According to Proposition on P 196,

$$\sigma_{a_1 \cdots a_k} \in H_{2k(n-k)-2\sum a_i}(G(k,n))$$

$\Rightarrow$ By Poincare duality, $\tilde{\sigma}_{a_1 \cdots a_k} \in H^{2\sum a_i}(G(k,n))$. In our case

$$\tilde{\sigma}_{a_1 \cdots a_k} \in H^{(k+1)(k+2)}(G(k+1,n+2)), \quad \to \text{Poincare dual}$$

The cohomology in complementary dimension is likewise generated by Schubert cycles σ_b with $\sum b_i = (k+1)(n-k+1) - (k+1)(k+2)/2$; the intersection pairing is

$$\#(\sigma_{a_1 \cdots a_{k+1}} \cdot \sigma_{b_1 \cdots b_{k+1}}) = \begin{cases} 1, & \text{if } a_i + b_{k+2-i} = n-k+1 \text{ for all } i, \\ 0, & \text{otherwise.} \end{cases}$$

See P198

To find the class of $\Sigma_{k,n}$, accordingly, we have to evaluate the intersection numbers $\#(\Sigma_{k,n} \cdot \sigma_b)$ for all such $b = (b_1 \cdots b_{k+1})$ with $\sum b_i = (k+1)(n-k+1) - (k+1)(k+2)/2$.

Note that $\tilde{\Sigma}_{k,n} \in H^{(k+1)(k+2)}(G(k+1,n+2))$ and so

$$\tilde{\Sigma}_{k,n} = \sum x_{a_1 \cdots a_k} \tilde{\sigma}_{a_1 \cdots a_k} \quad \text{since } \tilde{\sigma}_{a_1 \cdots a_k}\text{'s generate}$$

$$(2\sum a_i = (k+1)(k+2))$$

$H^{(k+1)(k+2)}(G(k+1, n+2))$. $\Rightarrow$ To find $x_{a_1 \cdots a_k}$, we have to evaluate the intersection numbers $\#(\Sigma_{k,n} \cdot \sigma_b)$ for all $b = (b_1 \cdots b_{k+1})$ with $\sum b_i = (k+1)(n-k+1) - (k+1)(k+2)/2$.