

$\Rightarrow \tilde{\phi}(l_{x_3}) = x_3 \subset \tilde{\phi}(\sigma_{2,1}(p, h_{1,2})) = \overline{x_1, x_2} \Rightarrow x_i$ 's  
 lie on a line i.e. they are collinear  $\Rightarrow$  Contradiction.  $\Rightarrow l_{x_i}$ 's are not coplanar.  
 $\Rightarrow$  a line  $l$  meets all  $l_{x_i}$ 's  $\Leftrightarrow$  it passes the point  $p$ , (<sup>otherwise</sup> if  $l$  meets all  $l_{x_i}$ 's, then  
 $\langle p, l \rangle = \mathbb{P}^2 = \overline{l_{x_i}'s}$ )  $\quad \sqcup$

In the first case,  $V_2$  is contained in  $\tilde{\phi}(\sigma_{2,1}(h))$  — hence equal to — the Schubert cycle  $\sigma_{2,1}(h)$  of lines lying in  $h$ ; in the second case  $V_2$  is contained in  $\tilde{\phi}(\sigma_2(p))$ , and so equal to, the Schubert cycle  $\sigma_2(p)$  of lines through  $p$ .

$$\Gamma \quad V_2 \stackrel{?}{\subset} \tilde{\phi}(\sigma_{2,1}(h)).$$

$\forall x \in V_2 \Rightarrow l_x \cap l_{x_i} \neq \emptyset$  for all  $i$  since  $V_2 \subset$

$G \stackrel{?}{\cap} T_{x_i}$  by the argument above  $\Rightarrow$  By 1 above,

$l_x \subset h = \overline{p_{12}, p_{13}, p_{23}} \Rightarrow l_x \in \sigma_{2,1}(h) \Rightarrow \tilde{\phi}(l_x) = x \in$

$\tilde{\phi}(\sigma_{2,1}(h)) \Rightarrow V_2 \subset \tilde{\phi}(\sigma_{2,1}(h)) \Rightarrow$  Since  $\dim \tilde{\phi}(\sigma_{2,1}(h))$

$= 2$  by the result on p 256 ( $\sigma_{2,1}(h) = \mathbb{P}^2$ ),  $V_2 = \tilde{\phi}(\sigma_{2,1}(h))$ .

$$V_2 \stackrel{?}{\subset} \tilde{\phi}(\sigma_2(p))$$

$\forall x \in V_2 \Rightarrow$  Again  $l_x \cap l_{x_i} \neq \emptyset$  for all  $i$ .  $\Rightarrow l_x \ni p$

$\Rightarrow l_x \in \sigma_2(p) \Rightarrow x \in \tilde{\phi}(\sigma_2(p)) \Rightarrow V_2 \subset \tilde{\phi}(\sigma_2(p))$

$\Rightarrow$  Again  $V_2 = \tilde{\phi}(\sigma_2(p))$ .  $\quad \sqcup$

We will henceforth write the Schubert cycles on  $G$  simply as  $\sigma(p)$ ,  $\sigma(h)$ ,  $\sigma(l)$ , and  $\sigma(p, h)$ . In