Date

Abel's theorem in the case $g=1$ tells us that if D is the divisor (f) of a meromorphic function f on S, then $\mu(D)=0$. It is not hard to extend this statement to the case of genus g: If $D=(f)$, then the map

$$\psi : [\lambda_0, \lambda_1] \longrightarrow \mu((\lambda_0 f - \lambda_1))$$

from $\mathbb{P}^1$ to $J(S)$ is holomorphic, and since the holomorphic 1-forms dz_i on the complex torus $J(S)$ span the cotangent space at each point.

$$\psi^*(dz_i) \equiv 0 \Rightarrow \psi \text{ is constant}$$
$$\Rightarrow \mu(D) = \psi(0) = \psi(\infty) = 0.$$

$\ulcorner$ $\mu(D) = \psi(0)$. where 0 means the point $[1,0] \in \mathbb{P}^1$

$$\lambda_0 \frac{z-b}{z-a} - \lambda_1 = \frac{\lambda_0(z-b) - \lambda_1(z-a)}{z-a} \longrightarrow -\lambda_1 \quad \text{as } \lambda_0 \to 0$$

$\Rightarrow$ zero $\longrightarrow$ pole. in other words, $\lambda_0 f - \lambda_1$ becomes holomorphic more and more.

$$\psi(\infty) = \psi([0,1]) = \mu((\lambda_1)) = 0$$

Question:

Is roots of $\lambda_0 f - \lambda = 0$ holomorphic functions of λ_0 & λ_1?

Yes. again by the implicit function theorem (equivalently the inverse function theorem) generically.

Let $S = (F_1 = F_2 = \cdots = F_l = 0)$ in $\mathbb{P}^N$.

$$f = G/F$$

$$K : \mathbb{C}^N \longrightarrow \mathbb{C}$$

$$(z_1 \cdots z_n\ \lambda_0, \lambda_1) \longmapsto (F_1, F_2, \cdots F_l, \lambda_0 G - \lambda_1 F, \lambda_0, \lambda_1)$$

if $l = N-1$.