Again adding small vector to (B_{21}, C_{21}''), make $\{(B_1, C_1'), \cdots (B_{20}, C_{20}')$ $(B_{21}', C_{21}'')\}$ linearly independent set.

$$\Rightarrow \begin{pmatrix} A_1, & B_1, & C_1' \\ A_2, & B_2 & C_2' \\ \vdots & & \vdots \\ A_{20}, & B_{20} & \underline{C_{20}'} \\ A_{21}, & B_{21}, & C_{21} \\ \vdots & \vdots & \\ A_{100} & B_{100}, & C_{100} \end{pmatrix} \text{rank} \Rightarrow \begin{pmatrix} A_1, & B_1, & C_1' \\ A_2 & B_2 & C_2' \\ \vdots & \vdots & \vdots \\ A_{20}, & \underline{B_{20}} & \underline{C_{20}'} \\ A_{21}, & B_{21} & C_{21}'' \\ & \vdots & \vdots \\ A_{100}, & B_{100} & C_{100}'' \end{pmatrix} \text{rank}$$

$$\Rightarrow \begin{pmatrix} A_1, & B_1, & C_1' \\ \vdots & \vdots & \vdots \\ A_{20}, & B_{20}, & C_{20}' \\ A_{21}, & \underline{B_{21}',} & \underline{C_{21}''} \\ A_{22}, & B_{22} & C_{22}'' \\ \vdots & \vdots & \vdots \\ A_{100}, & B_{100}, & C_{100}'' \end{pmatrix} \Rightarrow \begin{pmatrix} A_1, & B_1, & C_1' \\ \vdots & & \\ A_{20}, & B_{20} & C_{20}' \\ \underline{A_{21},} & \underline{B_{21}'} & \underline{C_{21}''} \\ A_{22}, & B_{22}', & C_{22}''' \\ & \vdots & \\ A_{100}, & B_{100}', & C_{100}''' \end{pmatrix}$$

$$C_{21}'' = y_1 C_1' + \cdots + y_{20} C_{20}'$$
$$B_{21}' \neq y_1 B_1 + \cdots + y_{20} B_{20}'$$

$(B_{22}', C_{22}''') = z_1 (B_1, C_1') + \cdots + z_{21}(B_{21}', C_{21}'')$
where $(A_{22}, B_{22}, C_{22}) = z_1 (B_1, C_1) + \cdots + z_{21} (B_{21}, C_{21})$.
$(B_{23}', C_{23}''') = w_1 (B_1, C_1') + \cdots + w_{21}(B_{21}', C_{21}'')$
where $(A_{23}, B_{23}, C_{23}) = w_1 (B_1, C_1) + \cdots + w_{21} (B_{21}, C_{21})$

$\vdots$

Finally, consider $(A_{22}, B_{22}', C_{22}''')$. $(A_{23}, B_{23}', C_{23}''')$
$(A_{24}, B_{24}', C_{24}''')$
$\Rightarrow \quad (B_{22}', C_{22}''') = z_1 (B_1, C_1') + \cdots + z_{21} (B_{21}', C_{21}'')$
so, we have to choose $\underline{A_{22}' \neq z_1 A_1 + \cdots + z_{21} A_{21}}$ — ③