is well-defnd. $\Rightarrow [T] \in H^*(\mathcal{D}^*(M), d) \cong H^*_{DR}(M)$
$\Rightarrow$ T defines a cycle by de Rham isomorphism. ⌟

Now suppose $\sigma_1, \cdots \sigma_k$ are generic sections of E. Usin-g a partition of unity on M, we can then construct additional sections $\sigma_{k+1}, \cdots \sigma_n$ of E s.t together $\sigma_1(x), \cdots \sigma_n(x)$ span the fiber E_x of E over each point $x \in M$.

⌜ Since M is compact, $\exists$ finite # of open sets U_α's s.t on U_α, $\exists$ sections which span E_x for $x \in \overline{U_\alpha'} \subset U_\alpha$, $\{U_\alpha'\}$ covers M and the section vanish outside U_α'. ⌟

By the construction of Section 6 of Chapter 1, then, the sections $\sigma_1, \cdots \sigma_n$ give us a map

$$\iota : M \longrightarrow G(k, n).$$

⌜ See P207 and note P533 ⌟

In terms of a trivializaton of E, we can express $\sigma_1, \cdots \sigma_k \cdots \sigma_n$ as k-vectors $V_1, \cdots V_n$ of C^∞ functions; the map ι is given by

$$x \longrightarrow [(V_1(x), \cdots V_n(x))] \in G(k, n).$$

⌜ $\sigma_1 : M \longrightarrow E \qquad \sigma_1 : U \longrightarrow E|U.$

Let $e_1, \cdots e_k$ be a frame for $E|U$.

$\Rightarrow \quad \sigma_1 = a_{11} e_1 + a_{21} e_2 + \cdots + a_{k1} e_k$ ⌟