Date . .

$$\Rightarrow \quad f_* v = \sum a_i f_*\left(\frac{\partial}{\partial z_i}\right) \qquad \Rightarrow \quad f_*\left(\frac{\partial}{\partial z_i}\right) = \sum \frac{\partial f_i}{\partial z_k}\frac{\partial}{\partial w_k}$$

$$\overline{f_* v} = \overline{a_i}\left(\overline{\frac{\partial f_i}{\partial z_k}}\right)\left(\overline{\frac{\partial}{\partial w_k}}\right) = \overline{a_i}\,\frac{\partial \overline{f_i}}{\partial \overline{z_k}}\,\frac{\partial}{\partial \overline{w_k}}$$

$$f_* \overline{v} = f_*\left(\sum \overline{a_i}\frac{\partial}{\partial \overline{z_i}}\right) = \sum \overline{a_i}\, f_*\left(\frac{\partial}{\partial \overline{z_i}}\right)$$

$$f_*\left(\frac{\partial}{\partial \overline{z_i}}\right) = c_k \frac{\partial}{\partial w_k} + b_l \frac{\partial}{\partial \overline{w_l}}.$$

$$f_*\left(\frac{\partial}{\partial \overline{z_i}}\right)(w_k) = c_k = \frac{\partial}{\partial \overline{z_i}}(w_k \circ f) = \frac{\partial f_k}{\partial \overline{z_i}} = 0 \quad \text{since}$$
$f_k$ is holomorphic.

$$f_*\left(\frac{\partial}{\partial \overline{z_i}}\right)(\overline{w_l}) = b_l = \frac{\partial}{\partial \overline{z_i}}(\overline{w_l}\circ f) = \frac{\partial \overline{f_l}}{\partial \overline{z_i}}$$

$$\Rightarrow \quad f_*\left(\frac{\partial}{\partial \overline{z_i}}\right) = \frac{\partial \overline{f_l}}{\partial \overline{z_i}}\,\frac{\partial}{\partial \overline{w_l}}. \qquad \Rightarrow \quad f_* \overline{v} = \sum \overline{a_i}\,\frac{\partial \overline{f_l}}{\partial \overline{z_i}}\,\frac{\partial}{\partial \overline{w_l}}$$

$$\Rightarrow \quad f_* \overline{v} = \overline{f_* v}$$

$\Rightarrow \quad \langle \Omega_{\pi^* L}; v, \overline{v} \rangle = \langle \Omega_L; \pi_* v, \pi_* \overline{v} \rangle = \langle \Omega_L; \pi_* v, \overline{\pi_* v} \rangle \geq 0.$

Since $\Omega_L$ is positive definite, $\langle \Omega_L; \pi_* v, \overline{\pi_* v} \rangle = 0 \Leftrightarrow \pi_* v = 0.$

$$\tilde{U} = \{(z, l) \in U \times \mathbb{P}^{n-1} \mid z \in l\}$$

$$\Rightarrow \quad \begin{tikzcd} T'_{(0,l)}\tilde{U} \arrow[r, hook] \arrow[dr, "\pi_*"'] & T'_0 U \times T_l\mathbb{P}^{n-1} \arrow[d, "\pi_*"] & [-3em] \\ & T'_x U & \end{tikzcd} \qquad T_l\mathbb{P}^{n-1} \cong T E$$

$\Rightarrow \quad \pi_* v = 0 \Leftrightarrow v$ is tangent to $\mathbb{P}^{n-1}$

$\Rightarrow \quad v$ is tangent to $E$.

Thus $\Omega_{\pi^* L} = \begin{cases} \geq 0 & \text{everywhere} \\ > 0 & \text{on } \tilde{M} - E \\ > 0 & \text{on } T'_x(M)/T'_x(E) \text{ for all } x \in E, \end{cases}$