$$\Rightarrow \quad c_1(\mathbb{P}^n)^n = \sum_i \frac{(-(n+1)\alpha_i)^n}{\prod_{j\neq i}(\alpha_j-\alpha_i)} = (-1)^n(n+1)^n \sum_i \frac{\alpha_i^n}{\prod_{j\neq i}(\alpha_j-\alpha_i)}$$

$$= (n+1)^n.$$

$c_1(\mathbb{P}^n) = a\,\omega \in H^2(\mathbb{P}^n)$.

$\Rightarrow \quad c_1(\mathbb{P}^n)^n = a^n \omega^n$

$$\int_{\mathbb{P}^n} c_1(\mathbb{P}^n)^n = \int_{\mathbb{P}^n} a^n\omega^n = a^n \int_{\mathbb{P}^n} \omega^n = a^n = (n+1)^n$$

since $\int_{\mathbb{P}^n} \omega^n = 1$. ($\because \int_{\mathbb{P}^1} \omega = 1$ and $\omega^2 = 1 \in H^2(\mathbb{P}^2, \mathbb{Z})$ ... $\omega^n \in H^n(\mathbb{P}^n, \mathbb{Z})$.)

$\Rightarrow \quad a^n = (1+n)^n \Rightarrow \quad a = 1+n$.

$\Rightarrow \quad c_1(\mathbb{P}^n) = (n+1)\,\omega$.

Now to compute the rest of the Chern classes of $\mathbb{P}^n$ we need only evaluate the Chern numbers $c_1(\mathbb{P}^n)^{n-r} \cdot c_r(\mathbb{P}^n)$. By Bott residue applied to v,

$$c_1(\mathbb{P}^n)^{n-r} \cdot c_r(\mathbb{P}^n) = \sum_{i=0}^{n} \frac{\left(\sum_{j\neq i}(\alpha_j-\alpha_i)\right)^{n-r} \sum_{i\in I}^{\#I=r} \prod_{j\in I}(\alpha_j-\alpha_i)}{\prod_{j\neq i}(\alpha_j-\alpha_i)}$$

$$= \sum_{i=0}^{n} \frac{(-1)^{n-r}(n+1)^{n-r}\,\alpha_i^{n-r} \sum_{i\in I}^{\#I=r} \prod_{j\in I}(\alpha_j-\alpha_i)}{\prod_{j\neq i}(\alpha_j-\alpha_i)}.$$

Γ Let $P_1(A)$ be the trace of a matrix A.
Let $P_r(A) = \sum_{\#I=r} \det(A_{I,I})$, where $A_{I,J}$ denote the (I, J)th minor of A, see p 402.