$[a] \in E_r^{p,q}$ $\quad a \in F^pK^{p+q}$ s.t $da \in F^{p+r}K^{p+q+1}$
$[da] \Rightarrow da \in F^{p+r}K^{p+q+1}$ and $d(da) = 0 \in F^{p+2r+1}$
$K^{p+q+2} \Rightarrow d_r[a] = [da]$ is well-defined. ⅃

A computation - straightforward but messy - gives
$H^*(E_r) \cong E_{r+1}$.

Γ

$$E_r^{p-r,\,q+r-1} \xrightarrow{d_r} E_r^{p,q} \xrightarrow{d_r} E_r^{p+r,\,q-r+1}$$

$$\ker d_r = \frac{\{a \in F^pK^{p+q} ;\ da \in F^{p+r+1}K^{p+q+1} + d(F^{p+1}K^{p+q})\} + d(F^{p-r+1}K^{p+q-1})}{d(F^{p-r+1}K^{p+q-1}) + F^{p+1}K^{p+q}}$$

$$\frac{+ F^{p+1}K^{p+q}}{} = \frac{\{a \in F^pK^{p+q} ;\ da \in F^{p+r+1}K^{p+q+1}\} + d(F^{p-r+1}K^{p+q-1}) + F^{p+1}K^{p+q}}{d(F^{p-r+1}K^{p+q-1}) + F^{p+1}K^{p+q}}$$

by the same reason as the one on p423.

$$\operatorname{im} d_r = \frac{d(F^{p-r}K^{p+q-1}) \cap F^pK^{p+q} + d(F^{p-r+1}K^{p+q-1}) + F^{p+1}K^{p+q}}{d(F^{p-r+1}K^{p+q-1}) + F^{p+1}K^{p+q}}$$

$$\Rightarrow H^{p,q}(E_r) = \frac{\{a \in F^pK^{p+q} ;\ da \in F^{p+r+1}K^{p+q+1}\} + d(F^{p-r+1}K^{p+q-1}) + F^{p+1}K^{p+q}}{d(F^{p-r}K^{p+q-1}) \cap F^pK^{p+q} + d(F^{p-r+1}K^{p+q-1}) + F^{p+1}K^{p+q}}$$

Consider the map $h : \dfrac{\{a \in F^pK^{p+q} ;\ da \in F^{p+r+1}K^{p+q+1}\}}{(d(F^{p-r}K^{p+q-1}) + F^{p+1}K^{p+q}) \cap \{a \in F^pK^{p+q} ;\ da \in F^{p+r+1}K^{p+q+1}\}}$ $\;/\!/\; B$

$\longrightarrow H^{p,q}(E_r)$ defined by $[a] \longmapsto [a]$.

$\Rightarrow$ Since $\underset{\substack{\Uparrow \\ d(F^{p-r}K^{p+q-1})}}{dl} + \underset{\substack{\Uparrow \\ F^{p+1}K^{p+q}}}{x} = a \in \{a \in F^pK^{p+q} ;\ da \in F^{p+r+1}K^{p+q+1}\} = A$,

$dl \in F^pK^{p+q}$, and $[a] = 0$ in $H^{p,q}(E_r)$. $\Rightarrow h$ is well-defined.