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Note that if $\dim(*\Lambda \cap V^*_{n+1}) \geq l$, then $\dim(*\Lambda \cap V^*_n) \geq l-1$. For, suppose that $\dim(*\Lambda \cap V^*_n) < l-1$. $\Rightarrow$ There are two linearly independent vectors $\omega_1, \omega_2 \in *\Lambda \cap V^*_{n+1}$. s.t $\omega_1, \omega_2 \notin *\Lambda \cap V^*_n$.

$\Rightarrow$ Since $\omega_1 - \alpha e_{n+1}$; $\omega_2 - \beta e_{n+1} \in *\Lambda \cap V^*_n$,

$\exists\ r$ s.t $r(\omega_1 - \alpha e_{n+1}) - (\omega_2 - \beta e_{n+1}) \in *\Lambda \cap V^*_n$ and $r\alpha = \beta$, where $\alpha \neq 0$.

$\Rightarrow \omega_2 - r\omega_1 \in *\Lambda \cap V^*_n$.

$\Rightarrow \dim(*\Lambda \cap V^*_{n+1}) \leq 1 + \dim(*\Lambda \cap V^*_n) \Rightarrow \divideontimes$

From the observation above,

$$\dim(*\Lambda \cap V^*_{k+a_{m_{(p+1)}}-1-a^*_{a_{m_{p+1}}-1}}) \geq a_{m_{(p+1)}}-1.$$

$$\Rightarrow \dim(*\Lambda \cap V^*_{k+a_{m_{p+1}}-2-a^*_{a_{m_{p+1}}-2}}) \geq a_{m_{p+1}}-2$$

$$\vdots$$

$$\dim(*\Lambda \cap V^*_{k+i-a^*_i}) \geq i.$$

where $a_{m_p} < i \leq a_{m_{(p+1)}}$

Thus we proved the desired.

$\Rightarrow *\sigma_a = \sigma_{a^*}$ where a^* is the smallest sequence s.t $a^*_{a_i} \geq i$ for all i, since

$*\sigma_a \subset \sigma_{a^*}$ and $\dim(*\sigma_a) = \dim(\sigma_a) = \dim \sigma_{a^*}$.

For example, $*\sigma_2 = \sigma_{1,1}$, $*(\sigma_{2,1,1}) = \sigma_{3,1}$.