See p42.

Claim $H^p(M, \mathcal{A}^{r,s}(E)) = 0$ for $p > 0$.

pf) Given any locally finite cover $\underline{\mathcal{U}} = \{U_\alpha\}_{\alpha \in I}$ of M, we can find a partition of unity subordinate to $\underline{\mathcal{U}}$; i.e

C^∞ – functions ρ_α on M s.t $\sum \rho_\alpha \equiv 1$ and $\mathrm{supp}(\rho_\alpha) \subset U_\alpha$.

Now given $\sigma \in Z^p(\underline{\mathcal{U}}, \mathcal{A}^{r,s}(E))$, we define $\tau \in C^{p-1}(\underline{\mathcal{U}}, \mathcal{A}^{r,s}(E))$ by setting.

$$\tau_{\alpha_0 \cdots \alpha_{p-1}} = \sum_{\beta \in I} \rho_\beta \, \sigma_{\beta, \alpha_0, \cdots \alpha_{p-1}},$$

where the section $\rho_\beta \sigma_{\beta, \alpha_0 \cdots \alpha_{p-1}}$ extends to $U_{\alpha_0} \cap \cdots \cap U_{\alpha_{p-1}}$ by zero; one verifies that $\delta\tau = \sigma$.

In the case $p=1$, explicitly:

$$\sigma = \{ \sigma_{UV} \in \mathcal{A}^{r,s}(E)(U \cap V) \};$$

$$\sigma_{UV} + \sigma_{VW} + \sigma_{WU} = 0 \quad \text{in } U \cap V \cap W.$$

Set $\tau_U = \sum_V \rho_V \sigma_{VU}$; then

$$(\delta\tau)_{UV} = -\tau_U + \tau_V = -\sum_W \rho_W \sigma_{WU} + \sum_W \rho_W \sigma_{WV}$$

$$= \sum_W \rho_W \sigma_{UV} = \sigma_{UV}.$$

See the Remark below.
In general, sheaves that admit partitions of unity [more precisely, for any $U = \bigcup U_\alpha$, maps $\eta_\alpha : \mathcal{F}(U_\alpha) \longrightarrow \mathcal{F}(U)$ s.t the support of $(\eta_\alpha \sigma)$