Remark. In the class (C'), Definitions 1 and 1' coincide: the (C')-plurisubharmonic functions can be identified with the $\mathbb{R}^2$-subharmonic functions.

Proposition 1. A C^n-plurisubharmonic function is $\mathbb{R}^{2n}$-subharmonic.

In fact (1b) implies that each of the distributions $\frac{\partial^2 V}{\partial z_p \partial \bar{z}_q}$ is positive, and we have then

$$\Delta V = 4 \sum \frac{\partial^2 V}{\partial z_k \partial \bar{z}_k} \geq 0.$$

Γ $\vec{\lambda} = (\lambda_1, \cdots \lambda_n)$, $\lambda_p \neq 0$, $\lambda_i = 0$ for $i \neq p$.

$$\Rightarrow \quad T(V, \vec{\lambda}) = \sum \frac{\partial^2 V}{\partial z_p \partial \bar{z}_q} \lambda_p \bar{\lambda}_q$$

$$= \frac{\partial^2 V}{\partial z_p \partial \bar{z}_p} |\lambda_p|^2 \text{ positive, so } \frac{\partial^2 V}{\partial z_p \partial \bar{z}_p} \text{ is positive.}$$

$$\Rightarrow \quad \Delta V = 4 \sum \frac{\partial^2 V}{\partial z_k \partial \bar{z}_k} \text{ is positive.}$$

$\lrcorner$

More precisely:

Theorem 1. V is plurisubharmonic in a domain D of $\mathbb{C}^n \Leftrightarrow$ it is $\mathbb{R}^{2n}$-subharmonic and remains $\mathbb{R}^{2n}$-subharmonic in the nbd of the origin with respect to the variables z_k' after every change of variables $z \longrightarrow A(z')$

$$z_k - z_k^\circ = \sum a_k^j z_j', \quad z^\circ \in D$$

in which the a_k^j are arbitrary constants subject to the condition