

pencil $X_{p_2} = \sigma(p_2) \cap H$ of lines of X through p_2 .

Let $h_1 = h_2'$, and choose $p_3, p_4 \in h_1, p_1$ so that $\{p_1, p_2, p_3, p_4\}$ forms a tetrahedron. \Rightarrow Clearly

$$p_i = \bigcap_{j \neq i} h_j, \quad h_1 = \overline{p_2, p_3, p_4}, \quad h_2 = \overline{p_1, p_3, p_4} \text{ \& \& } h_3 = \overline{p_1, p_2, p_4}$$

$$h_4 = \overline{p_1, p_2, p_3}$$

and $h_1 = h_2' \Rightarrow h_1 = h_1 \cap h_2'$ is a α -plane.

But anyway, we can show that, as for h_1 , $h_3 \cap h_2'$ contains p_3' , $h_4 \cap h_2' \ni p_4' \Rightarrow h_2' \ni p_1', p_3', p_4'$.

Similarly, $h_3' \ni p_1', p_2', p_4'$, $h_4' \ni p_1', p_2', p_3'$.

$$\Rightarrow p_1' \in h_2' \cap h_3' \cap h_4'$$

Since $\{h_i'\}$ is linearly independent, $\{p_i'\} = h_2' \cap h_3' \cap h_4'$.

In general, $p_i' = \bigcap_{j \neq i} h_j'$ i.e., the points $\{p_i'\}$ are

the vertices of the tetrahedron T' having sides $\{h_i'\}$.

□

The line complex X thus associates to any tetrahedron T in P^3 a "dual" tetrahedron T^* both inscribed in and circumscribed about T .

¶ I think, the definition of 'inscribe' is as follows:

Given a tetrahedron T with sides h_1, h_2, h_3, h_4 and vertices

$$p_i = \bigcap_{j \neq i} h_j,$$