$$0\begin{vmatrix} z_{11}, & z_{12} \\ z_{21}, & z_{22}\end{vmatrix} + 1\begin{vmatrix} 0, & z_{12} \\ 1, & z_{22}\end{vmatrix}$$

$\Rightarrow p(\Lambda) \perp (0, 0, \cdots 0, 1)$. $\Rightarrow p(\Lambda)$ lies in the hyperplane $z_6 = 0$. In general, from

$$\det\begin{pmatrix} \Lambda_I \\ e_1 \\ \vdots \\ e_{n-k}\end{pmatrix} = 0,$$ we get a hyperplane in $\mathbb{P}^{\binom{n}{k}-1}$.

We can always find, for $\Lambda \neq \Lambda' \in G(k,n)$, an $(n-k)$-plane V_{n-k} such that $\Lambda \cap V_{n-k} \neq (0)$, $\Lambda' \cap V_{n-k} = (0)$, so p is 1-1; and since, in each open set $U_I = \{\Lambda : |\Lambda_I| \neq 0\}$ the Euclidean coordinates on $G(k,n)$ described above appear as

$$a_{jk} = \frac{|\Lambda_{I-j+k}|}{|\Lambda_I|},$$

the map p has nonzero differential. Thus the Plücker mapping is an embedding.

If $\Lambda \neq \Lambda' \in G(k,n)$, $\exists$ a vector $v \in \mathbb{C}^n$, s.t $v \in \Lambda$, and $v \notin \Lambda'$. $\Rightarrow \exists$ $V_{n-k} = \langle v, w_1, \cdots w_{n-k-1}\rangle$ s.t $\langle V_{n-k} \cup \Lambda'\rangle = \mathbb{C}^n$. $\Rightarrow V_{n-k} \cap \Lambda' = (0)$.
$\Rightarrow V_{n-k} \cap \Lambda \neq (0)$.
$\Rightarrow$ Suppose that H is the hyperplane containing $p(\Lambda)$ & $p(\sigma_1(V))$. $\Rightarrow$ As we saw above, the hyperplane H does not contain Λ', since $\det\begin{pmatrix}\Lambda_I' \\ V_{n-k}\end{pmatrix} \neq 0$.