「 In the assertion 3. if $V \subset \mathbb{C}^n$ does not contain the line $z_1 = \cdots = z_{n-1} = 0$, $V \xrightarrow{\pi} \pi(V)$ is proper.
$\Rightarrow$ By the proper mapping theorem, $\pi(V)$ is an analytic subvariety of $\mathbb{C}^{n-1}$.

If we prove the assertion 1 by using the proper mapping theorem, since the assertion 2 is proved by the assertions 1 & 3 as above, all the assertions follow from the proper mapping theorem.

Note: $V \xrightarrow{\pi} \pi(V) \xrightarrow{\pi} \pi^2(V)$ where

① each π is proper.

③ $\pi^2(V)$ contains a nbd of 0.

② $V = V_1 \cup \cdots \cup V_l$, where V_i's are irreducible at 0.

$\Rightarrow$ Each $\pi(V_i)$ is irreducible at 0. $\Rightarrow$ By P12 & the assertion 3 (P13), $\pi(V_i)$ is the zero locus of an irreducible $f_i \in \mathcal{O}_{n-1}$, and, up to multiplication by units, $\pi(V) = \{f_1 \cdots f_l = 0\}$ uniquely.

Note: $\pi : V^{\subset \mathbb{C}^n} \longrightarrow \pi(V)^{\subset \mathbb{C}^{n-1}}$

① Suppose that $\pi(V) = W_1 \cup W_2 \cup \cdots \cup W_l$, each W_i is irreducible at 0 and $W_i \not\subset W_j$, $i \neq j$.

② Assume that $\pi(V) = W_1 \cup W_2 \cup \cdots \cup W_l$ is unique.

③ Assume that $V = V_1 \cup V_2 \cup \cdots \cup V_k$ where V_i's are irreducible at 0 and each $\pi(V_i)$ does not contain a nbd of 0.

Then, if $V = V_1' \cup \cdots \cup V_n'$ where V_i's are irredu-