Since $F^{p+1}A^{p+q+1} = \{ \varphi = \sum_{\#I \geq p+1} \varphi_{IJ}(x,y)\, dx_I \wedge dy_J \}$,

if $d_0 (\sum_{\#I=p} \eta_I \wedge dx_I + F^{p+1}A^{p+q}) = 0$, then

$\sum_{\#I=p} d_y \eta_I \wedge dx_I = 0 \Rightarrow d_y \eta_I = 0$ for each I.

$\Rightarrow \eta_I \in Z^q_{DR}(F_x)$ see P488 note for more.

Intuitively, we may think of $E_1^{p,q}$ as the p-forms on B with values in the bundle $H^q_{DR}(F)$ whose fibers are

$$H^q_{DR}(F)_x = H^q_{DR}(F_x).$$

$$\begin{array}{ccccc} E_0^{p,q-1} & \xrightarrow{d_0} & E_0^{p,q} & \xrightarrow{d_0} & E_0^{p,q+1} \\ \| & & \| & & \| \\ \dfrac{F^pA^{p+q-1}}{F^{p+1}A^{p+q-1}} & \longrightarrow & \dfrac{F^pA^{p+q}}{F^{p+1}A^{p+q}} & \longrightarrow & \dfrac{F^pA^{p+q+1}}{F^{p+1}A^{p+q+1}} \end{array}$$

$$E_1^{p,q} = \frac{\{ \sum_{\#I=p} \eta_I \wedge dx_I \mid d_y \eta_I = 0 \} + F^{p+1}A^{p+q}}{\{ \sum_{\#I=p} d_y \psi_I \wedge dx_I \mid \psi_I(x,y,dy),\ (q-1)\text{-form on } F_x \} + F^{p+1}A^{p+q}}$$

$$\doteq \{ \sum_{\#I=p} \bar{\eta}_I \wedge dx_I \} + F^{p+1}A^{p+q}, \quad \bar{\eta}_I \in H^p_{DR}(F_x).$$

We now compute $d_1 \bar{\varphi}$. For φ as above with $d_0\varphi = d_y \varphi = 0$,

$$d_1 \bar{\varphi} = \overline{d\varphi}.$$