By an abuse of notation, we denote by T_ϵ the distribution on $\mathbb{R}^n$ defined by the function $T_\epsilon(x)$.

The following formal properties of the T_ϵ's will be proved:

1. $(T_\varphi)_\epsilon = T_{\varphi_\epsilon}$ for $\varphi(x) \in C^\infty(\mathbb{R}^n)$.
2. $T_\epsilon(\psi) = T(\psi_\epsilon)$ for $\psi(x) \in C_c^\infty(\mathbb{R}^n)$.
3. $(DT)_\epsilon = D(T_\epsilon)$ for $D = \partial^\alpha/\partial x^\alpha$.

Proof of 1. For $\psi \in C_c^\infty(\mathbb{R}^n)$,

$$(T_\varphi)_\epsilon(\psi) = \int_{\mathbb{R}^n} T_{\varphi_y}(\chi_\epsilon(x-y))\,\psi(x)\,dx$$

$$= \int\int \varphi(y)\,\chi_\epsilon(x-y)\,\psi(x)\,dx\,dy$$

$$= T_{\varphi_\epsilon}(\psi)$$

by interchanging the order of integration.

∵ $(T_\varphi)_\epsilon$ is the distribution on $\mathbb{R}^n$ defined by $(T_\varphi)_\epsilon(x)$

$$\Rightarrow (T_\varphi)_\epsilon(\psi) = \int (T_\varphi)_\epsilon(x)\,\psi(x)\,dx = \int\int \varphi(y)\,\chi_\epsilon(x-y)\,\psi(x)\,dy\,dx \quad (*)$$

$$\left(\because (T_\varphi)_\epsilon(x) = \int \varphi(y)\,\chi_\epsilon(x-y)\,dy\right)$$

$$= \int\int \varphi(y)\,\chi_\epsilon(x-y)\,\psi(x)\,dx\,dy$$ by Fubini's theorem since ψ has the compact support.

$$= \int \varphi(y) \int \chi_\epsilon(x-y)\,\psi(x)\,dx\,dy$$

φ_ϵ is the distribution on $\mathbb{R}^n$ defined by $\varphi_\epsilon(x)$ i.e.,

$$\varphi_\epsilon(x) = \int \varphi(y)\,\chi_\epsilon(x-y)\,dy.$$

$\Rightarrow$ From $(*)$, $(T_\varphi)_\epsilon(\psi) = \int \varphi_\epsilon(x)\,\psi(x)\,dx = T_{\varphi_\epsilon}(\psi)$. We don't