sible. $\Rightarrow f_i \cdot h_0 = g_i \cdot h_0 / h_i = g_i \cdot l_1/l_2$, where l_1/l_2 is holomorphic. $\Rightarrow$ Let $l = l_1/l_2$. $\Rightarrow f_i \cdot h_0 = g_i \cdot l$.
$\Rightarrow l$ is not divisible by k, otherwise, $l_1 = l' k l_2$, and l_1 is divisible by k, which is absurd. ⅃

Thus no function vanishing at p can divide all the functions $h_0, h_0 f_i$.

⸢ In the above, we used the fact that $\mathcal{O}$ is UFD. From the above, we know that h_0 & $h_0 f_i$ have no common factor which is not a unit. ⅃

It follows that the locus $\cap (\tilde{f}_i)$ contains no divisors, i.e., that a rational map f is defined away from a subvariety of codimension 2 or more.

⸢ From the above, we know that $\{\tilde{f}_i\}$ has no common factor. $\Rightarrow \cap (\tilde{f}_i)$ contains no divisors, and so on. ⅃

Conversely, if $V \subset M$ is any analytic subvariety of codimension at least 2, $f: M - V \longrightarrow \mathbb{P}^n$ a holomorphic map, then by the Levi theorem from Section 2 of Chapter 3 the pull-back to $M-V$ of the Euclidean coordinate functions $x_i = X_i / X_0$, $i = 1, \cdots n$, on $\mathbb{P}^n$ extend to meromorphic functions f_i on M: the map $f = [1, f_1(z), \cdots f_n(z)]$ is thus rational.