Now we have by the Lefschetz decomposition

$$h^{p,p+j} = \sum_{i=0}^{p} \dim P^{i,i+j} ;$$

Γ $$H^{p,p+j} = \bigoplus_{k=0}^{p} L^k P^{p-k,\,p-k+j}$$ put. $p-k=i$.

From $H^{2p+j} = \bigoplus_{k=0}^{p} L^k P^{2p+j-k}$

$$P^{2p+j-k} = \bigoplus_{l+m=2p+j-k} P^{l,m} \qquad L^k P^{p-k,\,p-k+j} \subset H^{p,p+j}$$

$$\Rightarrow \quad H^{p,p+j} \supset \bigoplus L^k P^{p-k,\,p-k+j}$$

Given $v \in H^{p,p+j}$, $v = L^k \xi_{2p+j-k}$. $\xi_{2p+j-k} \in P^{2p+j-k}$.
Since $\xi_{2p+j-k} \in H^{p-k,\,p-k+j}$, $\xi_{2p+j-k} \in P^{p-k,\,p-k+j}$.
$\Rightarrow$ $v \in \bigoplus L^k P^{k-k,\,p-k+j}$ $\lrcorner\!\lrcorner$

Thus, along a vertical line in the Hodge diamond,

$$\sum_{i=0}^{p} (-1)^i \dim P^{i,i+j} = (-1)^p h^{p,p+j} + 2\sum_{i=0}^{p-1} (-1)^i h^{i,i+j}$$

and we can write, finally,

$$I(M) = \sum_{\substack{p+q=2n \\ p\le q}} (-1)^p h^{p,q} + 2 \sum_{\substack{p+q\equiv 0(2) \\ p+q<2n \\ p\le q}} (-1)^p h^{p,q}$$

or $$I(M) = \sum_{\substack{p+q\equiv 0(2) \\ p\le q}} (-1)^p h^{p,q},$$

the last equality holding by virtue of the duality $h^{p,q} = h^{2n-p,\,2n-q}$.

Γ ① $$\sum_{i=0}^{p} (-1)^i \dim P^{i,i+j} = (-1)^0 \dim P^{0,j} + (-1)^1 \dim P^{1,1+j} + (-1)^2 \dim P^{2,2+j} + \cdots$$

$$= (-1)^p h^{p,p+j} + 2\sum_{i=0}^{p-1} (-1)^i h^{i,i+j}.$$