$$(K-1)\ \tilde{P}(\eta,\ \Theta_t\wedge\eta,\cdots\Theta_t)$$
$$= t\,(K-1)K\ \tilde{P}(\eta,\ \eta\wedge\Theta_t-\Theta_t\wedge\eta,\ \Theta_t,\cdots\Theta_t).$$

Similarly, by (*),

$$\tilde{P}(\tilde{\theta}\wedge\eta,\ \Theta_t,\cdots\Theta_t)-(K-1)\ \tilde{P}(\eta,\ \tilde{\theta}\wedge\Theta_t,\cdots\Theta_t)$$
$$= -\tilde{P}(\eta\wedge\tilde{\theta},\ \Theta_t,\cdots\Theta_t)-(K-1)\ \tilde{P}(\eta,\ \Theta_t\wedge\tilde{\theta},\cdots\Theta_t),$$

and so

$$-K\ \tilde{P}(\tilde{\theta}\wedge\eta+\eta\wedge\tilde{\theta},\cdots\Theta_t\cdots)=K(K-1)\tilde{P}(\eta,\ \Theta_t\wedge\tilde{\theta}-\tilde{\theta}\wedge\Theta_t,\cdots\Theta_t).$$

Let $A_1=\eta,\quad \theta=\tilde{\theta}\quad A_2=\cdots=A_K=\Theta_t.\quad P=\tilde{P}$

$$\sum_i (-1)^{d_1+\cdots+d_{i-1}}\ \tilde{P}(\eta,\cdots\tilde{\theta}\wedge\Theta_t,\cdots\Theta_t)$$

$$= \tilde{P}(\eta\wedge\tilde{\theta},\cdots\Theta_t)-\tilde{P}(\eta,\ \tilde{\theta}\wedge\Theta_t,\cdots\Theta_t)-\tilde{P}(\eta,\Theta_t,\tilde{\theta}\wedge\Theta_t,\Theta_t)-\cdots$$

$$= \tilde{P}(\tilde{\theta}\wedge\eta,\ \Theta_t,\cdots\Theta_t)-(K-1)\ \tilde{P}(\eta,\ \tilde{\theta}\wedge\Theta_t,\cdots\Theta_t)$$

$$= \sum_i (-1)^{d_1+\cdots+d_i}\ \tilde{P}(\eta,\cdots\Theta_t\wedge\tilde{\theta},\cdots\Theta_t)$$

$$= -\tilde{P}(\eta\wedge\tilde{\theta},\cdots\Theta_t)-\tilde{P}(\eta,\ \Theta_t\wedge\tilde{\theta},\cdots\Theta_t)-\cdots$$

$$= -\tilde{P}(\eta\wedge\tilde{\theta},\cdots\Theta_t)-(K-1)\tilde{P}(\eta,\ \Theta_t\wedge\tilde{\theta},\cdots\Theta_t)$$

$$\Rightarrow\ \tilde{P}(\tilde{\theta}\wedge\eta+\eta\wedge\tilde{\theta},\cdots\Theta_t)=(K-1)\ \tilde{P}(\eta,\ \tilde{\theta}\wedge\Theta_t-\Theta_t\wedge\tilde{\theta},\cdots\Theta_t)$$

$$\Rightarrow -K\tilde{P}(\tilde{\theta}\wedge\eta+\eta\wedge\tilde{\theta},\cdots\Theta_t)=K(K-1)\tilde{P}(\eta,\ \Theta_t\wedge\tilde{\theta}-\tilde{\theta}\wedge\Theta_t,\cdots\Theta_t).$$

Here again we used the fact that $\tilde{P}$ is symmetric