$\Rightarrow$ $V \cap V_3 = f_L(L)$ is well-defined, since for every point $x \in L$, V is in $T_x(X)$, and V is tangent to X at x.

Suppose $V = \overline{L, L'}$ $\Rightarrow$ $L' \cap L \neq \phi$ $\Rightarrow$ $L' \in B_L$ if $L' \subset X$. $\Rightarrow$ $L' \subset T_x(X)$ for all $x \in L$
$\Rightarrow$ L' passes all $x \in L$ $\Rightarrow$ $L = L'$ $\Rightarrow$ contradiction
$\Rightarrow$ L' can not lie on X. $\Rightarrow$ E_L is isormorphic to B_L even for L special.

The image Q of F, on the other hand, is readily seen to be a quadric surface — in fact, it is just the intersection of $V_3 \subset \mathbb{P}^5$ with the quadric hypersurface

$$\bigcup_{x \in L} T_x(\ \ X).$$

$\ulcorner$ First , since $\dim T_x(X) = 3$, and x varies over L, $\dim \bigcup_{x \in L} T_x(X) = 4$.

$\tilde{f}_L(F) = \{ \overline{L, L'} \cap V_3 \mid L' \in F \}$
$\Rightarrow$ $p \in \tilde{f}_L(F)$ $\Rightarrow$ $\overline{p, x, L} \cap V_3 \ni p$ $\Rightarrow$ $\tilde{f}_L(F) \subset V_3 \cap (\bigcup_{x \in L} T_x(X))$. Conversely, $p \in V_3 \cap T_x(X)$ $\Rightarrow$ $\overline{p, x, L} \cap V_3 \ni p$. since $\overline{p, x}$ represents a normal vector L_p, $\overline{p, x} \in F$ $\Rightarrow$ $p \in \tilde{f}_L(F)$.
($\because$ $\overline{p, x} \subset T_x(X)$).