Suppose $D \sim D'$. $\Rightarrow$ $\exists$ a meromorphic function h on M s.t $D = D' + (h=0)$.

$\Rightarrow$ $\pi_1^{-1} D = \pi_1^{-1} D' + (h \circ \pi)$ generically. $\Rightarrow$

$\pi_1^{-1} D = \pi_1^{-1} D' + (h \circ \pi_1)$. Again $\pi_2(\pi_1^{-1} D) = \pi_2(\pi_1^{-1} D') + (h \circ \pi_1 \circ \pi_2^{-1})$ generically.

$\Rightarrow$ $h \circ \pi_1 \circ \pi_2^{-1}$ can be extended to N as a meromorphic function. $\Rightarrow$ $\pi_2(\pi_1^{-1} D) \sim \pi_2(\pi_1^{-1} D')$

Thus the total transform map preserves linear equivalence.

$\pi : \widetilde{\mathbb{P}}^2 \longrightarrow \mathbb{P}^2$ is the blow-up of $\mathbb{P}^2$ at $p \in \mathbb{P}^2$.

Consider two conics C_1, C_2 s.t $C_1 \ni p$ $C_2 \not\ni p$.

$\Rightarrow$ Clearly $C_1 \sim C_2$ $\Rightarrow$ By P130 & P134, $[\pi^* C_1] = [\pi^* C_2]$ $\Rightarrow$ $\pi^* C_1 \sim \pi^* C_2$. $\Rightarrow$ $\pi^* C_1 = \widetilde{C}_1 + E$.

$\Rightarrow$ $\pi^* C_2 - \widetilde{C}_1 \sim E$ $\Rightarrow$ $\pi_*(\pi^* C_2 - \widetilde{C}_1) = C_2 - C_1$

$\pi_* E = p$ $\Rightarrow$ $C_2 - C_1 \nsim p = \pi_* E$.

Thus the proper transform does not preserve linear equivalence. ┘

"Comments on the proper transform".

$f : M \longrightarrow N$ birational, $D \subset M$ an effective divisor. $\Rightarrow$ $\exists$ birational inverse $g : N \longrightarrow M$ s.t $g \circ f$ & $f \circ g$ are identities as rational maps.

$$N \xrightarrow{g} M \supset D.$$

$\Rightarrow \exists$ open set U in M s.t $U \cap D = \{h = 0\}$.