Clearly $\frac{\partial \eta}{\partial z}(0) \doteq h(0) \neq 0$. η is a coordinate, and $d\eta = h(z)dz = \omega$.
$\Rightarrow F(\eta) = \frac{a_{-2}}{\eta^2} + a_0 + a_1 z + \cdots \Rightarrow$ After multiplying by a constant and adding a constant
$\Rightarrow F(\eta) = \frac{1}{\eta^2} + [1]$." So the authors maybe misunderstood I think.

「 We may write dF/ω as

$$\frac{dF}{\omega} = \frac{a_{-3}}{z^3} + \frac{a_{-2}}{z^2} + \frac{a_{-1}}{z} + a_0 + a_1 z + \cdots$$
$$= \frac{a_{-3}}{z^3} + \frac{a_{-2}}{z^2} + \frac{a_{-1}}{z} + a_0 + [1], \quad a_{-3} \neq 0$$

Since $F(z) = \frac{1}{z^2} + [1]$, and ω is nonzero everywhere,

$$\mathrm{Res}\left(\frac{dF}{\omega}\cdot\omega\right) = 0 = a_{-1}$$
$$\Rightarrow \frac{dF}{\omega} = \frac{a_{-3}}{z^3} + \frac{a_{-2}}{z^2} + a_0 + [1]$$

Let $\lambda = a_{-3}^{-1}$, $\lambda' = -\lambda a_{-2}$, $\lambda'' = -a_0\lambda$.

$$a_{-3}^{-1}\frac{dF}{\omega} - \lambda a_{-2}F + \lambda'' = F' = \frac{1}{z^3} + [1].$$ 」

The map $\iota_L : S \longrightarrow \mathbb{P}^2$ associated to the line bundle $L = [3p]$ can thus be given by

$$q \longmapsto [1, F(q), F'(q)].$$

「 Recall from P 222 that $H^0(S, \mathcal{O}(L))$ corresponds to meromorphic functions on S holomorphic on $S - \{p\}$ and of order ≥ -3 at p and any such function is uniquely determined by its principal part.
$\Rightarrow$ If $(s_0 = 0) = 3p$, $\{s_0, F\cdot s_0, F'\cdot s_0\}$ spans $H^0(S, \mathcal{O}(L))$.

$$\iota_L : S \longrightarrow \mathbb{P}^2$$
$$q \longmapsto [s_0(q), F(q)s_0(q), F'(q)\cdot s_0(q)]$$
$$= [1, F(q), F'(q)].$$ 」