

two points. Second:

$H \cap \bar{V}_6$  is a smooth quadric of dimension 4.

$$\begin{aligned} & T_{p_1}(H) \cap T_{p_{11}}(H) \cap H \cap \bar{V}_6 \\ &= T_{p_1}(H \cap \bar{V}_6) \cap T_{p_{11}}(H \cap \bar{V}_6) \cap (H \cap \bar{V}_6) \text{ by the argument above.} \end{aligned}$$

Note: ①  $T_{p_1}(H \cap \bar{V}_6) \cap (H \cap \bar{V}_6)$  is the cone through  $p_1$  over a smooth quadric of dim. 2, by P234 \*.

$$\begin{aligned} \text{② } IP_{11}^{n-3} \cap \bar{V}_6 &\subset T_{p_1}(H) \cap T_{p_{11}}(H) \cap \bar{V}_6 \\ &= T_{p_1}(H \cap \bar{V}_6) \cap T_{p_{11}}(H \cap \bar{V}_6) \end{aligned}$$

is an 1-plane disjoint from  $\overline{p_1 p_{11}}$  since  
 $\dim IP_{11}^{n-3} + \dim \bar{V}_6 = n-3+5 = n+2 > n+1 = \dim P^{n+1}$  and  
 $\dim (T_{p_1}(H \cap \bar{V}_6) \cap T_{p_{11}}(H \cap \bar{V}_6)) = 3$ .

③  $T_{p_1}(H \cap \bar{V}_6) \cap T_{p_{11}}(H \cap \bar{V}_6) \cap (H \cap \bar{V}_6)$  is a quadric in the 3-plane  $T_{p_1}(H \cap \bar{V}_6) \cap T_{p_{11}}(H \cap \bar{V}_6)$  with singular set  $\overline{p_1 p_{11}}$ . See P822 note.

$$\begin{aligned} \Rightarrow T_{p_1}(H \cap \bar{V}_6) \cap T_{p_{11}}(H \cap \bar{V}_6) \cap (H \cap \bar{V}_6) &\stackrel{\cap (IP_{11}^{n-3} \cap \bar{V}_6)}{=} T_{p_1}(H) \cap T_{p_{11}}(H) \cap H \\ \cap \bar{V}_6 \cap IP_{11}^{n-3} &= IP_{11}^{n-3} \cap H \cap \bar{V}_6 = H_{11} \cap \bar{V}_6 \text{ is a smooth} \\ \text{quadric of dimension 0.} &\Rightarrow IP_{11}^{n-3} \cap H \cap \bar{V}_6 \text{ is a set} \\ \text{of distinct two points.} &\Rightarrow IP_{11}^{n-3} \cap H \cap \bar{V}_6 = \{p_{11}, p_{12}\}. \end{aligned}$$

Third:

$H \cap \bar{V}_8$  is a smooth quadric of dim. 6

$$\begin{aligned} & T_{p_1}(H) \cap T_{p_{11}}(H) \cap T_{p_{111}}(H) \cap H \cap \bar{V}_8 \\ &= T_{p_1}(H \cap \bar{V}_8) \cap T_{p_{11}}(H \cap \bar{V}_8) \cap T_{p_{111}}(H \cap \bar{V}_8) \cap (H \cap \bar{V}_8) \end{aligned}$$

by the argument above again.

Note: (i)  $\{p_1, p_{11}, p_{111}\}$  is a set of linearly independent points in  $\bar{V}_8$ , since ①  $p_1$  &  $p_{11}$  are linearly indep<sup>det</sup>,