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th quadric in V_4. $\Rightarrow$ Choose a V_2 so that $V_2 \cap (V_4 \cap F)$ is smooth $\Rightarrow$ Choose generic $p \notin F$ in V_2. $\Rightarrow$ Here implicitly, we used Bertini's theorem.

□

Say $\Lambda \in \omega_F \cap \sigma_{3,2,1}$, i.e., Λ is a 2-plane containing p, having a line in common with V_2, lying in V_4, and meeting F in a line. (See Figure 18.)

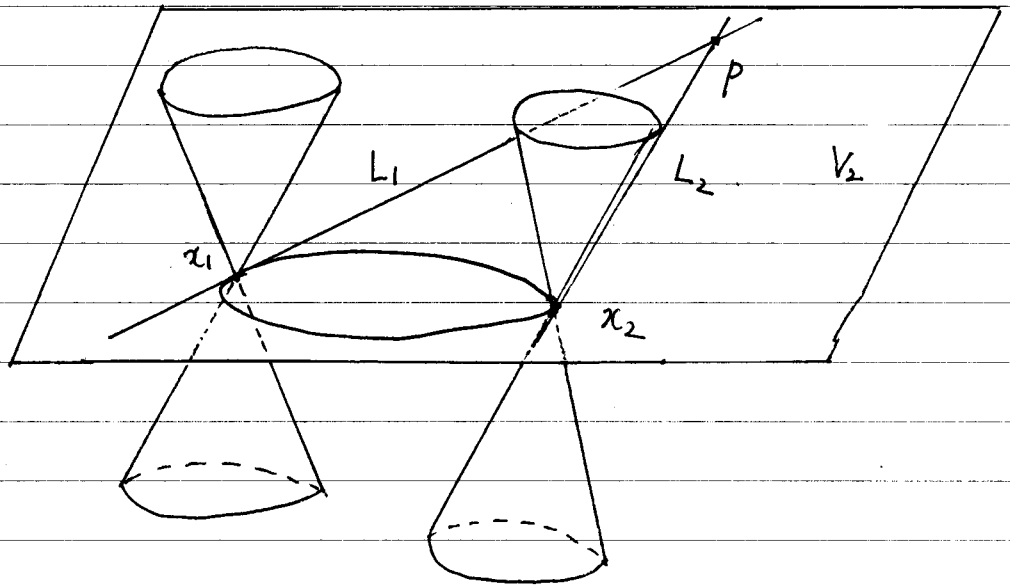

Figure 18.

Then the line $\Lambda \cap V_2$ can meet C in only one point, hence meet be one of the two tangent lines L_1, L_2 to C through p in V_2.