$f^*(\mathcal{O}_W)+I/I \cong \mathbb{C} \subset \mathcal{O}/I$. Since $[\mathcal{O}/I, \mathbb{C}] = d$, in an abstract way, we may put this way.

$I_0 = \mathbb{C} \subset I_1 \subset I_2 \subset \cdots \subset I_\ell = \mathcal{O}/I$. where

$I_i = \langle a_i, I_{i-1} \rangle$ = the ideal generated by a_i and $I_{i-1} = \mathcal{O}/I$, since I_{i-1} contains a unit.

$\Rightarrow$ Clearly, I_ℓ is a complex vectorspace over $I_0 = \mathbb{C}$ of $\dim \prod_{i=1}^{\ell} l_i$. $l_i = \dim I_i/I_{i-1}$. There is an error, I_i may not contain an unit, so I_i/I_{i-1} need not be an field. I can not find a way to get $\dim_{\mathbb{C}}(\mathcal{O}/I) = d$. Let's see sometime later.

Summarizing: Let $D_i = (f_i)$ be n divisors given in some small nbd U of the origin in $\mathbb{C}^n$ with $\bigcap_i D_i = \{0\}$. Then the local intersection number has the following interpretations:

(1) Analytic: The formula

$$(D_1 \cdots D_n)_{\{0\}} = \mathrm{Res}_{\{0\}}\left(\frac{df_1}{f_1} \wedge \cdots \wedge \frac{df_n}{f_n}\right)$$

was taken as our definition.

(2) Topological: Setting $f = (f_1, \cdots f_n) : U^* \longrightarrow \mathbb{C}^n - \{0\}$,

$$(D_1 \cdots D_n)_{\{0\}} = \text{degree}(f).$$

Equivalently, $f : U \longrightarrow W$ is a finite open surjective holomorphic mapping for some nbd W of the origin in $\mathbb{C}^n$. Since f is orientation preserving, $(D_1 \cdots D_n)_{\{0\}}$ is the sheet number of f.

(3) Algebraic: If $\mathcal{O}$ is the local ring at the origin and $I \subset \mathcal{O}$ the ideal generated by the f_i, then