the induced automorphism is the hyperelliptic involution.
$\Rightarrow$ Let $\tilde{g}$ be the induced hyperelliptic involution.
$\Rightarrow$ By P254, for all $\omega \in H^0(B, \Omega^1)$, $\tilde{g}^* \omega = -\omega$.
$\Rightarrow$ Since $J(B) = \dfrac{H^{1,0}(B)^*}{H_1(B, \mathbb{Z})}$,
$\tilde{g}$ induces an isomorphism of $J(B)$ defined by
$(z_1, z_2) + \Lambda \longmapsto (-z_1, -z_2) + \Lambda$.
$\Rightarrow$ The induced isomorphism of $J(B)$ is expressed as
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

I do not what is the relation between a given isomorphism and the induced isomophism on B vice versa. But it seems to me that for $g = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$g \circ \rho = m_2$. $\Rightarrow$ ρ is not a multiplication by two.
More information on Torelli's theorem, I guess.

One point that emerges from this discussion is this: since ρ is surjective, the quadric line complex X is determined by the curve B.

Pf Since $g \circ \rho = m_2$, $\rho = g^{-1} \circ m_2$ and ρ is surjective
($\because$ m is isomophic and m_2 is surjective).