$\Rightarrow \dim(\Lambda \cap V_{n-k+i-a_i}) \geq i$ $\quad = \overline{V}_{n-k+i-a_i}$

If $i < \alpha$, $\quad \pi(V_{n-k+i-a_i}) = V_{n-k+i-a_i}$ and $\pi(\Lambda) = \overline{\Lambda}$.

$\Rightarrow \dim(\overline{\Lambda} \cap \overline{V}_{n-k+i-a_i}) \geq i$.

If $i > \alpha$. $\quad \dim(\Lambda \cap V_{n-k+i+1-a_{i+1}}) \geq i+1$.

$\Rightarrow \pi(V_{n-k+i+1-a_{i+1}}) = \overline{V}_{n-k+i-a_{i+1}} = \overline{V}_{n-k+i-a'_i}$

$\Rightarrow$ Since $\Lambda \supset L$ and $V_{n-k+i+1-a_{i+1}} \supset L$,

$\quad \dim(\overline{\Lambda} \cap \overline{V_{n-k+i-a_{i+1}}}) = \dim(\overline{\Lambda} \cap \overline{V_{n-k+i-a'_i}}) \geq i$.

$\Rightarrow \overline{\Lambda} \in \sigma_{a_1 \cdots \hat{a}_\alpha \cdots a_k}(V)$

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Thus we have the

Reduction Formula I. For any three indices $0 \leq \alpha, \beta, \gamma \leq k$ with $\alpha + \beta + \gamma = 2k+1$,

$$\#(\sigma_a \cdot \sigma_b \cdot \sigma_c)_{G(k,n)} = \begin{cases} 0 & \text{if } a_\alpha + b_\beta + c_\gamma > n-k \\ \#(\sigma_{a-a_\alpha} \cdot \sigma_{b-b_\beta} \cdot \sigma_{c-c_\gamma})_{G(k-1,n-1)} & \text{if } a_\alpha + b_\beta + c_\gamma = n-k \end{cases}$$

Pf Here $a - a_\alpha = a_1, a_2, \cdots \hat{a}_\alpha \cdots a_k$. $b - b_\beta = b_1 \cdots \hat{b}_\beta \cdots b_k$.

$\#(\sigma_a \cdot \sigma_b \cdot \sigma_c) = \#(\sigma_a \cap \sigma_b \cap \sigma_c) = \#(\sigma_{a-a_\alpha} \cap \sigma_{b-b_\beta} \cap \sigma_{c-c_\gamma})$

$= \#(\sigma_{a-a_\alpha} \cdot \sigma_{b-b_\beta} \cdot \sigma_{c-c_\gamma})$ assuming that $\sigma_a \cap \sigma_b \cap \sigma_c$ meet transversely.

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Note that in case we take $\beta = \gamma = k$, this reduction applies if $a_1 = n-k$; in case we take $\gamma = k$,