$$\begin{vmatrix} 2 & 4 & 0 \\ 0 & 2 & 4 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 2 & -4 \\ 0 & 2 & 4 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -4 \\ 2 & 4 \end{vmatrix} = 16.$$

$$(2x^2-2x+4)\,g(x) - 2(x-1)\,f(x) =$$

$$(2x^2-2x+4)(x^2+x+2) - 2(x-1)(x^3+x^2+4x+4)$$

$$= \cancel{2x^4} - \cancel{2x^3} + 4x^2 + \cancel{2x^3} - \cancel{2x^2} + \cancel{4x} + 4x^2 - \cancel{4x} + 8$$

$$- \cancel{2x^4} - \cancel{2x^3} - 8x^2 - \cancel{8x} + \cancel{2x^3} + \cancel{2x^2} + \cancel{8x} + 8 = 16.$$

$\Rightarrow \quad \alpha(x) = -2(x-1). \quad \beta(x) = 2x^2-2x+4. \quad \gamma = 16.$

$(\alpha, \beta, \gamma) = 2$ in $R = \mathbb{Z}$ and 2 is not a unit in $\mathbb{Z}$.

$\Rightarrow$ By P 343 Whitney Theorem 8B. $f(z_n)$ & $g(z_n)$ are Weierstrass polynomials in $\mathcal{O}_{n-1}[z_n]$ and $\exists$ $\alpha(z_n)$, $\beta(z_n)$ s.t $\deg\alpha < \deg g$. $\deg\beta < \deg f$

$$\alpha f + \beta g = \gamma.$$

Next, we have to divide α, β & γ by a greatest common divisor $d \in \mathcal{O}_{n-1}$, since α, β are relatively prime in $\mathcal{O}_{n-1}[z_n]$.

$\Rightarrow$ We have $\alpha', \beta' \in \mathcal{O}_{n-1}[z_n]$ and $\gamma' \in \mathcal{O}_{n-1}$ s.t

$$\alpha' f + \beta' g = \gamma' \quad \text{and} \quad (\alpha', \beta', \gamma') = 1 \text{ in } \mathcal{O}_n$$

(which means $\alpha', \beta', \& \gamma'$ are relatively prime in $\mathcal{O}_n$).

$\Rightarrow$ Claim: $\pi(W) = (\gamma' = 0)$.