Away from E, π is one to one, i.e. π does not affect on multiplicities. $\Rightarrow$ $(\pi^*\omega) = \pi^*(\omega)$, i.e, $(\pi^*\omega=0) = \pi^*(\omega=0)$.
Recall that the definition of a divisor is a formal sum of hypersurfaces in $\tilde{M}_x$.

$\Rightarrow$ $D = \sum a_i V_i$. $\Rightarrow$ $\pi^*\omega = 0$ on E with multiplicity $(n-1)$.
$\Rightarrow$ For some i, $V_i = E$ and $a_i = n-1$.
$\Rightarrow$ $\pi^*\omega = 0$ on $\pi^*(\omega=0)$ with multiplicity 1.
For references of a divisor, see P130 ~ P131, specially ⊛. $\Rightarrow$ $K_{\tilde{M}} = [(\pi^*\omega=0)] = [\pi^*(\omega=0) + (n-1)E]$
$= [\pi^*(\omega=0)] + (n-1)[E] = \pi^*[\omega=0] + (n-1)E = \pi^* K_M + (n-1)E$.
In a nbd of E, the local defining function for $(\pi^*\omega=0)$ is $\pi^*(f)\cdot z_i^{n-1}$
In a nbd away from E, the local defining function is $\pi^* f = 0$.

Thus the formula is proved under the assumption that M has a meromorphic n-form; this is the easiest way to see the result.
To prove the lemma in general, we let $\underline{\mathcal{U}} = \{U_0, U_\alpha\}_\alpha$ be an open coordinate cover of M with $x \in U_0$, $x \notin U_\alpha$ and all sets U_α having nonempty intersection with U_0 lying in one coordinate patch with coordinates $z_1, z_2 \cdots z_n$. Let
$$\underline{\tilde{\mathcal{U}}} = \{\tilde{U}_\alpha = \pi^{-1}U_\alpha,\ \tilde{U}_i = \pi^{-1}U_0 \cap (l_i \neq 0)\}$$
be a corresponding cover for $\tilde{M}$; we compute the transition functions $\{g_{ij}, g_{i\alpha}, g_{\alpha\beta}\}$ for $K_{\tilde{M}}$ in terms of coordinates $z(i)_j$ on $\tilde{U}_i$ and $w_{i,\alpha} = \pi^* w_{i,\alpha}$ on $\tilde{U}_\alpha$, where $\{w_{i,\alpha}\}_i$ are coordinates on U_α in M. First we have in $\tilde{U}_1 \cap \tilde{U}_2$

" all sets U_α <u>having nonempty intersection</u>)