

(C) Suppose now that the divisors $D_i = (f_i)$ meet at a finite number of points P_j interior to U . The total number of intersections of the D_i in U is defined by

$$(D_1 \cdots D_n)_U = \sum_U (D_1 \cdots D_n)_{P_j}.$$

We shall prove

The total intersection number is invariant under continuous deformation of the D_i .

Proof. We assume that $f_{t,i}(z) \in \mathcal{O}(\bar{U})$ is continuous in t and has divisor $D_i(t)$ with $f_{0,i} = f_i$ and $D_i(0) = D_i$.

Since $\sum_i |f_i(z, t)|^2 \geq C > 0$ for $z \in \partial U$ and $|t| < \varepsilon$, the divisors $D_i(t)$ will meet at isolated points interior to U . The total intersection number

$$(D_1(t) \cdots D_n(t))_U$$

is on the one hand an integer and on the other hand, by the continuity principle, continuous in t . Consequently, it is constant. Q.E.D.

$$\text{If } (D_1, \cdots, D_n)_U = \sum_U (D_1, \cdots, D_n)_{P_j} = \sum_U \text{Res}_{P_j} \left(\frac{df_1}{f_1} \wedge \cdots \wedge \frac{df_n}{f_n} \right)$$

$$= \sum_{P_j \in f_0^{-1}(0)} \text{Res}_{P_j} \left(\frac{df_1}{f_1} \wedge \cdots \wedge \frac{df_n}{f_n} \right) = \lim_{t \rightarrow 0} \sum_{P_{j,t} \in f_t^{-1}(0)} \text{Res}_{P_{j,t}} \left(\frac{df_{t,1}}{f_{t,1}} \wedge \cdots \wedge \frac{df_{t,n}}{f_{t,n}} \right)$$

$$= \lim_{t \rightarrow 0} \sum_{P_{j,t}} (D_1(t), \cdots, D_n(t))_{P_{j,t}} = (D_1(t), \cdots, D_n(t))_U \Rightarrow$$