

This implies that if $\pi_2^{-1}(\epsilon)$ meets $\tilde{V} \cap \tilde{W}$ at (p, ϵ) transversely, then V meets $W + \epsilon$ at p transversely.

Given any vector $v \in T_p \Delta'$, since $T_p V + T_p(W + \epsilon) = T_p \Delta'$, $\exists \alpha(t) \in V \quad \beta(t) \in W + \epsilon$ s.t. $\alpha(0) = \beta(0) = p$, and $\alpha'(0) + \beta'(0) = v$.

Let $\gamma(t) = \alpha(t) - \beta(t) + \epsilon \Rightarrow \gamma(0) = \epsilon$
 $\Rightarrow (\alpha(t), \gamma(t)) \in \tilde{W}$ since $\alpha(t) - \gamma(t) = \beta(t) - \epsilon \in W$

$\Rightarrow (\alpha'(0), \gamma'(0)) = (\alpha'(0), v) \in T_{(p, \epsilon)}(\tilde{V} \cap \tilde{W})$

\Rightarrow Since $T_{(p, \epsilon)} \pi_2^{-1}(\epsilon) = (T_p \Delta', 0)$,

$\pi_2^{-1}(\epsilon)$ meets $\tilde{V} \cap \tilde{W}$ transversely at (p, ϵ) , when V and $W + \epsilon$ meet transversely at p . \square

The intersection $\tilde{V} \cap \tilde{W} \subset \Delta' \times \Delta'$ is an analytic variety of dimension n , and so the projection $\pi_2: \tilde{V} \cap \tilde{W} \rightarrow \Delta'$ expresses $\tilde{V} \cap \tilde{W}$ as a branched μ -sheeted cover of Δ' ; accordingly, we see that for $\epsilon \in \Delta'$ lying outside an analytic subvariety of Δ' , the varieties V and $W + \epsilon$ will meet transversely in μ points in Δ' . The number μ is called the intersection multiplicity of V and W at 0 and is written

$$\mu = m_0(V \cdot W).$$