relatively prime, then f gives a map of S to $\mathbb{P}^1$ by $p \longmapsto [g(p), h(p)]$.

$\ulcorner$ $S \hookrightarrow \mathbb{P}^N$. $\Rightarrow$ $\exists$ a global meromorphic function on $\mathbb{P}^N$ $\Rightarrow$ Take the restriction to S $\Rightarrow$ We get a global meromorphic function on S $\Rightarrow$ $\mathcal{M}(S) \neq \phi$.

$$\begin{array}{ccc} S & \longrightarrow & \mathbb{P}^1 \\ \rotatebox{90}{$\in$} & & \\ p & \longmapsto & [g(p), h(p)]. \qquad f = \dfrac{g}{h}. \end{array}$$

well-defined. $\lrcorner$

Let $f: S \longrightarrow \mathbb{P}^1$ be such a map: on $\mathbb{P}^1$ we have $\chi(\mathbb{P}^1) = 2 = -\deg K_{\mathbb{P}^1}$

and so $\chi(S) = n \cdot \chi(\mathbb{P}^1) - \sum_{p \in S} (v(p) - 1)$

$= -n \cdot \deg K_{\mathbb{P}^1} - \sum_{p \in S} (v(p) - 1)$

$= -\deg K_S$.

$\ulcorner$ $\chi(\mathbb{P}^1) = -\deg K_{\mathbb{P}^1}$ by P216.

By $\chi(S) = n \cdot \chi(S') - \sum_{q \in S} (v(q) - 1)$.

$\chi(S) = n \cdot \chi(\mathbb{P}^1) - \sum_{p \in S} (v(p) - 1) = -n \cdot \deg K_{\mathbb{P}^1} - \sum_{p \in S} (v(p) - 1) = -\deg K_S$, by $\deg K_S = n \cdot \deg K_S + \sum_{p \in S} (v(p) - 1)$. $\lrcorner$

Thus for any S, $\deg K_S = -\chi(S) = 2g - 2$, and the Riemann-Hurwitz formula is established.

$\ulcorner$ From the above, $\deg K_S = -\chi(S) = 2g - 2$ $\lrcorner$

"Comment: We did not use the Gauss-Bonnet theorem."