

angle that the tangent  $T_v$  makes with the line  $L$ .

We shall show now that the Reiss relation is sufficient. The polynomials  $f(x, y)$  of degree  $n$  form a vector space of dimension  $(n+1)(n+2)/2$ .

$$\begin{aligned} \mathbb{F} \quad \binom{n+2}{n} &= \frac{(n+2)!}{n! 2!} = \frac{(n+1)(n+2)}{2} = \dim H^0(\mathbb{P}^2, \mathcal{O}(n+2)) \text{ by P166} \\ [X_0, X_1, X_2] &\longleftrightarrow \left( \frac{X_1}{X_0}, \frac{X_2}{X_0} \right) \longleftrightarrow (x, y). \quad \square \end{aligned}$$

Those of the form  $g(x, y) x^3$  ( $\deg(g) = n-3$ ) form a vector space of dimension  $(n-2)(n-1)/2$ .

$$\mathbb{F} \quad \binom{n-3+2}{n-3} = \frac{(n-1)!}{(n-3)! 2!} = \frac{(n-1)(n-2)}{2} = \dim H^0(\mathbb{P}^2, \mathcal{O}(n-3)) \quad \square$$

The quotient space  $V$  has dimension

$$\frac{n^2+3n+2}{2} - \frac{n^2-3n+2}{2} = 3n.$$

$$\mathbb{F} \quad \{f(x, y)\} \supset \{g(x, y) x^3\} \Rightarrow V = \frac{\{f\}}{\{g \cdot x^3\}} \text{ has dimension}$$

$$3n = \frac{n^2+3n+2}{2} - \frac{n^2-3n+2}{2} \quad \square$$

Finding a curve  $C$  of degree  $n$  and with prescribed second-order behavior at points  $p_i$  on the line  $\{x=0\}$  is equivalent to finding a suitable point in the projective space  $\mathbb{P}(V) \cong \mathbb{P}^{3n-1}$ . Each second-order arc element imposes three linear conditions, and so there are  $3n$  conditions in