to W_1 at a smooth point $F = l_1 + l_2$ is just the plane $H \subset W$ of conics passing through the point $p = l_1 \cap l_2 \in \mathbb{P}^2$, while the tangent cone to W_1 at a double point $F = 2l$ is the locus of conics tangent to l.

Γ We may assume that $F = X_0^2 + X_1^2$, $l_1 = X_0 + iX_1$, $l_2 = X_0 - iX_1$, and $p = [0, 0, 1]$.

$$W = \left\{ \begin{pmatrix} X_0 & X_3 & X_4 \\ X_3 & X_1 & X_5 \\ X_4 & X_5 & X_2 \end{pmatrix} \;\middle|\; X_i \in \mathbb{C} \right\} \Big/ \mathbb{C}^*$$

$$= \{ [X_0, X_1, X_2, X_3, X_4, X_5] \} = \mathbb{P}^5.$$

$\Rightarrow$

$$W_1 = \{ Q \in W \mid \det Q = 0 \}.$$

$$\Rightarrow 0 = \begin{vmatrix} X_0 & X_3 & X_4 \\ X_3 & X_1 & X_5 \\ X_4 & X_5 & X_2 \end{vmatrix} = X_0 X_1 X_2 - X_0 X_5^2 - X_3^2 X_2 + 2 X_3 X_4 X_5 - X_4^2 X_1 .$$

F is expressed as $\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$ in W

i.e. $F = [1, 1, 0, 0, 0, 0]$.