

Refer to the argument on P236, i.e. the n -planes in F through p are exactly the n -planes spanned by p together with $(n-1)$ -planes in \tilde{F} .

Thus \overline{pq} must pass through x_i , which is impossible since $x_i \in \overline{pq} \subset U_i$ ($x_i \notin U_i$). \Rightarrow We see that $F \cap U_i$ is smooth, and $T_{x_i}(Q) \cap Q$ is the cone through x_i over C_i .

\Rightarrow

Λ must therefore intersect U_i , $i=1$ or 2 , in one of the two tangent lines L_{i1} , L_{i2} to C_i through p ; i.e., Λ must be one of the four 2 -planes Λ_{ij} spanned by L_i and L_{ij} .

If $\Lambda \cap F$ is a line $\Rightarrow \Lambda \cap Q = \Lambda \cap F \cap V_u = \Lambda \cap F$ is a line since $\Lambda \subset V_u$. $\Rightarrow \Lambda \cap Q \subset T_{x_i}(Q)$
 $\Rightarrow \Lambda \cap Q \subset \Lambda \cap T_{x_i}(Q) \cap Q \Rightarrow \exists$ a point $q \in C_i$ s.t. $q \in \Lambda$. $\Rightarrow \overline{pq} \subset \Lambda$ since Λ is a plane
 $\Lambda \cap U_i$ is a line since, if $\Lambda = U_i$, then $F \cap \Lambda = F \cap U_i \Rightarrow$ double line = smooth conic ($\Rightarrow \Lambda \cap U_i = \overline{pq}$). \Rightarrow

If $\overline{pq} \cap C_i = \{q_1, q_2\}$, $q_1 \neq q_2$, then $\Lambda \cap C_i = \{q_1, q_2\} \Rightarrow \Lambda \cap F \cap U_i = \{q_1, q_2\} = \Lambda \cap U_i$ which is $\hat{\vee}$ one-point set \Rightarrow contradiction.