sversely almost everywhere and such that

$$\partial C = A - A'.$$

The proof that $\#(A\cdot V) = \#(A'\cdot V)$ then proceeds exactly as at the beginning of this section. Consequently V defines a linear functional

$$H_{2n-2k}(M, \mathbb{Z}) \longrightarrow \mathbb{Z};$$

the corresponding cohomology class $\eta_V \in H^{2n-2k}(M)$ is the fundamental class of V.

$\ulcorner$

$$L_V : H_{2n-2k}(M, \mathbb{Z}) \longrightarrow \mathbb{Z}$$

$$\alpha \longmapsto L_V(\alpha) = \#(V\cdot A)$$

where $\alpha = [A]$

$\Rightarrow$ Since $H^{2n-2k}(M, \mathbb{Z}) \overset{\phi}{\cong} \mathrm{Hom}\,(H_{2n-2k}(M, \mathbb{Z}), \mathbb{Z})$,

$\exists\ \eta_V \in H^{2n-2k}(M, \mathbb{Z})$ s.t $\phi(\eta_V) = L_V$. $\lrcorner$

Note: When we speak of the fundamental class of a variety $V \subset M$ we may also refer to its Poincare dual – that is; the element of homology given by the linear functional

$$H_{DR}^{2k}(M) \longrightarrow \mathbb{C}$$

$$[\varphi] \longmapsto \int_V \varphi.$$

$\ulcorner$

$$\int_V \varphi = \#(\bar{V}\cdot B), \quad B \text{ is a } (2n-2k) \text{ cycle}$$

Poincare-dual to φ and $\bar{V}$ is a $2k$ cycle.