$\{$ lines in $\mathbb{P}^n\}$ $\xleftrightarrow[\text{onto}]{1-1}$ $G(2, n+1)$.

$U_I \cap \tau(W) = \{ \Lambda : \ f(v_1) = 0 \ \& \ f(v_2) = 0 \ \& \ f(v_1+v_2) = 0 \}$ where $f$ is a defining homogeneous polynomial for $W$. $\Rightarrow$ $\tau(W)$ is an analytic subvariety of $G(2, n+1)$, and so $\tau(W)$ can be considered as a cyle in homology group by P61. Note.

Let $g_1(\Lambda) = f(v_1)$, $g_2(\Lambda) = f(v_2)$ $g_3(\Lambda) = f(v_1+v_2)$ or.

$$g_1(z_{11}, z_{12}, \cdots z_{1,i_1-1}, \ z_{1,i_1+1}, \cdots z_{1,i_2-1}, z_{1,i_2+1}, \cdots z_{1,n+1}, z_{21}, \cdots)$$
$$= f(z_{11}, z_{12}, \cdot 1; 0, z_{1,n+1})$$
$$g_2(z_{11} \cdots \ z_{2,n+1}) = f(z_{21}, z_{22}, \cdot 0, 1, z_{2n+1})$$
$$g_3(z_{11} \cdots \ z_{2,n+1}) = f(z_{11}+z_{21}, \cdot 1; 1 \cdot z_{1,n+1}+z_{2,n+1}).$$

$$\begin{pmatrix} \frac{\partial g_1}{\partial z_{11}}, & \cdots & \frac{\partial g_1}{\partial z_{2,n+1}} \\ \frac{\partial g_2}{\partial z_{11}}, & \cdots & \frac{\partial g}{\partial z_{2,n+1}} \\ \frac{\partial g_3}{\partial z_{11}}, & \cdots & \frac{\partial g}{\partial z_{2,n+1}} \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial W_1}(z_1), \frac{\partial f}{\partial W_2}(z_1) \cdot \hat{} \ \hat{} & \frac{\partial f}{\partial W_{n+1}}, & 0 \cdots \hat{}\hat{} \ 0 \\ 0. \quad 0 \cdots & 0, & \frac{\partial f}{\partial W_1}(z_2) \cdot \hat{}\hat{} \cdot \frac{\partial f}{\partial W_{n+1}} \\ \frac{\partial f}{\partial W_1}(z_{11}+z_{21}), & \frac{\partial f}{\partial W_{n+1}}, & \frac{\partial f}{\partial W_1} \quad \frac{\partial f}{\partial W_{n+1}} \end{pmatrix}$$

Note that $\frac{\partial f}{\partial W_i}$'s are homogeneous polynomials of degree 1. For exaple. if $f(W_1 \cdots W_{n+1})$
$= W_1^2 + W_2^2 + \cdots + W_n^2$,

$$\frac{\partial f}{\partial W_i} = 2W_i \quad \Rightarrow \quad \begin{pmatrix} 2z_{11}, & 2z_{12} & \cdots & 0 & \cdots & 0 \\ 0 & 0 & & 2z_{21} & \cdots & \\ 2z_{11}+2z_{21}, & \cdots & & & & \end{pmatrix}$$

$$\frac{\partial f}{\partial W_1}(z_{11}+z_{21}, \cdots \ ) = \frac{\partial f}{\partial W_1}(z_{11}) + \frac{\partial f}{\partial W_1}(z_{21})$$

since $\frac{\partial f}{\partial W_1}$ is linear. $\Rightarrow$ $\underset{\text{in general.}}{\text{rank}} \begin{pmatrix} 2z_{11} \\ \\ 2z_{11}+2z_{21} \end{pmatrix} = 3$