curve $D \in |\mathcal{J}_{P_0}(4)|$ containing a point $q \in L_{ij}$ other than p_i and p_j has two double points and one single point of intersection with L_{ij}, and so contains L_{ij}.

Γ $D \in |\mathcal{J}_{P_0}(4)|$ has two double points at p_i & p_j, and a single point of intersection with L_{ij}. But since $\deg D = 4$, $\#(D \cdot L_{ij}) \doteq 4$ if D does not contain L_{ij}. $\Rightarrow$ D contains L_{ij}. See P64. $\lrcorner$

Also, while the proper transform of the linear system $|\mathcal{J}_{P_0}(4)|$ in $\mathbb{P}^2$ has intersection number 2 with each E_i, it does not cut out a complete linear system in E_i: fixing one tangent line to a curve $D \in |\mathcal{J}_{P_0}(4)|$ at p_i determines the other, and so f maps E_i two-to-one onto a line in $\mathbb{P}^3$.

Γ $\tilde{D} \sim \pi^* D - 2E_1 - 2E_2 - 2E_3$

$\Rightarrow$ $\tilde{D} \cdot E_i = 2$ since $E_i \cdot E_j = -1$ if $i = j$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ " $= 0$ otherwise

$$\pi^* D \cdot E_i = 0 . \quad \text{by } P426$$

Given a curve $D \in |\mathcal{J}_{P_0}(4)|$,

$D = \{ f + \lambda g = 0 \}$, where f, g represent C and C' respectively.