$\Rightarrow \quad {}^t g\, Y = X. \quad \Rightarrow$ Obviously ${}^t g$ is holomorphic

$${}^t g : \mathbb{C}^k_{U_{\lambda'}} \longrightarrow \mathbb{C}^k_{U_\lambda}$$

The quotient bundle $Q = \mathbb{C}^n/S$ is called the universal quotient bundle on $G(k,n)$. Note that under the identification $* : G(k,n) \longrightarrow G(n-k,n)$, the universal subbundle on $G(n-k,n)$ corresponds to the dual of the universal quotient bundle in $G(k,n)$, and likewise $Q \longrightarrow G(n-k,n)$ pulls back to the dual $S^* \longrightarrow G(k,n)$.

Pf. Let $* = f$.

$$\begin{array}{ccc} f^*S & \longrightarrow & S \\ \downarrow & & \downarrow \\ G(k,n) & \xrightarrow{f} & G(n-k,n) \end{array}$$

Then $f^*S \cong Q^*$.

To show the above, we will describe the universal quotient bundle $Q \longrightarrow G(k,n)$.

In general, suppose F is a subbundle of E.

$$\Rightarrow \quad \begin{array}{ccccc} U_\beta \times \mathbb{C}^n & \xleftarrow{\varphi_\beta} & E|_{U_\alpha} = E|_{U_\beta} & \xrightarrow{\varphi_\alpha} & U_\alpha \times \mathbb{C}^n \\ & & \cup & & \cup \\ U_\beta \times \mathbb{C}^k & \longleftarrow & F|_{U_\alpha} & \longrightarrow & U_\alpha \times \mathbb{C}^k \end{array}$$

$$\mathbb{C}^k \hookrightarrow \mathbb{C}^n$$
$$" \{ (\underbrace{* \cdots *}_{k}, 0 \cdots 0) \in \mathbb{C}^n \}$$