┌ We may assume a fixed line $l_0 = (X_0 = 0)$.

Let $\{l_\lambda\} = \{(a_0(\lambda) X_0 + a_1(\lambda) X_1 + a_2(\lambda) X_2 = 0)\}_{\lambda \in \mathbb{P}^1}$.

$\Rightarrow$ Since l_λ is a pencil, $\exists$ a point $p = [(b_0, b_1, b_2)]$, s.t $a_0(\lambda) b_0 + a_1(\lambda) b_1 + a_2(\lambda) b_2 = 0$.

In other words, p is the base point of $\{l_\lambda\}$.

Note that $\{[a_0, a_1, a_2] \mid b_0 a_0 + a_1 b_1 + a_2 b_2 = 0\} \underset{\text{onto}}{\overset{1-1}{\longleftrightarrow}} \{l_\lambda\}$.

① $p \notin l_0$

$\Rightarrow b_0 \neq 0$. $\Rightarrow$ $a_0 = -\frac{b_1}{b_0} a_1 - \frac{b_2}{b_0} a_2$.

$$\begin{pmatrix} -\frac{b_1}{b_0} a_1 - \frac{b_2}{b_0} a_2, & \frac{a_1}{2}, & \frac{a_2}{2} \\ \frac{a_1}{2} & 0 & 0 \\ \frac{a_2}{2} & 0 & 0 \end{pmatrix}$$ has rank 2

since a_1 and a_2 can not be zero at the same time.

$\Rightarrow$ The pencil $\{l_0 + l_\lambda\}$ misses W_2 altogether.

② $p \in l_0$

$\Rightarrow$ $b_0 = 0$ $\Rightarrow$ $b_1 a_1 + b_2 a_2 = 0$.

$\{[a_0, a_1, a_2] \mid b_1 a_1 + b_2 a_2\} \ni [1, 0, 0]$

If either a_1 or a_2 is not zero,

$$\begin{pmatrix} a_0 & \frac{a_1}{2} & \frac{a_2}{2} \\ \frac{a_1}{2} & 0 & 0 \\ \frac{a_2}{2} & 0 & 0 \end{pmatrix}$$ has rank 2.