

$\Rightarrow$  Up to translation,  $\rho^* \mathbb{H} = m_2^* \mathbb{H}$ .

If we let  $z_1 = x_1 + i x_3$ , and  $z_2 = x_2 + i x_4$ , then

$$\begin{aligned} m_2^* (dx_1 \wedge dx_3 + dx_2 \wedge dx_4) \\ = 4(dx_1 \wedge dx_3 + dx_2 \wedge dx_4) \quad \text{and} \\ \rho^* (dx_1 \wedge dx_3 + dx_2 \wedge dx_4) = \rho^* \left( \frac{1}{2i} d\bar{z}_1 \wedge dz_1 + \frac{1}{2i} d\bar{z}_2 \wedge dz_2 \right) \\ = 4 \left( \frac{1}{2i} d\bar{z}_1 \wedge dz_1 + \frac{1}{2i} d\bar{z}_2 \wedge dz_2 \right). \end{aligned}$$

Assume that  $\rho$  is represented by  $\begin{pmatrix} a & b \\ c & e \end{pmatrix}$ .

$$\begin{aligned} \Rightarrow \rho^* dz_1 &= a dz_1 + b dz_2 & \rho^* dz_2 &= c dz_1 + e dz_2 \\ \rho^* d\bar{z}_1 &= \bar{a} d\bar{z}_1 + \bar{b} d\bar{z}_2 & \rho^* d\bar{z}_2 &= \bar{c} d\bar{z}_1 + \bar{e} d\bar{z}_2 \end{aligned}$$

$\Rightarrow$  By simple computations, we obtain

$$\begin{aligned} |a|^2 + |c|^2 &= 4 & |b|^2 + |e|^2 &= 4 \\ a\bar{b} + c\bar{e} &= 0 \end{aligned}$$

Since  $\rho(\Lambda) \subset \Lambda$ , where  $\Lambda = \mathbb{C}^2 / \Lambda$ ,  $a, b, c$  and  $e$  are integers.  $\Rightarrow g \circ \rho = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = m_2$ .

The possible choices of  $g$  are

$$\begin{aligned} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}. \end{aligned}$$

$$\Rightarrow g^* \omega = \pm \omega. \quad \omega = dx_1 \wedge dx_3 + dx_2 \wedge dx_4$$

$$\Rightarrow \text{If } g^* \omega = -\omega, \text{ then } (g \circ \rho)^* \omega = m_2^* \omega$$

$$\Rightarrow -4\omega = 4\omega \Rightarrow \text{Contradiction.}$$

$\Rightarrow$  If  $g^* \omega = \omega$ , and  $g$  is not the identity, then  $g$  induces an automorphism of  $B$ , and by the assumption,