$$\varphi\, \bar\partial \overset{=0}{\beta} = \lim_{\epsilon\to 0} \int_{\mathbb{C}^n - \{\|x\|\le\epsilon\}} \bar\partial(\varphi\beta) = \lim_{\epsilon\to 0} \int_{\mathbb{C}^n - \{\|x\|\le\epsilon\}} d(\varphi\beta)$$

Stokes' theorem

$$= -\lim_{\epsilon\to 0} \int_{\|x\|=\epsilon} \varphi\beta = -\varphi(0) = -\delta_0(\varphi) \quad \Rightarrow \quad \bar\partial T_\beta = \delta_0.$$

$\Rightarrow \quad d\,T_\beta = \partial T_\beta + \bar\partial T_\beta = \bar\partial T_\beta$ since $\partial T_\beta(\varphi) = T_\beta(\partial\varphi) = 0$ for all $\varphi \in C_c^\infty(\mathbb{C}^n)$, ($\because$ $\partial\varphi\wedge\beta$ is of type $(n+1, q)$).

$\Rightarrow \quad d\,T_\beta - T_{d\beta} = R(\beta) = \bar\partial T_\beta = \delta_0. \quad \Rightarrow \quad R(\beta) = \delta_0.$ $\lrcorner$

Explicitly, for $\varphi \in C_c^\infty(\mathbb{C}^n)$,

$$\varphi(0) = \int_{\mathbb{C}^n} \bar\partial\varphi\wedge\beta$$

and just as in the one-variable case this formula may be extended to non compactly supported forms to obtain

$$\varphi(0) = \int_{B[r]} \bar\partial\varphi\wedge\beta + \int_{\partial B[r]} \varphi\beta,$$

where $B[r] = \{z\in\mathbb{C}^n : \|z\| \le r\}$ is the ball of radius r in $\mathbb{C}^n$.

$\ulcorner$ I think if $\bar\partial T_\beta = \delta_0$, $\varphi(0) = -\int_{\mathbb{C}^n} \bar\partial\varphi\wedge\beta$, for

$$\bar\partial T_\beta(\varphi) = (-1)^{2n-1+1}\, T_\beta(\bar\partial\varphi) = T_\beta(\bar\partial\varphi) = \int_{\mathbb{C}^n} \beta\wedge\bar\partial\varphi$$

$$= (-1)^{2n-1}\int_{\mathbb{C}^n} \bar\partial\varphi\wedge\beta = -\int_{\mathbb{C}^n} \bar\partial\varphi\wedge\beta = \delta_0(\varphi) = \varphi(0).$$

$\lrcorner$

In case $\varphi \in \mathcal{O}(\mathbb{C}^n)$ is holomorphic, this reduces to the Bochner - Martinelli formula

$$\varphi(0) = \int_{\|z\|=r} \varphi(z)\, \beta(z,\bar z).$$