$(d(dT_\omega - T_{d\omega}))(\eta) = \pm (dT_\omega - T_{d\omega})(d\eta) = \pm d(d\eta) = 0$

$\Rightarrow d(dT_\omega - T_{d\omega}) = 0 \Rightarrow dT_\omega - T_{d\omega} \in \ker d.$

$\Rightarrow$ This implies that $\omega \longmapsto T_\omega$ induces a map

$$H^*_{DR}(M) \longrightarrow H^*(\mathcal{D}^*(M), d)$$

We will prove that this mapping is an isomorphism. By de Rham's theorem the same is true of the mapping

$$H^*(M, \text{sing}) \longrightarrow H^*(\mathcal{D}^*(M), d)$$

from the cohomology of piecewise smooth singular chains into the cohomology of currents. If Γ is a piecewise smooth $(n-p)$-cycle, then there will be a smooth, closed p-form ψ such that the equation of currents

$$T_\Gamma = T_\psi + dR$$

will be satisfied. Although we will not prove it, one may think of R as the current defined by a $(p-1)$-form η that is integrable on M, C^∞ on $M - \Gamma$, and where $d\eta = -\psi$ on $M - \Gamma$.

$\ulcorner$ We had better define T_ω by $T_\omega(\varphi) = \int_M \omega \wedge \varphi$, essentially the same as $T_\omega(\varphi) = \omega \wedge \varphi$.

$T_\Gamma(\varphi) = \int_\Gamma \varphi = \underbrace{\int_M \psi \wedge \varphi}_{= T_\psi(\varphi)}$ by Poincare duality.