

$$l \cap l_x \Rightarrow l \in \sigma(p) \Rightarrow \sigma(l_x) \subset \bigcup_{p \in l_x} \sigma(p).$$

$$\Rightarrow \sigma(l_x) = \bigcup_{p \in l_x} \sigma(p)$$

$$\forall l \in \sigma(l_x) \Rightarrow l \cap l_x \neq \emptyset \Rightarrow \text{Let } h = \overline{l, l_x},$$

if $l \neq l_x$, choose a α -plane $h \supset l$ if $l = l_x$.

$$\Rightarrow \sigma(h) \ni l \Rightarrow \sigma(l_x) \subset \bigcup_{h \supset l_x} \sigma(h). \dots \textcircled{1}$$

$$\forall l \in \sigma(h), h \supset l_x \Rightarrow l, l_x \subset h \Rightarrow l \cap l_x \neq \emptyset$$

since h is a α -plane. $\Rightarrow l \in \sigma(l_x) \Rightarrow \sigma(h) \subset \sigma(l_x).$

$$\Rightarrow \bigcup_{h \supset l_x} \sigma(h) \subset \sigma(l_x). \dots \textcircled{2} \Rightarrow \text{By } \textcircled{1} \text{ \& } \textcircled{2}, \sigma(l_x) = \bigcup_{h \supset l_x} \sigma(h).$$

$$G \cap \bigcap_{x \in L} T_x(G) = \bigcap_{x \in L} (G \cap T_x(G)) = \bigcap_{x \in L} \sigma(l_x)$$

$$\forall l \in \bigcap_{x \in L} \sigma(l_x)$$

$$\textcircled{1} \quad l \ni p_L = \bigcap_{x \in L} l_x. \Rightarrow \text{O.K. since } l \in \sigma(p_L).$$

$$\textcircled{2} \quad l \not\ni p_L \Rightarrow \forall x, l \cap l_x \neq \emptyset \Rightarrow \exists p_1 \neq p_2 \quad p_i \neq p_L$$

s.t. $p_1 = l \cap l_{x_1}, \quad p_2 = l \cap l_{x_2}. \Rightarrow \text{Since } p_1, p_2 \in h_L,$

$$l = \overline{p_1, p_2} \subset h_L \Rightarrow l \in \sigma(h_L).$$

$$\Rightarrow l \in \sigma(p_L) \cup \sigma(h_L) \Rightarrow \bigcap_{x \in L} \sigma(l_x) \subset \sigma(p_L) \cup \sigma(h_L).$$

$$\textcircled{1} \quad \text{If } l \in \sigma(p_L), \text{ then } l_x \cap l \neq \emptyset \text{ for all } x \Rightarrow$$

$$l \in \bigcap_{x \in L} \sigma(l_x) \Rightarrow \sigma(p_L) \subset \bigcap_{x \in L} \sigma(l_x)$$

$$\textcircled{2} \quad \text{If } l \in \sigma(h_L), \quad l \cap l_x \neq \emptyset \text{ for all } l_x \subset h_L,$$