3)}{2}$$

Consider a hyperplane $A_i \subset \mathbb{P}^N$ containing $P_1, P_2, \cdots \hat{P_i} \cdots P_K$, which is a curve of degree n, i.e., $A_i \cap i_n(\mathbb{P}^2)$.
$\Rightarrow$ A_i must contain P_i since $A_i + B$ must contain P_i and B can not contain P_i. $\Rightarrow$ Since $i_n(P_1), \cdots i_n(P_K)$ lie on $H = \mathbb{P}^{N-1}$, $\{ i_n(P_1), \cdots i_n(P_K) \cdots i_n(P_N) \}$ is linearly dependent. $\lrcorner$

We have now illustrated the application of the residue theorem to points arising as intersections of plane algebraic curves having no multiple components. Multiple components arises naturally when we wish to know not only about the position in $\mathbb{P}^2$ of the points of intersection, but also about high-order infinitesimal behavior.

For example, let $L \subset \mathbb{P}^2$ be a line. If we mark points $p_1, p_2, \cdots p_n$ on L, then it is trivially possible to find an algebraic curve C of degree n passing through the