$$f_3 = g_2 h_{32} + g_3 h_{33} + g_4 h_{34}$$
$$f_4 = g_2 h_{42} + g_3 h_{43} + g_4 h_{44}$$
$$\Rightarrow \quad df_2 = h_{22}\, dg_2 + h_{23}\, dg_3 + h_{24}\, dg_4 \qquad \text{on } D_i$$
$$df_3 = h_{32}\, dg_2 + h_{33}\, dg_3 + h_{34}\, dg_4$$
$$df_4 = h_{42}\, dg_2 + h_{43}\, dg_3 + h_{44}\, dg_4$$

$$\Rightarrow df_2 \wedge d\bar{f_2} \wedge df_3 \wedge d\bar{f_3} \wedge df_4 \wedge d\bar{f_4}$$
$$= \left| \det \begin{pmatrix} h_{22}, & h_{23}, & h_{24} \\ h_{32}, & h_{33}, & h_{34} \\ h_{42}, & h_{43}, & h_{44} \end{pmatrix} \right|^2 dg_2 \wedge d\bar{g_2} \wedge dg_3 \wedge d\bar{g_3} \wedge dg_4 \wedge d\bar{g_4}.$$

for, consider
$$z_1 = a_{11} w_1 + \cdots + a_{1n} w_n$$
$$z_2 = a_{21} w_1 + \cdots + a_{2n} w_n$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$z_n = a_{n1} w_1 + \cdots + a_{nn} w_n$$
a_{ij}'s complex constants

$$\Rightarrow \quad dz_1 = a_{11}\, dw_1 + \cdots + a_{1n}\, dw_n$$
$$\vdots \qquad \vdots$$
$$dz_n = a_{n1}\, dw_1 + \cdots + a_{nn}\, dw_n$$
$$\Rightarrow \quad dz_1 \wedge \cdots \wedge dz_n = \det(a_{ij})\, dw_1 \wedge \cdots \wedge dw_n$$
$$d\bar{z}_1 \wedge \cdots \wedge d\bar{z}_n = \overline{\det(a_{ij})}\, d\bar{w}_1 \wedge \cdots \wedge d\bar{w}_n$$
$$\Rightarrow \quad dz_1 \wedge d\bar{z}_1 \wedge \cdots \wedge dz_n \wedge d\bar{z}_n = \epsilon\, dz_1 \wedge \cdots \wedge dz_n \wedge d\bar{z}_1 \wedge \cdots \wedge d\bar{z}_n$$
$$= \epsilon\, |\det(a_{ij})|^2\, dw_1 \wedge \cdots \wedge dw_n \wedge d\bar{w}_1 \wedge \cdots \wedge d\bar{w}_n$$
$$= |\det(a_{ij})|^2\, dw_1 \wedge d\bar{w}_1 \wedge \cdots \wedge dw_n \wedge d\bar{w}_n.$$

The computations above imply that the way of determining an orientation on D_i is independent of the choice of a frame for E provided $e_1 = \sigma_1, \cdots e_{i-1} = \sigma_{i-1}$. $\lrcorner\!\rfloor$

Example. We can now make a second computation for the