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$\Rightarrow$ We obtain the following exact sequence of sheaves over U,

$$0 \longrightarrow \mathcal{F} \longrightarrow \mathcal{F} \oplus \mathcal{O}^{(k_0)} \Big/ \mu\left(\mathcal{O}^{(k_1)} / \partial \mathcal{O}^{(k_2)}\right) \longrightarrow \mathcal{G} \to 0$$

$$\mu : \mathcal{O}^{(k_1)} / \partial \mathcal{O}^{(k_2)} \longrightarrow \mathcal{F} \oplus \mathcal{O}^{(k_0)}.$$

$\Rightarrow$ We have local extensions. Now choose a sufficiently fine covering $\underline{U} = \{U_\alpha\}$ and corresponding local extensions

$(\mathcal{E}_\alpha)$ $\qquad 0 \longrightarrow \mathcal{F}|_{U_\alpha} \longrightarrow \mathcal{E}_\alpha \longrightarrow \mathcal{G}|_{U_\alpha} \longrightarrow 0$

(we didn't use the property of local rings) P698 P699

In $U_\alpha \cap U_\beta$, by the same arguments on ~ note, there will be a commutative diagram

$$\begin{array}{ccccccccc}
0 & \longrightarrow & \mathcal{F}|_{U_\alpha \cap U_\beta} & \longrightarrow & \mathcal{E}_\alpha|_{U_\alpha \cap U_\beta} & \longrightarrow & \mathcal{G}|_{U_\alpha \cap U_\beta} & \longrightarrow & 0 \\
 & & \| & & \downarrow \varphi_{\alpha\beta} & & \| & & \\
0 & \longrightarrow & \mathcal{F}|_{U_\alpha \cap U_\beta} & \longrightarrow & \mathcal{E}_\beta|_{U_\alpha \cap U_\beta} & \longrightarrow & \mathcal{G}|_{U_\alpha \cap U_\beta} & \longrightarrow & 0
\end{array}$$

since, over U_α and U_β, we may have different projective resolutions of $\mathcal{G}$, i.e.,

over U_α

$$\mathcal{O}^{(k_2)} \longrightarrow \mathcal{O}^{(k_1)} \longrightarrow \mathcal{O}^{(k_0)} \longrightarrow \mathcal{G} \to 0$$

over U_β

$$\mathcal{O}^{(k_2')} \longrightarrow \mathcal{O}^{(k_1')} \longrightarrow \mathcal{O}^{(k_0')} \longrightarrow \mathcal{G} \longrightarrow 0.$$