$$\Rightarrow \quad \sigma^*_{k-i+1,\dots k-i+1} = \begin{vmatrix} \sigma^*_{k-i+1}, & \cdots & \sigma^*_{k-i+j} \\ \sigma^*_{k-i}, & \cdots & \sigma^*_{k-i+j-1} \\ \vdots & & \vdots \\ \sigma^*_{k-i-j+2} & \cdots & \sigma^*_{k-i+1} \end{vmatrix}$$

by the naturality of the Poincare isomorphism

$$\Rightarrow \quad f^*\sigma^*_{k-i+1,\dots} = \begin{vmatrix} f^*\sigma^*_{k-i+1}, & \cdots, & f^*\sigma^*_{k-i+j} \\ f^*\sigma^*_{k-i}, & \cdots, & f^*\sigma^*_{k-i+j-1} \\ \vdots & & \vdots \\ f^*\sigma^*_{k-i-j+2}, & \cdots, & f^*\sigma^*_{k-i+1} \end{vmatrix}$$

$$= \begin{vmatrix} C_{k-i+1}(E), & \cdots, & C_{k-i+j}(E) \\ \vdots & & \\ C_{k-i-j+2}(E), & \cdots, & C_{k-i+1}(E) \end{vmatrix}$$

by $f^*\sigma_r^* = C_r(E)$.

$\lrcorner\!\rfloor$

Finally, we will specialize our general Gauss-Bonnet formula to obtain a more classical form, and also to explain the terminology. Suppose that M is of real dimension $2n$, $E \longrightarrow M$ of complex rank n, and σ a global C^∞ section of E having nondegenerate zeros at points $p_\nu \in M$. For each ν, let $e_1, \cdots e_n$ be a frame for E around p_ν, $x = (x_1, \cdots x_{2n})$ oriented real coordinates on M centered around p_ν, and write

$$\sigma(x) = \sum (a^\nu_{\alpha k} + \sqrt{-1}\, b^\nu_{\alpha k}) \cdot x_\alpha \cdot e_k(x) + [2], \quad a^\nu_{\alpha k}, b^\nu_{\alpha k} \in \mathbb{R}.$$