$\dim(\Lambda \cap V_{n-k+k-i+2}) = \dim(\Lambda \cap V_{n-i+2}) \overset{?}{\geq} k-i+2.$

Suppose the inquality does not hold. $\Rightarrow$ Since $\dim(\Lambda \cap V_{n-i+2}) \geq k-l \geq k-i+j. \Rightarrow j=1$ or 0

(i) $j=1$ and $\dim(\Lambda \cap V_{n-i+2}) = k-i+1.$

$\Rightarrow k-i+1 \geq k-l \Rightarrow l \geq i-1$, and $l \leq i-1$

$\Rightarrow l = i-1.$

$V_{n-i+2} = \{e_{i-1}, e_i, \cdots e_n\}$. Assume that $\sigma_1, \cdots \sigma_i$ lin. indept. without loss of generality

Suppose $\sigma_1 = * \sigma_2 + \cdots + * \sigma_{i-1} + a\sigma_i$

① $a \neq 0 \Rightarrow \sigma_1, \sigma_2 \cdots \sigma_{i-1}$ are linearly independent since $\dim \overline{\sigma_1(x) \cdots \sigma_{i-1}(x)} = i-1.$

$$\begin{array}{l} \Rightarrow \sigma_1 = \sigma_1 + 0 \\ \sigma_2 = 0 + \sigma_2 + \cdot \\ \vdots \\ \sigma_{i-1} = 0 + 0 \cdots + \sigma_{i-1} + 0 \\ \sigma_i = *\sigma_1 + \quad *\sigma_{i-1} + 0 \\ \vdots \end{array}$$

(column labels $i-1$, i above)

$$\Rightarrow L(x) = \begin{array}{c c} & \begin{array}{ccccc} & i-1 & i & i+1 & \end{array} \\ \begin{array}{c} \\ \\ \\ i-1 \\ i \\ i+1 \\ \\ n \end{array} & \left(\begin{array}{ccccc} (1, 0 \cdots\cdots & 0 & * & * & *) \\ (0, 1 & 0 & * & * & *) \\ \vdots & \vdots & \vdots & \vdots & \\ (0, 0 \cdots & 1 & * & * & *) \\ (0, 0 \cdots & 0 & 0 & * & *) \\ (0\ 0 & 0 & 0 & * & *) \\ \vdots & \vdots & \vdots & & \\ (0 & 0 & 0 & * & *) \end{array} \right) \end{array}$$

$\Rightarrow \dim(V_{n-i+2} \cap \Lambda) \geq k-l+1 = k-(i-1)+1 = k-i+2$

$\Rightarrow$ Contradiction

② $a=0$. $\Rightarrow$ Say, $\sigma_1, \sigma_3, \cdots \sigma_i$ are linear indepent.