$$\cong \bigoplus_{p \in Z} \mathrm{Ext}^n_{\mathcal{O}_p}(\mathcal{O}_{Z,p}, \Omega^n_p)$$

## Explanation of the General Global Duality Theorem

We have now found duality theorem for the coherent sheaf cohomology $H^q(M, \mathcal{F})$ in the two cases where $\mathcal{F} \cong \mathcal{O}(F)$ is locally free and $\mathcal{F} = \mathcal{O}_Z$ with $\dim Z = 0$ and $\mathcal{O}_Z = \mathcal{O}/I$ with $I$ a sheaf of regular ideals. These represent the two extremes of a general duality theorem for $H^q(M, \mathcal{F})$, which will now be explained.

$$\mathcal{F} \cong \mathcal{O}(F). \implies H^q(M, \mathcal{F}) \cong H^{n-q}(M, \Omega^n(F^*))$$

Kodaira - Serre duality p153. & p102

The steps are the following:

1. Given modules $L, M, N$ over the local ring $\mathcal{O} = \mathcal{O}_n$, the pairing

$$\mathrm{Hom}_{\mathcal{O}}(L, M) \otimes_{\mathcal{O}} \mathrm{Hom}_{\mathcal{O}}(M, N) \longrightarrow \mathrm{Hom}_{\mathcal{O}}(L, N)$$

induces a pairing, called the Yoneda pairing

$$\mathrm{Ext}^p_{\mathcal{O}}(L, M) \otimes_{\mathcal{O}} \mathrm{Ext}^q_{\mathcal{O}}(M, N) \longrightarrow \mathrm{Ext}^{p+q}_{\mathcal{O}}(L, N)$$

having associativity and graded commutativity properties analogous to the usual cup product. This is a formal exercise using the four-part proposition.