goes as follows: we first claim that for a generic $x \in G$, the surface

$$\mathcal{U} = T_x(G) \cap X \subset \mathbb{P}^5$$

is smooth — this fact will emerge in a moment. Granting this, we recall

$$G \cap T_x(G) = \bigcup_{p \in l_x} \sigma(p)$$

so that the curves

$$X_p = \sigma(p) \cap F \subset \mathcal{U}$$

form a linear system on $\mathcal{U}$ without base point.

$\sqsubset$ See P158. for $G \cap T_x(G) = \bigcup_{p \in l_x} \sigma(p)$.

$$\mathcal{U} = T_x(G) \cap X = T_x(G) \cap G \cap F = \bigcup_{p \in l_x} \sigma(p) \cap F$$

$$= \bigcup_{p \in l_x} (\sigma(p) \cap F) = \bigcup_{p \in l_x} X_p.$$

$\dim \mathcal{U} = 4 + 3 - 5 = 2$. $\dim X_p = 1$.

$\Rightarrow X_p$ is a divisor on $\mathcal{U}$.

Since x is a generic point in G, $x \notin F$.

$\bigcap_{p \in l_x} \sigma(p) \ni x$, since $p \in l_x$, $\bigcap_{p \in l_x} \sigma(p) = \{x\}$.

$\Rightarrow F \cap \bigcap_{p \in l_x} \sigma(p) = \bigcap_{p \in l_x} X_p = \phi \Rightarrow \{X_p\}_{p \in l_x}$ has no base point.

We may assume that $l_x = \{[*, *, 0, 0]\}$.