

$p_0 \in U$, $p'_0 \in U'$ and $\varphi(p_0) = \varphi(p'_0)$, which is absurd. $\Rightarrow d = 1$. \Rightarrow Since φ is one to one, φ is biholomorphic. Thus $N \xrightarrow{\pi} M$ is the blow-up of M at x_0 .

Let's return to P750.

We first compute the degree of the hypersurface $V_C \subset \mathbb{P}^5$ for smooth C : to do this, let $L \subset W$ be a generic line and $\{C_\lambda\}_{\lambda \in \mathbb{P}^1}$ the pencil of conics it represents. The curves $\{C_\lambda\}$ then cut out on C a linear system of degree 4 without base points.

$\Gamma^*(C \cdot C_\lambda) = 4$ Let $(\sigma_\lambda) = C_\lambda$, $\sigma_\lambda \in H^0(\mathbb{P}^2, \mathcal{O}(H+H))$

$\Rightarrow \{\sigma_\lambda|_C\}$ is a vector ^{sub}space of $H^0(C, \mathcal{O}(4H))$.

Choose distinct 4 points on C .

\Rightarrow Since $\dim H^0(\mathbb{P}^2, \mathcal{O}(2H)) = 6$, we can find a conic passing those 4 points. \Rightarrow Since $L = \{C_\lambda\}$ is a generic pencil, there is no base point.

The corresponding map expresses C as a 4-sheeted cover of \mathbb{P}^1 ; and by the Riemann-Hurwitz formula, the number of branch points of this map is

$$b = 2g(C) - 2 - 4(2g(\mathbb{P}^1) - 2) = 6.$$