Before giving some examples, we need one lemma. A map

$$j : \mathcal{L}^* \longrightarrow \mathcal{K}^*$$

between complexes of sheaves is a quasi-isomorphism if it induces an isomorphism on cohomology sheaves:

$$j_* : \mathcal{H}^q(\mathcal{L}^*) \longrightarrow \mathcal{H}^q(\mathcal{K}^*), \quad q \geq 0.$$

Lemma. If $j : \mathcal{L}^* \longrightarrow \mathcal{K}^*$ is a quasi-isomorphism, then the induced map on hypercohomology

$$j_* : \mathbb{H}^*(X, \mathcal{L}^*) \longrightarrow \mathbb{H}^*(X, \mathcal{K}^*)$$

is an isomorphism.

Proof. Clearly j induces mappings on the spectral sequences, and

$$j_* : H^p(X, \mathcal{H}^q(\mathcal{L}^*)) \longrightarrow H^p(X, \mathcal{H}^q(\mathcal{K}^*))$$

is an isomorphism by our assumption. It is a reasonably obvious general fact that a map between filtered complexes that induces an isomorphism on any term $\{E_r\}$ in the spectral sequences necessarily induces an isomorphism on the total cohomology. Q.E.D.

Γ

$$\begin{tikzcd}
\mathcal{K}^0 \arrow[r] & \cdots & \mathcal{K}^q \arrow[r, "d"] & \mathcal{K}^{q+1} \\
\mathcal{L}^0 \arrow[r] \arrow[u, "j"] & \cdots & \mathcal{L}^q \arrow[r, "d"] \arrow[u, "j"] \arrow[ur, phantom, "\circlearrowright"] & \mathcal{K}^{q+1} \arrow[u]
\end{tikzcd}$$

$$C^p(\underline{U}, \mathcal{K}^q) \xleftarrow{\ j\ } C^p(\underline{U}, \mathcal{L}^q)$$

Since j commutes with d & δ, j induces mappings on