The transition functions g_{UV} of E relative to these trivializations will then look like

$$g_{UV}(x) = \begin{array}{c|c} & \begin{array}{cc} \ell & k-\ell \end{array} \\ \ell & \left(\begin{array}{c|c} h_{UV}(x) & K_{UV}(x) \\ \hline 0 & j_{UV}(x) \end{array}\right) \end{array}$$

h_{UV} : transition funs of F.

$$\varphi_U : E_U \longrightarrow U \times \mathbb{C}^k. \qquad \varphi_V : E_V \longrightarrow V \times \mathbb{C}^k.$$

$$\varphi_U|_{F_U} : F_U \longrightarrow U \times \mathbb{C}^\ell.$$

$$\begin{array}{ccc} U\cap V \times \mathbb{C}^k & \xrightarrow{\varphi_V \circ \varphi_U^{-1} = g_{VU}} & V \cap U \times \mathbb{C}^k \\ \cup & & \cup \\ U \cap V \times \mathbb{C}^\ell & \xrightarrow{(\varphi_V|_{F_V}) \circ (\varphi_U|_{F_U})^{-1} = h_{VU}} & V \cap U \times \mathbb{C}^\ell \end{array}$$

$$g_{UV} = g_{VU}^{-1}. \qquad \underset{\displaystyle h_{VU}(x)}{\underset{\shortparallel}{g_{VU}(x)}}\big|(\mathbb{C}^\ell) \subset \mathbb{C}^\ell$$

$$g_{UV}(x) \left(\begin{array}{c} \left(\begin{array}{c} * \\ * \\ \vdots \\ * \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right) \begin{array}{c} | \\ \ell \\ | \\ \\ \\ \\ \\ \end{array} \end{array} \right) = \left(\begin{array}{c} * \\ * \\ * \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right) \begin{array}{c} | \\ \ell \\ | \\ \\ \\ \\ \\ \end{array}$$

$\Rightarrow$ This proves why we have zero matrix on the left below.

The maps j_{UV} are transition functions for the quotient bundle E/F given by $(E/F)_x = E_x/F_x$.

$$g_{UV}(x)\, g_{VU}(x) = I = \left(\begin{array}{c|c} h_{UV} & K_{UV} \\ \hline 0 & j_{UV} \end{array}\right) \left(\begin{array}{c|c} h_{VU} & K_{VU} \\ \hline 0 & j_{VU} \end{array}\right)$$

$$= \left(\begin{array}{c|c} h_{UV} h_{VU} & * \\ \hline 0 & j_{UV} j_{VU} \end{array}\right) \Rightarrow \quad j_{UV}(x)\, j_{VU}(x) = I$$

In the similar way, we can show that $j_{UV}\, j_{VW}\, j_{WU} = I$.