lines. $\Rightarrow$ $\deg(T_x(X)\cap X) = 4$ $\Rightarrow$ Since, by the assumption, $T_x(X)\cap X$ has no multiple components, $T_x(X)\cap X$ must consist of four lines. But by the argument above,

$$l_x \cap \mathcal{S} \xleftrightarrow{1-1} \text{Set of } \underset{\displaystyle T_x(X)\cap X}{\overset{\parallel}{\quad}} \text{ lines in } X \text{ through } x$$

thus $\#(l_x \cap \mathcal{S}) = 4$, since the number of lines of $T_x(X)\cap X$. $\Rightarrow$ $\deg \mathcal{S} = \#(l_x\cap\mathcal{S}) = 4$.

Now, all the assumptions made about the generic line l_x of our complex are in fact the case, but their verifications are best left until we know more about the complex. There is one point worth mentioning now, which will emerge from this computation once we have established that $\mathcal{S}$ is quartic: Since $T_x(X)\cap X$ can never contain more than four lines, for any point $p\in\mathcal{S}-R$ the line l_x held in common by the two pencils in X through p - that is, the line of intersection of the two hyperplanes comprising the locus of X_p - can lie on at most two lines in X not on $\sigma(p)$.

$X_p = F\cap\sigma(p) = L_1\cup L_2$. $L_1\cap L_2 \ni x$ $L_1 \neq L_2$.
$\Rightarrow$ Since $X\cap T_x(X) =$ The locus of lines in X