For each $q \in L$, $T_q(G) = \{ {}^tXQq = 0 \}$ and $T_q(F) = \{ {}^tXQ'q = 0 \}$, where $G = \{ {}^tXQX = 0 \}$ and $F = \{ {}^tXQ'X = 0 \}$. $\Rightarrow$ At $q \in L$, $T_q(X) = \{ {}^tXQq = {}^tXQ'q = 0 \}$.
Choose two distinct points $q_0, q_1 \in L$.
$\Rightarrow$ For $q \in L$, $q = \lambda q_0 + q_1$, $\lambda \in \mathbb{P}^1$.
$\Rightarrow$ $T_q(X) = \{ {}^tXQ(\lambda q_0 + q_1) = {}^tX Q'(\lambda q_0 + q_1) = 0 \}$.
$\Rightarrow$ $\lambda\, {}^tXQq_0 + {}^tXQq_1 = 0$ and $\lambda\, {}^tXQ'q_0 + {}^tXQ'q_1 = 0$.
$\Rightarrow$ $\lambda : 1 = {}^tXQq_1 : -{}^tXQq_0 = {}^tXQ'q_1 : -{}^tXQ'q_0$
$\Rightarrow$ $({}^tXQq_0)({}^tXQ'q_1) = ({}^tXQq_1)({}^tXQ'q_0)$
$\Rightarrow$ Let $\tilde{Q} = \{ X \mid ({}^tXQq_0)({}^tXQ'q_1) - ({}^tXQq_1)({}^tXQ'q_0) = 0 \}$.
$\Rightarrow$ $\tilde{Q}$ is a quadric in $\mathbb{P}^5$. Clearly any $X \in \bigcup_{x \in L} T_x(X)$ lies in $\tilde{Q}$.
If $\bigcup_{x \in L} T_x(X)$ is in a hyperplane H, $L \subset H \subset \mathbb{P}^5$.

$\cancel{(\bigcup_{x \in L} T_x(X)) \cap X \subset H \cap X}$ ~~which~~ is a surface
$\Rightarrow$ $\cancel{\bigcup_{x \in L} (T_x(X) \cap X)}$ is the set of all lines in X meeting L.

then $\bigcup_{x \in L} T_x(X) \cap X = \bigcup_{x \in L} (T_x(X) \cap X)$
$= \bigcup_{L' \in B_L} L' \cup L \subset H \cap X \Rightarrow \overline{f(\bigcup_{L' \in B_L} L')}$ = the closure of $f(\bigcup_{L' \in B_L} L') = E_L$ lies in $H \cap V_3 = \mathbb{P}^2$.
$\Rightarrow$ E_L is a plane curve $\Rightarrow$ By the Plücker formula, $g(E_L) = g(B_L) = 2 \neq \frac{(5-1)(5-2)}{2} = 6$ which is impossible. $\Rightarrow$ $\bigcup_{x \in L} T_x(X) \not\subset H$. $\Rightarrow$ $\bigcup_{x \in L} T_x(X)$