

$$\omega = \frac{\sqrt{-1}}{2} \left( \sum_{i,j} h_{i\bar{j}} dz_i \wedge d\bar{z}_j \right)$$

is real if  $\bar{h}_{ij} = h_{j\bar{i}}$ , strictly positive if the matrix  $h_{i\bar{j}}$  is positive definite, and closed exactly when the corresponding hermitian metric

$$ds^2 = \sum_{i,j} h_{i\bar{j}} dz_i d\bar{z}_j$$

is Kähler. The powers  $\omega^p$  of a Kähler form define closed, positive  $(p,p)$  currents.

$\square$   $d\omega^p = 0$  since  $d\omega = 0$ . See P110.

$$\Rightarrow (\sqrt{-1})^{n-p} \text{ constant } \int_M \omega \wedge \eta \wedge \bar{\eta} = \pm 1 \text{ Constant } \int_M \sum |\eta_i|^2 \Phi(z)$$

$\wedge \bar{\Phi}(z) \Rightarrow (\pm 1) \omega^p$  is positive  $(p,p)$  current.  $\square$

3. A real function  $\varphi \in L^1(M, \text{loc})$  is said to be plurisubharmonic in case  $\sqrt{-1} \partial \bar{\partial} \varphi$  is positive  $(1,1)$  current. Here the derivatives are taken in the sense of distributions. Plurisubharmonic functions define potentials essentially suitable for complex function theory.

Lemma ( $\partial \bar{\partial}$ -Poincaré lemma). Let  $T$  be a closed, positive  $(1,1)$ -current. Then locally

$$T = \sqrt{-1} \partial \bar{\partial} \varphi$$

for a real plurisubharmonic function  $\varphi$ , which is unique