

$\Rightarrow \psi = \varphi_v \circ \varphi_u^{-1}$. This proves that $\{(\alpha|_u(\sigma_i^*), \alpha|_u(\sigma_j^*))\}$ defines a section of E over S .

$$\begin{aligned} p_g(S) &= h^{2,0}(S) = h^{0,2}(S) = \dim H^{0,2}(S) \\ &= \dim H^2(S, \mathcal{O}) = 0 \Rightarrow H^2(S, \mathcal{O}) = E_2^{2,0} = 0 \text{ if we take } \mathcal{L} = \mathcal{O}. \end{aligned}$$

I don't understand what "sufficiently ample" means. Maybe??

$$S \subset \mathbb{P}^N \rightarrow (i^*[H])^k = \mathcal{L} \text{ for } k \geq k_0, \quad k_0 \text{ some positive integer} \Rightarrow H^q(S, \mathcal{L}) = 0 \text{ for } q > 0.$$

\Rightarrow

Example

Take $S = \mathbb{P}^2$, so that the $p_g = 0$ condition is satisfied. Then there exists a rank-two holomorphic vector bundle $E \rightarrow \mathbb{P}^2$ and section $s \in H^0(\mathbb{P}^2, \mathcal{O}(E))$ that defines any given Z .

$$\Gamma \quad p_g(S) = \dim H^{2,0}(\mathbb{P}^2) = 0 \text{ by P49.} \quad \Rightarrow$$

If Z is nonempty, then we claim that s is unique up to a constant, and the vector bundle E is not a global extension

$$0 \rightarrow L \rightarrow E \rightarrow L' \rightarrow 0$$

of line bundles on \mathbb{P}^2 . Thus, in this manner