$$(\theta_{ji}+\theta)\wedge De_i\otimes\sigma + d(\theta_{ji}+\theta)\otimes e_i\otimes\sigma - (\theta_{ji}+\theta)\wedge(e_i\otimes\theta\sigma)$$

$$= d(\theta_{ji}+\theta)\otimes e_i\otimes\sigma - (\theta_{ji}+\theta)\wedge\theta_{ik}\, e_k\otimes\sigma$$

$$+ d(\theta_{ji}+\theta)\otimes e_i\otimes\sigma - (\theta_{ji}+\theta)\wedge e_i\otimes\theta\sigma$$

$$= 2\, d(\theta_{ji}+\theta)\otimes(e_i\otimes\sigma) - (\theta\wedge\theta)_{jk}\, e_k\otimes\sigma - \cancel{\theta\wedge\theta_{jk}\, e_k\otimes\sigma}$$

$$- \theta_{ji}\wedge\theta\, e_i\otimes\sigma - \theta\wedge\theta\, e_i\otimes\sigma$$

$$= 2\, d(\theta_{ji}+\theta)\otimes(e_i\otimes\sigma) - (\theta\wedge\theta)_{jk}\, e_k\otimes\sigma - \theta\wedge\theta\, e_i\otimes\sigma$$

$$= (d\theta_{ji} - (\theta\wedge\theta)_{ji})\, e_{\hat{i}}\otimes\sigma + (d\theta - \theta\wedge\theta)\, e_i\otimes\sigma$$

$$+ d(\theta_{ji}+\theta)\otimes e_i\otimes\sigma. \quad = (\Theta_E)_{ji}\, e_i\otimes\sigma + \Theta_L\, e_i\otimes\sigma$$

$$+ d(\theta_{ji}+\theta)\otimes e_i\otimes\sigma.$$

Wrong

$$D_{E\otimes L}(e_i\otimes\sigma) = De_i\otimes\sigma + e_i\otimes D\sigma$$

$$= \theta_{ij}\, e_j\otimes\sigma + e_i\otimes\tau\sigma$$

$$\Rightarrow D_{E\otimes L}(\theta_{ji}+\tau)\, e_i\otimes\sigma$$

$$= d(\theta_{ji}+\tau)\otimes e_i\otimes\sigma - (\theta_{ji}+\tau)\wedge D_{E\otimes L}(e_i\otimes\sigma)$$

$$= d(\theta_{ji}+\tau)\otimes e_i\otimes\sigma - (\theta_{ji}+\tau)\wedge(\theta_{ik}\, e_k\otimes\sigma + e_i\otimes\tau\sigma)$$

$$= d(\theta_{ji}+\tau)\otimes e_i\otimes\sigma - \theta_{ji}\wedge\theta_{ik}\, e_k\otimes\sigma - (\tau\wedge\theta_{ik})\, e_k\otimes\sigma$$

$$- (\theta_{ji}\wedge\tau)\otimes e_i\otimes\sigma - \tau\wedge\tau\, e_i\otimes\sigma$$

$$= (d\theta_{ji} - (\theta\wedge\theta)_{ji})\, e_i\otimes\sigma + (d\tau - \tau\wedge\tau)\otimes e_i\otimes\sigma$$

$$- (\underbrace{\tau\wedge\theta_{ik} + \theta_{jk}\wedge\tau}_{=0})\otimes e_k\otimes\sigma$$

$$= (\Theta_E)_{ji}\, e_i\otimes\sigma + \Theta_L\, e_i\otimes\sigma$$

$$\Rightarrow \Theta_{E\otimes L} = \begin{pmatrix} (\Theta_E)_{11} & (\Theta)_{12} & \cdots & (\Theta)_{1r} \\ \vdots & \vdots & & \vdots \\ (\Theta_E)_{r1} & \cdots & & (\Theta)_{rr} \end{pmatrix} + \begin{pmatrix} \Theta_L & & \Theta_L \\ \vdots & & \vdots \\ \Theta_L & & \Theta_L \end{pmatrix}$$

MOOKEUK CO