Thus we can conclude that we have to choose not any three coefficients $a_\alpha, b_\beta, c_\gamma$ with $a_\alpha + b_\beta + c_\gamma \geq 2(n-k)+1$.

We have to impose the following conditions on a_α, b_β, c_γ.

$$a_\alpha > a_{\alpha+1}, \quad b_\beta > b_{\beta+1}, \quad c_\gamma > c_{\gamma+1}.$$

$\Rightarrow$ We get the right formula. See Note P521. ♪

For the purposes of this formula, we may set $a_0 = b_0 = c_0 = n-k$ formally; thus in case we take $\gamma = \beta = 0$, this reduction applies if $a_k \neq 0$, and if we take $\gamma = 0$, it applies in case $a_i + b_{k-i} \geq n-k+1$ for some i.

If $\gamma = \beta = 0$. ① $\alpha > k \Rightarrow$ Clearly, $\#(\sigma_a \cdot \sigma_b \cdot \sigma_c) = 0$

② $\alpha = k$. $a_k + b_0 + c_0 = a_k + 2(n-k) \geq 2(n-k)+1$.

$\Rightarrow a_k \neq 0$.

We have to show that

$$\#(\sigma_a \cdot \sigma_b \cdot \sigma_c)_{G(k,n)} = \#(\sigma_{a_1-1, \cdots a_k-1,} \cdot \sigma_b \cdot \sigma_c)_{G(k,n-1)}.$$

Since $a_k \neq 0$, $a^*_{a_k} \geq k$ and $a^*_1 \geq a^*_{a_k}$.

$\Rightarrow a_1^* = k. = n-(n-k) \quad b^*_{n-k} \quad c^*_{n-k}$.

$\Rightarrow$ By the note above, $\beta = \gamma = n-k$, $a_1^* = k$ can be applied. in the following situation.

$\Rightarrow \#(\sigma_a \cdot \sigma_b \cdot \sigma_c)_{G(k,n)} = \#(\sigma_{a^*} \cdot \sigma_{b^*} \cdot \sigma_{c^*})_{G(n-k,n)}$

$= \#(\sigma_{a^* - a^*_1} \cdot \sigma_{b^* - b^*_{n-k}} \cdot \sigma_{c^* - c^*_{n-k}})$ if $\underset{a_1^{*}{}^{\shortparallel}}{a^*_{a_k}} + b^*_{n-k} + c^*_{n-k} = k$

$b^*_{n-k} = c^*_{n-k}$ must be 0 since $a^*_1 + b^*_{n-k} + c^*_{n-k} = k$