① $x \in \overline{D} - D$. Suppose $x \in \Delta^n - V$. $\Rightarrow \exists$ open U s.t $U \cap \Delta^n - V = \phi$. $U \cap D \neq \phi \Rightarrow$ Since D is closed in U, $x \in D \Rightarrow$ Contradiction $\Rightarrow x \in V$.

Assume $x \in U_1$. $x \notin D \cap U_1 \neq \phi$, $x \in O$ is an open set in $U_1 \Rightarrow$ O is open in Δ^n. $\Rightarrow O \cap D \neq \phi$
$\phi \neq O \cap D \cap U_1 = O \cap D \Rightarrow x$ is a limit point of $D \cap U_1$ in $U_1 \Rightarrow x \in \overline{D \cap U_1} \Rightarrow \overline{D} \subset \overline{D \cap U_1} \cup \overline{D \cap U_2}$.
$\Rightarrow \overline{D} = \overline{D \cap U_1} \cup \overline{D \cap U_2}$, since, clearly, $\overline{D \cap U_1} \cup \overline{D \cup U_2} \subset \overline{D}$.

② $x \in \Delta^n \Rightarrow$ If $\exists$ open set U s.t $U \cap \overline{D} = \phi$, then O.K. If not so, $x \in \overline{D \cap U_1}$ in U_1.
$\Rightarrow$ Since $\overline{D \cap U_1}$ is analytic in U_1, $\exists$ O open set in U_1 s.t $O \cap \overline{D \cap U_1} = (f_1 = \cdots = f_l = 0)$, where $f_1 \cdots f_l$ holomorphic on O. $\Rightarrow (f_1 = \cdots = f_l = 0) = U_1 \cap O \cap \overline{D} = \overline{D} \cap O$.
$\Rightarrow \overline{D}$ is analytic in Δ^n.

In case $D \not\subset U_1 \cup U_2$, since the problems are related to V, nothing changes outside $U_1 \cup U_2$, and we don't need to worry anything. //

The essential case is thus when $n=2$ and $V = \{ z_1 = z_2 = 0 \}$ is the origin. We shall prove the result in this situation, from which the general conclusion may be drawn by analogy.

Let $\Delta' = \{ |z_1| < 1, \ |z_2| < 1, \ z_1 \neq 0 \} \cong \Delta^* \times \Delta$.