

This contradicts to the fact $\dim H^0(\mathbb{P}^2, \mathcal{O}(H)) = 3$ since l_1, l_2, l_3, l_4 are linearly independent.

(ii) $|f_{\Lambda_0}(2)|$ satisfies case 2.

$$\Rightarrow \dim |f_{\Lambda_0}(2)| = 5 - 4 + 0 = 1 \quad \text{Contradiction.}$$

$$\textcircled{2} \quad \dim |f_{\Lambda_0}(2)| = 2$$

$$\Rightarrow \dim |f_{\Lambda_0}(2)| \geq \dim |f_{P_0''}(2)| = \dim |f_{P_0}(3)| \geq 2.$$

Since clearly $\sigma \in H^0(\mathbb{P}^2, f_{P_0''}(2)) \Rightarrow \sigma \in H^0(\mathbb{P}^2, f_{\Lambda_0}(2))$ and $H^0(\mathbb{P}^2, f_{P_0''}(2))$ corresponds to $H^0(\mathbb{P}^2, f_{P_0}(3))$.

$\Rightarrow \exists$ linearly independent $\tau_1, \tau_2, \tau_3 \in H^0(\mathbb{P}^2, f_{\Lambda_0}(2))$.

s.t. $\tau_1 = l' l_1, \tau_2 = l' l_2, \tau_3 = l' l_3, l'$ a fixed line and $\{\tau_1 l'_1, \tau_2 l'_2, \tau_3 l'_3\}$ spans $H^0(\mathbb{P}^2, f_{P_0}(3))$. -- *

(i) If $l' \cap \Lambda_0 \neq \Lambda_0$, then \exists a point $p \in \Lambda_0$ s.t. l_1, l_2, l_3 pass through p .

Since $\dim H^0(\mathbb{P}^2, f_P(H)) = 2$, $\{l_1, l_2, l_3\}$ is not linearly independent. \Rightarrow Contradiction

(ii) If $l' \supset \Lambda_0$, consider $\{l' l'_1, l' l'_2, l' l'_3\} \subset H^0(\mathbb{P}^2, f_{\Lambda_0}(2))$.
Since $\{l' l'_1, l' l'_2, l' l'_3\}$ span $H^0(\mathbb{P}^2, f_{P_0}(3))$,
($\because \{l_1, l_2, l_3\}$ is linearly independent, and each $l' l'_i$ passes