

must lie in a linear space of dimension $d+k-1$.

$\Rightarrow f(L)$ must lie in a linear space of dim $2+1-1=2$. \square

Now $p \in \overline{f(u), f(u')} \subset V_2$, and any line through p in V_2 must intersect $f(L)$ twice, so that any point of \mathbb{P}^5 lying on a chord of S lies on infinitely many chords of S .

\square Any line l through p in V_2 must intersect $f(L)$ twice, maybe tangent to one point, since $l \cong \mathbb{P}^1$ and $f(L)$ has degree 2.

Let p be a point of \mathbb{P}^5 lying on a chord of S .

$\Rightarrow \exists u, u' \in \mathbb{P}^2$ s.t. $p \in \overline{f(u), f(u')} \Rightarrow \exists V_2 \supset \overline{f(u), f(u')}$

\Rightarrow Any line l through p lying in V_2 must intersect $f(L)$, where $L = \overline{u, u'}$.

$\Rightarrow \exists q_1, q_2 \in f(\overline{u, u'}) = f(L)$ s.t. $\overline{q_1, q_2} \ni p$.
and $q_1, q_2 \in S$, i.e. p is in a chord $\overline{q_1, q_2}$ of S . \square

In particular, if we let L_0 be the line $(s=0)$ in \mathbb{P}^2 , and let $u_0 = L_0 \cap L$, then the line

$\overline{f(u_0), p} \subset \mathbb{P}^5$ is a chord of S .

\square $p \in \overline{q_1, q_2}$, $q_1, q_2 \in S \Rightarrow$ By the previous result, (i.e. any line through p must intersect $f(L)$ twice, and the line is a chord of S), $\overline{f(u_0), p}$ is a chord