and likewise for V'.

$\ulcorner$ In the proof of Lefschetz hyperplane theorem, P157.

$$H^p(\mathbb{P}^n, \Omega^q_{\mathbb{P}^n}) \longrightarrow H^p(V, \Omega^q_V)$$

is isomorphic for $p+q \le n-2$ and injective for $p+q = n-1$.

Let $p = 1, 2$, $q = 0$, $n \ge 4$.

$\Rightarrow$ We get $H^p(\mathbb{P}^n, \mathcal{O}) \cong H^p(V, \mathcal{O})$, $p = 1, 2$, and by P49 & P118, $\Rightarrow \overset{\|}{0}$

$$0 \to \mathbb{Z} \longrightarrow \mathcal{O} \longrightarrow \mathcal{O}^* \longrightarrow 0$$ is exact, see P37.

$$\underset{\overset{\|}{0}}{H^1(V, \mathcal{O})} \longrightarrow H^1(V, \mathcal{O}^*) \xrightarrow{\cong} H^2(V, \mathbb{Z}) \longrightarrow \underset{\overset{\|}{0}}{H^2(V, \mathcal{O})}$$

$$\mathbb{Z} = H^2(\mathbb{P}^n, \mathbb{Z}) = H^2(V, \mathbb{Z}) \cong H^1(V, \mathcal{O}^*) = \mathrm{Pic}(V)$$ $\lrcorner$

Thus, if $\varphi : V \longrightarrow V'$ is biholomorphic, $\varphi^* K_{V'} = K_V$

$\Rightarrow \varphi^*(H|_{V'}) = H|_V$, so V and V' are projectively isomorphic.

$\ulcorner$ Clearly $\varphi^* K_{V'} = K_V$. $\Rightarrow \varphi^* K_{V'} = \varphi^*[(d-n-1)H]|_V$

$$\Rightarrow \quad \varphi^* H|_V \overset{?}{=} \varphi^* H|_{V'} \qquad \overset{\|}{\varphi^*[(d-n-1)H]|_{V'}}.$$

$\mathbb{Z} = H^2(\mathbb{P}^n, \mathbb{Z}) = H^2(V, \mathbb{Z}) \cong H^1(V, \mathcal{O}^*)$ implies that every divisor of V is a multiple of $V \cap H$ where H is a hyperplane in $\mathbb{P}^n$. Similarly for V'.

$\Rightarrow \varphi^*(H|_V) = d_0(H|_{V'})$