$\Rightarrow \; S(\varphi) = \int_{f^{-1}(f(U'))=U'} f^*(\varphi) = \int_{f(U')} \varphi .$

$\Rightarrow$ By the argument on P391,

$$\Theta(S, q) = \text{mult}_q(f(U')) = 1$$

since $\Theta(S, q) = \Theta(S|_{f(U')}, q)$ by the definition ($\because r \to 0$).
Let $\{q : q = f(p),\ p \in M^*,\ f$ has maximum rank at $p\} = L$. L is clearly open, and dense since some holomorphic relation holds otherwise. $\Rightarrow$ By Sard's theorem, $f(M-L)$ is a set of measure zero.

Let $\Theta(S, q, r) = g_r(q)$.
For each $r \in (0, \infty)$, $\quad g_r : f(M) \longrightarrow [0, \infty)$.
Claim: $\quad g^{-1}[a, \infty) = \bigcap_r g_r^{-1}[a, \infty)$.

where $\quad g(q) = \lim_{r\to 0} g_r(q)$, where, since $g_r(q)$ is increasing of r for each q, $\exists$ a limit point $g(q) \geq 0$.
($g(q) = \Theta(S, q)$; "see p390 lemma")
Let $x \in g^{-1}[a, \infty)$. $\Rightarrow a \leq g(x) < \infty$
$\Rightarrow$ For every r, $\quad g(x) \leq g_r(x) \quad \Rightarrow \quad a \leq g_r(x) < \infty$
$\Rightarrow \quad x \in g_r^{-1}[a, \infty)$, $\Rightarrow \quad x \in \bigcap g_r^{-1}[a, \infty)$.
$\Rightarrow \quad g^{-1}[a, \infty) \subset \bigcap g_r^{-1}[a, \infty)$.
$x \in \bigcap g_r^{-1}[a, \infty)$. $\quad \Rightarrow \quad a \leq g_r(x) < \infty$
$\Rightarrow \quad \lim g_r(x) \geq a \quad \Rightarrow \quad a \leq g(x) < \infty \Rightarrow x \in g^{-1}[a, \infty)$
$\Rightarrow \quad \bigcap g_r^{-1}[a, \infty) \subset g^{-1}[a, \infty)$, $\Rightarrow$ Claim is proved.