of the geometry of blow-ups later on in the chapter on surfaces; for the time being, we have enough information to proceed to the proof of the embedding theorem.

Proof of the Kodaira Theorem.

Again, let $L \longrightarrow M$ be a positive line bundle on the compact complex manifold M. We want to prove that there exists k_0 such that

1. The restriction map
$$H^0(M, \mathcal{O}(L^k)) \xrightarrow{r_{x,y}} L_x^k \oplus L_y^k$$
is surjective for all $x \neq y \in M$, $k \geq k_0$; and
2. The differential map
$$H^0(M, \mathcal{J}_x(L^k)) \xrightarrow{d_x} T_x^{*\prime} \otimes L_x^k$$
is surjective for all $x \in M$, $k \geq k_0$.

To prove assertion 1, let $\tilde{M} \xrightarrow{\pi} M$ denote the blow-up of M at both x and y, $E_x = \pi^{-1}(x)$ and $E_y = \pi^{-1}(y)$ the exceptional divisors of the blow-up; for notational convenience, let E denote the divisor $E_x + E_y$ and $\tilde{L} = \pi^* L$. (Here we are tacitly assuming that $n = \dim(M) \geq 2$; in case M is a Riemann surface, all the arguments that follow will be valid for $\tilde{M} = M$, $\pi = id$.)

Consider the pullback map on sections

$$\pi^* : H^0(M, \mathcal{O}_M(L^k)) \longrightarrow H^0(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{L}^k)).$$

$\tilde{M} \xrightarrow{\pi} M$