impossible since $s = 0 = s'$ on $Z$. ⌟

If $E$ is a global extension of line bundles, then $\mathcal{L} \cong \mathcal{O}(n)$ and $\mathcal{L}' \cong \mathcal{O}(n')$, since $\mathrm{Pic}(\mathbb{P}^2) \cong \mathbb{Z}$.
Now $\mathrm{Ext}^1(\mathbb{P}^2; \mathcal{L}', \mathcal{L}) \cong H^1(\mathbb{P}^2, \mathcal{O}(n-n')) = 0$.

⌜ Suppose $E$ is a global extension of some line bundles, i.e. $E = \mathcal{O}(n) \oplus \mathcal{O}(n')$. Here $\mathcal{L}$ is different from the trivial bundle above. (The trivial bundle is used to construct $E$.)
By the result on P206,

$$\mathrm{Ext}^1(\mathbb{P}^2; \mathcal{O}(n'), \mathcal{O}(n)) = H^1(\mathbb{P}^2, \mathcal{O}(n')^* \otimes \mathcal{O}(n))$$
$$= H^1(\mathbb{P}^2, \mathcal{O}(n-n')) = 0 \text{ by P156.}$$

⌟

Also, $n+n' = 0$, since $c_1(E) = 0$. Thus

$$\mathcal{E} \cong \mathcal{O}(n) \oplus \mathcal{O}(-n),$$

where $n \geq 0$, and this is a contradiction, since any section of $\mathcal{E}$ is either nowhere zero $(n=0)$ or else vanishes on a curve $(n>0)$. Q.E.D.

⌜ Given a section $\sigma$ of $\mathcal{E}$, then $\sigma = (\sigma_1, \sigma_2)$.

$\Rightarrow$ $\sigma_2 = 0$ since $\mathcal{O}(-n)$ is negative, $n > 0$

$\sigma_2$ is constant if $n = 0$.

$\Rightarrow$ (i) $n = 0$

$\sigma_1, \sigma_2$ are constant $\Rightarrow$ $\sigma$ is constant

(ii) $n > 0$.