

$$\begin{array}{ccc}
 H^0(E, \mathcal{O}_E(\tilde{L}^k - E)) & \xrightarrow{\phi} & H^0(E, \mathcal{O}_E(\tilde{L}^k)) \otimes H^0(E, \mathcal{O}_E(-E)) \\
 \downarrow \sigma & \xrightarrow{\quad} & 1 \otimes 0
 \end{array}$$

where $1: E \longrightarrow \tilde{L}^k$
 $\alpha \longrightarrow 1$

$\Rightarrow \phi$ is well-defined and $H^0(E, \mathcal{O}_E(\tilde{L}^k)) = H^0(\mathbb{P}^{n-1}, \mathcal{O}_{\mathbb{P}^{n-1}}) \cong \mathbb{C}$.
 $\Rightarrow \dim H^0(E, \mathcal{O}_E(\tilde{L}^k - E)) = \dim(H^0(E, \mathcal{O}_E(\tilde{L}^k)) \otimes H^0(E, \mathcal{O}_E(-E)))$
 $\Rightarrow \phi$ is isomorphic.

$\Rightarrow H^0(E, \mathcal{O}_E(\tilde{L}^k - E)) = L_x^k \otimes H^0(E, \mathcal{O}_E(-E)) = L_x^k \otimes T_x^{*'} \text{ by P185 (**).}$

$$\begin{array}{ccc}
 H^0(\tilde{M}, \mathcal{O}_{\tilde{M}}(\tilde{L}^k - E)) & \xrightarrow{\gamma_E} & H^0(E, \mathcal{O}_E(\tilde{L}^k - E)) \\
 \cong \uparrow \pi^* & & \parallel \\
 H^0(M, \mathcal{I}_X(L^k)) & \xrightarrow{dx} & T_x^{*'} \otimes L_x^k
 \end{array}$$

L_x^k -valued maps which are tensor product of sections of $[-E]$ and sections of L^k . See note P466 ~ P467. \Downarrow

On \tilde{M} , there is an exact sequence

$$0 \longrightarrow \mathcal{O}_{\tilde{M}}(\tilde{L}^k - 2E) \longrightarrow \mathcal{O}_{\tilde{M}}(\tilde{L}^k - E) \xrightarrow{\gamma_E} \mathcal{O}_E(\tilde{L}^k - E) \longrightarrow 0.$$

Again, choose k_1 such that $L^{k_1} + K_M^*$ is positive on M and k_2 such that $\tilde{L}^{k_1} - (n+1)E$ is positive on \tilde{M} for $k' \geq k_2$. [See P187 for existence of k_2] \Downarrow

For $k \geq k_0 = k_1 + k_2$

$$\mathcal{O}_{\tilde{M}}(\tilde{L}^k - 2E) = \mathcal{O}_{\tilde{M}}((\tilde{L}^{k_1} + \tilde{K}_M^*) \otimes (\tilde{L}^{k'} - (n+1)E)).$$