given by

$$z \longmapsto [1, \wp(z), \wp'(z)]$$

embeds $\mathbb{C}/\Lambda$ as the locus of the polynomial $f(x,y) = y^2 - 4x^3 + g_2 x + g_3$.

「 $\wp(z)$, $\wp'(z)$, 1 $\overbrace{}^{\text{correspond to}}$ generators of $H^0(S, \Omega^1(3p))$ with $h^0(S, \Omega^1(3p)) = 3$. see p222 ~ p223. 」

The differential $\omega = dz$ on $C = \mathbb{C}/\Lambda$ is the Poincaré residue

$$\omega = R\left(\frac{dx \wedge dy}{f(x,y)}\right) = \frac{dx}{\frac{\partial f}{\partial y}} = \frac{dx}{y},$$

and the inverse of ψ is the Abelian integral

$$\psi^{-1}(p) = \int_{p_0}^{p} \frac{dx}{y} \qquad (\text{mod periods})$$

where we take in this case $p_0 = \psi(0) = [0, 0, 1] \in \mathbb{P}^2$.

「 $R\left(\frac{dx \wedge dy}{f(x,y)}\right) = -\frac{dx}{\frac{\partial f}{\partial y}} = -\frac{dx}{2y}\Big|_{\text{on } f=0}$ see p147.

$$\psi^*(dx) = a(z)\, dz.$$

$$\psi^*(dx)\left(\frac{\partial}{\partial z}\right) = a(z) = dx\left(\psi_*\left(\frac{\partial}{\partial z}\right)\right)$$

But since $\psi(z) = (\wp(z), \wp'(z))$, $\psi_*\left(\frac{\partial}{\partial z}\right) = \wp'(z)\frac{\partial}{\partial x} + \wp''(z)\frac{\partial}{\partial y}$. $\Rightarrow$ $dx\left(\psi_* \frac{\partial}{\partial z}\right) = \wp'(z)$

$$\Rightarrow \psi^*(dx) = \wp'(z)\, dz \quad \Rightarrow \quad \psi^*\left(-\frac{dx}{2y}\right) = -\frac{\wp'(z)\, dz}{2\wp'(z)} = -\frac{dz}{2} = -\frac{\omega}{2}.$$

$$\Rightarrow \psi^*\left(\frac{dx}{y}\right) = \omega$$