$\sum_{\alpha<\beta} \Lambda_{\alpha\beta} \otimes u_\alpha \wedge u_\beta$ .

$\Rightarrow \quad i(u_\gamma^*)(u_\alpha \wedge u_\beta) = ?$

$\Rightarrow \quad \langle i(u_\gamma^*)(u_\alpha \wedge u_\beta), \ \bar{z} \rangle = \langle u_\alpha \wedge u_\beta, \ u_\gamma^* \wedge \bar{z} \rangle$

Here $\bar{z} \in U^*$. $\Rightarrow \quad \bar{z} = \sum a_\sigma u_\sigma^*$

$\Rightarrow \quad \langle u_\alpha \wedge u_\beta, \ u_\gamma^* \wedge \bar{z} \rangle = \langle u_\alpha \wedge u_\beta, \ u_\gamma^* \wedge a_\sigma u_\sigma^* \rangle$

$= a_\sigma \langle u_\alpha \wedge u_\beta, \ u_\gamma^* \wedge u_\sigma^* \rangle$

$= a_\sigma \ \delta_{\alpha\gamma} \delta_{\beta\sigma}$ or $a_\sigma \ \delta_{\alpha\sigma} \delta_{\beta\gamma}$.

$\Rightarrow \quad i(u_\alpha^*)(u_\alpha \wedge u_\beta) = u_\beta$ and

$i(u_\beta^*)(u_\beta \wedge u_\alpha) = u_\alpha$. "

$i(u_\gamma^*)(u_\alpha \wedge u_\beta) = 0$ if $\alpha \neq \beta \neq \gamma, \ \alpha \neq \gamma$.

$i(u_\alpha^*)(\sum \Lambda_{\alpha\beta} \otimes u_\alpha \wedge u_\beta) = \sum_\beta \Lambda_{\alpha\beta} \otimes . u_\beta = 0$

$\Rightarrow \quad \Lambda_{\alpha\beta} = 0$ for all $\beta$ and a fixed $\alpha$.

$\Rightarrow$ Consequently, $\Lambda_{\alpha\beta} = 0$ for all $\alpha, \beta$.

$\Rightarrow$ Similarly, we get the same results for the other factors of $\Lambda$. $\Rightarrow \quad \Lambda \in \Lambda^k W$. ⅃

It is easy to see that $W$ is the minimal such subspace. Q. E. D.

⌈ Suppose $W' \subset W$ is a subspace s.t $\Lambda$ is in the image of $\Lambda^k W' \longrightarrow \Lambda^k V$.

$\Rightarrow$ We may express $W = W' \oplus W''$ &

$V = W \oplus V' = W' \oplus W'' \oplus V' = W' \oplus T$

where $T = W'' \oplus V'$.

In the proof that $\Lambda$ lies in $\Lambda^k W$, as we saw above, $i(u_\alpha^*) \Lambda = 0$ for all $\alpha$.