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We are using the different convention from p72.
 $e_1, e_2 \dots e_s$ $De_i = \theta_{ij} e_j$

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$$D_E(e_j) = D_s(e_j) + A(e_j) = \theta_{\lambda j} e_{s+\lambda} + A_{s+j} A_{\lambda, j} e_{s+\lambda}.$$

$$\Rightarrow D_E(e_1) = \theta_{11} e_1 + \theta_{21} e_2 + \dots + \theta_{s1} e_s + A_{11} e_{s+1} + \dots + A_{rs1} e_r$$

$$D_E(e_2) = \theta_{12} e_1 + \theta_{22} e_2 + \dots + \theta_{s2} e_s + A_{12} e_{s+1} + \dots + A_{rs2} e_r$$

\vdots

$$\Rightarrow \theta_E = \begin{pmatrix} \theta_s & {}^t \bar{A} \\ A & \theta_Q \end{pmatrix}$$

$$\Rightarrow \Theta_E = d\theta_E + \theta_E \wedge \theta_E$$

$$\Rightarrow \begin{pmatrix} \theta_s & {}^t \bar{A} \\ A & \theta_Q \end{pmatrix} \begin{pmatrix} \theta_s & {}^t \bar{A} \\ A & \theta_Q \end{pmatrix} = \begin{pmatrix} \theta_s \wedge \theta_s - {}^t \bar{A} \wedge A, & * \\ * & -A \wedge {}^t \bar{A} + \theta_Q \wedge \theta_Q \end{pmatrix}$$

$$\Rightarrow \Theta_E|_S = d\theta_s + \theta_s \wedge \theta_s - {}^t \bar{A} \wedge A$$

$$\Theta_E|_Q = d\theta_Q + \theta_Q \wedge \theta_Q - A \wedge {}^t \bar{A}$$

$$\Rightarrow \Theta_E|_S = \theta_s - {}^t \bar{A} \wedge A \Leftrightarrow \theta_s = \Theta_E|_S + {}^t \bar{A} \wedge A$$

$$\Theta_E|_Q = \theta_Q - A \wedge {}^t \bar{A} \Leftrightarrow \theta_Q = \Theta_E|_Q + A \wedge {}^t \bar{A}$$

$$\Rightarrow \theta_Q \geq \Theta_E|_Q \Rightarrow \theta_s \leq \Theta_E|_S \quad *$$