Let W be the set of all $\phi \in \mathcal{D}(\Omega)$ that satisfy

(4) $\qquad |\phi(x_m)| < m^{-1} |\phi_m(x_m)| \qquad (m = 1, 2, 3, \cdots).$

Since each K contains only finitely many x_m, it is easy to see that $\mathcal{D}_K \cap W \in \tau_K$.

$\ulcorner$ Since $\{x_m\}$ has no limit point in Ω, each K contains only finitely many x_m.
Suppose we have K which contains x_1, x_2.
$\phi \in \mathcal{D}_K \cap W. \quad \Rightarrow \quad \phi(x_1), \ \phi(x_2).$
$V_N = \{ \psi \in \mathcal{D}_K : \ \|\psi\|_N < \frac{1}{N} \}. \quad \Rightarrow \quad |\psi(x)| < \frac{1}{N}.$

$$|\phi(x_1) + \psi(x_1)| < \frac{1}{m} |\phi_1(x_1)| \qquad m = 1$$
$$|\phi(x_2) + \psi(x_2)| < \frac{1}{m} |\phi_2(x_2)|. \qquad m = 2$$

$\Rightarrow \quad |\phi(x_i) + \psi(x_i)| < |\phi(x_i)| + |\psi(x_i)|. \qquad i = 1, 2.$
For N sufficiently large, since $|\phi_1(x_1)|$ & $\frac{1}{2}|\phi_2(x_2)|$ are already determined, $|\phi(x_i)| + |\psi(x_i)| < \frac{1}{i} |\phi_i(x_i)|$.

$\Rightarrow \quad \phi + V_N \subset \mathcal{D}_K \cap W \quad \Rightarrow \quad \mathcal{D}_K \cap W$ is open in $\mathcal{D}_K$
$\Rightarrow \quad \mathcal{D}_K \cap W \in \tau_K.$ $\lrcorner$

Thus $W \in \beta$.

$\ulcorner \quad W = \{ \phi \in \mathcal{D}(\Omega) : \ |\phi(x_m)| < m^{-1} |\phi_m(x_m)| \}$
$\phi, \ \phi' \in W, \quad |(t\phi + (1-t)\phi')(x_m)| \leq t|\phi(x_m)| + (1-t) |\phi'(x_m)| < t m^{-1} |\phi_m(x_m)| + (1-t) m^{-1} |\phi_m(x_m)| = m^{-1} |\phi_m(x_m)|$
$\Rightarrow \quad t\phi + (1-t)\phi' \in W \qquad W$ convex
$|\alpha\phi(x_m)| \leq |\phi(x_m)| \leq m^{-1} |\phi_m(x_m)|$ for $|\alpha| \leq 1$
$\Rightarrow \quad \alpha W \subset W \quad \Rightarrow \quad W$ balanced
$\Rightarrow \quad W \in \beta$ $\lrcorner$