a coordinate chart, we have maps

$$\varphi_{U*}: T_x(M) \longrightarrow T_{\varphi(x)}(\mathbb{C}^n) \cong \mathbb{C}\left\{\frac{\partial}{\partial x_i}, \frac{\partial}{\partial y_i}\right\} \cong \mathbb{C}^{2n}.$$

for each $x \in U$, hence a map

$$\varphi_{U*}: \bigcup_{x\in U} T_x(M) \longrightarrow U \times \mathbb{C}^{2n}.$$ giving

$T(M) = \bigcup_{x\in M} T_x(M)$ the structure of a complex vector bundle called the complex tangent bundle. Transition functions for $T(M)$ are given by

$$J_{U,V} = J_R(\varphi_U \varphi_V^{-1}).$$

$$T\mathbb{C}^{2n} \xrightarrow{(\varphi_U \circ \varphi_V^{-1})_*} T\mathbb{C}^n$$

$$\begin{pmatrix} \frac{\partial u_i}{\partial x_j}, & \frac{\partial u_i}{\partial y_j} \\ \frac{\partial v_i}{\partial x_j}, & \frac{\partial v_i}{\partial y_j} \end{pmatrix}$$

with respect to $\left\{\frac{\partial}{\partial x_i}, \frac{\partial}{\partial y_i}\right\}$, $\left\{\frac{\partial}{\partial u_i}, \frac{\partial}{\partial v_i}\right\}$

$$(\varphi_U \varphi_V^{-1})_*\left(\frac{\partial}{\partial x_j}\right) \qquad (\varphi_U \varphi_V^{-1})_*\left(\frac{\partial}{\partial y_j}\right)$$

$$= \sum \circ \frac{\partial}{\partial u_i} + \sum \circ \frac{\partial}{\partial v_i}$$

Now for each $x \in M$,

$$T_x(M) = T_x'(M) \oplus T_x''(M),$$ where $T_x'(M)$ holomorphic, $T_x''(M)$ anti-holomorphic

The subspaces $\{T_x'(M) \subset T_x(M)\}$ form a subbundle $T'(M) \subset T(M)$, called the holomorphic tangent bundle. Transition functions for $T'(M)$ are given by

$$J_{U,V} = J_{\mathbb{C}}(\varphi_U \varphi_V^{-1})$$ w.r.t. $\left\{\frac{\partial}{\partial z_i}\right\}$ & $\left\{\frac{\partial}{\partial w_i}\right\}$