$$[\Lambda, \bar\partial] = -\frac{i}{2} D'^{*}.$$

This identity follows from the analogous identity $[\Lambda, \bar\partial] = -\frac{i}{2}\partial^{*}$ on scalar forms $A^{p,q}(M)$, which we have already proved. To see this, pick a local frame $\{e_\alpha\}$ for E; if $\theta = \theta' + \theta''$ is the connection matrix for D in terms of $\{e_\alpha\}$, we can write, for $\eta \in A^{p,q}(E)$,

$$\eta = \sum_\alpha \eta_\alpha \otimes e_\alpha, \quad \eta_\alpha \in A^{p,q}(M)$$

$$\bar\partial\eta = \sum_\alpha \bar\partial\eta_\alpha \otimes e_\alpha + \sum_{\alpha,\beta} (\eta_\alpha \wedge \theta''_{\alpha\beta}) \otimes e_\beta$$

$$\Lambda\eta = \sum_\alpha \Lambda(\eta_\alpha) \otimes e_\alpha,$$

so
$$[\Lambda, \bar\partial]\eta = \sum [\Lambda, \bar\partial]\eta_\alpha \otimes e_\alpha + [\Lambda, \theta'']\eta$$
$$= \sum -\frac{i}{2}\partial^{*}\eta_\alpha \otimes e_\alpha + [\Lambda, \theta'']\eta.$$

⌈ I think the right basic identity is

$$[\Lambda, \bar\partial] = -i D'^{*} \quad \text{according to } p\,111.$$

$$[\Lambda, \bar\partial]\eta = (\Lambda\bar\partial - \bar\partial\Lambda)\eta$$
$$= \Lambda(\bar\partial\eta) - \bar\partial(\Lambda\eta)$$

$$= \Lambda\left(\sum \bar\partial\eta_\alpha \otimes e_\alpha + \sum (\eta_\alpha \wedge \theta''_{\alpha\beta}) \otimes e_\beta\right) - \bar\partial\left(\sum \Lambda(\eta_\alpha) \otimes e_\alpha\right)$$

$$= \sum \Lambda(\bar\partial\eta_\alpha) \otimes e_\alpha + \sum \Lambda(\eta_\alpha \wedge \theta''_{\alpha\beta}) \otimes e_\beta - \sum \bar\partial\Lambda(\eta_\alpha) \otimes e_\alpha$$

$$- \sum \Lambda(\eta_\alpha) \wedge \theta''_{\alpha\beta} \otimes e_\beta$$

$$= \sum [\Lambda, \bar\partial](\eta_\alpha) \otimes e_\alpha + \sum \Lambda(\eta_\alpha \wedge \theta''_{\alpha\beta}) \otimes e_\beta - \sum \Lambda(\eta_\alpha) \wedge \theta''_{\alpha\beta}$$