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$\Rightarrow \quad * \sigma_{1,1,\cdots 1} = \sigma_r$. See p 200.

The Gauss-Bonnet formula gives us a relatively concrete interpretation of Chern classes in general as follows. Let M be a compact, oriented manifold, $E \to M$ a complex vector bundle of rank k, and $\sigma = (\sigma_1 \cdots \sigma_k)$ k global C^∞ sections of E. We define the degeneracy set $D_i(\sigma)$ to be the set of points $x \in M$, where $\sigma_1 \cdots \sigma_i$ are linearly dependent, i.e.,

$$D_i(\sigma) = \{x ;\ \sigma_1(x) \wedge \cdots \wedge \sigma_i(x) = 0\}.$$

We say that the collection σ of sections is generic if, for each i, σ_{i+1} intersects the subspace of E spanned by $\sigma_1, \cdots \sigma_i$ transversely – so that $D_{i+1}(\sigma)$ is, away from $D_i(\sigma)$, a submanifold of codimension $2(k-i)$ – and if, moreover, integration over $D_{i+1}(\sigma) - D_i(\sigma)$ defines a closed current as discussed in Section 1 of this chapter.

「 $D_i(\sigma) = \{x ;\ \sigma_1(x) \wedge \cdots \wedge \sigma_i(x) = 0\}$ is closed in M

Let $E_i = \{(x, v) \in E|_{M - D_i(\sigma)} \mid v$ is a linear combination of $\sigma_1(x), \cdots$, and $\sigma_i(x)$. $\}$

$\Rightarrow$ Consider $\sigma_{i+1} : M - D_i(\sigma) \longrightarrow E|_{M-D_i(\sigma)}$.

$\Rightarrow$ $\sigma_{i+1}^{-1}(E_i) = D_{i+1}(\sigma) - D_i(\sigma)$ and σ_{i+1} intersects E_i transversely, where E_i is a submanifold of $E|_{M-D_i(\sigma)}$ of codimension $(k-i)$. (real codimension is $2(k-i)$.)

$\Rightarrow$ By the well-known theorem ("sort of" implicit function theorem), $D_{i+1}(\sigma) - D_i(\sigma)$ is a submanifold of $M - D_i(\sigma)$ of real codimension $2(k-i)$. See M. Hirsch p22. Th3.3 」