~~Since St($\nu_{\alpha i}$) is path-connected~~

pf) For each vertex $\nu_\alpha \in K$, consider an open set, called the star of ν_α,

$St(\nu_\alpha) = (\overset{\circ}{\bigcup_{\substack{\sigma \text{ simplex} \\ \sigma \ni \nu_\alpha}} \sigma})$ = the interior of the union of all simplices in K having ν_α as a vertex.

$\Rightarrow$ $\underline{\mathcal{U}} = \{ U_\alpha = St(\nu_\alpha) \}$ is an open covering of M

Note ! If $\nu_{\alpha_0}, \nu_{\alpha_1}, \cdots \nu_{\alpha_p}$ are the vertices of a p-simplex in our decomposition,

$\bigcap_{i=0}^{p} St(\nu_{\alpha_i})$ is non-empty and connected, otherwise empty. (open ball of dim n)

Obivously, for each α_i, $St(\nu_{\alpha_i})$ is a n-dim. open ball.

Since $(\overset{\circ}{\bigcap_{i=1}^{m} A_i}) = \bigcap_{i=1}^{m} \overset{\circ}{A_i}$, $\bigcap_{i=0}^{p} St(\nu_{\alpha_i}) = (\overset{\circ}{\bigcap_{i=0}^{p} \overline{st(\nu_{\alpha_i})}})$

where $\overline{st(\nu_{\alpha_i})}$ = closure of $st(\nu_{\alpha_i})$ = closed n-ball.

Then $\bigcap_{i=0}^{p} \overline{st(\nu_{\alpha_i})} = \bigcup_{\substack{\sigma \supset [\nu_{\alpha_0}, \cdots \nu_{\alpha_p}] \\ \sigma \ n\text{-simplex}}} \sigma$ which is path-connected.

and the interior is path-connected, too, since the interior of $[\nu_{\alpha_0} \cdots \nu_{\alpha_p}]$ is contained in the interior.

, and $\bigcup \overset{\circ}{\sigma_i} \cup [\nu_{\alpha_0} \overset{\circ}{\cdots} \nu_{\alpha_p}] = \bigcap_{i=0}^{p} St(\nu_{\alpha_i})$

Here $[\nu_{\alpha_0}, \overset{\circ}{\cdots} \nu_{\alpha_p}]$ means the interior of $[\nu_{\alpha_0}, \cdots \nu_{\alpha_p}]$ ~~as~~ in the p-simplex $[\nu_{\alpha_0} \ldots \nu_{\alpha_p}]$.

Thus a p-cochain α of the sheaf $\mathbb{Z}$ associates to every $(\alpha_0, \cdots \alpha_p)$,