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previous result. $\frac{\sqrt{-1}}{\pi}\partial\bar\partial \log|z_n| = T_{\{z_n=0\}}$. $\Rightarrow$ RHS = LHS. ⅃

One may suspect that a general closed, positive current should be somewhere between the smooth currents and those supported by analytic varieties. This turns out to be basically true, and in order to describe what is known, we show how a closed, positive current $T \in \mathcal{D}'^{p,p}(U)$ an open set U in $\mathbb{C}^n$ has associated to each point p, a Lelong number

$$\Theta(T, p) \geq 0,$$

which is identically zero for smooth currents, and where at the other extreme

$$\Theta(T_Z, p) = \mathrm{mult}_p(Z)$$

gives the multiplicity of an analytic variety Z at a point.

For simplicity we assume $U = \mathbb{C}^n$, p is the origin, and we use the notations $B[r] = \{z \in \mathbb{C}^n : \|z\| \leq r\}$,

$\chi(r)$ = characteristic function of $B[r]$,

$B[r, R] = \{z \in \mathbb{C}^n : r \leq \|z\| \leq R\}$. $(r < R)$, $\omega = \frac{\sqrt{-1}}{2}(\sum_i dz_i \wedge d\bar z_i)$.

As mentioned above, by taking monotone limits the current T may be defined on suitable L^1-forms, such as $\chi(r)\,\omega^{n-p}$. In case $T = T_Z$,

$$T_Z(\chi(r)\,\omega^{n-p}) = \int_{Z \cap B[r]} \omega^{n-p} = \text{volume of } (Z \cap B[r])$$

by the Wirtinger theorem from section 2 of Chapter 0.

⌜ See P31. We have $f_n(r) < f_{n+1}(r) < \cdots < \chi(r)$

$\Rightarrow \int_{Z \cap B[r]} \omega^{n-p} = \lim_{n\to\infty} \int_Z f_n(r)\,\omega^{n-p}$ by Lebesgue. R.C.A. Rudin P22 monotone convergence theorem. ⅃