Since

$$\varepsilon_j = \begin{vmatrix} m_{j-1} & m_j \\ n_{j-1} & n_j \end{vmatrix} = m_{j-1}n_j - m_jn_{j-1},$$

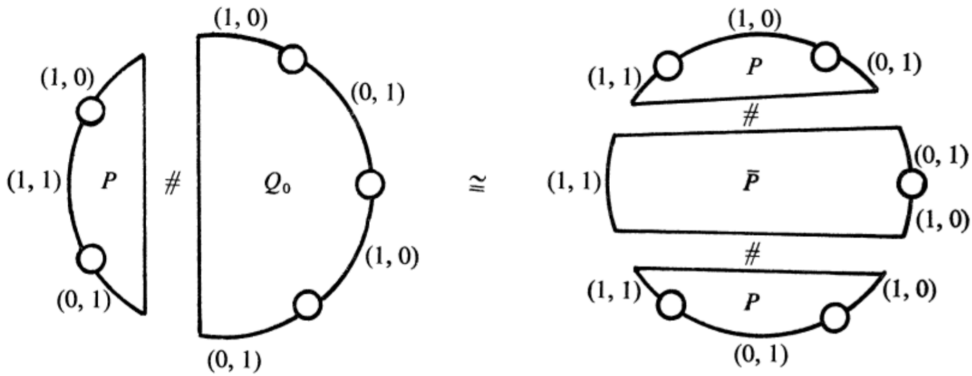
we have $m_{j-1}n_j = \varepsilon_j + m_jn_{j-1}$, and in particular, $|m_{j-1}|\ |n_j| \leqq 1 + |m_j|\ |n_{j-1}|$. But,

$$(|n_j|+1)(|n_{j-1}|+1) \leqq 1+|m_j|\ |n_{j-1}|.$$

This yields a contradiction as $|n_{j-1}|>0$ and $|m_j|>0$. This completes the proof.

Without normalizing, what this actually proved is that for adjacent pairs (m_i, n_i) and (m_{i-1}, n_{i-1}) one can always find a different pair (m_j, n_j) equal to either $\pm(m_i, n_i)$, $\pm(m_{i-1}, n_{i-1})$ or $(\varepsilon m_i \pm m_{i-1}, \varepsilon n_i \pm n_{i-1})$, $t>4$. We wish to thank George Cooke for supplying part of the computation above.

5.8. Remark. The decomposition of M as a connected sum is not unique. In particular the following diagram answers in the affirmative a question of Milnor [3]:

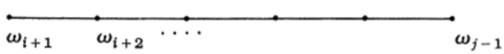


$$CP^2 \# S^2 \times S^2 \cong CP^2 \# (-CP^2) \# (CP^2)$$

5.9. Remark. We have already observed that the submanifold $W_{i-1,i+1}$ is a D^2-bundle over S_i with characteristic class

$$\omega_i = \varepsilon_i\varepsilon_{i+1}\begin{vmatrix} m_{i-1} & m_{i+1} \\ n_{i-1} & n_{i+1} \end{vmatrix}.$$

In general the manifold $W_{i,j}$ is the result of the linear plumbing (in the sense of Hirzebruch [2]) according to the graph



5.10. Remark. $P \# \bar{P} = \{(1, 1), (1, 0), (1, 1), (2, 3)\}$ *is a simple example of an action of a connected group on a manifold that is a connected sum, with the property that there is no invariant* 3-*sphere separating the components of the connected sum.*